Applications of relative entropy to entanglement, chaos and holography

New Frontiers in String Theory 2018, Kyoto

Tomonori Ugajin (OIST)

This talk is based on

・"Modular Hamiltonian of excited states, OPE blocks and emergent bulk fields" 1705.07899 with Gabor Sarosi

・ "Chaos and relative entropy" 1805. 01051 with Gabor Sarosi, Yuya Nakagawa

・A CFT computation of an entanglement measure for mixed states"

To appear soon, with Tadashi Takayanagi, and Koji Umemoto

Also 1705.01486 1611.02959 1603.03057

Relative entropy

- Relative entropy between two density matrices is defined by

$$S(\rho||\sigma) = \mathrm{tr}\,(\rho\log\rho) - \mathrm{tr}\,(\rho\log\sigma)$$

- This quantity measures the distance between the two.

- A generalization of free energy

- Positive definite.

- =0 iff $\rho = \sigma$.

Plan of this talk

RE has nice properties such as positivity , monotonicity.

These general properties have been efficiently used to prove remarkable theorems in QFT. Ex: ANEC, A theorem.

There are (infinitely)many other applications, since it provides a natural metric structure on spaces of density matrices.

In this talk I would like to talk about several problems, in which actual computations of relative entropy is needed, rather than general

properties, mainly in conformal field theory.

Perturbative calculations of relative entropy and bulk dynamics

Mainly based on the Work with G Sarosi

arXiv :1705.01486

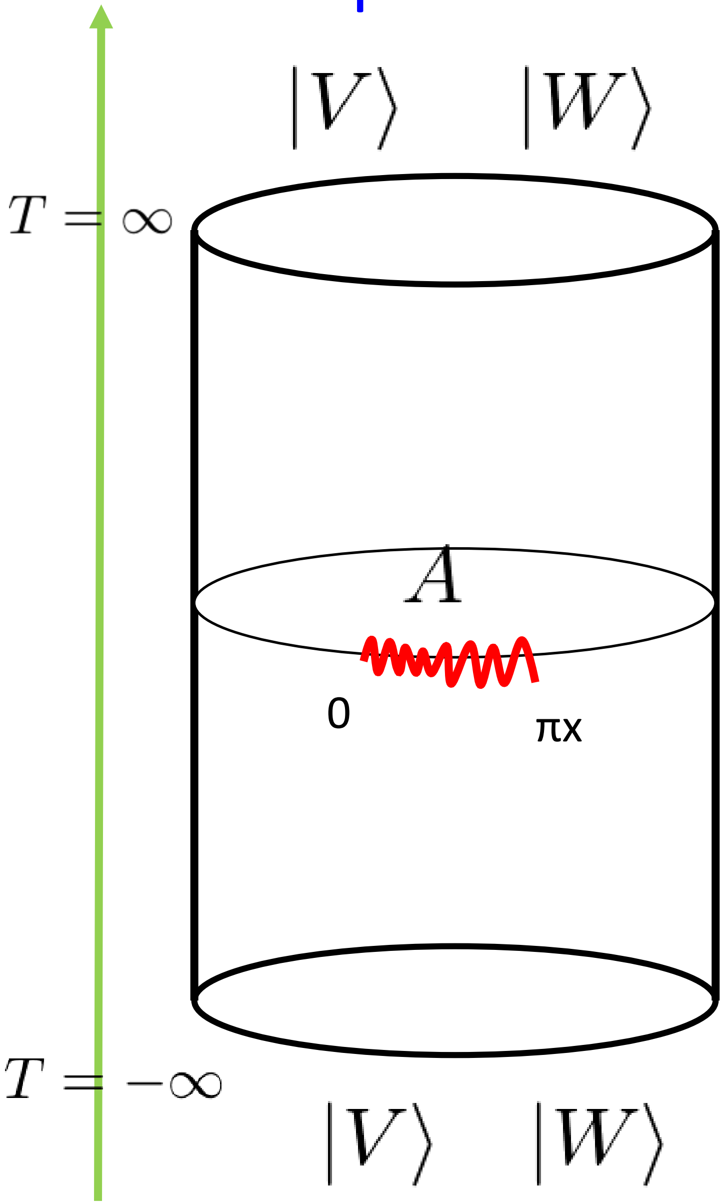
"Modular Hamiltonians of excited states, OPE blocks and emergent bulk fields"

See also

Faukner, Haehl, Hijano, Parrikar, Rabideau, Van Raamsdonk

"Nonlinear Gravity from Entanglement in Conformal Field Theories"

Set up



- A CFT on a Cylinder $\mathbb{R}\times S^{d-1}$
- A subsystem $[0,\theta_0]\times S^{d-2}$
- Excited states $|V\rangle$ $|W\rangle$ at $T=\pm\infty$
- Reduced density matrices $\rho_V=\mathrm{tr}_{A_c}|V\rangle\langle V|$

$$\rho_W=\mathrm{tr}_{A_c}|W\rangle\langle W|$$

Relative entropy in the Ising CFT

- In some cases, one can calculate relative entropy exactly. [Nakagawa, TU]

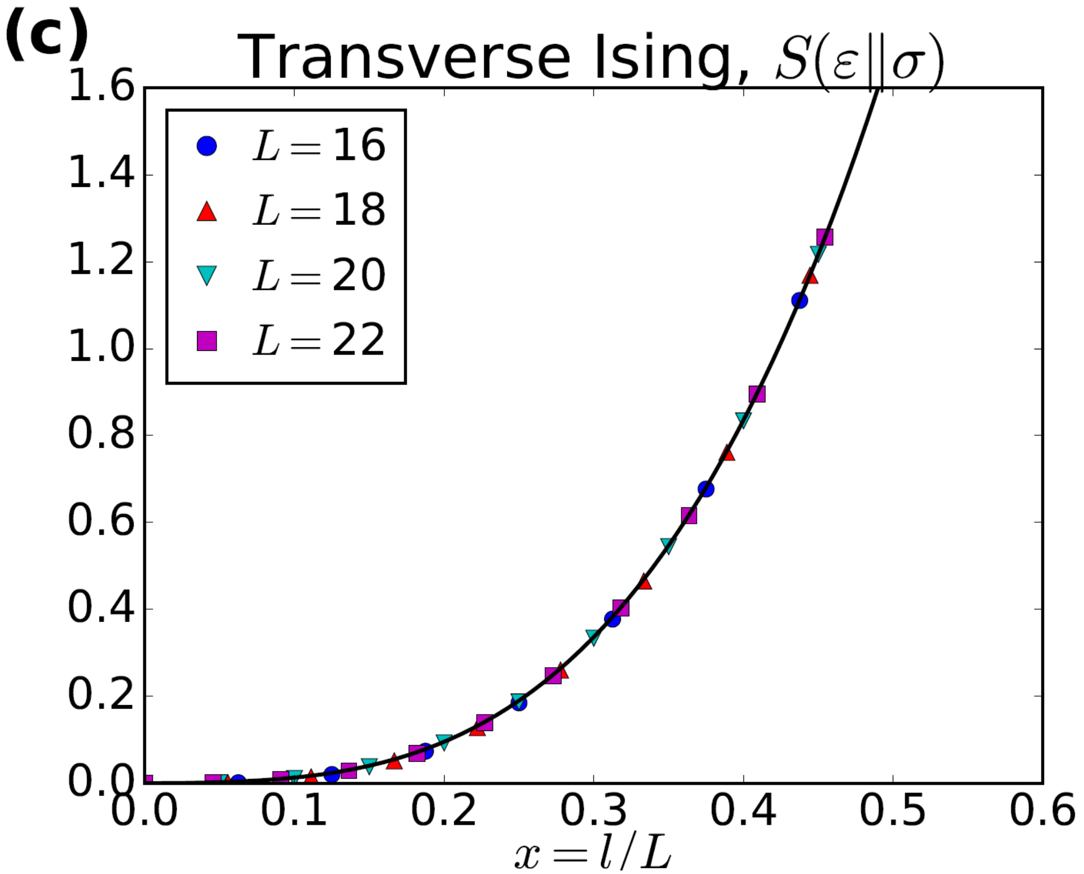
- Consider the Ising CFT, and take two primary states, $|\epsilon\rangle$ $|\sigma\rangle$.

  (ϵ : the energy operator, σ : the spin operator)

$$S(\rho_\epsilon||\rho_\sigma) = 2\log\left(\sin\theta_0\right) + \frac{5}{4}\left(1-\theta_0\cot\theta_0\right) + 2\psi_0\left(\frac{1}{2\sin\theta_0}\right) + 2\sin\theta_0$$

Relative entropy in the Ising CFT

The analytic result nicely matches with the numerical result in the corresponding spin Chain model. [Nakagawa, TU]



Two ways to calculate relative entropy

There are mainly two ways to compute relative entropy

1. Replica trick: $$S(\rho_V||\rho_W) = \lim_{n\to 1}\frac{1}{n-1}\left(\mathrm{tr}\rho_V^n - \mathrm{tr}\rho_V\rho_W^{n-1}\right)$$ [Lashkari]

For global states, each term can be written as a 2n point function.

Example: In the Ising CFT, we can use it to get the exact answers.

2. Perturbative approach:
$$S(\rho||\sigma) = S(\rho_0+\delta\rho||\rho_0+\delta\sigma)$$
$$= \sum S_{n,m}(\delta\rho^n\delta\sigma^n)$$

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Perturbative calculation of relative entropy

- In the calculations of relative entropy, the key is the calculations of modular Hamiltonian, $K_\rho = -\log\rho$, since

$$S(\rho||\sigma) = \mathrm{tr}\,[\rho\log\rho] - \mathrm{tr}\,[\rho\log\sigma]$$
$$= -\langle\rho K_\rho\rangle + \langle\rho K_\sigma\rangle$$

K_ρ is highly non local for generic excited states.

However for vacuum reduced density matrix $\rho_0 = \mathrm{tr}_{A^c}|0\rangle\langle 0|$, K_{ρ_0} has a nice local expression in CFT so it can be a convenient starting point of the point of the perturbation.

Sketch of the argument

- Step 1: Write $\rho = \rho_0 + \delta\rho$

- Step2: Expand the modular Hamiltonian.

$$K_\rho = K_0 + \sum_{n=1}^{\infty} (-1)^n \delta K^{(n)}$$

- Step 3: $S(\rho) = \langle \rho K_\rho \rangle \quad = \sum_{n=1}^{\infty} \delta S^{(n)}(\delta\rho)$

- Step4: Combine then to get $S(\rho||\sigma) = S(\rho_0 + \delta\rho||\rho_0 + \delta\sigma)$

Step1: The expression of $\delta\rho$

In our case of interest, it turned out that $\delta\rho$ can decomposed into contributions of primary operators in the CFT.

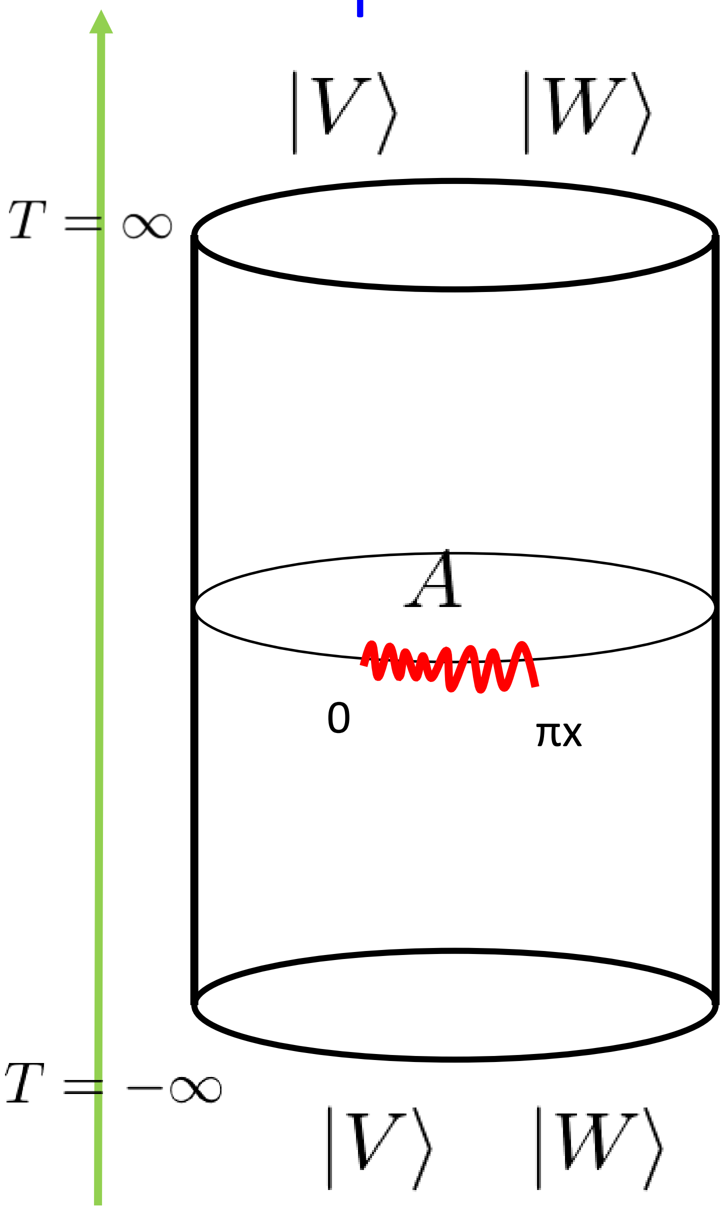
$$\begin{aligned}\rho_V &= \mathrm{tr}_{A^c}|V\rangle\langle V| \\ &= \rho_0 + e^{-\pi K}\sum_{\mathcal{O}_k} C_{VV}^{\mathcal{O}_k} B_{\mathcal{O}_k}(\theta_0,-\theta_0)e^{\pi K}\end{aligned}$$

1pt function: $C_{VV}^{\mathcal{O}_k} = \langle V|\mathcal{O}_k|V\rangle$

OPE block

The OPE block $B_{\mathcal{O}_k}(\theta_0,-\theta_0)$ of a primary $\mathcal{O}_k$ is summing up all descendants of it.

Set up



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- Reduced density matrices

$$\rho_V=\mathrm{tr}_{A_c}|V\rangle\langle V|$$

$$\rho_W=\mathrm{tr}_{A_c}|W\rangle\langle W|$$

Step2: Expanding the log

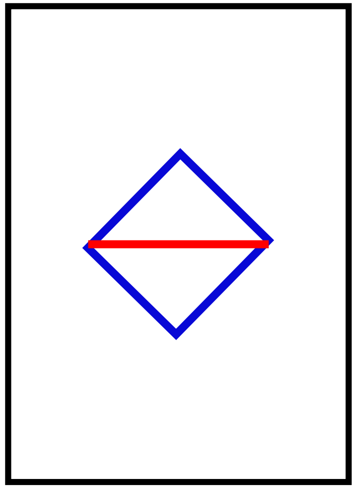
- We can the modular Hamiltonian by using the formula,

$$-\log\rho=\int_0^\infty d\beta\left(\frac{1}{\beta+\rho}-\frac{1}{\beta+1}\right)$$

The result is

$$K_\rho=K_0+\sum_{n=1}^{\infty}(-1)^n\int_{-\infty}^{\infty}ds_1...ds_n\mathcal{K}_n(s_1,...,s_n)\prod_{k=1}^{n}(e^{-\left(\frac{is_k}{2\pi}+\frac{1}{2}\right)K_0}\delta\rho e^{\left(\frac{is_k}{2\pi}-\frac{1}{2}\right)K_0})$$

The parameter s is the Rinder time in the causal diamond.



Expanding the log

- The explicit expression of the kernel is

$$\mathcal{K}_n(s_1, \ldots, s_n) = \frac{(2\pi)^2}{(4\pi)^{n+1}} \frac{i^{n-1}}{\cosh\frac{s_1}{2}\cosh\frac{s_n}{2}\prod_{k=2}^{n}\sinh\frac{s_k - s_{k-1}}{2}}$$

- Some special cases:

$$\mathcal{K}_1(s_1) = \frac{1}{(2\cosh\frac{s_1}{2})^2},$$

$$\mathcal{K}_2(s_1, s_2) = \frac{1}{16\pi} \frac{i}{\cosh\frac{s_1}{2}\cosh\frac{s_2}{2}\sinh\frac{s_2 - s_1}{2}}$$

CFT results

- Using the general formula, we derived the (formal) perturbative expression of the excited state mH. The leading term is

$$K_V = K + \sum_{k \neq vac} C^{\mathcal{O}_k}_{VV} \int_{-\infty}^{\infty} \frac{ds}{\cosh^2 \frac{s}{2}} B_{\mathcal{O}_k}(\tau - \pi + is, \hat{\tau} - \pi + is) + \cdots$$

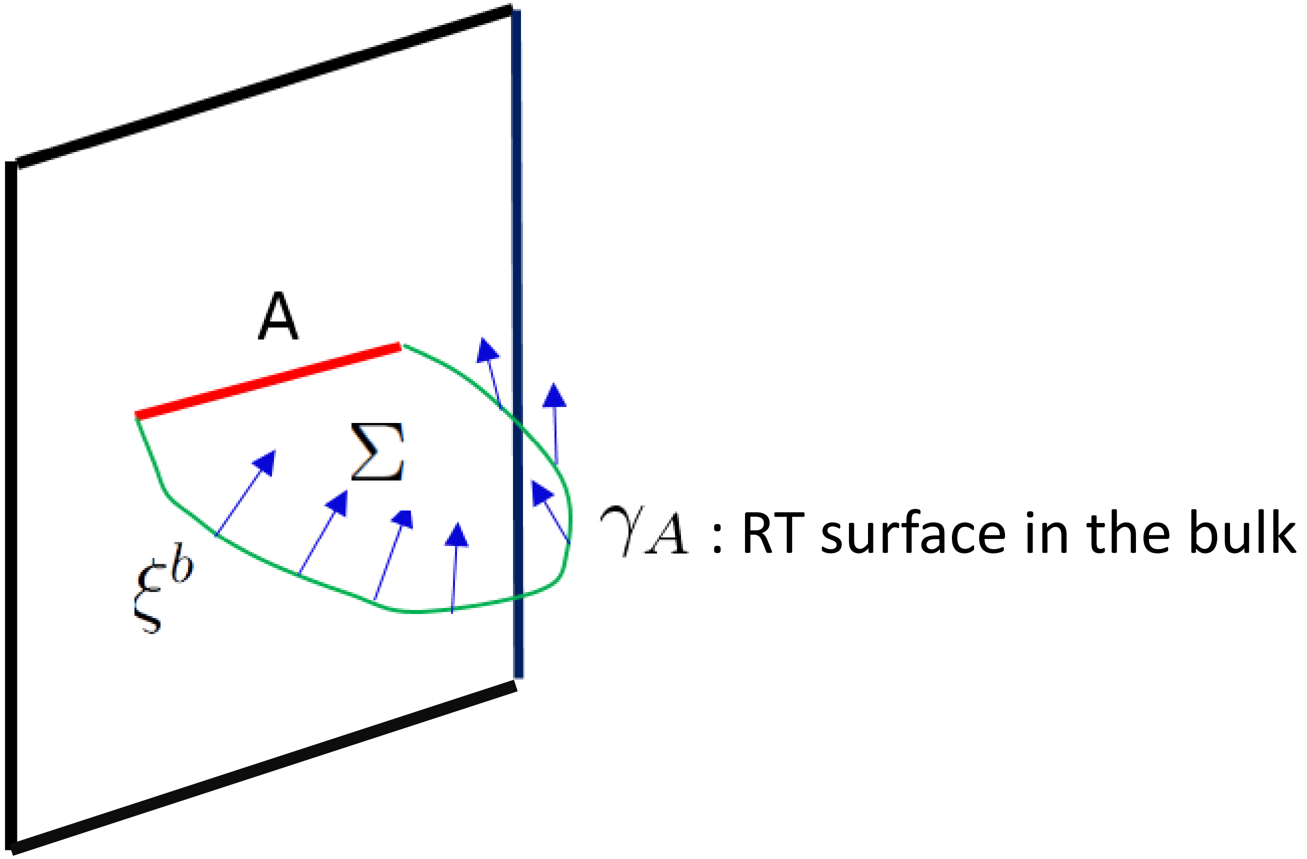
and for entanglement entropy, $\delta S_V = \sum_{m=2}^{\infty} \delta S_V^{(m)}$

$$\delta S_V^{(2)} = -\sum_k (C^k_{VV})^2 \int \frac{ds}{8\cosh^2 \frac{s}{2}} \mathcal{F}_k(\tau, \hat{\tau}, \tau - \tau_s, \hat{\tau} - \tau_s)$$

Where F is the 4pt conformal block of the primary $\mathcal{O}_k$

Holographic rewriting of the CFT results

We also found the leading part of the entanglement entropy as well as mH can be re written in terms of holographic variables.



Fisher information =Canonical energy

$$S^{(2)}_{\mathcal{O}}(\rho_V) = \langle V|\mathcal{O}|V\rangle^2 \int \frac{ds}{\cosh^2 \frac{s}{2}} \mathcal{F}(\tau, \hat{\tau}, \tau - \tau_s, \hat{\tau} - \tau_s)$$

$$= -2\pi \int_\Sigma d\Sigma^a \xi^b T_{ab}(\langle V|\phi|V\rangle)$$

Holographic Studies:

[Nozaki Numasawa Prudenziatti Takayanagi]

[Lin marcori Ooguri Stoica]

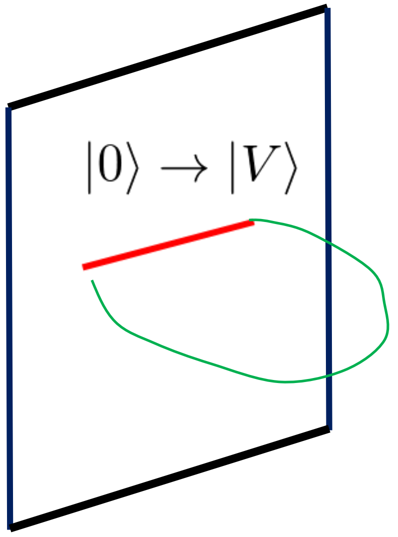
[van Raamsdonk, Lashkari]

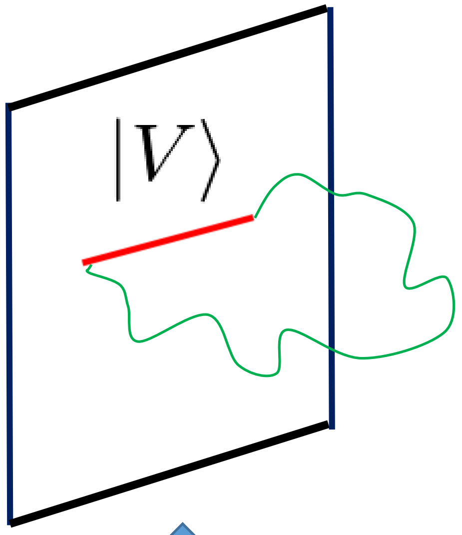
ϕ is the bulk field which correspond to the CFT primary $\mathcal{O}$,

$T_{ab}(\langle V|\phi|V\rangle)$ is the bulk stress tensor of the field.

A proof of Fisher information = Canonical energy

Fisher information= Canonical energy





$$\frac{\delta A}{4G} = \Delta\langle K_0\rangle + S^{(2)}(\rho_V)$$

Back reaction

HKLL

$$\{\langle V|O|V\rangle, \cdots\} \quad \rightarrow \quad \{\langle V|\phi|V\rangle, \cdots\}$$

The excitation turns on non trivial CFT 1pt functions

Bulk profile

This formula captures the first non trivial back reaction effect in the bulk spacetime.

Rewriting modular Hamiltonian

We can also rewrite the CFT expression of the modular Hamiltonian,

$$K_V = 2\pi K - \quad C^{\mathcal{O}}_{VV} \int_{-\infty}^{\infty} ds \frac{B_{\mathcal{O}}(\tau - \pi + is, \hat{\tau} - \pi + is)}{(\cosh \frac{s}{2})^2}$$

Then, we get

$$K_V = 2\pi \left(K - \int_{\Sigma} d\Sigma^a \xi^b T_{ab}(\phi) \right) + 2\pi \int_{\Sigma} d\Sigma^a \xi^b T_{ab}(\phi - \langle \phi \rangle_V) + \delta S_V^{(2)} + \cdots$$

The first term canbe identified with the area operator, and the second term is the mH of the bulk excited state dual to the CFT state $|V\rangle$. => It has the expected bulk form of modular Hamiltonian, conjectured by JLMS.

Cubic order term of EE

- Similarly we can evaluate the cubic order term of the EE

$$\delta S_V^{(3)} = -(C_{VV}^{\mathcal{O}})^3 \int_{-\infty}^{\infty} ds_1 ds_2 \mathcal{K}_2(s_1, s_2)$$
$$\times \frac{i(s_2 - s_1)}{2\pi} \langle B_{\mathcal{O}}(\tau - \tau_{s_1}, \hat{\tau} - \tau_{s_1}) B_{\mathcal{O}}(\tau - \tau_{s_2}, \hat{\tau} - \tau_{s_2}) B_{\mathcal{O}}(\tau, \hat{\tau}) \rangle_{\Sigma_1}$$

In the small subsystem size limit we can evaluate the integral,

$$\delta S_V^{(3)} = (2\theta_0)^{3\Delta} (C_{VV}^{O})^3 C_{OOO} \frac{\Gamma(\frac{1+\Delta}{2})^3}{12\pi\Gamma(\frac{3+3\Delta}{2})}.$$

This again agree with the holographic calculation[Casini, Galante Myers], in the presence of the bulk cubic interaction. $\mathcal{L}_{bulk} = (\partial\phi)^2 - \kappa\phi^3$

Summary so far

- We developed a way to compute relative entropy perturbatively in CFT.

- We then rewrite the CFT results at leading order holographically. They have expected form, namely canonical energy for entanglement

entropy, JLMS form for modular Hamiltonian.

We did not use the RT formula in order to derive the holographic expression. If we combine these results with the RT formula, we can derive bulk gravitational eom, beyond the linearized level.

Comments [TU, in progress]

- So far, we have been talking about the perturbative expansion of relative entropy

using the vacuum modular flow.

This scheme also allows us to perturbatively compute other quantum theoretic quantities that can not be computed by replica trick.

Quasi entropy is one of them. $S_\gamma = \mathrm{tr}\ \left[\rho_0^{1-\gamma}\rho^\gamma\right]$

Using the Dunford representation , we can expand it,.

$$S_\gamma = \int_C \frac{dz}{2\pi i}\mathrm{tr}\left[\frac{\rho_0^{1-\gamma}}{z-\rho}\right] z^\gamma$$

Comments

- This allows us to expand this similarly,

$$S_\gamma = \sum_{n=0}^{\infty} \int ds_1 \cdots ds_{n-1} \mathcal{K}_n^\gamma(s_1, \cdots s_{n-1}) \mathrm{tr} \left[e^{-2\pi K} \prod_{k=1}^{n} e^{iKs_k} \tilde{\delta}\rho e^{-iKs_k} \right]$$

$$\mathcal{K}_n^\gamma(s_1, \cdots s_{n-1}) = \left(\frac{-i}{4\pi} \right)^n \frac{(s_1 + 2\pi i\gamma) \sin \pi\gamma}{\sinh\left(\frac{s_1 + 2\pi i \gamma}{2}\right) \prod_{k=2}^{n-1} \sinh\left(\frac{s_k - s_{k-1}}{2}\right) \sinh\left(\frac{s_{n-1}}{2}\right)}$$

- Looks possible to rewrite the quadratic part holographically for any perturbation.

A CFT computation of an entanglement measure for mixed states

Work in progress with Tadashi Takayanagi, Koji Umemoto

1807.@@@@@

Motivation

- Consider a bipartite system $H = H_A \otimes H_B$

- Question: For a given state, how do we evaluate entanglement between A and B?

▪For pure states $|\Psi\rangle_{AB}$ entanglement entropy is the unique measure
$\Rightarrow$ it counts # of Bell pairs, LOCCs are reversible.

▪For mixed states, ρ_{AB} EE and I_{AB} are not good measures.
Also, negativity is not faithful, it can fail to detect the entanglement.

Motivation(2)

- Fact: We know the form of states with no entanglement: Separable states

$$\sigma_{AB} = \sum_a p_a \rho_A^a \otimes \rho_B^a \qquad \sum_a p_a = 1$$

Any entanglement measure has to vanish for this class of states .

- Idea: For a given mixed state ρ_{AB} the relative entropy between it and a separable state σ_{AB} measures the entanglement in ρ_{AB} .

⇒Relative entropy of entanglement(REE) [Vedral Plenio Rippin Knight]

$$E_R(\rho_{AB}) = \inf_{\sigma_{AB}\in \mathrm{Sep}} S(\rho_{AB}||\sigma_{AB})$$

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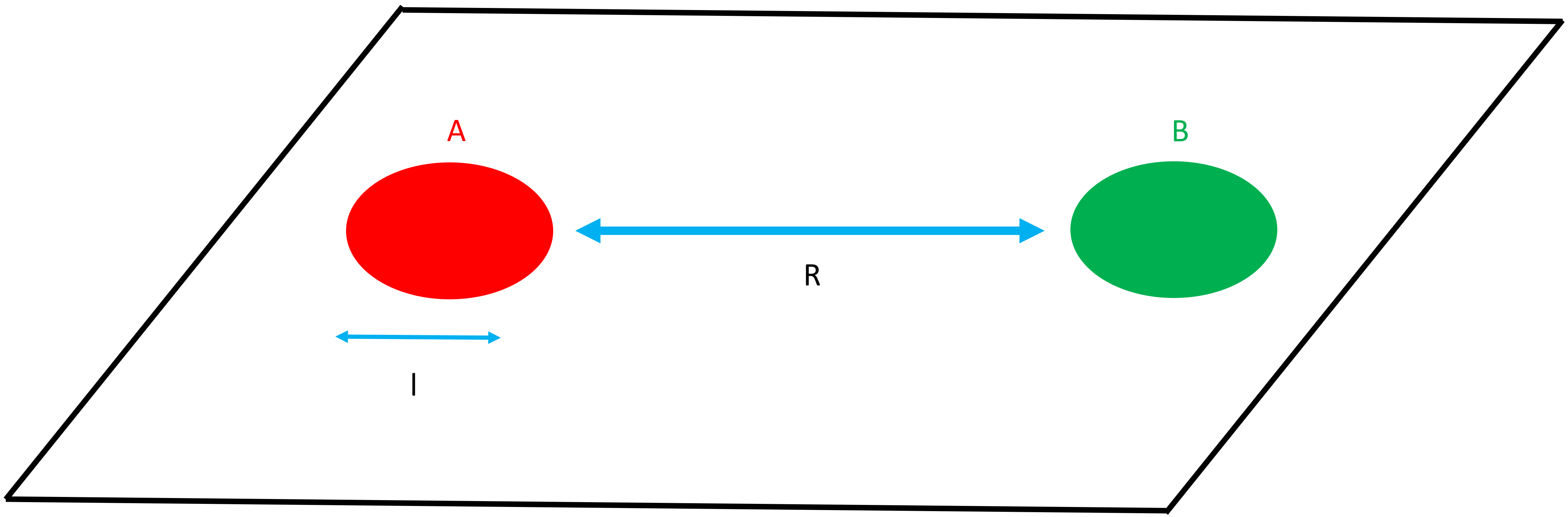
$\Rightarrow$Relative entropy of entanglement(REE) [Vedral Plenio Rippin Knight]

$$E_R(\rho_{AB}) = \inf_{\sigma_{AB} \in \mathrm{Sep}} S(\rho_{AB} || \sigma_{AB})$$

We would like to calculate this in CFT

Set up



$\rho^0_{AB} = \mathrm{tr}_{(AB)^c}|0\rangle\langle 0|$

$E_R(\rho^0_{AB})$ in the large distance limit. $l/R \to 0$

Set up(2)

- In this limit, mutual information behaves like

$$I(\rho^0_{AB}) \simeq (l)^{4\Delta}\frac{\Gamma(\frac{3}{2})\Gamma(2\Delta+1)}{2\Gamma(2\Delta+\frac{3}{2})}\langle O_A O_B\rangle^2 \equiv a_{2\Delta}\left(\frac{l}{R}\right)^{4\Delta}$$

where O is the lightest primary operator.

- Since $I_{AB}(\rho^0_{AB}) = S(\rho^0_{AB}||\rho^0_A \otimes \rho^0_B)$ $\Rightarrow$ $\begin{cases} \rho^0_{AB} \sim \rho^0_A \otimes \rho^0_B \\ \sigma_{AB} \sim \rho^0_A \otimes \rho^0_B \end{cases}$

we can pertubatively compute the RE $S(\rho^0_{AB}||\sigma_{AB})$

Computation of the relative entropy

$$S(\sigma_{AB}||\rho^0_{AB}) = -S(\sigma_{AB}) + \text{tr}\,\sigma_{AB}K^0_{AB}$$

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$$S(\sigma_{AB}||\rho^0_{AB}) = -S(\sigma_{AB}) + \mathrm{tr}\,\sigma_{AB}K^0_{AB}$$

$$\begin{aligned}-S(\sigma_{AB}) = &-\sum_a p_a\left(\langle K^0_A\rho^a_A\rangle + \langle K^0_B\rho^a_B\rangle\right)\\ &+ a_\Delta\,(l)^{2\Delta}\left[\left(\sum_a p_a\langle\rho^a_A O\rangle\right)^2 + \left(\sum_a p_a\langle\rho^a_B O\rangle\right)^2\right]\\ &+ a_{2\Delta}\,(l)^{4\Delta}\left[\sum_a p_a\langle\rho^a_A O_A\rangle\langle\rho^a_B O_B\rangle - \left(\sum_a p_a\langle\rho^a_A O\rangle\right)\left(\sum_a p_a\langle\rho^a_B O\rangle\right)\right]^2\end{aligned}$$

Similar to MI calculations of excited states found in [TU 2016]

Computation of the relative entropy

$$S(\sigma_{AB}||\rho^0_{AB}) = -S(\sigma_{AB}) + \text{tr}\ \sigma_{AB} K^0_{AB}$$

Modular Hamiltonian of the two disjoint subsystems:

$$K^0_{AB} = K^0_A + K^0_B + \tilde{K}^0_{AB}$$

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Modular Hamiltonian of the two disjoint subsystems:

$$K^0_{AB} = K^0_A + K^0_B + \tilde{K}^0_{AB}$$

Again found in [TU, 2016]

$$\tilde{K}^0_{AB} = -2a_{2\Delta}\, l^{4\Delta}\langle O_A O_B\rangle O_A O_B + I_{AB} \qquad \frac{l}{R} \to 0$$

The Result

$$S(\sigma_{AB}||\rho^0_{AB}) = l^{4\Delta}\left(\langle O_A O_B\rangle - \sum_a p_a \langle \rho^a_A O_A\rangle\langle\rho^a_B O_B\rangle\right)^2$$

From the expression we can find the separable state which minimize the relative entropy,

$$\sigma_{AB} = \lim_{x\to\infty}\left[\left(1 - \frac{l^{2\Delta}\langle O_A O_B\rangle}{x^2}\right)\rho^0_A\rho^0_B + \frac{l^{2\Delta}\langle O_A O_B\rangle}{x^2}\rho^1_A\rho^1_B\right]$$

$$\mathrm{tr}[\rho^1_A O_A] = \mathrm{tr}[\rho^1_B O_B] = l^{-\Delta}x$$

The result(2)

- This results in ER=0, meaning VRDM is indistinguishable from the separable state.

- This indicates that if we only take the effect of lightest primary into account, there is no entanglement.

- This is very much in contrast to the fact that MI is non vanishing in the limit.

･MI in the limit   is purely coming from classical correlations .

An intuition for the result.

- There is an intuitive way to understand there is a separable state which make the RE

vanish.

- In the large distance limit, the algebra on the disjoint regions is

$$\mathcal{A}_{A\cup B} = \{1_A \otimes 1_B,\ 1_A \otimes O_B,\ O_A \otimes 1_B,\ O_A \otimes O_B\}$$

- Only the non trivial observable of the algebra is the two point function: $\mathrm{tr}\left[\rho_{AB} O_A O_B\right]$

- The separable state reproduces the 2pt function.

- These two matrices are indistinguishable=> The RE is vanishing.

The effect of next lightest primary

- So far we have been only taking account of the lightest primary.

- In order to complete the story, we need to argue that we can actually suppress the effects of higher dimensional operators in the RE, and the separable state σ_{AB} still minimize the RE.

In CFTs with few primaries (ex: the Ising model) we can explicitly show this.

$\Rightarrow$There is not entanglement in the class of theories.

However in CFTs with a gravity dual, it is not true. In these theories, we can not Supress the contributions of multi trace operators.

The effect of next lightest primary

- The conjectural picture is that, entanglement structure might be highly depending on the spectrum, even in the large distance limit.

- Stating this explicitly is beyond our current computational technique.

Scrambling and relative entropy

"Chaos and relative entropy"

Work in progress with Y. Nakagawa, G Sarosi

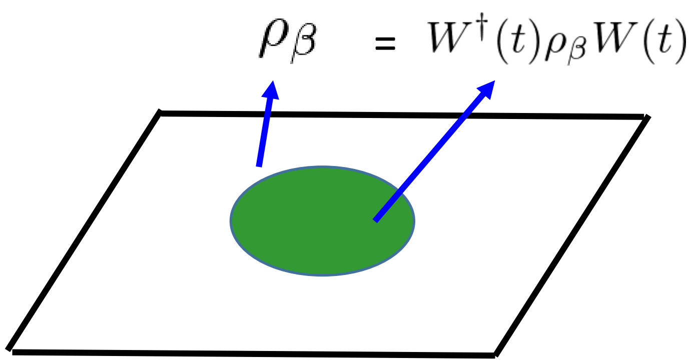
Scrambling

Scrambling refers to the phenomena of quick delocalization of quantum information in thermal states. [Sekino Susskind] [Lashkari Stanford et al]....

The typical time scale of these phenomena is the scrambling time $t_{sc} = \beta \log c$

In a chaotic system, any local subregion A due to the delocalization, the thermal RDM ρ_β and its perturbation $W^\dagger(t)\rho_\beta W(t)$ by a local operator W(t) become indistinguishable after the scrambling time.

Conjecture: Relative entropy can diagnose scrambling.



Set up.

We start from a TFD state, and perturb it
By a local operator W(t).

$$|\Psi_1\rangle = \quad |TFD\rangle = \frac{1}{\sqrt{Z(\beta)}}\sum_n e^{-\frac{\beta}{2}E_n}|E_n\rangle_L|E_n\rangle_R$$

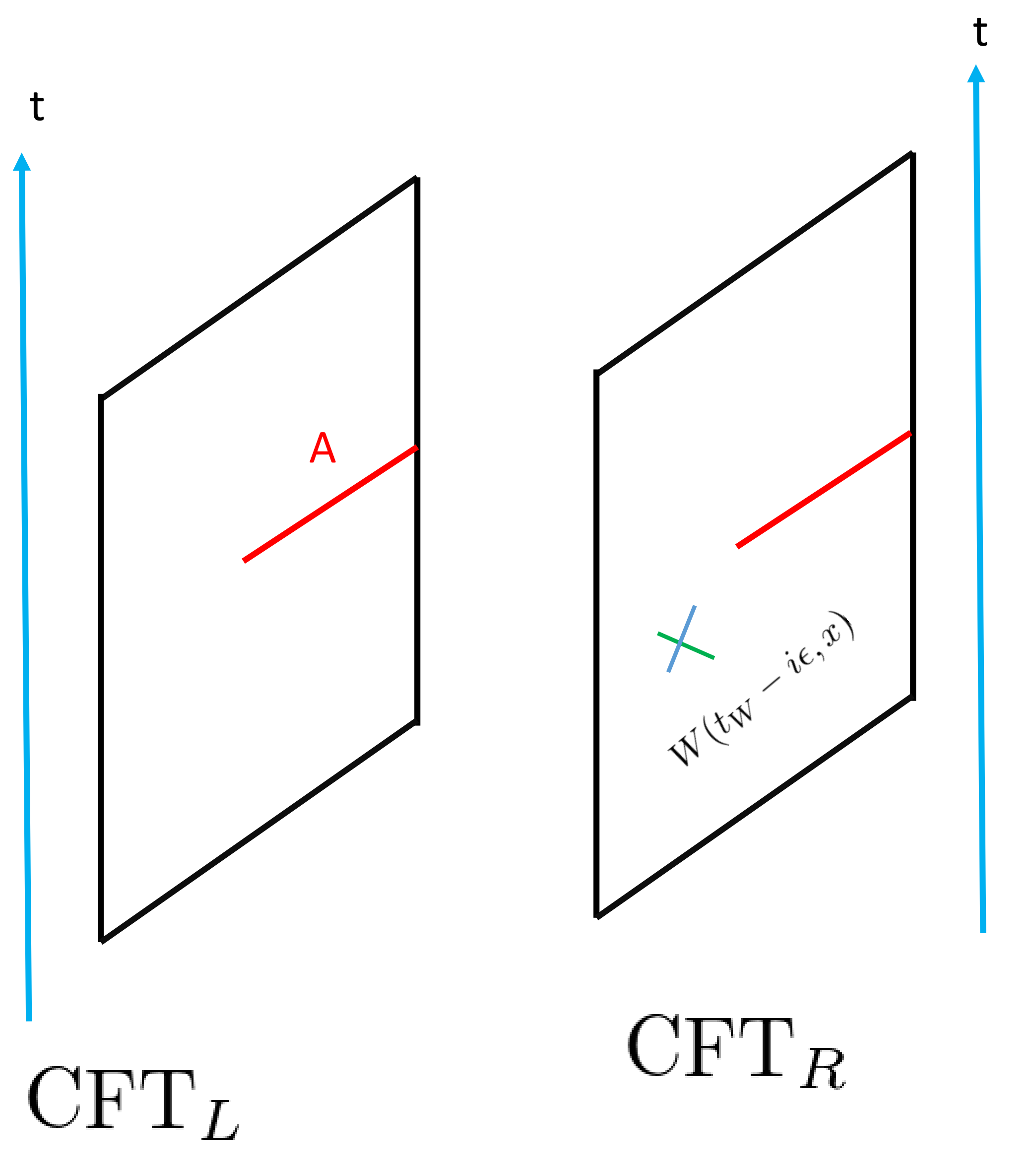
$$|\Psi_2\rangle = W(t_W - i\epsilon, x)|TFD\rangle$$

Evolve both CFTs forward in time.

Subsystem A: half spaces in both CFTs

Relative entropy between two RDMS.

$$S(\rho_1||\rho_2)$$



The relative entropy in the large C limit

In the large central charge limit, we can use the gravity description to calculate the relative entropy.

A TFD state with a local operator insertion $W(t_W - i\epsilon, x)|TFD\rangle$ has a simple gravity dual : a BTZ black hole with a shock wave.

$$ds^2 = -\frac{4}{(1+uv)^2}dudv + \left(\frac{1-uv}{1+uv}\right)^2 dy^2 + 4\delta(u)h(y)du^2$$

The entanglement entropy part can be easily computed via the RT formula.

The modular Hamiltonian has a local expression, and it is universal (theory independent.)

The relative entropy in the large C limit

- We first computed the relative entropy in the large C limit, where we can use the gravity description, as well as the RT formula.
- When $t \gg \log \frac{\beta^2}{\epsilon}$ the relative entropy decays exponentially,

$$S(\rho_1||\rho_2) \sim ce^{-\frac{t}{\beta}}$$

- The relative entropy becomes O(1) at the scrambling time $t_{sc} = \beta \log c$. After this,the large c approximation breaks down, and quantum corrections to the RT formula become important, in order to further follow the time evolution.

Some Numerics in spin chain models.

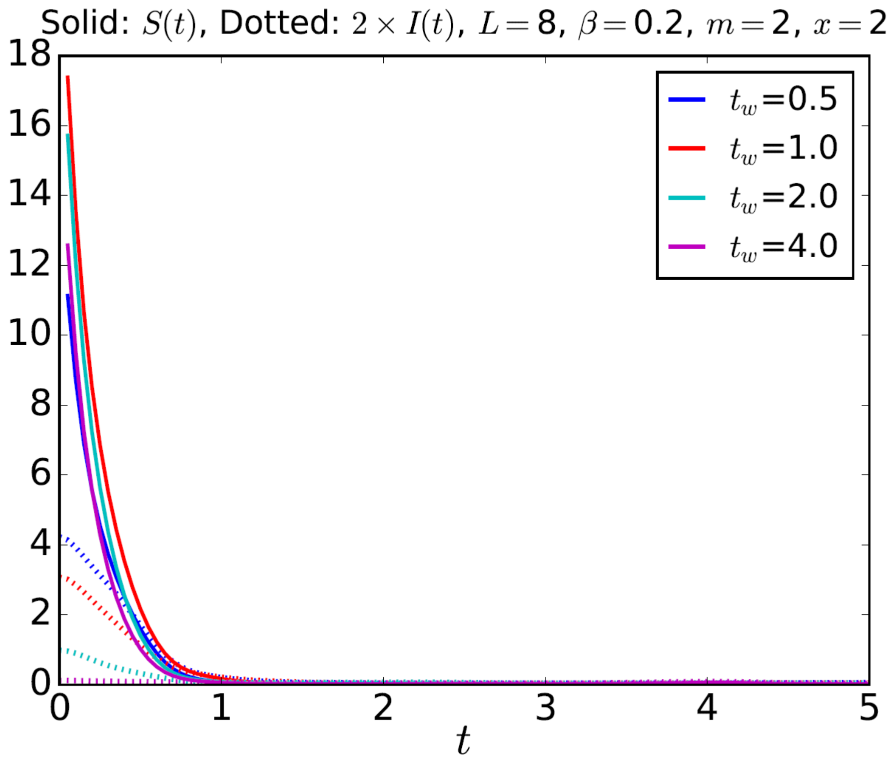
In order to further follow the time evolution of the relative entropy after the scrambling time, we performed numerical calculations in several spin chain models.

$$H = -\sum_{i=1}^{N} (Z_i Z_{i+1} + g X_i + h Z_i)$$

The Hamiltonian is integrable when $h = 0$.

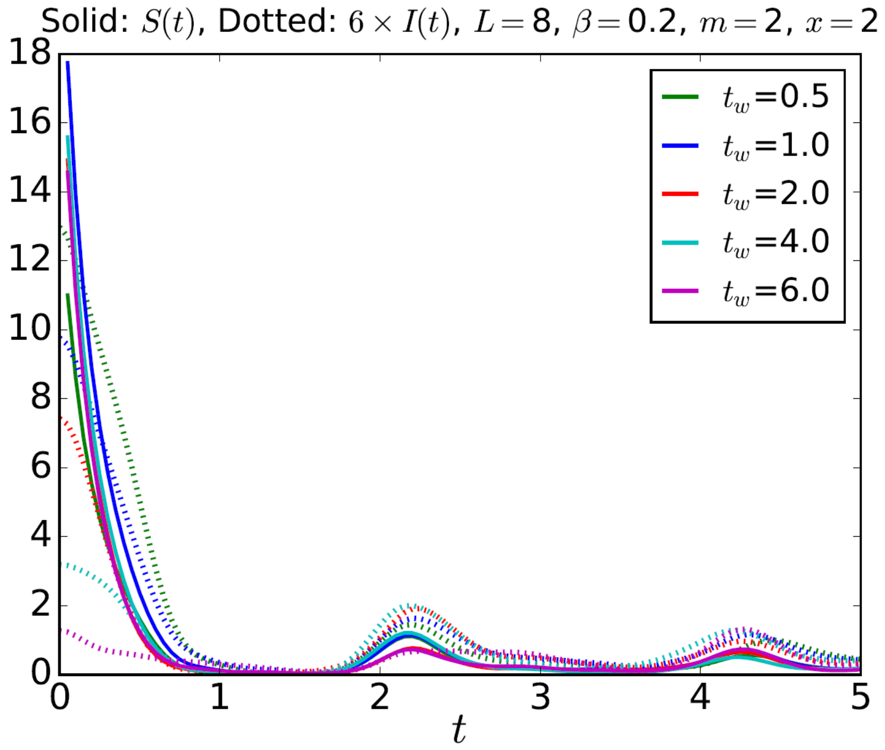
A chaotic point (g,h)= (-1.05, 0.5).

Non integrable case



No oscillations in the late time regime

Integrable case



Oscillations in the late time regime

In non chaotic systems, we observe recursions of the relative entropy , however in chaotic systems, there are no recursions.

Conclusions

- We developed a way to perturbatively calculate it in CFT .

We also found a bulk expression of it by rewriting the CFT answer.

- In the second part, we used RE to compute an entanglement measure in CFT

In the final part of this talk, I explained the application of relative entropy to the physics of scrambling .