New Frontiers in String Theory 2018

Based on 1712.02647 with P. Narayan

SUSY SYK Model with Global Symmetry

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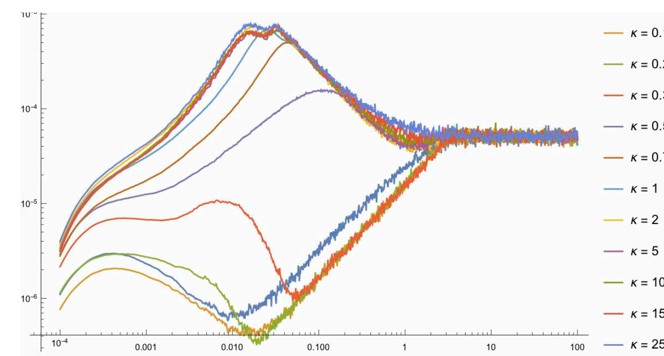
Quantum Chaos

- ✳ Quantum Chaos can be diagnosed by Lyapunov exponent in Out-of-time-ordered Correlators (OTOCs). Hanada, Yoshida Talks + Qi Talk

$$\langle [\mathcal{O}(t), \mathcal{O}(0)]^2 \rangle \sim 1 - \frac{1}{N} e^{\lambda t}$$

- ✳ Spectral form factor(SFF) exhibits random matrix behavior at late time.
【Hanada, Shenker, Stanford, Tezuka et al】【 Garcia-Garcia, Tezuka et al】
【Nosaka, Rosa, JY】



- ✳ Connection between exponential growth at (relatively) early time and random matrix behavior at late time is not so clear.
- ✳ Talk on numerical study on scrambling time and Thouless time by Tomoki Nosaka in July 31st (5th week).
- ✳ Today, we talk about OTOCs at early time.

Maximally Chaotic SYK Model

✴ Bound of Quantum Chaos (Lyapunov exponent).

$$\lambda \leqq \frac{2\pi}{\beta}$$ 【Maldacena, Shenker, Stanford】

✴ Black hole is maximally chaotic. $\lambda = \frac{2\pi}{\beta}$ 【Shenker, Stanford】

✴ Dual field theory would also exhibit the saturation of chaos bound of OTOCs.

- ✓ Difficult to calculate four point functions in strong coupling limit.
- ✓ Difficult to capture long time behavior of OTOCs.

✴ Sachdev-Ye-Kitaev Model 【Sachdev, Ye】【Kitaev】

- ✓ Melonic dominance: solvable at strong coupling limit
  Klebanov's talk
- ✓ Analytic continuation/Real-time formulation: OTOCs

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Review: Sachdev-Ye-Kitaev Model

- Quantum Mechanics of N Majorana Fermions: $\chi^i \quad (i = 1, 2, \cdots, N)$
  Sachdev, Ye; Kitaev; Polchinski, Rosenhaus; Jevicki, Suzuki, JY; Maldacena, Stanford

$$S = \int d\tau \left[\frac{1}{2}\chi^i \partial_\tau \chi^i + i^{\frac{q}{2}} J_{i_1 \ldots i_q} \chi^{i_1} \ldots \chi^{i_q} \right]$$

- SYK Model has emergent reparametrizaiton symmetry at strict strong coupling limit.

$$\Psi(\tau_1, \tau_2) = \frac{1}{N}\chi^i(\tau_1)\chi^i(\tau_2) \longrightarrow [f'(\tau_1)]^{\frac{1}{q}} \Psi(f(\tau_1), f(\tau_2))[f'(\tau_2)]^{\frac{1}{q}}$$

- At finite (large) coupling, the reparametrization symmetry is broken spontaneously and explicitly, and this leads to Schwarzian action for low energy effective action. Blommaert, Mertens Talks + Qi Talk

- Spectrum: Light modes + Infinite tower of massive modes
  - ✓ Light mode: the leading exponential growth with $\lambda = 2\pi/\beta$
  - ✓ Massive mode: $1/\beta J$ correction to Lyapunov exponent

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Generalization of SYK Models

- ✳ Complex SYK Model, Flavor 【Sachdev】【Davison, Jensen, Sachdev et al】【Bulycheva】【Gross, Rosenhaus】【JY】
- ✳ Lattice and higher dimension 【Gu, Qi, Stanford】【Berkooz, Narayan et al】【Murugan, Stanford, Witten】
- ✳ Supersymmetry 【Fu, Gaiotto, Maldacena, Sachdev】【Murugan, Stanford, Witten】【Peng, Spradlin, Volovich】【JY】【Narayan, JY】【Bulycheva】
- ✳ Some generalizations can easily be incorporated into "internal space" of bi-local field $\Psi^{ab}(\tau_1,\tau_2)=\frac{1}{N}\chi_i^a(\tau_1)\chi_i^b(\tau_2)$ $\quad M\ll N\ (a,b=1,2,\cdots,M)$
  - ✓ Replica space, Flavor, Thermofield Double, Superspace
- ✳ Today's Questions :
  - Additional emergent symmetry and effective action for light modes
  - Their contributions to OTOCs (and Lyapunov exponent)

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“To Gauge, or Not To Gauge”

- ✳ Can we gauge SYK model by O(N)/U(N) gauge field?
- ✳ In Tensor Model, useful to restrict large numbers of light modes 【Minwalla et al】【Klebanov et al】
- ✳ In SYK model, random interaction is not gauge invariant:

$$J_{ijkl}\chi_i\chi_j\chi_k\chi_l$$

- ✳ Maybe, one can consider the random coupling constant as a dynamical variable. 【Polchinski et al】【Nishinaka, Terashima】
- ✳ At T=0, fix gauge: A=0
- ✳ At T≠0, non-trivial holonomy along thermal circle
- ✳ In this talk, we will not consider O(N)/U(N) gauge field.

Complex SYK Model (or Flavor)

- ✴ One can gauge global U(1) symmetry of complex SYK model or SO(M) symmetry of generalized SYK model with flavor 【Sachdev】【Davison, Jensen, Sachdev et al】【Gross, Rosenhaus】【Bulycheva】【JY】
  - ✓ U(1) (or SO(M)) Singlet sector
  - ✓ Only (super-)reparametrizaion light modes appear
- ✴ One can consider background gauge field.
  - ✓ At T≠0, chemical potential or non-trivial boundary condition.
  - ✓ Two-point function need not to be anti-symmetric. $\langle \bar{\chi}_i(\tau_1)\chi^i(\tau_2)\rangle \sim \dfrac{\mathrm{sgn}\,(\tau_{12})+\epsilon}{|\tau_{12}|^{2\Delta}}$
- ✴ Emergent reparametrization and ("local") U(1) symmetry are broken spontaneously and explicitly. (e.g. in complex SYK model)

$$\Psi(\tau_1,\tau_2) \longrightarrow e^{i\phi(\tau_1)}\Psi(\tau_1,\tau_2)e^{-i\phi(\tau_2)}$$

- ✴ Chemical potential change the emergent reparametrization, and the two light modes are coupled. 【Davison, Jensen, Sachdev et al】

$\mathcal{N}$=1 SUSY SYK Model

Auxiliary Boson

* ✳ Superfield with flavor index: $\psi^{i\alpha}(\tau,\theta) \equiv \chi^{i\alpha}(\tau) + \theta b^{i\alpha}(\tau)$

Majorana fermion

  - ✓ Large N, finite q : $i = 1, 2, \cdots, N \qquad \alpha = 1, 2, \cdots, q$
  - ✓ Auxiliary Boson: random average is ok (cf. tensor model)

$$S = \int d\tau d\theta \left[-\frac{1}{2}\psi^{i\alpha}\mathtt{D}\psi^{i\alpha} + i^{\frac{q-1}{2}} J_{i_1\cdots i_q}\psi^{i_1\alpha_1}\cdots\psi^{i_q\alpha_q}\epsilon_{\alpha_1\cdots\alpha_q}\right]$$

【Narayan, JY】

  - ✓ q: odd (measure: Grassmaniann odd)
  - ✓ SO(q) Global Symmetry
- ✓ In this talk,
  - ✓ No gauge field
  - ✓ SO(q) invariant ansatz for two point function $\langle\psi^{i\alpha_1}\psi^{i\alpha_2}\rangle \sim \delta^{\alpha_1\alpha_2}$

Emergent Symmetry

- After random average, one can write collective action of bi-local superfield in large N:

$$\mathbf{\Psi}^{\alpha_1\alpha_2}(\tau_1,\theta_1;\tau_2,\theta_2) \equiv \frac{1}{N}\sum_{i=1}^{N}\psi^{i\alpha_1}(\tau_1,\theta_1)\psi^{i\alpha_2}(\tau_2,\theta_2)$$

- At strict strong coupling limit, emergent Super-reparametrization and ("local") SO(q) symmetry

$$\mathbf{\Psi}(\tau_1,\theta_1;\tau_2,\theta_2) \longrightarrow \mathbf{\Psi}_{(f,y)}(\tau_1,\theta_1;\tau_2,\theta_2) \equiv [\mathtt{D}_1 y_1]^{\frac{1}{q}}\mathbf{\Psi}(f_1,y_1;f_2,y_2)[\mathtt{D}_2 y_2]^{\frac{1}{q}}$$

$$\mathbf{\Psi}(\tau_1,\theta_1;\tau_2,\theta_2) \longrightarrow \boldsymbol{g}(\tau_1,\theta_1)\mathbf{\Psi}(\tau_1,\theta_1;\tau_2,\theta_2)\boldsymbol{g}^{-1}(\tau_2,\theta_2)$$

- At finite coupling constant, the emergent symmetries are broken spontaneously and explicitly to OSp(1|2) and (Global) SO(q)

- Low energy effective action: Super-Schwarzian + action for a super-particle on group manifold

$$S_{\text{eff}} \equiv -\frac{N\alpha_{\text{SDiff}}}{J}\int d\tau d\theta\, 2\left[\frac{\mathtt{D}^4 y}{\mathtt{D}y} - 2\frac{\mathtt{D}^2 y \mathtt{D}^3 y}{[\mathtt{D}y]^2}\right] - \frac{N\alpha_{\text{SO(q)}}}{J}\int d\tau d\theta\, \frac{1}{2\mathrm{k}}\mathrm{tr}\left[\boldsymbol{\mathcal{J}}\mathtt{D}\boldsymbol{\mathcal{J}} + \frac{1}{\mathrm{k}}\boldsymbol{\mathcal{J}}^3\right]$$

- 4 types of light modes: (B/F) reparametrization + (B/F) SO(q)

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Correction to Leading Lyapunov Exponent

* $1/\beta J$ Correction to the Leading Lyapunov Exponent

$$\lambda_L = \lambda_L^{(0)} + \frac{1}{\beta J}\lambda_L^{(1)} + \cdots$$

➡ Leads to Correction to the Exponential Growth

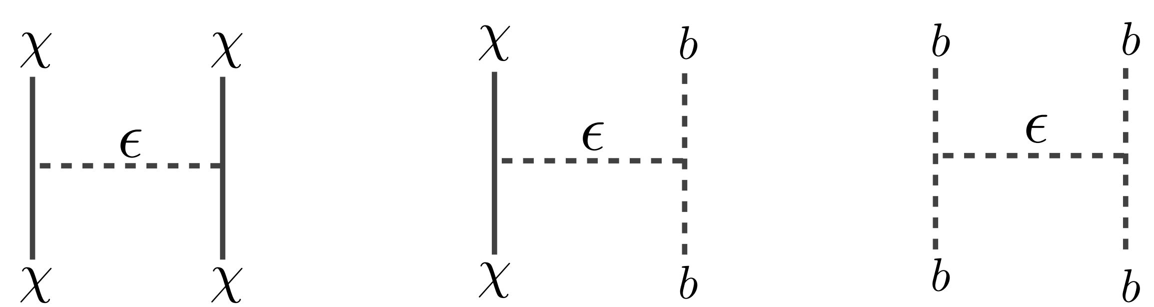
$$e^{\lambda_L t} = e^{\lambda_L^{(0)} t} + \frac{\lambda_L^{(1)}}{\beta J} t\, e^{\lambda_L^{(0)} t} + \cdots$$

Light Mode Contribution
e.g. Super-reparametrization light modes and SO(q) light modes

Non-zero Mode Contribution
Summation of all contributions from Infinite Tower of Massive Modes

Out-of-time-ordered Correlators

- Bi-local superfield can be decomposed into singlet, anti-symmetric and symmetric representation of SO(q). Up to quadratic level of bi-locals, these are decoupled.
- SO(q) Singlet Channel of **Bosonic** bi-locals ($\chi\chi$ and bb) :
  - (bosonic) reparametrization light mode give maximal Lyapunov exponent ($\lambda = \frac{2\pi}{\beta}$) in OTOCs of bosonic bi-locals.



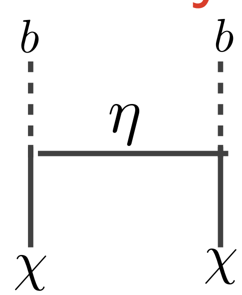
$$\sim \; e^{\frac{2\pi}{\beta}t}$$

【Maldacena, Stanford】
【Gross, Rosenhaus】
【Bulycheva】
【JY】
【Peng, Spradlin, Volovich】
【Narayan, JY】

  - Non-zero modes give $1/\beta J$ correction: $\frac{t}{\beta J}e^{\frac{2\pi}{\beta}t}$
  - Maximally Chaotic OTOCs

Out-of-time-ordered Correlators

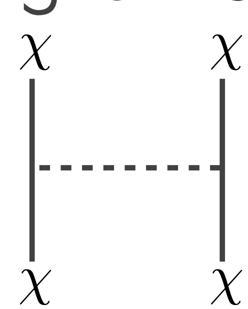
- SO(q) Singlet Channel of **Fermi** bi-locals (χb):
  - (Fermi) Super-reparametrization light mode give a half of the maximal Lyapunov exponent ($\lambda = \frac{\pi}{\beta}$) in OTOCs of Fermi bi-locals.



$\sim \ e^{\frac{\pi}{\beta}t}$ 【Narayan, JY】

  - Non-zero modes give $1/\beta J$ correction: $\frac{t}{\beta J} e^{\frac{\pi}{\beta}t}$

- SO(q) Anti-symmetric Channel of **Bosonic** bi-locals:
  - (Bosonic) SO(q) light mode give linear growth $(\lambda = 0)$in one OTOCs



$\sim t$

【JY】

【Minwalla et al】(for tensor model)

【Narayan, JY】

Summary of Chaotic Behavior

		Zero Mode	Non-Zero Mode
Singlet	$\langle\chi^i\chi^i\ \chi^j\chi^j\rangle$ $\langle\chi^i\chi^i\ b^jb^j\rangle$ $\langle b^ib^i\ b^jb^j\rangle$	$Je^{\frac{2\pi}{\beta}t}$	$\frac{t}{\beta}e^{\frac{2\pi}{\beta}t}$
	$\langle b^i\chi^i\ b^j\chi^j\rangle$	$Je^{\frac{\pi}{\beta}t}$	$\frac{t}{\beta}e^{\frac{\pi}{\beta}t}$
Anti	$\langle\chi^i\chi^i\ \chi^j\chi^j\rangle$	Jt	$\frac{t}{\beta}$
	$\langle\chi^i\chi^i\ b^jb^j\rangle$ $\langle b^ib^i\ b^jb^j\rangle$ $\langle b^i\chi^i\ b^j\chi^j\rangle$	No Growth	
Sym		No Growth	

✳ Lyapunov exponent from W_N vacuum Block： $\lambda_L^{(s)} = \frac{2\pi}{\beta}(s-1) \qquad (s = 1, 2, 3, \cdots)$

【Perlmutter, 1602.08272】

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Future Directions

- Thermofield Dynamics of SYK models (in progress)
- $\mathcal{N}$=4 SUSY SYK Model

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どうもありがとうございます。

(Thank you very much)