Space-filling branes & gaugings

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arXiv:1610.07975, 1704.08566 with D. Lombardo and S. Risoli

Outline

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Introduction

We are all familiar with the idea that fluxes in $\mathcal{N} = 1$ models generate a superpotential

Example: CY O3-orientifold of IIB with NS and RR 3-form fluxes turned on compatibly with susy results in $\mathcal{N} = 1$ supergravity with GVW superpotential

$$W = \int (\mathcal{F}_3 - iS\mathcal{H}_3) \wedge \Omega$$

We also know that fluxes induce RR tadpoles

The IIB theory has a Chern-Simons term

$$\int C_4 \wedge \mathcal{H}_3 \wedge \mathcal{F}_3$$

implying that $\mathcal{H}_3$ and $\mathcal{F}_3$ act as sources for the RR 4-form C_4

This source can be compensated by changing the net amount of D3-branes in the orientifold theory

Introduction

Consider a specific model: Type-IIB on a $T^6/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ orientifold

Antoniadis, Dudas, Sagnotti (1999)

Antoniadis, D'Appollonio, Dudas, Sagnotti (2000)

We take $T^6 = \bigotimes_{i=1}^{3} T^2_{(i)}$, coordinates (x^i, y^i)

The two $\mathbb{Z}_2$'s act as $(-,-,+)$ and $(+,-,-)$ on the coordinates of the 2-tori

There are seven complex moduli: axion-dilaton S, three complex Kähler moduli T^i (made of C_4's and the volumes of the tori) and three complex structure moduli U^i (which are the τ's of the tori)

Moduli space $(\mathrm{SL}(2,\mathbb{R})/\mathrm{SO}(2))^7$. Symmetry $\mathrm{SL}(2,\mathbb{Z})^7$

Introduction

Turn on fluxes $\mathcal{H}_3$ and $\mathcal{F}_3$, indices on the three different tori

Superpotential is cubic in U moduli

Fluxes induce tadpoles for D3 branes

This can be generalised to contain the non-geometric $\mathcal{Q}$ and $\mathcal{P}$ fluxes

Shelton, Taylor, Wecht (2005)

Aldazabal, Cámara, Font, Ibáñez (2006)

Fluxes belong to the $(\mathbf{2},\mathbf{2},\mathbf{2},\mathbf{2},\mathbf{2},\mathbf{2},\mathbf{2})$ of $\mathrm{SL}(2,\mathbb{R})^7$

Including all possible fluxes one ends up with a superpotential which is cubic in U and T moduli

Aldazabal, Cámara, Rosabal (2009)

Aldazabal, Andres, Cámara, Graña (2010)

What about tadpoles?

Fluxes & the embedding tensor

Form the supergravity point of view, fluxes induce a gauging, which is described in terms of the embedding tensor

de Wit, Samtleben, Trigiante (2002)

If the ungauged theory has a global symmetry G, the embedding tensor belongs to a specific representation of G

In our $\mathcal{N} = 1$ model the global symmetry of the ungauged supergravity theory is $\mathrm{SL}(2, \mathbb{R})^7$, and as we have just mentioned the embedding tensor belongs to the $(\mathbf{2}, \mathbf{2}, \mathbf{2}, \mathbf{2}, \mathbf{2}, \mathbf{2}, \mathbf{2})$

The components of this embedding tensor are all the geometric and non-geometric fluxes that can be turned on compatibly with the orientifold

Fluxes & the embedding tensor

Actually the $\mathcal{H}_3$ and $\mathcal{F}_3$ fluxes can be turned on also in the $\mathcal{N} = 2$ theory, that is before the orientifold projection

In this case you can't include D3-branes, which means you must impose the Bianchi identity

$\mathcal{H}_3 \wedge \mathcal{F}_3 = 0$

The Bianchi identity is a manifestation of the fact that for $\mathcal{N} > 1$ the embedding tensor must satisfy a quadratic constraint

The quadratic constraint of the orbifold $\mathcal{N} = 2$ theory is not required by $\mathcal{N} = 1$ supersymmetry, and this manifests itself in the fact that consistently one can add D3-branes in the orientifold theory so that the Bianchi identity is no longer satisfied

Fluxes & the embedding tensor

Consider now as a simpler example the $\mathcal{N}=8$ theory in four dimensions

In this case the embedding tensor is in the $\mathbf{912}$ of $\mathrm{E}_{7(7)}$

while the quadratic constraint is in the $\mathbf{8645} \oplus \mathbf{133}$

The theory can be truncated to the $\mathcal{N}=4$ theory with global symmetry $\mathrm{SL}(2,\mathbb{R}) \times \mathrm{SO}(6,6)$

Decomposing the embedding tensor one finds

$$\mathbf{912} = (\mathbf{3},\mathbf{32}) \oplus (\mathbf{2},\mathbf{220}) \oplus (\mathbf{1},\mathbf{352}) \oplus ....$$

Only the second representation survives the projection, that is the embedding tensor of the half-maximal theory is in the $(\mathbf{2},\mathbf{220})$

What happens to the quadratic constraint?

Fluxes & the embedding tensor

After the truncation, the quadratic constraint is projected on

$(\mathbf{3},\mathbf{495})\oplus(\mathbf{1},\mathbf{2079})\oplus(\mathbf{1},\mathbf{462})$

But only the first two are required by $\mathcal{N}=4$ supersymmetry!

Dibitetto, Guarino, Roest (2011)

We find that the $(\mathbf{1},\mathbf{462})$ is the representation that contains the space-filling branes that preserve the same supersymmetry of the truncation to $\mathcal{N}=4$, which can therefore be added to cancel the tadpoles induced by the fluxes

We also show that the same applies for any theory in any dimension

What are the branes? most of them are exotic branes!

Exotic branes

IIB 7-brane solution (Einstein frame)

$$ds^2 = \eta_{\mu\nu} dx^\mu dx^\nu + H(r) dz d\bar{z}$$

where

$$H(r) = \frac{1}{2\pi}\log(\frac{\mu}{r}) \qquad z = re^{i\theta}$$

and $\tau = C + ie^{-\phi}$ is an analytic function of z

Simplest solution (D7-brane) is

$$\tau = \frac{i}{2\pi}\log(\frac{\mu}{z})$$

This gives

$$e^{-\phi} = H(r) \qquad C = \theta/(2\pi)$$

Greene, Shapire, Vafa, Yau (1990)

Gibbons, Green, Perry (1996)

Exotic branes

The solution has monodromy

$C \to C + 1$

Ok because IIB string theory has a non-perturbative symmetry $SL(2, \mathbb{Z})$

Under $g = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$, the scalar τ transforms as

$\tau \to (a\tau + b)/(c\tau + d)$

Monodromy of D7 generated by the matrix

$M_{D7} = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$

Exotic branes

S-duality: $S = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$

It maps $\tau \to -1/\tau$

If one performs as S-duality transformation on the D7 solution, the (Einstein) metric does not change, while the monodromy becomes

$-1/\tau \to -1/\tau + 1$

So the real part of $-1/\tau$

$\mathrm{Re}(-1/\tau) = -C/(C^2 + e^{-2\phi})$

is shifted by one

The monodromy matrix is

$M_{SD7} = S M_{D7} S^{-1} = \begin{pmatrix} 1 & 0 \\ -1 & 1 \end{pmatrix}$

Exotic branes

NS5-brane solution in the string frame:

$$ds^2 = \eta_{\mu\nu} dx^\mu dx^\nu + H(r) dy^m dy^m$$

$$H_{mnp} = \epsilon_{mnpq} \partial_q H(r) \qquad e^\phi = H^{1/2}(r)$$

We want to T-dualise along the transverse directions 8 and 9. So we first have to smear the NS5 along these directions

After smearing, the harmonic function becomes logarithmic. The equation for B_{89} becomes

$$\partial_x B_{89} = \partial_y H(r) \qquad \partial_y B_{89} = -\partial_x H(r)$$

which in polar coordinates becomes

$$\frac{1}{r} \partial_\theta B_{89} = -\partial_r H(r)$$

Hence B_{89} depends linearly on θ

If one rotates around the brane: $B_{89} \to B_{89} + 1$

Exotic branes

Define the complex field

$\rho = B_{89} + i\sqrt{\det G}$

T-duality acts on ρ exactly as the IIB $SL(2,\mathbb{Z})$ S-duality acts on τ:

$\rho \to (a\rho + b)/(c\rho + d)$

In particular, the T-duality along the 8 and 9 direction is generated by the matrix

$$T = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$

This corresponds to $\rho \to -1/\rho$

Hence one ends up with a solution with monodromy

$\beta^{89} \to \beta^{89} + 1$

where $\beta^{89} = \mathrm{Re}(-1/\rho) = -B_{89}/(B_{89}^2 + \det G)$

Exotic branes

Exactly as in the SD7 solution the dilaton is not well-defined, here the metric is not well-defined

The resulting brane is globally non-geometric (T-fold)

de Boer, Shigemori (2010)

In general, using S and T duality one finds all the solutions of branes of codimension 2 of this type

Lozano-Tellechea, Ortín (2001)

Using the dualities, one derives the tension of all such branes as functions of the string coupling and the compactification radii

Obers, Pioline, Rabinovici (1998)

de Boer, Shigemori (2013)

Exotic branes

Instead of writing the tension, we write the mass of the particle in three dimensions that arises from wrapping

D7: $g_s^{-1} R_3 \ldots R_9$

SD7: $g_s^{-3} R_3 \ldots R_9$

NS5: $g_s^{-2} R_3 \ldots R_7$

T-fold: $g_s^{-2} R_3 \ldots R_7 R_8^2 R_9^2$

In particular, the square dependence on the radius implies that the tension of the exotic brane diverges in the decompactification limit

Exotic branes

We associate to each brane the potential under which the brane is electrically charged

Notation for the potentials: if tension scales like g_s^{-n}

$n = 1, 2, 3, 4...$: potentials $C, D, E, F, ...$

D7:	$g_s^{-1} R_3 ... R_9$	$\rightarrow$	$C_{03456789}$	of C_8
SD7:	$g_s^{-3} R_3 ... R_9$	$\rightarrow$	$E_{03456789}$	of E_8
NS5:	$g_s^{-2} R_3 ... R_7$	$\rightarrow$	D_{034567}	of D_6
T-fold:	$g_s^{-2} R_3 ... R_7 R_8^2 R_9^2$	$\rightarrow$	$D_{03456789,89}$	of $D_{8,2}$

Bergshoeff, Ortín, FR (2011)

The branes are also denoted as $7_1,\ 7_3,\ 5_2,\ 5_2^2,\ ...$

de Boer, Shigemori (2013)

Exotic branes

Under T-duality, for D-branes one has

$$g_s^{-1}: \quad C_{7\,x} \;\rightarrow\; C_7 \qquad C_8 \;\rightarrow\; C_{8\,x}$$

while for the NS5 one gets the chain

$$g_s^{-2}: \quad D_6 \;\rightarrow\; D_{6\,x,x} \;\rightarrow D_{6\,xy,xy}$$

Universal T-duality rule: Lombardo, FR, Risoli (2016)

$$g_s^{-n} \quad : \qquad \underbrace{a, a, ..., a}_{p} \overset{T_a}{\longleftrightarrow} \underbrace{a, a, ..., a}_{n-p}$$

For the SD7, with $n = 3$:

$E_{7\,x}$, 6-brane of tension $g_s^{-3} R_x$ in IIB mapped to $E_{7\,x,x}$, 6-brane of tension $g_s^{-3} R_x^2$ in IIA, potential $E_{8,1}$

E_8, 7-brane of tension g_s^{-3} in IIB mapped to $E_{8\,x,x,x}$, 7-brane of tension $g_s^{-3} R_x^3$ in IIA, potential $E_{9,1,1}$

Exotic branes

Using S and T duality recursively, one gets a full classification in terms of mixed-symmetry potentials

This is actually more general than codimension 2, it also works for codimension 1 (domain walls) and codimension 0 (space-filling) branes

Bergshoeff, FR (2011)

We are interested in space-filling branes in various dimensions

Exotic branes

To remember:

- Exotic branes charged under mixed-symmetry potentials
- relation between structure of indices and tension of the brane
- Universal T-duality rule for mixed-symmetry potentials
- Alphabetical vs n
- S-dual of C_8 is E_8

Space-filling branes

In $D = 8$ the D7 and its S-dual are space-filling

These two branes preserve the same supersymmetry

One can construct bound states which are still 1/2-BPS (familiar in F-theory)

Performing two T-dualities in the internal directions, which we call x and y, we have

$$7_1: \quad C_{01234567} \quad \text{of} \quad C_8 \quad \rightarrow \quad 9_1: \quad C_{0234567xy} \quad \text{of} \quad C_{10}$$

$$7_3: \quad E_{01234567} \quad \text{of} \quad E_8 \quad \rightarrow \quad 7_3^{(0,2)}: \quad E_{01234567xy,xy,xy} \quad \text{of} \quad E_{10,2,2}$$

Therefore we find that the $7_3^{(0,2)}$ brane preserves the same susy of the D9

Space-filling branes

We now move to $D = 7$. In seven dimensions there are three different $7_3^{(0,2)}$ branes corresponding to the potentials

$E_{0123456xyz,xy,xy}$ $E_{0123456xyz,xz,xz}$ $E_{0123456xyz,yz,yz}$

Obviously all these branes preserve the same susy because each of them preserves the same susy of the D9

If we now perform two T-dualities in x and y to go back to the D7 wrapping the z direction, these potentials are mapped to

$E_{0123456z}$ $E_{0123456xyz,yz,z}$ $E_{0123456xz,xz,z}$

So together with E_8 we get $E_{9,2,1}$ potential, that is a $7_3^{(1,1)}$ brane

Space-filling branes

We see that we get 2 branes in $D = 8$ and 4 branes in $D = 7$. This is completely general: we get a factor of two each time we go one dimension down

This means that in four dimensions we get 32 3-branes

These branes are precisely the branes in the $(\mathbf{1}, \mathbf{462})$ of $\mathrm{SL}(2, \mathbb{R}) \times \mathrm{SO}(6, 6)$ of the $\mathcal{N} = 4$ theory

This explains why the quadratic constraint in this representation survives the truncation to $\mathcal{N} = 4$ although it is not required by susy

Space-filling branes

The same applies for truncations of theories with less supersymmetry

In particular, we find that the 3-branes that preserve the same supersymmetry as the O7/O3 orientifold of IIB on $T^6/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ are in the representations of $\mathrm{SL}(2,\mathbb{R})^7$ made of three triplets and four singlets

More precisely, we get 16 representations of this kind, which are those with either zero or two triplets of $\mathrm{SL}(2,\mathbb{R})_{U^i}$

Remarkably, these are exactly the representations of the tadpole conditions, that is the representations of the Bianchi identities of the $\mathcal{N}=2$ theory that survive the orientifold truncation but are not required by $\mathcal{N}=1$ susy

Lombardo, FR, Risoli (2017)

IIB/O3 $SL(2,\mathbb{R})^7$ rep	potential	component	#	# reps
$(\mathbf{3}_{T_1},\mathbf{3}_{T_2},\mathbf{3}_{T_3})$	C_4	C_4	1	1
	$E_{8,4}$	$E_{4\,x^i y^i x^j y^j, x^i y^i x^j y^j}$	3	
	$G_{10,6,2,2}$	$G_{10,6,x^i y^i,x^i y^i}$	3	
	$I_{10,6,6,6}$	$I_{10,6,6,6}$	1	
$(\mathbf{3}_{T_i},\mathbf{3}_{U_j},\mathbf{3}_{U_k})$	$E_{8,4}$	$E_{4\,x^i y^i x^j y^k, x^i y^i x^j y^k}$	2	3
		$E_{4\,x^i y^i x^j x^k, x^i y^i x^j x^k}$	1	
		$E_{4\,x^i y^i y^j y^k, x^i y^i y^j y^k}$	1	
	$G_{10,6,2,2}$	$G_{10,6,x^j y^k,x^j y^k}$	2	
		$G_{10,6,x^j x^k,x^j x^k}$	1	
		$G_{10,6,y^j y^k,y^j y^k}$	1	
$(\mathbf{3}_{T_i},\mathbf{3}_{U_i},\mathbf{3}_{U_j})$	$E_{9,2,1}$	$E_{4\,x^i y^i x^j y^j y^k, x^i y^k, x^i}$	1	6
		$E_{4\,x^i y^i x^j y^j y^k, y^i y^k, y^i}$	1	
		$E_{4\,x^i y^i x^j y^j x^k, x^i x^k, x^i}$	1	
		$E_{4\,x^i y^i x^j y^j x^k, y^i x^k, y^i}$	1	
	$G_{10,5,4,1}$	$G_{10,x^j y^j x^k y^k x^i, x^j y^j x^k x^i, x^k}$	1	
		$G_{10,x^j y^j x^k y^k x^i, x^j y^j y^k x^i, y^k}$	1	
		$G_{10,x^j y^j x^k y^k y^i, x^j y^j x^k y^i, x^k}$	1	
		$G_{10,x^j y^j x^k y^k y^i, x^j y^j y^k y^i, y^k}$	1	
$(\mathbf{3}_S,\mathbf{3}_{T_j},\mathbf{3}_{T_k})$	C_8	$C_{4\,x^j y^j x^k y^k}$	1	3
	E_8	$E_{4\,x^j y^j x^k y^k}$	1	
	$E_{10,4,2}$	$E_{10,x^i y^i x^j y^j, x^j y^j}$	2	
	$G_{10,4,2}$	$G_{10,x^i y^i x^j y^j, x^j y^j}$	2	
	$G_{10,6,6,2}$	$G_{10,6,6,x^i y^i}$	1	
	$I_{10,6,6,2}$	$I_{10,6,6,x^i y^i}$	1	
$(\mathbf{3}_S,\mathbf{3}_{U_j},\mathbf{3}_{U_k})$	$E_{10,4,2}$	$E_{10,x^j y^k x^i y^i, x^j y^k}$	2	3
		$E_{10,x^j x^k x^i y^i, x^j x^k}$	1	
		$E_{10,y^j y^k x^i y^i, y^j y^k}$	1	
	$G_{10,4,2}$	$G_{10,x^j y^k x^i y^i, x^j y^k}$	2	
		$G_{10,x^j x^k x^i y^i, x^j x^k}$	1	
		$G_{10,y^j y^k x^i y^i, y^j y^k}$	1	

Conclusions

- We have found a relation between the quadratic constrains of the embedding tensor and the space-filling branes that all preserve the same supersymmetry
- In particular, this explains how the Bianchi identities of the $\mathcal{N} = 2$ theory can be uplifted generating tadpoles for these branes
- There is an elegant relation between these results and the structure of the supersymmetry algebra. The number of truncations to the theory with half the supersymmetry is always given by the number of vector central charges
- We have a universal T-duality rule for all the branes in string theory
- It would be extremely interesting to have some understanding of the physics of exotic branes