

How General is Holography?

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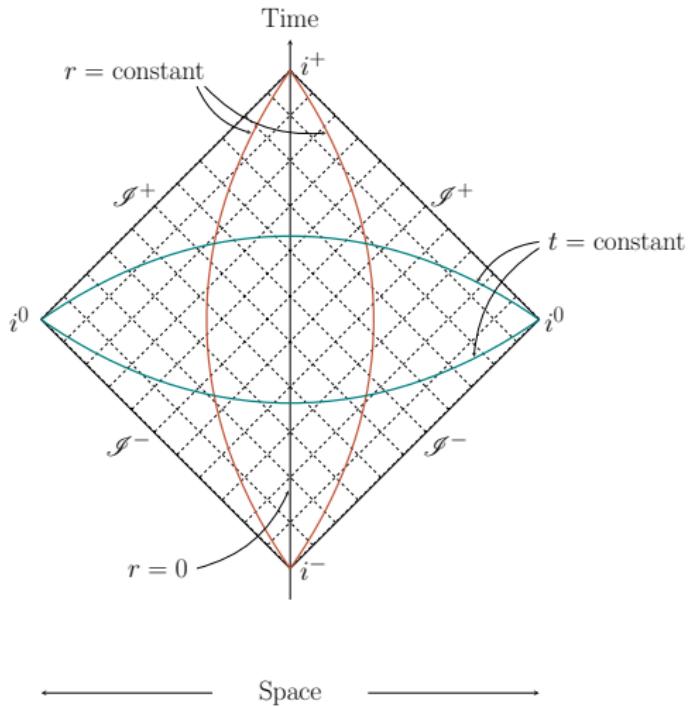
New Frontiers in String Theory One Day Conference
Yukawa Institute for Theoretical Physics, Kyoto University
July 12th, 2018



European Research Council
Established by the European Commission

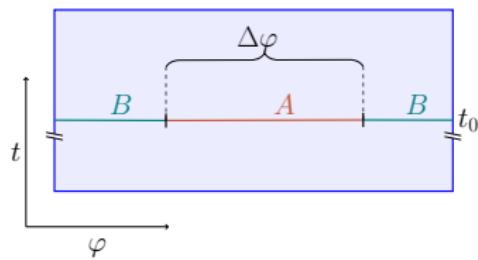
Introduction

Flat Space Holography



Entanglement Entropy and the Replica Trick

CFT₂ and gca₂/bms₃

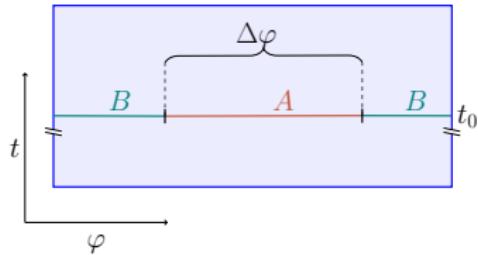


CFT₂

gca₂/bms₃

Entanglement Entropy and the Replica Trick

CFT₂ and $\mathfrak{gca}_2/\mathfrak{bms}_3$

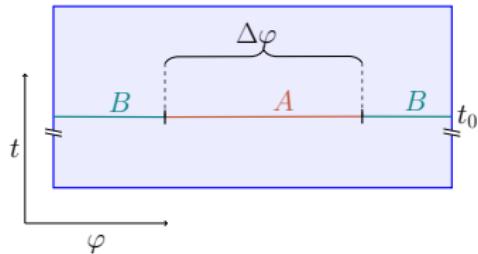


$$\begin{aligned} S_{\text{EE}} &= -\text{tr}_A \rho_A \log \rho_A = -\lim_{n \rightarrow 1} \partial_n \text{Tr}_A \rho_A^n \\ &\propto \lim_{n \rightarrow 1} \partial_n \langle \Phi_n(\varphi_1) \Phi_{-n}(\varphi_2) \rangle_{\mathbb{C}}^n \end{aligned}$$

CFT₂ $\mathfrak{gca}_2/\mathfrak{bms}_3$

Entanglement Entropy and the Replica Trick

CFT₂ and gca₂/bms₃



$$S_{\text{EE}} = \frac{c}{6} \ln \left[\frac{L}{\pi \epsilon} \sin \left(\frac{\pi \Delta \varphi}{L} \right) \right]$$

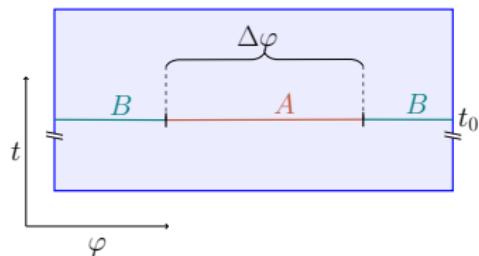
$$+ \frac{\bar{c}}{6} \ln \left[\frac{L}{\pi \epsilon} \sin \left(\frac{\pi \Delta \varphi}{L} \right) \right]$$

CFT₂

gca₂/bms₃

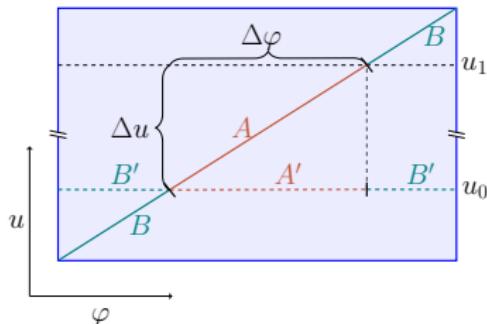
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$$S_{EE} = \frac{c}{6} \ln \left[\frac{L}{\pi \epsilon} \sin \left(\frac{\pi \Delta \varphi}{L} \right) \right]$$

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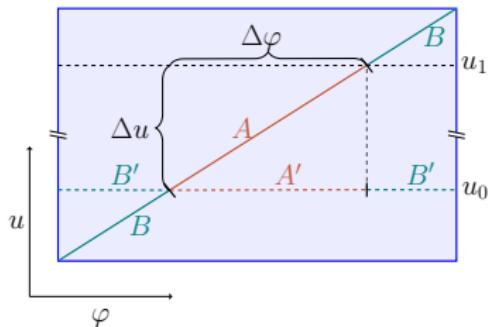
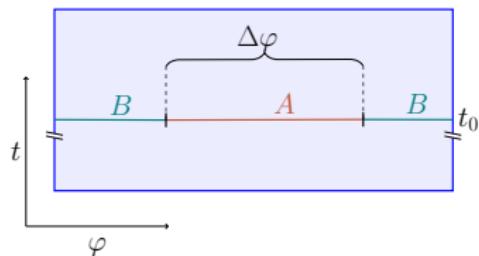


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CFT₂ and $\mathfrak{gca}_2/\mathfrak{bms}_3$



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$$S_{\text{EE}} = -\text{tr}_A \rho_A \log \rho_A = -\lim_{n \rightarrow 1} \partial_n \text{Tr}_A \rho_A^n$$

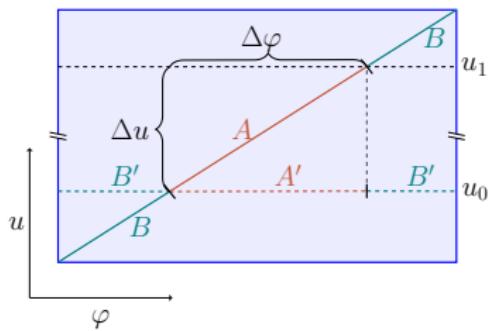
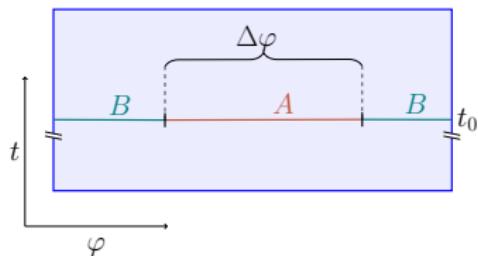
$$\propto \lim_{n \rightarrow 1} \partial_n \langle \Phi_n(\varphi_1, u_1) \Phi_{-n}(\varphi_2, u_2) \rangle_C^n$$

CFT₂

$\mathfrak{gca}_2/\mathfrak{bms}_3$

Entanglement Entropy and the Replica Trick

CFT₂ and $\mathfrak{gca}_2/\mathfrak{bms}_3$



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$$S_{\text{EE}} = \frac{c_L}{6} \ln \left[\frac{L}{\pi \epsilon} \sin \left(\frac{\pi \Delta \varphi}{L} \right) \right] + \frac{\pi}{6L} c_M \Delta u \cot \left(\frac{\pi \Delta \varphi}{L} \right).$$

CFT₂

$\mathfrak{gca}_2/\mathfrak{bms}_3$

Holographic Entanglement Entropy

Wilson Lines



$$W_{\mathcal{R}}(C) = \text{Tr}_{\mathcal{R}} \left[\mathcal{P} \exp \left(\int_C \mathcal{A} \right) \right] = \int \mathcal{D}U \exp (-S(U; \mathcal{A})_C)$$

$$S_{\text{EE}} = -\log [W_{\mathcal{R}}(C)]$$



Ammon, M., Castro, A., and Iqbal, N. (2013).

Wilson Lines and Entanglement Entropy in Higher Spin Gravity.

JHEP, 1310:110.



de Boer, J. and Jottar, J. I. (2014).

Entanglement Entropy and Higher Spin Holography in AdS_3 .

JHEP, 04:089.

$\mathfrak{sl}(2, \mathbb{R})$ and \mathfrak{vir}

$$[L_n, L_m] = (n - m)L_{n+m} + \frac{c}{12}n(n^2 - 1)\delta_{n+m}$$

$$[\bar{L}_n, \bar{L}_m] = (n - m)\bar{L}_{n+m} + \frac{\bar{c}}{12}n(n^2 - 1)\delta_{n+m}$$

$$[L_n, \bar{L}_m] = 0$$

AdS_3

Flat Space

Flat Space Holography

Basic Properties of the Wilson Line

ULB

4

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$$\mathfrak{sl}(2, \mathbb{R}) \oplus \mathfrak{sl}(2, \mathbb{R})$$

↓

$$W_{\mathcal{R}}^c(C) \times W_{\mathcal{R}}^{\bar{c}}(C)$$

↓

$$S_{EE} = -\log [W_{\mathcal{R}}^c(C)] - \log [W_{\mathcal{R}}^{\bar{c}}(C)]$$

AdS₃

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$\mathfrak{isl}(2, \mathbb{R})$ and \mathfrak{bms}_3

$$[L_n, L_m] = (n - m)L_{n+m} + \frac{c_L}{12}n(n^2 - 1)\delta_{n+m}$$

$$[L_n, M_m] = (n - m)M_{n+m} + \frac{c_M}{12}n(n^2 - 1)\delta_{n+m}$$

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AdS₃

Flat Space

Flat Space Holography

Basic Properties of the Wilson Line

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AdS₃

$$\mathfrak{sl}(2, \mathbb{R}) \in_{ad} (\mathfrak{sl}(2, \mathbb{R}))_{Ab}$$

✗

$$W_{\mathcal{R}}^{c_L}(C) \times W_{\mathcal{R}}^{c_M}(C)$$

? ↓ ?

$$S_{EE} = -\log [W_{\mathcal{R}}^{c_L}(C)] - \log [W_{\mathcal{R}}^{c_M}(C)]$$

Flat Space



Assumption

$$U = U_L U_M \quad \text{for} \quad U \in ISL(2, \mathbb{R})$$

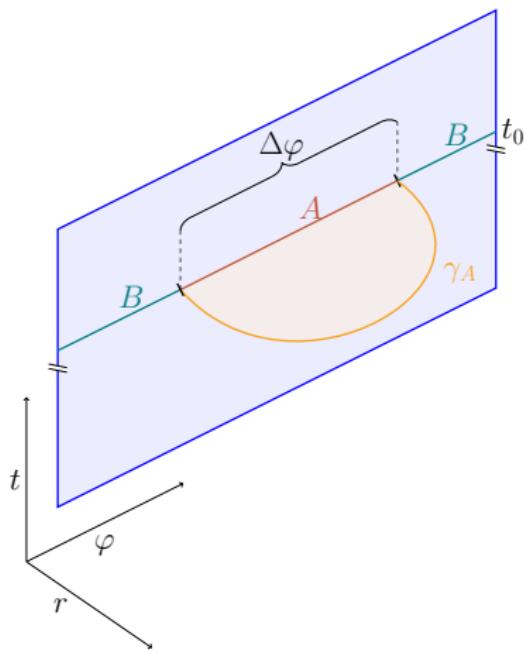
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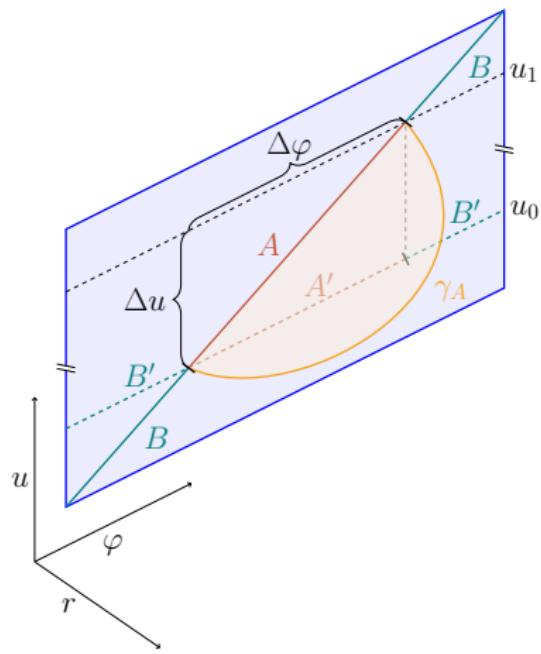
$$\begin{array}{c} \Downarrow \checkmark \\ W_{\mathcal{R}}^{c_L}(C) \times W_{\mathcal{R}}^{c_M}(C) \\ \Downarrow \\ S_{EE} = -\log [W_{\mathcal{R}}^{c_L}(C)] - \log [W_{\mathcal{R}}^{c_M}(C)] \end{array}$$

Flat Space Holography

Entangling Intervals



AdS₃



Flat Space

Flat Space Holography

Metric, Connection and Holographic Entanglement Entropy



$$ds^2 = \mathcal{M} du^2 - 2 du dr + 2\mathcal{N} du d\varphi + r^2 d\varphi^2$$

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$\mathfrak{isl}(2, \mathbb{R})$ Chern-Simons Gauge Field

$$\mathcal{A} = b^{-1}(\mathrm{d} + a)b \quad \text{with} \quad b = e^{\frac{r}{2}\mathbb{M}_{-1}}$$

$$a = \left(\mathbb{M}_1 - \frac{\mathcal{M}}{4} \mathbb{M}_{-1} \right) \mathrm{d}u + \left(\mathbb{L}_1 - \frac{\mathcal{M}}{4} \mathbb{L}_{-1} - \frac{\mathcal{N}}{2} \mathbb{M}_{-1} \right) \mathrm{d}\varphi$$

$$ds^2 = \mathcal{M} du^2 - 2 du dr + 2\mathcal{N} du d\varphi + r^2 d\varphi^2$$

$\mathfrak{sl}(2, \mathbb{R})$ Chern-Simons Gauge Field

$$\mathcal{A} = b^{-1}(d+a)b \quad \text{with} \quad b = e^{\frac{r}{2}\mathbb{M}_{-1}}$$

$$a = \left(\mathbb{M}_1 - \frac{\mathcal{M}}{4}\mathbb{M}_{-1} \right) du + \left(\mathbb{L}_1 - \frac{\mathcal{M}}{4}\mathbb{L}_{-1} - \frac{\mathcal{N}}{2}\mathbb{M}_{-1} \right) d\varphi$$

- $\mathcal{M} > 0, \mathcal{N} \neq 0$: Flat Space Cosmologies

Flat Space Holography

Metric, Connection and Holographic Entanglement Entropy



$$ds^2 = -du^2 - 2du dr + r^2 d\varphi^2$$

$\mathfrak{sl}(2, \mathbb{R})$ Chern-Simons Gauge Field

$$\mathcal{A} = b^{-1}(d+a)b \quad \text{with} \quad b = e^{\frac{r}{2}\mathbb{M}_{-1}}$$

$$a = \left(\mathbb{M}_1 + \frac{1}{4}\mathbb{M}_{-1} \right) du + \left(\mathbb{L}_1 + \frac{1}{4}\mathbb{L}_{-1} \right) d\varphi$$

- ▶ $\mathcal{M} > 0, \mathcal{N} \neq 0$: Flat Space Cosmologies
- ▶ $\mathcal{M} = -1, \mathcal{N} = 0$: Global Flat Space

Flat Space Holography

Metric, Connection and Holographic Entanglement Entropy



$$ds^2 = -2du dr + r^2 d\varphi^2$$

$\mathfrak{isl}(2, \mathbb{R})$ Chern-Simons Gauge Field

$$\mathcal{A} = b^{-1}(\mathrm{d} + a)b \quad \text{with} \quad b = e^{\frac{r}{2}\mathbb{M}-1}$$

$$a = \mathbb{M}_1 \mathrm{d}u + \mathbb{L}_1 \mathrm{d}\varphi$$

- ▶ $\mathcal{M} > 0, \mathcal{N} \neq 0$: Flat Space Cosmologies
- ▶ $\mathcal{M} = -1, \mathcal{N} = 0$: Global Flat Space
- ▶ $\mathcal{M} = \mathcal{N} = 0$: Null-Orbifold

$$ds^2 = -du^2 - 2du dr + r^2 d\varphi^2$$

$\mathfrak{sl}(2, \mathbb{R})$ Chern-Simons Gauge Field

$$\mathcal{A} = b^{-1}(d+a)b \quad \text{with} \quad b = e^{\frac{r}{2}\mathbb{M}_{-1}}$$

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$$ds^2 = -du^2 - 2du dr + r^2 d\varphi^2$$

Flat Space Holographic Entanglement Entropy

$$S_{EE} = \frac{c_L}{6} \ln \left[r_0 \sin \left(\frac{\Delta\varphi}{2} \right) \right] + \frac{c_M}{12} \cot \left(\frac{\Delta\varphi}{2} \right) \Delta u.$$

- ▶ $\mathcal{M} > 0, \mathcal{N} \neq 0$: Flat Space Cosmologies
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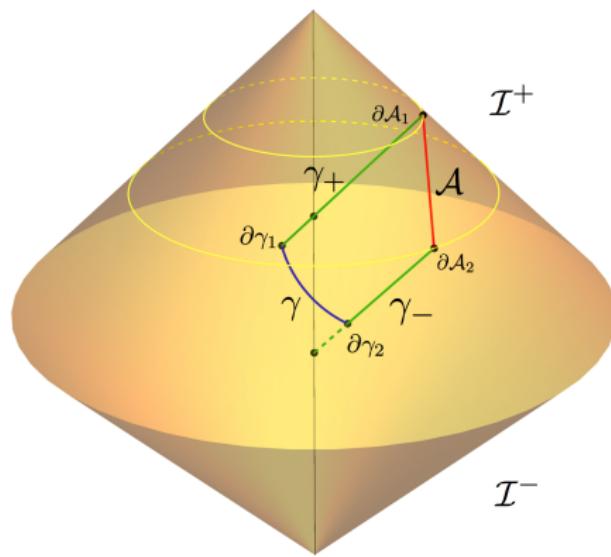
Bagchi, A., Basu, R., Grumiller, D., and Riegler, M. (2015).

Entanglement entropy in Galilean conformal field theories and flat holography.

Phys. Rev. Lett., 114(11):111602.

Flat Space Holography

Geometric Picture



Hongliang Jiang, Wei Song, and Qiang Wen.

Entanglement Entropy in Flat Holography.

JHEP, 07:142, 2017.

Conclusions and Outlook

The Frog, the Bird and the Wizard



- ▶ Holographic intuition coming from AdS/CFT also seems to be applicable to flat space holography.

Conclusions and Outlook

The Frog, the Bird and the Wizard



- ▶ Holographic intuition coming from AdS/CFT also seems to be applicable to flat space holography.
- ▶ Extension to multipartite entanglement entropy?

Conclusions and Outlook

The Frog, the Bird and the Wizard



- ▶ Holographic intuition coming from AdS/CFT also seems to be applicable to flat space holography.
- ▶ Extension to multipartite entanglement entropy?
- ▶ Extension to higher-dimensions?

Thank you for your attention!

