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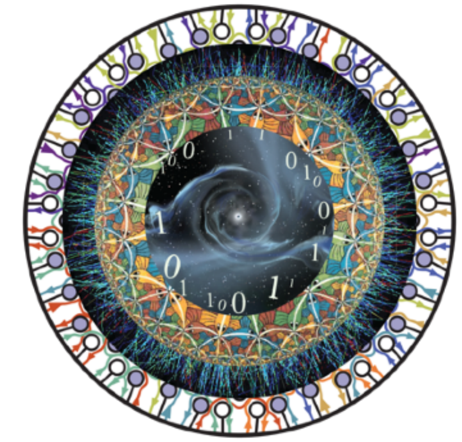

THE FLUID MANIFESTO REDUX

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July 6, 2018

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Related work

- ★ Crossley, Glorioso, Liu, Gao
- ★ Jensen, Marjieh, Pinzani-Fokeeva, Yarom
- ★ HLR + Geracie, Narayan, Ramirez

REFERENCES

Haehl, Loganayagam, MR (HLR)

- ✤ The Fluid Manifesto (basic philosophy): [1510.02494]
- ✤ Classification of solutions to hydro axioms: [1412.1090] [1502.00636]
- ✤ Dissipative hydrodynamic actions: [1511.07809] & [1803.11155]
- ✤ Origins in Schwinger-Keldysh: [1610.01940]
- ✤ Thermal Equivariance: [1610.01941]
- ✤ Overview of related works: [1701.07896]
- ✤ Inflow mechanism [1803.08490]

Related work

- ★ Hydrodynamic effective actions: Crossley, Glorioso, Liu [1511.03646]
- ★ Second Law: Glorioso, Liu [1612.07705]
- ★ Hydrodynamic effective actions II: Crossley, Glorioso, Liu [1701.07817]
- ★ Superspace formalism: Gao, Liu [1701.07445]

- ★ Jensen, Pinzani-Fokeeva, Yarom [1701.07436]
- ★ Inflow type picture + hydro actions: Jensen, Marjieh, Pinzani-Fokeeva, Yarom [1803.07070] & [1804.04654]

INSPIRATION

millennial developments in hydrodynamics

- Nickel, Son (2011)
- Dubovsky, Hui, Nicolis + Son (2011)
- Romatschke (2009), Bhattacharyya (2012)
- Banerjee, Bhattacharya, Bhattacharyya, Jain, Minwalla, Sharma (2012)
- Jensen, Kaminski, Kovtun, Meyer, Ritz, Yarom (2012)
- Jensen, Loganayagam, Yarom (2012-13)
- Bhattacharyya (2013-2014)

topological field theories and equivariance

- Cordes, Moore, Ramgoolam (1994)
- Blau, Thompson (1991, 1996)
- Vafa, Witten (1994)
- Dijkgraaf, Moore (1996)

stochastic and statistical dynamics

- Martin, Siggia, Rose (1973)
- Mallick, Moshe, Orland (2010)
- Kovtun, Moore, Romatschke (2014)
- Gaspard (2012)

Prologue

What we set out to do…

MOTIVATION

The Fluid Manifesto: Questions

- ✦ Effective field theories for dissipative systems out-of-equilibrium?
- ✦ Why doubled degrees of freedom? (microscopics: Schwinger-Keldysh)
- ✦ Where does dissipation come from?
- ✦ Boundary conditions that pick out dissipation & forward arrow of time?
- ✦ What is the role of entropy and second law?
- ✦ How does one understand Jarzynski relation (non-eq second law)?
- ✦ Role of emergent symmetries in hydrodynamics?
- ✦ Relevance for black hole physics via fluid/gravity?

HLR '15

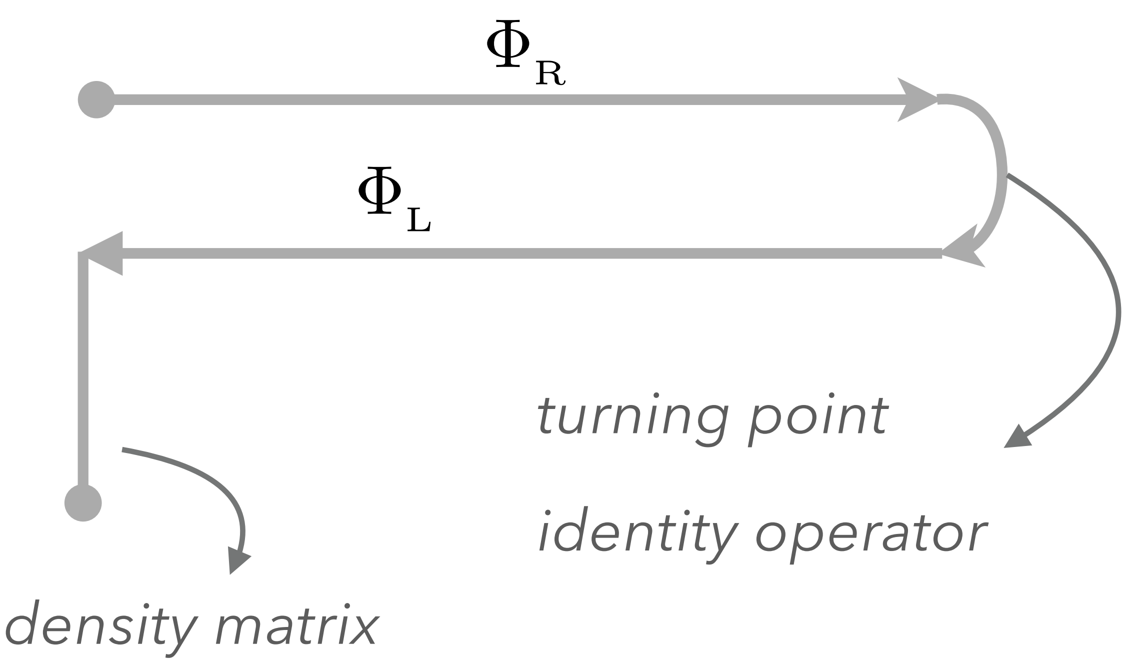
MOTIVATION

- Hydrodynamic effective field theories are source deformed topological sigma models.
- Doubling: classical hydrodynamic modes + attendant fluctuations.
- Emergent low energy $U(1)_T$ thermal diffeomorphism gauge symmetry.
- Dissipation: CPT symmetry breaking by vev for thermal field strength.
- Arrow of time: sign of the vev.
- Entropy is Noetherian and produced by inflow from superspace.
- Jarzynski arises as a BRST Ward identity.

Act I

Microscopic description and constraints on low energy dynamics

THERMAL SCHWINGER-KELDYSH FORMALISM



Generating functional (time ordered)

$$\mathcal{Z}_T[\mathcal{J}_{\rm R}, \mathcal{J}_{\rm L}] = {\rm Tr}\left(U[\mathcal{J}_{\rm R}]\,\hat{\rho}_T\,(U[\mathcal{J}_{\rm L}])^\dagger\right)$$

$$S_{SK} = S[\Phi_{\rm R}] - S[\Phi_{\rm L}]$$

$$\delta S_{SK} = \int d^dx\,\sqrt{-g}\,(\mathcal{J}_{\rm R}\,\mathbb{O}_{\rm R} - \mathcal{J}_{\rm L}\,\mathbb{O}_{\rm L})$$

SK unitarity (topological limit): $$\langle \mathcal{T}_{SK} \prod_k \left(\mathbb{O}^{(k)}_{\rm R}(t_k) - \mathbb{O}^{(k)}_{\rm L}(t_k)\right)\rangle = 0$$

Weldon '05

Fluctuation-dissipation: $$\langle \mathcal{T}_{SK} \prod_k \left(\mathbb{O}^{(k)}_{\rm R}(t_k) - \mathbb{O}^{(k)}_{\rm L}(t_k - i\,\beta)\right)\rangle = 0$$

Keldysh (light-cone) basis: $$\mathbb{O}_{dif} \equiv \mathbb{O}_{\rm R} - \mathbb{O}_{\rm L}\ , \qquad \mathbb{O}_{av} \equiv \frac{1}{2}\,(\mathbb{O}_{\rm R} + \mathbb{O}_{\rm L})$$

✦ Furthermore, a *largest time* and *thermal smallest time* equations hold.

HYDRODYNAMICS: TRANSPORT, FLUCTUATIONS

- ✦ Hydrodynamics: low energy dynamics of conserved currents in near equilibrium situations.

- ✦ Transport is captured by response functions: these are the first non-trivial correlators involving 1-average and rest difference operators.

- ✦ KMS relations relate response functions to fluctuations, e.g., and embody the fluctuation-dissipation theorem:

$$\text{Tr}\left(\widehat{\mathbb{A}}(t_{_A})\,\widehat{\mathbb{B}}(t_{_B})\hat{\rho}_{_T}\right) = \text{Tr}\left(\widehat{\mathbb{B}}(t_{_B} - i\,\beta)\,\widehat{\mathbb{A}}(t_{_A})\hat{\rho}_{_T}\right)$$

$$\Longrightarrow \quad \langle\{\widehat{\mathbb{A}}, \widehat{\mathbb{B}}\}\rangle = -\coth\left(\frac{1}{2}\,\beta\,\omega_{_B}\right)\,\langle[\widehat{\mathbb{A}}, \widehat{\mathbb{B}}]\rangle$$

- ✦ Look to constructing an effective field theory that captures all hydrodynamic transport & attendant fluctuations.

CONSTRAINTS ON LOW ENERGY DYNAMICS

- Topological (BRST super) symmetries are efficient ways to encode SK + KMS constraints.

HLR '15

Crossley, Glorioso, Liu '15

Unitarity → SK BRST supercharges

KMS/FDT → thermal diffeomorphism gauge symmetry

$$\mathcal{Q}^2 \sim 0\,, \qquad \bar{\mathcal{Q}}^2 \sim 0\,, \qquad \{\mathcal{Q}, \bar{\mathcal{Q}}\} \sim i\,\beta\,\partial_t \equiv i\,\pounds_\beta$$

- Precedent: Langevin dynamics of a Brownian particle

Martin, Siggia, Rose (1973) *Parisi Sourlas (1982)*

- Direct implementation of MSR type logic in hydrodynamics

Kovtun, Moore, Romatschke (2014)

TOPOLOGICAL GAUGE SYMMETRY

- Work in superspace $\mathbb{R}^{d|2}: \quad \{\sigma^a, \theta, \bar\theta\} \equiv z^I$

- Gauge potential with 2 BRST charges efficiently encoded in to superfield super-one-form.

$$\mathring{\mathcal{A}} = \mathring{\mathcal{A}}_I \, dz^I = \mathring{\mathcal{A}}_a \, d\sigma^a + \mathring{\mathcal{A}}_\theta \, d\theta + \mathring{\mathcal{A}}_{\bar\theta} \, d\bar\theta \, .$$

ghost charge	Faddeev-Popov ghost triplet	Vafa-Witten ghost of ghost quintet	Vector quartet
2		ϕ	
1	G	η	λ_a
0	B	ϕ^0	$\mathcal{A}_a$ $\mathcal{F}_a$
-1	$\bar G$	$\bar\eta$	$\bar\lambda_a$
-2		$\bar\phi$	

$$\mathring{\mathcal{D}}_I = \partial_I + [\mathring{\mathcal{A}}_I, \ \cdot \ ] \, ,$$

$$\mathring{\mathfrak{F}}_{IJ} \quad \frac{1}{2}\,\delta_{IJ}) \Big(\partial_I \mathring{\mathcal{A}}_J - (-)^{IJ} \partial_J \mathring{\mathcal{A}}_I + [\mathring{\mathcal{A}}_I, \mathring{\mathcal{A}}_J] \Big)$$

- Gauge-covariant super derivations obey the BRST algebra and define charges charges that anti-commute to gauge transformations:

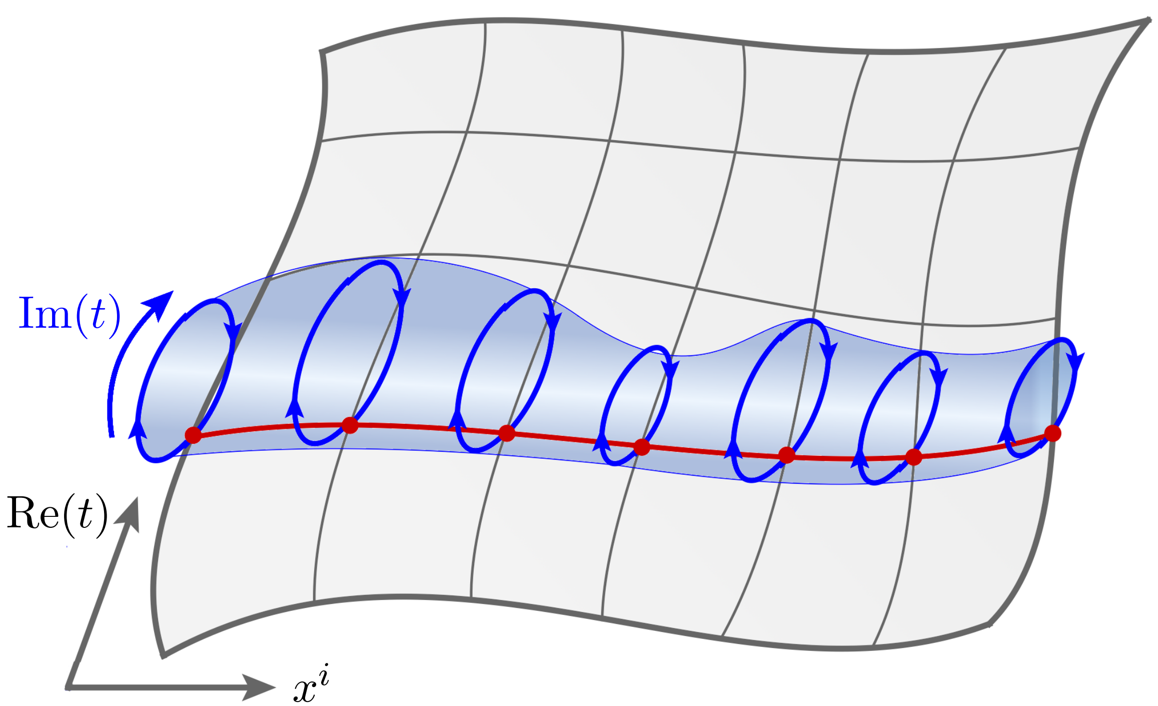
$$\mathring{\mathcal{D}}_{\bar\theta}^2 = \mathring{\mathcal{L}}_{\mathring{\mathfrak{F}}_{\bar\theta\bar\theta}} \, , \qquad \mathring{\mathcal{D}}_{\theta}^2 = \mathring{\mathcal{L}}_{\mathring{\mathfrak{F}}_{\theta\theta}} \, , \qquad \Big[\mathring{\mathcal{D}}_{\bar\theta}, \mathring{\mathcal{D}}_\theta \Big]_\pm = \mathring{\mathcal{L}}_{\mathring{\mathfrak{F}}_{\theta\bar\theta}} \, .$$

THE GAUGE ALGEBRA

- The gauge symmetry which the KMS conditions lead to is thermal diffeomorphisms (obtained in the high temperature/statistical limit).

Basic bracket relation for gauge algebra

$$(\Lambda_1, \Lambda_2)_{\boldsymbol{\beta}} = \Lambda_1 \pounds_{\boldsymbol{\beta}} \Lambda_2 - \Lambda_2 \pounds_{\boldsymbol{\beta}} \Lambda_1$$



View thermal circle fibred over spacetime with $\boldsymbol{\beta}^a$ picking out the choice of local intertial frame ($\boldsymbol{\beta}^a/||\boldsymbol{\beta}^a||$) and the local temperature ($||\boldsymbol{\beta}^a||$)

LOW ENERGY CONSTRAINTS II

- Effective dynamics constrained by

$$\mathcal{Q}^2 = -\mathring{\mathcal{F}}_{\bar{\theta}\bar{\theta}}|\pounds_{\boldsymbol{\beta}}\,, \qquad \bar{\mathcal{Q}}^2 = -\mathring{\mathcal{F}}_{\theta\theta}|\pounds_{\boldsymbol{\beta}}\,, \qquad \{\mathcal{Q},\bar{\mathcal{Q}}\} = -\mathring{\mathcal{F}}_{\theta\bar{\theta}}|\pounds_{\boldsymbol{\beta}}$$

HLR [1510.02494]

HLR	Crossley, Glorioso, Liu [1511.03646]
∗ Basic BRST charges $\mathcal{Q}_{SK}\,,\bar{\mathcal{Q}}_{SK}$ are nilpotent and arise from SK unitarity & are CPT conjugates. *HLR + Geracie, Narayan, Ramirez [1712.04459]*	∗ Single nilpotent BRST supercharge δ from SK unitarity.
∗ KMS conditions are implemented by gauge these BRST charges (equivariance).	∗ KMS condition is an involution (after combining with CPT) and gives another supercharge $\bar{\delta}$.
$\mathcal{Q}^2 = 0\,,\ \ \bar{\mathcal{Q}}^2 = 0\,,\ \ \{\mathcal{Q},\bar{\mathcal{Q}}\} = i\,\pounds_{\boldsymbol{\beta}}$	$\delta^2 = \bar{\delta}^2 = 0, \qquad \{\delta,\bar{\delta}\} = 2\,\tanh\left(\frac{i}{2}\,\beta\,\partial_t\right) \simeq i\,\beta\,\partial_t$

- This algebra is well known in the statistical mechanics literature in the context of stochastic Langevin dynamics.

Mallick, Moshe, Orland [1009.4800]

CPT SYMMETRY AND ITS BREAKING

- CPT symmetry can be implemented in the SK path integral by convolving usual transform with an exchange of L and R contours.

$$\mathcal{Z}_{SK}[\mathcal{J}_{\rm R}, \mathcal{J}_{\rm L}]^* = \mathcal{Z}_{SK}[\mathcal{J}_{\rm L}^*, \mathcal{J}_{\rm R}^*]$$

- Implemented in our construction as R-partity on superspace:

$$\bar{\theta}^{\rm CPT} = \theta\,, \qquad \theta^{\rm CPT} = \bar{\theta}$$

- Our formalism is CPT symmetric but needs a boundary condition to pick out a forward arrow of time.

- Hint: the universal gauge multiplet has exactly one component with vanishing ghost number which is CPT odd: the bottom component of $\mathring{\mathfrak{F}}_{\theta\bar{\theta}}$.

- Assumption: The topological gauge dynamics is consistent with the existence of a vacuum where $\langle \mathring{\mathcal{F}}_{\theta\bar{\theta}} | \rangle = -i$ & CPT is spontaneously broken.

CPT SYMMETRY AND ITS BREAKING

- Assumption: The topological gauge dynamics is consistent with the existence of a vacuum where $\langle \mathring{\mathcal{F}}_{\theta\bar{\theta}}| \rangle = -i$ & CPT is spontaneously broken.

  - Gauge dynamics is expected to be a topological BF theory which needs to be worked out explicitly.
  - Much of the analysis can be done by working with the MMO algebra where we freeze gauge dynamics by hand with a background gauge field, but our formalism has the advantage of allowing CPT symmetry to broken *dynamically*.
  - Additional evidence: Entropy conservation in superspace indicates absence of any obstacle (anomalies) to BF gauging thermal diffeomorphisms.

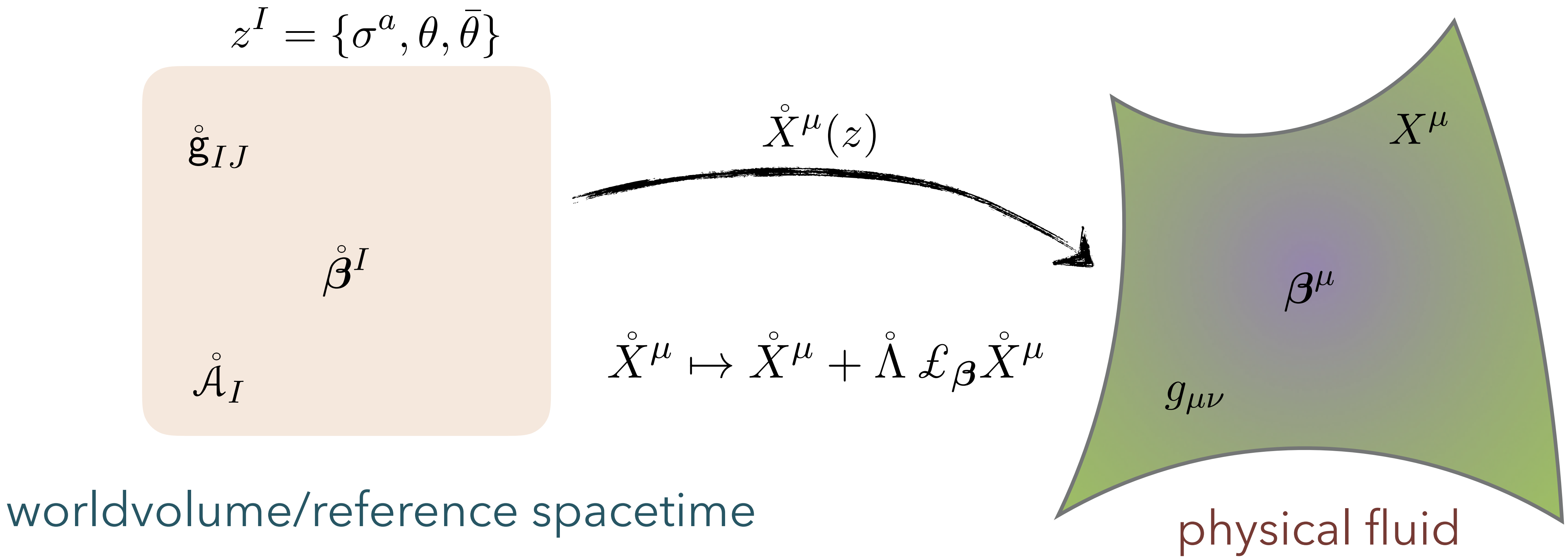
Act II

The hydrodynamic degrees of freedom

HYDRODYNAMIC FIELDS

- Hydrodynamics is usually phrased in terms of velocities and intensive thermodynamic variables (temperature, chemical potential).
- These obscure the basic information, since dynamical equations are conservation laws.
- Conservation laws are implied by symmetries (diffeo invariance leads to energy-momentum conservation) but generically do not imply Euler-Lagrange equations of motion.
- Earlier attempts use Lagrangian fluid variables but these obscure spacetime covariance. *Dubovsky, Hui, Nicolis + Son '11*
- Inspired by attempts to deconstruct holographic fluids, intuition from equilibrium partition functions, and D-brane effective actions one can identify the dynamical pions of hydrodynamics as spacetime coordinates viewed as a function of fiducial worldvolume data.

HYDRODYNAMIC SIGMA MODELS



$$\mathring{X}^\mu = X^\mu + \theta\, X^\mu_{\bar{\psi}} + \bar{\theta}\, X^\mu_{\psi} + \bar{\theta}\theta \left(\tilde{X}^\mu - \Gamma^\mu_{\rho\sigma}\, X^\rho_{\bar{\psi}}\, X^\sigma_{\psi}\right)$$

- ✦ Hydrodynamic action is a deformation of a topological field theory reflecting the near-topological nature of the observables.

$$\mathring{\mathrm{g}}_{IJ}(z) \;\to\; \mathring{\mathrm{g}}_{IJ}(z) + \bar{\theta}\,\theta\; \mathsf{h}_{IJ}(\sigma)$$

Act III

Hydrodynamic effective actions

BROWNIAN PARTICLE

✦ Brownian particle immersed in a fluid undergoes dissipative motion.

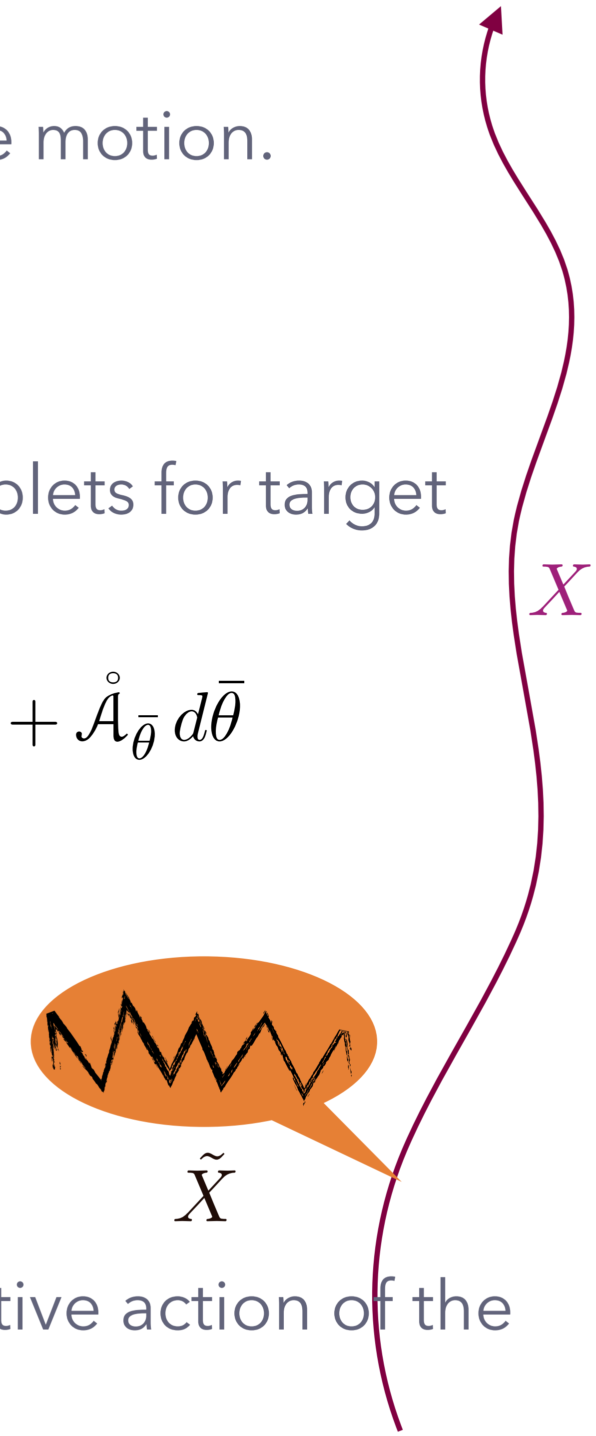
✦ Langevin effective action: worldvolume B0-brane theory.

✦ Data for the worldvolume theory: thermal equivariant multiplets for target space coordinate map and thermal gauge field data.

$$\mathring{X} = \{X, X_\psi, X_{\bar{\psi}}, \tilde{X}\} \qquad\qquad \mathring{\mathcal{A}} \equiv \mathring{\mathcal{A}}_t\, dt + \mathring{\mathcal{A}}_\theta\, d\theta + \mathring{\mathcal{A}}_{\bar{\theta}}\, d\bar{\theta}$$

✦ Effective action with the symmetries is simply.

$$S_{\text{B0}} = \int dt\, d\theta\, d\bar{\theta} \left\{ \frac{m}{2} \left(\mathring{\mathcal{D}}_t \mathring{X} \right)^2 - U(\mathring{X}) - i\, \nu\, \mathring{\mathcal{D}}_\theta \mathring{X} \mathring{\mathcal{D}}_{\bar{\theta}} \mathring{X} \right\}$$



✦ MSR action follows as the basic thermal gauge invariant effective action of the worldline theory after CPT breaking

$$(\mathring{\Lambda}, \mathring{X})_{\boldsymbol{\beta}} = \mathring{\Lambda}\, \pounds_{\boldsymbol{\beta}} \mathring{X} = \mathring{\Lambda}\, \Delta_{\boldsymbol{\beta}} \mathring{X} = \mathring{\Lambda}\, \boldsymbol{\beta} \frac{d}{dt} \mathring{X} \qquad\qquad \mathring{\mathcal{D}}_I = \partial_I + [\mathring{\mathcal{A}}_I, \ \cdot\ ]$$

Martin, Siggia, Rose (1973)
Parisi, Sourlas (1982)

EFFECTIVE ACTIONS

- Hydrodynamic effective actions can be constructed as superspace integrals

$$S_{wv} = \int d^d\sigma \, \mathcal{L}_{wv} \,, \qquad \mathcal{L}_{wv} = \int d\theta \, d\bar{\theta} \, \frac{\sqrt{-\mathring{\mathrm{g}}}}{\mathring{\mathbf{z}}} \, \mathring{\mathcal{L}}[\mathring{\mathrm{g}}_{IJ}, \boldsymbol{\beta}^a, \mathring{\mathfrak{D}}_I, \mathring{\mathfrak{g}}^{(\overline{\psi})}_{IJ}, \mathring{\mathfrak{g}}^{(\psi)}_{IJ}] \,,$$

$$\mathring{\mathbf{z}} = 1 + \mathring{\boldsymbol{\beta}}^I \, \mathring{\mathcal{A}}_I \qquad \mathring{\mathfrak{g}}^{(\overline{\psi})}_{IJ} \equiv \mathring{\mathcal{D}}_\theta \mathring{\mathrm{g}}_{IJ} \,, \qquad \mathring{\mathfrak{g}}^{(\psi)}_{IJ} \equiv \mathring{\mathcal{D}}_{\bar{\theta}} \mathring{\mathrm{g}}_{IJ}$$

HLR [1511.07809,1803.11155]

Symmetries constraining the action:

- Invariance under thermal diffeos and SK-KMS BRST symmetries.
- Target spacetime superdiffeomorphisms
- Restricted worldvolume diffeomorphisms
- CPT & ghost charge conservation.

Crossley, Glorioso, Liu [1511.03646,1701.07817]
Gao, Liu [1701.07445]
Jensen et al [1701.07436,1804.04654]

EFFECTIVE ACTIONS: EXAMPLES

- Ideal fluid is obviously captured by the pressure super-potential functional

$$\mathring{\mathcal{L}}^{(\text{ideal})} = \frac{\sqrt{-\mathring{\mathrm{g}}}}{\mathring{\mathbf{z}}}\; \mathring{\mathfrak{f}}(\mathring{T})$$

- Dissipative terms are captured by a an appropriate 4-tensor inspired coupling that involves the superderivatives of the metric:

$$\mathcal{L}_{\text{wv, diss}} = \int d\theta\, d\bar{\theta}\, \frac{\sqrt{-\mathring{\mathrm{g}}}}{\mathring{\mathbf{z}}} \left(-\frac{i}{4}\right) \mathring{\boldsymbol{\eta}}^{IJKL}\, \mathring{\mathfrak{g}}_{IJ}^{(\bar{\psi})}\, \mathring{\mathfrak{g}}_{KL}^{(\psi)}$$

$$\mathring{\boldsymbol{\eta}}^{IJKL} = \mathring{\zeta}(\mathring{T})\, \mathring{T}\, \mathring{P}^{IJ}\, \mathring{P}^{KL} + 2\, \mathring{\eta}(\mathring{T})\, \mathring{T}\, (-)^{K(I+J)}\, \mathring{P}^{K\langle I}\, \mathring{P}^{J\rangle L}$$

HLR [1511.07809]

- Positivity of entropy production follows on demanding the imaginary part is positive definite (which reduces us back to the remit of Bhattacharyya's theorem).

$$\Delta = \frac{1}{4}\, \boldsymbol{\eta}^{abcd}\, \pounds_{\boldsymbol{\beta}} \mathrm{g}_{ab}\, \pounds_{\boldsymbol{\beta}} \mathrm{g}_{cd} + \text{fluctuations} + \text{ghost-bilinears}$$

Glorioso, Liu [1612.07705]

Act IV

Entropy in hydrodynamics

& eightfold classification

ENTROPY IN EQUILIBRIUM

- The hydrostatic partition function on $\mathcal{M}_{\boldsymbol{\beta}}$ allows for computation of equal time correlation function of conserved currents.

Banerjee et al [1203.3544]
Jensen et al [1203.3566]

- Fluid view of such hydrostatic configurations when gradients are small:

➡ local temperature is fixed to be the size of the thermal circle.

➡ fluid velocity is oriented along the Killing field (the fibre).

$$\boldsymbol{\beta}^\mu = \frac{u^\mu}{T}$$

thermal vector

$$W[g_{\mu\nu}] = \int_{\mathcal{M}_{\boldsymbol{\beta}}} d^d x\, \sqrt{-g}\; P_s[\boldsymbol{\beta}^\alpha, g_{\mu\nu}]$$

- It then follows that the equilibrium entropy current is the Noether current for thermal diffeomorphisms (same argument as before).

$$(J^\mu_S)_{eq} = -\boldsymbol{\beta}_\nu\, T^{\mu\nu} + \boldsymbol{\beta}^\mu P_s[\boldsymbol{\beta}^\alpha, g_{\alpha\beta}] - \Theta^\mu_{PS} \qquad N^\mu = J^\mu_S + \boldsymbol{\beta}_\nu\, T^{\mu\nu}$$

$$\nabla_\mu J^\mu_S = 0 \implies \nabla_\mu N^\mu = 2\, T^{\mu\nu} \nabla_{(\mu} \boldsymbol{\beta}_{\nu)} = T^{\mu\nu} \pounds_{\boldsymbol{\beta}} g_{\mu\nu} = 0$$

HLR [1502.00636]

ENTROPY NEAR-EQUILIBRIUM

- **Bhattacharyya's Theorem:** There exists a hydrodynamic entropy current satisfying the local form of second law, as long as leading order dissipative terms are sign-definite (e.g., viscosities & conductivities are non-negative), provided hydrostatics is consistent with the equilibrium partition function.

Bhattacharyya [1403.7639]

- The off-shell adiabaticity equation encodes this and serves to classify hydrodynamic transport

$$\nabla_\mu N^\mu - \frac{1}{2} T^{\mu\nu} \pounds_{\boldsymbol{\beta}} g_{\mu\nu} = \Delta \geq 0 \qquad\qquad N^\mu = J^\mu_S + \boldsymbol{\beta}_\nu T^{\mu\nu}$$

- *Eightfold classification*: 1 class captures dissipation and entropy production, and is parameterized by a 4-tensor which gives a non-negative definite inner product on the space of symmetric two-tensors:

$$T^{\mu\nu}_{diss} = \frac{1}{2} \boldsymbol{\eta}^{(\mu\nu)(\rho\sigma)} \pounds_{\boldsymbol{\beta}} g_{\rho\sigma} \qquad\qquad \Delta = \frac{1}{4} \boldsymbol{\eta}^{(\mu\nu)(\rho\sigma)} \pounds_{\boldsymbol{\beta}} g_{\mu\nu} \pounds_{\boldsymbol{\beta}} g_{\rho\sigma}$$

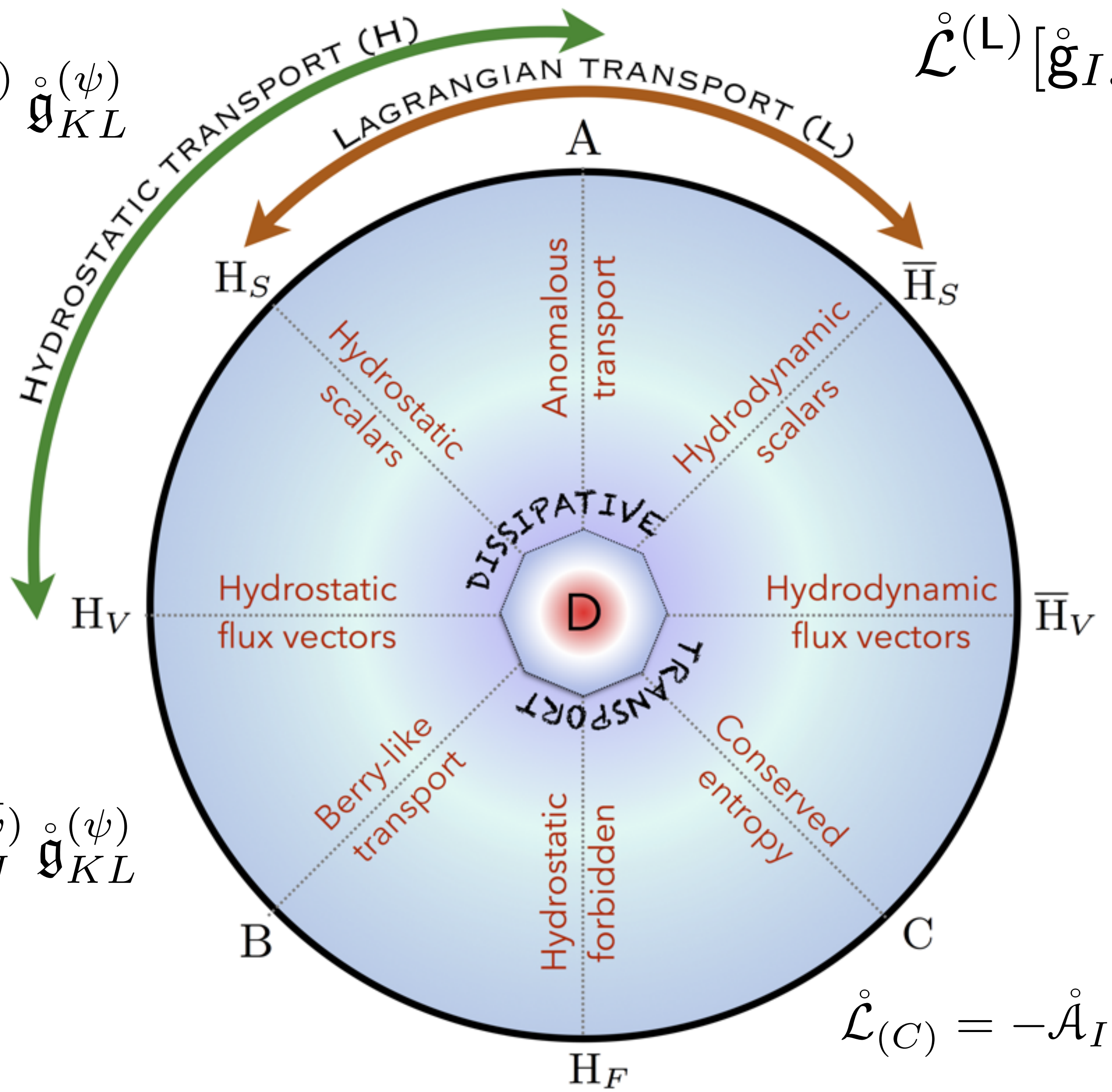
HLR [1502.00636]

cf., Glorioso, Liu [1612.07705]

EIGHTFOLD CLASSIFICATION OF TRANSPORT

$$\mathring{\mathcal{L}}_{(D)} = -\frac{i}{4}\, \mathring{\boldsymbol{\eta}}^{((IJ)|(KL))}\; \mathring{\mathfrak{g}}^{(\bar{\psi})}_{IJ}\; \mathring{\mathfrak{g}}^{(\psi)}_{KL}$$

$$\mathring{\mathcal{L}}^{(\mathsf{L})}[\mathring{\mathfrak{g}}_{IJ}, \mathring{\boldsymbol{\beta}}^I, \mathring{\mathfrak{D}}_J]$$



$$\mathring{\mathcal{L}}_{(B)} = \frac{1}{4}\, \mathring{\mathfrak{F}}_{\bar{\theta}\theta} \mathring{\boldsymbol{\eta}}^{[(IJ)|(KL)]}_{(B)}\; \mathring{\mathfrak{g}}^{(\bar{\psi})}_{IJ}\; \mathring{\mathfrak{g}}^{(\psi)}_{KL}$$

$$\mathring{\mathcal{L}}_{(C)} = -\mathring{\mathcal{A}}_I\, \mathring{\mathbf{N}}^I_{(C)}\,, \quad \mathring{\mathfrak{D}}_I\, \mathring{\mathbf{N}}^I_{(C)} = 0$$

$$\mathring{\mathcal{L}}_{(\overline{\mathrm{H}}_V)} = \frac{i}{4}\, \mathring{\mathfrak{C}}^{M(IJ)(KL)} \left[\left(\mathring{\mathfrak{D}}_M \mathring{\mathfrak{g}}^{(\bar{\psi})}_{IJ} + (\mathring{\mathfrak{F}}_{\theta M}, \mathring{\mathfrak{g}}_{IJ})_{\boldsymbol{\beta}}\right) \mathring{\mathfrak{g}}^{(\psi)}_{KL} - \mathring{\mathfrak{g}}^{(\bar{\psi})}_{IJ} \left(\mathring{\mathfrak{D}}_M \mathring{\mathfrak{g}}^{(\psi)}_{KL} + (\mathring{\mathfrak{F}}_{\bar{\theta} M}, \mathring{\mathfrak{g}}_{KL})_{\boldsymbol{\beta}}\right) \right]$$

HLR [1803.11155]

EIGHTFOLD CLASSIFICATION REDUX

- Details regarding anomalies etc need to be worked out (but it should work pretty much as in earlier analysis). *HLR [1312.0610]*

- Potentially refinement of the classification using CPT properties?

- Additional constraint on transport? *Jensen et al [1804.04654]*
  - eg., should a subset of Class B terms be forbidden based on CPT + constraints from demanding that the imaginary part of effective action is sign-definite?
  - The argument thus far is carried out in an unphysical corner of fluids where viscosity is set to zero.

- All transport admissible by Bhattacharyya's theorem + eightfold classification should be allowed (some interesting tests for hydrodynamic flux vectors).

Act V

Entropy production via inflow

INFLOW FROM DISSIPATIVE ACTIONS

- Thermal diffeomorphism symmetry implies that the *super-adiabaticity equation* is satisfied as a Bianchi identity:

$$\mathring{\mathcal{D}}_I \mathring{\mathbf{N}}^I - \frac{1}{2}\, \mathring{\mathbf{T}}^{IJ}\, \pounds_{\boldsymbol{\beta}} \mathring{\mathrm{g}}_{IJ} = 0\,.$$

- This in turn leads to the picture of entropy inflow:

$$\underbrace{\left(\mathring{\mathcal{D}}_a \mathring{\mathbf{N}}^a - \frac{1}{2}\, \mathring{\mathbf{T}}^{ab}\, \pounds_{\boldsymbol{\beta}}\, \mathring{\mathrm{g}}_{ab}\right)\Big|}_{\text{classical + fluctuations}} = -\underbrace{\left(\mathring{\mathcal{D}}_\theta \mathring{\mathbf{N}}^\theta + \mathring{\mathcal{D}}_{\bar\theta} \mathring{\mathbf{N}}^{\bar\theta} + \mathring{\mathbf{T}}^{a\theta}\, \pounds_{\boldsymbol{\beta}}\, \mathring{\mathrm{g}}_{a\theta} + \mathring{\mathbf{T}}^{a\bar\theta}\, \pounds_{\boldsymbol{\beta}}\, \mathring{\mathrm{g}}_{a\bar\theta} + \mathring{\mathbf{T}}^{\theta\bar\theta}\, \pounds_{\boldsymbol{\beta}}\, \mathring{\mathrm{g}}_{\theta\bar\theta}\right)\Big|}_{\text{entropy inflow}}$$

- Switching off the fluctuation fields leads to physical entropy flowing from superspace:

$$\Delta = -\left(\mathcal{D}_\theta \mathbf{N}^\theta + \mathcal{D}_{\bar\theta} \mathbf{N}^{\bar\theta}\right) + \text{ghosts bilinears}$$

HLR [1803.08490,1803.11155]
Jensen et al [1803.07070]

NO INFLOW OF ENERGY-MOMENTUM

- Target space diffeomorphisms ensure that the dynamical content of the effective action is simply super-energy momentum conservation.

$$\mathring{\mathfrak{D}}_I \left(\mathring{\mathbf{T}}^{IJ} \, \mathring{\mathfrak{D}}_J \mathring{X}^\mu \right) = 0$$

- This by itself would be problematic, since we would learn that the equations are contaminated by the presence of super-components which turn out to include physical degrees of freedom (not ghosts or fluctuations).

- However, superspace components of energy-momentum tensor conspire to mutually cancel out and do not modify dynamical equations.

$$\mathring{\mathfrak{D}}_a \left(\mathring{\mathbf{T}}^{ab} \, \mathring{\mathfrak{D}}_b \mathring{X}^\mu \right) \Big| \;\; = \nabla_\mu T^{\mu\nu} + \text{ghost bilinears} + \text{fluctuations} \;\; = 0$$

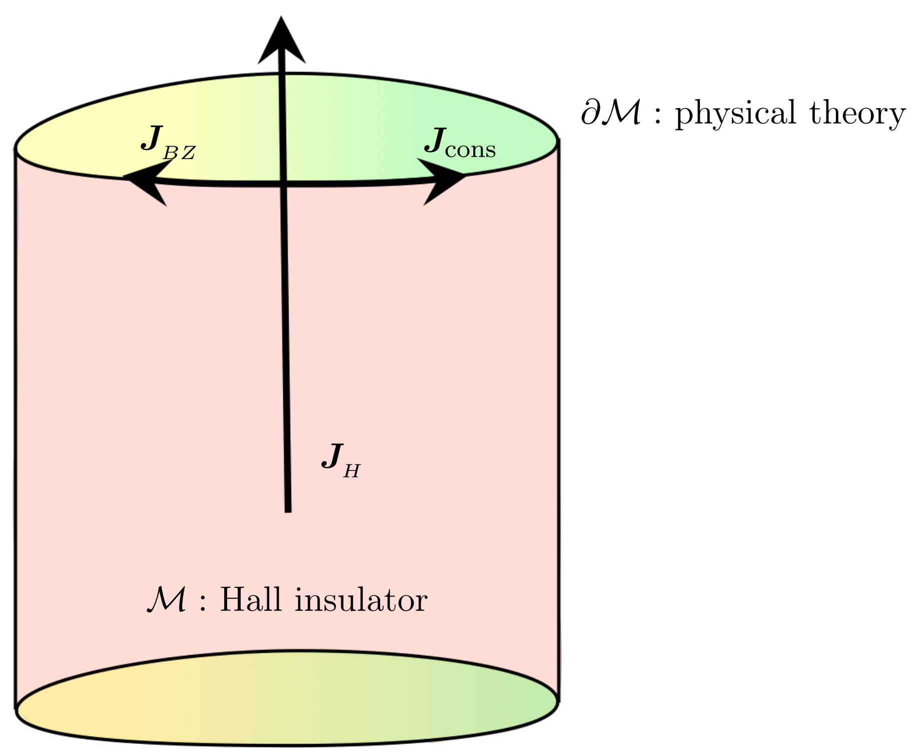
ENTROPY PRODUCTION VIA INFLOW

- For systems in local equilibrium the Noether current for thermal diffeomorphisms is the macroscopic free energy current (Legendre transform of entropy current).
- Local equilibrium is characterized by an emergent topological/BRST supersymmetry wherein diffeomorphisms along the Euclidean thermal circle are gauged (thermal equivariance).
- *Net entropy is conjugate to the gauged thermal diffeomorphisms & is conserved.*
- Physical entropy production happens by virtue of it being sourced in the superspace directions, i.e., there is an inflow of entropy from superspace.

$$\mathring{\mathcal{D}}_I \mathring{\mathbf{N}}^I - \frac{1}{2}\,\mathring{\mathbf{T}}^{IJ}\,\pounds_{\boldsymbol{\beta}}\mathring{\mathrm{g}}_{IJ} = 0 \qquad \Longrightarrow \qquad \mathcal{D}_a \mathbf{N}^a - \frac{1}{2}\,\mathbf{T}^{ab}\,\pounds_{\boldsymbol{\beta}}\mathbf{g}_{ab} = \Delta \geq 0$$

ENTROPY INFLOW

- While the inflow mechanism for entropy arises from the superspace, it is morally similar to the manner in which the inflow mechanism operates in the context of Hall insulators & chiral edge states ('t Hooft anomalies).



Callan, Harvey (1985)

anomaly inflow: coupling to a topological sector
with physical entropy being sourced in superspace

FLUCTUATION DISSIPATION AS CPT BREAKING

- Stochasticity and dissipation arises because of spontaneous CPT symmetry breaking.

- The Ward identities following from CPT convolved with a thermal gauge transformation results in the Jarzynski work relation for the Brownian particle

$$S_{\text{B0}} \mapsto S_{\text{B0}} - i \, \langle \mathring{\mathcal{F}}_{\theta\bar{\theta}} | \rangle \, \boldsymbol{\beta} \, (\Delta G + W) \quad \Longrightarrow \quad \langle e^{-\beta W} \rangle = e^{-\beta \, \Delta G}$$

Mallick, Moshe, Orland (2010)

- The CPT symmetry in our construction is implemented as R-parity in superspace and its breaking encoded in the vev for the ghost number zero field strength: $\langle \mathring{\mathcal{F}}_{\theta\bar{\theta}} | \rangle = -i$

- Expect similar statements to hold in hydrodynamic effective field theories.

Epilogue: What lies ahead…

LOOKING AHEAD…

- ✦ Near-equilibrium dynamics appears to be under control but what about non-equilibrium?

- ✦ Open quantum systems & renormalization

  Avinash, Jana, Loganayagam, Rudra [1704.08335]

- ✦ How does thermal equivariance extend to include non-stochastic fluctuations? Deformation quantization?

  Basart, Flato, Lichnerowicz, Sternheimer 1984

- ✦ Microscopic unitary which enforces fluctuation-dissipation etc., is upheld thanks to the BRST + thermal gauge symmetry. Lessons for gravity? Connections to SL(2,R) symmetry or temporal reparametrizations in discussions of chaos?

- ✦ What is the analogous story for higher out-of-time-order correlators?

- ✦ Are the similar statements for modular evolutions (equivalent in some contexts), and if so what does it imply for geometry = entanglement?

Thank You!