

# Compact stars and GWs: a progress report from Frankfurt

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*Nuclear Physics, Compact Stars,  
and Compact Star Mergers 2016*  
*NPCSM 2016, Oct.17-Nov.18, 2016, YITP, Kyoto, Japan*



# Plan of the talk

- \* Anatomy of GW signal: frequencies and EOS  
*see Takami's poster*
- \* Role of B-fields and EM counterparts  
*see Nissanke's and Zhang's talks*
- \* FRBs and "blitzars"  
*see Piran's, Sekiguchi's, and Tanaka's talks*
- \* Eccentric encounters and nucleosynthesis
- \* A new approach to relativistic hydrodynamics



# The two-body problem in GR

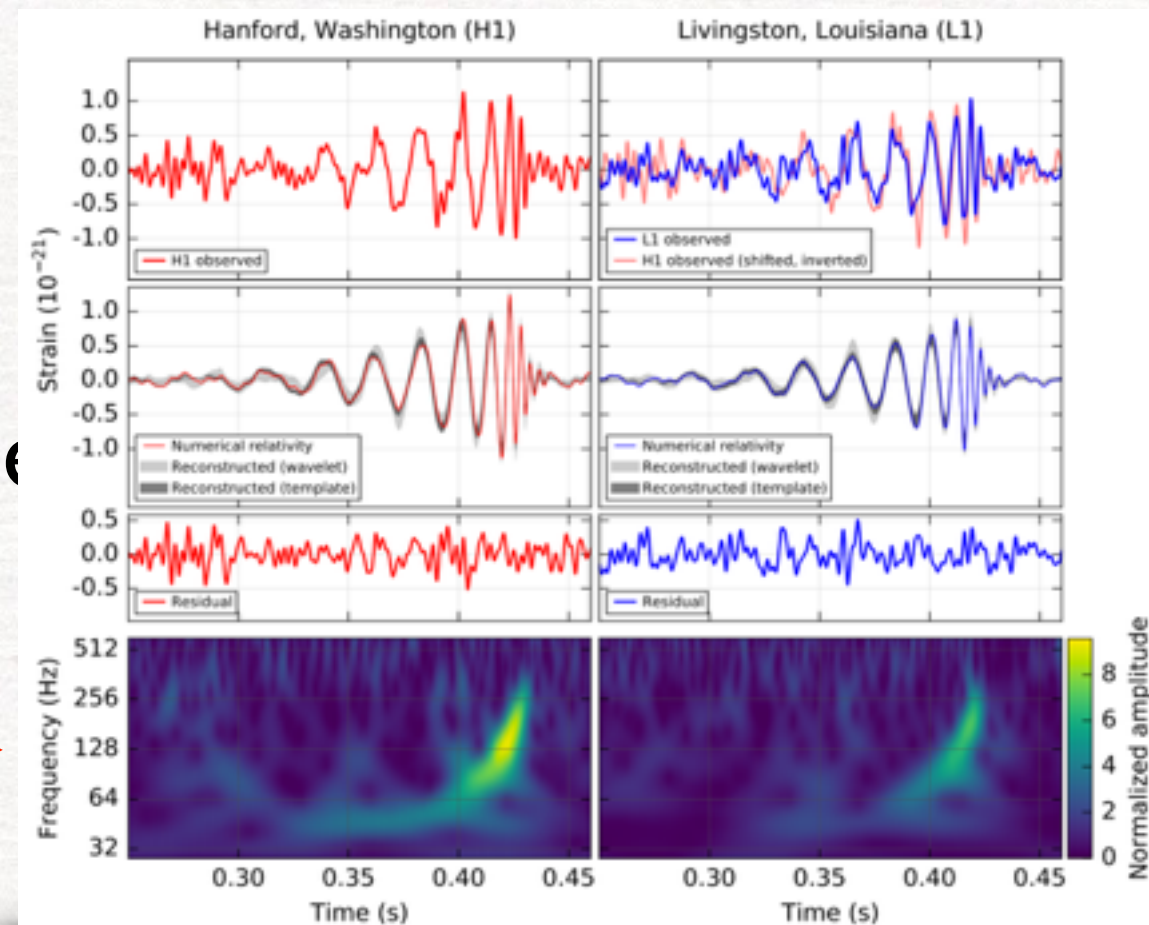
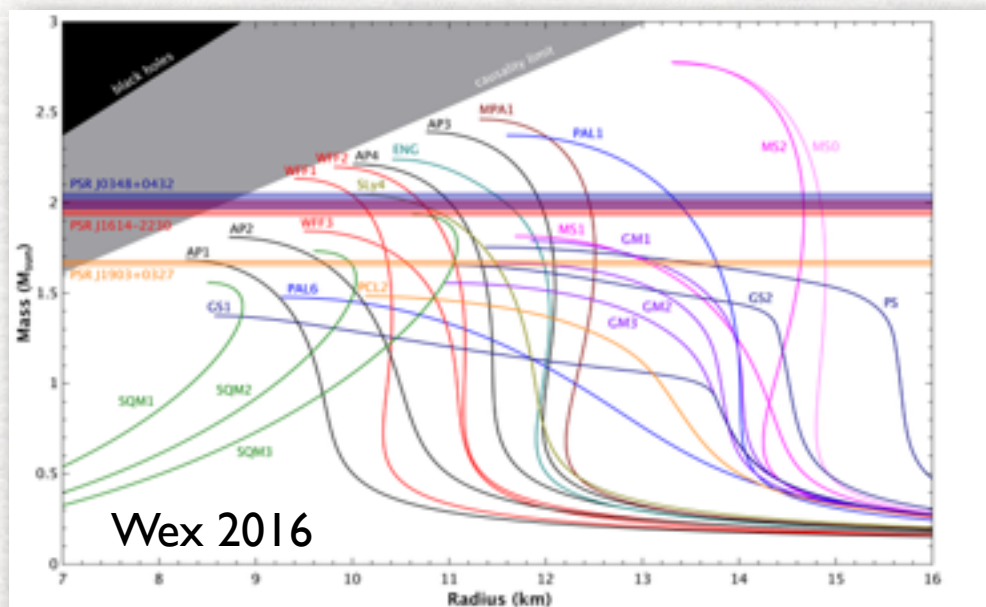
- For BHs we know what to **expect**:

$$\text{BH} + \text{BH} \longrightarrow \text{BH} + \text{GWs}$$

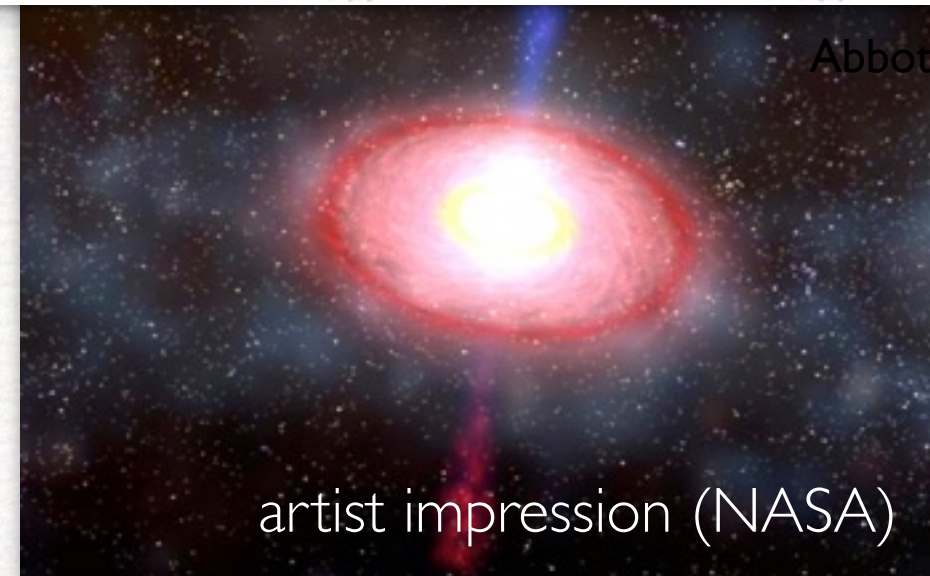
- For NSs the question is more **subtle**: hyper-massive neutron star (HMNS),

$$\text{NS} + \text{NS} \longrightarrow \text{HMNS} + \dots ? \longrightarrow$$

- **HMNS** phase can provide clear information on **EOS**



Abbott+ 2016



- **BH+torus** system may tell us on the central engine of **GRBs**



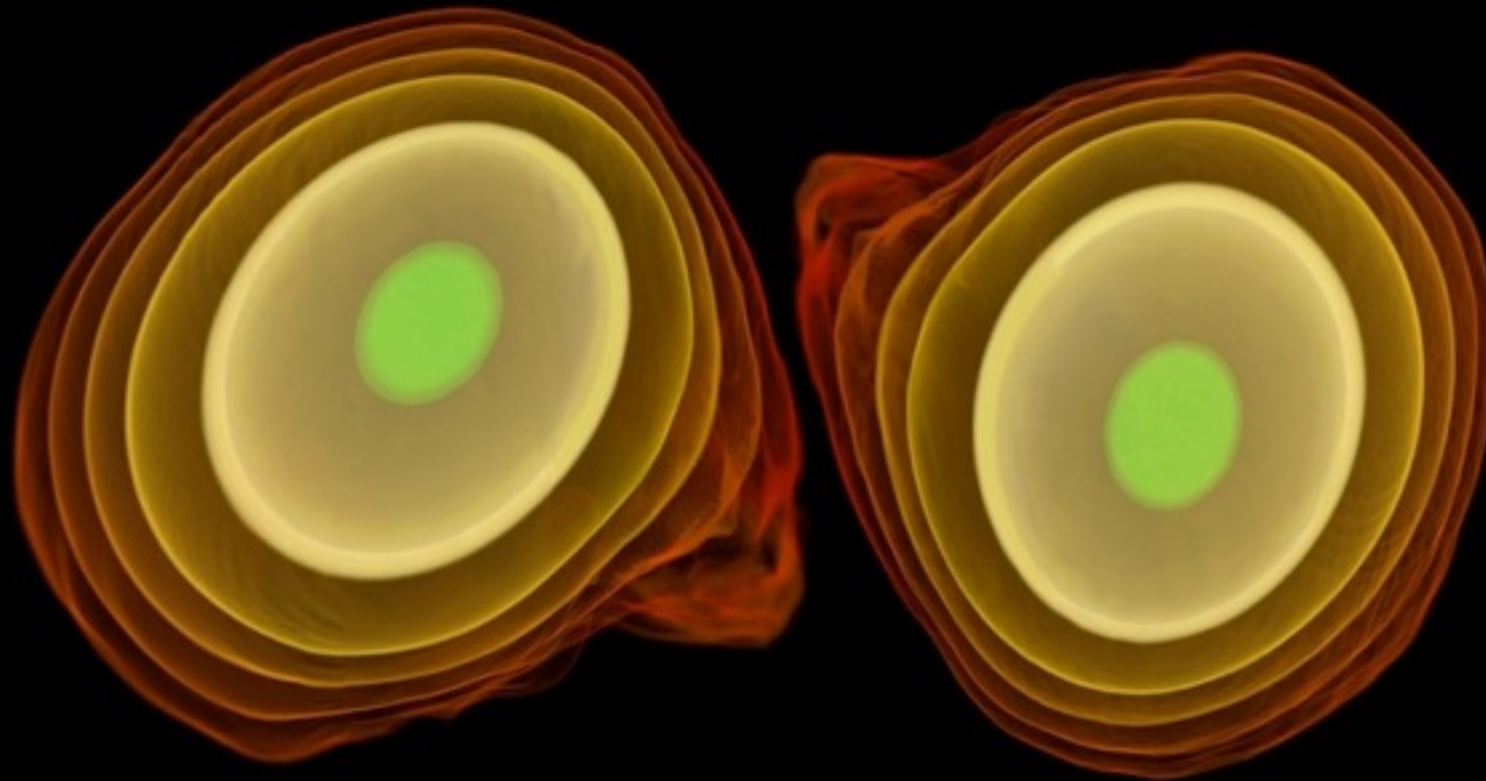


merger → HMNS → BH + torus

Quantitative differences are produced by:

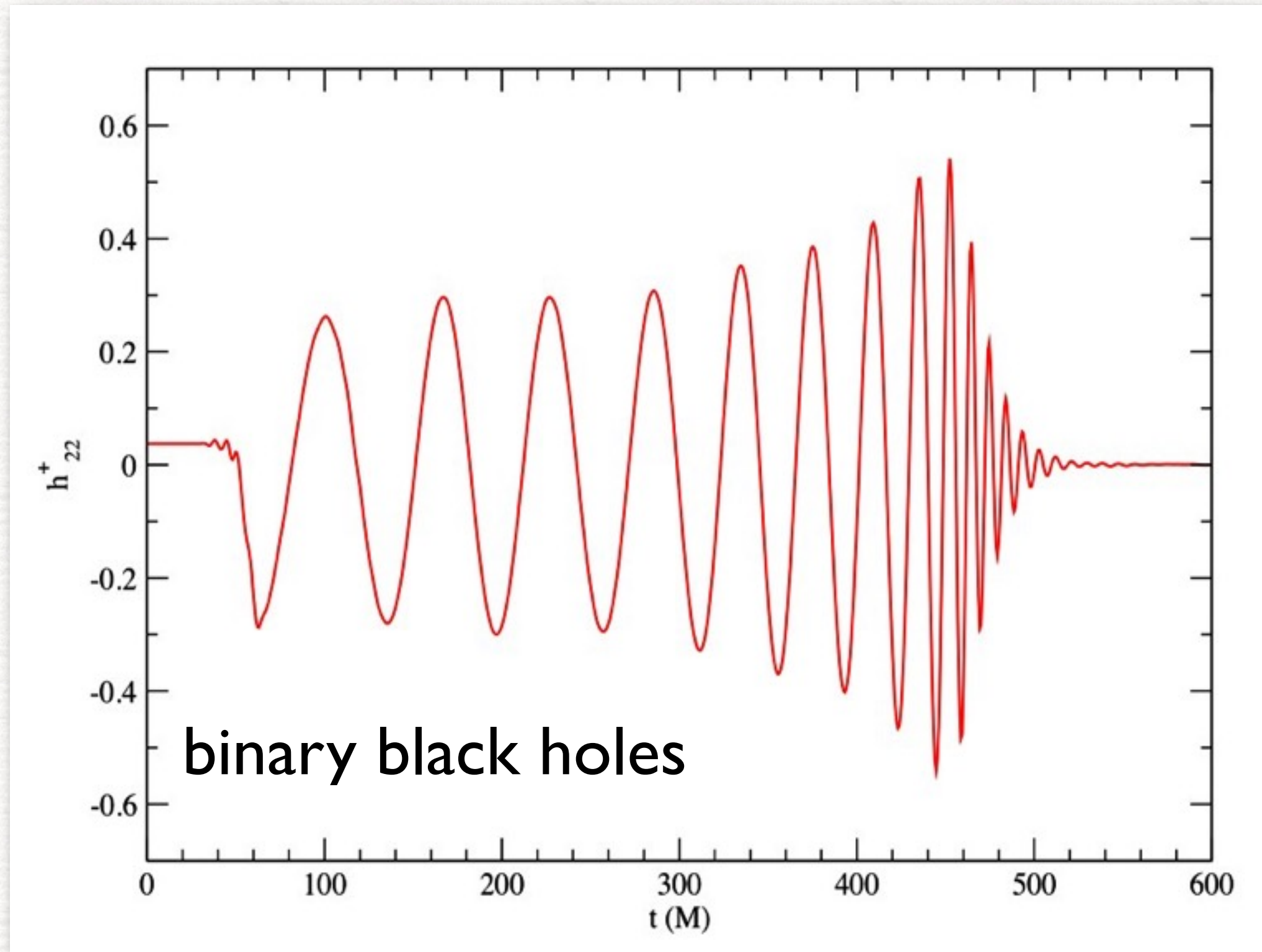
- total **mass** (prompt vs delayed collapse)
- mass **asymmetries** (HMNS and torus)
- soft/stiff **EOS** (inspiral and post-merger)
- **magnetic fields** (equil. and EM emission)
- **radiative** losses (equil. and nucleosynthesis)

# How to constrain the EOS

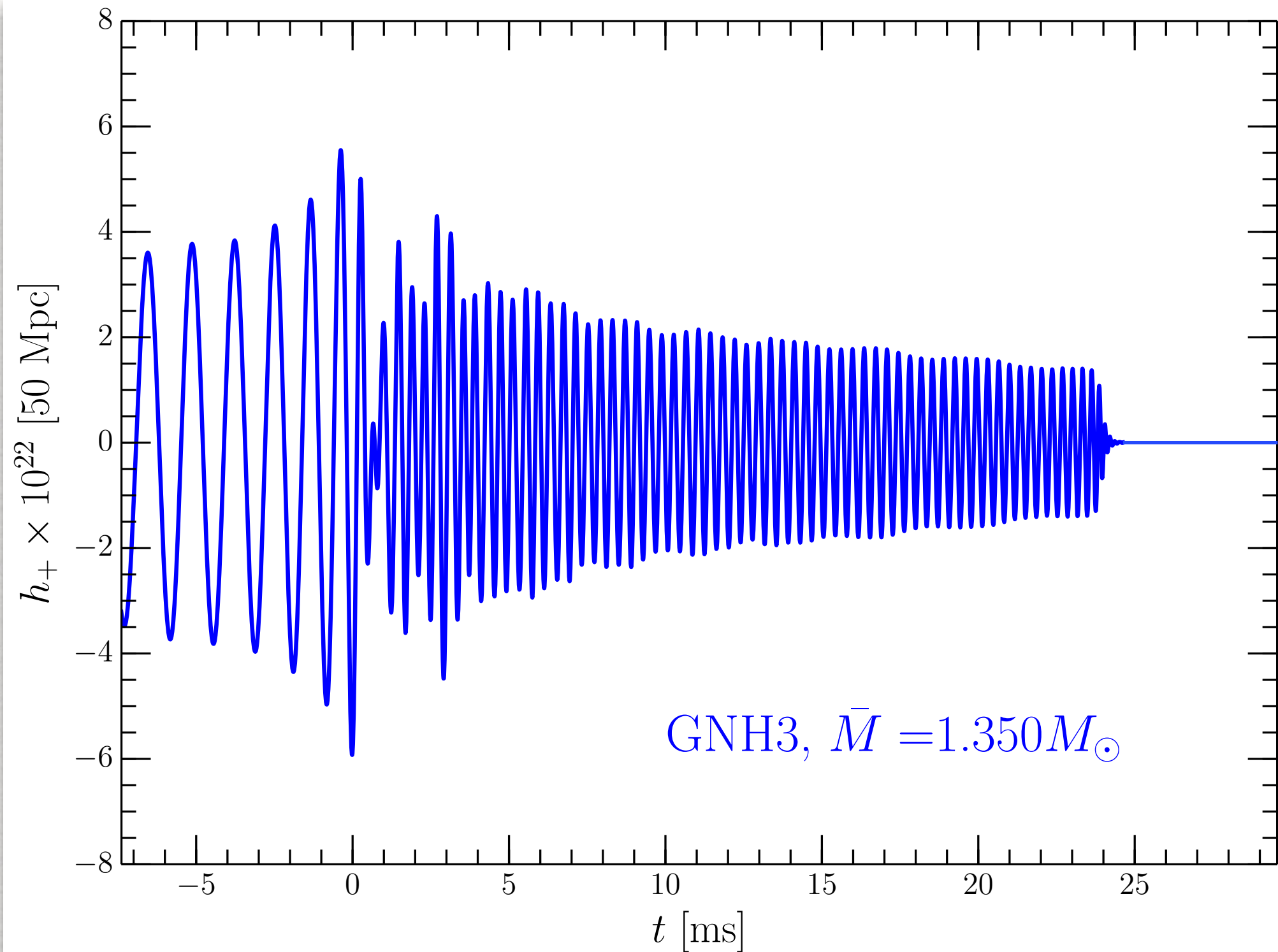




# Anatomy of the GW signal

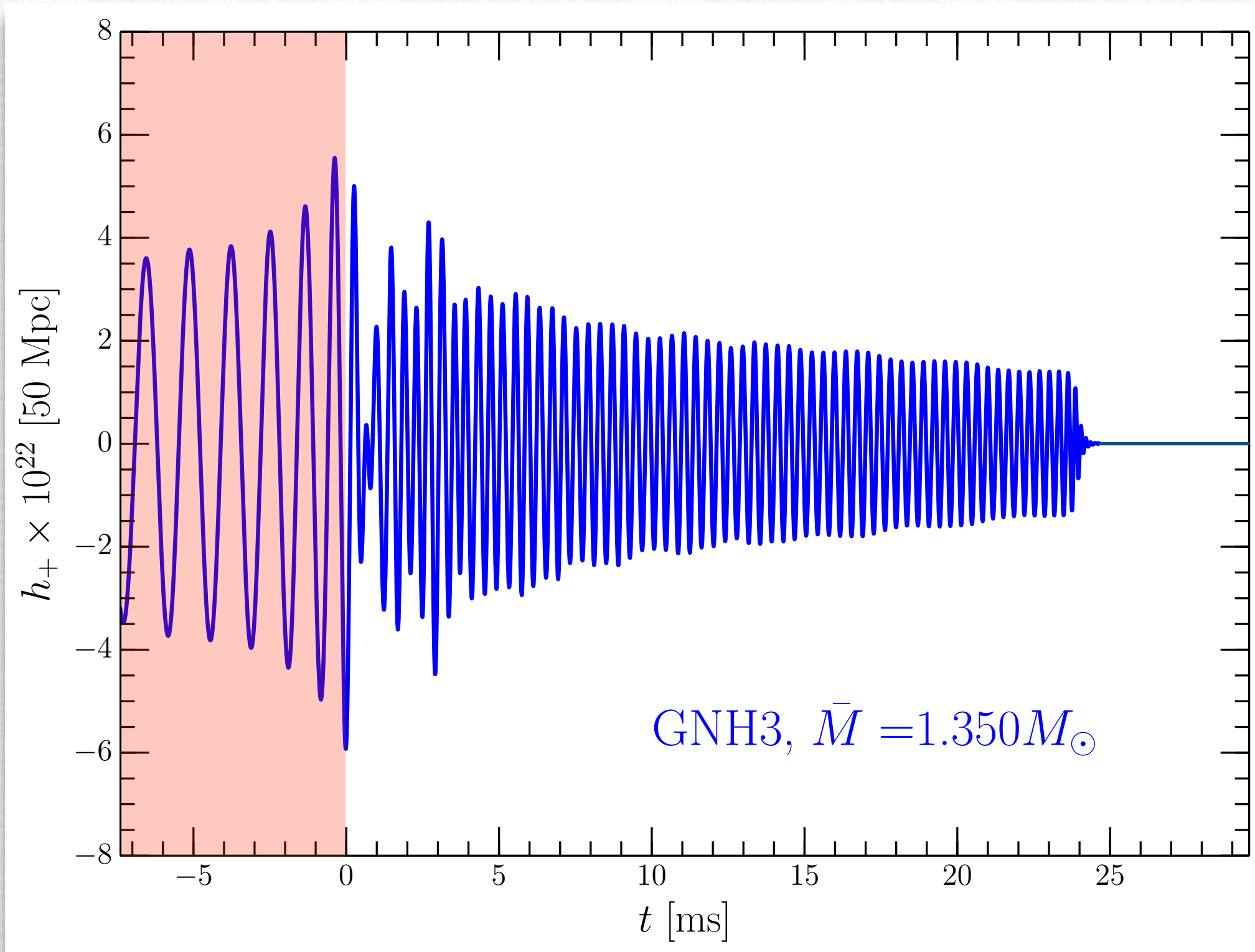


# Anatomy of the GW signal



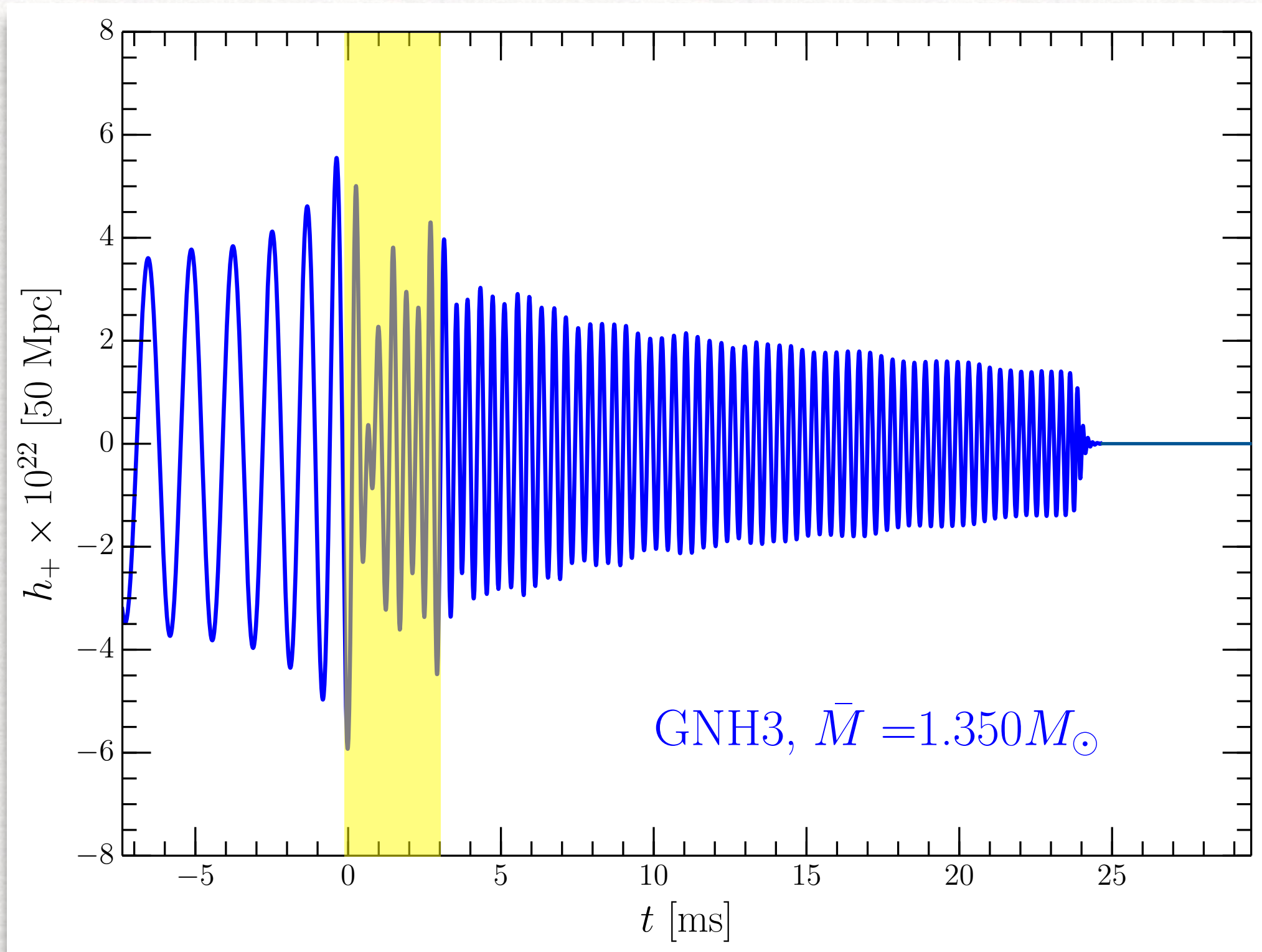


# Anatomy of the GW signal



**Inspiral:** well approximated by PN/EOB; tidal effects important

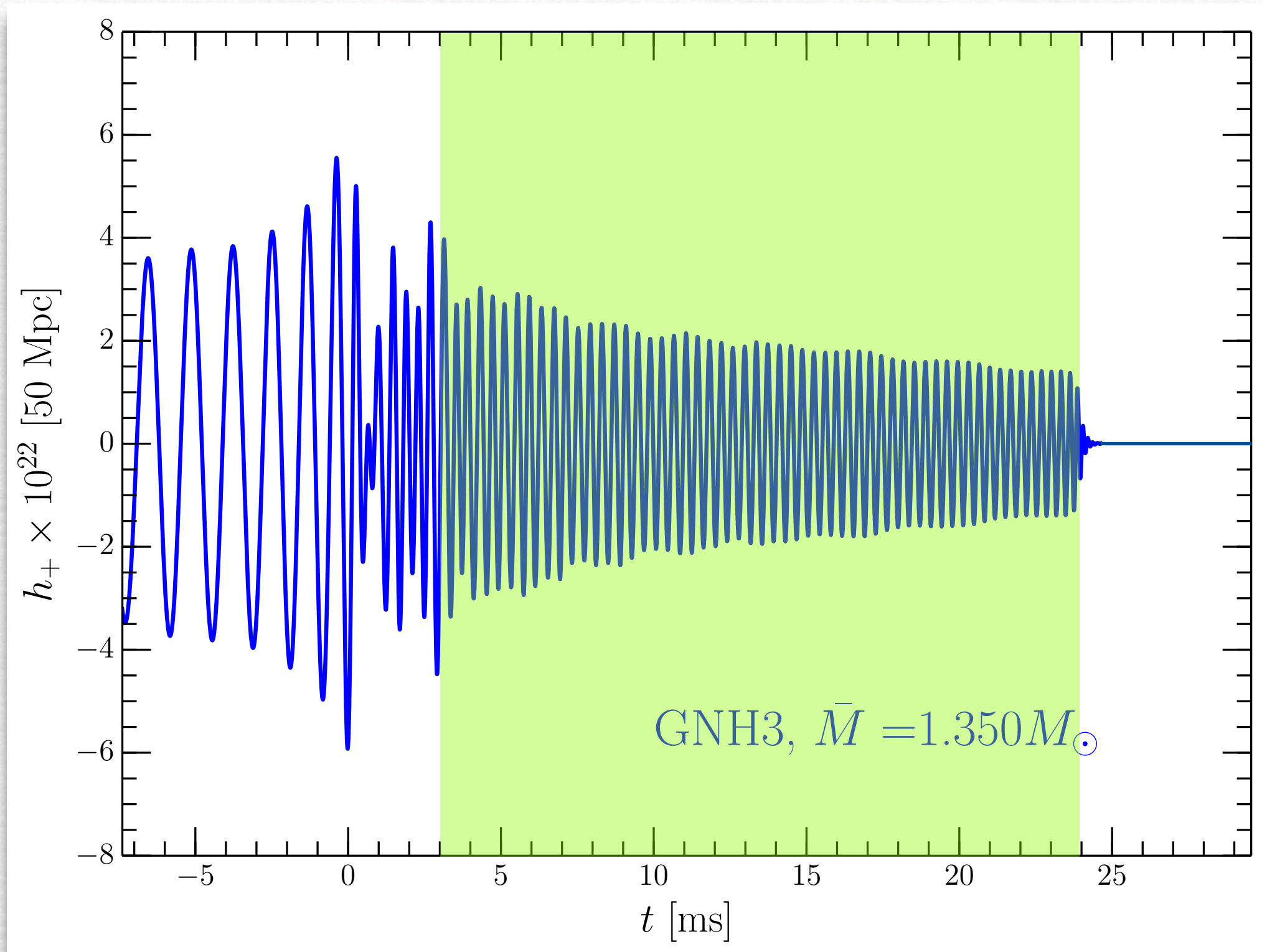
# Anatomy of the GW signal



**Merger:** highly nonlinear but analytic description possible

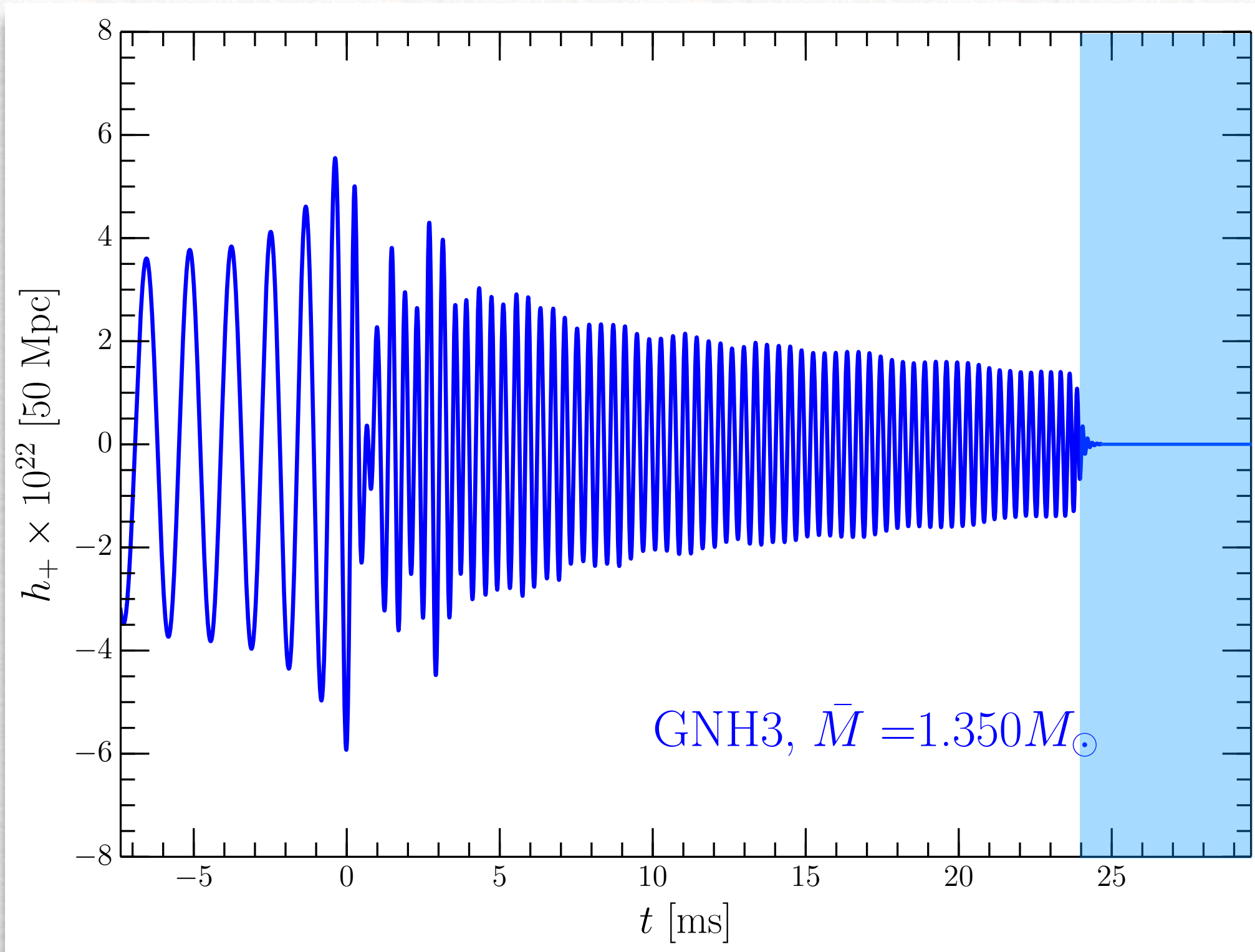


# Anatomy of the GW signal



**post-merger:** quasi-periodic emission of bar-deformed HMNS

# Anatomy of the GW signal

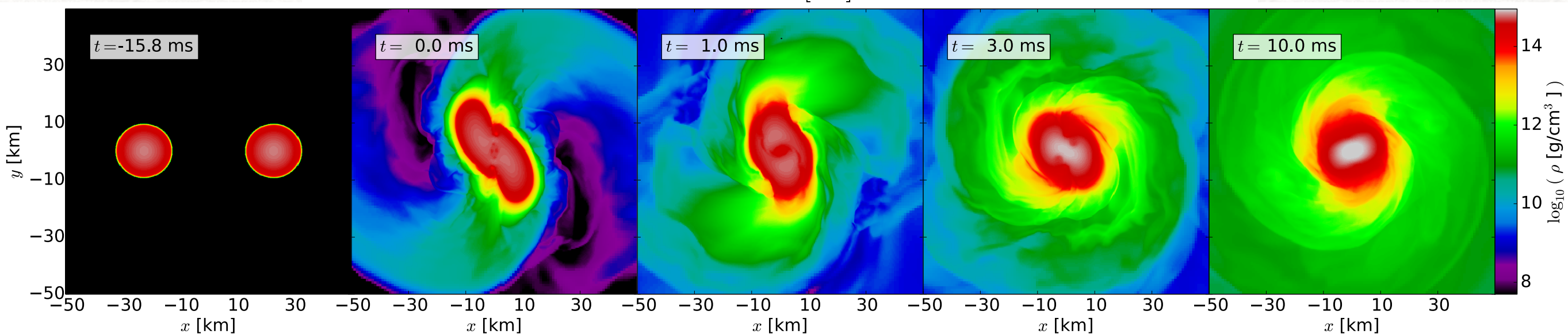
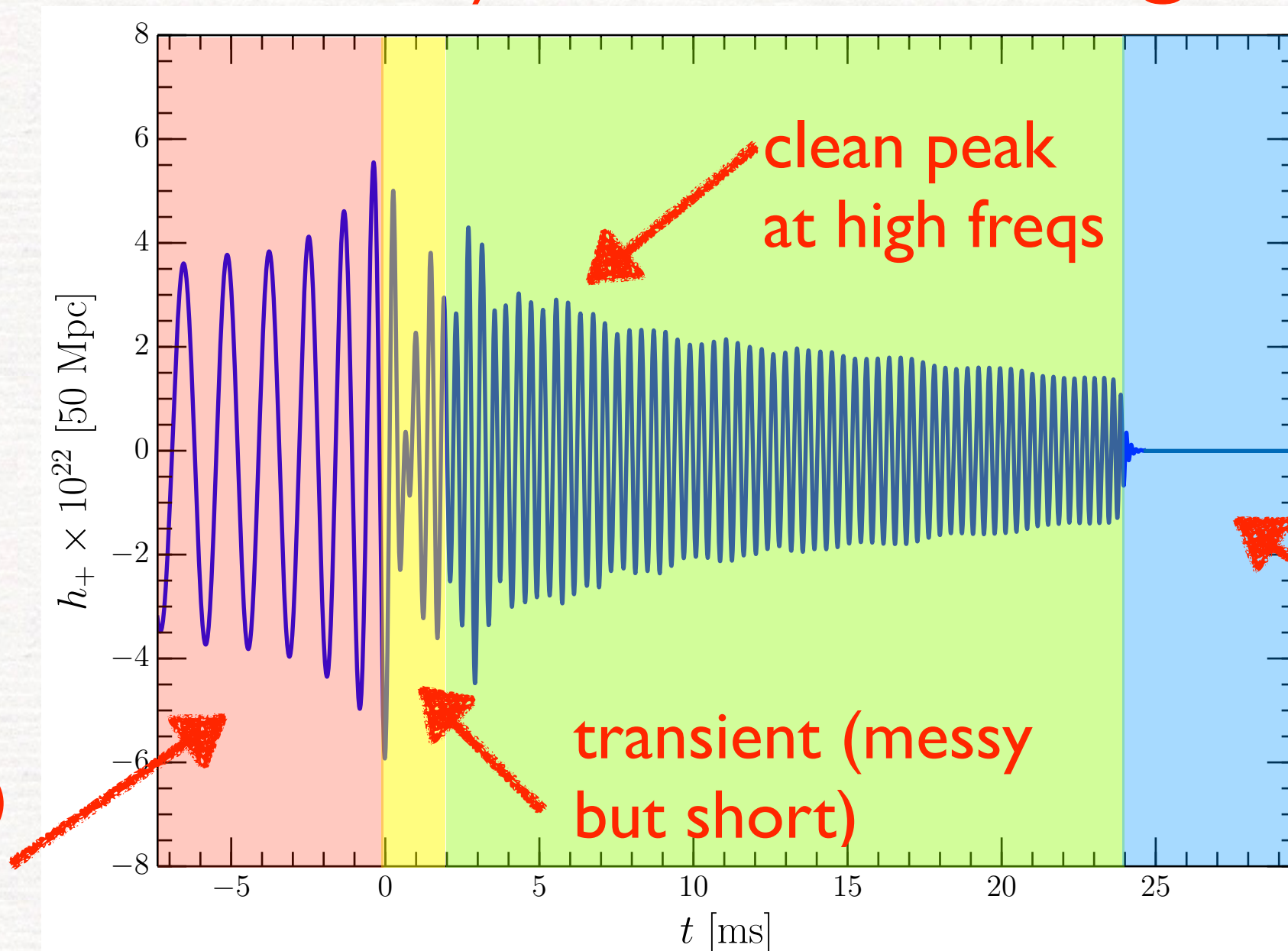


**Collapse-ringdown:** signal essentially shuts off.

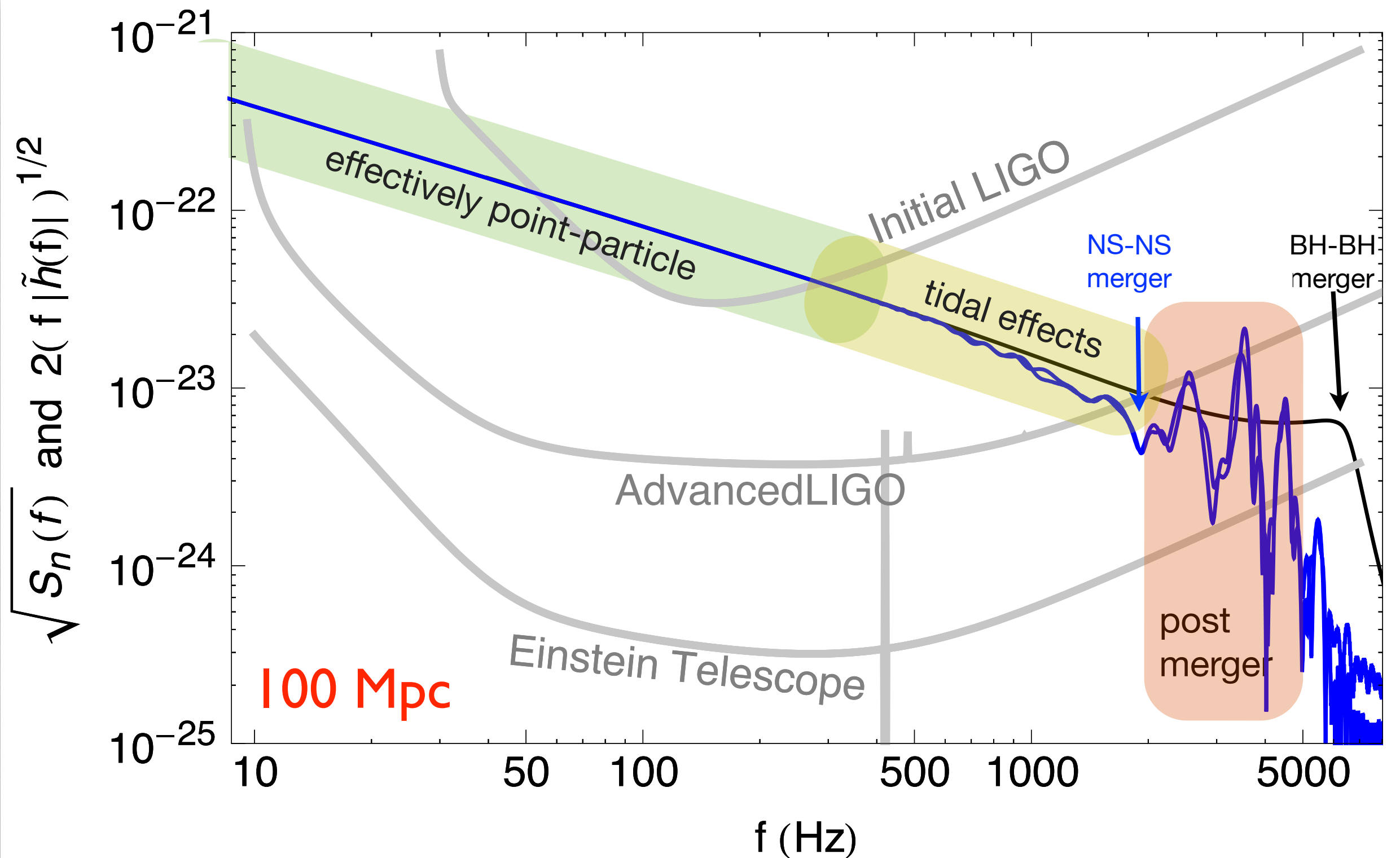


# Anatomy of the GW signal

Chirp signal  
(track from  
low to high  
frequencies)



# In frequency space

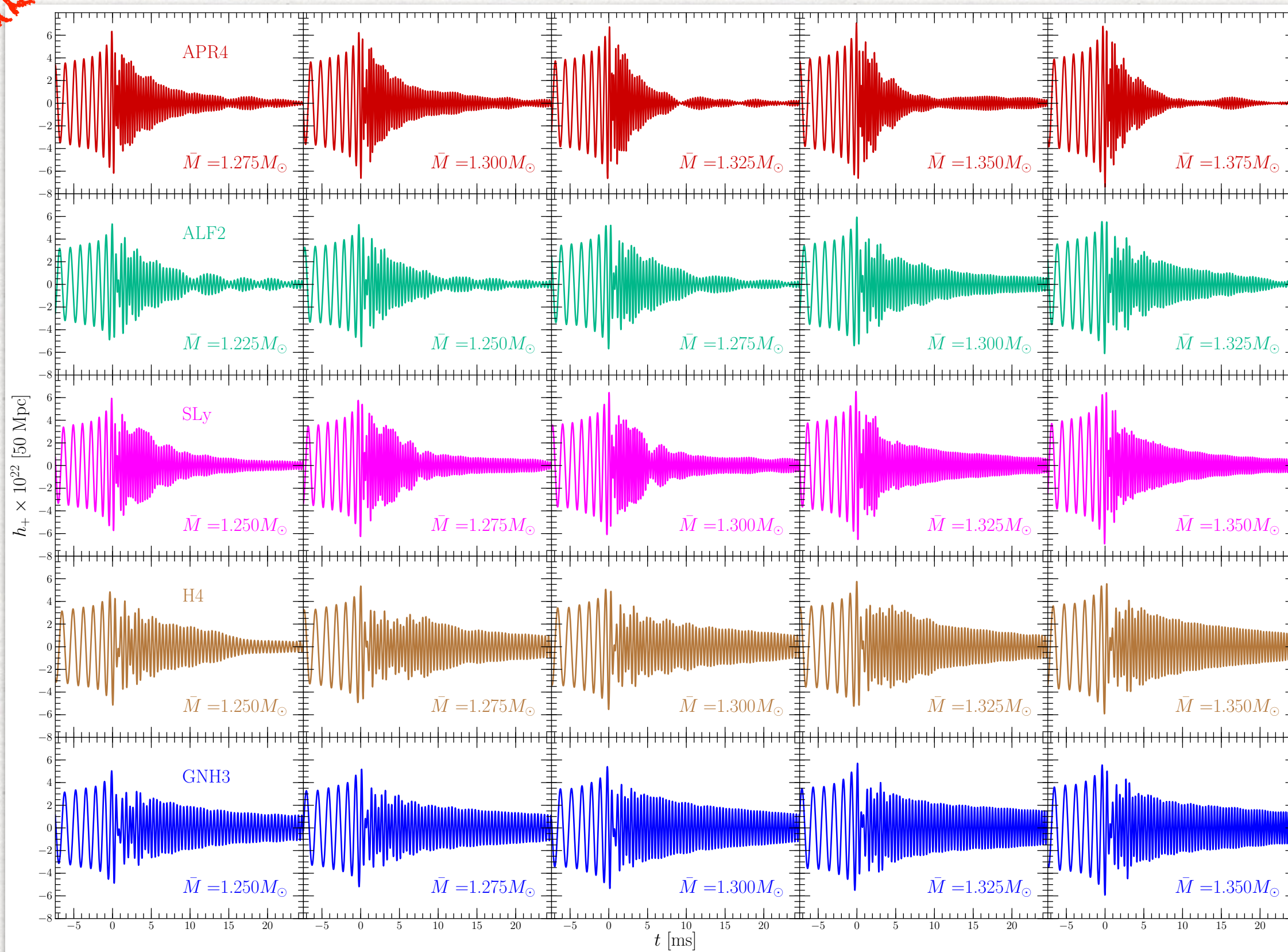




see Takami's poster

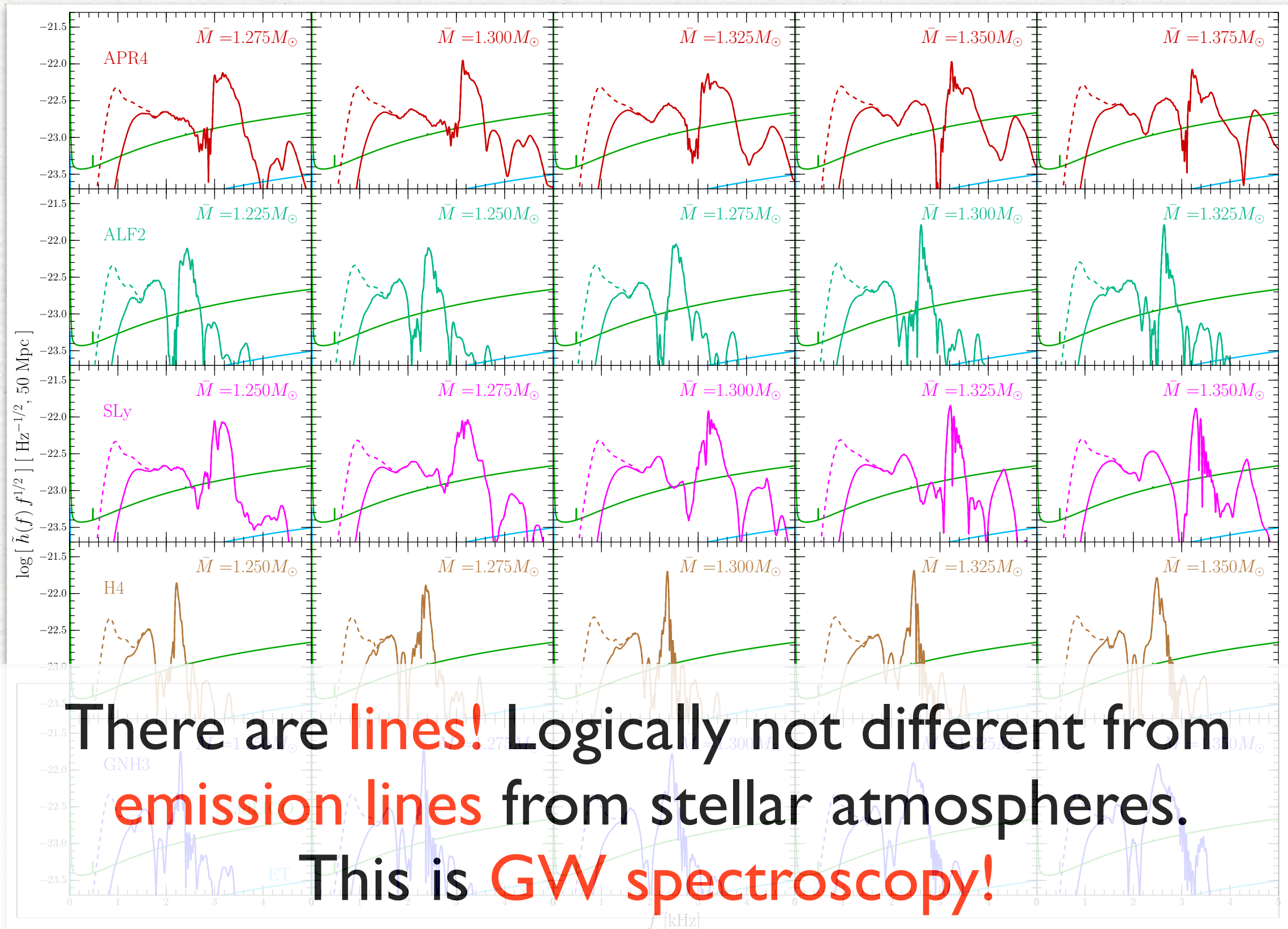
# Extracting information from EOS

Takami, LR, Baiotti (2014, 2015), LR+ (2016)



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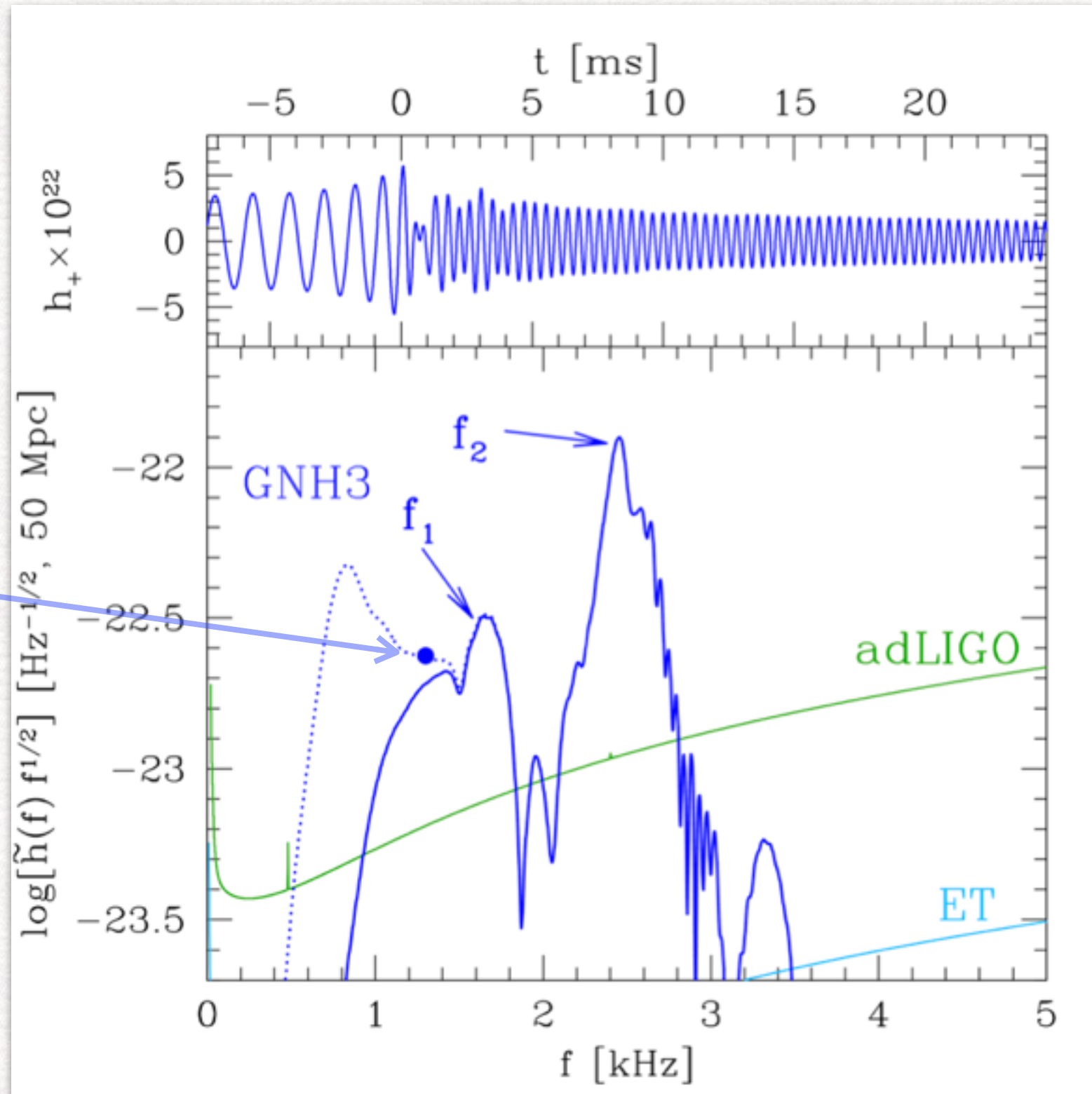




# A new approach to constrain the EOS

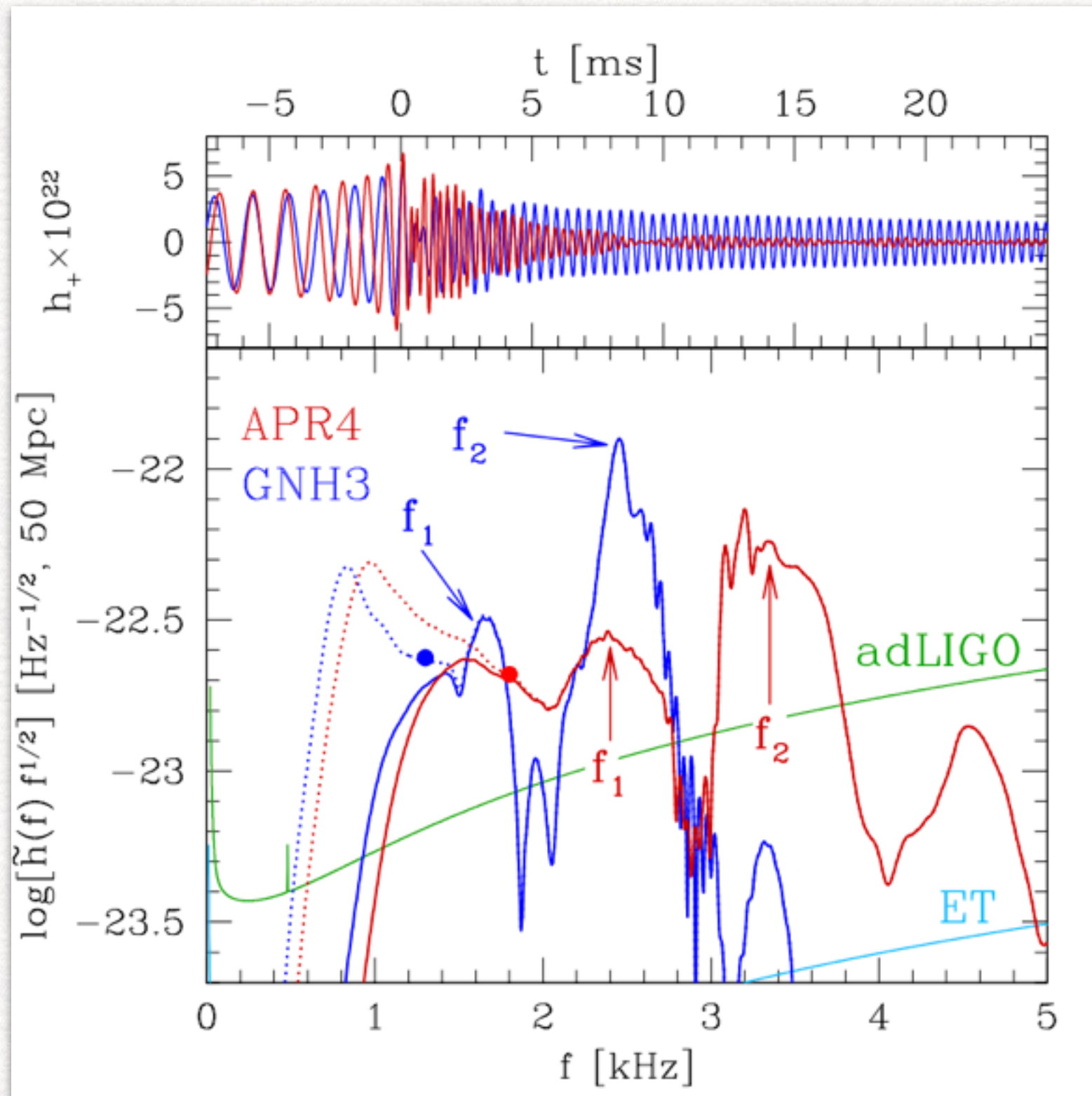
Oechslin+2007, Baiotti+2008, Bauswein+ 2011, 2012, Stergioulas+ 2011, Hotokezaka+ 2013, Takami 2014, 2015, Bernuzzi 2014, 2015, Bauswein+ 2015, LR+2016...

merger  
frequency



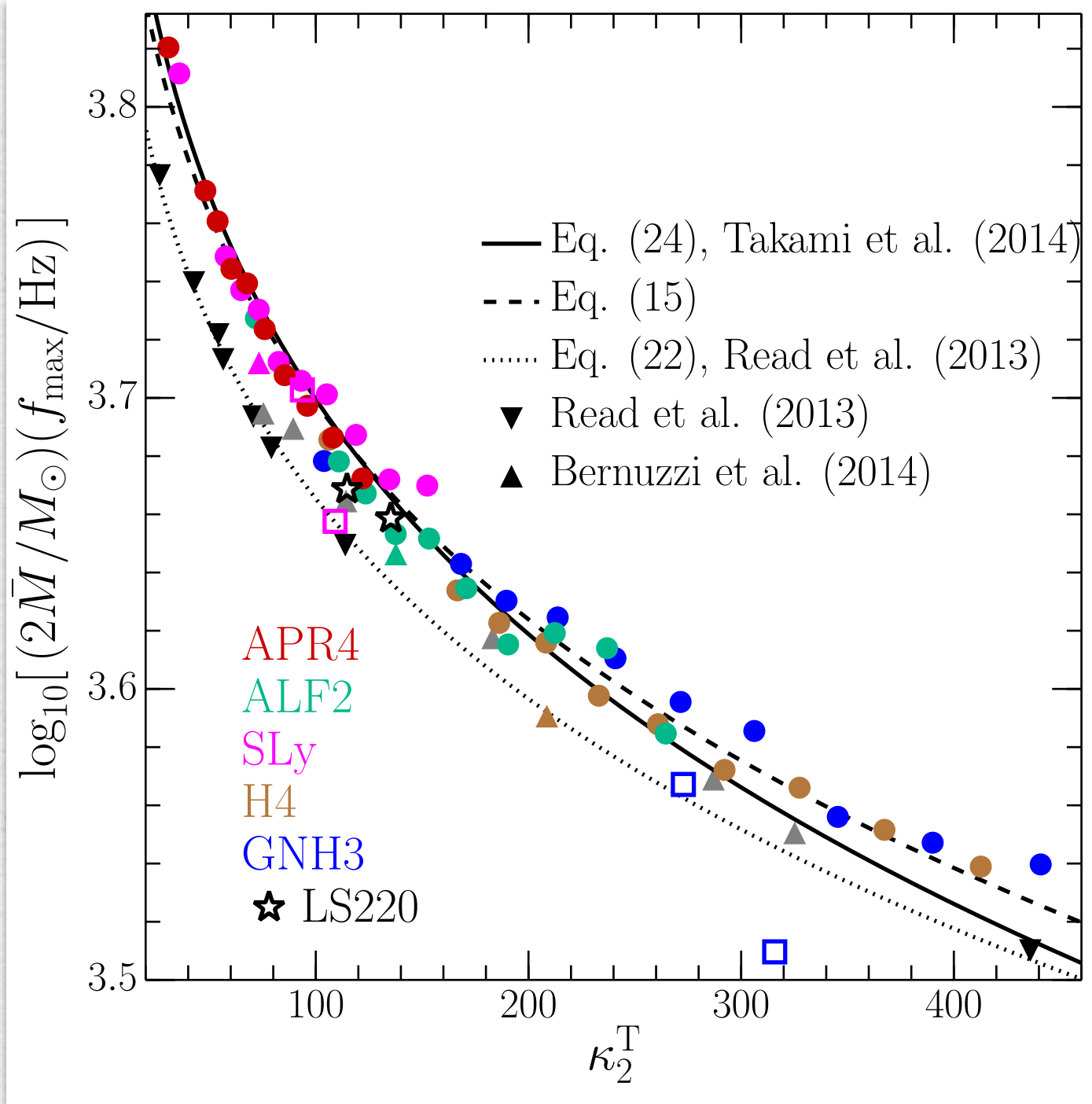
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# Quasi-universal behaviour: inspiral



“surprising” result: **quasi-universal** behaviour of GW frequency at amplitude peak (Read+2013)

Many other simulations have confirmed this (Bernuzzi+, 2014, Takami+, 2015, LR+2016).

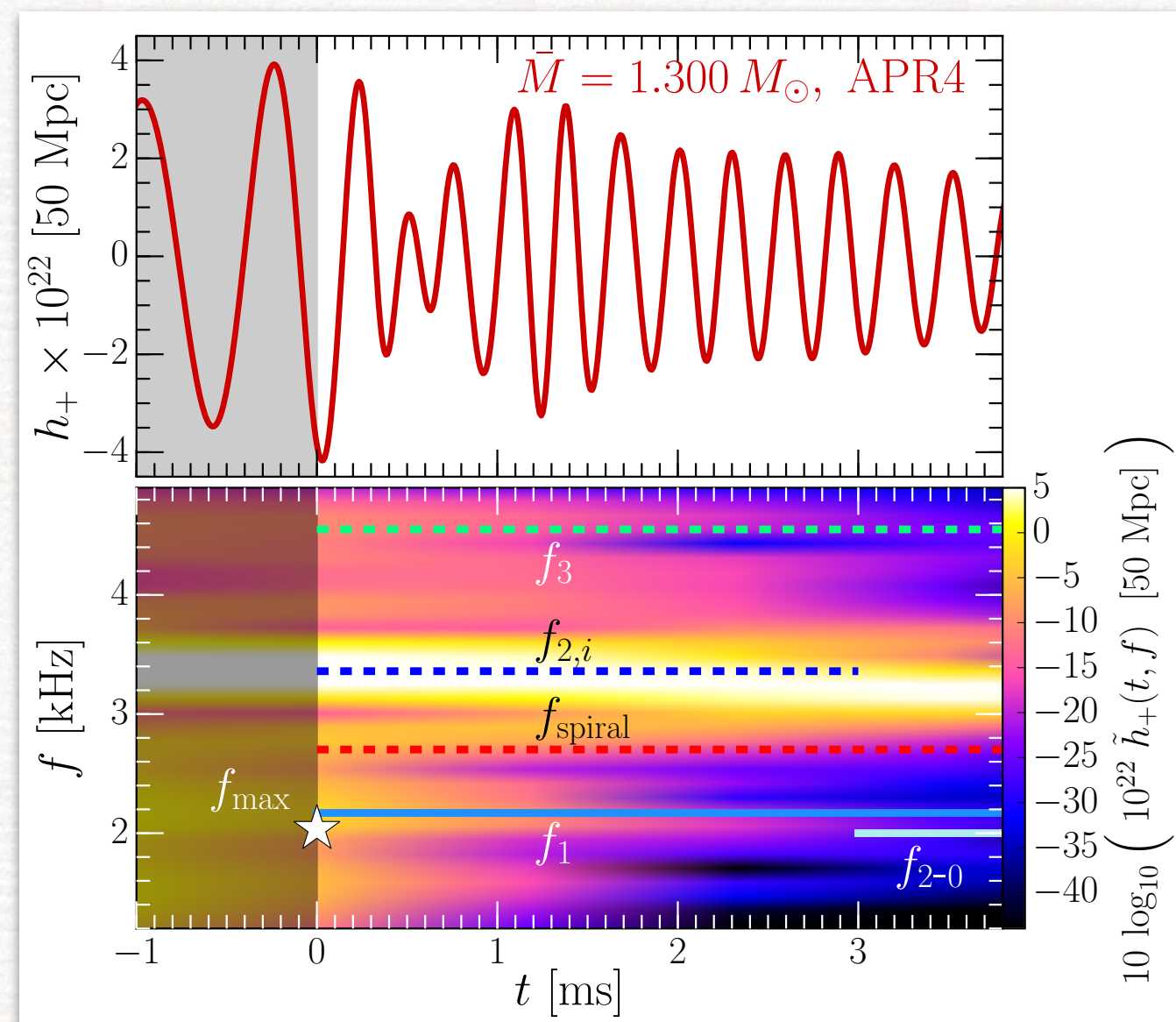
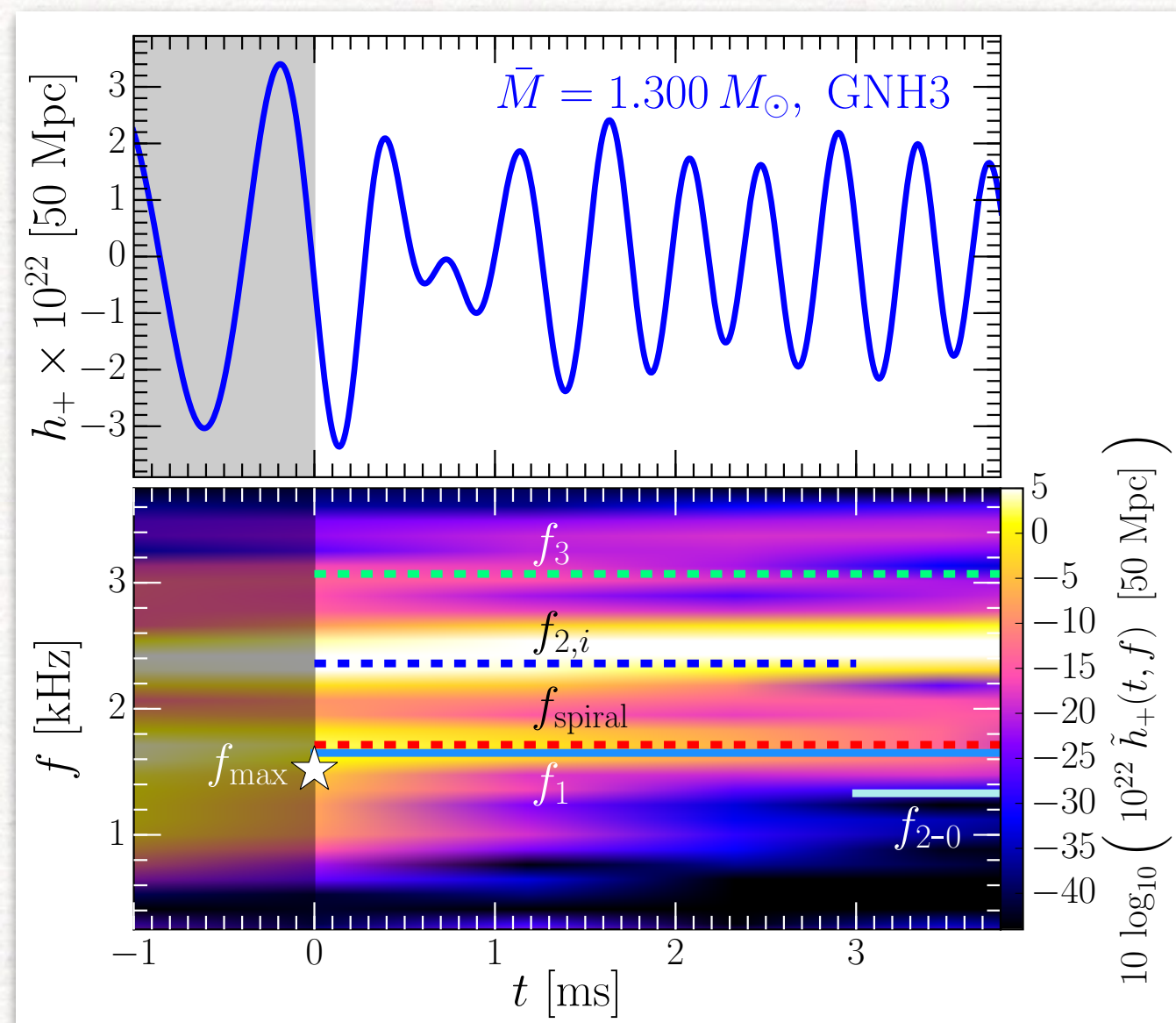
**Quasi-universal** behaviour in the **inspiral** implies that once  $f_{\max}$  is measured, so is tidal deformability, hence  $I, Q, M/R$

$$\Lambda = \frac{\lambda}{\bar{M}^5} = \frac{16}{3} \kappa_2^T \quad \text{tidal deformability or Love number}$$



# Understanding mode evolution

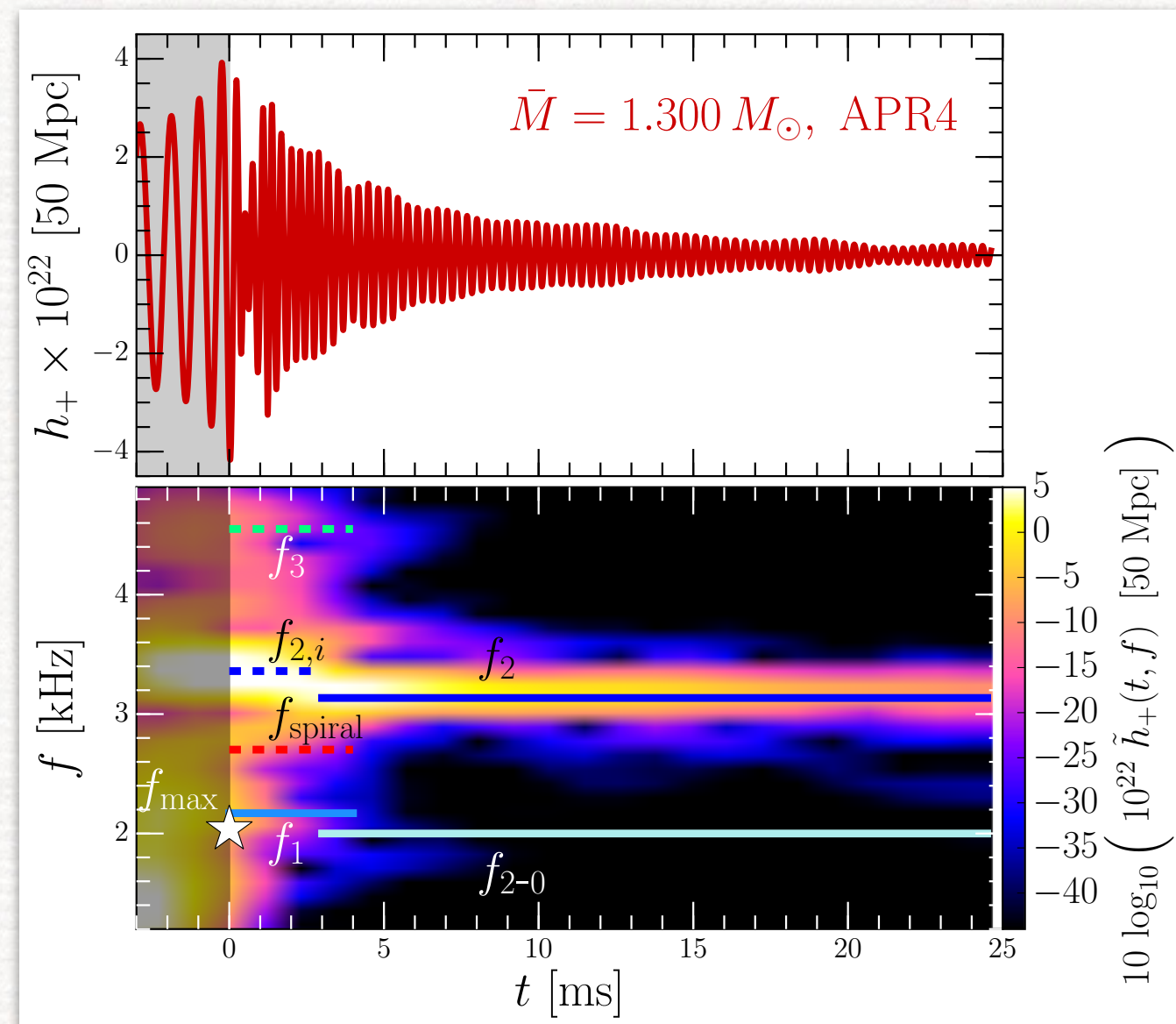
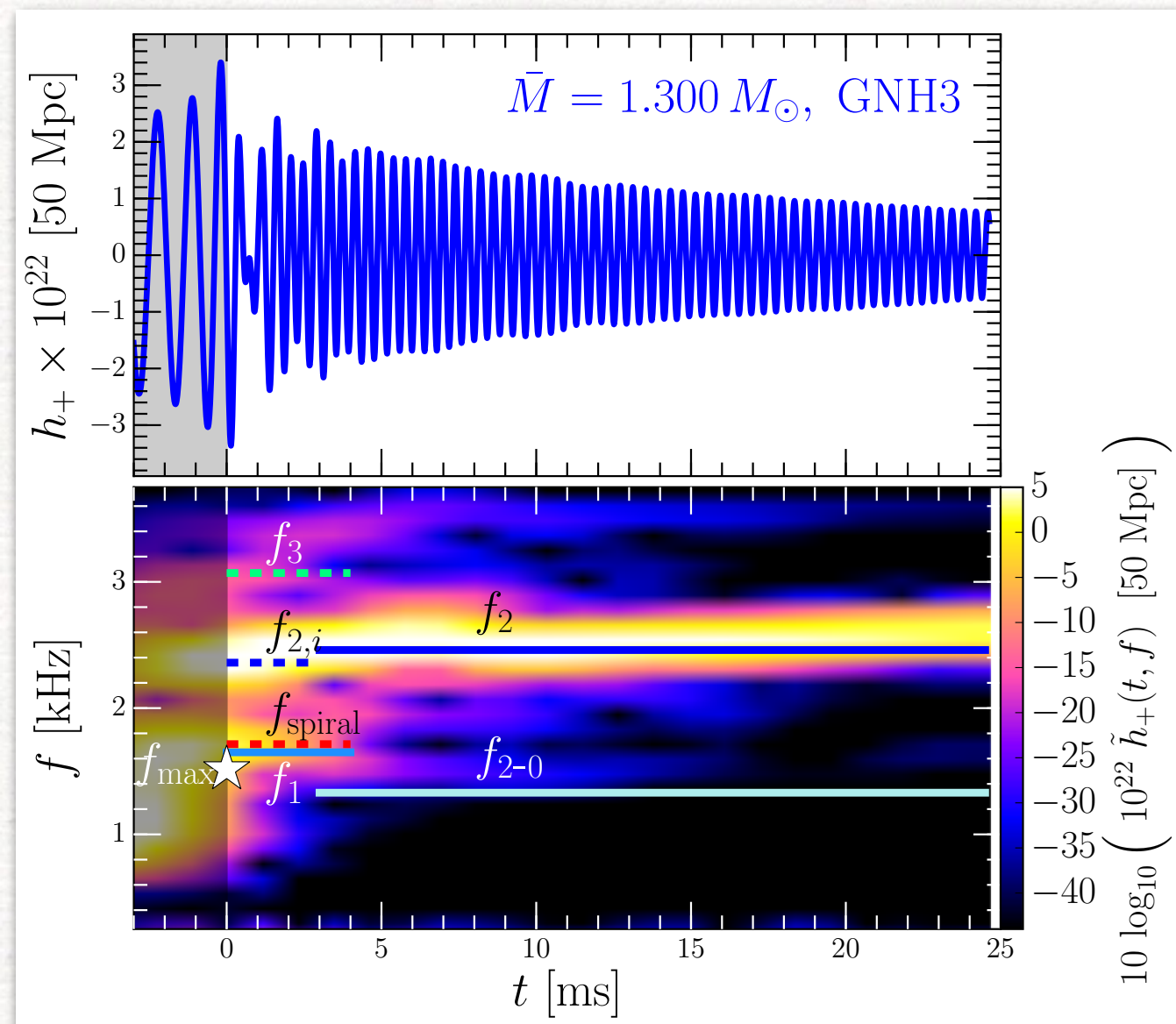
On a **short** timescale after the merger, it is possible to see the emergence of  **$f_1$** ,  **$f_2$** , and  **$f_3$** .



# Understanding mode evolution

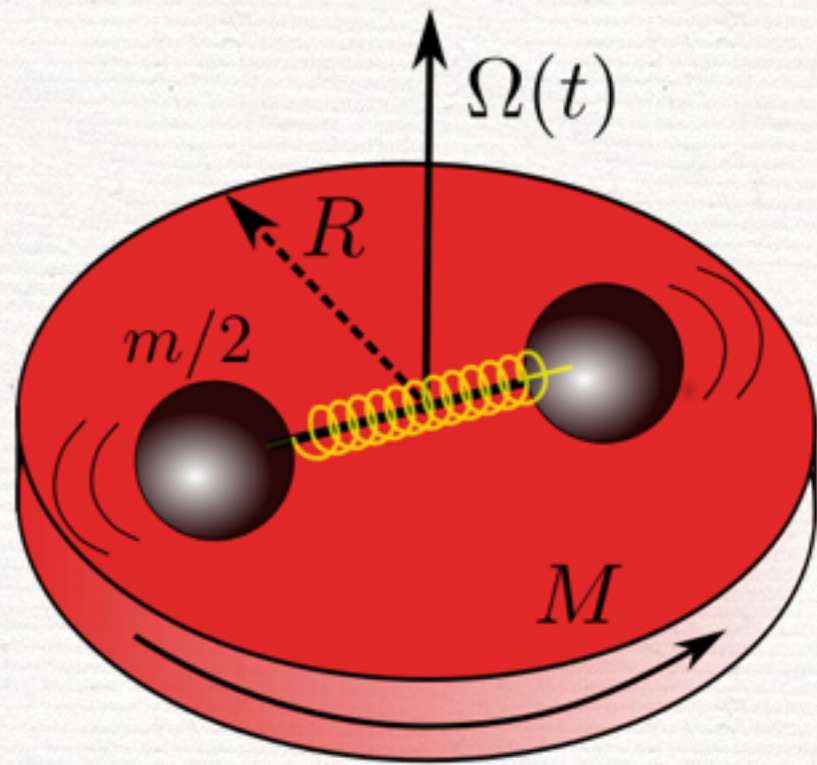
On a **long** timescale after the merger, only  **$f_2$**  survives.

What produces the short-lived  **$f_1$**  and  **$f_3$**  modes?



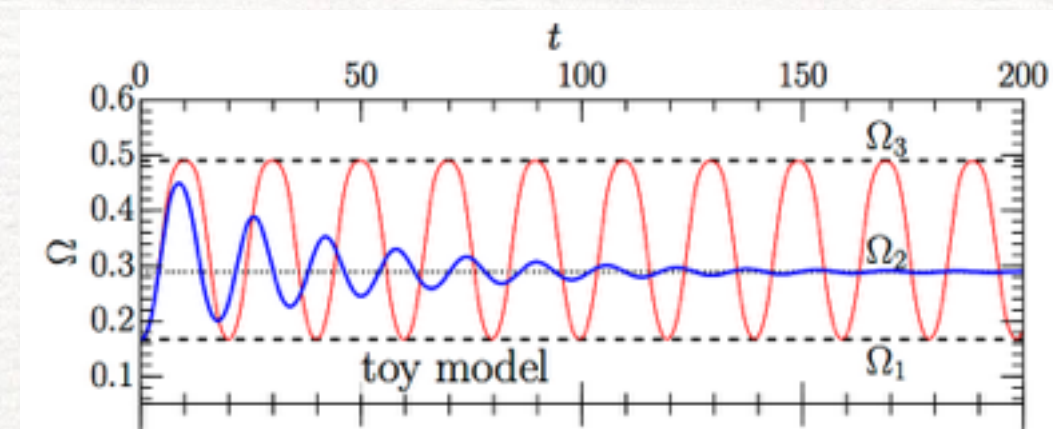


# A mechanical toy model for the $f_1, f_3$ peaks



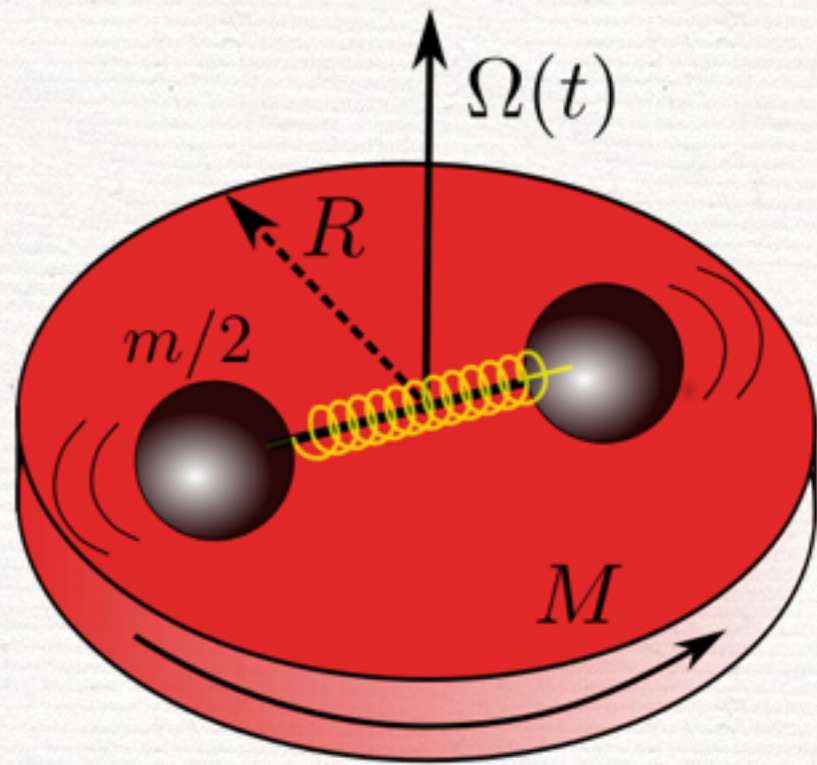
- Consider disk with 2 masses moving along a shaft and connected via a spring  $\sim$  HMNS with 2 stellar cores
- Let disk rotate and mass oscillate while conserving angular momentum

- If there is no friction, system will spin between: low freq ( $f_1$ , masses are far apart) and high ( $f_3$ , masses are close).
- If friction is present, system will spin asymptotically at  $f_2 \sim (f_1 + f_3)/2$ .



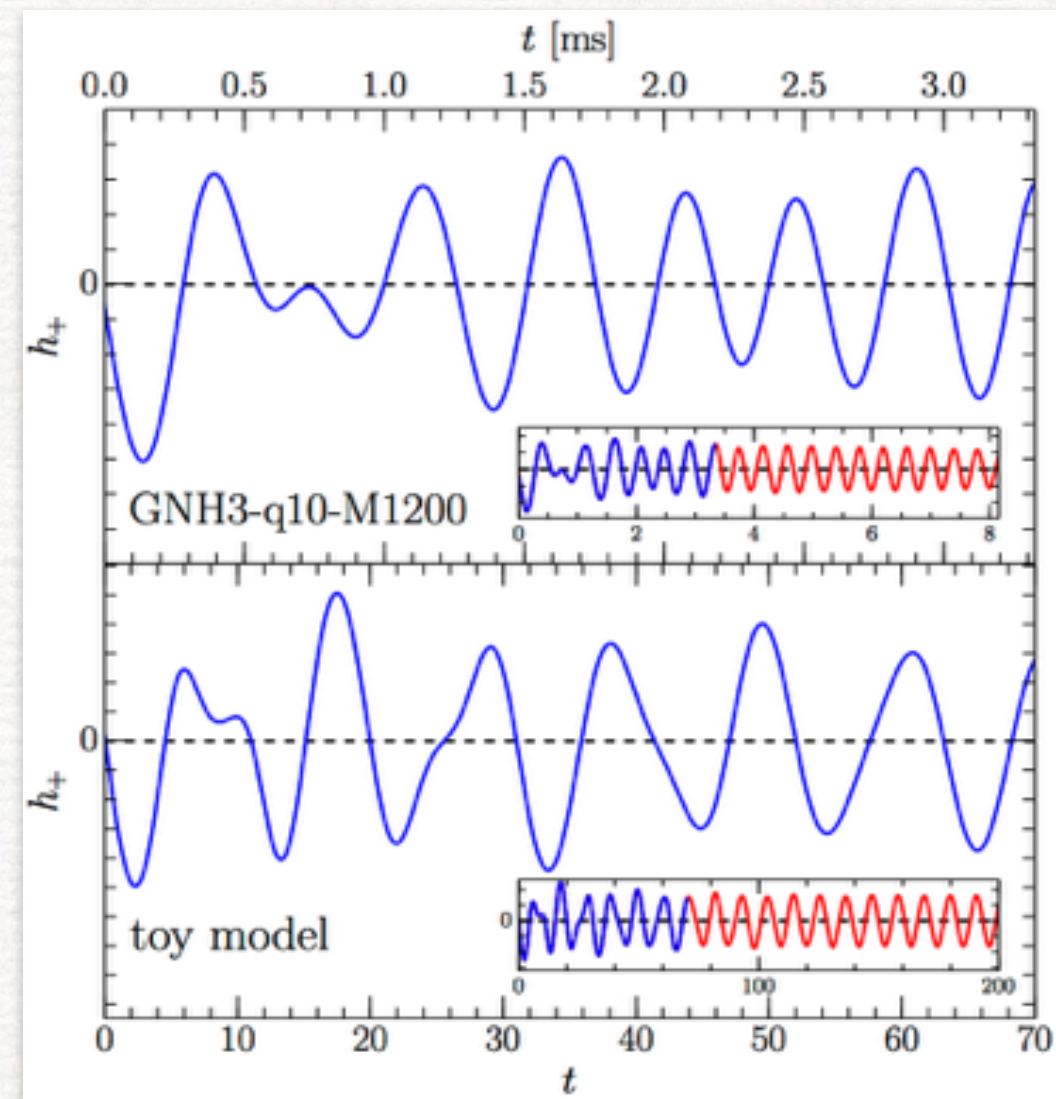


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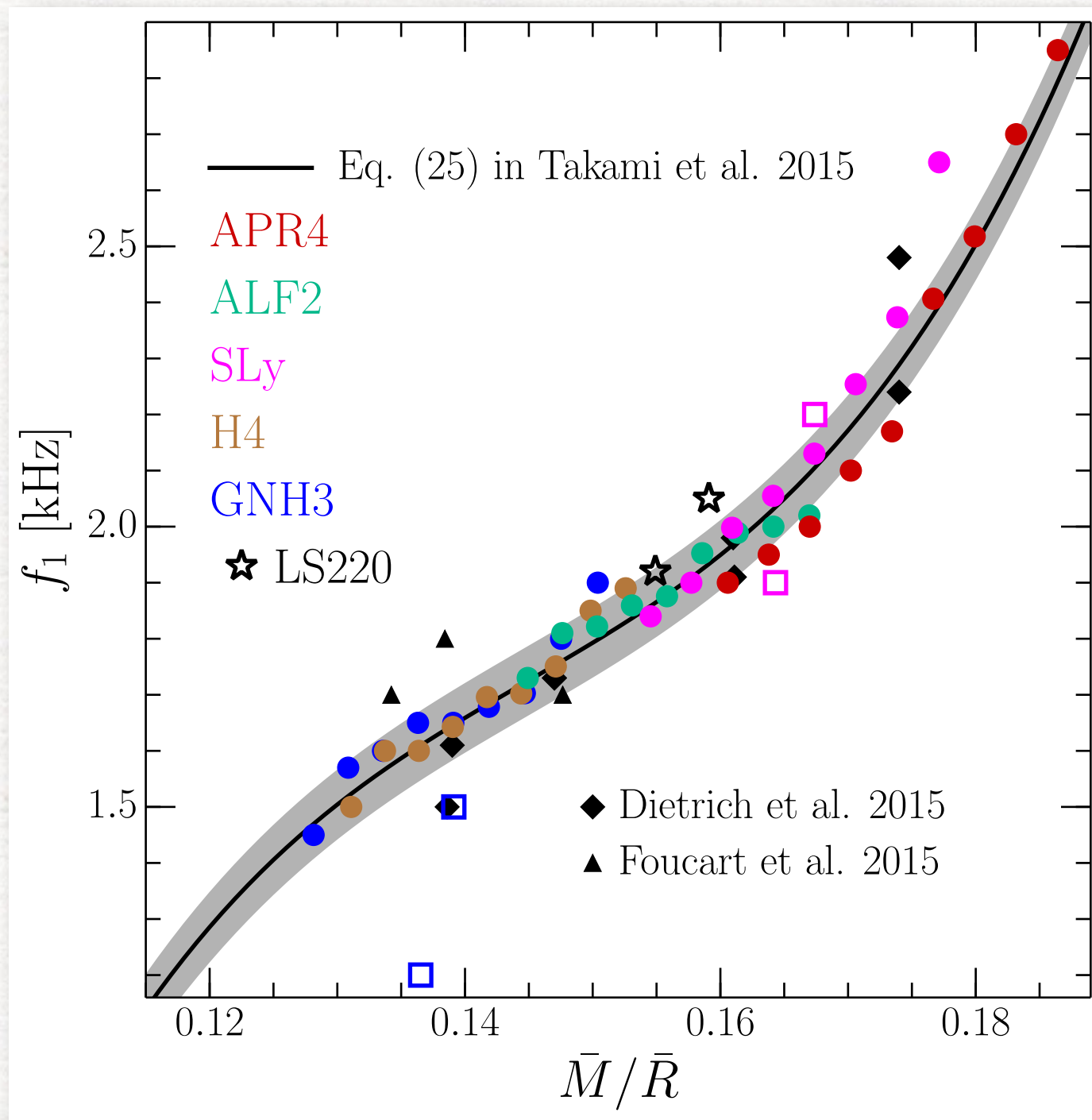


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- analytic model possible of post merger (see later).



# Quasi-universal behaviour: post-merger

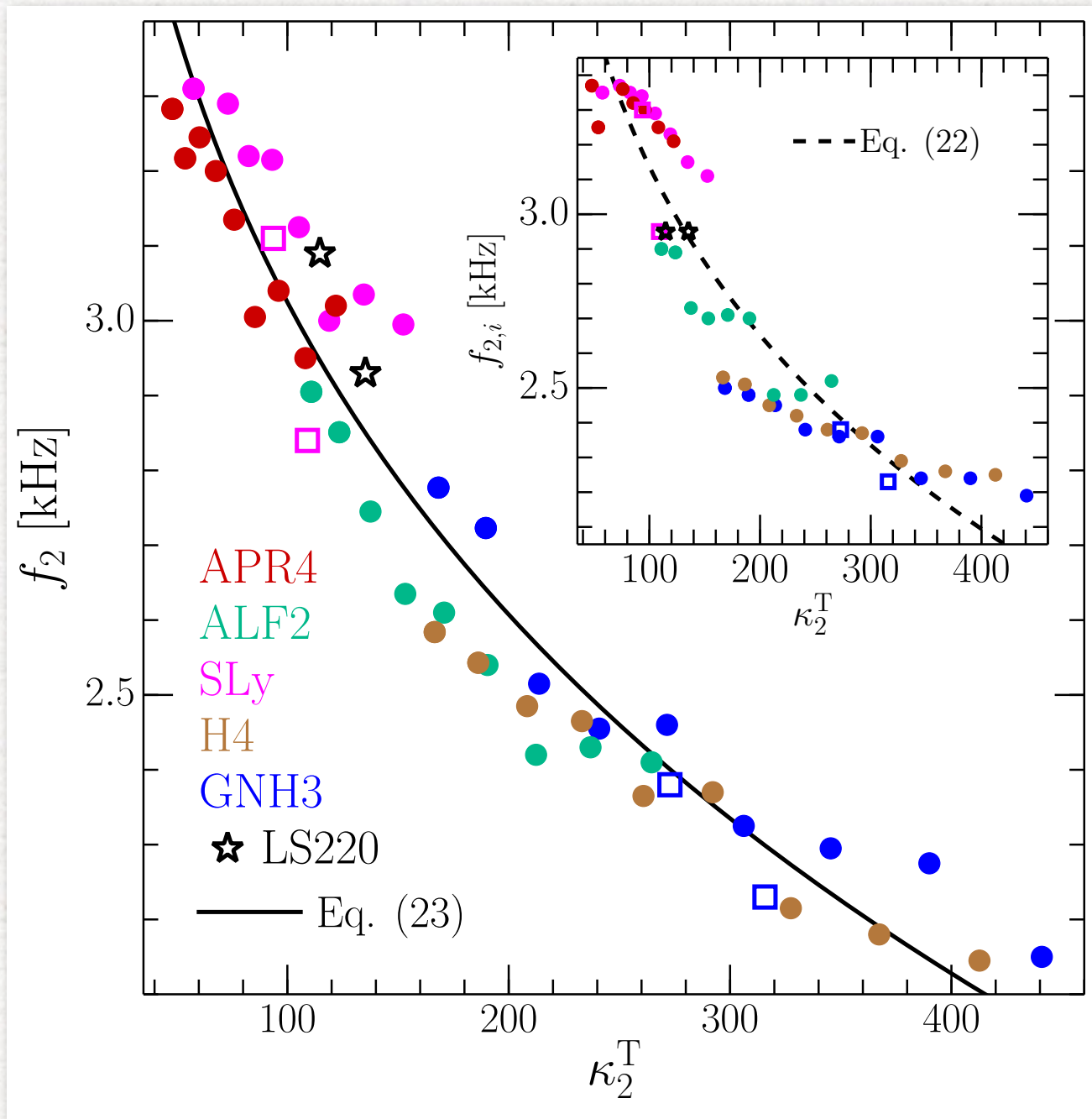


We have found **quasi-universal behaviour**: i.e., the properties of the spectra are only weakly dependent on the EOS.

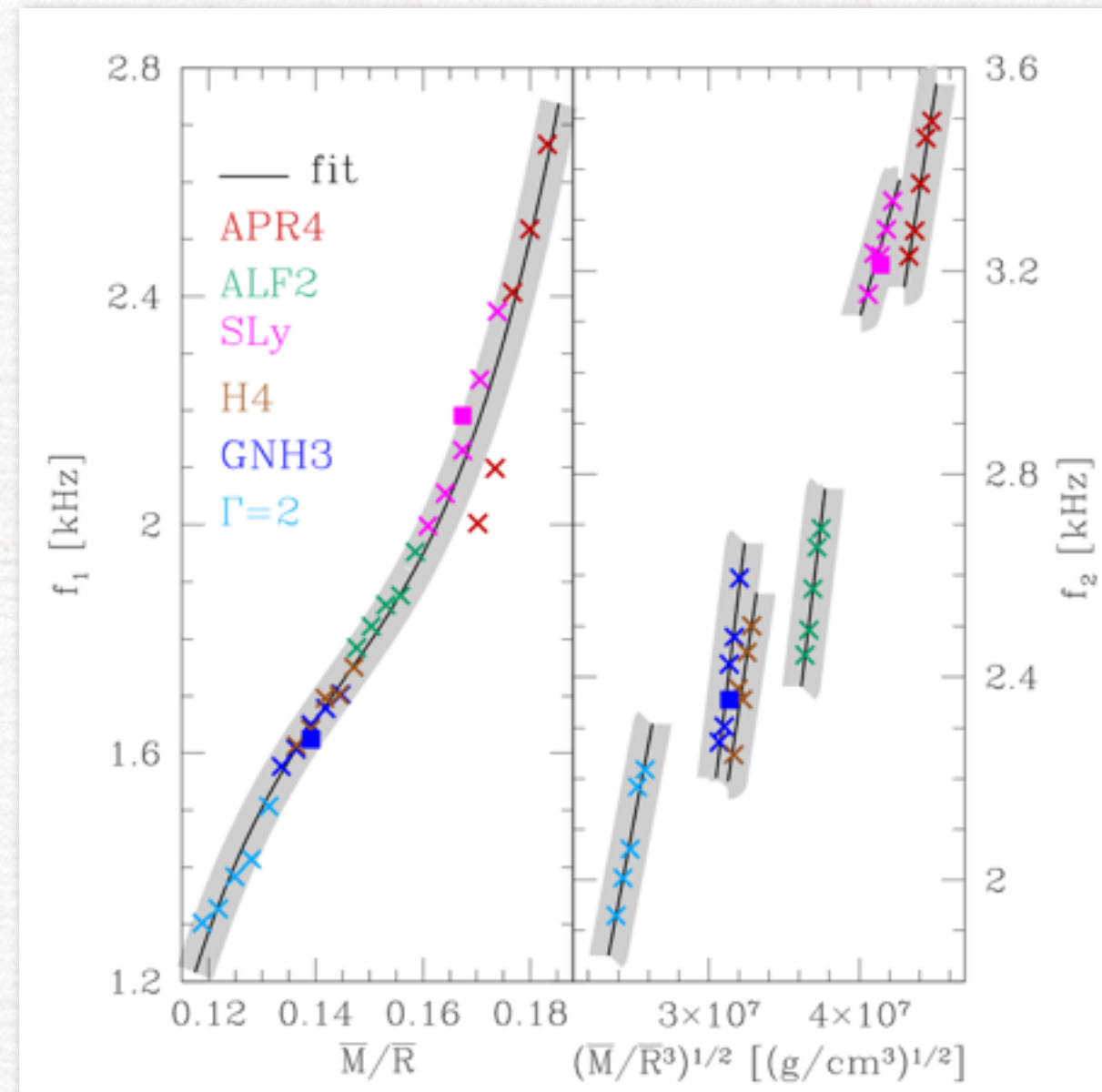
This has profound implications for the analytical modelling of the GW emission: “what we do for one EOS can be extended to all EOSs.”



# Quasi-universal behaviour: post-merger



Correlations with Love number found also for high frequency peak  $f_2$



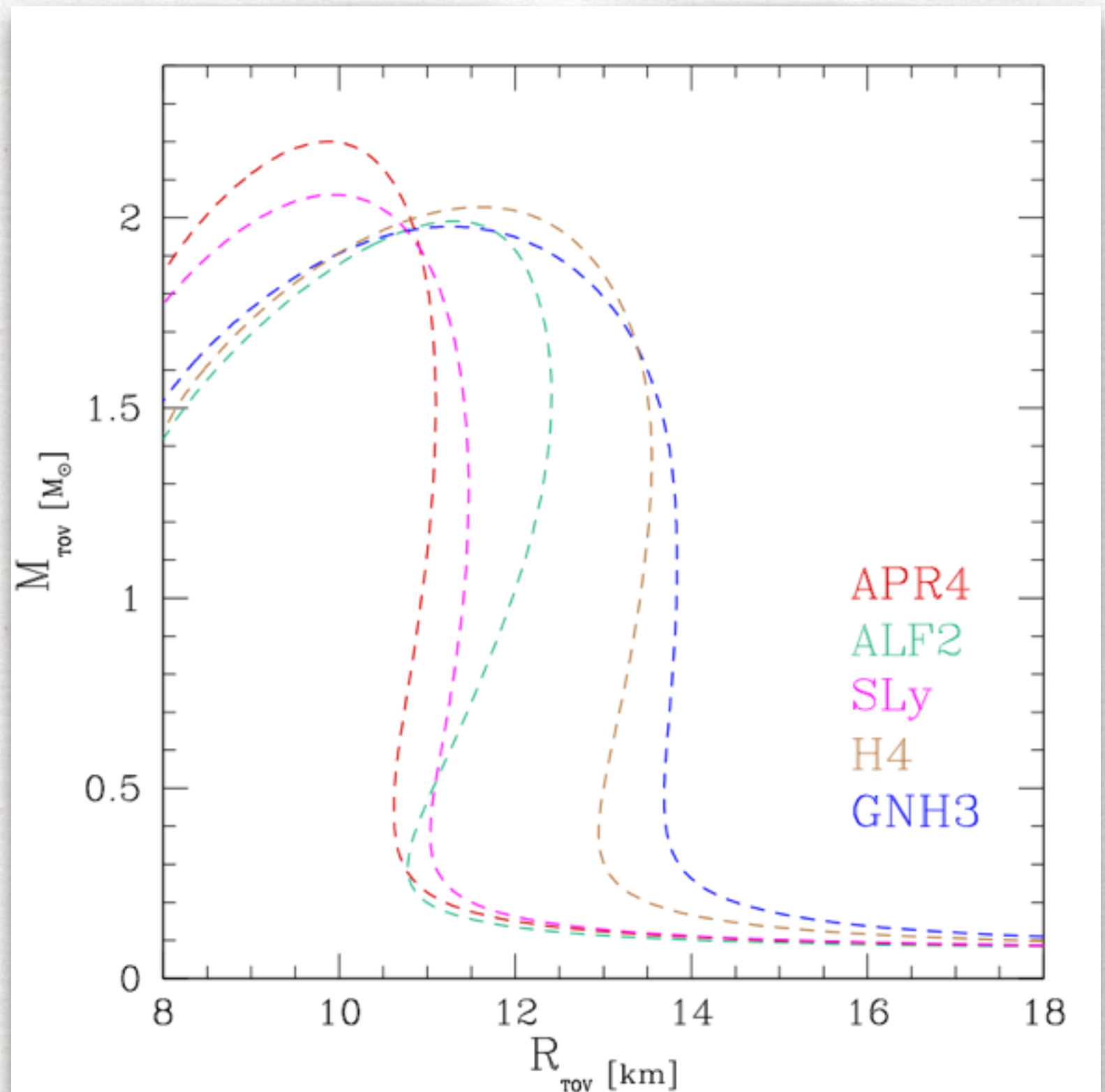
Correlations also with compactness  
These other correlations are **weaker** but equally useful.



# An example: start from equilibria

Assume that the GW signal from a binary NS is detected and with a SNR high enough that the two peaks are clearly measurable.

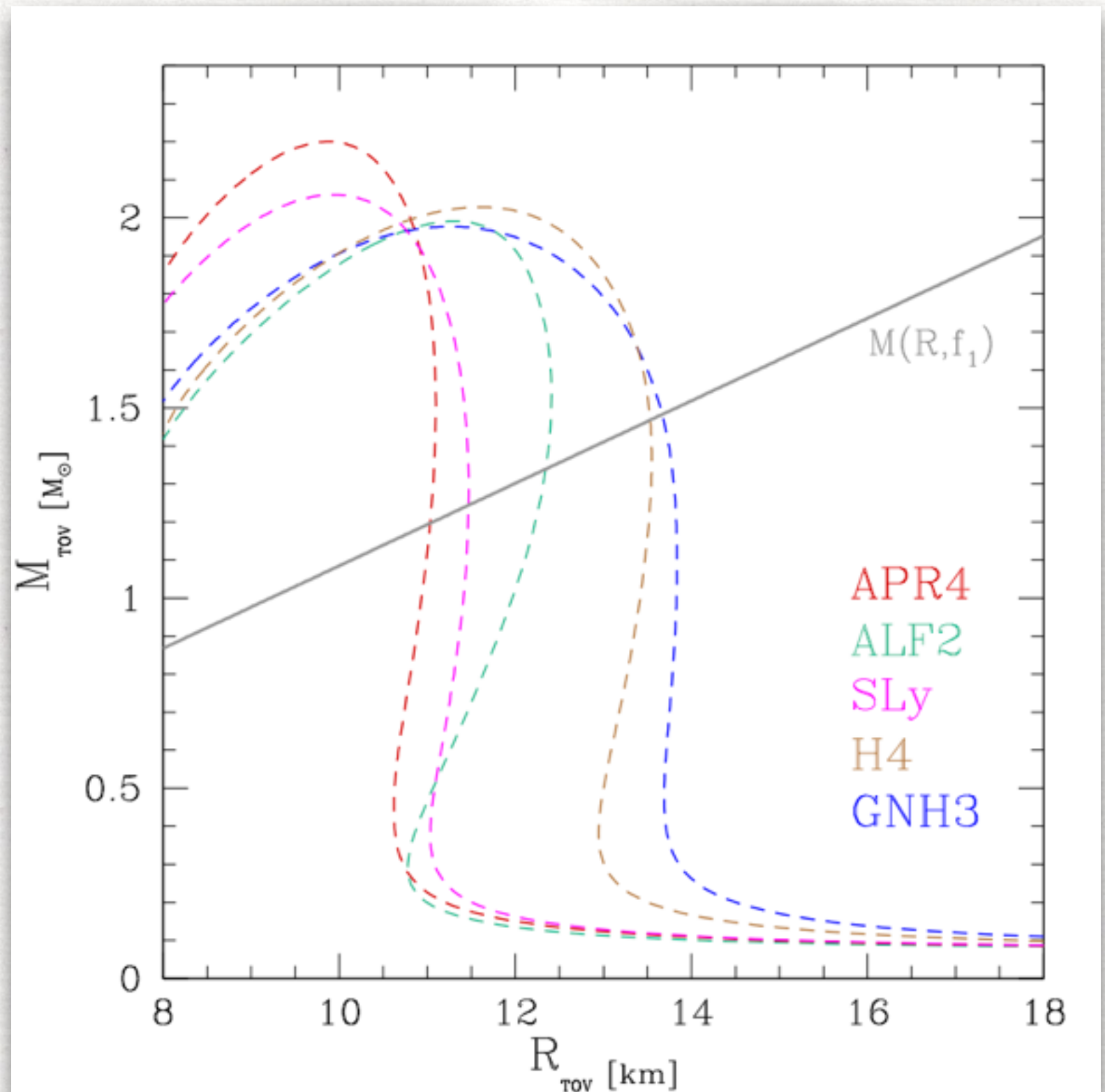
Consider your best choices as candidate EOSs



# An example: use the $M(R, f_1)$ relation

The measure of the  $f_1$  peak will fix a  $M(R, f_1)$  relation and hence a **single** line in the  $(M, R)$  plane.

All EOSs will have **one** constraint (crossing).

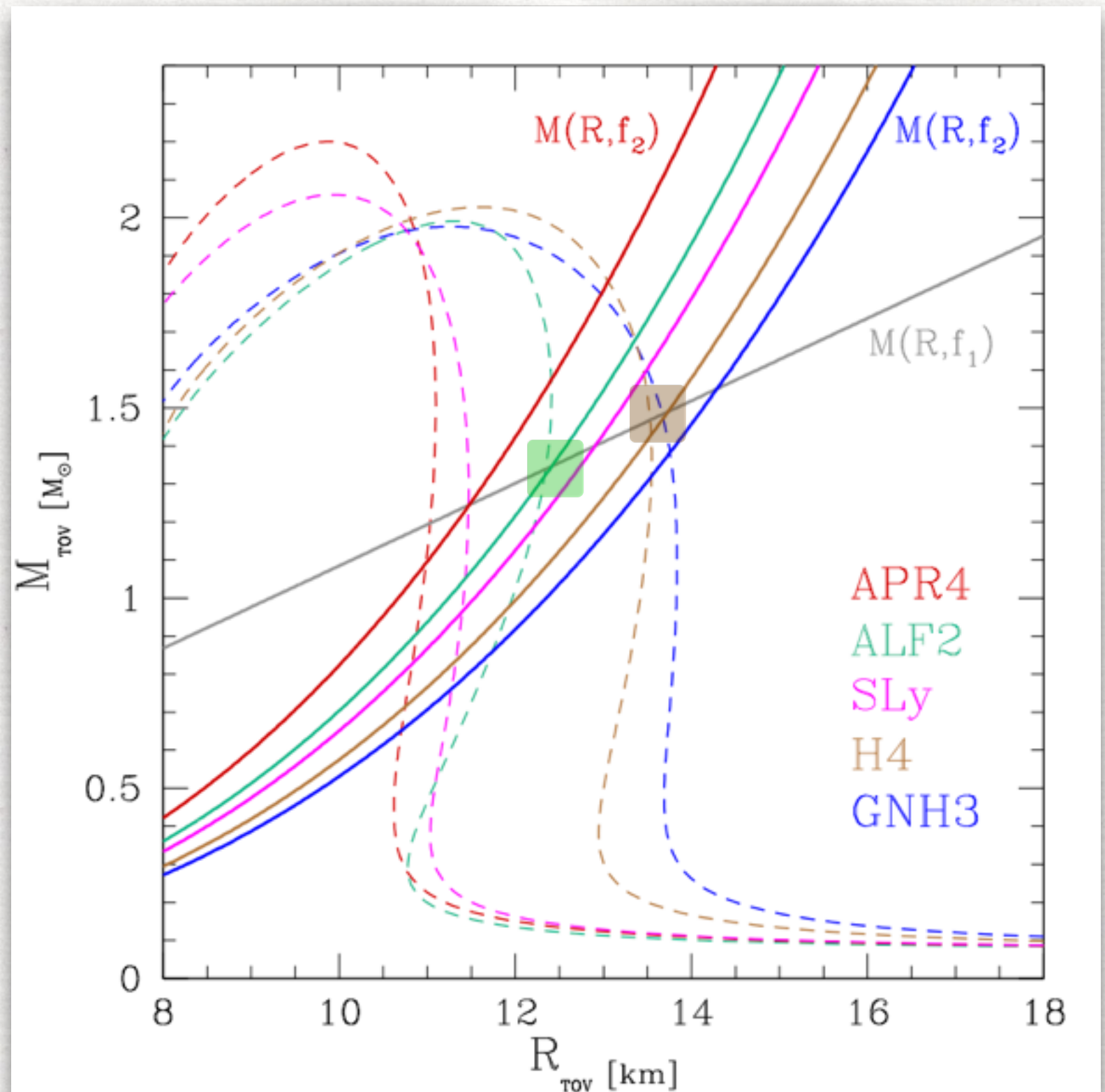




# An example: use the $M(R, f_2)$ relations

The measure of the  $f_2$  peak will fix a relation  $M(R, f_2, EOS)$  for each EOS and hence a **number** of lines in the  $(M, R)$  plane.

The right EOS will have **three** different constraints (APR, GNH3, SLy excluded)

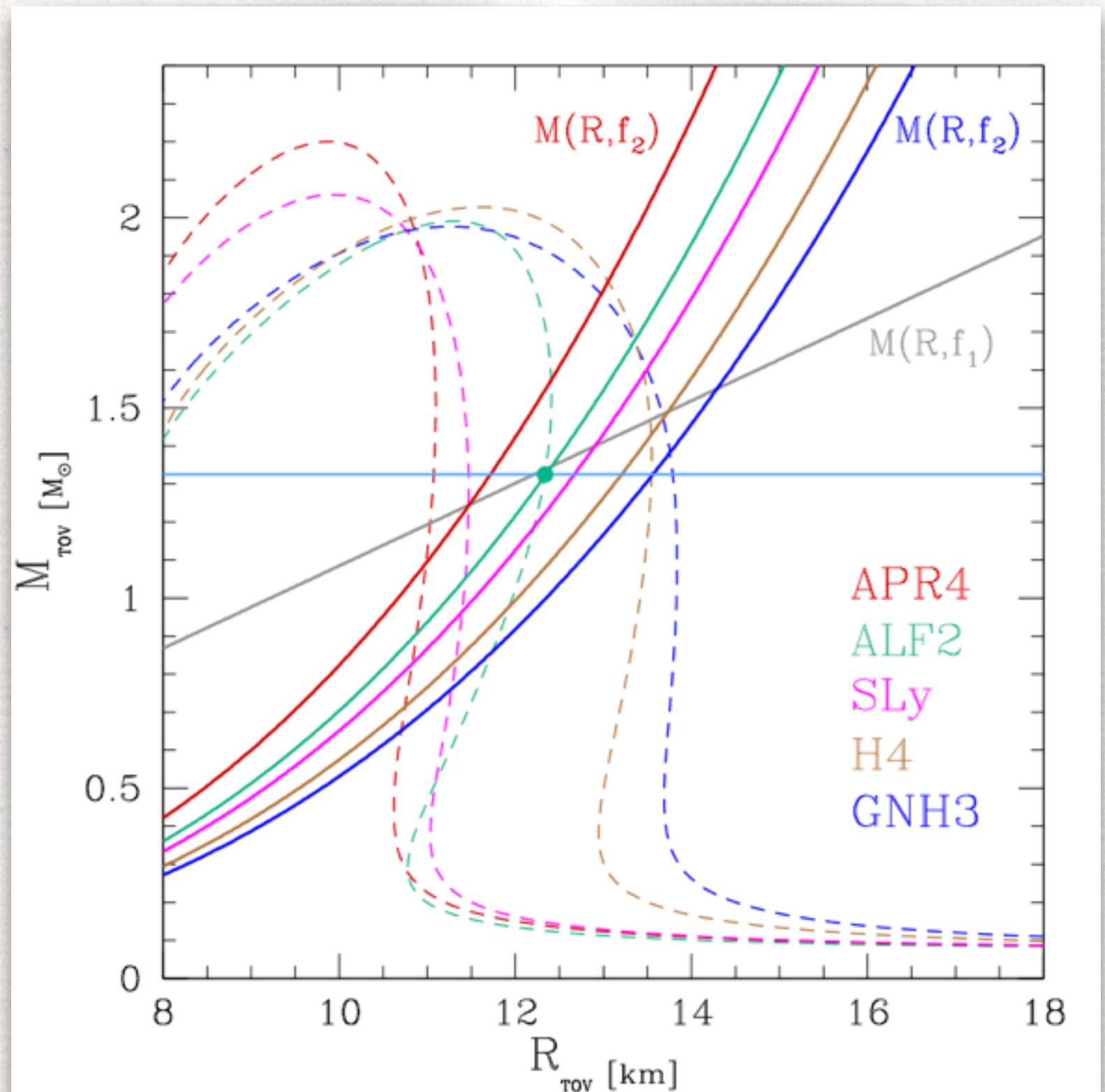




# An example: use measure of the mass

If the mass of the binary is measured from the inspiral, an additional constraint can be imposed.

The right EOS will have **four** different constraints. Ideally, a single detection would be sufficient.

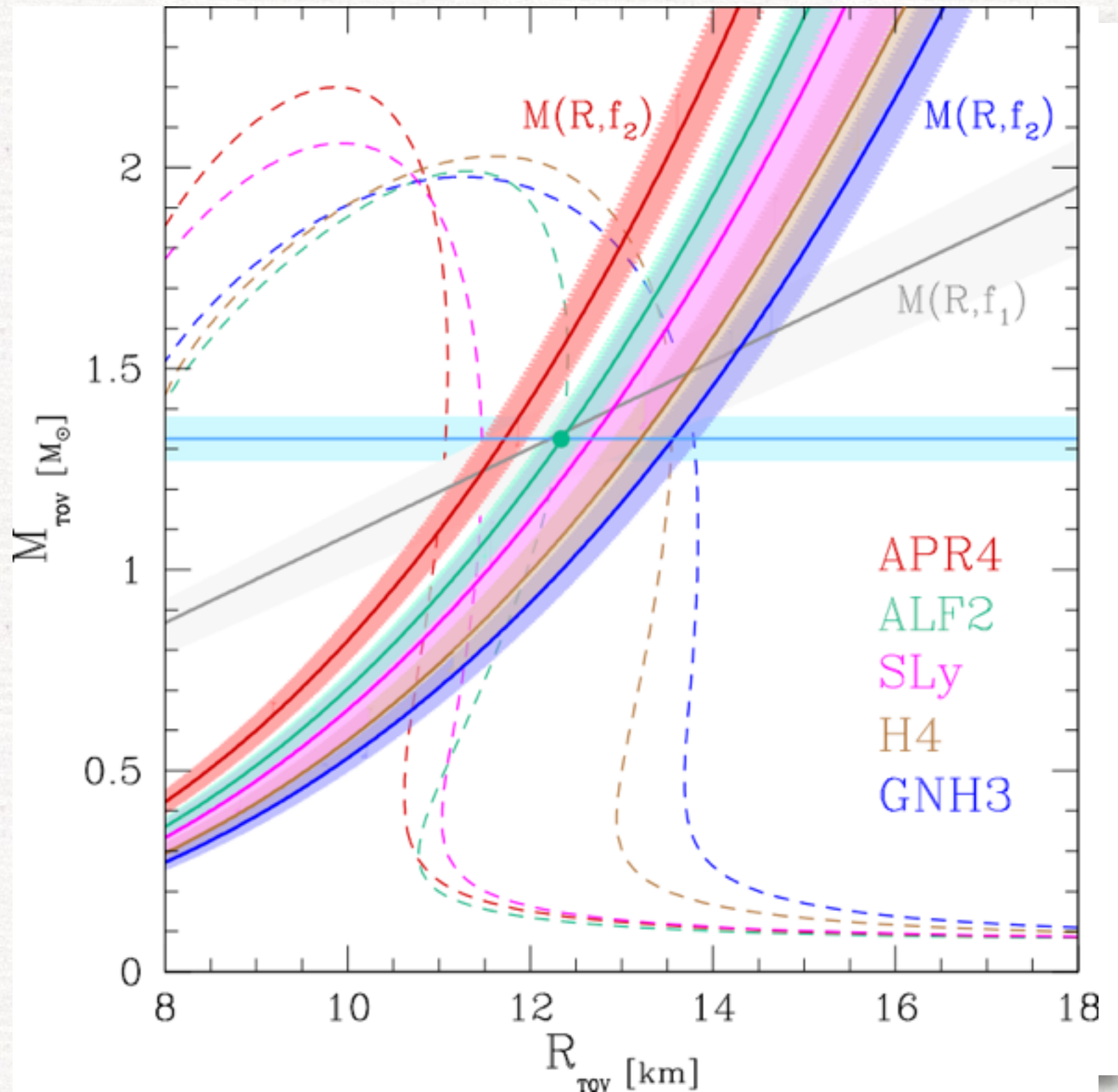


# This works for all EOSs considered

In reality things will be more complicated. The **lines** will be **stripes**; Bayesian probability to get precision on  $M$ ,  $R$ .

Some numbers:

- at 50 Mpc, freq. uncertainty from Fisher matrix is 100 Hz
- at SNR=2, the event rate is 0.2-2 yr<sup>-1</sup> for different EOSs.



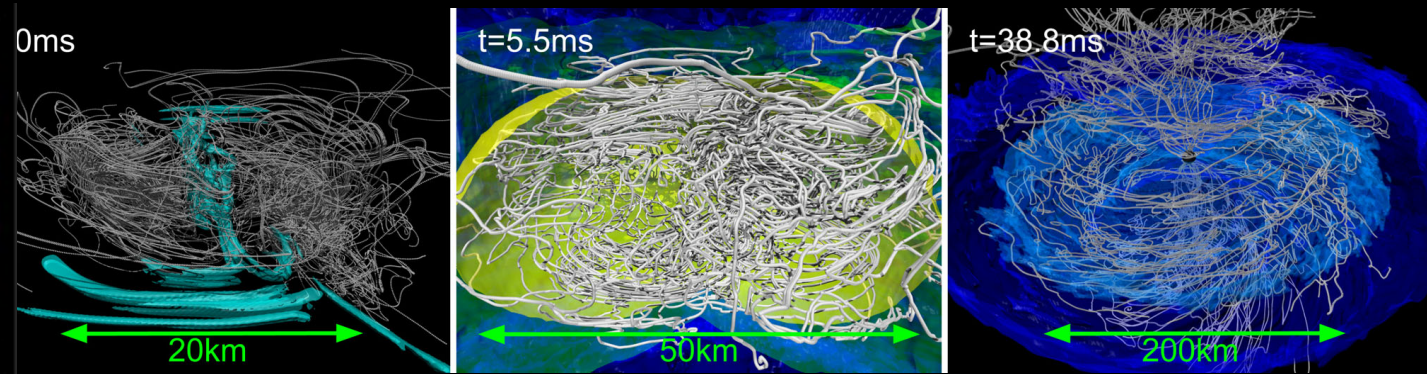


# The importance of B-fields

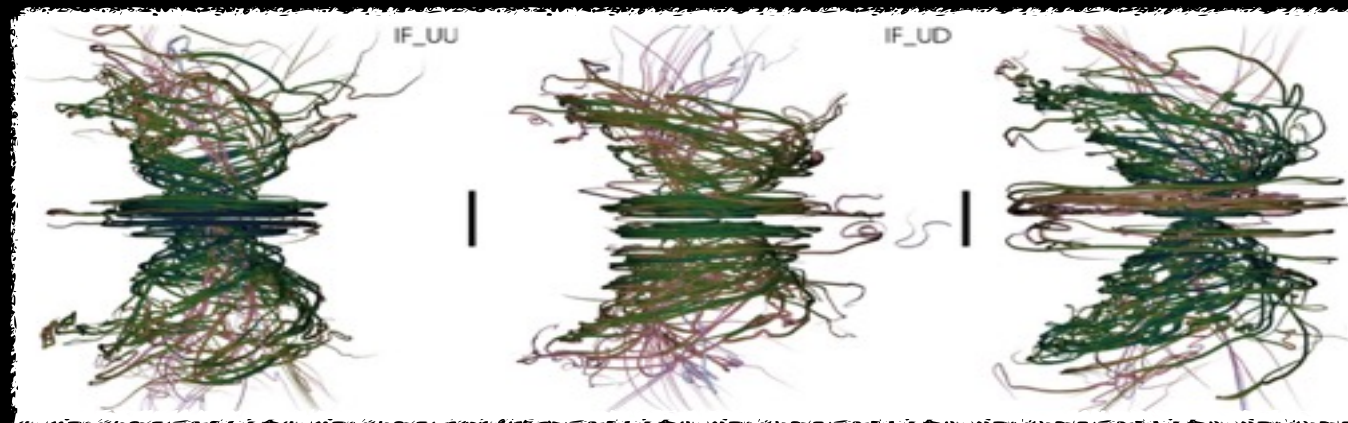
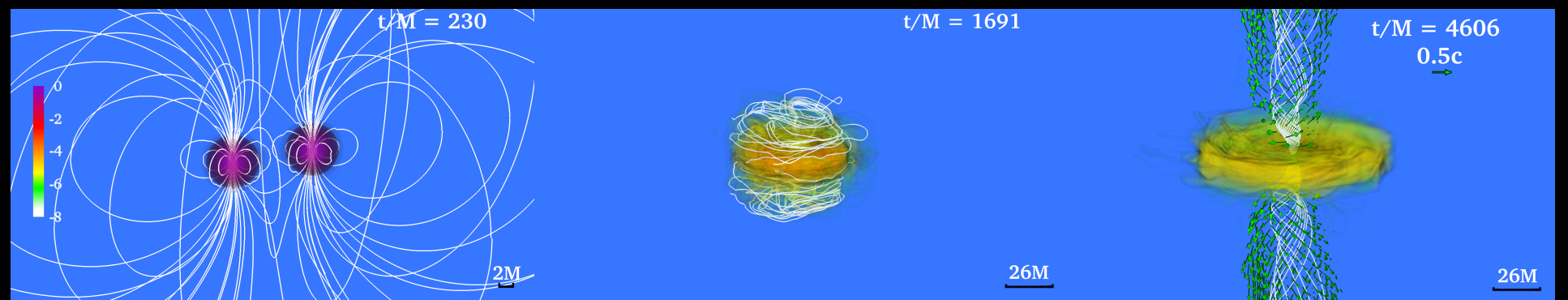
LR+ 2011



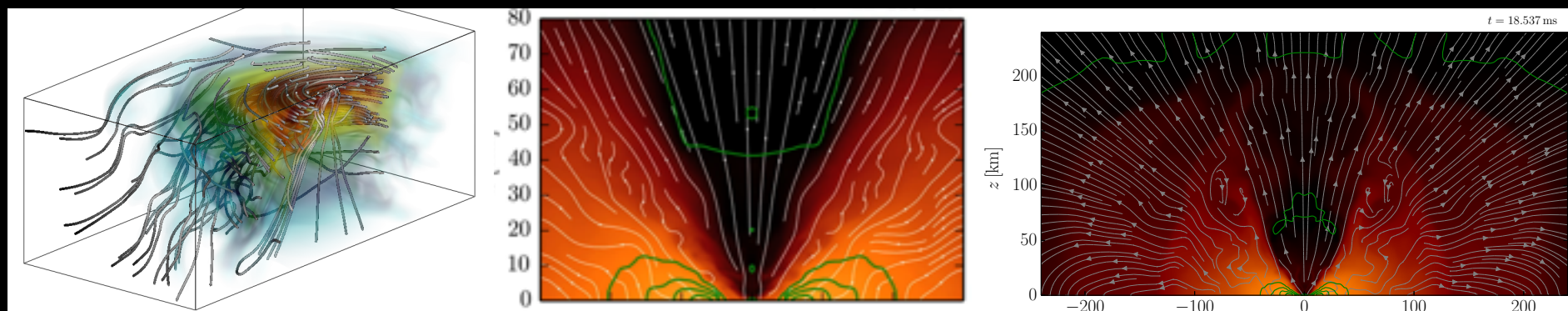
Kiuchi+ 2014



Ruiz+ 2016



Kawamura+2016







Dionysopoulou+ 2015



# Importance of B-fields

B-fields essential for EMCs. Most simulations use **ideal MHD**: infinite conductivity, magnetic field advected.

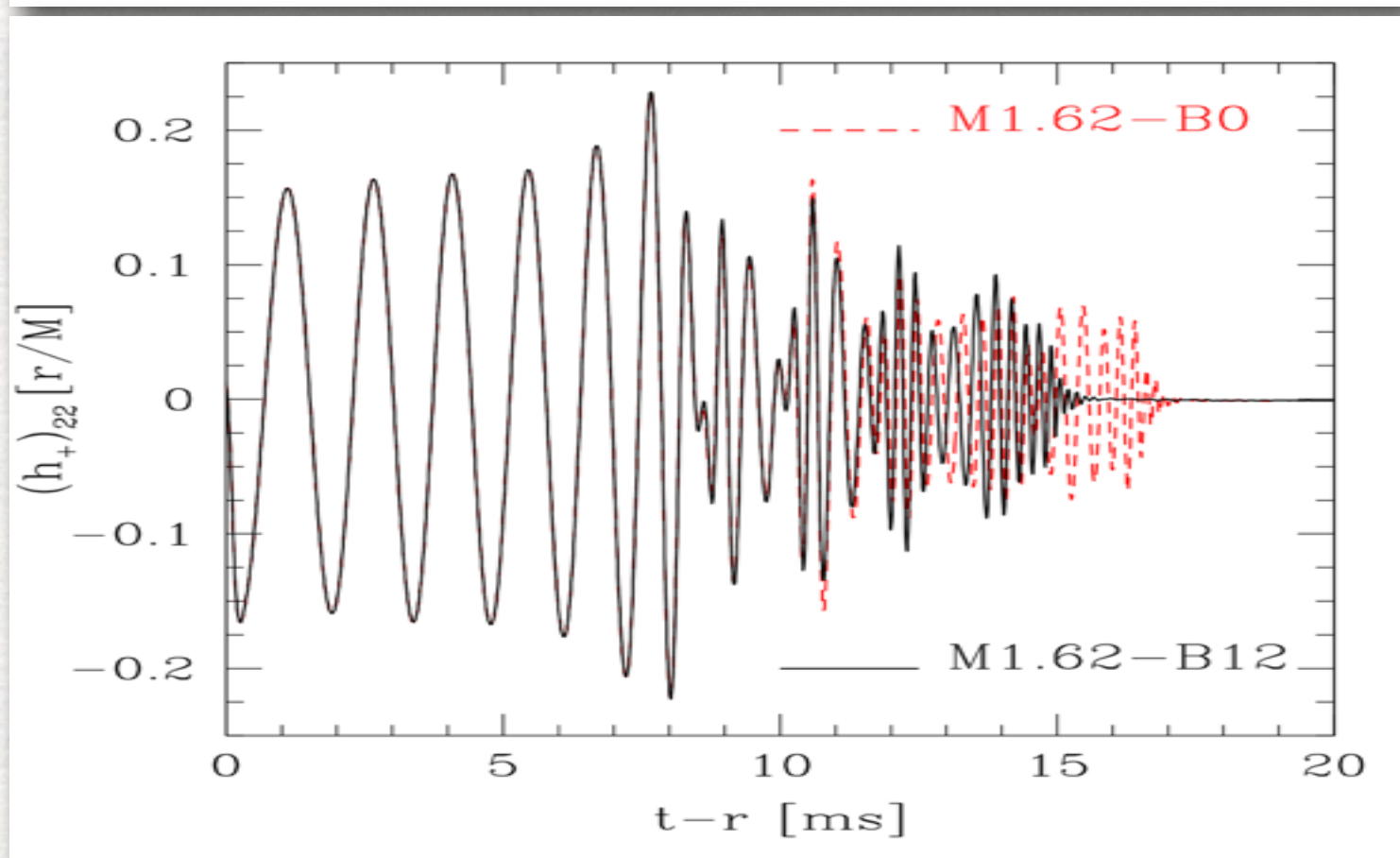
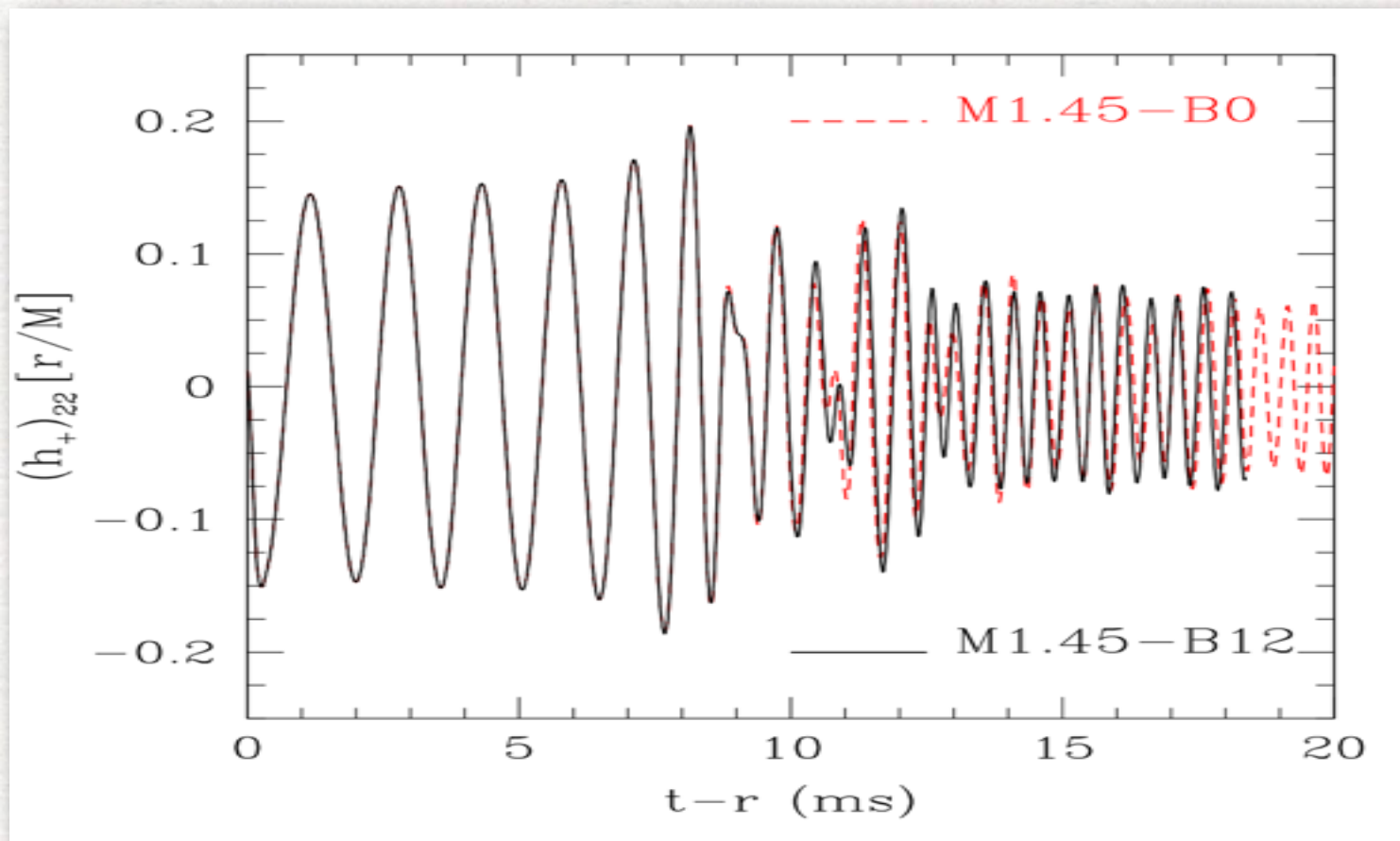
You can ask some simple questions.

- can B-fields be measured during the inspiral? 
- is EMC produced before merger? 
- do B-fields grow after merger and yield EMC? 
- does jet appear after BH formation and yield EMC? 

Last two questions are **incredibly hard** to answer; may require far more sophisticated numerics and microphysics



# Waveforms: comparing against magnetic fields



Compare B/no-B field:

- **inspiral** waveform is different but for unrealistic B-fields (i.e.  $B \sim 10^{17}$  G).

- **post-merger** waveform is different for all masses; strong B-fields delay the collapse to BH

Influence of B-fields on inspiral is **unlikely to be detected** for realistic fields

# Resistive Magnetohydrodynamics

Dionysopoulou, Alic, LR (2015)

- Ideal MHD is a good approximation in the inspiral, but not after the merger; match to **electro-vacuum** not possible.
- Main difference in resistive regime is the current, which is dictated by Ohm's law but microphysics is **poorly** known.
- We know conductivity  $\sigma$  is a **tensor** and proportional to density and inversely proportional to temperature.
- A simple prescription with scalar (isotropic) conductivity:

$$J^i = qv^i + W\sigma[E^i + \epsilon^{ijk}v_j B_k - (v_k E^k)v^i],$$

$\sigma \rightarrow \infty$  ideal-MHD (IMHD)

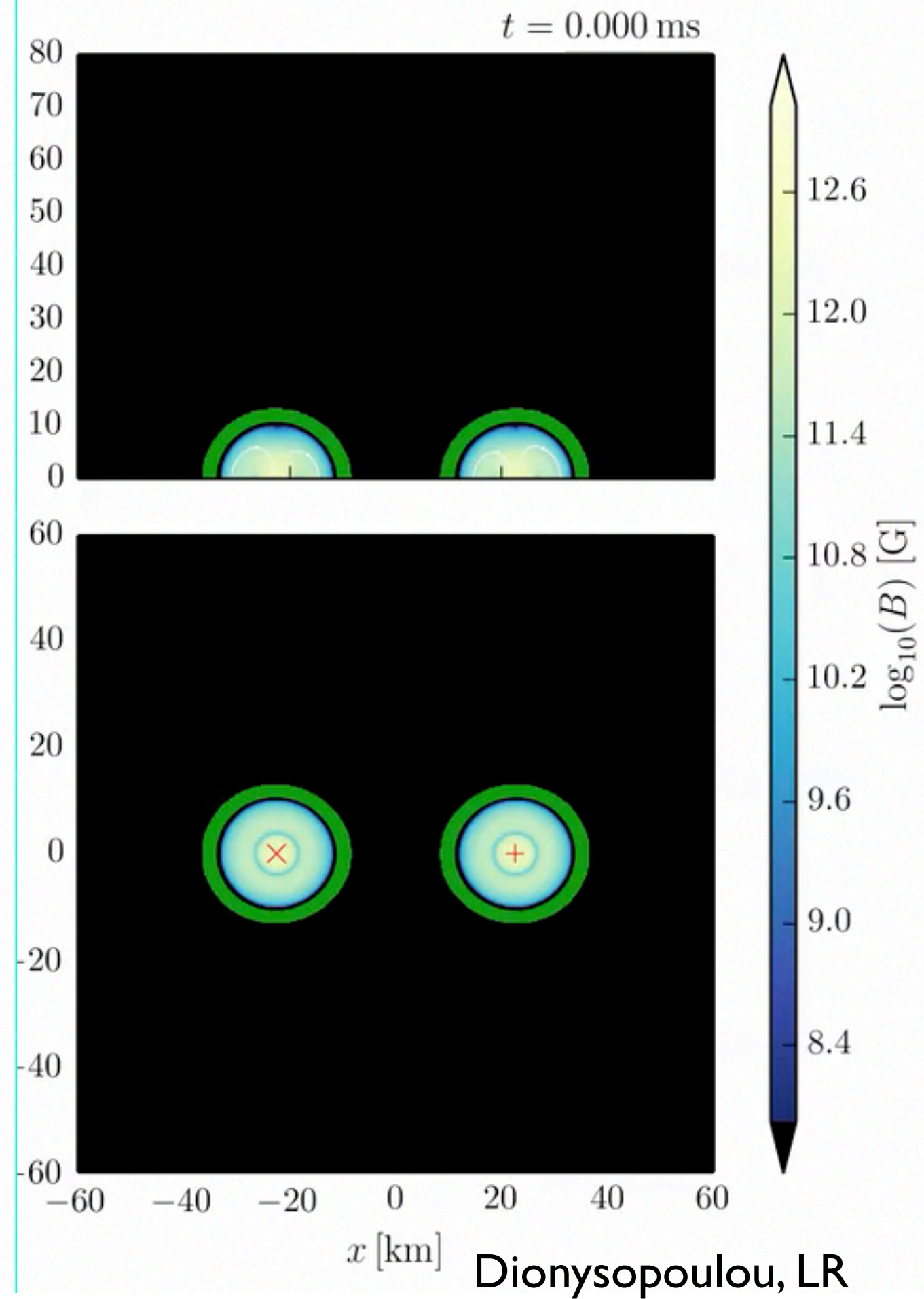
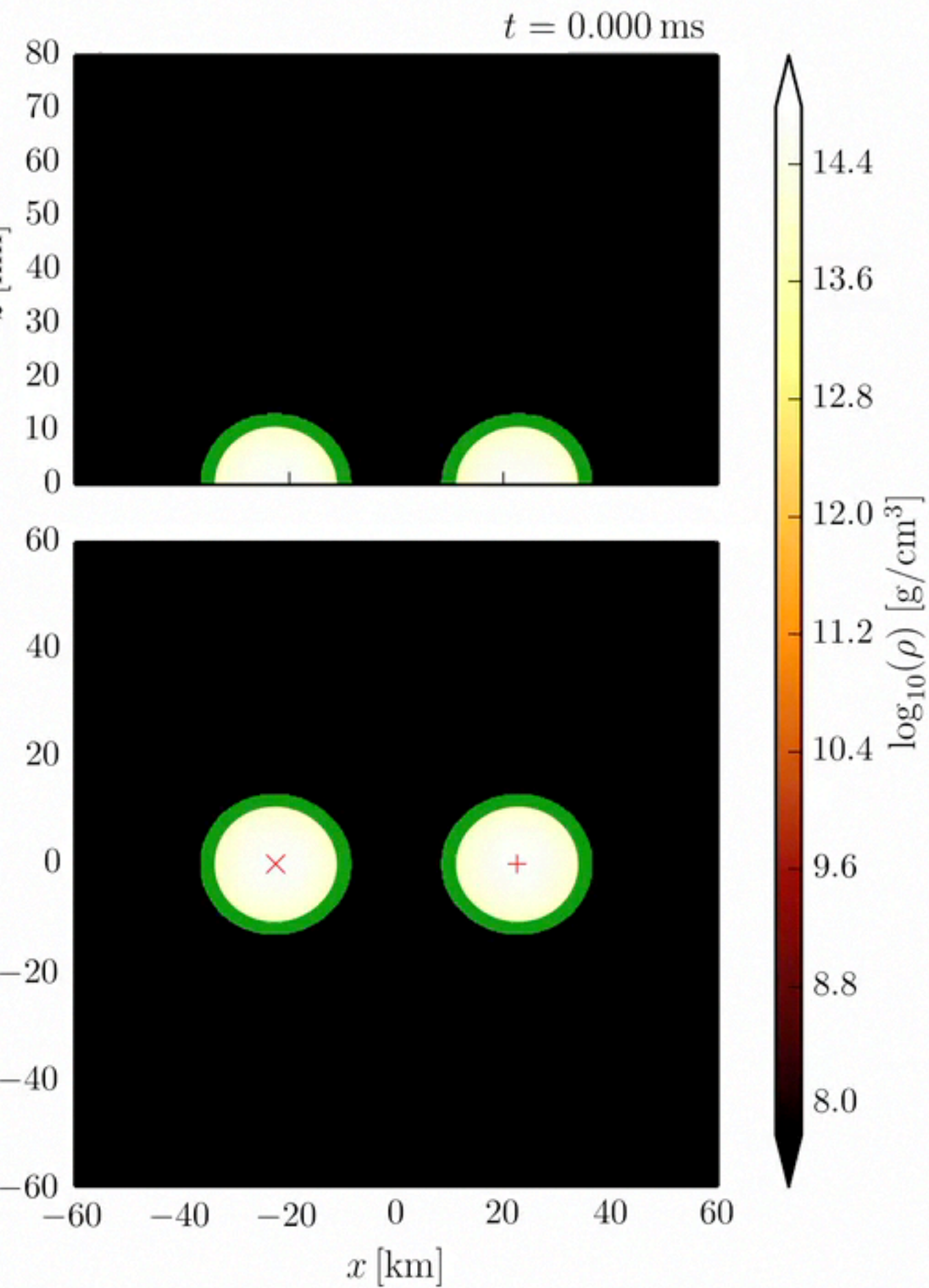
$\sigma \neq 0$  resistive-MHD (RMHD)

$\sigma \rightarrow 0$  electrovacuum

$$\sigma = f(\rho, \rho_{\min})$$

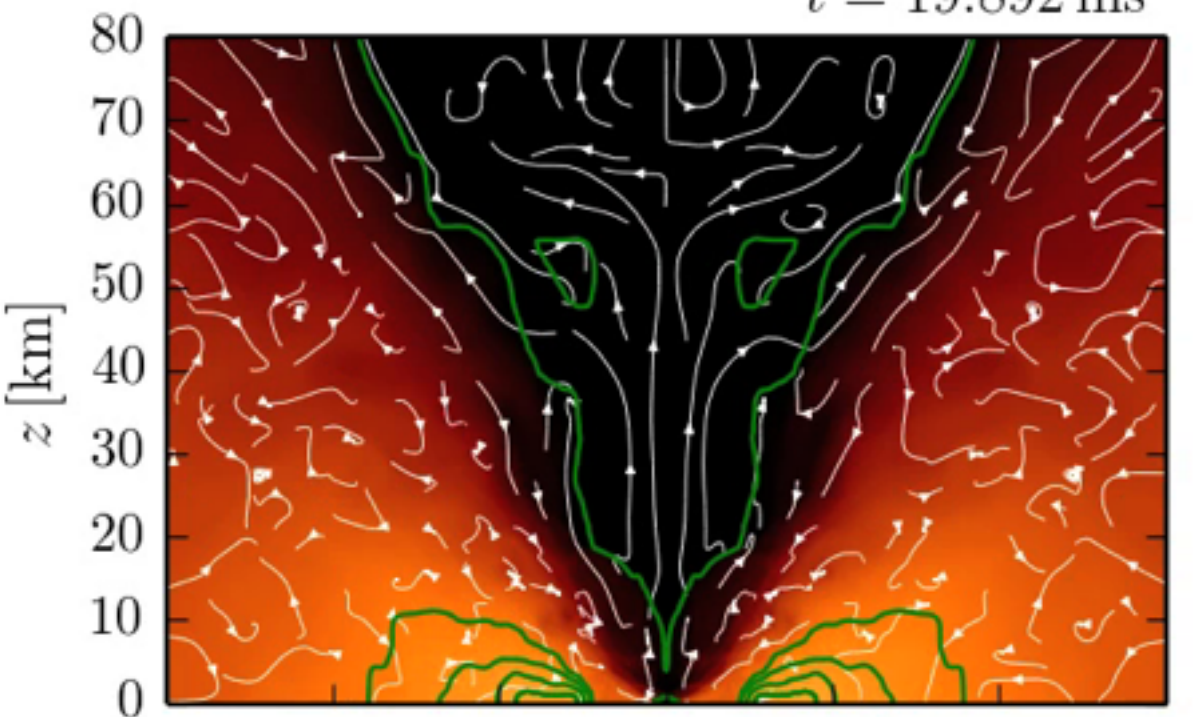
phenomenological prescription



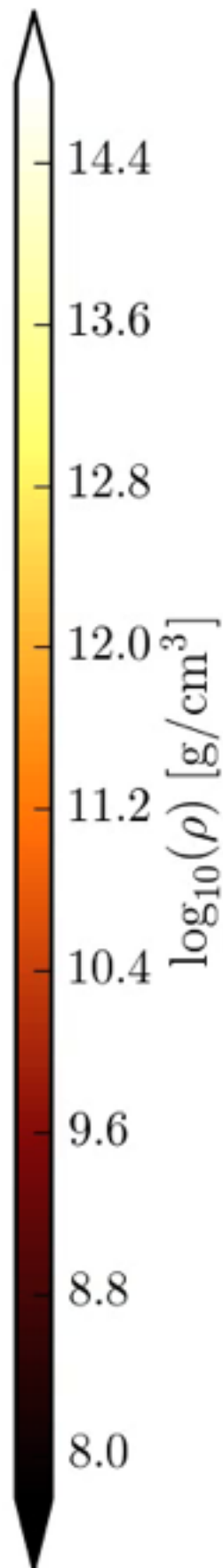
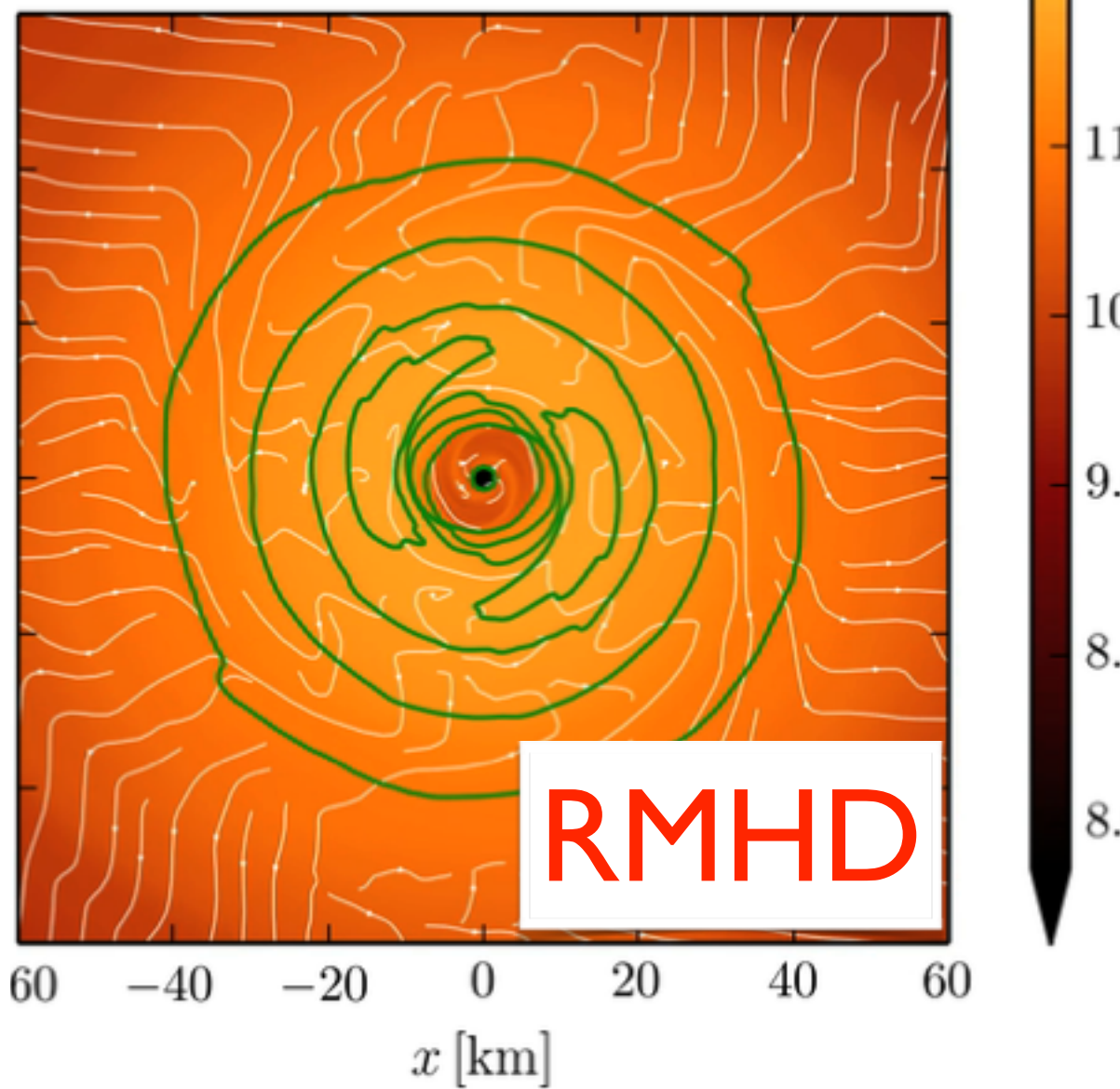
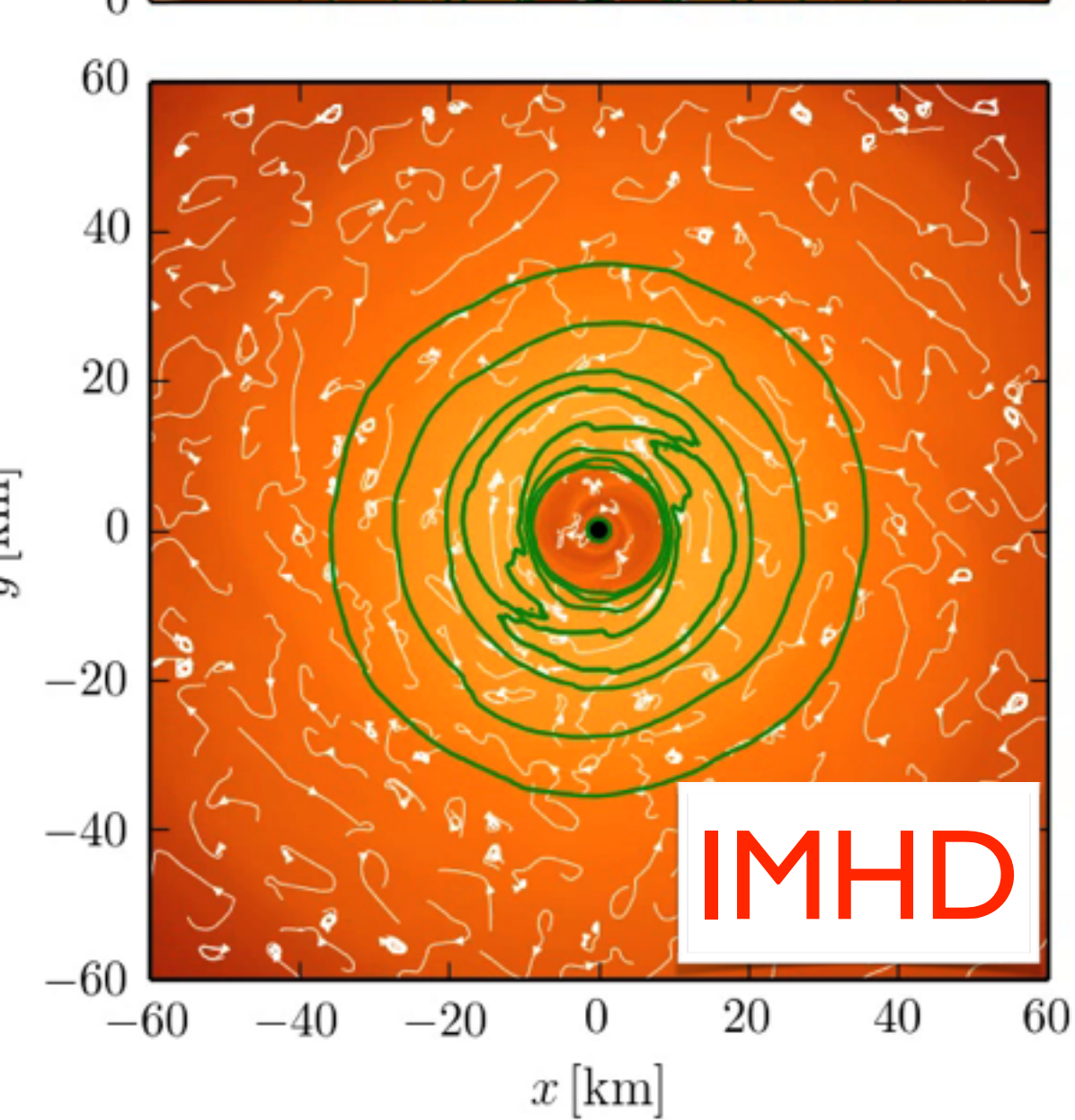
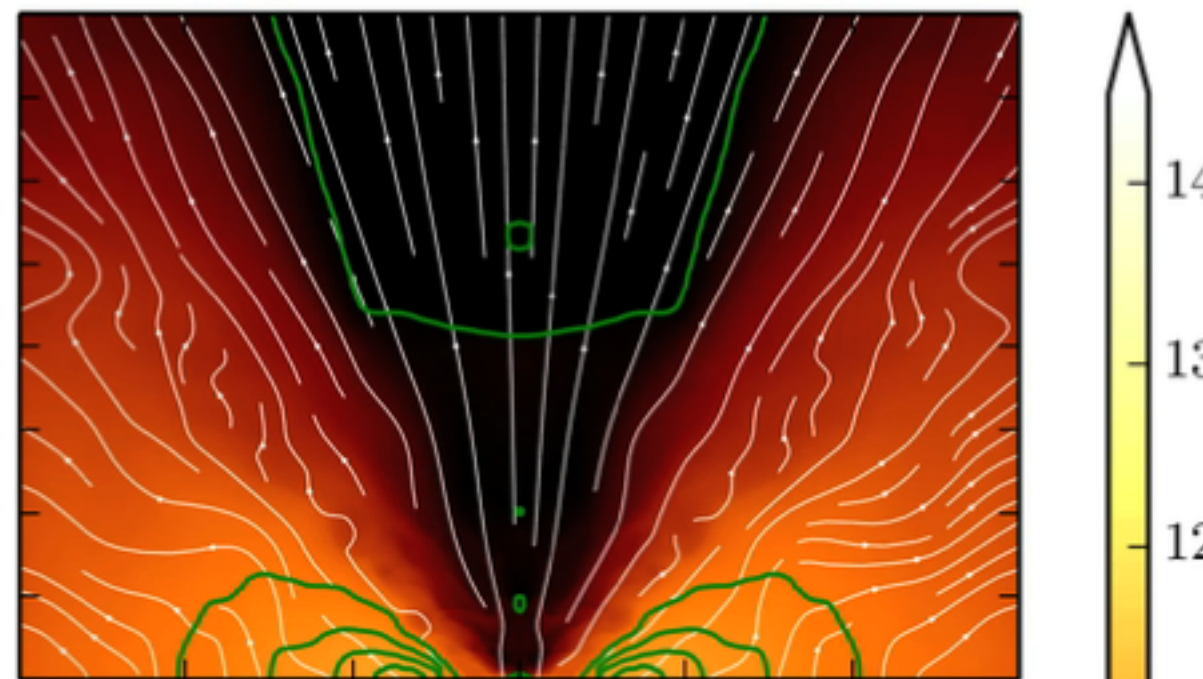




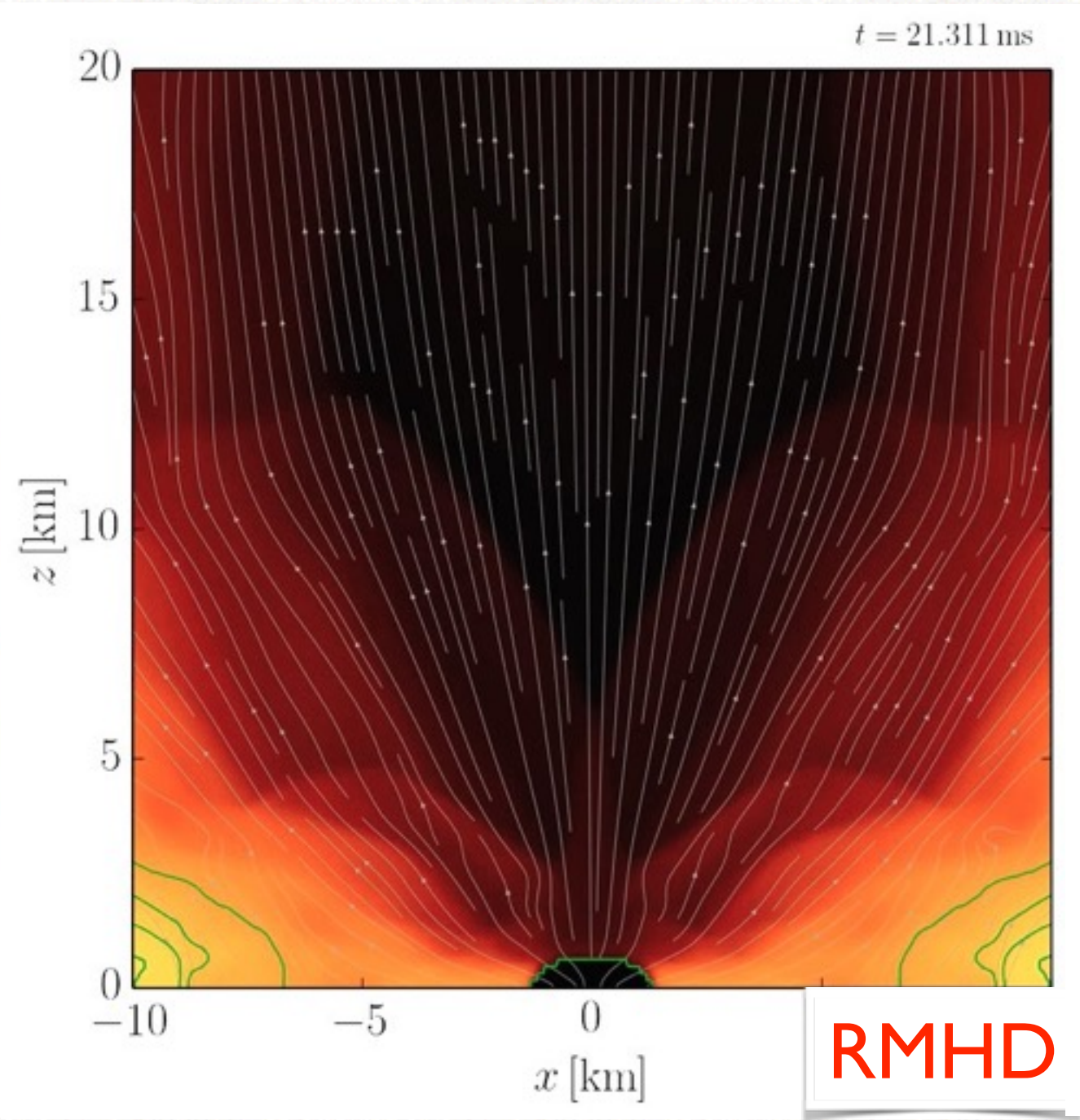
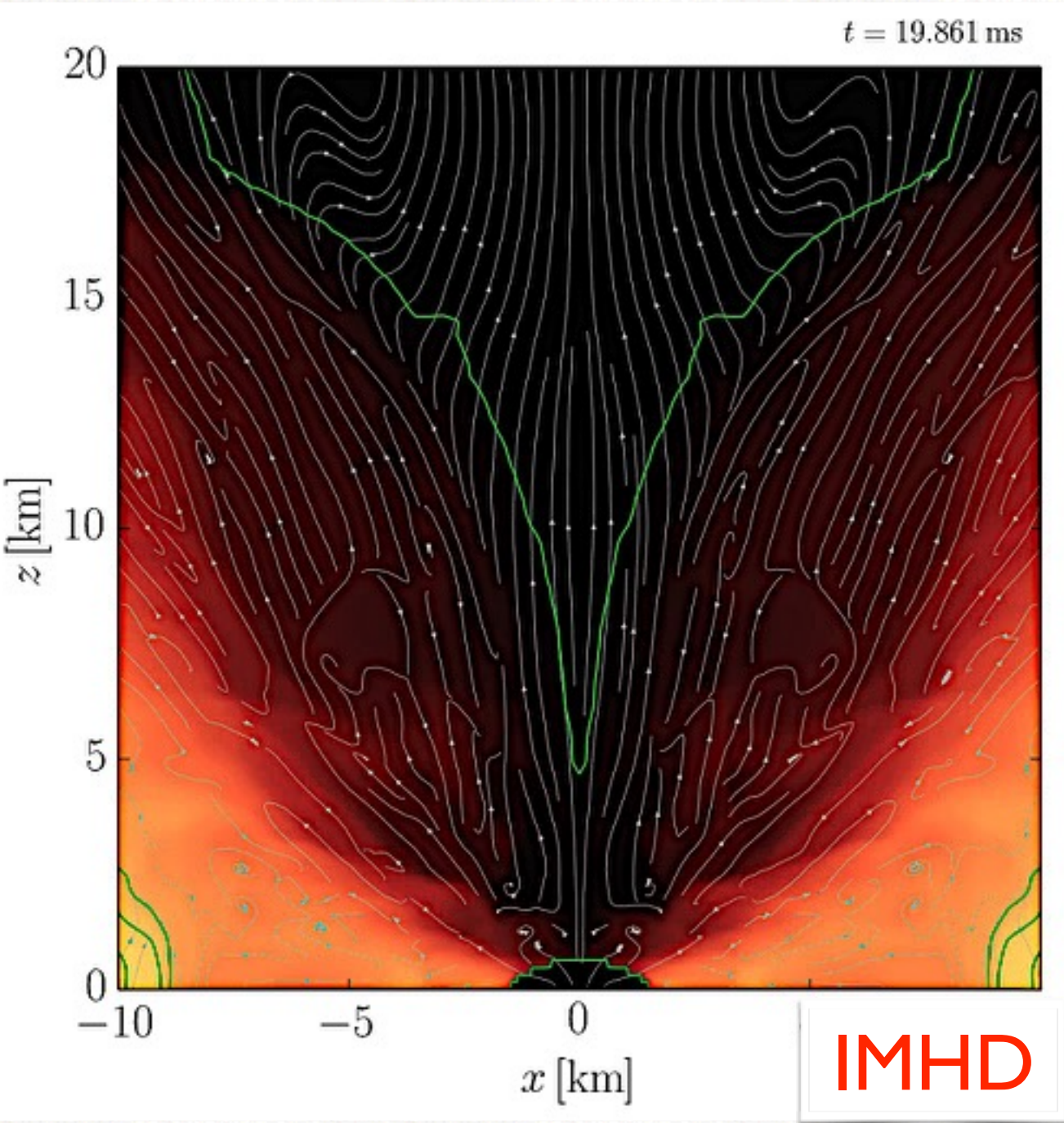
$t = 19.892$  ms



$t = 22.446$  ms







NOTE: the **magnetic jet structure** is **not** an **outflow**. It's a plasma-confining structure.

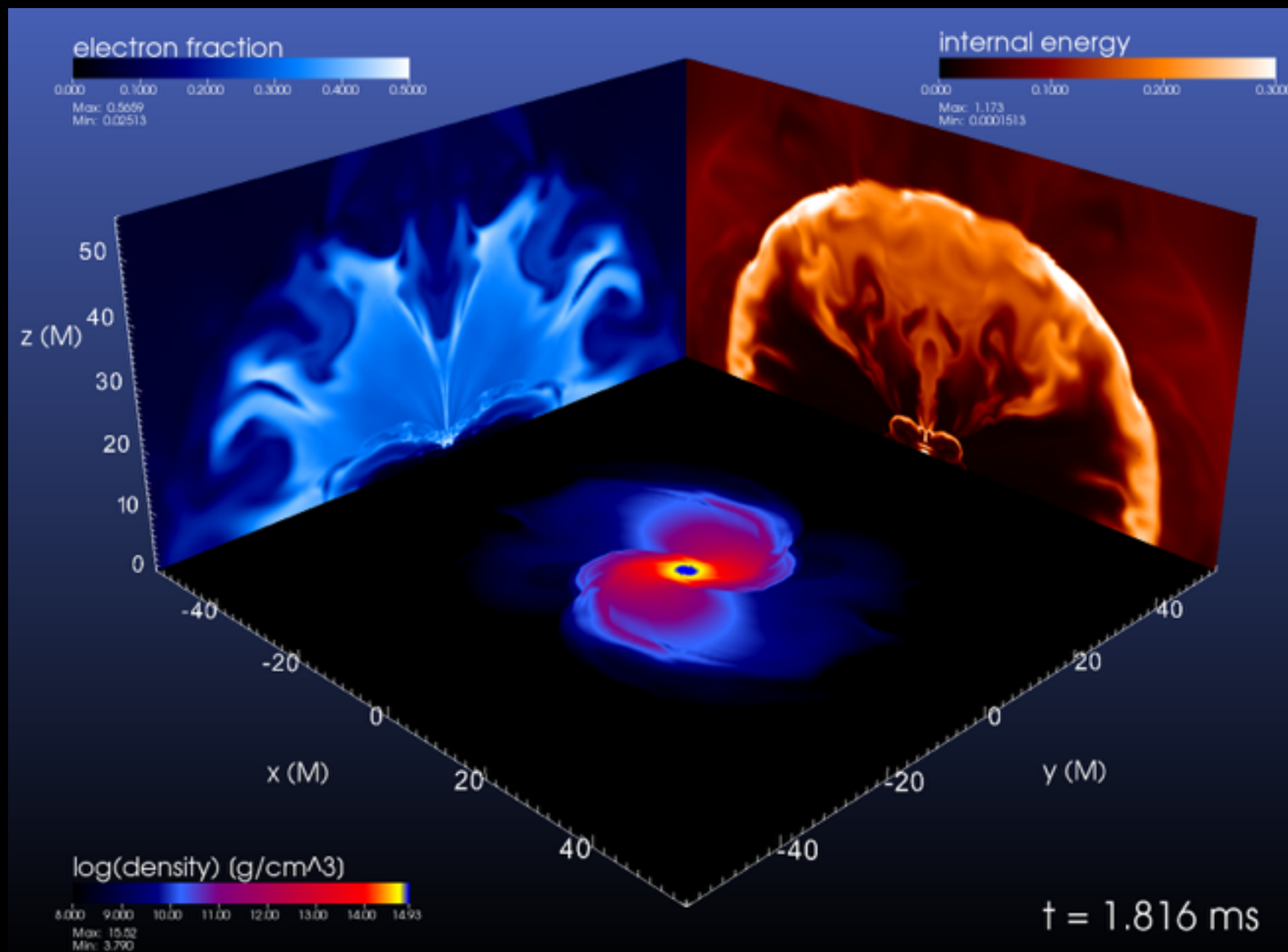
In **IMHD** the magnetic jet structure is present but less regular.

In **RMHD** it is more regular at all scales.



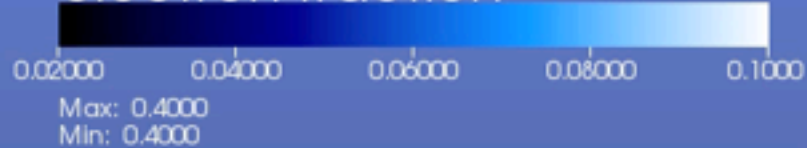
# Dynamically captured binaries and nucleosynthesis

see Piran's, Sekiguchi's, and Tanaka's talks

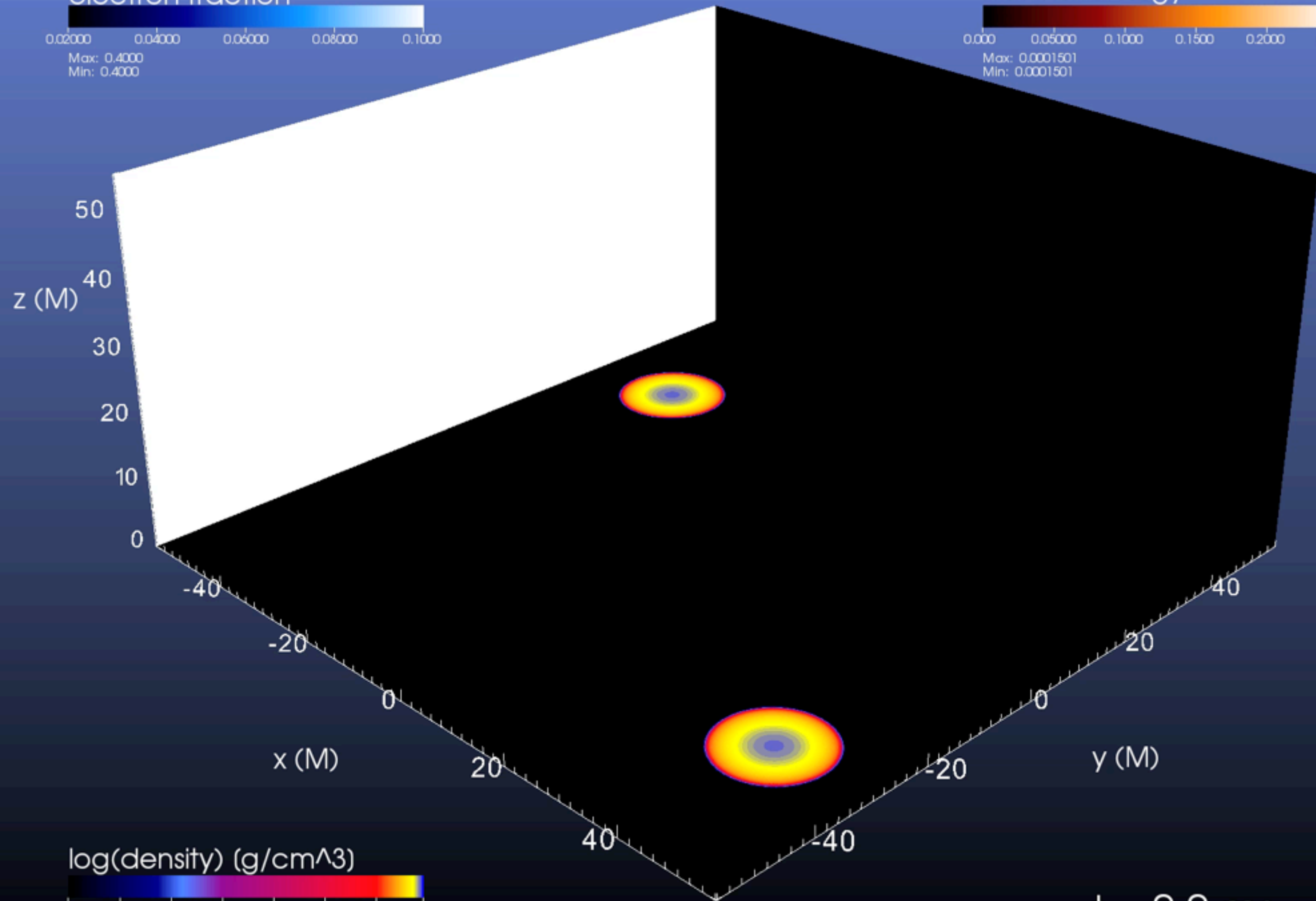
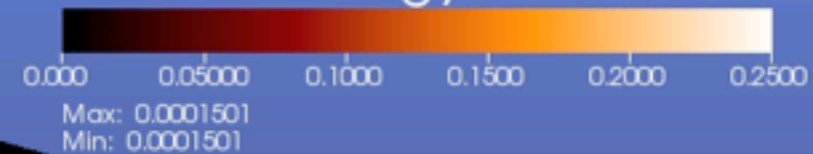




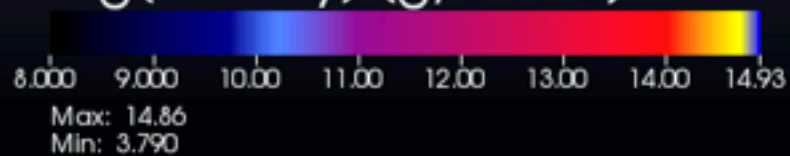
electron fraction



internal energy



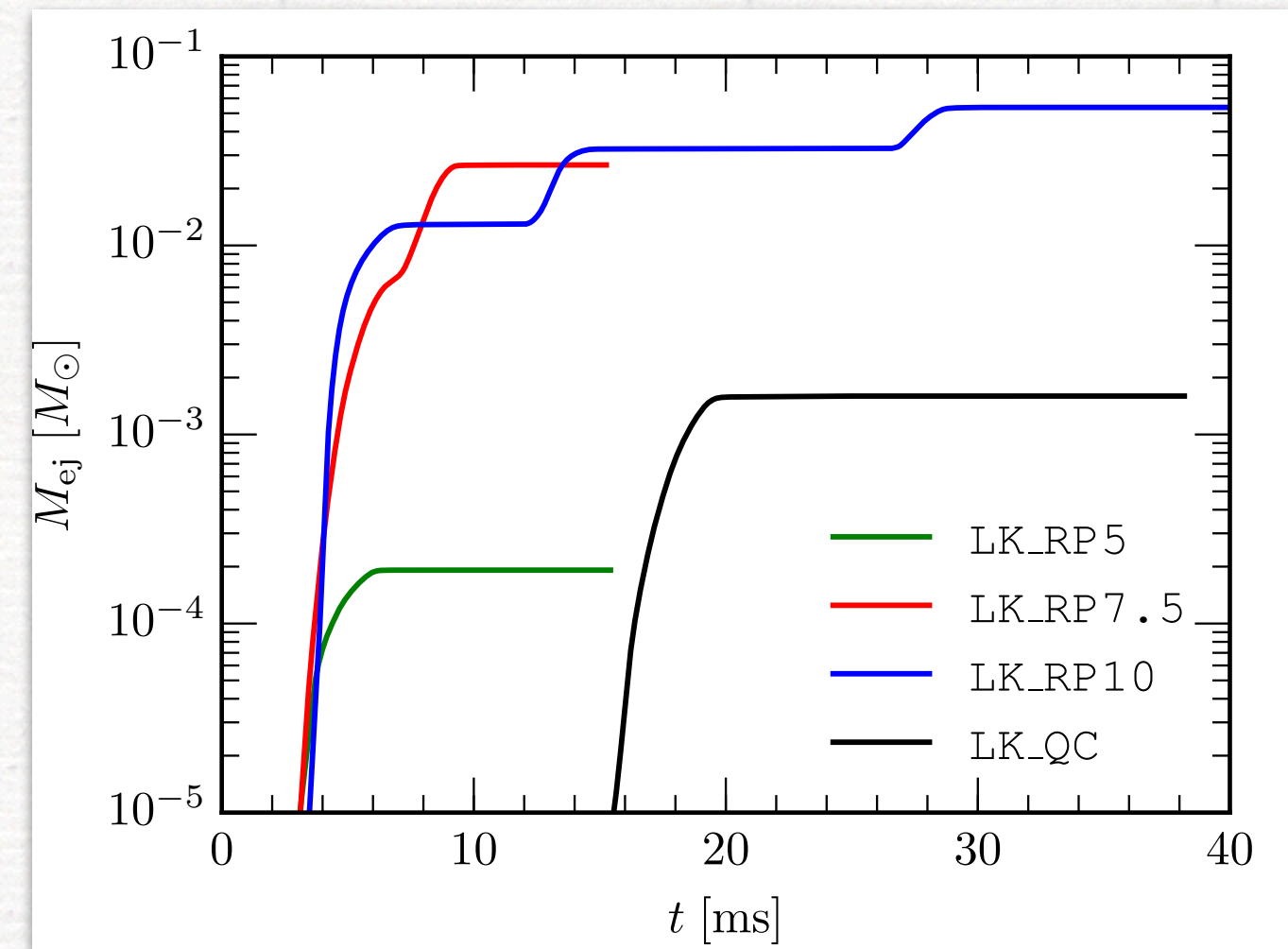
log(density) (g/cm<sup>3</sup>)



$t = 0.0$  ms

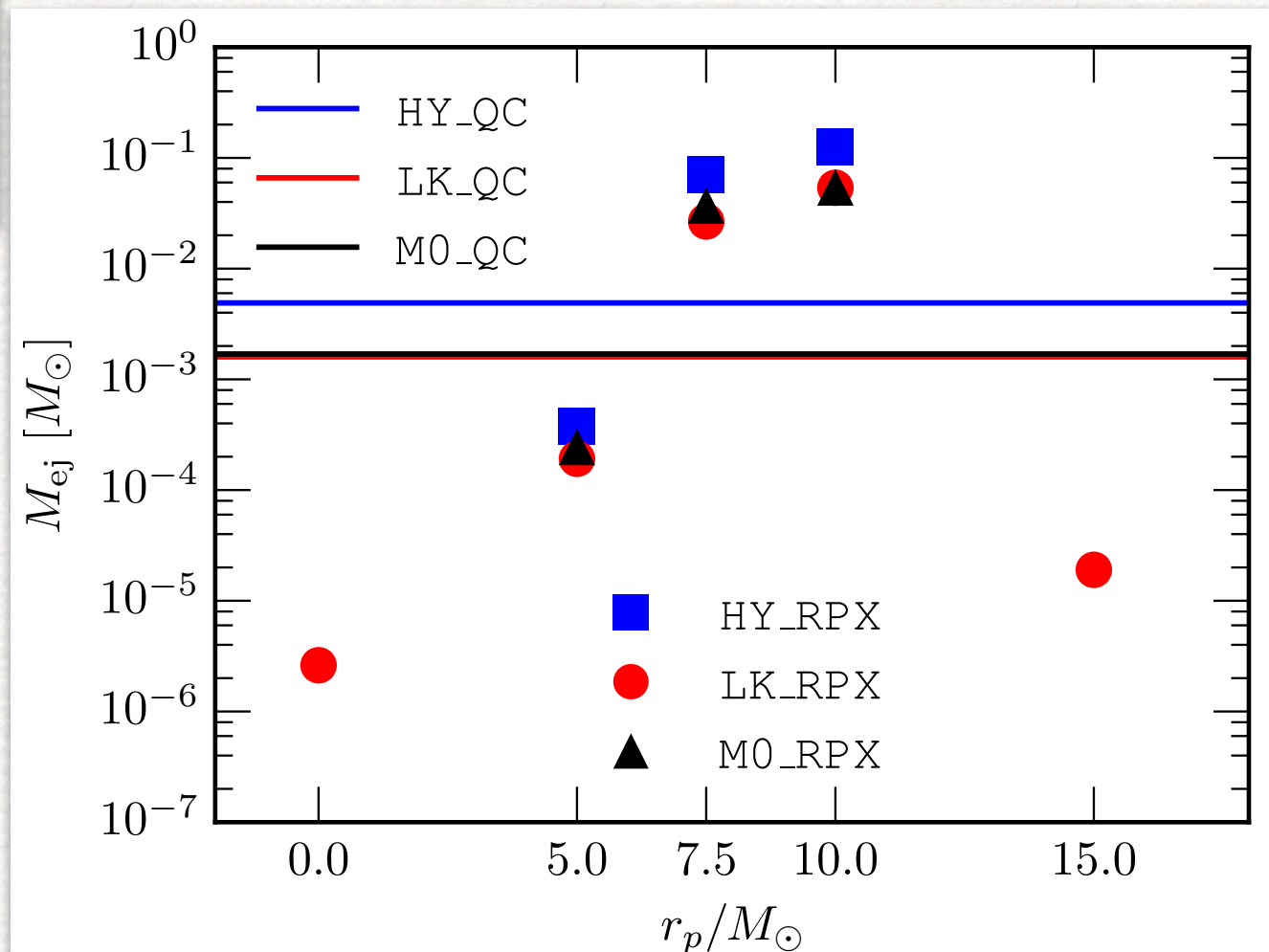
animations by J. Papenfort, L. Bovard, LR

# Mass ejection



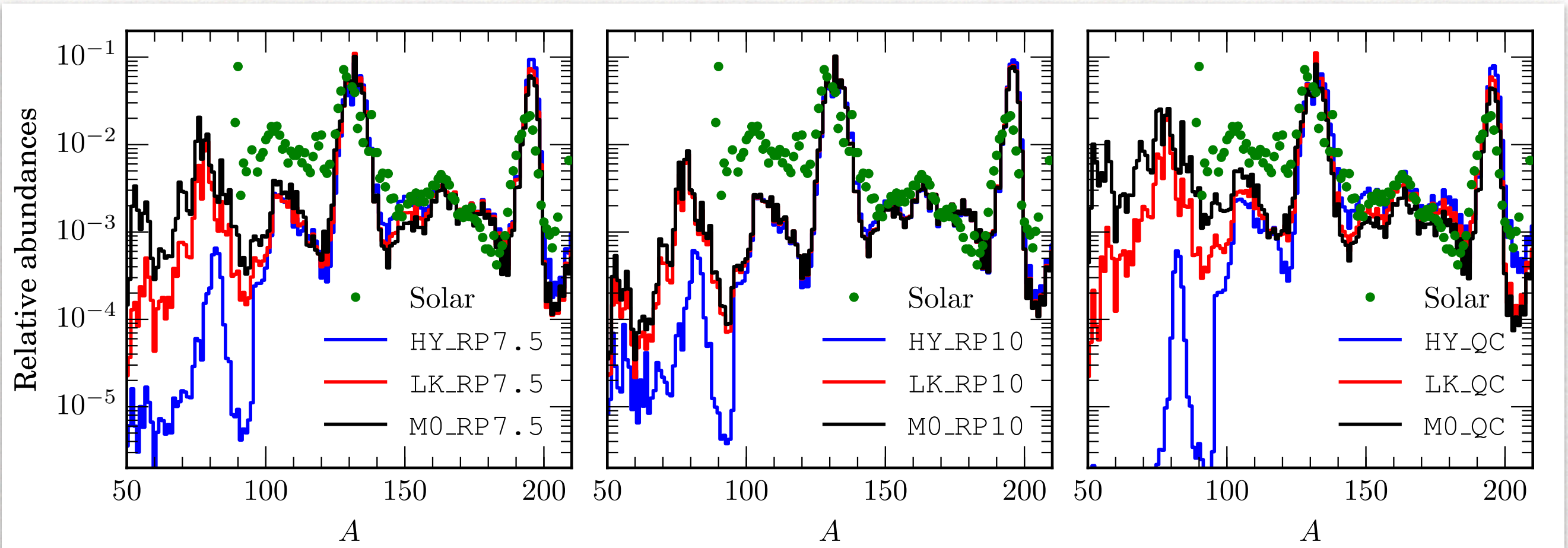
- Mass ejected depends on whether neutrino losses are taken into account (less ejected mass if neutrinos are taken into account)

- Mass ejected depends on impact parameter and takes place at each encounter.
- Quasi-circular binaries have smaller ejected masses (1-2 orders of magnitude)





# Nucleosynthesis



- Ejected matter undergoes **nucleosynthesis** as expands and cools.
- Abundance pattern for  $A > 120$  is robust and good agreement with solar (2nd and 3rd peak well reproduced)
- Abundances very **robust**: essentially the same for eccentric or quasi-circular binaries

# Macronova emission

Energy via radioactive decay of r-process nuclei powers transients in optical/near-infrared with **peak emission** after (Grossman+ 14)

$$t_{\text{peak}} = 4.9 \left( \frac{M_{\text{ej}}}{10^{-2} M_{\odot}} \right)^{1/2} \times \left( \frac{\kappa}{10 \text{ cm}^2 \text{ g}^{-1}} \right)^{1/2} \left( \frac{\langle v_{\infty} \rangle}{0.1 c} \right)^{-1/2} \text{ days},$$

The **peak bolometric luminosity** is estimated to be (“ectonova”)

$$L = 2.5 \times 10^{40} \left( \frac{M_{\text{ej}}}{10^{-2} M_{\odot}} \right)^{1-\alpha/2} \times \left( \frac{\kappa}{10 \text{ cm}^2 \text{ g}^{-1}} \right)^{-\alpha/2} \left( \frac{\langle v_{\infty} \rangle}{0.1 c} \right)^{\alpha/2} \text{ erg s}^{-1}.$$

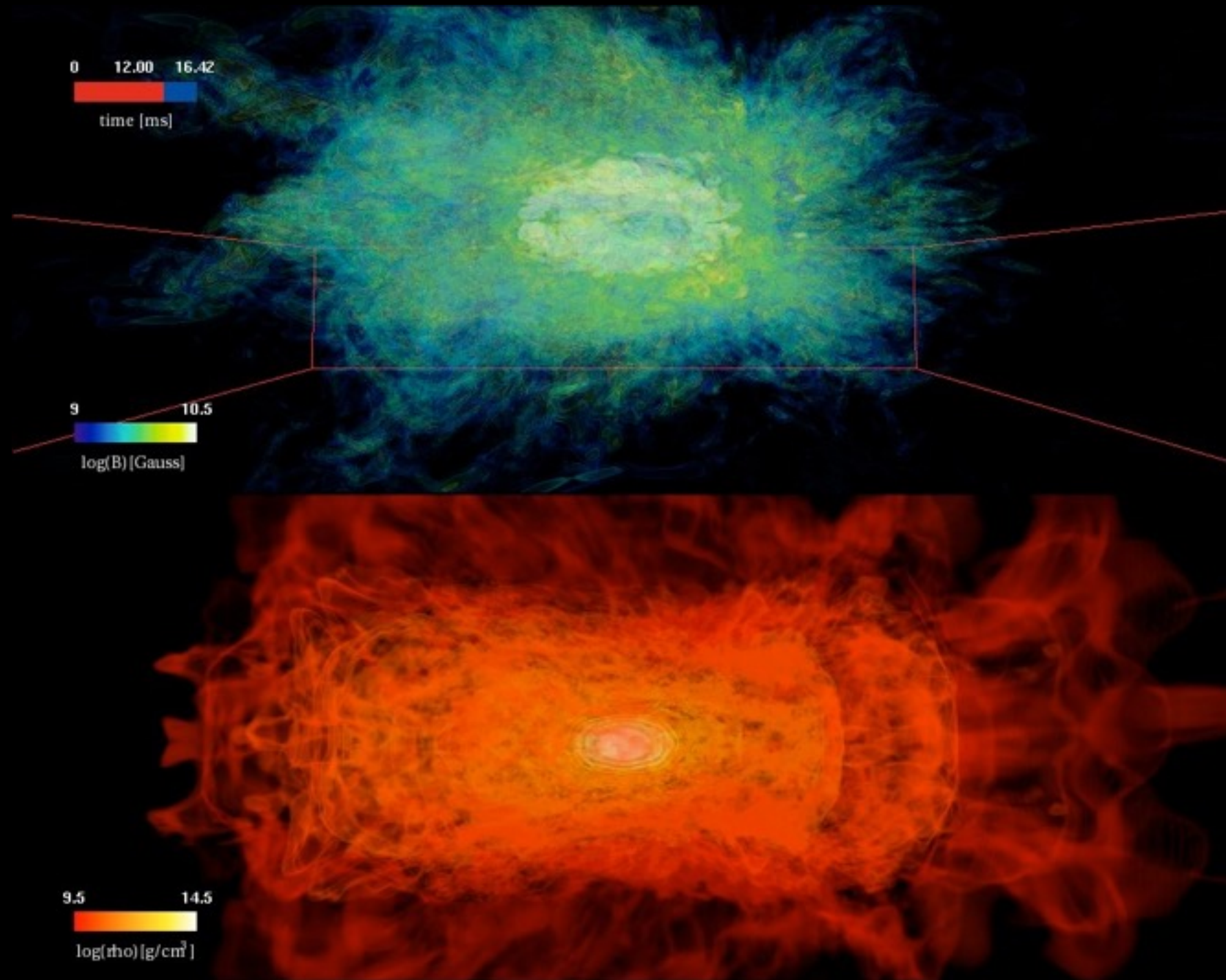
with radioactive energy release a power law  $\dot{\epsilon} = \dot{\epsilon}_0 (t/t_0)^{-\alpha}$ ,  $\alpha \simeq 1.3$

Eccentric binaries:  $\sim$  **4 times more luminous** than quasi-circular;  
**delayed peak emission:**  $\sim$  8 days (cf. 1.5 days)

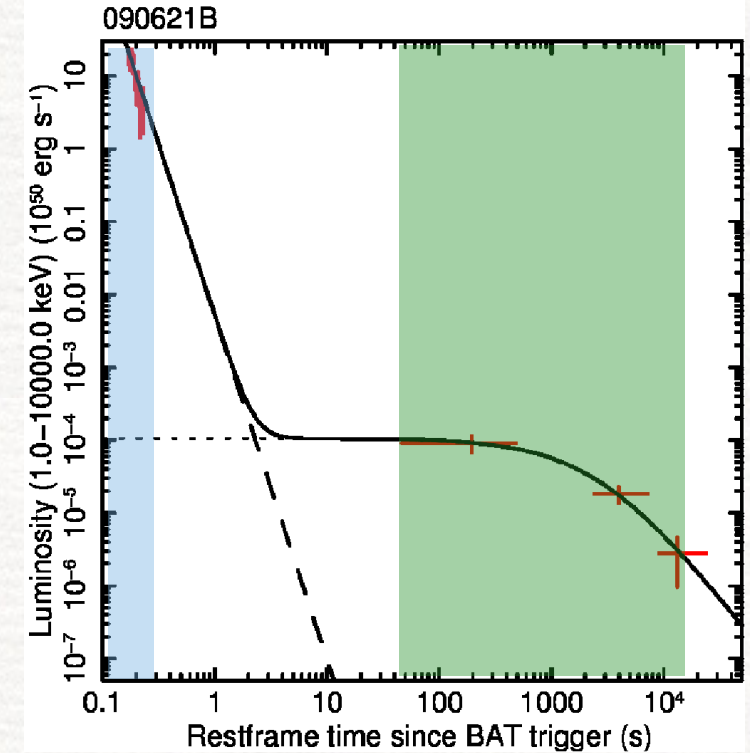
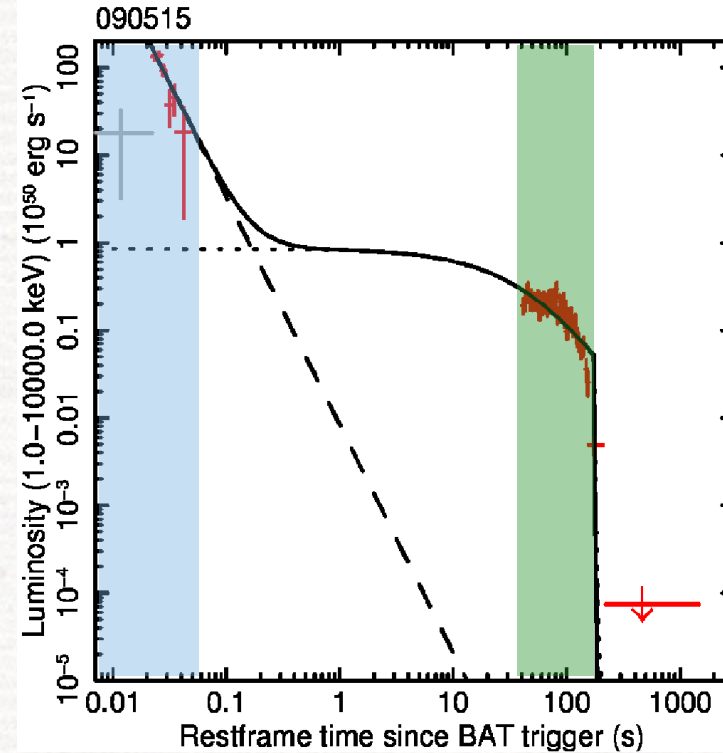
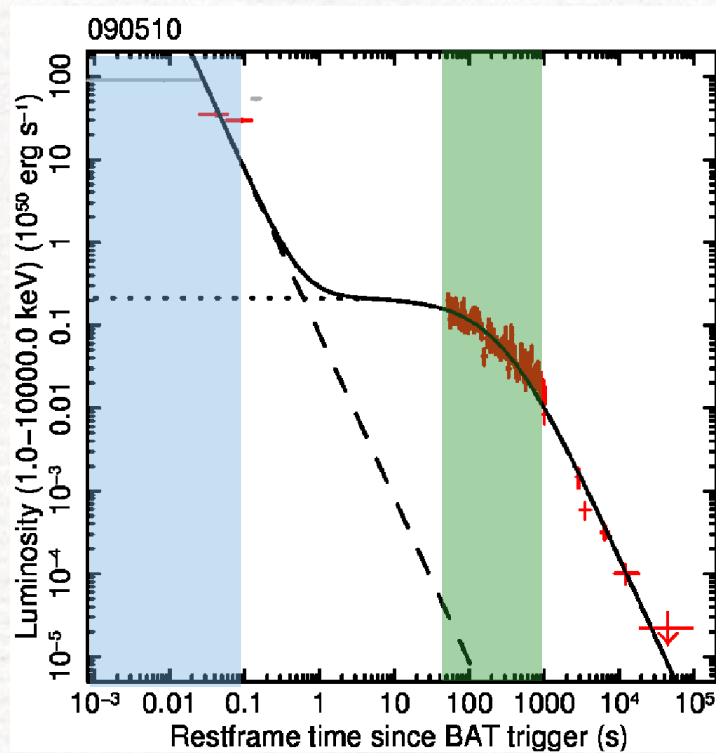


# X-ray emission

see Zhang's talk



# Do we understand X-ray afterglows?



- X-ray afterglows have been observed by Swift lasting as long as  $10^2$ – $10^4$  s (Rowlinson+ 13; Gompertz+ 13)
- The X-ray afterglow could also be produced by a “magnetically-driven” wind generated by differential rotation (Siegel+ 14)
- The X-ray afterglow could be produced by “proto-magnetar”: dipolar emission with  $L_x \sim 10^{49} \text{ erg s}^{-1}$  (Zhang & Mezsaros 01, Metzger+ 11, Zhang 13).



# The elephant in the room...

Magnetars are appealing for simplicity but not necessarily a solution

- differential rotation lost over Alfvén timescale:  $< \sim 10$  s; magnetically driven wind **can't explain** sustained emission for  $10^3$ - $10^4$  s
- X-ray plateaus **follow** the gamma emission, yet magnetar must come **before** the BH-torus.
- simulations do not show any sign of **jet**, which emerges only when **BH-torus** is produced.

Recap:

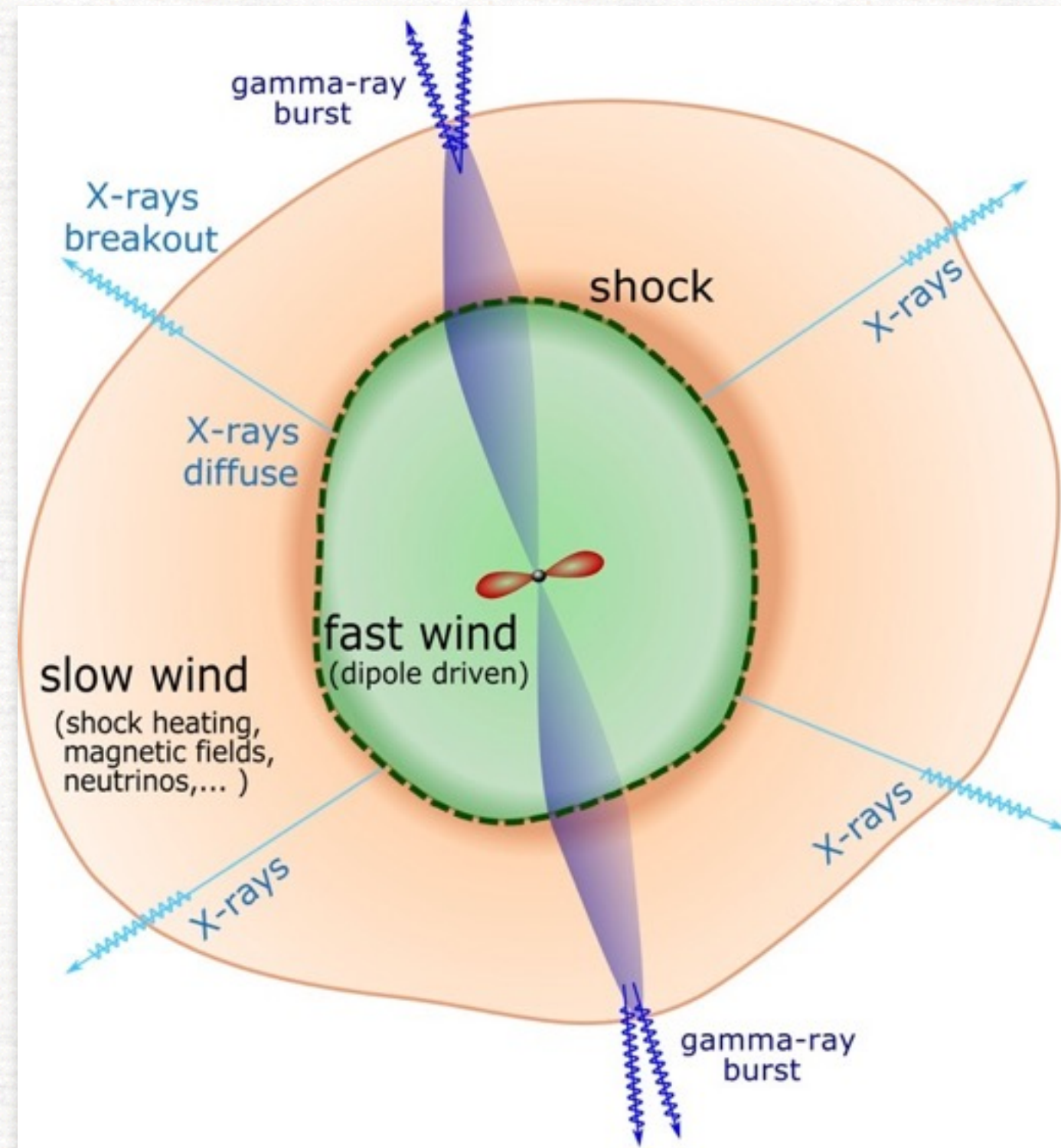
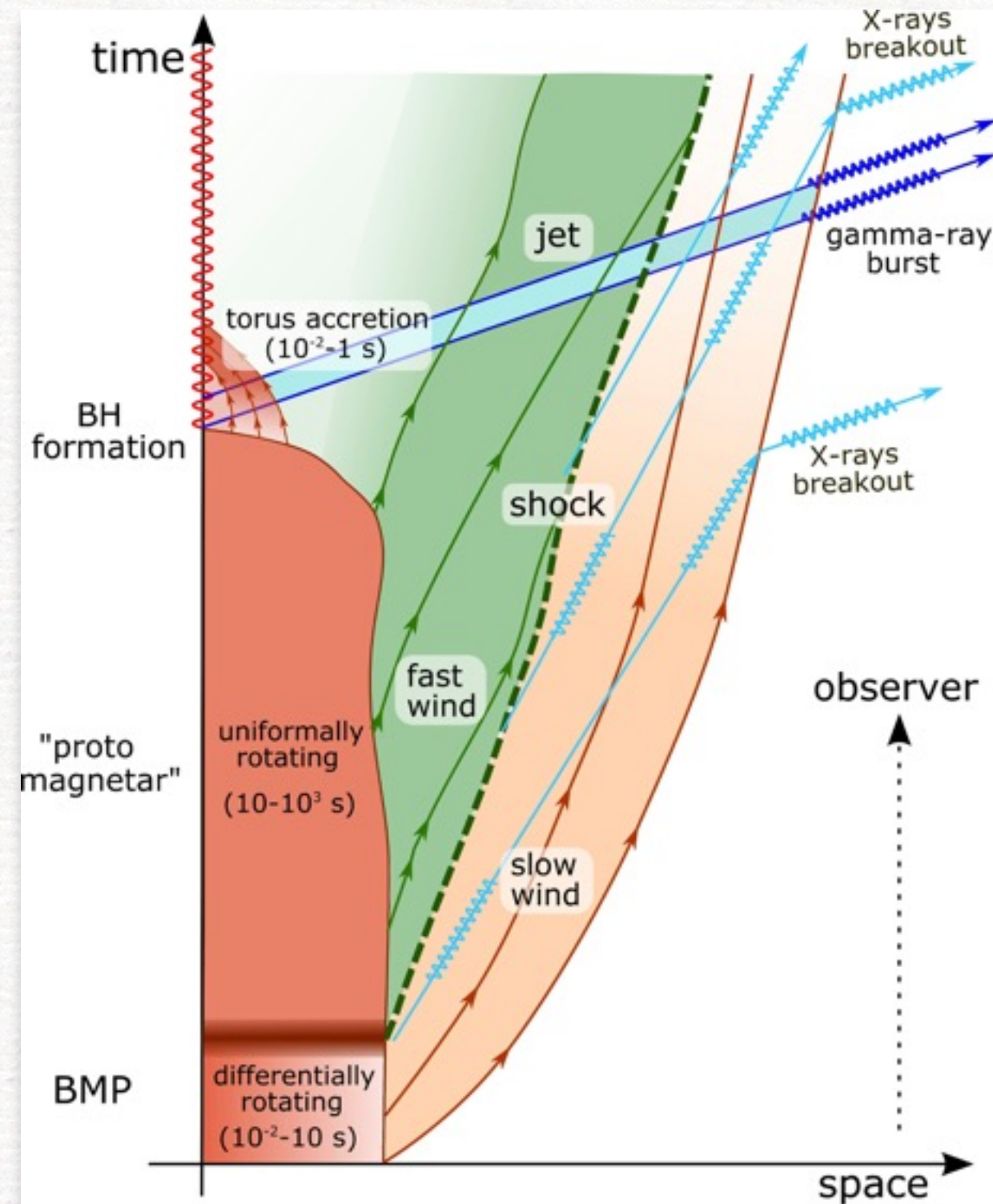
- X-rays produced by metastable magnetar
- gamma-rays produced by jet and BH-torus system

**Riddle:** How can the gammas arrive before the X-rays?



# A solution to the riddle?

LR, Kumar (2014) (also Ciolfi, Siegel 2014)





# A novel paradigm for GRBs?

LR, Kumar (2014)

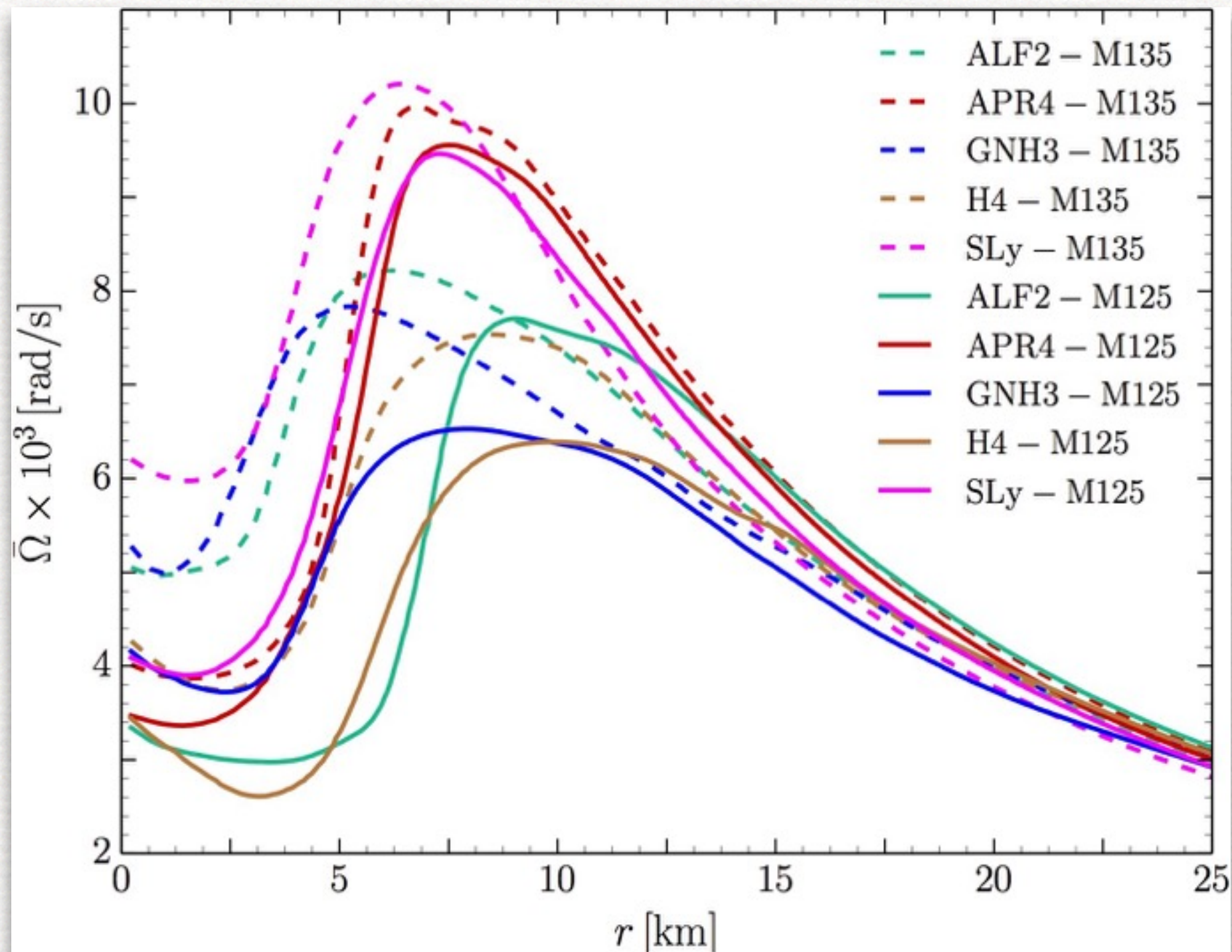
- ***solves the timescale riddle:*** X-ray luminosity is produced by HMNS and can last up to  $10^4$  s
- ***solves the timing riddle:*** X-ray emission is produced before gamma emission but propagates more slowly.
- ***consistent with simulations:*** slow wind is produced in many ways.
- ***unifying view with long GRBS:*** jet propagates in confining medium.
- ***predictions:*** X-ray emission possible before gamma; IC of thermal photons at break out.
- GW signal peak could be much ***earlier*** than gamma emission.
- ***potential problem:*** need a disk at collapse and this could be difficult (Margalit+15).



# An important requirement

Hanauske+ 2016

Angular velocity profiles of HMNS for different EOSs/masses:  
inner core uniformly rotating and “disk” on Keplerian orbits

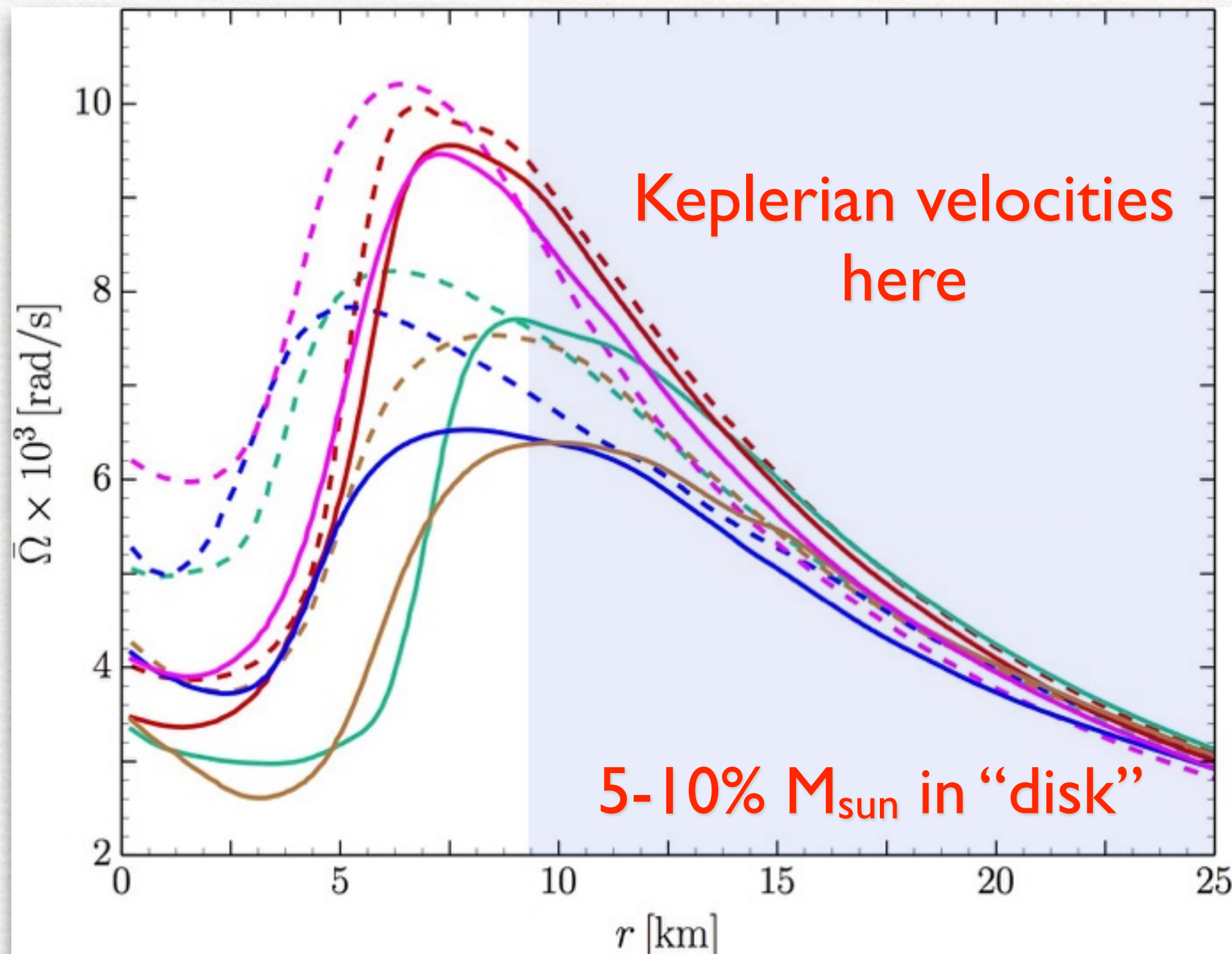




# An important requirement

Hanauske+ 2016

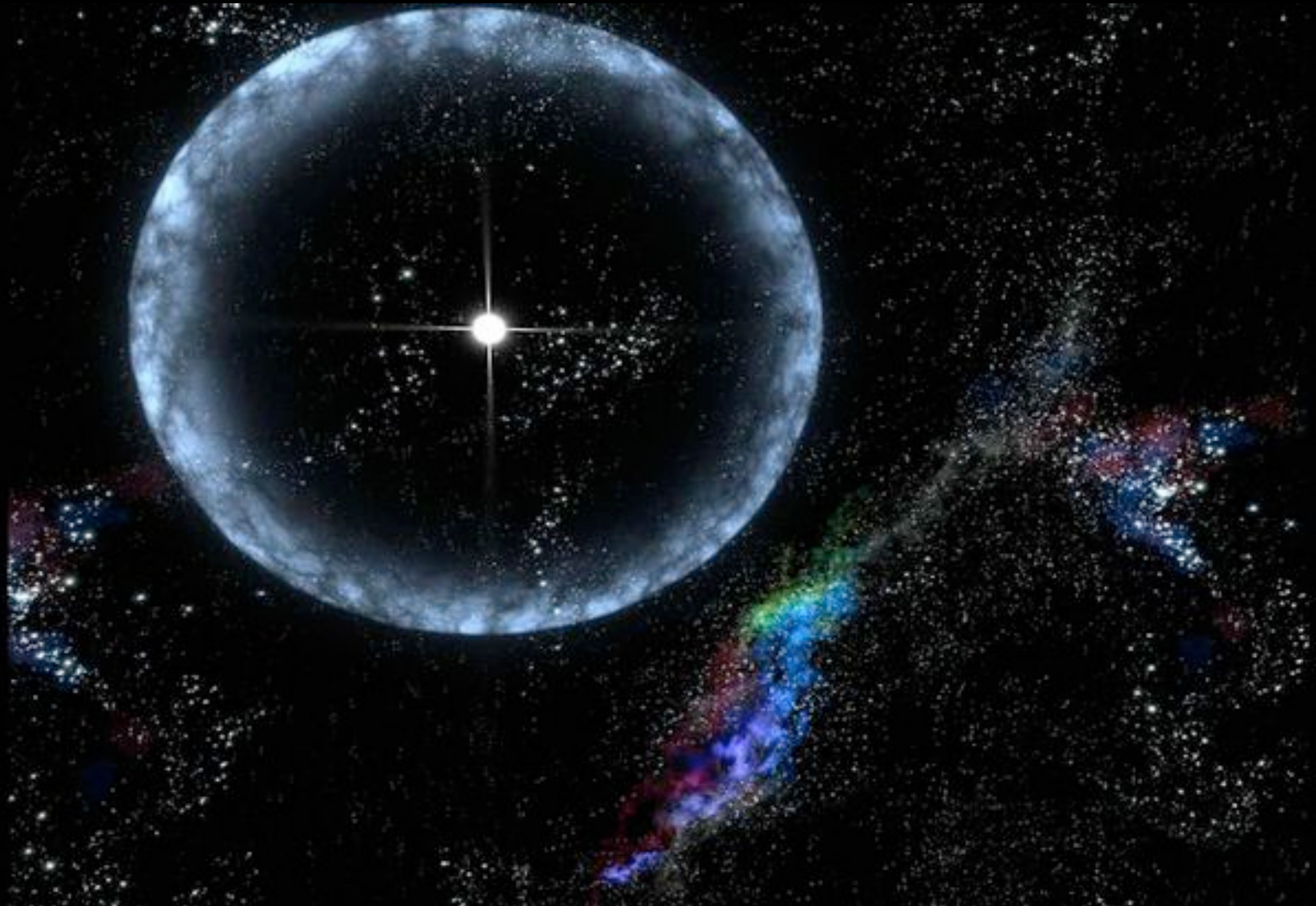
Angular velocity profiles of HMNS for different EOSs/masses: inner core uniformly rotating and “disk” on Keplerian orbits





# The riddle of FRBs

see Lorimer's and Zhang's talks





# Fast Radio Bursts

Several **fast radio bursts** (FRBs) have been discovered recently (Keane+ 2012, Thornton+ 2013, Spitler+ 2014):

- single bright, highly dispersed millisecond radio pulses;
- the high dispersion suggests sources at cosmological distances ( $z > 0.7$ ); expected rate:  $\simeq 0.1 \text{ deg}^{-2} \text{ day}^{-1} \sim 1\%$  that of SNe;
- assuming a cosmological distance, the luminosity is

$$L = 3 \times 10^{43} \left( \frac{\nu}{1.4 \text{ GHz}} \right)^{1+\alpha} \left( \frac{S_\nu}{1 \text{ Jy}} \right) \left( \frac{D_1}{11 \text{ Gpc}} \right)^2 \text{ erg sec}^{-1} .$$

- this luminosity is nine orders of magnitude larger than a giant kJy flare from Crab; over 1 ms this yields an energy which is a tiny fraction of the energy in a SN or GRB.



# A cartoon...

**1** Overweight neutron star collapses

But a neutron star can only enjoy retirement as a pulsar if it weighs less than two solar masses. If the core weighed more than two masses when it collapsed, the neutron star will be too heavy to support itself and it will immediately collapse to become a black hole

However, an overweight neutron star can delay its demise if it is spinning fast enough. With enough spin, centrifugal forces fling the star's material outwards – flattening it into a sort of squashed disk (called an oblate spheroid). This outward force is enough to counteract gravity's inward force and, for a while, the neutron star becomes a pulsar.

**2** Centrifugal forces prevent collapse

But it is living on borrowed time...

Because a pulsar's magnetic field isn't aligned with its spin axis, the vast magnetic field flails around in space – sapping the neutron star of rotational energy and behaving like a giant magnetic brake

**3** Spin slows

Magnetic field lines

Over the course of a few hundred million years, this, combined with the energy pumped out via the jets, slows the neutron star's rotation

Spin axis  
Magnetic field alignment

**4**

Without enough spin, the neutron star is at the mercy of its own crushing gravitational power. It takes less than a thousandth of a second for the neutron star to collapse to form a black hole

**5**

Black hole event horizon

Magnetic field severed

Anything caught on the wrong side of the black hole's event horizon (the point at which gravity becomes so extreme not even light can escape) is lost forever in a vortex of broken spacetime. Caught unawares by the sudden disappearance of its electromagnetic engine room, the vast magnetosphere finds itself cut off and adrift in space

**6**

Black hole

Magnetic field reconnects

**7**

Blast of radio waves

With the magnetic field suddenly severed, the magnetosphere seeks to reconnect itself. The field lines snap back violently (like when a fully stretched rubber band is cut) – creating an immensely powerful magnetic shock wave that blasts into space at almost the speed of light

This unleashes a surge of electromagnetic radiation (at radio wavelengths) that, in a fraction of second, carries as much energy into space as the Sun manages in a million years

A few billion years later, this energy will be detected on Earth as a brief flash of radio waves...



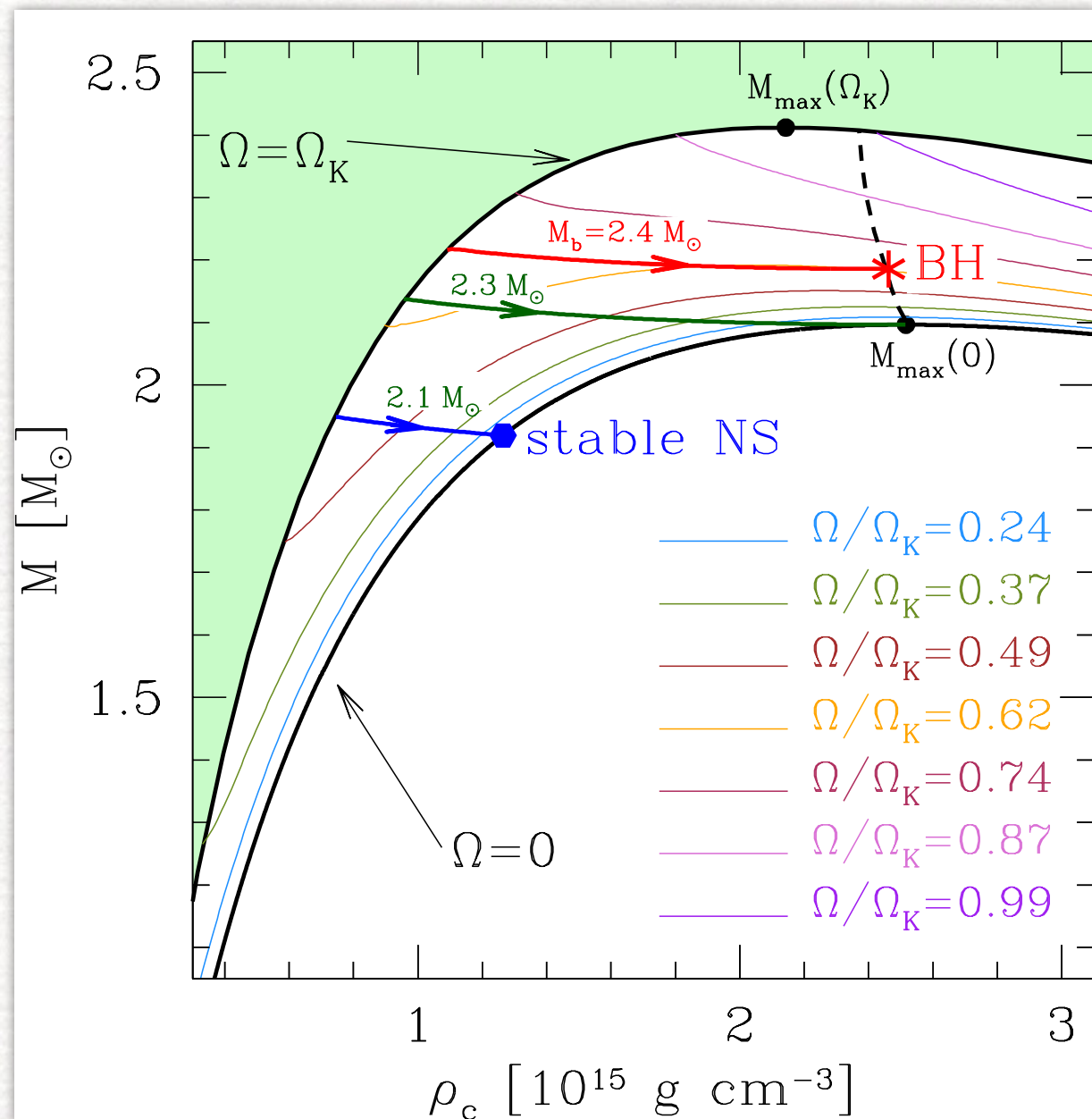
# “Blitzars”

Falcke, LR (13)

Use these constraints: **1)** signal on timescale  $\sim 1\text{ms}$ ; **2)** luminosity of  $10^{43}$  erg/s; **3)** **absence** of other emissions beside radio.

FRBs could be result of collapse of a **supramassive NS** to a BH,

i.e. NS whose large mass can be supported because in rotation.



A NS with mass  $M < M_{\max}(0)$  can support itself against collapse.

Any star with  $M > M_{\max}(0)$  can only collapse.

# Out of our rough estimates...

Falcke, LR+13

- Rate:

1% of core collapse SNe

- Luminosity for coherent curvature radiation (an upper limit?):

$$P_t \simeq 7.0 \times 10^{43} \eta_e \gamma f_{0.1}^2 \kappa_{\text{GJ}}^2 b_{12}^2 m_2 r_{10} \text{ erg s}^{-1}.$$

- Minimum frequency assuming coherent curvature radiation:

$$\nu_p = \frac{\omega_p}{2\pi} = \sqrt{\frac{eB\Omega}{2\pi^2 c m_e}} \simeq 38.6 f_{0.1}^{1/2} \kappa_{\text{GJ}}^{1/2} b_{12}^{1/2} m_2^{1/4} r_{10}^{-3/4} \text{ GHz}.$$

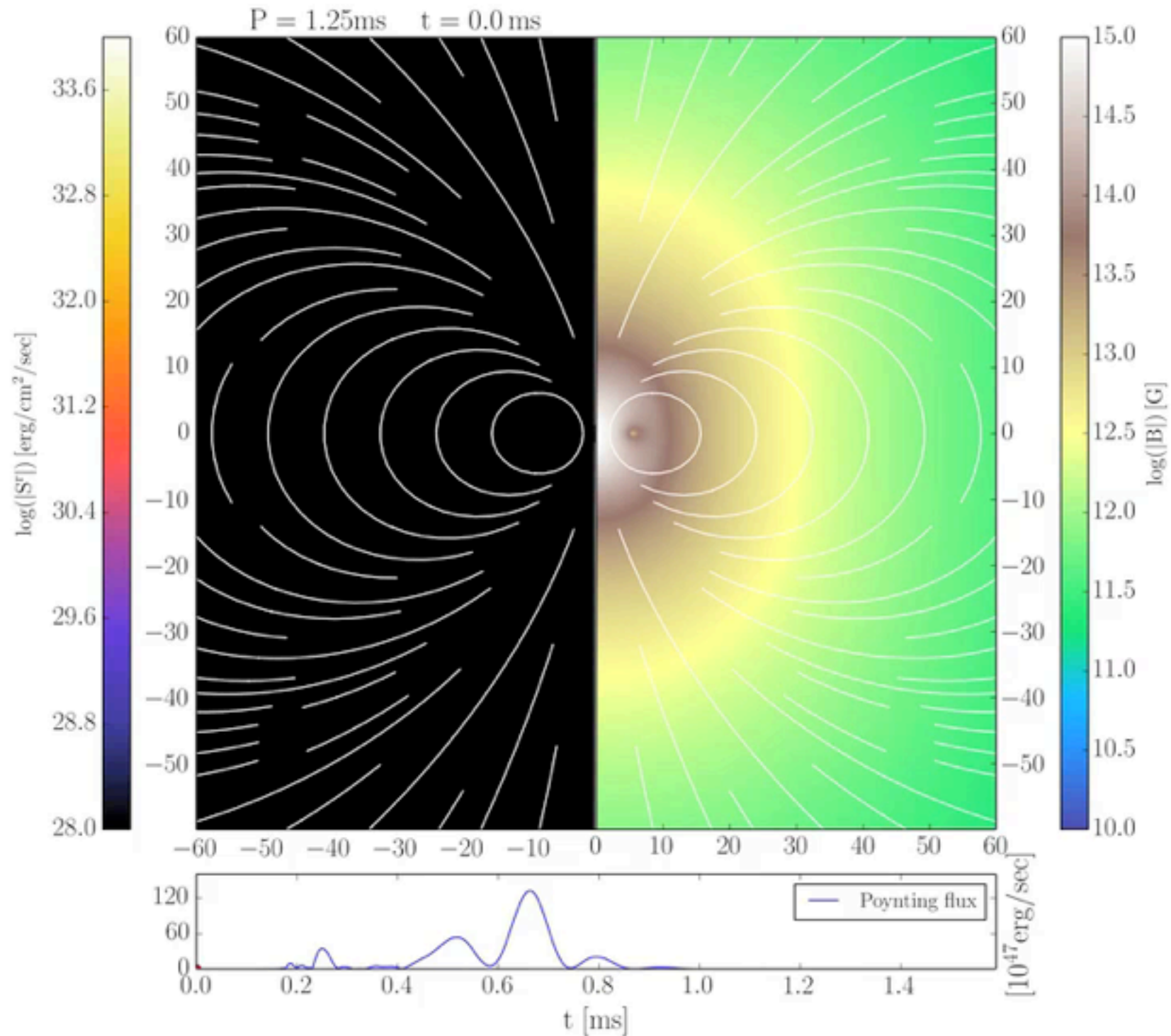
- Need relativistic particles but “reasonably” relativistic:

$$\gamma_{\text{min}} \gtrsim 175.3 f_{0.1}^{1/6} \kappa_{\text{GJ}}^{1/6} b_{12}^{1/6} m_2^{1/12} r_{10}^{1/12}.$$



# Overall dynamics

Most, Nathanail, LR 2016

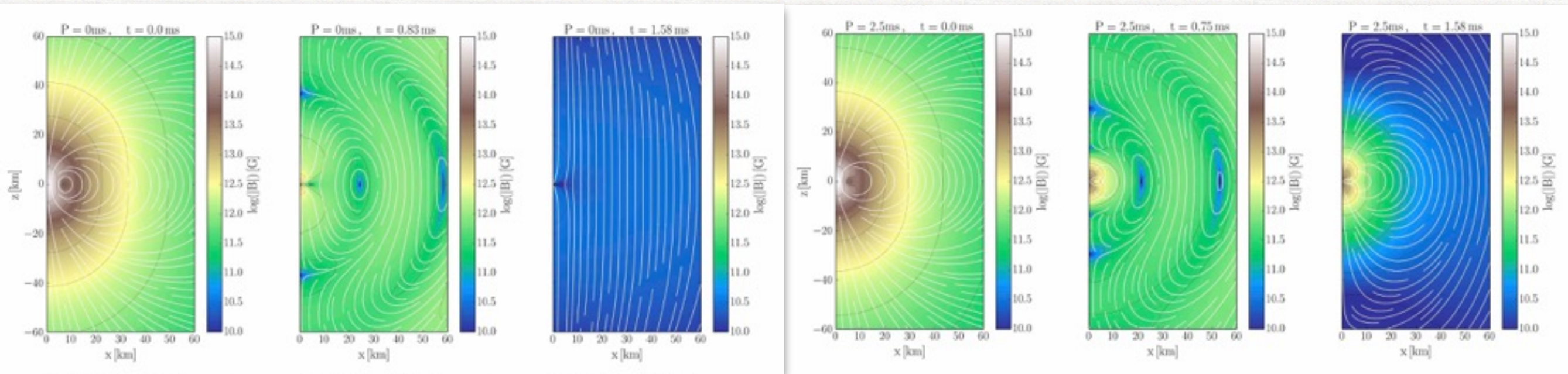


# Overall dynamics

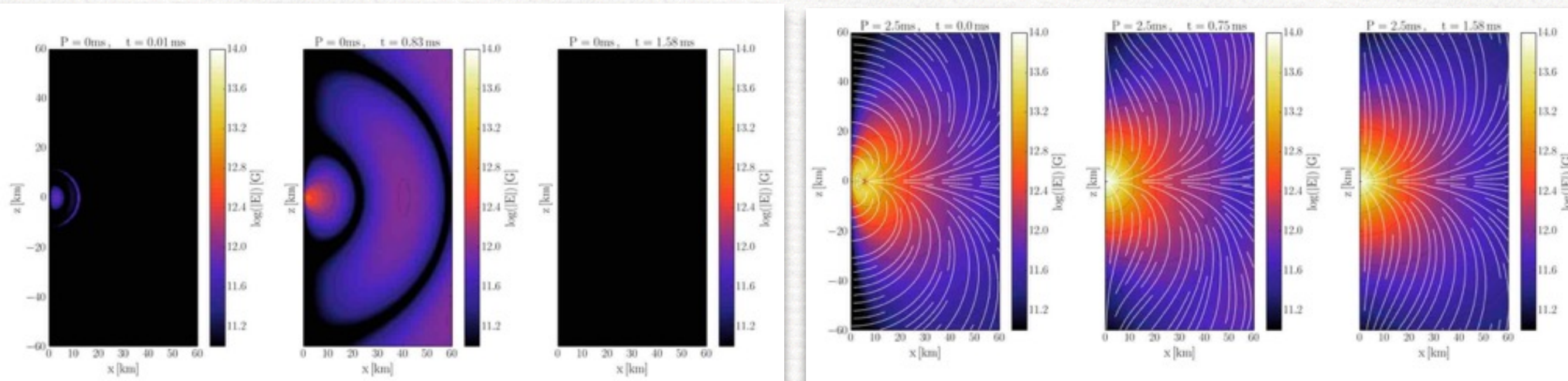
Most, Nathanail, LR 2016

nonrotating magnetised star

rotating magnetised star



B-field



Poynting flux

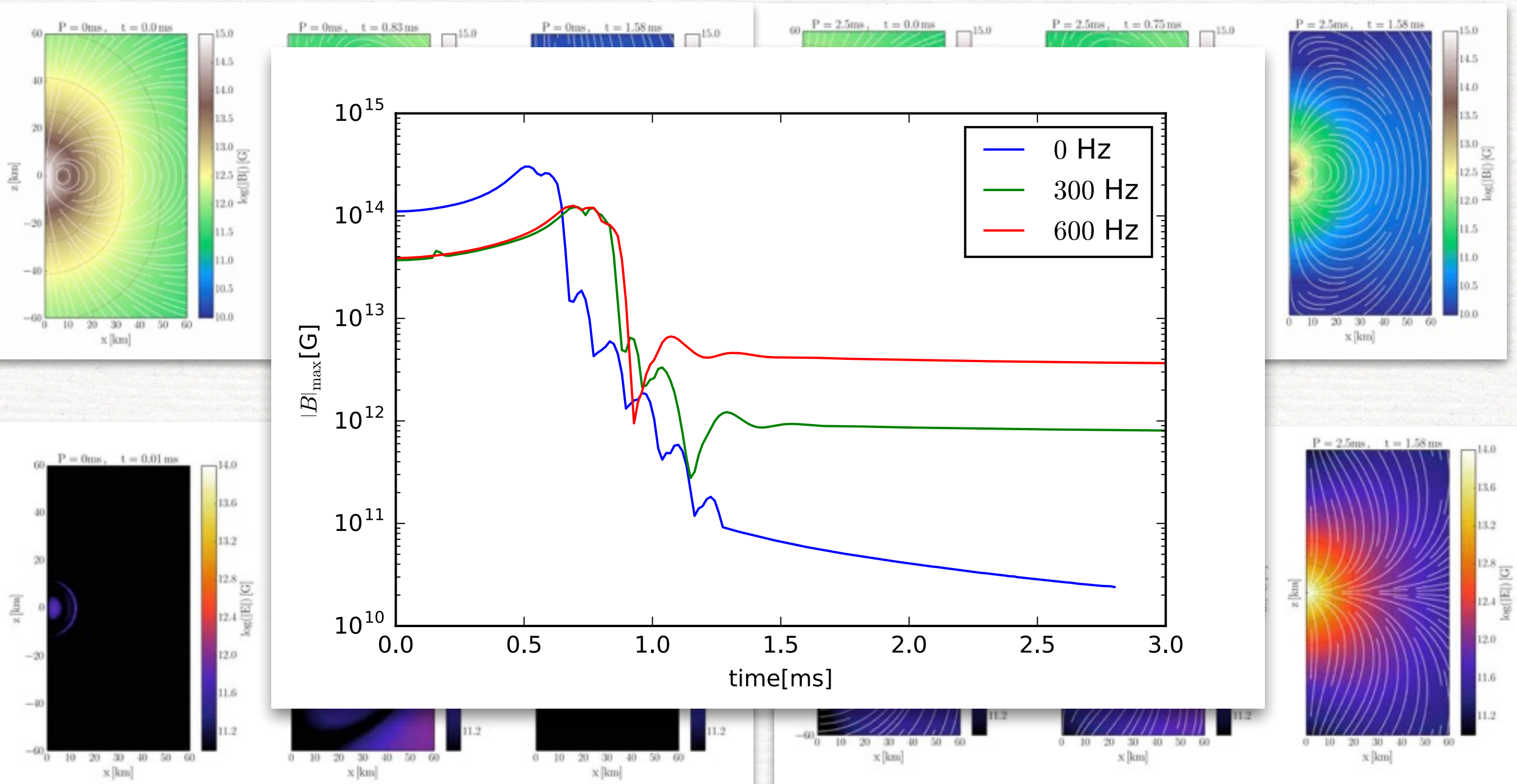


# Overall dynamics

Most, Nathanail, LR 2016

nonrotating magnetised star

rotating magnetised star



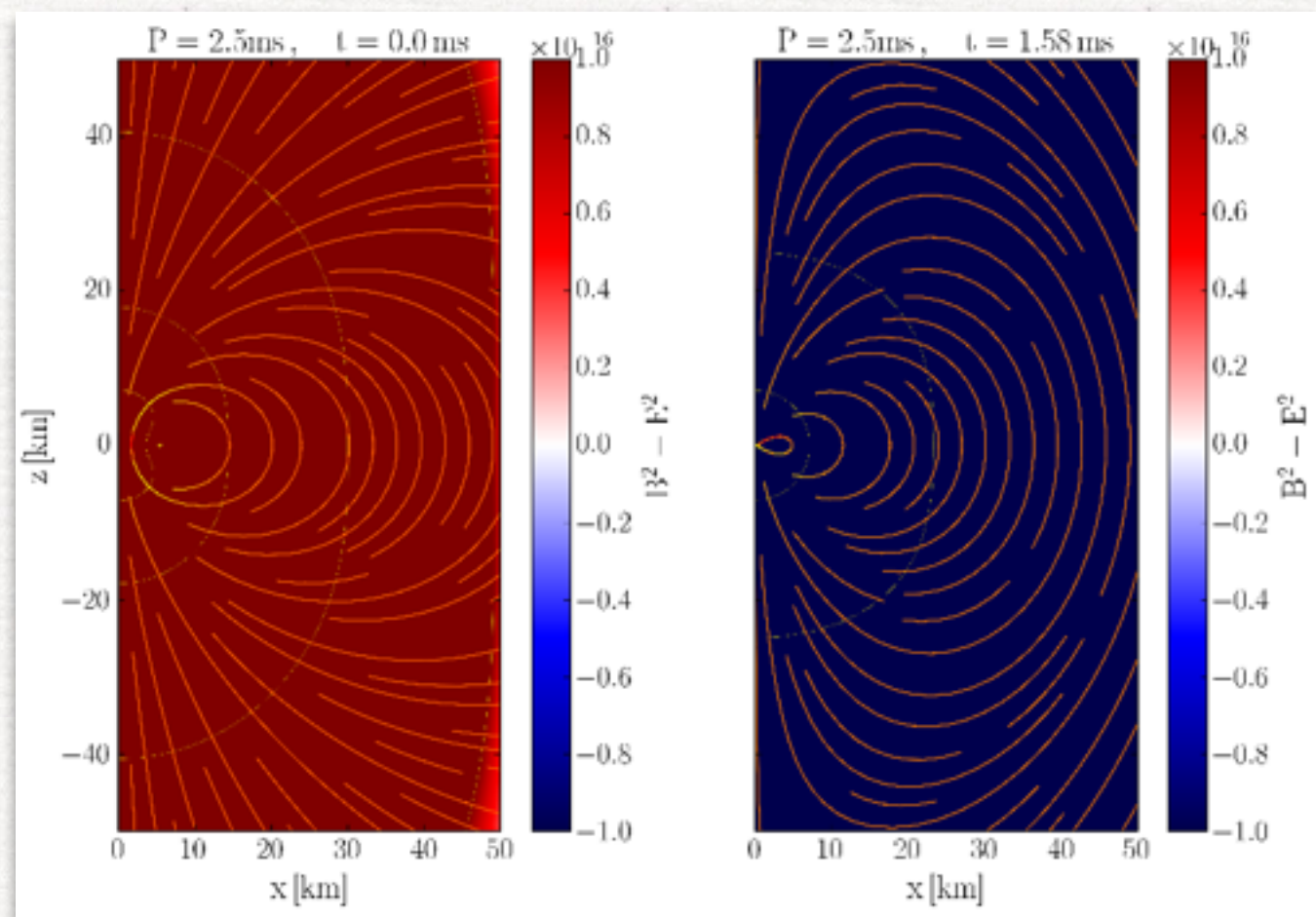
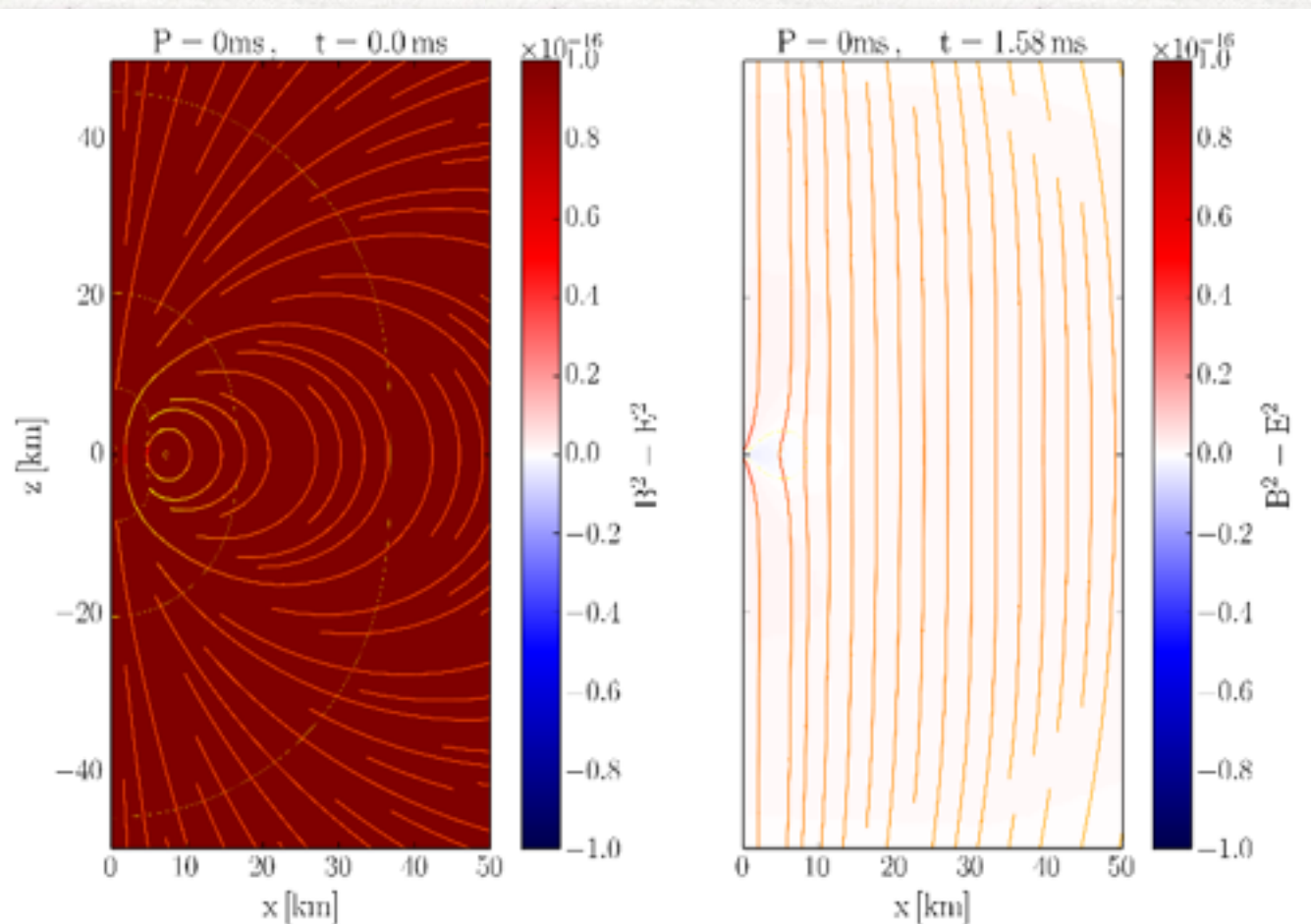
Poynting flux

# Collapse to what?

Nathanail, Most, LR 2016

nonrotating magnetised star

rotating magnetised star



$$\frac{1}{2} F^{\mu\nu} F_{\mu\nu} = B^2 - E^2 = 0$$

$$\frac{1}{2} F^{\mu\nu} F_{\mu\nu} = B^2 - E^2 < 0$$

collapse to **Schwarzschild BH**

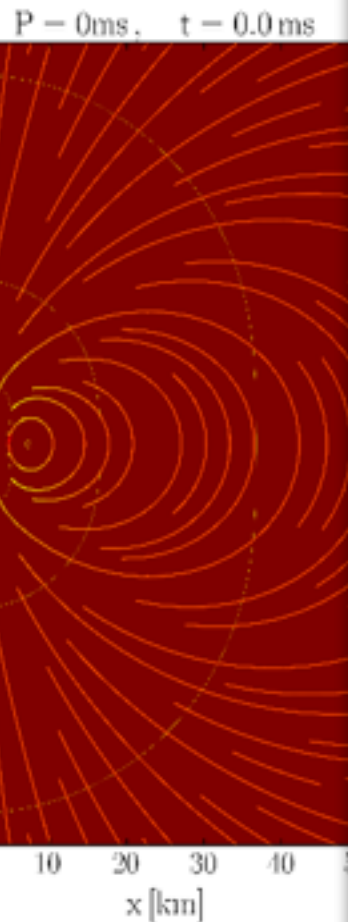
collapse to **Kerr-Newman BH**



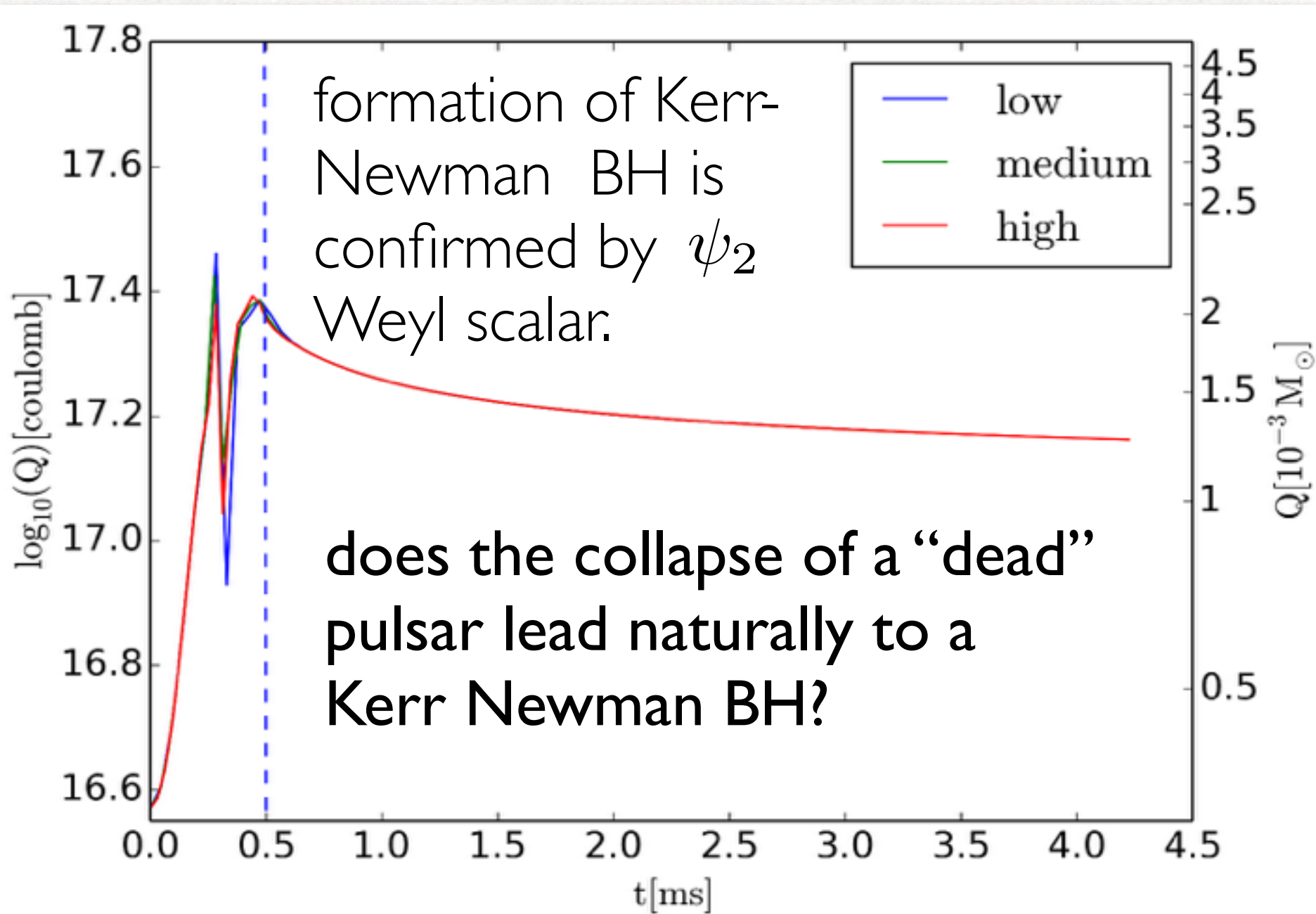
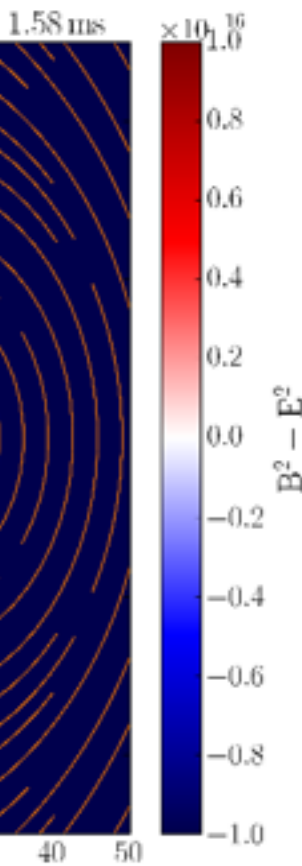
# Collapse to what?

Nathanail, Most, LR 2016

nonrotat



star



$$\frac{1}{2} F^{\mu\nu} F_{\mu\nu}$$

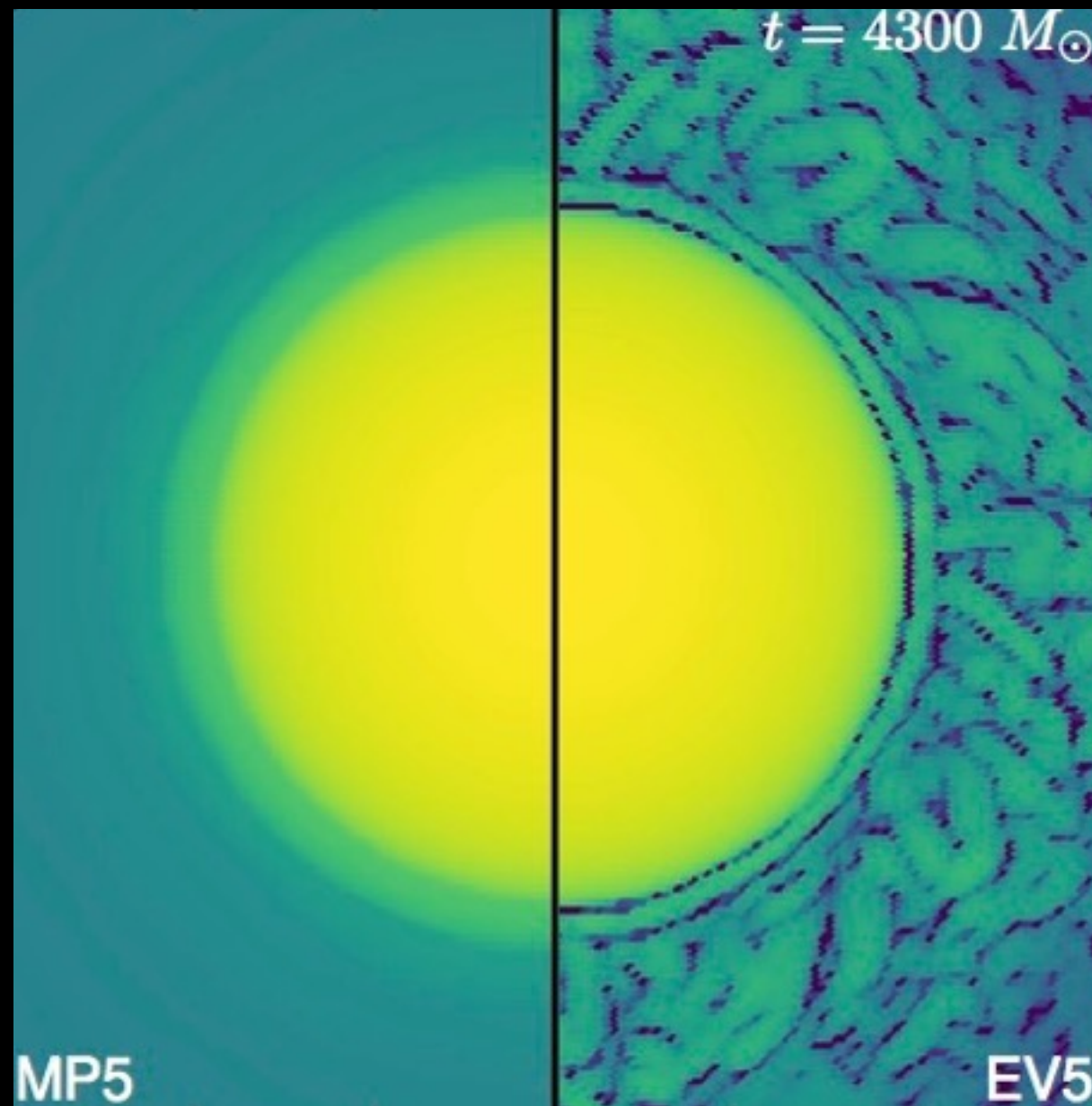
< 0

collapse to **Schwarzschild BH**

collapse to **Kerr-Newman BH**

# ELH: Entropy Limited Hydrodynamics

Guercilena, Radice, LR 2016



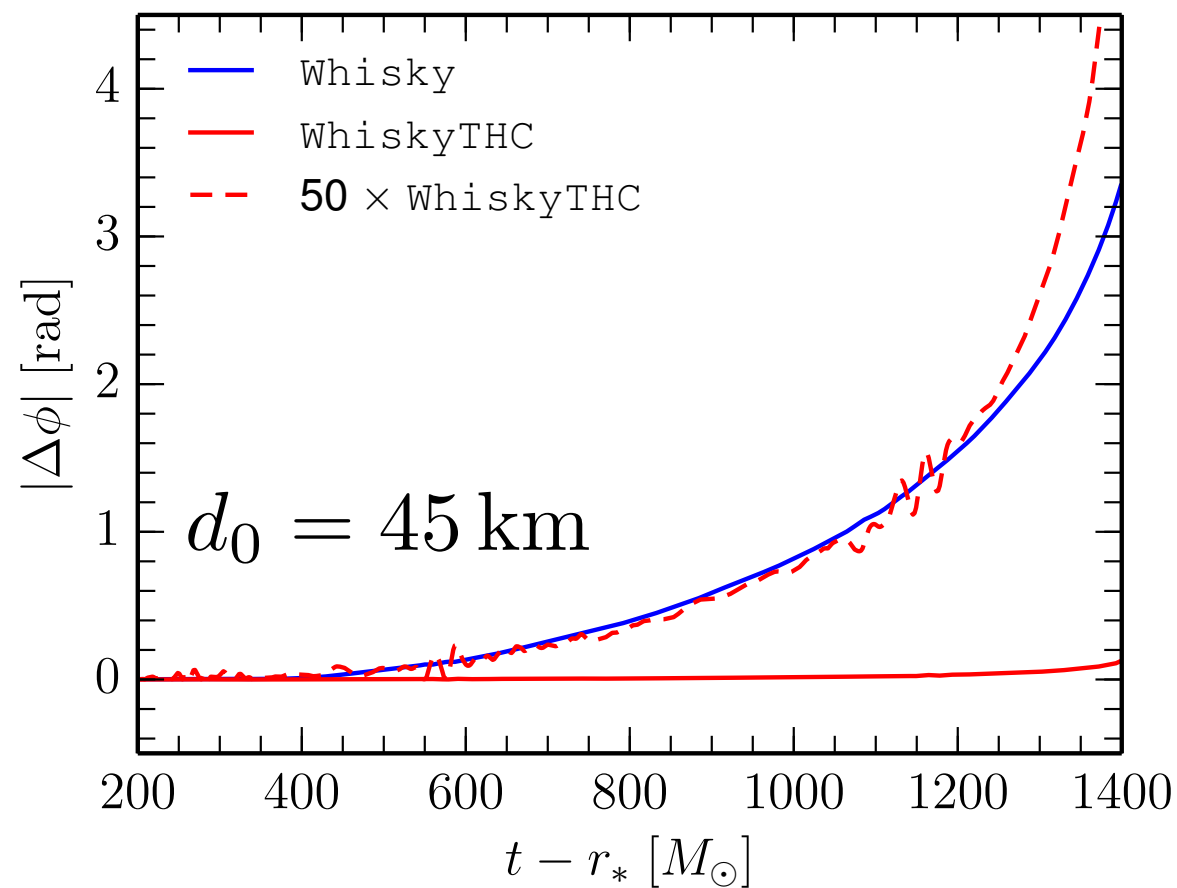


# The need for high accuracy

- Inspiral is the “cleanest” part of the problem: PN predicts **point-particle dynamics + tidal corrections**.
- These corrections come at very high (5) PN order and are therefore intrinsically small but near the merger.
- Computing these corrections is not trivial! Numerical errors and tidal corrections yield the same dynamics: **merger occurs earlier**.
- “Clean” high-order convergence is difficult to achieve: stellar surface reduces order to be **< 2**.
- High-order accuracy is now possible also for binary NSs: WhiskyTHC has convergence of **3.2**. (Radice+2013)

# WhiskyTHC: a high-order hydro code

Radice+ (2013a,b)

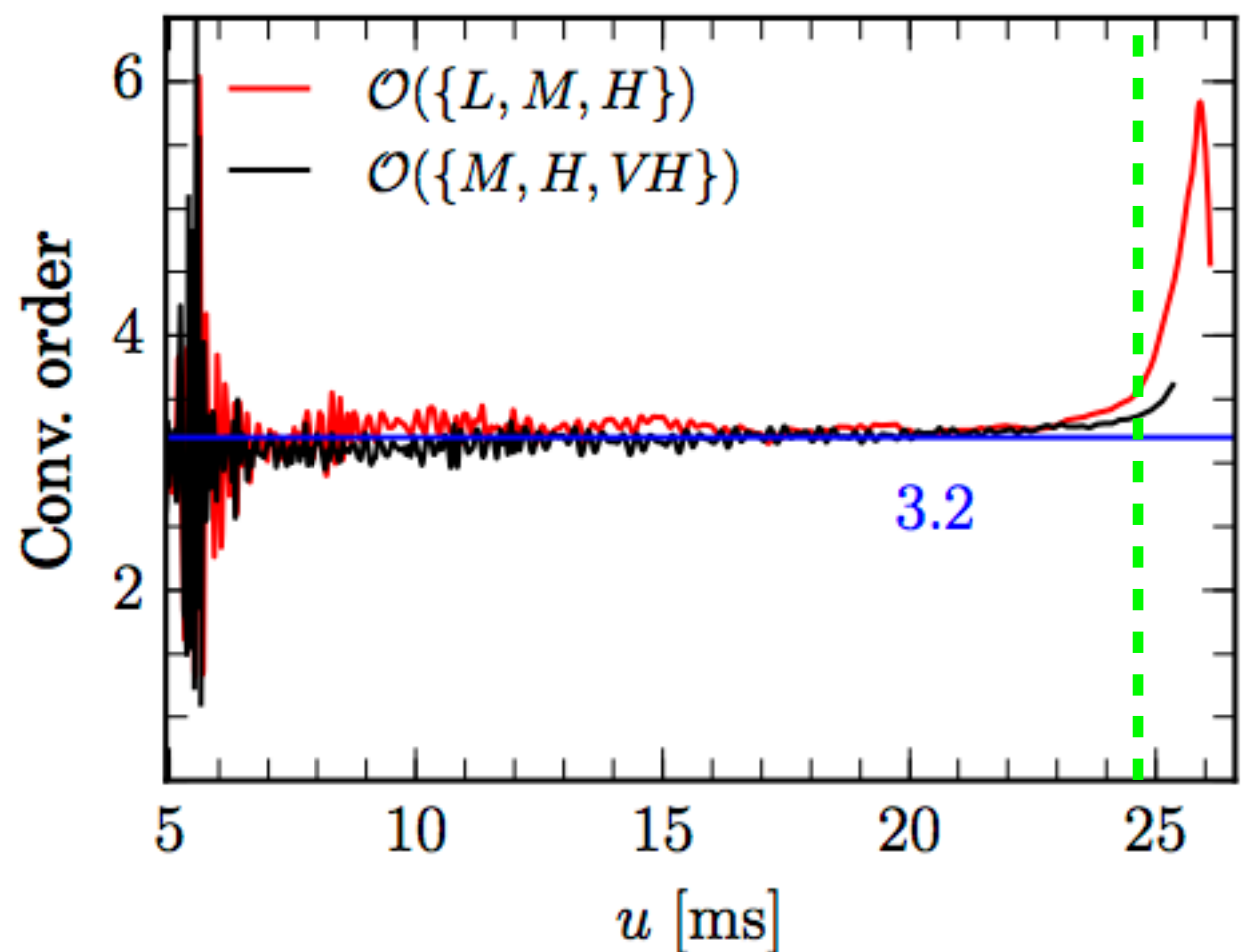


Computational saving best appreciated comparing phase error at the same resolution:  
Whisky (order  $\sim 1.8$ )  
WhiskyTHC (order  $\sim 3.2$ )

Clean convergence is essential for reliable results.

Rarely figures of this type are shown for BNSs.

$\{L, M, H, VH\} = \{370, 295, 215, 147\} \text{ m}$





# Entropy Limited Hydrodynamics: ELH

Guercilena, Radice, LR 2016

The equations of relativistic hydrodynamics are normally cast in a flux-conservative formulation of the type

$$\partial_t \mathbf{U} + \partial_i \mathbf{F}^i = \mathbf{S}.$$

**WhiskyTHC** uses finite-differences and a characteristics variables decomposition with Lax-Friedrichs flux-splitting for upwinding. The fluxes are reconstructed at high-order: 5 or 7.

In **ELH** the flux is expressed as a limiter, i.e.,

$$f_{i+1/2} = \theta f_{i+1/2}^{\text{HO}} + (1 - \theta) f_{i+1/2}^{\text{LF}},$$

where  $\theta \in [0, 1]$ ,  $f_{i+1/2}^{\text{HO}}$  is the standard high-order flux and

$$f_{i+1/2}^{\text{LF}} := \frac{1}{2} (f_i + f_{i+1}) - \frac{\alpha}{2} (u_i - u_{i+1})$$

The limiter should be unity in smooth flows and small in regions of large discontinuities, where entropy is generated

$$\theta = \min[\tilde{\theta}, 1 - \nu]$$

An effective definition is therefore

$$\nu = \max[c_1 |\mathcal{R}|, c_2 \lambda]$$

where

$$\mathcal{R} := \nabla_{\mu}(s\rho u^{\mu}) \geq 0$$

and  $s$  the specific entropy.

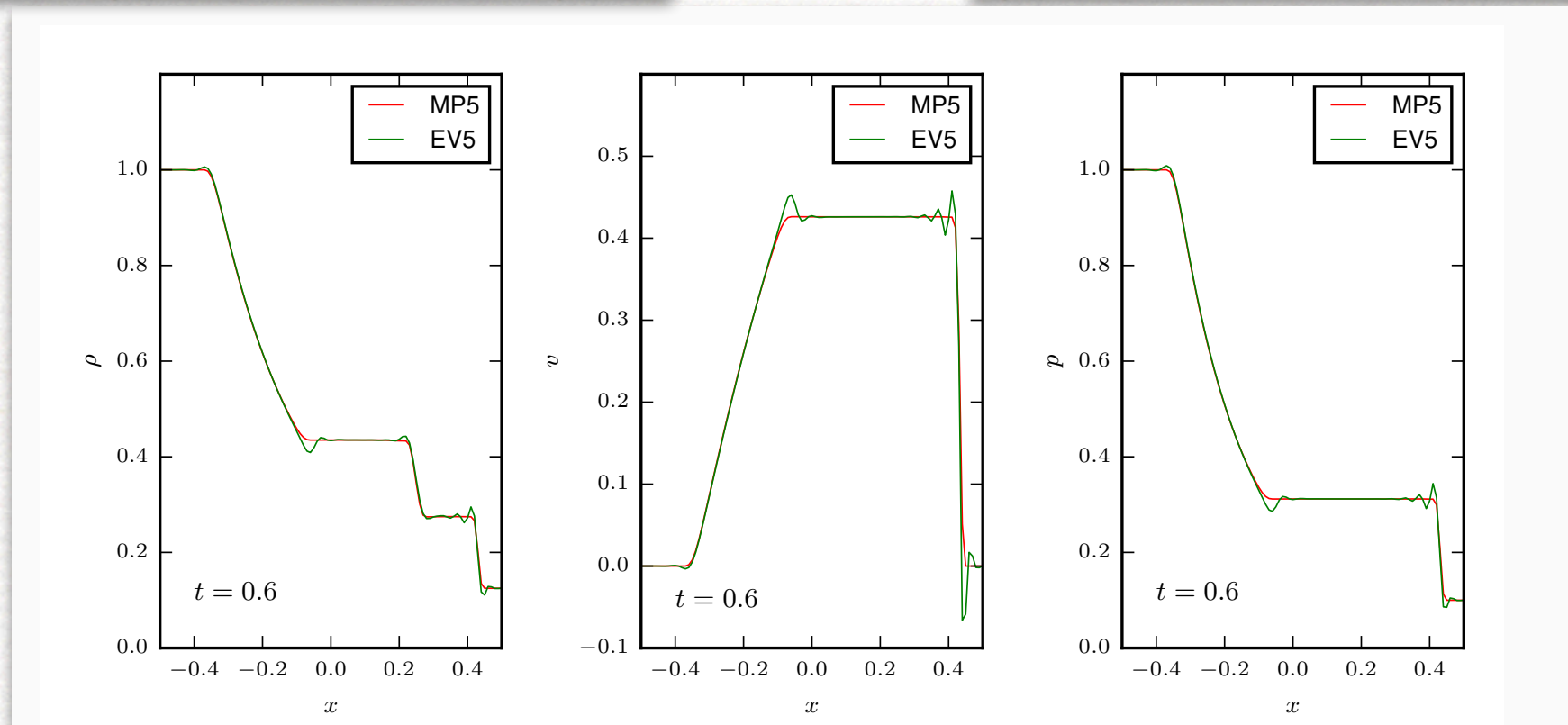
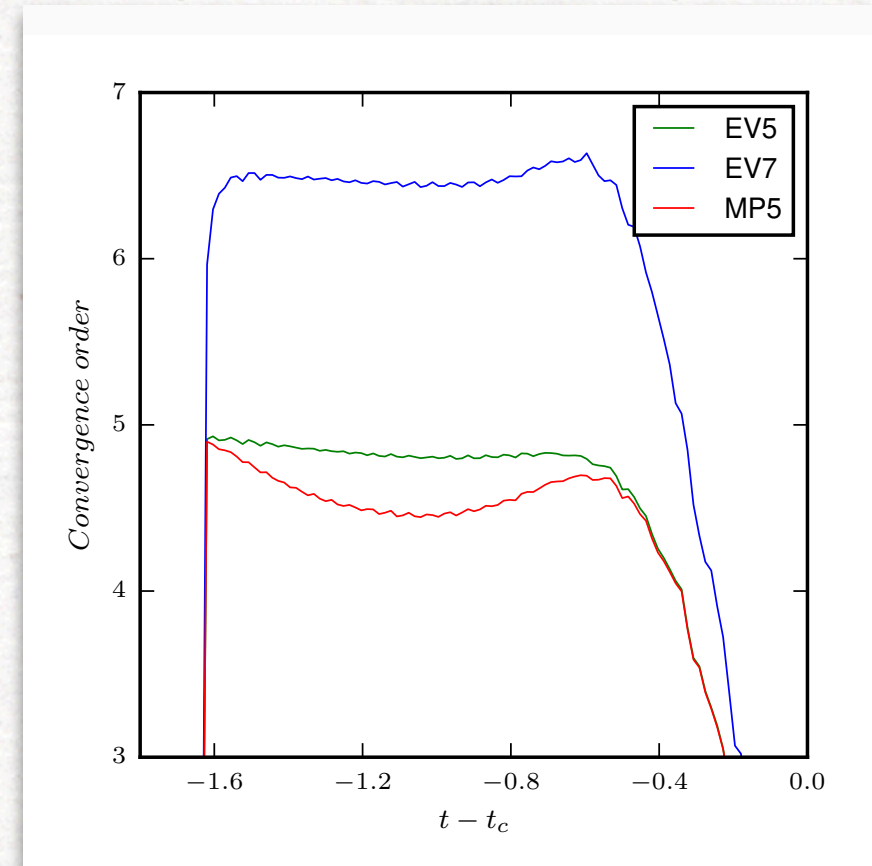
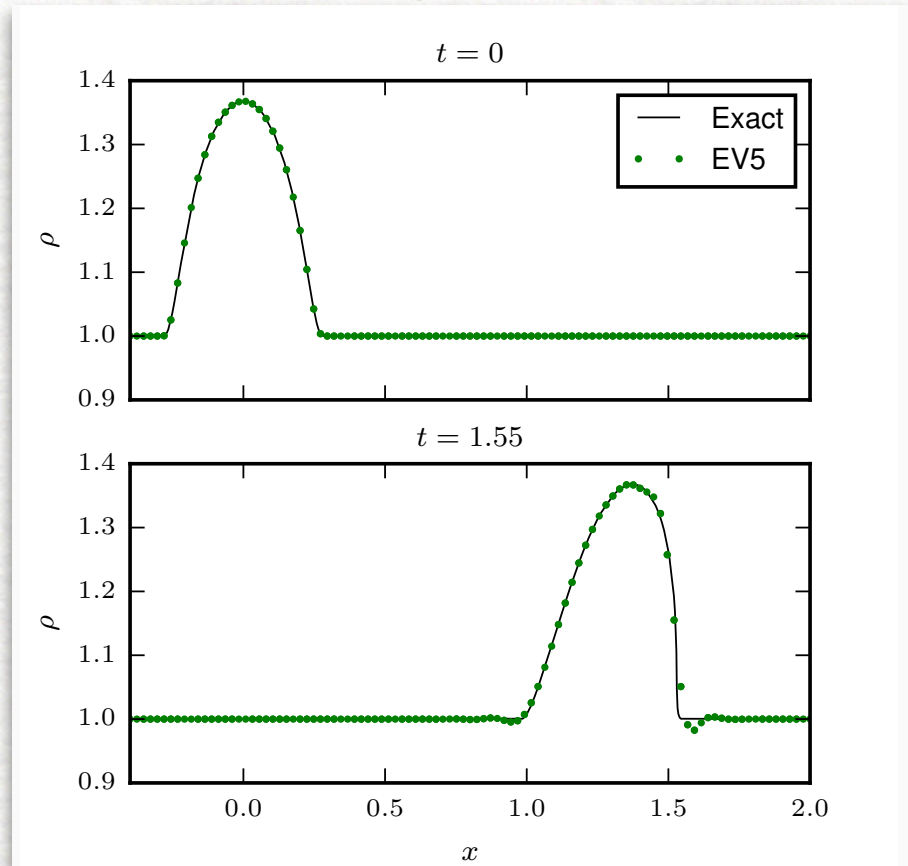
In other words, the flux is limited in those very localised regions where entropy is generated, i.e. **at shocks**.

The advantages are: **accuracy** (HO), **speed** (FD), **simplicity** and **extendability**.



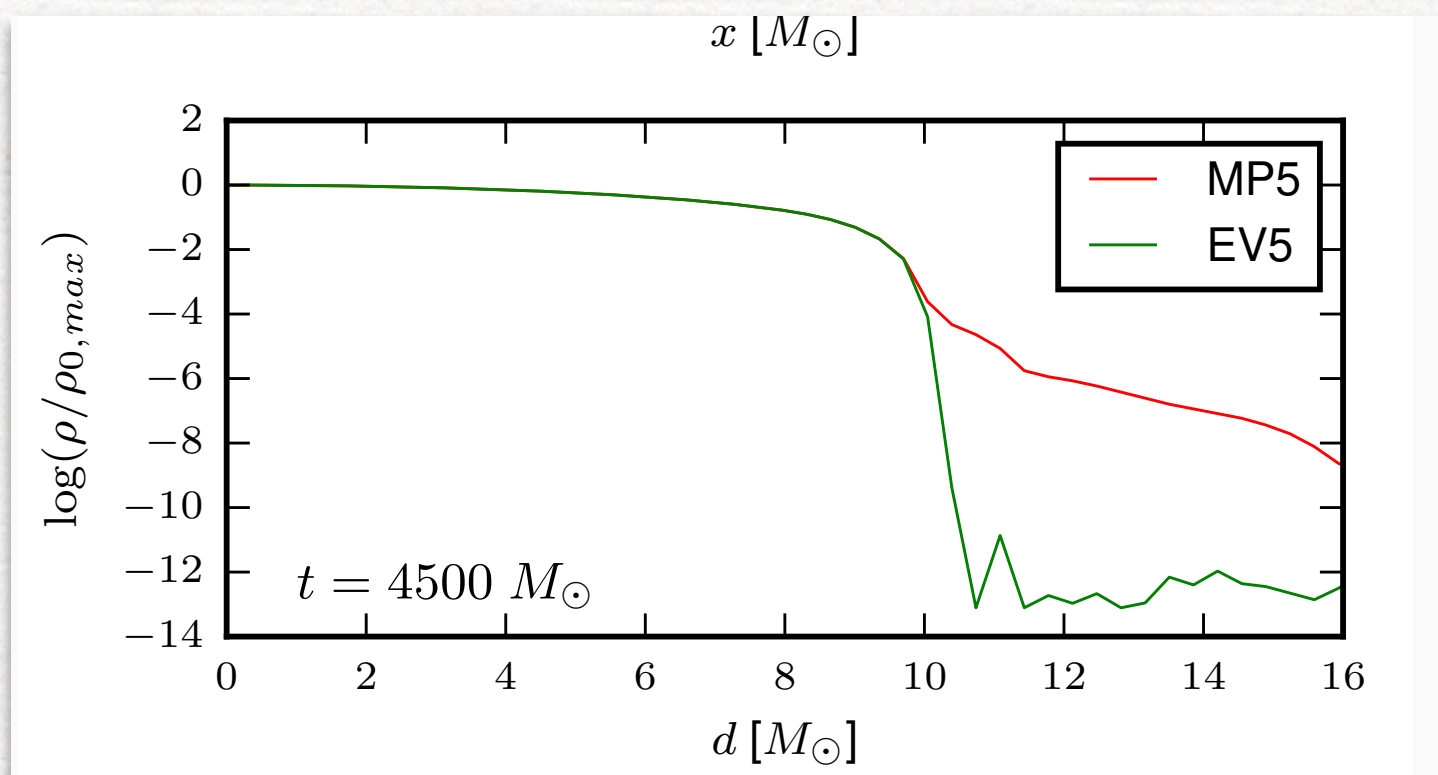
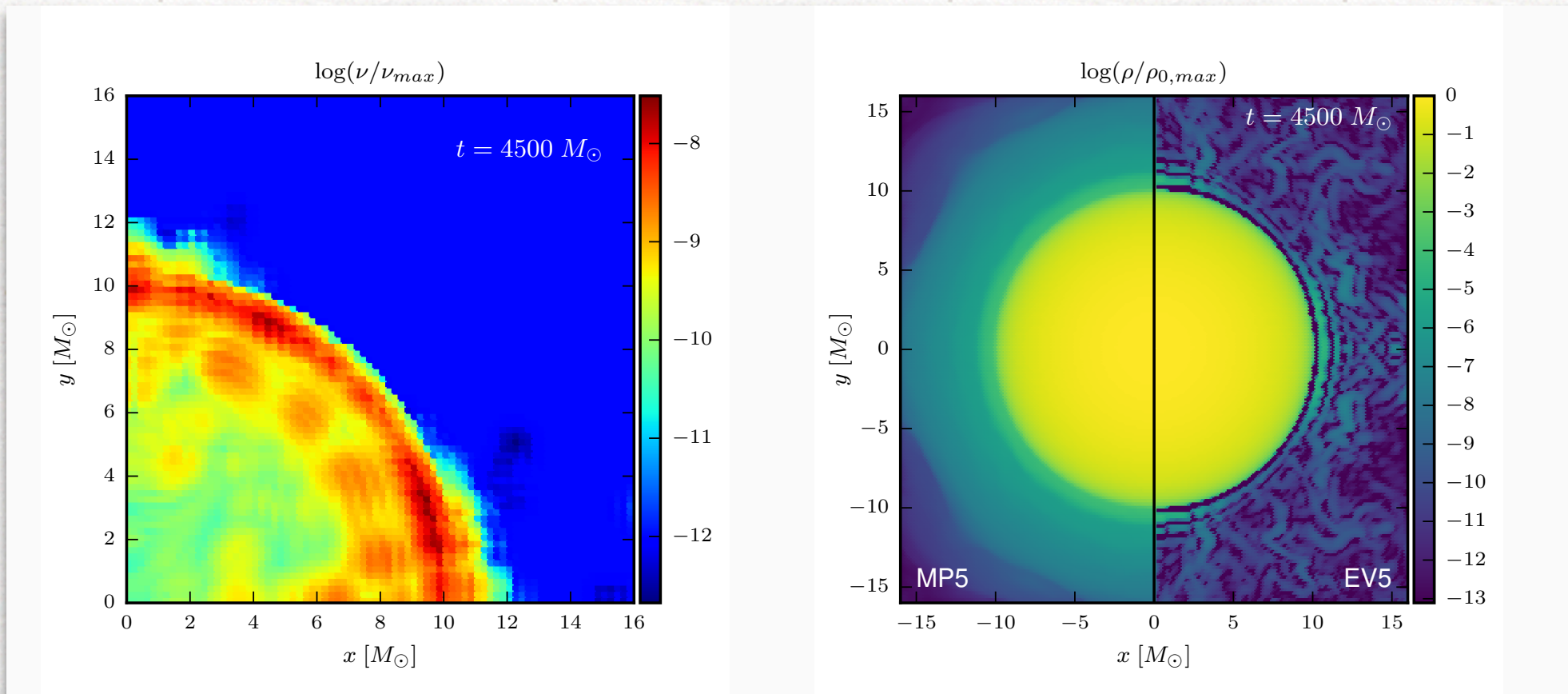
# Does it work?

Guercilena, Radice, LR 2016



# Does it work?

Guercilena, Radice, LR 2016



An exhaustive series of tests in full GR shows that ELH provides all of the advantages expected.



# Conclusions

- \* Spectra of post-merger shows clear peaks, some of which are **"quasi-universal"**. If observed, will set tight constraints on EOS
- \* Magnetic fields unlikely to be detected during the inspiral but **important** after the merger: instabilities and EM counterparts
- \* **Blitzars** are a simple manner to obtain an FRB phenomenology but may not be answer to the riddle.
- \* **Eccentric** binaries are rare but with larger ejected matter and macronova emission. "high-A" nucleosynthesis very robust.
- \* **Entropy Limited Hydrodynamics** is promising new approach to relativistic hydrodynamics and may become a new standard.