Conformal Blocks from AdS geodesics and its application

Kotaro Tamaoka (Osaka Univ.)

[1805.00217] to be published in JHEP w/ Mitsuhiro Nishida (GIST)
& [1803.10539] (PTEP) w/ Hayato Hirai and Tuyoshi Yokoya(Osaka)

see also Hirai's talk last week

18/07/20 New Frontiers in String Theory 2018

Bulk dual of conformal blocks?

Conformal blocks = fundamental objects in CFT
(partial waves)

Recently, turns out to be geodesic Witten diagrams

Hijano-Kraus-Perlmutter-Snively '15

Extension to the **bosonic** higher spin fields:
Nishida, KT, Chen, Kuo, Kyono, Castro, Llabrés, Rejon-Barrera, Dyer, Freedman, Sully,...

(blocks for global conformal symmetry in d-dim.)

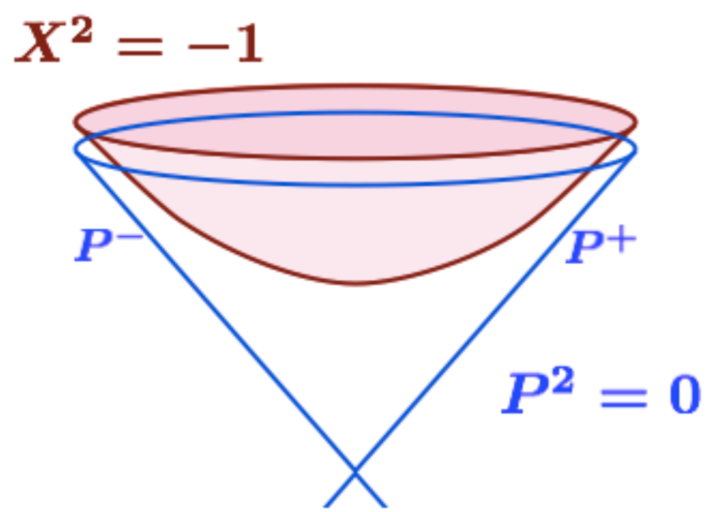
Q1. Fermionic sector ?

Q2. How useful to write it in the bulk language?

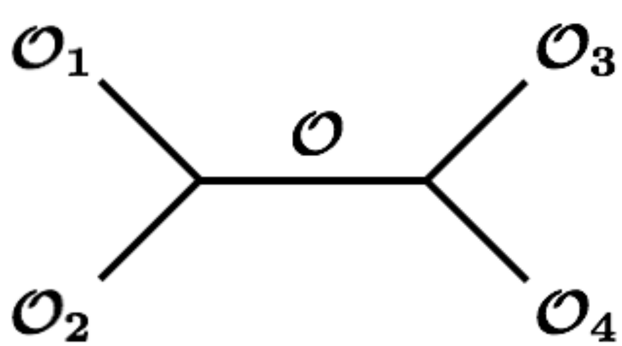
What we have done ①

CB=GWD for fermions [1805.00217]

Embedding formalism



CFT



conformal blocks (partial waves)

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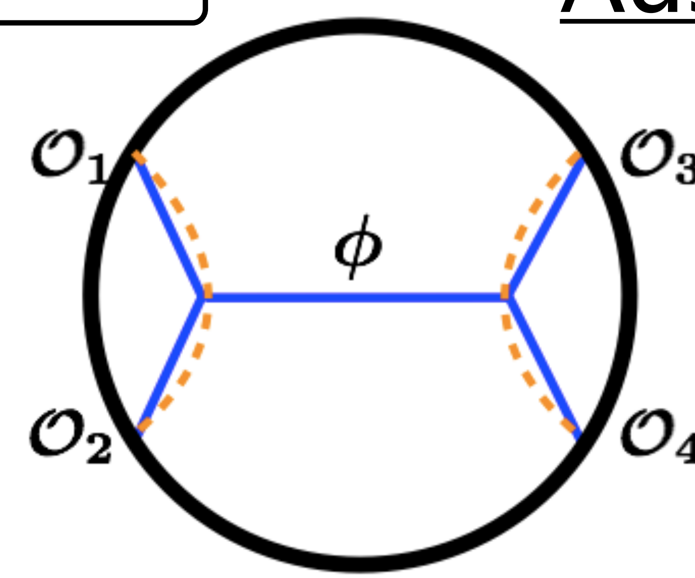
AdS



geodesic Witten diagram

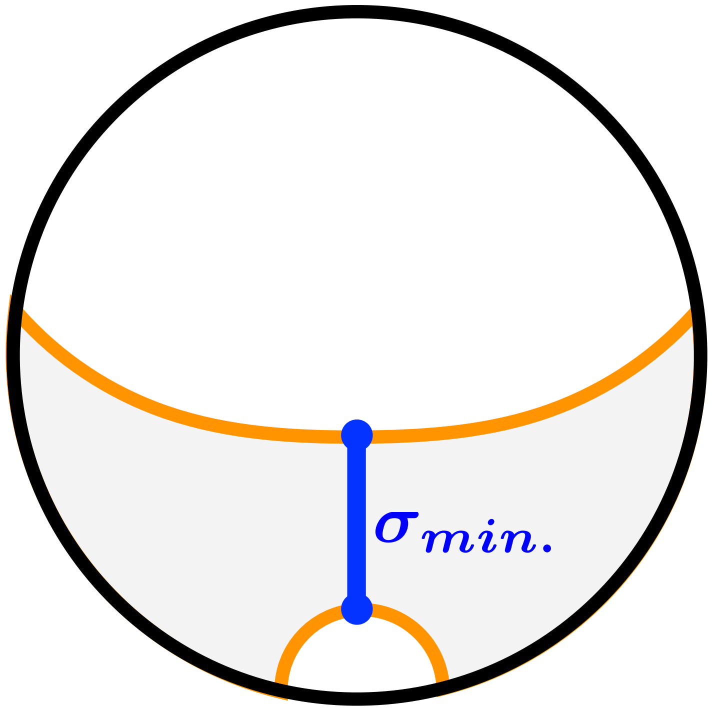
Hijano-Kraus-Perlmutter-Snively '15

What we have done ②

Entanglement wedge cross section from Holographic CFT$_2$

[1803.10539]

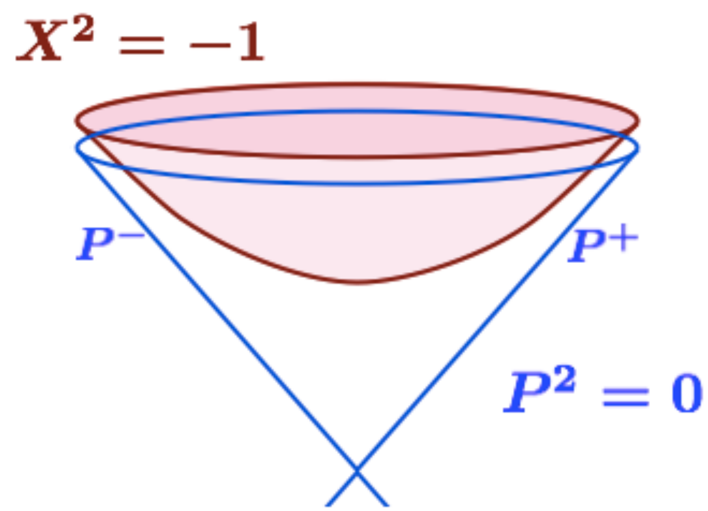
Consider Holographic CFT in two dimension
with the replica method & a channel including twist op.

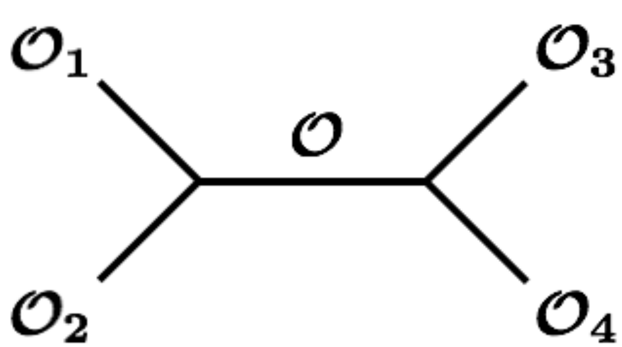


$$-\frac{\partial}{\partial n} G(u,v)\Big|_{n\to 1} = E_W$$

refer to Hirai's talk last week

2. Embedding formalism





1. conformal blocks (partial waves)

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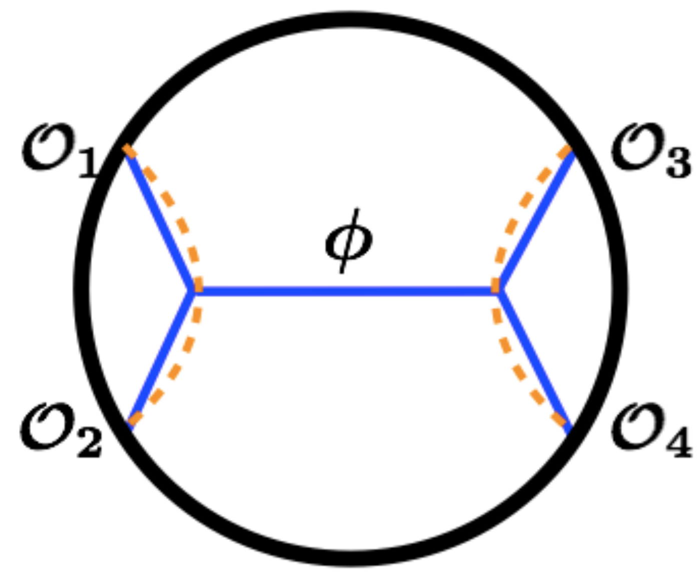


3. geodesic Witten diagram

→ 4. Applications

Conformal partial waves (blocks) = Basis of correlation functions

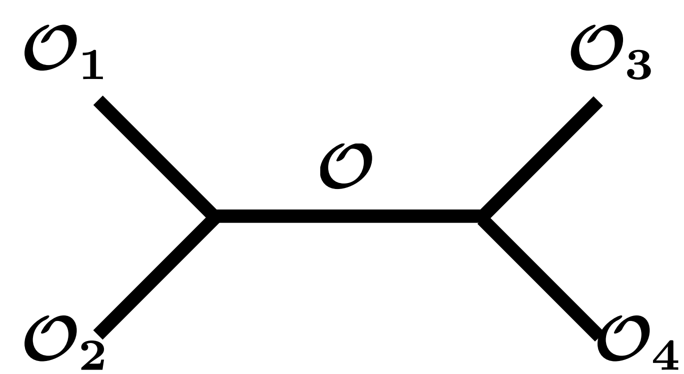
1,2,3pt function → fixed by Ward–Takahashi identity

(up to constants)

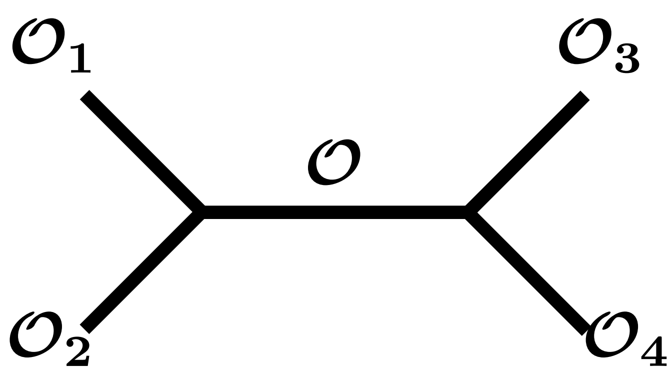
$$\langle \underbrace{\mathcal{O}_1\mathcal{O}_2}\,\underbrace{\mathcal{O}_3\mathcal{O}_4} \rangle$$

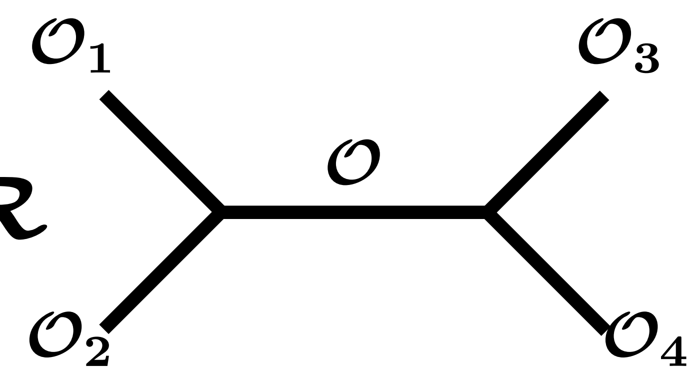
$$= \sum_{\mathcal{O}} C_{12\mathcal{O}} C_{34\mathcal{O}} \underline{W_{\mathcal{O}}^{12,34}(x_i)}$$

fancy notation:



Conformal partial waves (blocks)
= Eigen functions of conformal Casimir equation

L^2 

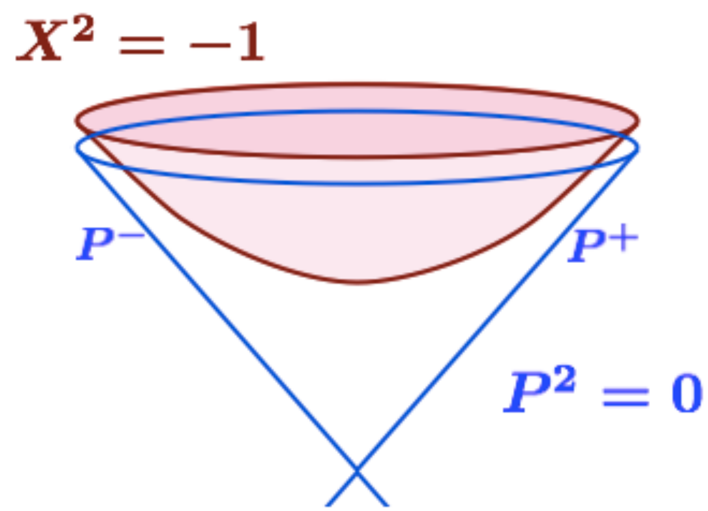
$= \; C_{\Delta,\mathcal{R}}$ 

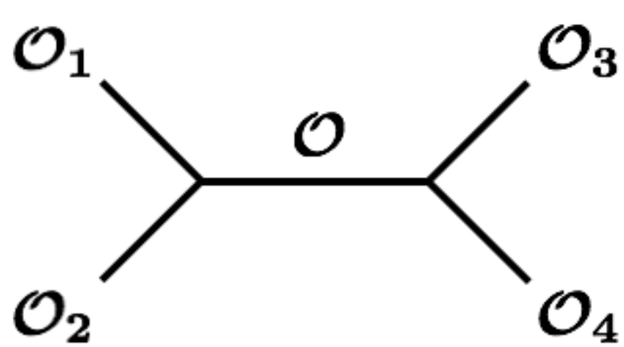
spin-l: $C_{\Delta,\ell} = \Delta(\Delta - d) + \ell(\ell + d - 2)$

Weyl fermion: $C_{\Delta,\frac{1}{2}} = \Delta(\Delta - d) + \frac{d}{8}(d-1)$

(c.f. L^2 in QM)

2. Embedding formalism





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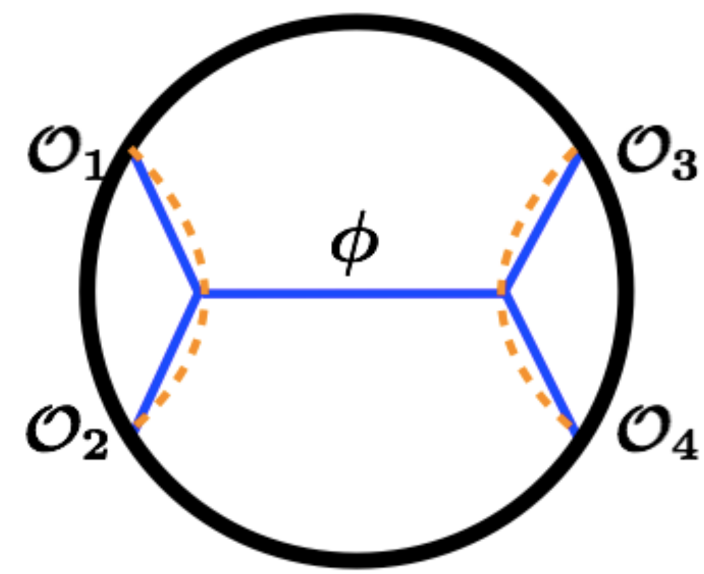


1. conformal blocks (partial waves)

3. geodesic Witten diagram

→ 4. Applications

2. Embedding AdS/CFT in to a flat space

AdS_{d+1} isometry / Conformal symmetry in d-dim

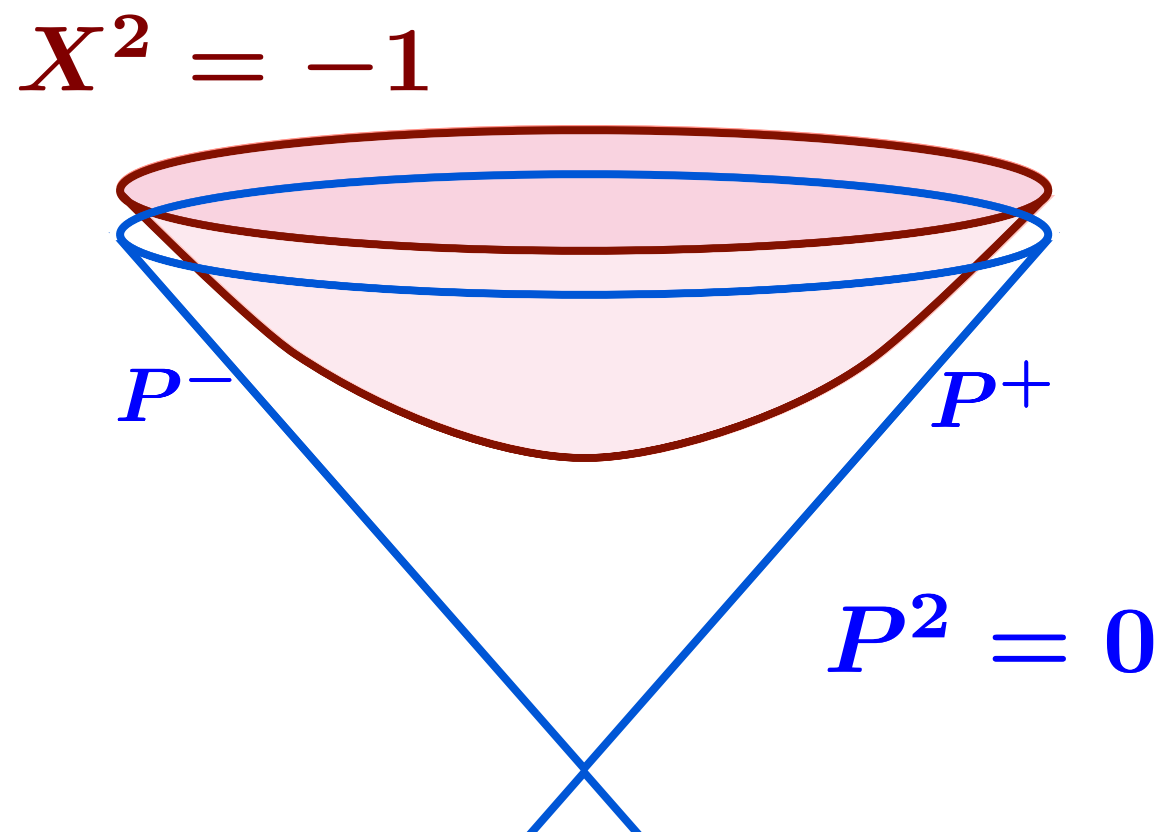
$$SO(d+1,1)$$

$$||$$

Lorentz symmetry in d+2-dim

2. Embedding AdS/CFT in to a flat space

$\mathbb{R}^{d+1,1}$

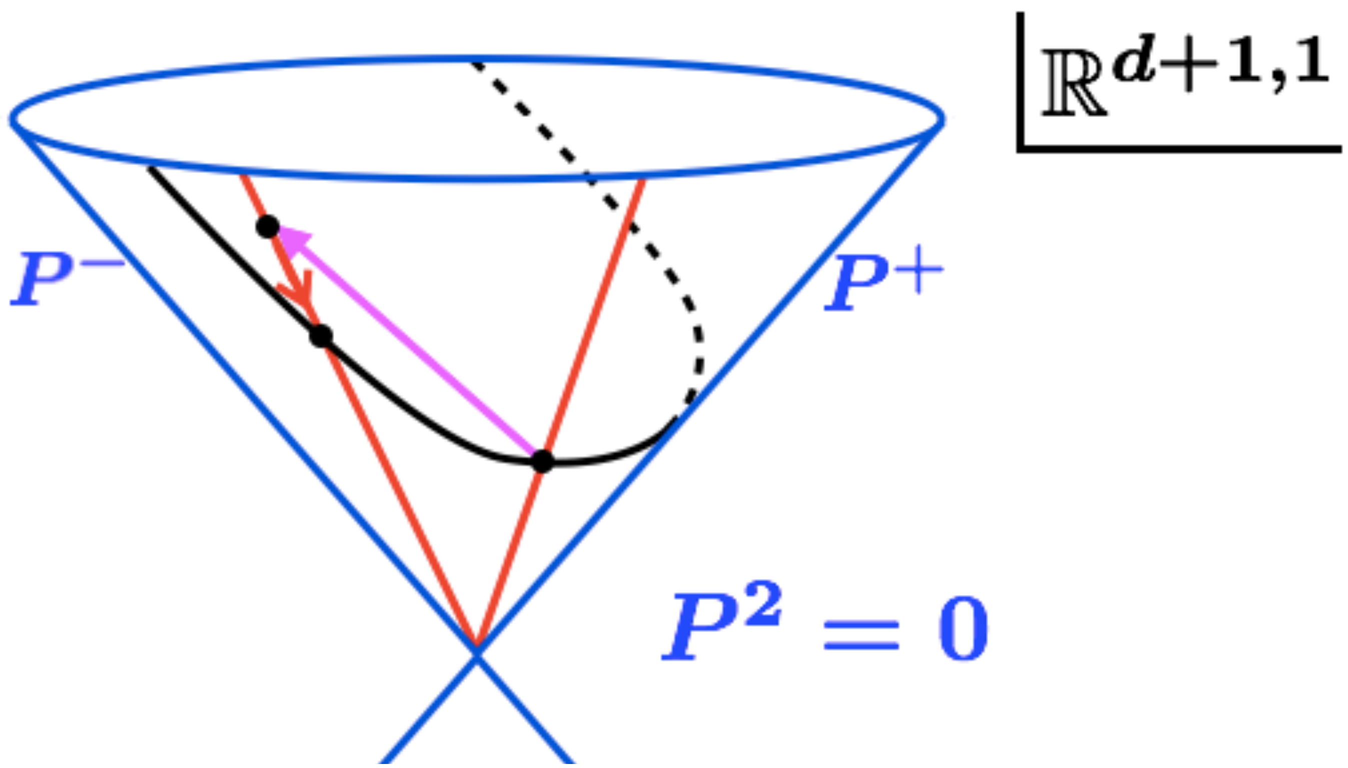


SO(d+1,1) transformation → linear

Dirac '36 Weinberg '10

see also Costa-Penedones-Poland-Rychkov '11

Embedding CFT (in detail)



$$P^2 = 0$$
$$\lambda P^A \sim P^A$$

Primary operator:

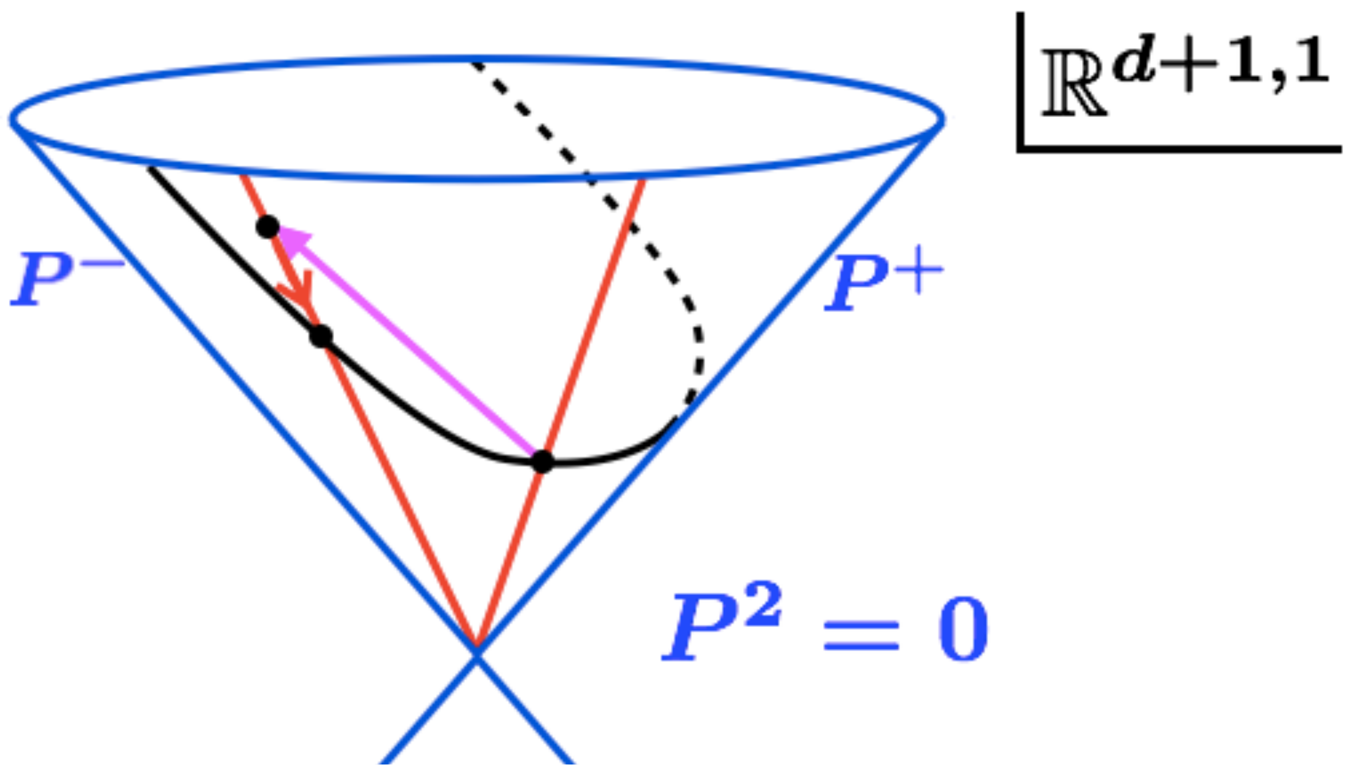
$$\Phi(\lambda P) = \lambda^{-\Delta}\Phi(P)$$

- Poincare Section ("gauge fixing")

$$P^A = (P^+, P^-, P^a) = (1, x^2, x^a)$$

$$ds^2 = -dP^+ dP^- + \delta_{ab} dP^a dP^b \to \delta_{ab} dx^a dx^b$$

Example: 2pt function of scalar



$$P^2 = 0$$
$$\lambda P^A \sim P^A$$

Primary operator:

$$\Phi(\lambda P) = \lambda^{-\Delta}\Phi(P)$$

$$\langle \Phi(\lambda_1 P_1)\Phi(\lambda_2 P_2)\rangle = \lambda_1^{-\Delta}\lambda_2^{-\Delta}\langle \Phi(P_1)\Phi(P_2)\rangle$$

$$\Rightarrow \quad \langle \Phi(P_1)\Phi(P_2)\rangle = \frac{1}{(-2P_1\cdot P_2)^{\Delta}}$$
$$= \frac{1}{|x_{12}|^{2\Delta}}$$

Dirac fermions (CFT$_{2n}$ and AdS$_{2n+1}$)

CFT	AdS
$P^A \Gamma_A \Psi(P) = 0$ Weinberg '10 Simmons-Duffin '12	$(X^A \Gamma_A - \Gamma_{\rm chiral}) \Psi(X) = 0$ Nishida-KT '18

★ Reproduce right propagators in physical space

$$G_{b\partial}^{\Delta,\frac{1}{2}}(X_1;P_2) = \frac{\langle \bar{S}_b \Pi_+ S_\partial \rangle}{(-2X_1\cdot P_2)^{\Delta+\frac{1}{2}}} \to \frac{(z\gamma^0 + x_{12}^a \gamma_a)\mathcal{P}_+}{\sqrt{z}} \left(\frac{z}{z^2+|x_{12}|^2} \right)^{\Delta+\frac{1}{2}}$$

Here we used "index free notation"

(Π&$\mathcal{P}$: chiral projection)

Quadratic Casimir & Free E.O.M.

$$-\frac{1}{2}L^{AB}L_{AB}\Phi = \nabla^2\Phi = m^2\Phi = C_{\Delta,0}\Phi$$

$$-\frac{1}{2}L^{AB}L_{AB}\Psi = \left[(\Gamma^A\nabla_A)^2 + \frac{1}{4}R\right]\Psi = C_{\Delta,\frac{1}{2}}\Psi$$

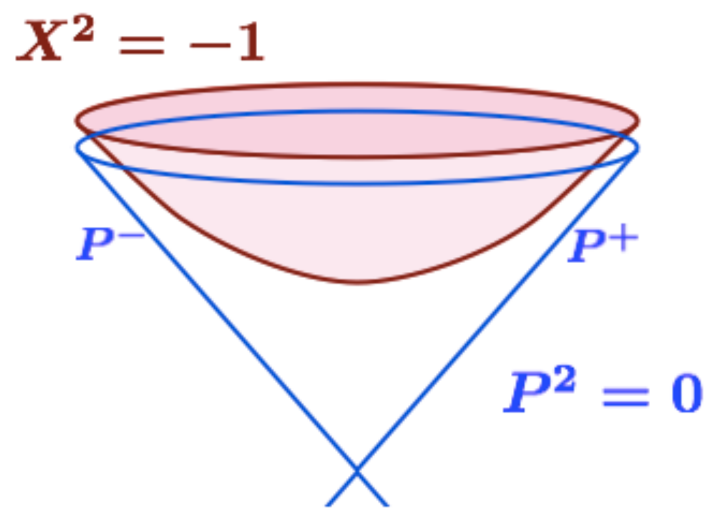
(↓spin)

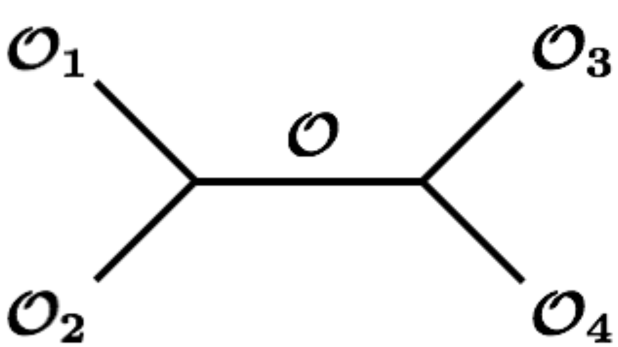
generator of SO(d+1,1): $L_{AB} = X_A\partial_B - X_B\partial_A + \Sigma_{AB}$

covariant derivative: $\nabla_A = G_{AB}\partial^B + X^B\Sigma_{AB}$

induced metric: $G_{AB} = \eta_{AB} + X_A X_B$

2. Embedding formalism





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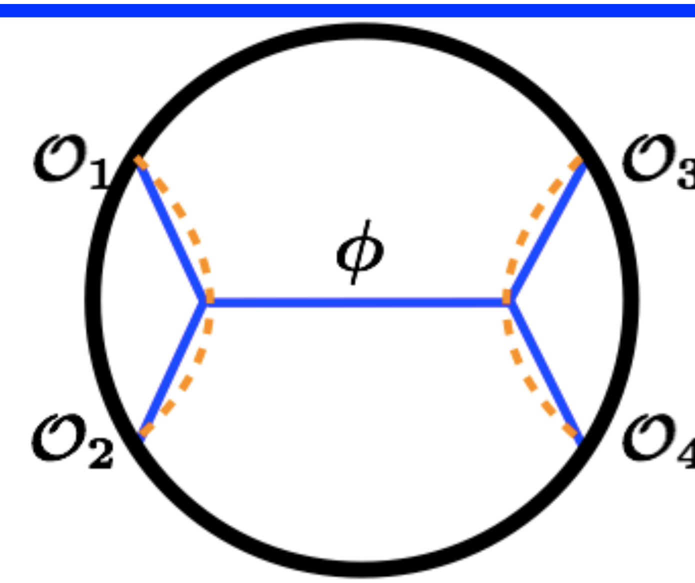
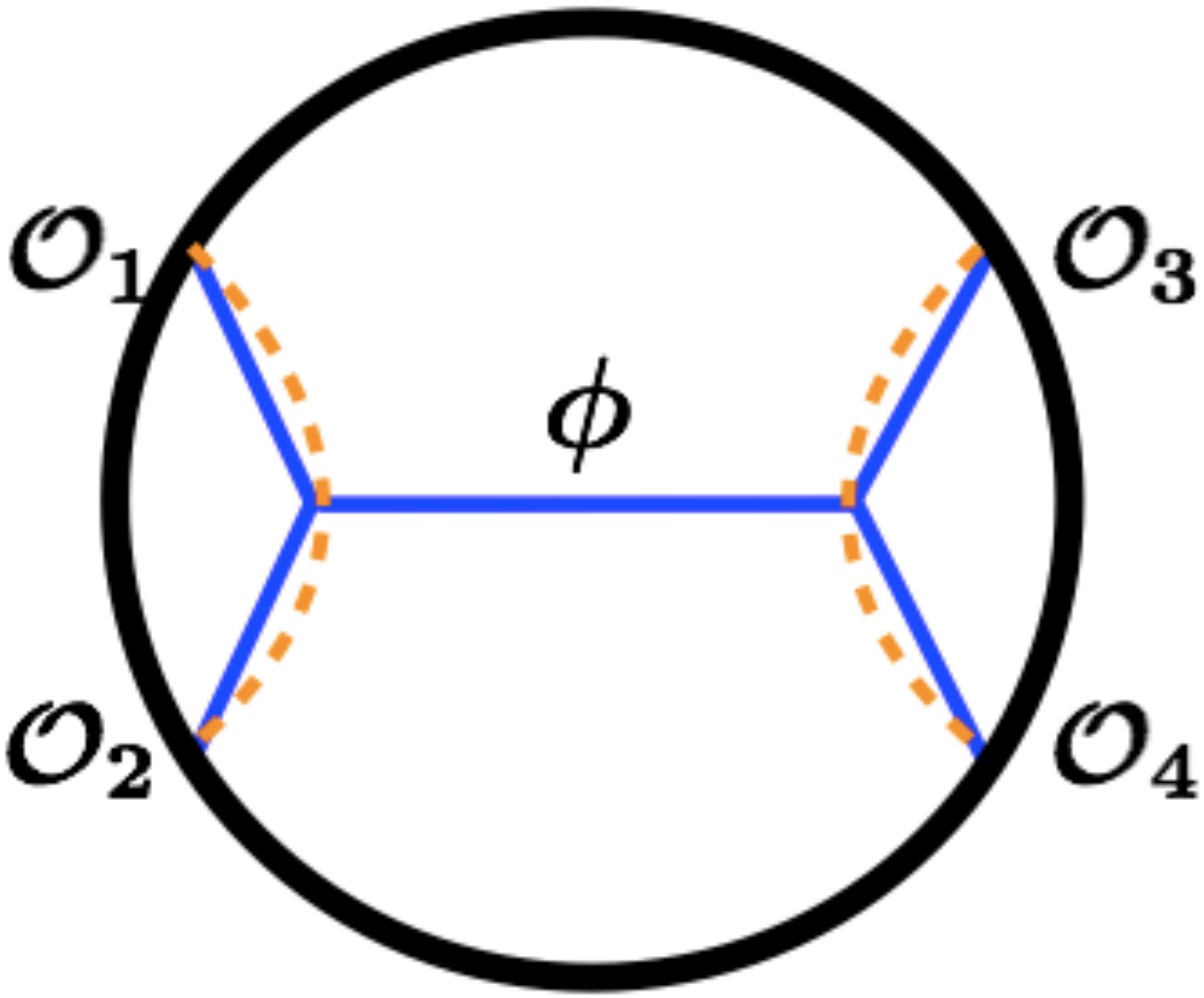


1. conformal blocks

3. geodesic Witten diagram

→ 4. Applications

Geodesic Witten diagrams



Originally proposed by

Hijano-Kraus-Perlmutter-Snively '15

External higher spinning fields:

Nishida-KT '16
Chen-Kuo-Kyono '17
Dyer-Freedman-Sully '17 …

Internal fermions:

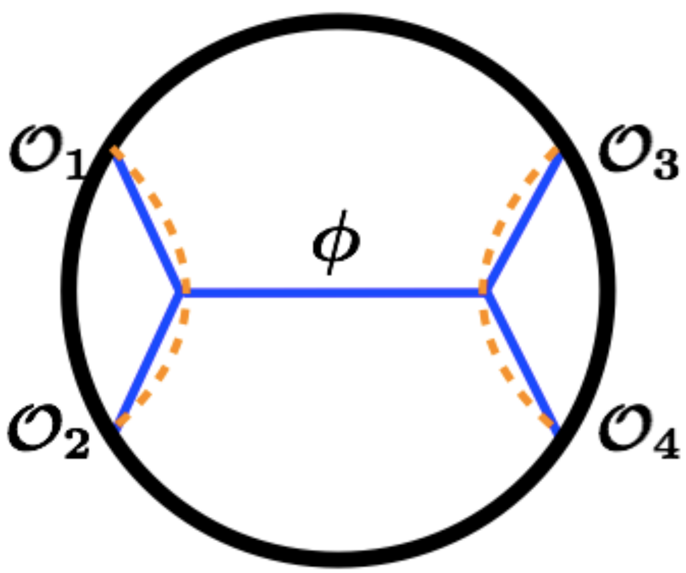
Nishida-KT '18

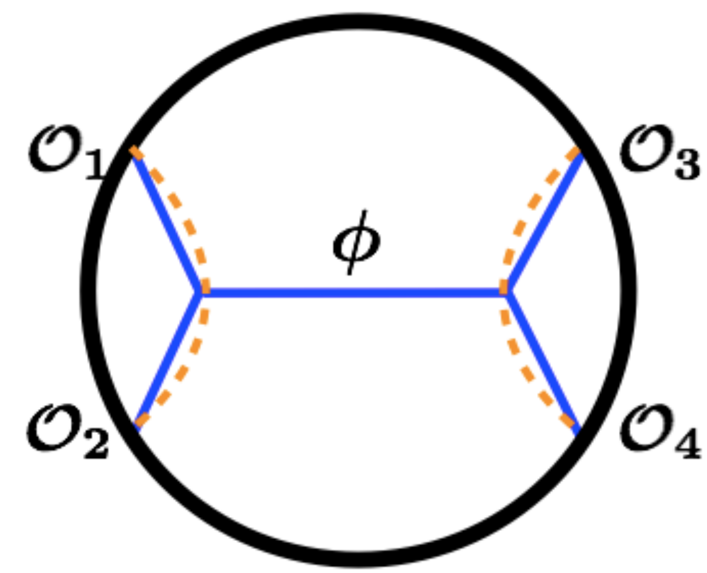
$$\int_{-\infty}^{\infty} d\lambda_1 \int_{-\infty}^{\infty} d\lambda_2 \, G_{b\partial}^{\Delta_1}(X(\lambda_1); P_1) G_{b\partial}^{\Delta_2}(X(\lambda_1); P_2) \times G_{bb}^{\Delta}(X(\lambda_1), X(\lambda_2)) G_{b\partial}^{\Delta_3}(X(\lambda_2); P_3) G_{b\partial}^{\Delta_4}(X(\lambda_2); P_4)$$

G_{bb} :bulk-bulk propagator

$G_{b\partial}$:bulk-boundary propagator

Geodesic Witten diagrams satisfy conformal Casimir equation

L^2 

$=$ $C_{\Delta,\mathcal{R}}$ 

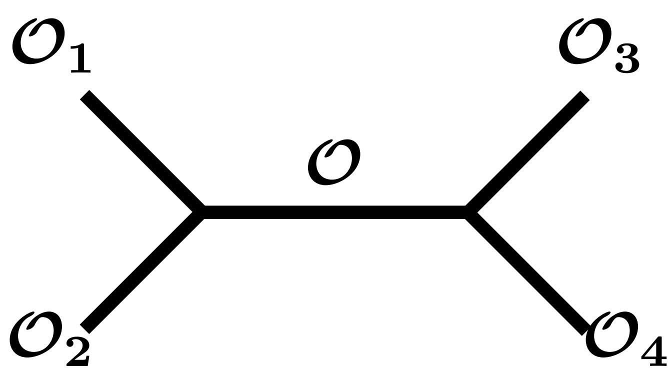
$$L^2 \equiv (L_{P_1} + L_{P_2})^2$$

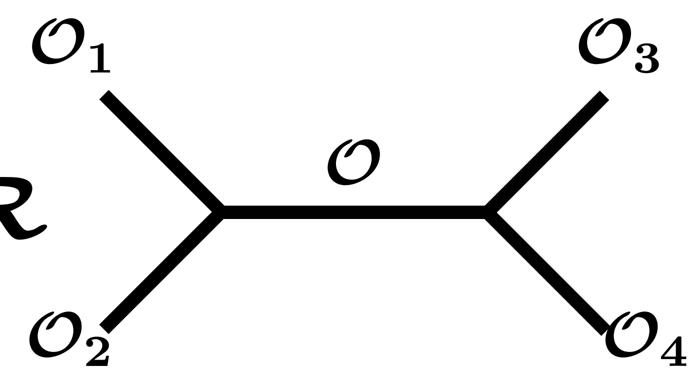
spin-l: $C_{\Delta,\ell} = \Delta(\Delta - d) + \ell(\ell + d - 2)$

Weyl fermion: $C_{\Delta,\frac{1}{2}} = \Delta(\Delta - d) + \frac{d}{8}(d-1)$

Conformal partial waves satisfy conformal Casimir equation

(recap)

L^2 

$=$ $C_{\Delta,\mathcal{R}}$ 

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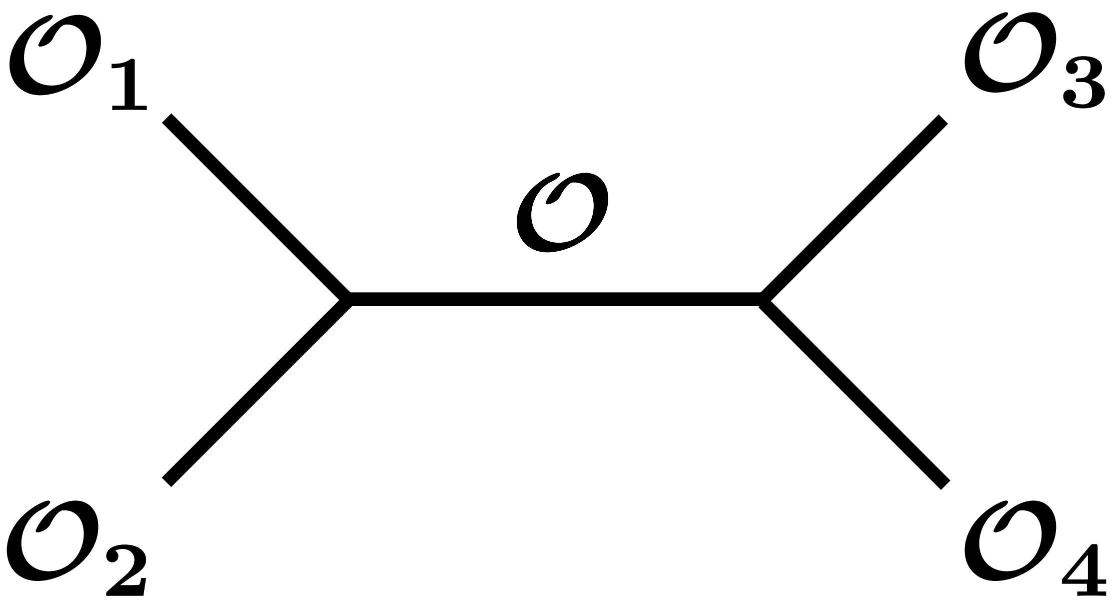
These are the same!

Conformal blocks
(partial waves)

Geodesic Witten diagrams



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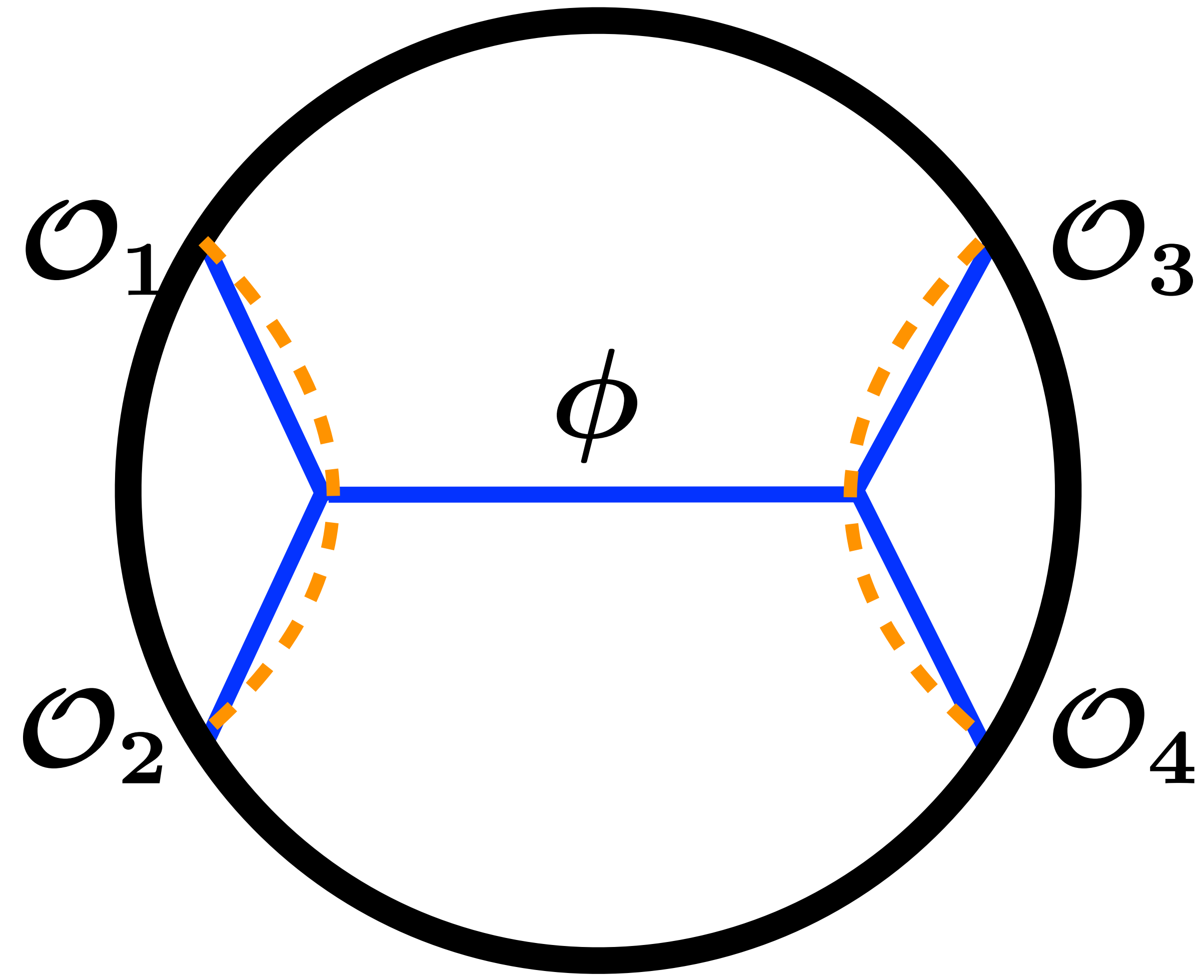
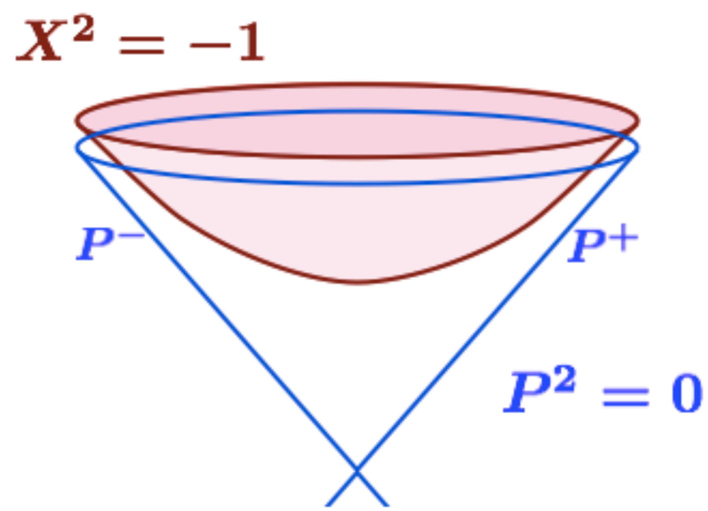
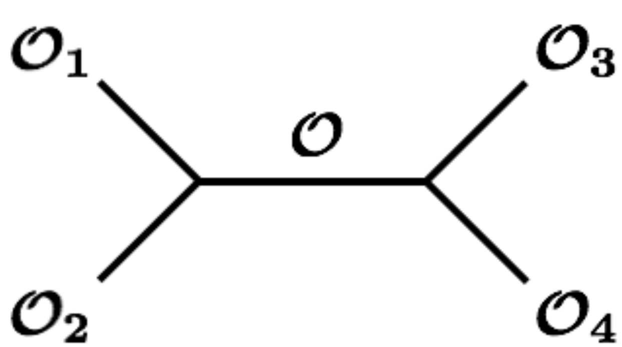


Boundary condition? : OK

2. Embedding formalism





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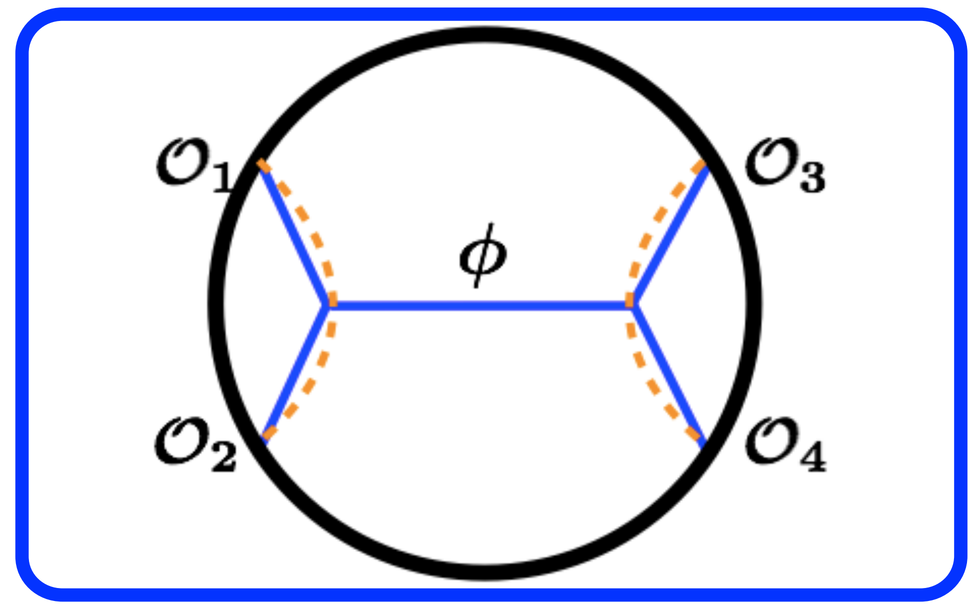


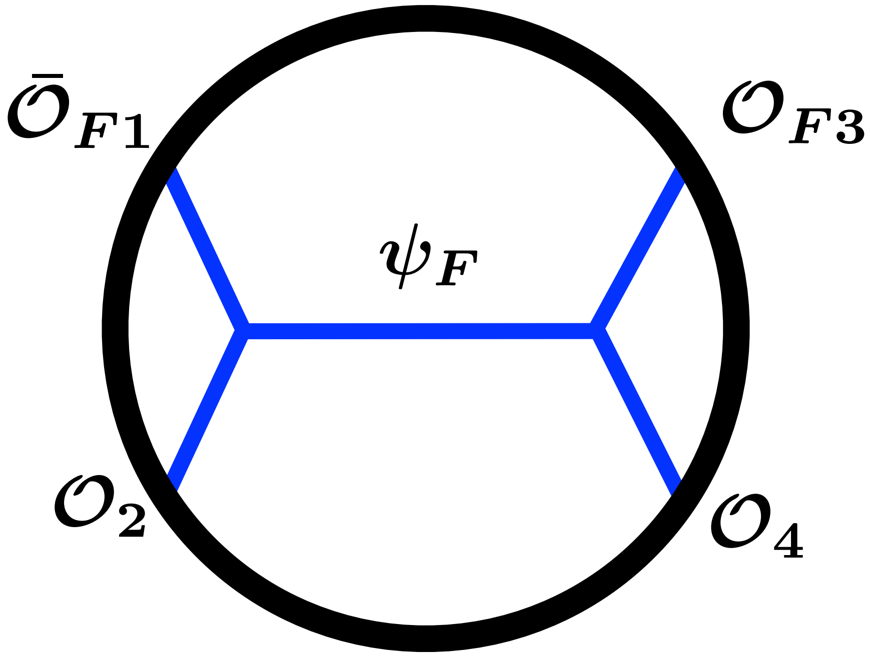
1. conformal blocks

3. geodesic Witten diagram

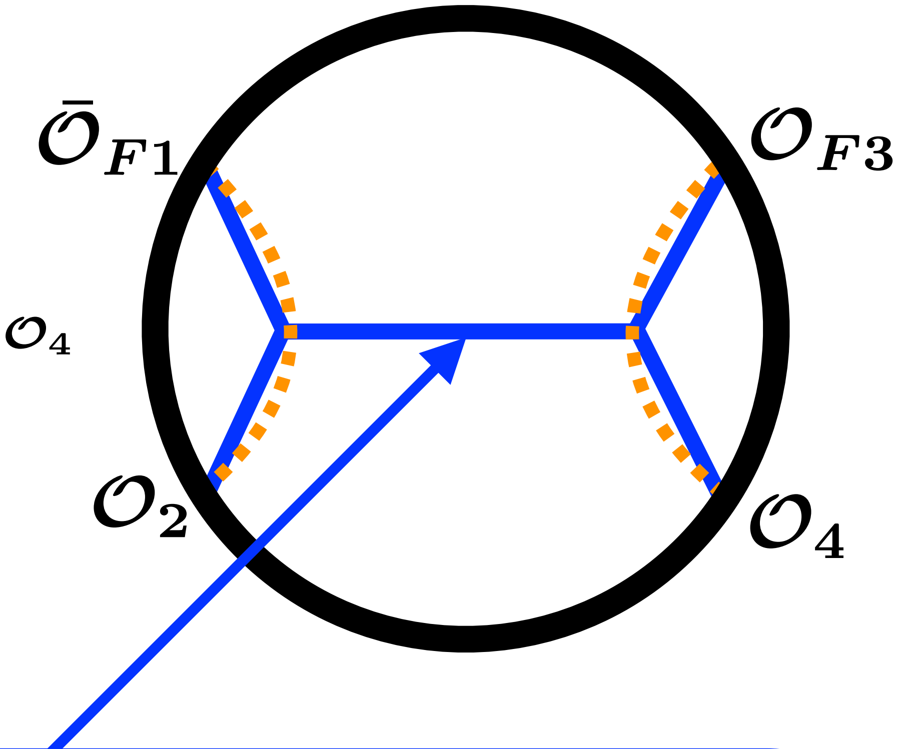
→ 4. Applications (GWD decomposition & EWCS)

GWD decomposition of Witten diagrams

c.f. Chen's talk last week



$$= \sum_{\mathcal{O}} C_{\mathcal{O}_1\mathcal{O}_2}{}^{\mathcal{O}} C_{\mathcal{O}\mathcal{O}_3\mathcal{O}_4}$$



<u>fermion exchange</u>

split rep. ⇒

<u>physical poles:</u>

Single trace: $\mathcal{O}_F$ (ψ_F)

Double trace:

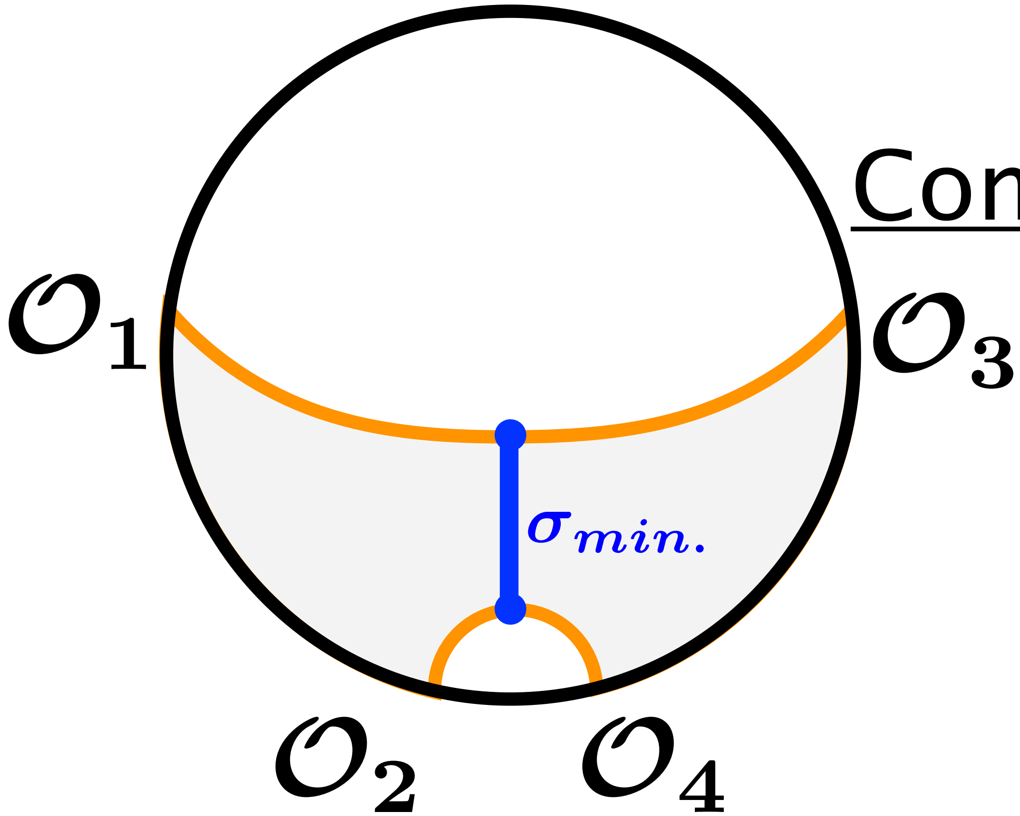
$[\bar{\mathcal{O}}_{F1}\mathcal{O}_2]_{2n}$ $[\mathcal{O}_{F3}\mathcal{O}_4]_{2n}$

<u>Nishida-KT '18</u>

[Umemoto-Takayanagi '17], [Nguyen-Devakul-Halbasch-Zaletel-Swingle '17]

Entanglement wedge cross section from Holographic CFT_2

refer to Hirai's talk last week



Consider a channel including twist op.

$$-\frac{\partial}{\partial n} G(u,v)\Big|_{n\to 1} = E_W$$

∵) For Holographic CFT, reduce to $-\frac{\partial}{\partial n} G_{\sigma_n}(u,v)\Big|_{n\to 1}$

$$G_{\sigma_n} \sim e^{-\frac{c}{12}\left(n-\frac{1}{n}\right)\sigma_{min}}$$

→ Can also apply to the static BTZ blackholes

Discussion

Geodesic Witten diagrams

- 3pt Interaction in the embedding space

- SUSY, Defects, Many points, …

Entanglement wedge cross section

- Calculation of the (twist #)=1 channel

  for more generic CFT_2