Non Abelian Vortices and Aharonov-Bohm Effect in Dense QCD

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$$\Delta_{a\alpha} \sim \epsilon_{abc}\epsilon^{\alpha\beta\gamma}\langle q_b^{\beta} \mathcal{C} q_c^{\gamma}\rangle$$

Color Index

Flavor Index

Charge Conjugation

$$U_c \, \Delta \, U_F^{\dagger} e^{i\theta_{\rm B}}$$

Z_3 Z_3

$SU(3)_c$ $SU(3)_F$ $U(1)_{\rm B}$

$$G = \frac{U(1)_B \times SU(3)_C \times SU(3)_F}{Z_3 \times Z_3}$$

Δ

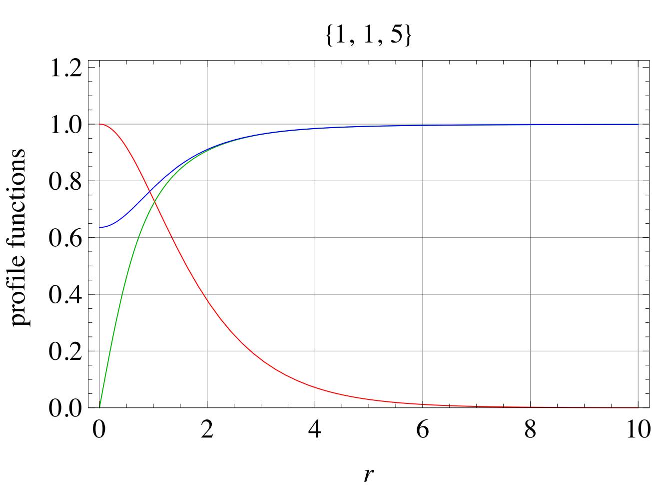
$$H = \frac{SU(3)_{C+F}}{Z_3}$$

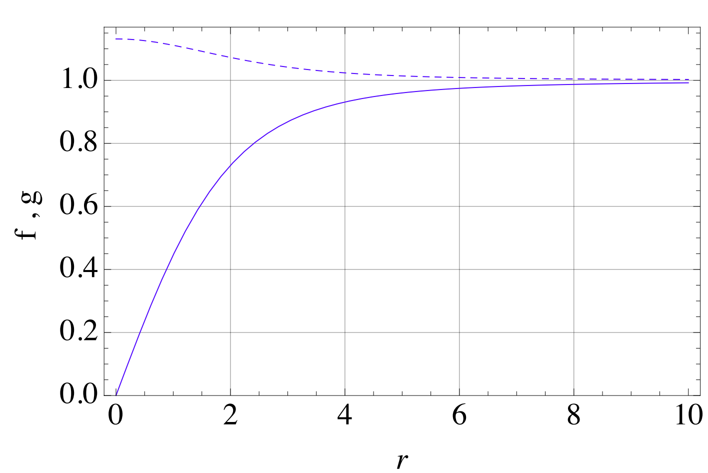
$$OPS = \frac{SU(3)\times U(1)}{Z_3} = U(3)$$

$$\Pi_1(OPS) = Z$$

$$\Phi(r,\theta) = \Delta_{\rm CFL} \begin{pmatrix} e^{i\theta} f(r) & 0 & 0 \\ 0 & g(r) & 0 \\ 0 & 0 & g(r) \end{pmatrix}, \qquad A_i(r) = -\frac{1}{3g_s}\frac{\epsilon_{ij}x_j}{r^2}A(r)\begin{pmatrix} 2 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}. \quad A(0)=0, \qquad A(R)=1.$$

$$f(0) = g'(0) = 0, \;\; f(R) = g(R) = 1$$





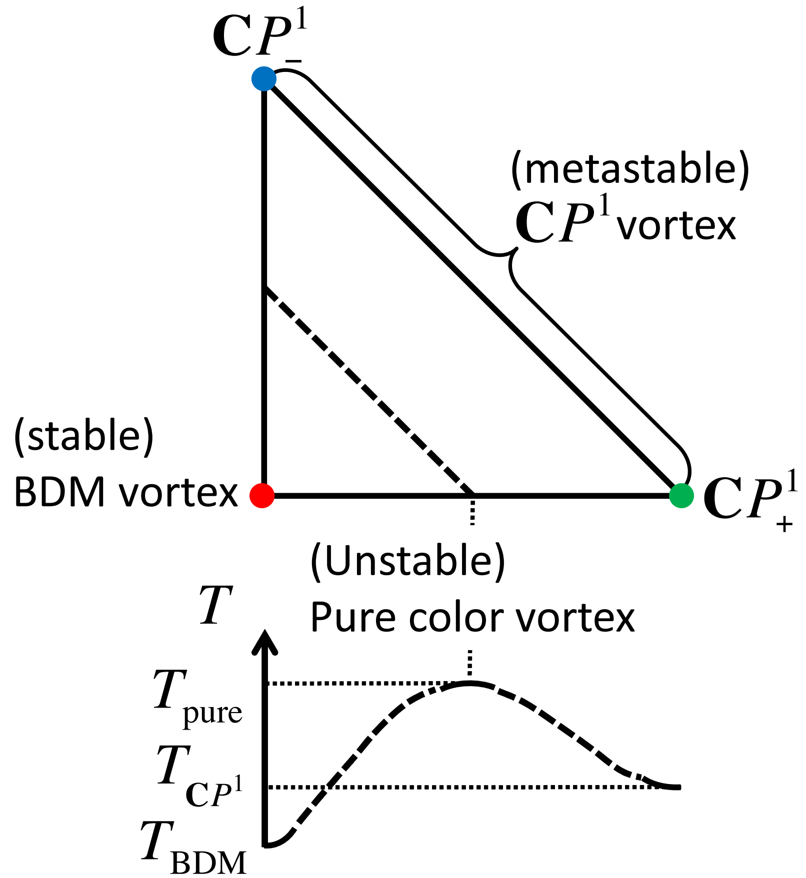
$$SU(3)_{C+F} \xrightarrow{\Delta_{\rm CFL}} SU(2) \times U(1)$$

$$CP^2 \text{ Moduli}$$

Electro Magnetic Interaction

$$D_\mu\Phi = \partial_\mu\Phi - ig_s A^a_\mu \tau^a \Phi - ieB_\mu \Phi Q, \tau^a \in SU(3)_C$$

$$D_\mu\Phi_{diag} = [\partial_\mu - i(gA^a_\mu H^a - eQ_{em}B_\mu)]\Phi_{diag}, H^a \in \{\tau^3, \tau^8\}$$



$$Q_{em} = \frac{1}{3}\begin{pmatrix} 2 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

$U(1)_{\rm em}$ generator

$$A^M_\mu = \frac{g_s}{g_M}A^8_\mu - \frac{\eta e}{g_M}B_\mu, \quad A^q_\mu = \frac{\eta e}{g_M}A^8_\mu + \frac{g_s}{g_M}B_\mu$$

Massive gauge field

Mass less Gauge field

$\eta = \frac{2}{\sqrt{3}}, \; g_M^2 = g_s^2 + \eta^2 e^2$

$$B_\mu = \frac{g_s}{g_M}A^q_\mu - \frac{\eta e}{g_M}A^M_\mu \qquad q_{eff} = \frac{qg_s}{\sqrt{g_s^2 + \eta^2 e^2}}$$

Vortex Configurations:

BDM Vortex:

$$\Phi(r,\theta) = \Delta_{\rm CFL}\begin{pmatrix} e^{i\theta}f(r) & 0 & 0 \\ 0 & g(r) & 0 \\ 0 & 0 & g(r) \end{pmatrix}, A^{\rm M}_i(r)\mathrm{T}^8 = -\frac{\epsilon_{ij}x_j}{3g_{\rm M}r^2}[1-h(r)]\begin{pmatrix} 2 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}.$$

[Balachandran,et al. 2005]

CP¹ Vortex:

$$\Phi(r,\theta) = \Delta_{\rm CFL}\begin{pmatrix} g(r) & 0 & 0 \\ 0 & e^{i\theta}f(r) & 0 \\ 0 & 0 & g(r) \end{pmatrix}, A^{\rm M}_i(r)\mathrm{T}^8 = \frac{1}{6g_{\rm M}}\frac{\epsilon_{\rm ijx_j}}{\rm r^2}[1-\mathrm{h(r)}]\begin{pmatrix} 2 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}, \quad A^3_i(r)\mathrm{T}^3 = -\frac{1}{2g_{\rm s}}\frac{\epsilon_{\rm ijx_j}}{\rm r^2}[1-\mathrm{h(r)}]\begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}.$$

[Nitta,et al. 2012]

Aharonov-Bohm Scattering

$$(\not\partial - iq\not B_\mu + iM_{e/\mu_e})\,\psi_{e/\mu_e} = 0$$

Electric Charge

$$\frac{d\sigma}{d\vartheta} = \frac{\sin^2(\pi\varphi)}{2\pi k \sin^2(\frac{\vartheta}{2})}, \varphi = \frac{q}{2\pi} \times \text{Flux.}$$

Scattering Angle

$$\varphi = \left| \frac{q}{2\pi} \oint B_\mu \cdot dl \right| = \left| \frac{\eta\, e\, q}{2\pi g_{\rm M}} \oint A^{\rm M} \cdot dl \right|$$

$$\varphi^{\rm BDM}_{e/\mu_e} = \frac{2e^2}{3g_s^2 + 2e^2} \qquad \varphi^{\mathbb{C}P^1}_{e/\mu_e} = \frac{e^2}{3g_s^2 + 2e^2}. \qquad \varphi^{\rm BDM}_{\Sigma} = \frac{2e^2}{3g_s^2 + 2e^2}, \varphi^{\mathbb{C}P^1}_{\Sigma} = \frac{e^2}{3g_s^2 + 2e^2} \qquad \Sigma^{AB}_{\rm cfl} = \Phi^{\dagger\,{\rm L}}_{Aa}\Phi^{\rm R}_{aB}$$

$$\Sigma^{AB}_{\rm cfl} \sim \epsilon^{ACD}\epsilon^{BEF}\bar{q}_{\rm L(a}{}^{C}\bar{q}^{D}_{\rm Lb)}q_{\rm R(a}{}^{E}q_{\rm Rb)}{}^{F} \qquad \Sigma'_{\rm cfl} = e^{ieQ\alpha}\Sigma_{\rm cfl}e^{-ieQ\alpha}$$

$$\begin{pmatrix} 0 & \Sigma^{1\,+}_{\rm cfl} & \Sigma^{2\,+}_{\rm cfl} \\ \Sigma^{1\,-}_{\rm cfl} & 0 & 0 \\ \Sigma^{2\,-}_{\rm cfl} & 0 & 0 \end{pmatrix}$$

Relaxation rate for particles scattering off vortices:

Vortex density: $n_v \simeq 1.9 \times 10^{19} \left(\frac{1\,ms}{P_{rot}}\right)\left(\frac{\mu/3}{300MeV}\right)\left(\frac{R}{10km}\right)^2$

$$s_0 = \frac{1}{\tau_v} = \frac{n_v}{k_F}\sin^2\pi\phi$$

Relaxation rate for Coulomb interaction:

$$\frac{1}{\tau_q} = \frac{6\zeta(3)}{\pi^2}\tilde{\alpha}T$$

The temperature T_F below which flux tubes dominate the relaxation of deviations from thermal equilibrium is determined as

$$T_f = \frac{\pi^2}{6\zeta(3)}\frac{\sin^2(\pi\phi)}{\tilde{\alpha}}\frac{n_v}{k_F},$$

Thank you