Non-Gaussianity in CMB temperature fluctuations from bended cosmic (super-) strings

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JCAP10,003 (2009), arXiv:0811.4698 [astro-ph], + in preparation
**Conventional (Field Theoretic) Cosmic Strings**

are line-like object formed in the early universe through spontaneous symmetry breaking.

Typical scale is the energy per unit length, called “tension” \( \mu \), which is directly related to the phase transition scale \( \mu \sim M_{\text{PT}}^2 \).

\[ \text{Ex) } G\mu_{\text{GUT}} = \frac{M_{\text{GUT}}^2}{M_{\text{pl}}^2} = 10^{-6} \]

It is recently shown that the defects formation at the end of inflation is a generic feature of particle physics motivated scenario.

[Jeannerot, Rocher and Sakellariadou, PRD 68, 103514 (‘03)]
Also, cosmic strings have attracted interested of string cosmology community. The brane inflation model motivated by *string theory* may produce another class of string object, that could be fundamental-, D-string.

\[ \text{COSMIC SUPERSTRINGS} \]

[Sarangi and Tye, PLB 536, 185 (’02), Dvali and Vilenkin, JCAP 0403, 010 (’04), Davis and Kibble (’05), Copeland and Kibble (’09), Majumdar (’08),…]

\[ \text{COSMIC SUPERSTRINGS} \] have properties different from conventional strings. One of the observationally interesting differences is concerning the **INTERCOMMUTING PROBABILITY “P”**.

\[ \begin{align*}
\text{• field theoretic strings} & \quad : \quad P = 1 \\
& \quad \text{[Eto, Hashimoto, Marmorini, Nitta, Ohashi and Vinci, PRL 98,091602 ]} \\
\text{• cosmic superstrings} & \quad : \quad P = 10^{-3} – 10^{-1} \\
& \quad \text{[ Jackson, Jones and Polchinski, JHEP 10,013 (’05) ]}
\end{align*} \]
The COSMIC (SUPER-)STRINGS are smoking gun for STRING COSMOLOGY.

From now on, we focus on the temperature fluctuations due to cosmic strings and investigate the features of cosmic superstrings.

[Ringeval, Sakellariadou and Bouchet, JCAP 0702,023,(2007)]
High precision simulation of CMB map induced by strings

Fraisse, Ringeval, Spergel and Bouchet, PRD78, 043535('08)

ΔT/T [Gμ]

Nambu-Goto simulation

One-point PDF of ΔT/T

Authors recommend

No loop

All structure

string configuration

θres = 0.42'

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The non-Gaussian Universe@YITP
• Non-Gaussianity due to strings

- The one-point probability distribution function (PDF) of $\Delta T/T$ has LARGE NON-GAUSSIANITY!

NG may help us distinguish cosmic string signals from other secondary effects and enhance their observability!

- ✓ Non-Gaussian tails
- ✓ Negative SKEWNESS
• Non-Gaussianity due to strings

**SKEWNESS**

is a measure of the *asymmetry* of the PDF and simplest statistic characterizing the NG.

\[
g_1 = \frac{(\Delta - \bar{\Delta})^3}{\sigma_\Delta^3}, \quad \Delta \equiv \Delta T/T.\]

Komatsu and Spergel, PRD 63, 063002 (‘01)

\[
\sigma_\Delta = 12.4 \text{ G}\mu \quad g_1 = -0.24
\]

Fraisse et al.

<table>
<thead>
<tr>
<th>SKEWNESS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gaussian best fit</td>
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<tr>
<td>One-point PDF of $\Delta T/T$</td>
</tr>
<tr>
<td>Negative</td>
</tr>
<tr>
<td>LARGE non-Gaussianity !!!</td>
</tr>
</tbody>
</table>

[cf. Hindmarsh, Ringeval and Suyama, PRD 80, 083501 (2009)

\[|f_{NL}|(k^{-1}\text{correlation length})=10^3 !!!\]
Purpose

➢ To interpret the numerical results with a simple model of string network

➢ To discuss the origin of the non-Gaussianity (NG-tails, skewness)

➢ To study the dependence of the intercommuting probability “P” for cosmic superstrings

➢ To calculate power-(/bi-) spectrum with “P”
Signal from cosmic strings
Signal from cosmic strings

When a photon passes by a moving straight string, it feels a discontinuities of gravitational potential across the string segment.

$$\Delta T_{GKS} \frac{G \mu}{T} (n) = 4\pi \frac{v}{\sqrt{1 - v^2}} \alpha_{seg} G \mu$$

A photon ray is scattered by segments many times ($\tau_{opt} \sim 16 P^{-1/2}$) and total $\Delta T/T$ would behave like a random walk $\rightarrow$ Gaussian distribution.
Signal from cosmic strings

Contributions from kinks

\[
\frac{\Delta T_{\text{kink}}}{T}(n) = -4G\mu\alpha_{\text{kink}} \times \log \left( \frac{\text{impact parameter}}{\text{distance between kinks}} \right)
\]

Since a photon ray has to pass sufficiently nearby a kink to have a large $\Delta T/T$, the differential cross section for the large $\Delta T/T$ becomes small.

→ Kinks would produce a non-Gaussian!
• PDF due to straight segments and kinks

We found

- Gaussian part $\rightarrow$ frequent scattering by long straight segments
- Non-Gaussian tail $\rightarrow$ rare and close encounter with kinks

[ Takahashi, Naruko, Sendouda, DY, Yoo, Sasaki, JCAP10, 003 ('09)]

NOTICE

The obtained PDF is symmetric for negative and positive $\Delta T/T$ and cannot produce the non-zero skewness!
• Signal from “bended” cosmic strings

Scaling Eq. for the RMS velocity

\[
\frac{dv_{\text{rms}}}{dt} = (1 - v_{\text{rms}}^2) \left[ \frac{k}{R} - 2H v_{\text{rms}} \right]
\]

Hubble friction

Curvature acceleration

with

\[
\frac{k}{R} = \frac{1}{a v_{\text{rms}} (1 - v_{\text{rms}}^2)} \left\langle \hat{r} \cdot \frac{1}{\epsilon} \left( \frac{r'}{\epsilon} \right)' \right\rangle.
\]

\[
\frac{1}{a} \left\langle \hat{r} \cdot \frac{1}{\epsilon} \left( \frac{r'}{\epsilon} \right)' \right\rangle \approx \frac{\dot{r}_0 \cdot \ddot{r}_0}{a} \approx 2H v_{\text{rms}}^2 (1 - v_{\text{rms}}^2) \neq 0
\]

✓ The nontrivial correlation between the velocities and the string curvature exists if the Hubble expansion exists.
• Signal from “bended” cosmic strings

If there are the correlations between the velocities and string curvatures, there would be a nonzero expectation value of the asymmetry of the PDF, namely **SKEWNESS**.
Monte Carlo Study for non-Gaussianity of PDF
• Setup for MC

✓ Bended segments are assumed to be located randomly between the LSS and the present time in consistent with the VOS model.

✓ consider the next-leading order of $\sigma$:

$$X^\perp(\sigma) = \delta + \left(\frac{dX^\perp}{d\sigma_{1c}}\right)_0 \sigma + \frac{1}{2} \left(\frac{d^2X^\perp}{d\sigma_{1c}^2}\right)_0 \sigma^2$$

Higher order terms are treated as perturbations from the uniformly moving straight segments.

✓ The correlations are taken into account:

1. The correlation between the velocities and the string curvature

$$\dot{r}_0 \cdot \ddot{r}_0 = 2aHv_{\text{rms}}^2 \left(1 - v_{\text{rms}}^2\right)$$

2. Light-cone effect

$$X(\sigma) = X_0 - \left[r'_0 + \left(\frac{d\eta_{1c}}{d\sigma}\right)_0 \dot{r}_0\right] \sigma - \left(\frac{d^2\eta_{1c}}{d\sigma^2}\right)_0 \dot{r}_0 \sigma^2 + O(\sigma^3)$$

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• PDF of $\Delta T/T$ due to “bended” cosmic strings

\[ P=1 \]
\[ \sigma=14.0 \, G\mu \]
\[ g_1=-0.136 \]

Kinks produce the NG tails.

The correlations between velocities and string curvatures produce the asymmetry, namely the skewness.

Good agreement with numerical simulation!
• PDF of $\Delta T/T$ due to “bended” cosmic super-strings

[ DY, Sendouda, Yoo, Takahashi, Naruko, Sasaki, in prep.]

$P=0.1$

\[
\sigma = 31.3 \, G\mu
\]

$g_1 = -0.036$
• PDF of $\Delta T/T$ due to “bended” cosmic super-strings

As $P$ decreases, the standard deviation $\sigma$ increases and the contribution of the non-Gaussianity are suppressed!

$P=0.01$

$\sigma=59.0 \ G\mu$

$g_1=-0.021$
• Dependence of P

For the sufficient small P, the skewness is < (a few) \times 10^{-2}.

\[ g_1(P) \approx 0.11P^2 - 0.23P - 0.02 \]

\[ \sigma = \begin{cases} 14.0 \mu \text{G} \\ -0.136 \end{cases} \quad \text{for } P = 1 \]

\[ \sigma = \begin{cases} 17.7 \mu \text{G} \\ -0.105 \end{cases} \quad \text{for } P = 0.5 \]

\[ \sigma = \begin{cases} 31.3 \mu \text{G} \\ -0.036 \end{cases} \quad \text{for } P = 0.1 \]

\[ \sigma = (\text{a few}) \times 10^{-2} \quad \text{for } P \text{ small} \]

[DY, Sendouda, Yoo, Takahashi, Naruko, Sasaki, in prep.]
1-point PDF has the **negative skewness** with the reasonable assumptions for the correlation between velocities and string curvatures.

As $P$ decreases, the standard deviation $\sigma$ increases and the contribution of the non-Gaussianity, namely the NG tails and the skewness, are suppressed!

For the sufficiently small $P$, our results suggest that the skewness is $< (a \ few) \times 10^{-2}$!
Power-(/bi-) spectrum from cosmic superstrings
• Segment approach

➢ To compute the angular power spectrum due to cosmic (super-)strings analytically, we use "segment formalism", which corresponds to "halo formalism" for the SZ effect.


For small scale, the Fourier component of $\Delta T/T$ can be decomposed into each contribution of each string segment;

$$a_{CS}(l, \{\Theta_i, \, z_i\}) = \sum_{i \in \text{segments}} a_i(l, \Theta_i; \, z_i)$$

\[
\begin{align*}
\text{"i" : index characterizing each string segment} \\
\text{"}\Theta_i\text{" : parameter vector which represents the configuration of the } i\text{-th string}
\end{align*}
\]

\[
a(l, \Theta; \, z) = \int d^2x^\perp \frac{\Delta T}{T}(x^\perp, \Theta; \, z)e^{-i l \cdot x^\perp}
\]

in flat sky approximation
Then, the angular power spectrum of the total $\Delta T/T$ can be reduced to

$$C_{CS}(\ell) \approx \int \frac{d\phi_\ell}{2\pi} \left\langle a_{CS}(1, \{\Theta_i, z_i\}) a_{CS}(1, \{\Theta_j, z_j\})^* \right\rangle$$

$$\equiv C_{CS}^{1\,\text{seg}}(\ell) + C_{CS}^{2\,\text{seg}}(\ell),$$

Can be neglected

Assuming that the segment description is valid, $a_i$ and $a_j$ are exactly independent!

Contribution of the 1-segment correlation ("Poisson" term)

Contribution of the segment-segment correlation

Number density of segment with parameter $\Theta$

with

$$G(\ell, \Theta; z) = \int \frac{d\phi_\ell}{2\pi} \left| a(1, \Theta; z) \right|^2$$
• Power spectrum of $\Delta T/T$ due to cosmic superstrings

We consider exactly straight string segments.

For $l < (\text{correlation length at LSS})^{-1}$, $l^2C_l$ has a plateau.

As $P$ decrease, the amplitude increase.

For large $l$, the $l^2C_l$ behaves as $l^{-1}$.

$C_l \propto l^{-1.0}$

$C_l \propto l^{-0.89}$

$\propto l^{-1}$

$P=10^{-4}$

$P=10^{-3}$

$P=10^{-2}$

$P=10^{-1}$

$P=1$

[ DY, Takahashi, Sendouda, Yoo, in prep. ]

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• Primary vs cosmic superstrings as secondary

As $P$ decreases, the amplitude from CS increases and the upper bound of $G\mu$ must be smaller!!!
• Summary

- We gave the “new” method to calculate the power spectrum of $\Delta T/T$ due to cosmic (super-)strings {	extit{analytically}}, so-called “segment formalism”.

- For the straight segments, the small scale angular power spectrums, in general, behave as $l^{-1}$ for large $l$ and have a plateau for small $l$.

- While for the exact straight segments the bispectrum is found to be zero, the bispectrum would have nonzero value if the correlations between the velocities and string curvatures are taken into account. (in progress)
THANK YOU!