# Emergent Gauge Theory

(Based on work with JiaHui Huang, Minkyoo Kim, Laila Tribelhorn and Jaco Van Zyl)

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# Motivation

AdS/CFT offers a non-perturbative definition of quantum gravity in negatively curved spaces (in terms of large $N$ CFT).

Many deep puzzles arise from the study of black holes - can we answer them using AdS/CFT?

Experience with 1/2 BPS sector suggests that operators dual to black holes have dimension $\sim N^2$

For even the strict large $N$ limit we need to learn how to handle non-planar diagrams, a problem of considerable complexity.

# Motivation

The usual approach to $1/N$ can't be used. Summing only planar diagrams gives a very poor approximation.

Some things we might like to understand:

1. Is there a systematic $1/N$ expansion for correlation functions of operators of dimension of order $N^2$?
2. How does the large $N$ limit of these correlators simplify?

What tools can we develop to answer these questions?

# Summary

We will study small deformations of 1/2 BPS sector: simplest non-trivial example.

Use underlying symmetries in a novel way: group representation theory organizes and sums the complete set of ribbon graphs in both free and (sometimes) interacting CFT.

Exhibit a hidden simplicity that implies something of planar integrability survives in these large $N$ but non-planar limits.

Concretely: decoupled sectors at large $N$ are equivalent to planar $\mathcal{N} = 4$ SYM with coupling $g_{YM}^2$ and gauge group $U(N_{\text{eff}})$

# Approach: Symmetries

Due to enormous number of fields involved summing planar diagrams *does not* give an accurate description of large $N$ dynamics. Not known if there is a class of diagrams that can be summed: sum everything.

Perhaps the sum simplifies when $N \to \infty$?

Idea is to use group theory to use permutation symmetries in the problem in a novel way.

For example, operators constructed from $Z$ and $Y$ complex adjoint scalars are invariant under permutations of the $Z$ and $Y$ fields separately.

We can also describe the most general multi-trace operator constructed using $Z$ and $Y$ fields using a permutation

# Approach: Permutation Language

$$Y^{i_1}_{i_{\sigma(1)}} \cdots Y^{i_m}_{i_{\sigma(m)}} Z^{i_{m+1}}_{i_{\sigma(m+1)}} \cdots Z^{i_{m+n}}_{i_{\sigma(m+n)}}$$

Two operators, corresponding to the permutations $\sigma_1$ and $\sigma_2$ are equivalent if

$$\sigma_1 = \gamma\sigma_2\gamma^{-1} \qquad \gamma \in S_n \times S_m$$

$\Rightarrow$ operators can be identified with elements of the coset

$$S_{n+m}/S_n \times S_m$$

This identification provides efficient ways to count the number of gauge invariant operators and this counting comes out correctly.

# Approach: Representation Language

Taking a Fourier transform on the coset we trade the permutation (defining the operator) for an irrep $R$ of $S_{n+m}$ and an irrep $(r, s)$ of $S_n \times S_m$ (accounts for symmetry we must mod out).

When modding by $S_n \times S_m$ we restrict $S_{n+m}$ to $S_n \times S_m$ and hence $R$ to $(r, s)$. We need multiplicity labels, $\alpha, \beta$.

Restricted Schur basis

$$\chi_{R,(r,s)\alpha\beta}(Z, Y)$$

This description generalizes to include any numbers of adjoint scalars, adjoint fermions, covariant derivatives, field strengths.

# Approach: Results

The language of representations provides a basis for the local gauge invariant operators.

Computation of correlators is reduced to linear algebra: computing two point function = multiplying two projectors and taking a trace.

$$\langle \chi_{R,(r,s)\alpha\beta} \chi^\dagger_{T,(t,u)\gamma\delta} \rangle = \frac{\delta_{RT}\delta_{rt}\delta_{su}\delta_{\alpha\gamma}\delta_{\beta\delta}\mathrm{hooks}_R f_R}{\mathrm{hooks}_r \mathrm{hooks}_s}$$

At one loop level

$$N_{R,(r,s)\mu_1\mu_2;T,(t,u)\nu_1\nu_2} \propto$$
$$\mathrm{Tr}\left(\left[(1,m+1),P_{R,(r,s)\mu_1\mu_2}\right] I_{R'T'}\left[(1,m+1),P_{T,(t,u)\nu_2\nu_1}\right] I_{T'R'}\right)$$

We will see that this approach to the problem gives a useful way to think about the computation, allowing us to exhibit hidden simplicity.

# Problem Statement

The LLM geometries are 1/2 BPS and result from backreaction of a condensate of giant graviton branes. We will study excitations of these geometries, using the CFT. This entails studying correlators of operators with a dimension $\sim N^2$.

The excitations we study are open string excitations of the underlying giant graviton branes. The dynamics of the open strings will give rise to an emergent gauge theory. In how much detail can we explore this emergent theory?

Balasubramanian, Berenstein, Feng and Huang, [hep-th/0411205]

We will argue that the planar limit of the emergent gauge theory is planar $\mathcal{N} = 4$ super Yang-Mills with gauge group $U(N_{\rm eff})$.

# Problem Statement: LLM Geometry

We will study excitations of the LLM geometries.

Lin, Lunin, Maldacena, [hep-th/0409174]

$$
\begin{aligned}
ds^2 &= -y(e^G + e^{-G})(dt + V_i dx^i)^2 + \frac{1}{y(e^G + e^{-G})}(dy^2 + dx^i dx^i) \\
&\quad + ye^G d\Omega_3 + ye^{-G} d\tilde{\Omega}_3
\end{aligned} \qquad (1)
$$

$$
z = \frac{1}{2}\tanh(G) \qquad y\partial_y V_i = \epsilon_{ij}\partial_j z \qquad y(\partial_i V_j - \partial_j V_i) = \epsilon_{ij}\partial_y z \qquad (2)
$$

$$
\partial_i\partial_i z + y\partial_y \frac{\partial_y z}{y} = 0. \qquad (3)
$$

Regularity forces $z = \pm\frac{1}{2}$ on $y = 0$ plane where $(ye^G)(ye^{-G}) = y^2 = 0$.

# Problem Statement: Boundary Condition

$$\partial_i \partial_i z + y \partial_y \frac{\partial_y z}{y} = 0. \qquad (4)$$

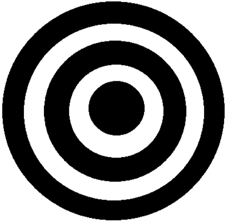


Figure: A coloring of the LLM plane determines $z$. The black regions are sources of five form flux.

The geometries are 1/2 BPS and are the result of backreaction from a condensate of giant graviton branes.

# Problem Statement: Schur Polynomial

The CFT operators dual to the LLM geometries are known - they are given by Schur polynomials $\chi_B(Z)$.

Corley, Jevicki, Ramgoolam, [hep-th/0111222]

Berenstein, [hep-th/0403110]

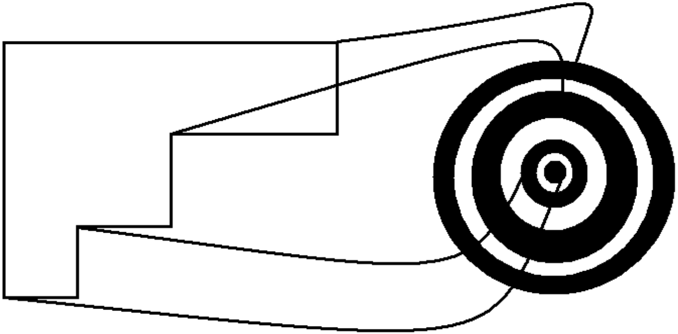
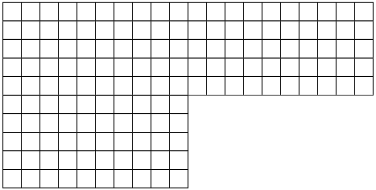
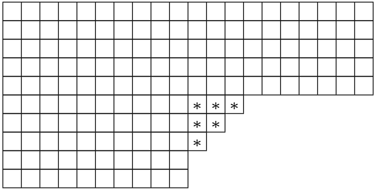


Figure: The coloring of the LLM plane determines both $z$ and $B$.

# Excitations



Excitations are described by adding boxes to $B$.




Add only at one inward pointing corner.

# Schur Polynomial

Schur polynomial:

$$\chi_R(Z) = \frac{1}{n!} \sum_{\sigma \in S_n} \chi_R(\sigma) Z^{i_1}_{i_{\sigma(1)}} \cdots Z^{i_n}_{i_{\sigma(n)}}$$

$$\chi_R(\sigma) = \mathrm{Tr}(\Gamma_R(\sigma)) = \sum_i \langle R, i | \Gamma_R(\sigma) | R, i \rangle$$

Restricted Schur polynomial:

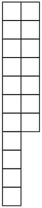
$$\chi_{R,(r,s)\alpha\beta}(Z, Y) = \frac{1}{n!m!} \sum_{\sigma \in S_{n+m}} \chi_{R,(r,s)\alpha\beta}(\sigma) Y^{i_1}_{i_{\sigma(1)}} \cdots Y^{i_m}_{i_{\sigma(m)}} Z^{i_{m+1}}_{i_{\sigma(m+1)}} \cdots Z^{i_{n+m}}_{i_{\sigma(n+m)}}$$

$$\chi_{R,(r,s)\alpha\beta}(\sigma) = \sum_i \langle (r,s)\alpha, i | \Gamma_R(\sigma) | (r,s)\beta, i \rangle$$

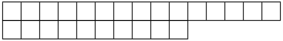
# How to read a Young diagram

Each box is a field (adjoint scalar, fermion, field strength or covariant derivative)

Columns are giant gravitons (expanded into $S^5$; constrained by stringy exclusion principle)



Rows are dual giants (expanded into $AdS_5$; number constrained $\leq N$)



# Strategy

Our goal is to argue that the planar limit of the emergent gauge theories agrees with the planar limit of $\mathcal{N} = 4$ super Yang-Mills theory. We do this in three steps

1. Construct a bijection between operators in planar $\mathcal{N} = 4$ super Yang-Mills and the planar limit of emergent gauge theory.
2. Argue that planar three point functions in planar emergent gauge theory vanish, so OPE coefficients match.
3. Argue that the spectrum of anomalous dimensions matches the spectrum of $\mathcal{N} = 4$ super Yang-Mills.

# Background independence

Classify ingredients of the computation as background independent / dependent.

Background independent: something that takes the same value on any inward pointing corner or even in the absence of a background, i.e. planar limit of original CFT.

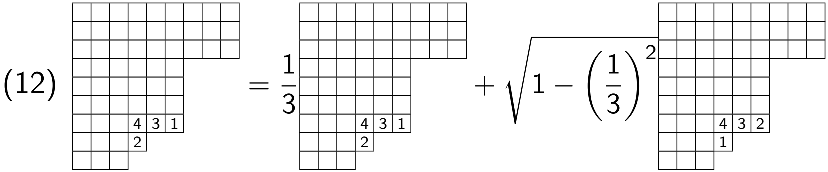
Take the same value regardless of which collection of branes are excited, hence the name “background independent”.

Background dependent: does depend on the collection of branes we excite.

# Background independent

$$R = \ydiagram{3,1}$$

$$(12)\ \ytableaushort{431,2} = \frac{1}{3}\ \ytableaushort{431,2} + \sqrt{1-\left(\frac{1}{3}\right)^2}\ \ytableaushort{432,1}$$

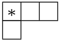



Action of the symmetry subgroup; projection operators; intertwining operators,.... = any symmetric group data.

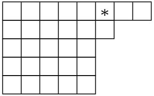
# Background dependent

There are two background dependent quantities that will plays a role:

The factor of a box in row $i$ and column $j$: $(= N - i + j)$



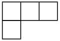
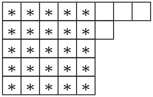

$\rightarrow \quad N$




$\rightarrow \quad N + 5$

All $N$ dependence comes from factors of the excitation.

Ratios of hooks

$R =$  $\qquad +R =$ 


$$\frac{\text{hooks}_{+R}}{\text{hooks}_{+r}} = \alpha^4 \frac{\text{hooks}_R}{\text{hooks}_r}$$

Can absorb $\alpha$ into field normalization - ignore this factor.

# Bijection

Operators in the planar $\mathcal{N} = 4$ super Yang-Mills (dimension $J$ obeys $\frac{J^2}{N} \ll 1$ - this means that $R$ has at most order $\sqrt{N}$ boxes)

$$O_A = \sum_{R,r,s,\alpha,\beta} a^{(A)}_{R,(r,s),\alpha,\beta} \chi_{R,(r,s)\alpha\beta}(Z, Y, X, \cdots)$$

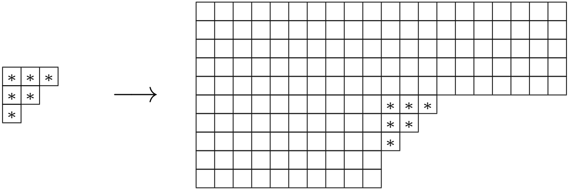
Operator in the planar emergent gauge theory

$$O_{+A} = \sum_{R,r,s,\alpha,\beta} a^{(A)}_{R,(r,s),\alpha,\beta} \chi_{+R,(+r,s)\alpha\beta}(Z, Y, X, \cdots)$$

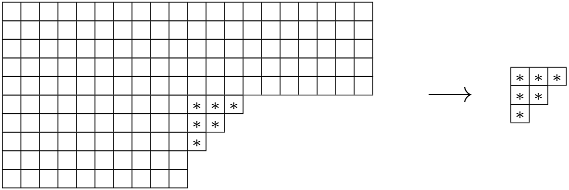
The coefficients appearing in the two sums are the same.

# Bijection

$N = 4 \quad SYM \ (R)$ $\qquad$ $Emergent \quad YM \ (+R)$



$Emergent \quad YM \ (+R)$ $\qquad$ $N = 4 \quad SYM \ (R)$



# Reminder: Correlator

$$\langle \chi_{R,(r,s)\alpha\beta} \chi^\dagger_{T,(t,u)\gamma\delta} \rangle = \frac{\delta_{RT}\delta_{rt}\delta_{su}\delta_{\alpha\gamma}\delta_{\beta\delta}\mathrm{hooks}_R f_R}{\mathrm{hooks}_r \mathrm{hooks}_s}$$

Bhattacharyya, Collins, dMK, [arXiv:0801.2061 [hep-th]]

Free field theory result.

All ribbon graphs were summed. Exact so valid for operators of any dimension.

All $N$ dependence is inside the product of factors $f_R$.

Obtained by taking a product of two projectors and taking a trace.

# Correlators: Free Field Theory

Planar correlator:

$$\langle O_A(x_1) O_B(x_2)^\dagger \rangle = \sum_{R,r,s,\alpha} \frac{a^{(A)}_{R,(r,s),\alpha,\beta} a^{(B)*}_{R,(r,s),\alpha,\beta} \mathrm{hooks}_R f_R}{\mathrm{hooks}_r \mathrm{hooks}_s} \frac{1}{|x_1 - x_2|^{2J}}$$

Emergent gauge theory correlator:

$$\langle \cdots \rangle_B = \frac{\langle \cdots \rangle}{f_B} |x_1 - x_2|^{2|B|}$$

$$\langle O_{+A}(x_1) O_{+B}(x_2)^\dagger \rangle_B = \sum_{R,r,s,\alpha} \frac{a^{(A)}_{R,(r,s),\alpha,\beta}\, a^{(B)*}_{R,(r,s),\alpha,\beta}\, \mathrm{hooks}_{+R}\, f_{+R}}{f_B\, \mathrm{hooks}_{+r} \mathrm{hooks}_s} \frac{1}{|x_1 - x_2|^{2J}}$$

# Correlators: Free Field Theory

From planar correlator:

$$\frac{\mathrm{hooks}_R}{\mathrm{hooks}_r} \times f_R$$

From emergent gauge theory correlator:

$$\frac{\mathrm{hooks}_{+R}}{\mathrm{hooks}_{+r}} \times \frac{f_{+R}}{f_B}$$

Explicit computation shows that

$$\frac{f_{+R}}{f_B} = f_R\big|_{N\to N_{\mathrm{eff}}}$$

$$\frac{\mathrm{hooks}_{+R}}{\mathrm{hooks}_{+r}} = \frac{\mathrm{hooks}_R}{\mathrm{hooks}_r}$$

# Value of $N_{\text{eff}}$

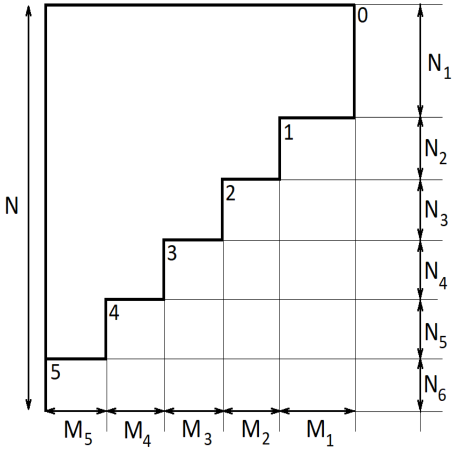



Figure: At corner 2, $N_{\text{eff}} = N - N_1 - N_2 + M_3 + M_4 + M_5$.

# Correlators

$$\langle O_A(x_1) O_B(x_2)^\dagger \rangle = F_{AB}(N) \frac{1}{|x_1 - x_2|^{2J}}$$

$$\langle O_A(x_1) O_B(x_2)^\dagger \rangle_B = F_{AB}(N_{\rm eff}) \frac{1}{|x_1 - x_2|^{2J}} \left(1 + O\left(\frac{1}{N}\right)\right)$$

Planar three point functions of CFT single traces vanish. Planar three point functions of emergent gauge theory single traces vanish. OPE coefficients of both agree in free field theory.

We **CONJECTURE** that this is a property of the full interacting theory.

# Anomalous dimensions

$$DO_{+R,(+r,s)\mu_1\mu_2}(Z,Y) =$$

$$\sum_{T,(t,u)\nu_1\nu_2} N_{+R,(+r,s)\mu_1\mu_2;+T,(+t,u)\nu_1\nu_2} O_{+T,(+t,u)\nu_1\nu_2}(Z,Y)$$

where

$$N_{+R,(+r,s)\mu_1\mu_2;+T,(+t,u)\nu_1\nu_2} = -\frac{g_{YM}^2}{8\pi^2}\sum_{+R'} \frac{c_{+R,+R'} d_{+T} nm}{d_{+R'} d_{+t} d_u (n+m)}$$

$$\times\sqrt{\frac{f_{+T}\mathrm{hooks}_{+T}\mathrm{hooks}_{+r}\mathrm{hooks}_s}{f_{+R}\mathrm{hooks}_{+R}\mathrm{hooks}_{+t}\mathrm{hooks}_u}}$$

$$\times\mathrm{Tr}\left(\left[(1,m+1),P_{+R,(+r,s)\mu_1\mu_2}\right]I_{+R'+T'}\left[(1,m+1),P_{+T,(+t,u)\nu_2\nu_1}\right]I_{+T'+R'}\right)$$

De Comarmond, dMK, Jefferies, [arXiv:1012.3884 [hep-th]]

# Anomalous dimensions

$$c_{+R,+R'}\sqrt{\frac{f_{+T}}{f_{+R}}} = c_{R,R'}\sqrt{\frac{f_T}{f_R}}\Big|_{N\to N_{\rm eff}}$$

$$\frac{d_{+T}\,nm}{d_{+R'}d_{+t}d_u(n+m)}\sqrt{\frac{\mathrm{hooks}_{+T}\mathrm{hooks}_{+r}\mathrm{hooks}_s}{\mathrm{hooks}_{+R}\mathrm{hooks}_{+t}\mathrm{hooks}_u}}d_{+r'}$$

$$= \frac{d_T\,nm}{d_{R'}d_t d_u(n+m)}\sqrt{\frac{\mathrm{hooks}_T\mathrm{hooks}_r\mathrm{hooks}_s}{\mathrm{hooks}_R\mathrm{hooks}_t\mathrm{hooks}_u}}d_{r'}$$

$$\frac{\mathrm{Tr}\left(\left[(1,m+1),P_{+R,(+r,s)\mu_1\mu_2}\right]I_{+R'+T'}\left[(1,m+1),P_{+T,(+t,u)\nu_2\nu_1}\right]I_{+T'+R'}\right)}{d_{+r'}}$$

$$=\frac{\mathrm{Tr}\left(\left[(1,m+1),P_{R,(r,s)\mu_1\mu_2}\right]I_{R'T'}\left[(1,m+1),P_{T,(t,u)\nu_2\nu_1}\right]I_{T'R'}\right)}{d_{r'}}$$

In the end: $N\to N_{\rm eff}$ in planar anomalous dimensions.

# Summary

So far we have managed to do the following three things:

1. Constructed a bijection between operators in planar $\mathcal{N} = 4$ super Yang-Mills and the planar limit of emergent gauge theory.
2. Argued that planar three point functions in FREE planar emergent gauge theory vanish, so OPE coefficients match. Conjectured this is true when interactions are included.
3. Argued that the planar spectrum of anomalous dimensions matches the spectrum of planar $\mathcal{N} = 4$ super Yang-Mills with gauge group $U(N_{\rm eff})$.

Conclusion: The planar limit of the emergent gauge theory is the planar limit of $\mathcal{N} = 4$ super Yang-Mills with gauge group $U(N_{\rm eff})$. It is an integrable system. dMK, Huang, Tribelhorn [arXiv:1806.06586 [hep-th]]

# Decoupling

Conclusion: The planar limit of the emergent gauge theory is the planar limit of $\mathcal{N}=4$ super Yang-Mills with gauge group $U(N_{\rm eff})$. It is an integrable system.

The planar emergent gauge theory is decoupled from the rest of the theory in the sense that starting in the Hilbert space of the emergent gauge theory will not evolve you out of this space.

Interactions between excitations of the planar limit of different emergent gauge theories are $1/N$ suppressed.

Interactions between delocalized excitations and the planar limit of an emergent gauge theory are $1/N$ suppressed.

# Weak coupling tests

Evidence supporting the above result:

1. Explicit computations of anomalous dimensions of the emergent gauge theory, to two loops, agree with $N \to N_{\rm eff}$ rule.
2. Using $su(2|2)$ symmetry two magnon $S$-matrix is determined. Agrees up to two loops with a computation performed in the emergent gauge theory.

dMK, Mathwin, van Zyl, [arXiv:1601.06914 [hep-th]]

# Weak coupling tests

Perform a coordinate space Bethe ansatz: M. Staudacher [hep-th/0412188]

$$\psi_{AB} \equiv \sum_{l_2>l_1} \psi_{AB}(l_1,l_2)\mathrm{Tr}(Z^{l_1}AZ^{l_2-l_1}BZ^{J-l_1-l_2})$$

Solve

$$D\Psi_{AB} = E\Psi_{AB}$$

with $D$ the two loop dilatation operator. Ansatz for wave function

$$\psi_{YX} = e^{i(p_1 l_1 + p_2 l_2)} + Ae^{i(p_2 l_1 + p_1 l_2)} + \delta_{l_2,l_1+1}\phi_{YX}(l_1)$$

The energy $E$ agrees with the planar two loop anomalous dimension after replacing $N \to N_{\rm eff}$. The $S$-matrix element $A$ agrees with the planar result after replacing $N \to N_{\rm eff}$.

# Strong coupling tests

Finite size giant magnon solution to NG in LLM background: $\phi = \sigma - \tau$, $t = \kappa\tau$, $r = r(\sigma)$, with $\phi$ and $r$ coordinates of the bubbling plane. Equation of motion can be integrated once

$$r'(\sigma) = \frac{\kappa r \sqrt{1 - \frac{r^2}{C^2}}}{\sqrt{(1-\kappa)^2 h^4(r) r^2 - (\kappa - (1-\kappa)V_\phi(r))^2}}$$

$C$ is an integration constant.

$$E = \frac{\sqrt{\lambda}}{2\pi}\int_{\sigma_{\rm min}}^{\sigma_{max}} d\sigma \frac{\partial L_{NG}}{\partial \dot{t}} \qquad J = \frac{\sqrt{\lambda}}{2\pi}\int_{\sigma_{\rm min}}^{\sigma_{max}} d\sigma \frac{\partial L_{NG}}{\partial \dot{\phi}},$$

$$E - J = \frac{\sqrt{\lambda}}{\pi} r_0 \sin\left(\frac{p}{2}\right) - 4\frac{\sqrt{\lambda}}{\pi} r_0 \sin^3(\frac{p}{2}) e^{-2\left(\frac{\pi}{\sqrt{\lambda} r_0 \sin(\frac{p}{2})} + 1\right)} + \cdots$$

Net effect $\sqrt{\lambda} \to \sqrt{\lambda} r_0$ which is $N \to N_{\rm eff}$.

dMK, Kim, Van Zyl, arXiv:1802.01367 [hep-th]

# Further strong coupling test

In Kim, Van Zyl, arXiv:1805.12460 [hep-th] semiclassical string solutions that live on white regions of the bubbling plane were constructed and studied.

Solutions are labeled by conserved charges $E$, $J$ and $S$.

Holographically dual to operators in $SL(2)$ sector of $\mathcal{N} = 4$ super-Yang Mills (made of covariant derivatives acting on complex scalar fields Z).

In an appropriate short string limit the $N \to N_{\rm eff}$ rule is again confirmed.

# Beyond the planar limit

The planar limit of the emergent gauge theory matches the planar limit of $\mathcal{N} = 4$ super Yang-Mills theory with gauge group $U(N_{\text{eff}})$.

Beyond the planar limit, these two theories differ: if the emergent gauge theory really was $\mathcal{N} = 4$ with $U(N_{\text{eff}})$ we would expect

1. Giant gravitons with momenta $\leq N_{\text{eff}}$.
2. Dual giant gravitons with an arbitrarily high momentum.

The momenta of giant gravitons is cut off well below $N_{\text{eff}}$. It is cut off by "the size of the corner". The momenta of dual giant gravitons is also cut off by "the size of the corner".

Dual giant gravitons and giant gravitons that have a momentum close to the cut off start to interact with excitations localized at adjacent corners.

# Summary and a Future Direction

We have studied the planar limit of the emergent gauge theory that arises by exciting the branes that backreact to produce the LLM geometry.

Using group representation theory techniques we have managed to learn enough about the dynamics to suggest its matches the planar limit of $\mathcal{N} = 4$ super Yang-Mills with gauge group $U(N_{\text{eff}})$.

Passes weak and strong coupling tests. (two loop anomalous dimensions, two loop $S$-matrix, finite size correction to energy using Nambu-Goto description)

Beyond the planar limit the emergent gauge theory differs from $\mathcal{N} = 4$ super Yang-Mills with gauge group $U(N_{\text{eff}})$.

Next: emergent gauge theory living on the intersection of giant graviton branes.

Thanks for your attention!