Open superstring field theory including the Ramond sector based on the supermoduli space

Tomoyuki Takezaki
The University of Tokyo, Institute of Physics

based on a paper in preparation

"New Frontiers in String Theory 2018"
17 Jul, 2018 in Kyoto

Formulations for open superstring field theory

The Berkovits formulation [Berkovits 1995]

- The action has Wess-Zumino-Witten-like gauge invariance.

$\rightarrow$ Ramond extension: [Kunitomo, Okawa 2015]

Homotopy algebra based construction [Erler, Konopka, Sachs 2013]

- The action exhibits the A_∞ structure.

$\rightarrow$ Ramond extension: [Erler, Okawa, TT 2016], [Konopka, Sachs 2016]

Sen's quantum BV master action [Sen 2015]

- Simpler worldsheet realization of interaction terms

$\rightarrow$ Applications to open SSFT: [Erler, Okawa, TT 2016], [Konopka, Sachs 2016]

Open SSFT based on the covering of supermoduli space

- Recently, a new approach to formulating NS sector of open superstring field theory based on the covering of the supermoduli space of super-Riemann surfaces was proposed [Ohmori, Okawa2017].
- We extend this approach and construct a gauge-invariant action including the Ramond sector up to quartic interactions.
- Our approach is based on the covering of the supermoduli space, and our action exhibits the A_∞ structure.
- We also construct an action based on the products with stubs. One of the advantages of our construction is that incorporation of stubs is straightforward.

Table of contents

- 1. A_∞ structure
- 2. Neveu-Schwarz sector [Ohmori, Okawa 2017]
- 3. Ramond sector
- 4. Open superstring field thoery with stubs

1. A_∞ structure

Witten's open bosonic string field theory [Witten 1985]

$$S = -\frac{1}{2}\langle\Psi, Q\Psi\rangle - \frac{g}{3}\langle\Psi, \Psi * \Psi\rangle\,,$$
$$\delta\Psi = Q\Lambda - g\Lambda * \Psi + g\Psi * \Lambda\,.$$

The star product $*$ is non-commuttative: $A * B \neq B * A$,
but associative: $(A * B) * C = A * (B * C)$.

The gauge invariance follows from

$$\langle A, B\rangle = (-1)^{AB}\langle B, A\rangle\,,$$
$$\langle QA, B\rangle = -(-1)^{A}\langle A, QB\rangle\,,$$
$$\langle A * B, C\rangle = \langle A, B * C\rangle\,,$$
$$Q^2 A = 0\,,$$
$$Q(A * B) = QA * B + (-1)^{A} A * QB\,,$$
$$(A * B) * C = A * (B * C)\,.$$

Actually, we can construct a gauge-invariant action based on a string product without associative two-string product:

$$S = -\frac{1}{2}\langle\,\Psi, Q\Psi\,\rangle - \frac{g}{3}\langle\,\Psi, V_2(\Psi,\Psi)\,\rangle - \frac{g^2}{4}\langle\,\Psi, V_3(\Psi,\Psi,\Psi)\,\rangle + \mathcal{O}(g^3)\,.$$

We assume V_2, V_3 are cyclic:

$$\langle\,V_2(A_1,A_2),A_3\,\rangle = \langle\,A_1, V_2(A_2,A_3)\,\rangle\,,$$
$$\langle\,V_3(A_1,A_2,A_3),A_4\,\rangle = -(-1)^{A_1}\langle\,A_1, V_3(A_2,A_3,A_4)\,\rangle\,.$$

The action is invariant up to $\mathcal{O}(g^3)$ under a gauge transformation

$$\begin{aligned}\delta\Psi = {}& Q\Lambda + gV_2(\Psi,\Lambda) - gV_2(\Lambda,\Psi) \\ & + g^2V_3(\Psi,\Psi,\Lambda) - g^2V_3(\Psi,\Lambda,\Psi) + g^2V_3(\Lambda,\Psi,\Psi) + \mathcal{O}(g^3)\,,\end{aligned}$$

if Q, V_2, and V_3 satisfy ...

A_∞ structure

$$
\begin{aligned}
0 &= Q^2 A_1 \\
0 &= QV_2(A_1, A_2) - V_2(QA_1, A_2) - (-1)^{A_1} V_2(A_1, QA_2) \\
0 &= QV_3(A_1, A_2, A_3) - V_2(V_2(A_1, A_2), A_3) + V_2(A_1, V_2(A_2, A_3)) \\
&\quad + V_3(QA_1, A_2, A_3) + (-1)^{A_1} V_3(QA_1, A_2, A_3) \\
&\quad + (-1)^{A_1 + A_2} V_3(A_1, A_2, QA_3)
\end{aligned}
$$

- These equations are extended to higher orders, and a set of these relations is called an A_∞ structure.
- The A_∞ structure is closely related to the decomposition of the moduli space of Riemann surfaces.
- The quantization of string field theory based on the Batalin-Vilkovisky formalism is straightforward if the theory has the A_∞ structure.

2. Neveu-Schwarz sector [Ohmori, Okawa 2017]

Two-string product (NS sector)

Disks with three NS punctures have one odd modulus, and we denote it by ζ.

In [Ohmori, Okawa 2017], an integral over the odd modulus is implemented by

$$X_N = \int d\zeta d\tilde{\zeta} \mathcal{X}_N(\zeta, \tilde{\zeta}) , \quad \mathcal{X}_N(\zeta, \tilde{\zeta}) = e^{-\tilde{\zeta}\beta_{-1/2} + \zeta G_{-1/2}} ,$$

where $\tilde{\zeta}$ is a Grassmann-even variable [Witten 2012]. In [Ohmori, Okawa 2017], the two-string product in the following form is introduced:

$$V_2(N_1, N_2) = \frac{1}{3}\Big({X_N}^{\star}(N_1 * N_2) + X_N N_1 * N_2 + N_1 * X_N N_2 \Big) .$$

$V_2(N_1, N_2)$ realizes cyclic A_∞ structure at this order, but not associative:

$$V_2(V_2(N_1, N_2), N_3) \neq V_2(N_1, V_2(N_2, N_3)) .$$

We need a three-string product to satisfy the 3rd order A_∞ relation.

The 3rd order A_∞ relation

We define $V_3(N_1, N_2, N_3)$ in terms of a Grassmann-odd operator Ξ:

$$\langle\, N_1, V_3(N_2, N_3, N_4)\,\rangle \equiv \langle\, g_1 \circ N_1(0)\, \Xi\, g_2 \circ N_2(0)\, g_3 \circ N_3(0)\, g_4 \circ N_4(0)\,\rangle_D\,.$$

where $\langle \ldots \rangle_D$ is the correlation function on the disk.

Furthermore, we define Grassmann-even operators X_t, X_s by

$$\langle\, N_1, V_2(V_2(N_2, N_3), N_4)\,\rangle \equiv \langle\, X_t\, g_1 \circ N_1(0)\, g_2 \circ N_2(0)\, g_3 \circ N_3(0)\, g_4 \circ N_4(0)\,\rangle_D\,,$$
$$\langle\, N_1, V_2(N_2, V_2(N_3, N_4))\,\rangle \equiv \langle\, X_s\, g_1 \circ N_1(0)\, g_2 \circ N_2(0)\, g_3 \circ N_3(0)\, g_4 \circ N_4(0)\,\rangle_D\,.$$

In this notation, the 3rd order A_∞ relation is expressed as

$$Q \cdot \Xi = X_t - X_s\,.$$

Rule of the game (NS sector)

The 3rd order A_∞ relation in terms of Ξ

$$Q \cdot \Xi = X_t - X_s \,.$$

We construct Ξ in the following steps:

1. define two-string product V_2 (already done),
2. calculate X_t and X_s from the definition of V_2,
3. pull out Q from $X_t - X_s$.

Since we assumed $\langle V_3(A_1, A_2, A_3), A_4 \rangle = -(-1)^{A_1} \langle A_1, V_3(A_2, A_3, A_4) \rangle$, the operator Ξ must be constructed to satisfy the cyclic equation

$$\omega \circ \Xi = -\Xi \,,$$

where $\omega(z) = e^{i\pi/2} z = iz$ is 90°-rotation on disks.

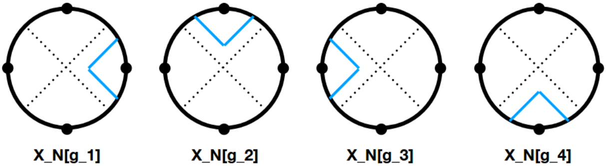
We define $X_N[g_i]$ for conformal transformation $g_i(\xi)$ by

$$X_N[g_i] = \int d\zeta d\tilde{\zeta} e^{-\tilde{\zeta}\beta_N[g_i]+\zeta G_N[g_i]}$$

where $\beta_N[g_i]$ and $G_N[g_i]$ are pull back of $\beta_{-1/2}$ and $G_{-1/2}$ by g_i. Then we can pull out X_N from each vertex operators.

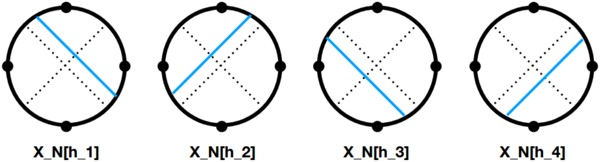
For example, we find

$$\langle\, X_N N_1, N_2 {*} (X_N N_3) {*} N_4 \,\rangle = \langle\, X_N[g_1]\, X_N[g_3]\, g_1 \circ N_1(0)\, g_2 \circ N_2(0)\, g_3 \circ N_3(0)\, g_4 \circ N_4(0) \rangle_D \,.$$



Step 2: calculate X_t and X_s (continued)

Furthermore, we introduce $X_N[h_i]$ for $(i = 1, 2, 3, 4)$ with $h_i(\xi)$



For example, we find

$$\langle\, (X_N N_1), N_2 X_N (N_3 N_4)\,\rangle = \langle\, X_N[g_1]\, X_N[h_3]\, g_1\circ N_1(0)\, g_2\circ N_2(0)\, g_3\circ N_3(0)\, g_4\circ N_4(0) \rangle_D \,.$$

Using this method, we find

$$X_t = \frac{1}{9}\Big(X_N[h_4] + X_N[g_2] + X_N[g_3] \Big)\Big(X_N[g_1] + X_N[h_2] + X_N[g_4] \Big)\,,$$
$$X_s = \frac{1}{9}\Big(X_N[h_1] + X_N[g_3] + X_N[g_4] \Big)\Big(X_N[g_2] + X_N[h_3] + X_N[g_1] \Big)\,.$$

Step 3: pull out Q from $X_t - X_s$

It is useful to introduce the following operator:

$$\Xi[a,b] = \int_0^1 dt' \int d\tilde{t}' \int d\zeta d\tilde{\zeta} \; e^{-\{Q', t'\zeta\beta_N[a]\}} \; e^{-\{Q', (1-t')\zeta\beta_N[b]\}} ,$$

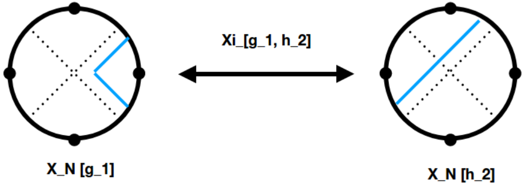
where Q' is the extended BRST operator [Witten 2012]:

$$Q' = Q + \tilde{t}'\partial_{t'} + \tilde{\zeta}\partial_{\zeta} .$$

The operator $\Xi[a,b]$ satisfies

$$Q \cdot \Xi[a,b] = X_N[a] - X_N[b] .$$

Therefore, we can pull out Q from the difference of X_N.



In [Ohmori, Okawa 2017], an operator Ξ satisfying the 3rd order A_∞ relation and the cyclic equation was given in the following form:

$$\Xi = \frac{1}{18}\Big(\Xi[h_2, h_4; h_3, h_1] + (17 \text{ terms}) \Big),$$

where

$$\Xi[a_1, a_2; b_1, b_2] = \frac{1}{2}\Big(\Xi[a_1, b_1] X_N[a_2] + (3 \text{ terms}) \Big).$$

- Ξ consists of an integration over one even modulus and two odd moduli.
- The three-string product V_3 constructed from Ξ covers the missing region of moduli space of disks with four NS punctures.

3. Ramond sector

Ramond string field

For the Ramond sector, it is known that the BRST cohomology on an appropriate subspace of the small Hilbert space reproduces the correct spectrum, and we use the string field restricted to this subspace.

The restriction on the Ramond field is characterized by $X_R Y_R \Psi_R = \Psi_R$, where

$$X_R = \int d\zeta d\tilde{\zeta} \mathcal{X}_R(\zeta, \tilde{\zeta}), \quad \mathcal{X}_R(\zeta, \tilde{\zeta}) = e^{-\tilde{\zeta}\beta_0 + \zeta G_0}, \quad Y_R = -c_0 \delta'(\gamma_0).$$

X_R commutes with Q, and X_R is BPZ even: $X_R^\star = X_R$. For restricted Ramond fields R_1, R_2, we use the inner product in the following form:

$$\langle R_1, Y_R R_2 \rangle .$$

Define X_t, X_s, and Ξ for NS/R sector

We consider two-string products $V_2(N_1, R_1)$, $V_2(R_1, N_1)$, $V_2(R_1, R_2)$, and three-string products

$$V_3(N_1, N_2, R_1)\,, V_3(N_1, R_1, N_2)\,, V_3(R_1, N_1, N_2)\,,$$
$$V_3(N_1, R_1, R_2)\,, V_3(R_1, N_1, R_2)\,, V_3(R_1, R_2, N_1)\,, V_3(R_1, R_2, R_3)\,.$$

We define Ξ, X_t, and X_s for NS/R sector. For example,

$$\langle\, R_1, Y_R V_3(N_2, R_3, N_4)\,\rangle \equiv \langle\, g_1 \circ R_1(0)\, \Xi_{RNRN}\, g_2 \circ N_1(0)\, g_3 \circ R_2(0)\, g_4 \circ N_2(0)\,\rangle_D\,,$$
$$\langle R_1, Y_R V_2(V_2(N_1, R_1), N_2)\rangle \equiv \langle\, (X_t)_{RNRN}\, g_1 \circ R_1(0)\, g_2 \circ N_1(0)\, g_3 \circ R_2(0)\, g_4 \circ N_2(0)\,\rangle_D\,,$$
$$\langle R_1, Y_R V_2(N_1, V_2(N_2, N_2))\rangle \equiv \langle\, (X_s)_{RNRN}\, g_1 \circ R_1(0)\, g_2 \circ N_1(0)\, g_3 \circ R_2(0)\, g_4 \circ N_2(0)\,\rangle_D\,.$$

Rule of the game (NS/R sector)

A_∞ relation for $V_3(N_1, R_1, N_2)$ in terms of Ξ_{RNRN}

$$Q \cdot \Xi_{RNRN} = (X_t)_{RNRN} - (X_s)_{RNRN} \,.$$

We construct Ξ_{RNRN} in the following steps:

1. define the two-string products V_2 for NS/R inputs,
2. calculate $(X_t)_{RNRN}$ and $(X_s)_{RNRN}$ from the definition of V_2 ,
3. pull out Q from $(X_t)_{RNRN} - (X_s)_{RNRN}$.

The cyclic equation $\langle\, V_3(N_1, R_1, N_2), R_2 \,\rangle = -(-1)^{N_1} \langle\, N_1, V_3(R_1, N_2, R_2) \,\rangle$ is translated into

$$\omega \circ \Xi_{RNRN} = -\Xi_{NRNR} \,,$$

Step 1: define the two-string products for NS/R inputs

Let us start form the NS-R-R interaction. Since disks with two R punctures and one NS puncture has no moduli, we can simply use the star product:

$$\alpha \langle \Psi_N, \Psi_R * \Psi_R \rangle ,$$

where α is non-zero constant to be determined. This term can be rewritten as

$$\alpha \langle \Psi_N, \Psi_R * \Psi_R \rangle = \alpha \langle \Psi_R, Y_R X_R(\Psi_N * \Psi_R) \rangle = \alpha \langle \Psi_R, Y_R X_R(\Psi_R * \Psi_N) \rangle .$$

Motivated by this equation, we define the following two-string products:

$$\begin{aligned} V_2(N_1, R_1) &= X_R(N_1 * R_1) , \\ V_2(R_1, N_1) &= X_R(R_1 * N_1) , \\ V_2(R_1, R_2) &= R_1 * R_2 . \end{aligned}$$

These two-string products have cyclic A_∞ structure at this order, but not associative except for $V_2(V_2(R_1, R_2), R_3) = V_2(R_1, V_2(R_2, R_3))$.

Step 2: calculate $(X_t)_{RNRN}$ and $(X_s)_{RNRN}$

Using the two-string products defined in the step 1, we find

$$\begin{aligned}\langle\, R_1, Y_R V_2(V_2(N_1, R_2), N_2)\,\rangle &= \langle\, R_1, Y_R X_R(\, X_R(N_1 * R_2) * N_2\,)\,\rangle \\ &= \langle\, R_1, X_R(N_1 * R_2) * N_2\,\rangle\,,\end{aligned}$$

and

$$\begin{aligned}\langle\, R_1, Y_R V_2(N_1, V_2(R_2, N_2))\,\rangle &= \langle\, R_1, Y_R X_R(\, N_1 * X_R(R_2 * N_2)\,)\,\rangle \\ &= \langle\, R_1, N_1 * X_R(R_2 * N_2)\,\rangle\,.\end{aligned}$$

Therefore, we have

$$(X_t)_{RNRN} = X_R[h_2], \quad (X_s)_{RNRN} = X_R[h_3]\,,$$

where $X_R[h_i]$ is pull back of X_R by h_i.

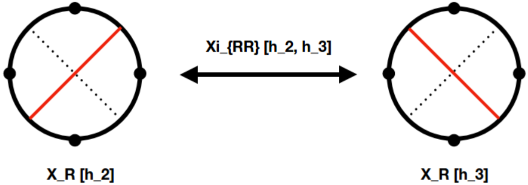
Step 3: pull out Q from $(X_t)_{RNRN} - (X_S)_{RNRN}$

The 3rd order A_∞ relation is $Q \cdot \Xi_{RNRN} = X_R[h_2] - X_R[h_3]$. We introduce

$$\Xi_{RR}[a,b] = \int_0^1 dt' \int d\tilde{t'} \int d\zeta d\tilde{\zeta}\, e^{-\{Q',t'\zeta\beta_R[a]\}}\, e^{-\{Q',(1-t')\zeta\beta_R[b]\}},$$

which satisfies

$$Q \cdot \Xi_{RR}[a,b] = X_R[a] - X_R[b].$$



$$\Xi_{RNRN} = \Xi_{RR}[h_2,h_3].$$

We can also construct Ξ_{NRNR} and show the equation $\omega \circ \Xi_{RNRN} = -\Xi_{NRNR}$.

For other NS and R combination, we can show that the following $\Xi_{NNRR}, \dots$ satisfies the 3rd order A_∞ relation and cyclic equations.

$$\Xi_{RRNN} = \frac{1}{3}\Big(\Xi_{RN}[h_2, h_1] + \Xi_{RN}[h_2, g_3] + \Xi_{RN}[h_2, g_4]\Big),$$
$$\Xi_{NRRN} = \frac{1}{3}\Big(\Xi_{NR}[g_1, h_3] + \Xi_{NR}[h_2, h_3] + \Xi_{NR}[g_4, h_3]\Big),$$
$$\Xi_{RNNR} = \frac{1}{3}\Big(\Xi_{NR}[h_4, h_1] + \Xi_{NR}[g_2, h_1] + \Xi_{NR}[g_3, h_1]\Big),$$
$$\Xi_{RRNN} = \frac{1}{3}\Big(\Xi_{NR}[h_1, h_2] + \Xi_{NR}[g_3, h_2] + \Xi_{NR}[g_4, h_2]\Big).$$

where

$$\Xi_{RN}[a,b] = \int_0^1 dt' \int d\tilde{t'} \int d\zeta d\tilde{\zeta}\; e^{-\{Q', t'\zeta\beta_R[a]\}}\; e^{-\{Q', (1-t')\zeta\beta_N[b]\}},$$
$$\Xi_{NR}[a,b] = \int_0^1 dt' \int d\tilde{t'} \int d\zeta d\tilde{\zeta}\; e^{-\{Q', t'\zeta\beta_N[a]\}}\; e^{-\{Q', (1-t')\zeta\beta_R[b]\}}.$$

- The three-string products constructed from $\Xi_{RNRN}, \dots$ covers the missing region of moduli space of disks with two NS and two R punctures.

4. Open superstring field theory with stubs

Open bosonic SFT with stubs - step1

We define the bosonic two-string product by attaching propagators of length w:

$$V_2^w(A_1, A_2) \equiv \; e^{-wL_0}\Big((e^{-wL_0}A_1) * (e^{-wL_0}A_2)\Big)\,.$$

V_2^w has the cyclic A_∞ structure at this order, but not associative.

Let us consider V_3^w. We define $\mathcal{B}_3^w, \mathcal{L}_t^w$, and $\mathcal{L}_s^w$ by

$$\begin{aligned}
\langle\, A_1, V_3^w(A_2, A_3, A_4)\,\rangle &\equiv \langle\, g_1\circ A_1(0)\, \mathcal{B}_3^w\, g_2\circ A_2(0)\, g_3\circ A_3(0)\, g_4\circ A_4(0)\,\rangle_D\,,\\
\langle\, A_1, V_2^w(V_2^w(A_2, A_3), A_4)\,\rangle &\equiv \langle \mathcal{L}_t^w g_1\circ A_1(0)\, g_2\circ A_2(0)\, g_3\circ A_3(0)\, g_4\circ A_4(0))\,\rangle_D\,,\\
\langle\, A_1, V_2^w(A_2, V_2^w(A_3, A_4))\,\rangle &\equiv \langle \mathcal{L}_s^w g_1\circ A_1(0)\, g_2\circ A_2(0)\, g_3\circ A_3(0)\, g_4\circ A_4(0))\,\rangle_D\,.
\end{aligned}$$

Rule of the game (open bosonic SFT with stubs)

The 3rd order A_∞ relation for V_3^w in terms of $\mathcal{B}_3^w$

$$Q \cdot \mathcal{B}_3^w = \mathcal{L}_t^w - \mathcal{L}_s^w \, .$$

We construct $\mathcal{B}_3^w$ in the following steps:

1. define the two-string product V_2^w (already finished),
2. calculate $\mathcal{L}_t^w$ and $\mathcal{L}_s^w$ from the definition of V_2^w ,
3. pull out Q from $\mathcal{L}_t^w - \mathcal{L}_s^w$.

The cyclic equation for V_3^w is translated into

$$\omega \circ \mathcal{B}_3^w = -\mathcal{B}_3^w \, .$$

Step 2 & 3

Using the definition of V_2^w, we have

$$\mathcal{L}_t^w = e^{-wL_0[g_1]}\, e^{-2wL_0[h_2]}\, e^{-wL_0[g_2]}\, e^{-wL_0[g_3]}\, e^{-wL_0[g_4]}\,,$$
$$\mathcal{L}_s^w = e^{-wL_0[g_1]}\, e^{-wL_0[g_2]}\, e^{-2wL_0[h_3]}\, e^{-wL_0[g_3]}\, e^{-wL_0[g_4]}\,.$$

We introduce an interpolation function $\mathcal{B}[a,b]$:

$$\mathcal{B}[a,b] = \int_0^1 dt \int d\tilde{t}\left(e^{\{Q',(1-t)(-2w)b_0[a]\}} + e^{\{Q',t(-2w)b_0[b]\}}\right),$$

where $\tilde{t}$ is a Grassmann-odd variable, and $Q' = Q + \tilde{t}\partial_t$. $\mathcal{B}[a,b]$ satisfies

$$Q \cdot \mathcal{B}[a,b] = e^{-2wL_0[a]} - e^{-2wL_0[b]}\,.$$

Then $\mathcal{B}_3$ is realized in the following form:

$$\mathcal{B}_3^w = \mathcal{B}[h_2,h_3]\, e^{-wL_0[g_1]}\, e^{-wL_0[g_2]}\, e^{-wL_0[g_3]}\, e^{-wL_0[g_4]}\,.$$

Rule of the game (open SSFT with stubs)

Let us consider open SSFT including the Ramond sector with stubs. We define Ξ^{w}_{RNRN}, $(X_t)^{w}_{RNRN}$, and $(X_s)^{w}_{RNRN}$ as in the case of OSSFT without stubs.

The 3rd order A_∞ relation in terms of Ξ^{w}_{RNRN}

$$Q \cdot \Xi^{w}_{RNRN} = (X_t)^{w}_{RNRN} - (X_s)^{w}_{RNRN} \,.$$

We construct Ξ^{w}_{RNRN} in the following steps:

1. define the two-string products,
2. calculate $(X_t)^{w}_{RNRN}$ and $(X_s)^{w}_{RNRN}$,
3. pull out Q from $(X_t)^{w}_{RNRN} - (X_s)^{w}_{RNRN}$.

The cyclic equation for $V_3(N_1, R_1, N_1)$ is translated into

$$\omega \circ \Xi^{w}_{RNRN} = -\Xi^{w}_{NRNR} \,.$$

Step 1 & 2

We define two-string products by replacing the star product $*$ with V_2^w:

$$V_2(N_1, N_2) = \frac{1}{3}\left(X_N^\star V_2^w(N_1, N_2) + V_2^w(X_N N_1, N_2) + V_2^w(N_1, X_N N_2)\right),$$
$$V_2(N_1, R_1) = X_R V_2^w(N_1, R_1),$$
$$V_2(R_1, N_1) = X_R V_2^w(R_1, N_1),$$
$$V_2(R_1, R_2) = V_2^w(R_1, R_2).$$

Since we just switched the star product $*$ to V_2^w, these two-string products have cyclic A_∞ structure at this order. Then we have

$$(X_t)^w_{RNRN} = X_R[h_2]\, e^{-wL_0[g_1]}\, e^{-2wL_0[h_2]}\, e^{-wL_0[g_2]}\, e^{-wL_0[g_3]}\, e^{-wL_0[g_4]},$$
$$(X_s)^w_{RNRN} = X_R[h_3]\, e^{-wL_0[g_1]}\, e^{-wL_0[g_2]}\, e^{-2wL_0[h_3]}\, e^{-wL_0[g_3]}\, e^{-wL_0[g_4]}.$$

Remember that $X_R[h_2] = (X_t)_{RNRN}$, $X_R[h_3] = (X_s)_{RNRN}$, therefore

$$(X_t)^w_{RNRN} = (X_t)_{RNRN}\mathcal{L}_t^w, \quad (X_s)^w_{RNRN} = (X_s)_{RNRN}\mathcal{L}_s^w.$$

Step 3: pull out Q from $(X_t)^w_{RNRN} - (X_s)^w_{RNRN}$

$$\begin{aligned}
&(X_t)^w_{RNRN} - (X_s)^w_{RNRN}\\
&= \Big((X_t)_{RNRN} e^{-2wL_0[h_2]} - (X_s)_{RNRN} e^{-2wL_0[h_3]} \Big)\\
&\qquad \times e^{-wL_0[g_1]}\, e^{-wL_0[g_2]}\, e^{-wL_0[g_3]}\, e^{-wL_0[g_4]}\\
&= \Big((X_t)_{RNRN}(e^{-2wL_0[h_2]} - 1) + (X_t)_{RNRN} - (X_s)_{RNRN} + (X_s)_{RNRN}(1 - e^{-2wL_0[h_3]}) \Big)\\
&\qquad \times e^{-wL_0[g_1]}\, e^{-wL_0[g_2]}\, e^{-wL_0[g_3]}\, e^{-wL_0[g_4]}
\end{aligned}$$

We introduce

$$\mathcal{B}[a,*] = \int_0^1 dt \int d\tilde{t}\, e^{\{Q',(1-t)(-2w)b_0[a]\}}\,, \quad \mathcal{B}[*,b] = \int_0^1 dt \int d\tilde{t}\, e^{\{Q',t(-2w)b_0[b]\}}\,,$$

which satisfy

$$Q \cdot \mathcal{B}[a,*] = e^{-2wL_0[a]} - 1\,, \quad Q \cdot \mathcal{B}[*,b] = 1 - e^{-2wL_0[b]}\,.$$

Step 3 (continued)

Then we have

$$
\begin{aligned}
&(X_t)^w_{RNRN} - (X_s)^w_{RNRN} \\
&= \Big((X_t)_{RNRN} Q \cdot \mathcal{B}[h_2, *] + Q \cdot \Xi_{RNRN} + (X_s)_{RNRN} Q \cdot \mathcal{B}[*, h_3] \Big) \\
&\qquad \times e^{-wL_0[g_1]}\, e^{-wL_0[g_2]}\, e^{-wL_0[g_3]}\, e^{-wL_0[g_4]} \\
&= Q \cdot \Big[\Big((X_t)_{RNRN}\, \mathcal{B}[h_2, *] + \Xi_{RNRN} + (X_s)_{RNRN}\, \mathcal{B}[*, h_3] \Big) \\
&\qquad \times e^{-wL_0[g_1]}\, e^{-wL_0[g_2]}\, e^{-wL_0[g_3]}\, e^{-wL_0[g_4]} \Big].
\end{aligned}
$$

Finally, we find

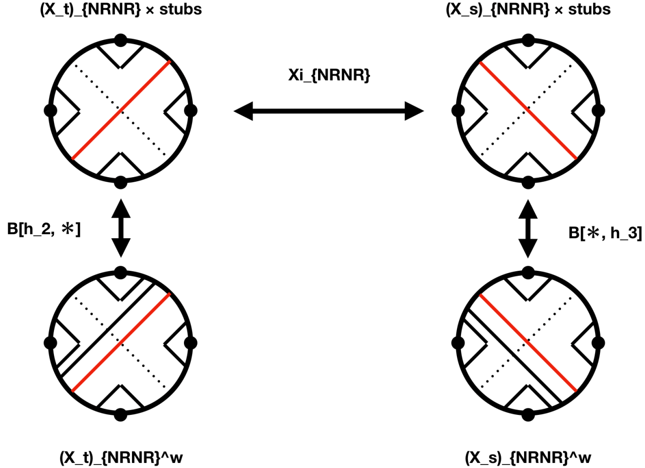
$$
\begin{aligned}
\Xi^w_{RNRN} = &\Big((X_t)_{RNRN}\, \mathcal{B}[h_2, *] + \Xi_{RNRN} + (X_s)_{RNRN}\, \mathcal{B}[*, h_3] \Big) \\
&\times e^{-wL_0[g_1]}\, e^{-wL_0[g_2]}\, e^{-wL_0[g_3]}\, e^{-wL_0[g_4]}
\end{aligned}
$$

We can also show that Ξ^w_{RNRN} satisfies the cyclic equation:

$$
\omega \circ \Xi^w_{RNRN} = -\,\Xi^w_{NRNR}
$$

We can construct Ξ^w_{ABCD} for other NS/R combinations in the similar way.

Illustration of Ξ^w_{RNRN}



4. Conclusion and Future directions

Conclusion and Future directions

Conclusion

We constructed open superstring field theory based on the supermoduli space

- including the Ramond sector
- with stubs
- up to quartic order

Future directions

- Extension to all order?
- Relation to other formulations?
  (especially Homotopy algebra based constructions)

Thank you for your attention!

∞. Back up

Ξ_{RNRN} is cyclic

We have

$$\Xi_{RNRN} = \Xi_{NRNR} = \Xi_{RR}[h_2, h_3]\,.$$

Let us consider the cyclic equation. We find

$$\begin{aligned}\omega \circ \Xi_{RNRN} &= \Xi_{RR}[h_3, h_4] = -\,\Xi_{RR}[h_4, h_3] \\ &= -\,\Xi_{RR}[h_2, h_3] = -\,\Xi_{NRNR}\,.\end{aligned}$$

Therefore, three-string products constructed from Ξ_{RNRN} and Ξ_{NRNR} has the cyclic A_∞ structure [1].

[1]We used antisymmetric property of Ξ_{RR}:

$$\Xi_{RR}[a, b] = -\,\Xi_{RR}[b, a]\,,$$

and $\beta_R[h_2] = \beta_R[h_4]$ which follows form the fact that β_0 is BPZ even.

$$\Xi_{RR}[h_2, h_3] = \int dt' \int d\tilde{t'} \int d\zeta d\tilde{\zeta}\; e^{-\{Q', t'\zeta\beta_R[h_2]\}}\; e^{-\{Q', (1-t')\zeta\beta_R[h_3]\}}$$

Interploation function

We can replace $\Xi_{RR}[h_2, h_3]$ with $\check{\Xi}[h_2, h_3]$.

$$\check{\Xi}_{RR}[h_2, h_3] = \frac{1}{4}\sum_{i=1}^{4}\Big(\Xi_{RR}[h_2, g_i] + \Xi_{RR}[g_i, h_3]\Big)$$

$\check{\Xi}[h_2, h_3]$ satisfies

$$\begin{aligned} Q \cdot \check{\Xi}_{RR}[h_2, h_3] =& X_R[h_2] - X_R[h_3]\,, \\ \check{\Xi}_{RR}[h_2, h_3] =& -\check{\Xi}_{RR}[h_3, h_2]\,. \end{aligned}$$

$\mathcal{B}_3^w$ is cyclic

We can show that $\mathcal{B}_3^w$ satisfies the cyclic equation. We find

$$\begin{aligned}
\omega \circ \mathcal{B}_3^w &= \omega \circ \Big(\mathcal{B}[h_2, h_3] e^{-wL_0[g_1]} e^{-wL_0[g_2]} e^{-wL_0[g_3]}\, e^{-wL_0[g_4]}\Big) \\
&= \mathcal{B}[h_3, h_4]\, e^{-wL_0[g_2]}\, e^{-wL_0[g_3]}\, e^{-wL_0[g_4]}\, e^{-wL_0[g_1]} \\
&= \mathcal{B}[h_3, h_2]\, e^{-wL_0[g_2]}\, e^{-wL_0[g_3]}\, e^{-wL_0[g_4]}\, e^{-wL_0[g_1]} \\
&= -\mathcal{B}[h_2, h_3]\, e^{-wL_0[g_1]}\, e^{-wL_0[g_2]}\, e^{-wL_0[g_3]}\, e^{-wL_0[g_4]} \\
&= -\mathcal{B}_3^w\,,
\end{aligned}$$

where we used $\mathcal{B}[a, b] = -\mathcal{B}[b, a]$ and

$$b_0[h_1] = b_0[h_3], \quad b_0[h_2] = b_0[h_4]\,,$$

which follows from the fact that L_0 is BPZ even. [2]

2

$$\mathcal{B}[h_3, h_4] = \int_0^1 dt \int d\tilde{t}\left(e^{\{Q', (1-t)(-2w) b_0[h_3]\}} + e^{\{Q', t(-2w) b_0[h_4]\}}\right),$$

Ξ^w_{ABCD} is cyclic

If $(X_t)_{ABCD}, (X_s)_{ABCD}$, and Ξ_{ABCD} satisy

$$\omega \circ (X_{t/s})_{ABCD} = (X_{s/t})_{DABC}\,, \quad \omega \circ \Xi_{ABCD} = -\,\Xi_{DABC}\,,$$

we can show that Ξ^w_{ABCD} satisfies the cyclic equation. We find

$$\begin{aligned}
\omega \circ \Xi^w_{ABCD} &= \omega \circ \Big[\Big((X_t)_{ABCD}\, \mathcal{B}[h_2, *] + \Xi_{ABCD} + (X_s)_{ABCD}\, \mathcal{B}[*, h_3] \Big) \\
&\qquad \times e^{-wL_0[g_1]}\, e^{-wL_0[g_2]}\, e^{-wL_0[g_3]}\, e^{-wL_0[g_4]} \Big] \\
&= \Big((X_s)_{DABC}\, \mathcal{B}[h_3, *] - \Xi_{DABC} + (X_t)_{DABC}\, \mathcal{B}[*, h_4] \Big) \\
&\qquad \times e^{-wL_0[g_2]}\, e^{-wL_0[g_3]}\, e^{-wL_0[g_4]}\, e^{-wL_0[g_1]} \\
&= \Big(-(X_s)_{DABC}\, \mathcal{B}[*, h_3] - \Xi_{ABCD} - (X_t)_{DABC}\, \mathcal{B}[h_2, *] \Big) \\
&\qquad \times e^{-wL_0[g_1]}\, e^{-wL_0[g_2]}\, e^{-wL_0[g_3]}\, e^{-wL_0[g_4]} \\
&= -\,\Xi^w_{DABC}\,.
\end{aligned}$$