Precision studies of AdS/CFT

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References

M. Mezei, S. S. Pufu and Y. Wang, *A 2d/1d Holographic Duality*, 1703.08749.

C.-M. Chang, M. Fluder, Y.-H. Lin and Y. Wang, *Spheres, Charges, Instantons, and Bootstrap: A Five-Dimensional Odyssey*, *JHEP* **03** (2018) 123, [1710.08418].

C.-M. Chang, M. Fluder, Y.-H. Lin and Y. Wang, *Romans Supergravity from Five-Dimensional Holograms*, 1712.10313.

M. Mezei, S. S. Pufu and Y. Wang, *5d Super-Yang-Mills, 3d Chern-Simons, and 6d* $(2,0)$ *Theories*, 1807.XXXXX.

Techniques to study superconformal fixed points

SUSY → Control over strongly-coupled dynamics

supersymmetric partition function from localization

$S^1 \times S^{d-1}$, S^d ...

protected operator algebra

chiral ring, chiral algebra, topological quantum mechanics, defects ...

superconformal bootstrap

stress-tensor bootstrap, flavor bootstrap, Mellin amplitudes ...

anomaly

conformal anomalies, t'Hooft anomalies ...

Motivation

Q1: Can we use these techniques in the large N limit to do precision studies of AdS/CFT?
Q2: Gain more insight about String/M-theory?
Q3: Non-SUSY AdS/CFT from embedding in dual pairs with SUSY?
Q4: Confirmations/Lessons for strongly coupled field theory phenomena?
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AdS/CFT with supersymmetry

***AdS*$_3$** Examples with $\mathcal{N} = (2,2)$ and $(4,4)$ SUSY well-studied. Little known about $\mathcal{N} = (1,1), (3,3)$ etc. See Massimo's and Matthias's talks for $\mathcal{N} = (4,4)$.

***AdS*$_4$** Examples with $\mathcal{N} = 2, 3, 4, 6, 8$ studied extensively. Though $\mathcal{N} = 1$ is little studied.

***AdS*$_5$** Lots of works for $\mathcal{N} = 1, 2$ and $\mathcal{N} = 4$, little for $\mathcal{N} = 3$. See Robert's and Charlotte's talks for $\mathcal{N} = 4$.

***AdS*$_6$** Only $\mathcal{N} = 1$ is possible See also Oren's talk.

***AdS*$_7$** Many works for $(2,0)$ but few for $(1,0)$. See also Eric's talk for $(2,0)$.

AdS/CFT with supersymmetry

Our focus today

AdS_3 Examples with $\mathcal{N} = (2,2)$ and $(4,4)$ SUSY well-studied. Little known about $\mathcal{N} = (1,1), (3,3)$ etc. See Massimo's and Matthias's talks for large $N = 4$.
AdS_4 Examples with $\mathcal{N} = 2,3,4,6,8$ studied extensively. Though $\mathcal{N} = 1$ is little studied.
AdS_5 Lots of works for $\mathcal{N} = 1,2$ and $\mathcal{N} = 4$, little for $\mathcal{N} = 3$. See Robert's and Charlotte's talks for $\mathcal{N} = 4$.
AdS_6 Only $\mathcal{N} = 1$ is possible See also Oren's talk.
AdS_7 Many works for $(2,0)$ but fewer for $(1,0)$. See also Eric's talk for $(2,0)$.

AdS_4/CFT_3

Mezei-Pufu-YW: arXiv:1703.08749

TQM from the 3d $N = 4$ SCFT

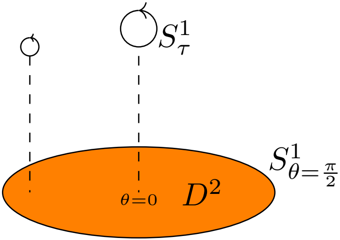
- 3d $\mathcal{N} = 4$ SCFT has a 1d topological sector with nontrivial operator algebra [Chester-Lee-Pufu-Yacoby, Beem-Rastelli-Peelaers, Dedushenko-Pufu-Yacoby]
- Defined by **cohomology** with respect to nilpotent $\mathcal{Q}_{1,2}$. Correlations function only depend on ordering on a line in $\mathbb{R}^3$.
- Higgs branch operators **twisted** by $SU(2)_H$ R-symmetry polarizations
- Higgs branch chiral ring $\rightarrow$ **noncommutative associative** deformation (also for Coulomb branch).
- Stereographic map: $\mathbb{R} \subset \mathbb{R}^3 \rightarrow S^1 \subset S^3$, the TQM can be formulated as a **gauged** QM (for certain 3d SCFTs).

Goal: Obtain a nontrivial 1d subsector of the large N 3d SCFTs

Review: 1d TQM on S^1 from 3d $\mathcal{N}=4$ SCFT on S^3

In [Dedushenko-Pufu-Yacoby]

- Localizing supercharge $\mathcal{Q}=\mathcal{Q}_1+\mathcal{Q}_2$ satisfies
$$\mathcal{Q}^2 \sim iP_\tau + R_C$$
- BPS locus is **covariantly constant** along τ, and parametrized by matter fields $Q, \tilde{Q}$ on S^1 at $\theta=\pi/2$
- Weighted by a gauged QM action where the gauge field $\mathcal{A}$ is **emergent**

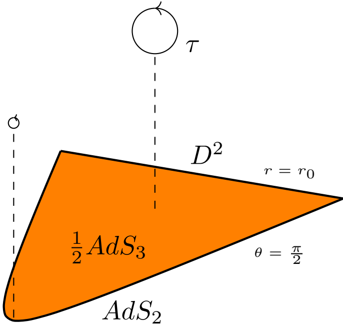


$$Z_{S^3} \sim \int D\mathcal{A} DQ\, D\tilde{Q}\, \exp\left[4\pi r \int d\varphi\, \tilde{Q}(\partial_\varphi + i\mathcal{A}_\varphi)Q\right]$$

2d protected sector of the $N = 4$ SUGRA on AdS_4

Goal: 1d TQM in 3d SCFT $\rightarrow$ 2d protected sector in AdS_4 SUGRA?
The strategy in [Mezei-Pufu-YW'17]

- Focus on matter multiplets in $\mathcal{N} = 4$ SUGRA
- Action with off-shell SUSY on manifold with **boundary**
- Show that on the BPS locus the fields are **covariantly constant** along τ
- Reduce the theory on AdS_4 to $\frac{1}{2}AdS_3$ and write the action as a **total derivative**
- Obtain **2d Yang-Mills** action on AdS_2 with appropriate **boundary terms** on S^1. **Gauge group** determined by the matter content.



Case-Study: ABJM and $\mathrm{Sdiff}(S^2)$

Focus on a familiar AdS_4/CFT_3 dual pair

$$\mathcal{N} = 8 \text{ ABJM at } k = 1 \xleftrightarrow{\text{Holographic dual}} \text{M theory on } AdS_4 \times S^7$$

The $k = 1$ ABJM theory has an equivalent/mirror description in terms of $\mathcal{N} = 4$ $U(N)$ vector multiplet coupled to one adjoint hyper X and one fundamental hyper Q [Kapustin, Bashkirov '10]

In [Mezei-Pufu-YW'17]

$$\text{1d TQM} \xleftrightarrow{\text{Holographic dual}} \text{2d YM with } \frac{1}{g_{\mathrm{YM}}^2} = \frac{N^{3/2}}{3\sqrt{2}} \text{ and } \mathfrak{g} = \mathrm{Sdiff}(S^2)$$

Case-Study: ABJM and $\mathrm{Sdiff}(S^2)$

Field theory side:

- TQM determined by matter content of 3d SCFT
- Solve large N limit analytically
- 2,3 and some 4-point correlators in the planar limit.

Gravity side:

- Lowest KK modes: $\mathcal{N} = 4$ SYM subsector of 4d $\mathcal{N} = 8$ SUGRA
- Localization to obtain bulk 2d action
- Explicit check of the 2d/1d proposal at the 2-derivative order
- Predictions for bulk higher derivatives interactions from field theory

Similar analysis for $\mathcal{N} = 4$ higher spin type AdS/CFT duals in the paper.

AdS_6/CFT_5

Chang-Fluder-Lin-YW: arXiv:1710.08418, arXiv:1712.10313

Local correlators in 5d $N = 1$ SCFT

- 5d SCFTs do not have weakly coupled UV descriptions but rather emerge as UV fixed points of IR free gauge theories. $g^2_{\mathrm{YM}} \sim \frac{1}{m_I}$. [Seiberg, Morrision, Intriligator ...]
- Global symmetries at the fixed point not manifest in the IR phase
- Localization computation for superconformal index and S^5 free energy [Kim-Kim-Lee, Kim-Kim, Hosomichi-Seong-Terashimam, Källén-Qiu-Zabine, Lockhart-Vafa ...]

Goal: Develop a method to compute local correlators for 5d SCFTs
How do we compute local correlators? Any? Simplest ones:

$$\langle T_{\mu\nu}(x) T_{\rho\sigma}(0) \rangle = \frac{C_T}{V^2_{\hat{\mathrm{S}}^4}} \frac{\mathcal{I}_{\mu\nu,\sigma\rho}(x)}{x^{10}},, \quad \langle J^a_\mu(x) J^b_\nu(0) \rangle = \frac{C_J}{V^2_{\hat{\mathrm{S}}^4}} \frac{\delta^{ab} I_{\mu\nu}(x)}{x^8}.$$

Local correlators in 5d $N = 1$ SCFT

In [Chang-Fluder-Ying-YW: 1710.08418]

- Look at quadratic **deformation** of S^5 partition function by squashing parameters a_i and flavor masses M^a.
- Classify supergravity counter-terms to make sure coefficients of deformations in F_{S^5} do not suffer from **ambiguities**
- Do conformal perturbation theory and solve **superconformal Ward identities**

$$F_{\mathrm{S}^5}\big|_{a_i^2} = -\frac{\pi^2 C_T}{1920}\left(\sum_{i=1}^3 a_i^2 - \sum_{i<j} a_i a_j\right)$$

$$F_{\mathrm{S}^5}\big|_{M^2} = \frac{3\pi^2 r^2 C_J}{256}\delta_{ab} M^a M^b$$

- New 5d superconformal anomaly, conformal bootstrap, non-BPS spectrum, symmetric enhancement, testing F-theorem, C-theorem ...

KK-reduction of type I' SUGRA on $AdS_6 \times_w M_4$

For AdS/CFT duals

$$C_T \leftrightarrow \frac{1}{G_N}, \quad C_J \leftrightarrow \frac{1}{e^2}$$

Goal: Perform the KK reduction of the 10d type I' SUGRA solution describing AdS_6/CFT_5 duals $\rightarrow$ fix G_N, e. Then compare with field theory answers C_T, C_J.

- Complication: **warped** AdS_6 solutions. **singularity** at the boundary of M_4
- Missing gauge fields for flavor symmetries: from D8 **flavor branes**
- Integrate over M_4 gives **finite** answer for 2-derivative couplings! (SUSY non-renormalization theorems)

$$\frac{1}{\kappa^2}\int_{AdS_6 \times M^4} \sqrt{G}(R + F^2 + \dots) \rightarrow \int_{AdS_6} \sqrt{g_6}(\frac{1}{G_N} R_6 + \frac{1}{e^2} F_6^2 + \dots)$$

Case-Study: Seiberg-Morrison theories and generalizations

In [Chang-Fluder-Ying-YW: 1712.10313]

- A class of AdS_6/CFT_5 duals (Seiberg-Morrison theories)
- SCFT described by $USp(2N)$ $\mathcal{N}=1$ SYM, 1 antisymmetric and $N_f \leqslant 7$ fundamental hypers. It has type I' SUGRA dual.
- Instanton contributions in F_{S^5} suppressed at large N. Solve matrix model to get analytic results

$$C_T = \frac{1152\sqrt{2}}{\pi\sqrt{8-N_f}}N^{\frac{5}{2}}, \quad C_J(E_{N_f+1}) = \frac{256\sqrt{2}}{3\pi\sqrt{8-N_f}}N^{\frac{3}{2}}. \tag{2.1}$$

- Matches with supergravity action on AdS_6 from massive IIA reduced on HS^4 with $D8$ flavor branes (tricky warped geometry).
- More general AdS_6/CFT_5 pairs [Bergman-Rodriguez Gomez;...] were also analyzed. Symmetry enhancement explained.

AdS_7/CFT_6

Mezei-Pufu-YW: arXiv:1807.XXXXX

3d Chern-Simons on S^3 from 6d $(2,0)$ SCFT on $S^1 \times S^5$

- The mysterious 6d $(2,0)$ SCFT is intrinsically non-Lagrangian. But on S^1 it is believed to reduce to 5d $\mathcal{N}=2$ SYM (ADE gauge group, higher derivative terms). In particular

$$\boxed{\text{6d } (2,0) \text{ ADE SCFT on } S^1 \times S^5 \leftrightarrow \text{5d ADE MSYM on } S^5}$$

- In [Kim-Kim: 1206.6339], they noticed a "coincidence": the S^5 partition function of 5d $U(N)$ MSYM contains a $U(N)$ CS matrix model

$$Z_{S^5} \sim \int d^N\lambda \, e^{i\pi k \sum_{i=1}^N \lambda_i^2} \prod_{i<j} \left[2\sinh\left[\pi(\lambda_i - \lambda_j)\right]\right]^2 .$$

- **Imaginary** CS level

$$k = i\frac{4\pi^2 R}{g_{\rm YM}^2}$$

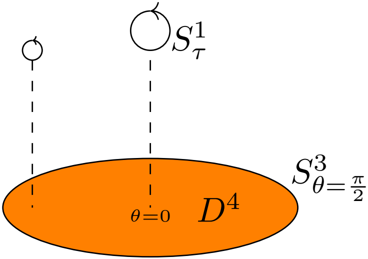
Goal: Take the coincidence seriously: find the 3d CS sector of 6d $(2,0)$ SCFT

In [Mezei-Pufu-YW: arXiv:1807.XXXXX]

- Localizing supercharge $\mathcal{Q}$ such that

$$\mathcal{Q}^2 = iP_\tau + R_{45}$$

  S^5 as $S^1 \times S^3$ fibered over an interval $\theta = [0, \frac{\pi}{2}]$.



- BPS locus parametrized by **emergent** gauge fields $\mathcal{A}$ on S^3 at $\theta = 0$ and 5d **instantons** located at the $\theta = \frac{\pi}{2}$
- Weighted by CS action

$$Z_{S^5} \sim \int D\mathcal{A} \exp \frac{ik}{4\pi} \int_{S^3} \mathrm{Tr}\left(\mathcal{A} d\mathcal{A} + \frac{2}{3}\mathcal{A}^3\right)$$

  with **imaginary** level $k \sim i\frac{R_6}{R_{S^5}}$.

$\frac{1}{8}$ BPS surface operators and knots on S^3

CS sector of 6d $(2,0)$ on $S^1 \times S^5$

- New SUSY observables beyond the superconformal index (SCI):

$$\boxed{\tfrac{1}{8}\text{ BPS surface operators in 6d }(2,0)\text{ SCFT}}$$
$$\downarrow$$
$$\boxed{\tfrac{1}{8}\text{ BPS Wilson loops on }S^5\text{ in 5d MSYM}}$$
$$\downarrow$$
$$\boxed{\text{knots on }S^3\text{ in 3d CS}}$$

- CS interpretation $\rightarrow$ These surface operators are **topological** (invariant under infinitesimal shape changes).
- Refining the **chiral algebra** limit of 6d SCI [Beem-Rastelli-van Rees, Bullimore-Kim].

Dual M2-branes in $AdS_7 \times S^4$

- Surface operator on Σ in 6d SCFT are dual to M2-branes on $\mathcal{M}$ in $AdS_7 \times S^4$ such that $\partial\mathcal{M} = \Sigma$ (winds in both ∂AdS_7 and S^4, complicated shape!)
- Need to minimize M2 brane action

$$S_{\rm M2} = \tau_{\rm M2} \int_{\mathcal{M}} d^3\sigma \sqrt{-g} + i\tau_{\rm M2} \int_{\mathcal{M}} A_3 + iS_{\rm WZW}$$

- Difficult. SUSY helps $\to$ **calibrated** submanifolds $\mathcal{M}$ (1st order differential equations). No log ambiguities.
- Need appropriate **counter-terms**
- Answer matches with field theory results computed from the 3d CS theory (topological!).

Outlook

AdS_4/CFT_3:

- Extract predictions for higher derivative couplings in M-theory (recall Eric's talk).
- Understanding the 2d protected sector of M-theory from topological twist [Kostello. . .]
- Generalization to other large N 3d SCFTs.
- embedding of SYK/tensor models?

AdS_6/CFT_5:

- Beyond two point functions
- Large N limit of other 5d SCFTs

AdS_7/CFT_6

- Relation to other 3d CS sectors of $(2,0)$ theory [Dimofte-Gaiotto-Gukov, Cordova-Jafferis, Lee-Yamazaki . . .]
- Identify the dual 4d protected sector of M-theory
- Extend the 6d analysis to $(1,0)$ theories

Thank you!