The Nuclear Equation of State

from laboratory to stars

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Outlook

1. What is an EOS ?
2. Relevance of the nuclear EOS.
3. Experimental and observational methods to constrain the EOS.
4. Microscopic many-body theories.
5. Choice of the force.
6. Comparison of the results.
7. The saturation point and around.
8. Higher density
9. Where do we stand ?

What is an EOS ?

In thermodynamics the simplest EOS is the equation that connects pressure with desnsity and temperature

$$P = P(\rho, T)$$

which can be derived from e.g. the free energy. However for the nuclear EOS it is essential to include also the proton fraction as a variable.
In astrophysics, in particular for Neutron Stars, it is mandatory to include leptons (electrons, muons), and it is also possible that 'exotic' components are present at macroscopic level, like mesons, hyperons, etc.

Relevance of the EOS

1. Heavy ion collisions. (H.I.)
2. Supernovae and Neutron Stars. (SN , NS)
3. Gravitational waves emission. (GW)

However the physical conditions are quite different in each case.

1. H.I. : small asymmetry, high temperature.
2. SN : high asymmetry and high temperature.
3. NS : high asymmetry and low temperature.
4. GW : very high density, asymmetry and temperature (NS mergers).

A microscopic theory must be able to treat all these physical situations.

Overview of experimental and observational constraints

1. Nuclear structure

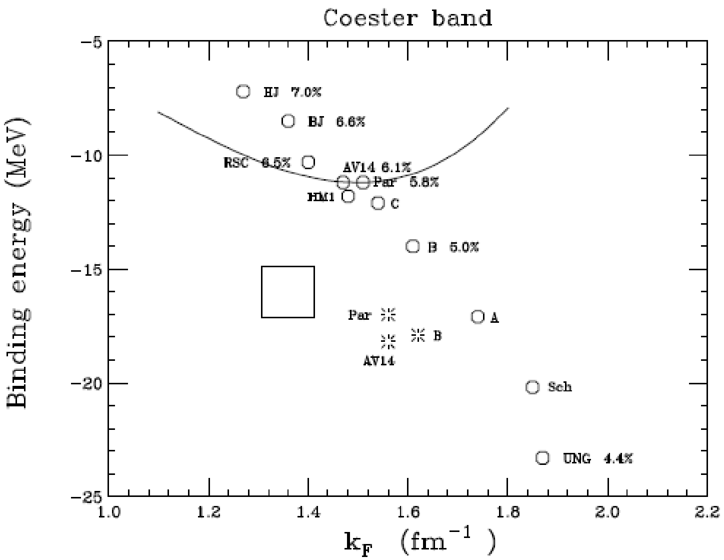
2. Heavy ions

3. Neutron Stars

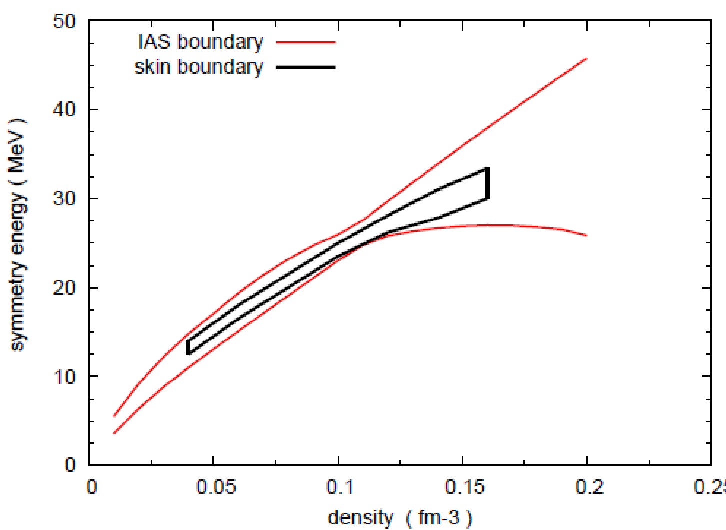
4. Gravitational waves

Nuclear Structure

1. Saturation point
2. Incompressibility
3. Symmetry energy at sub-saturation density

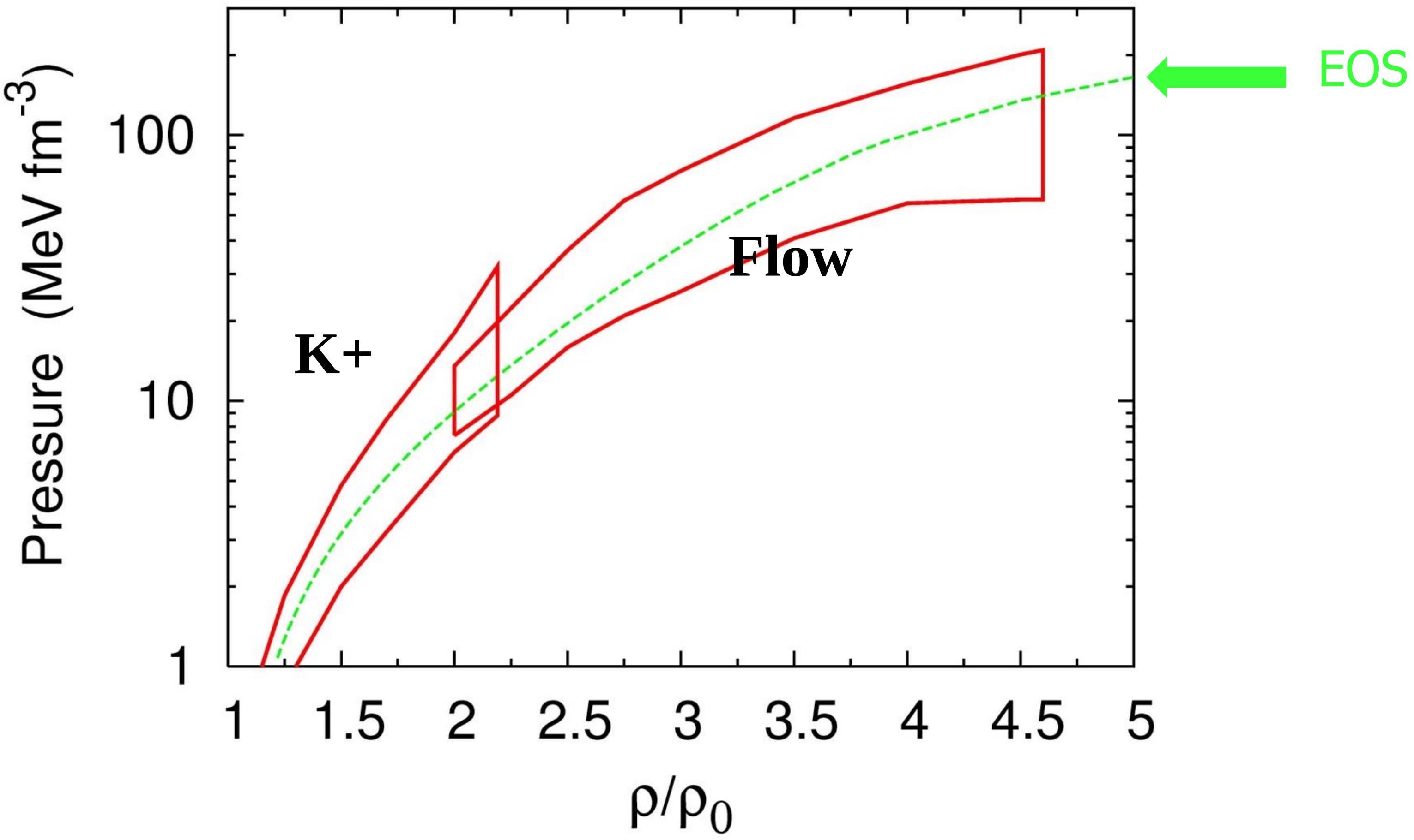


$$K_A = K_{vol} + K_{surf} A^{-1/3} + K_{curv} A^{-2/3} + (K_{sym} + K_{ss} A^{-1/3}) \left[\frac{N-Z}{A} \right]^2 + K_{Coul} \frac{Z^2}{A^{4/3}} + \cdots ,$$



Danielewicz & Lee
NPA922, 1 (2014).

SUPRA-SATURATION DENSITY CONSTRAINTS FROM HEAVY ION REACTIONS

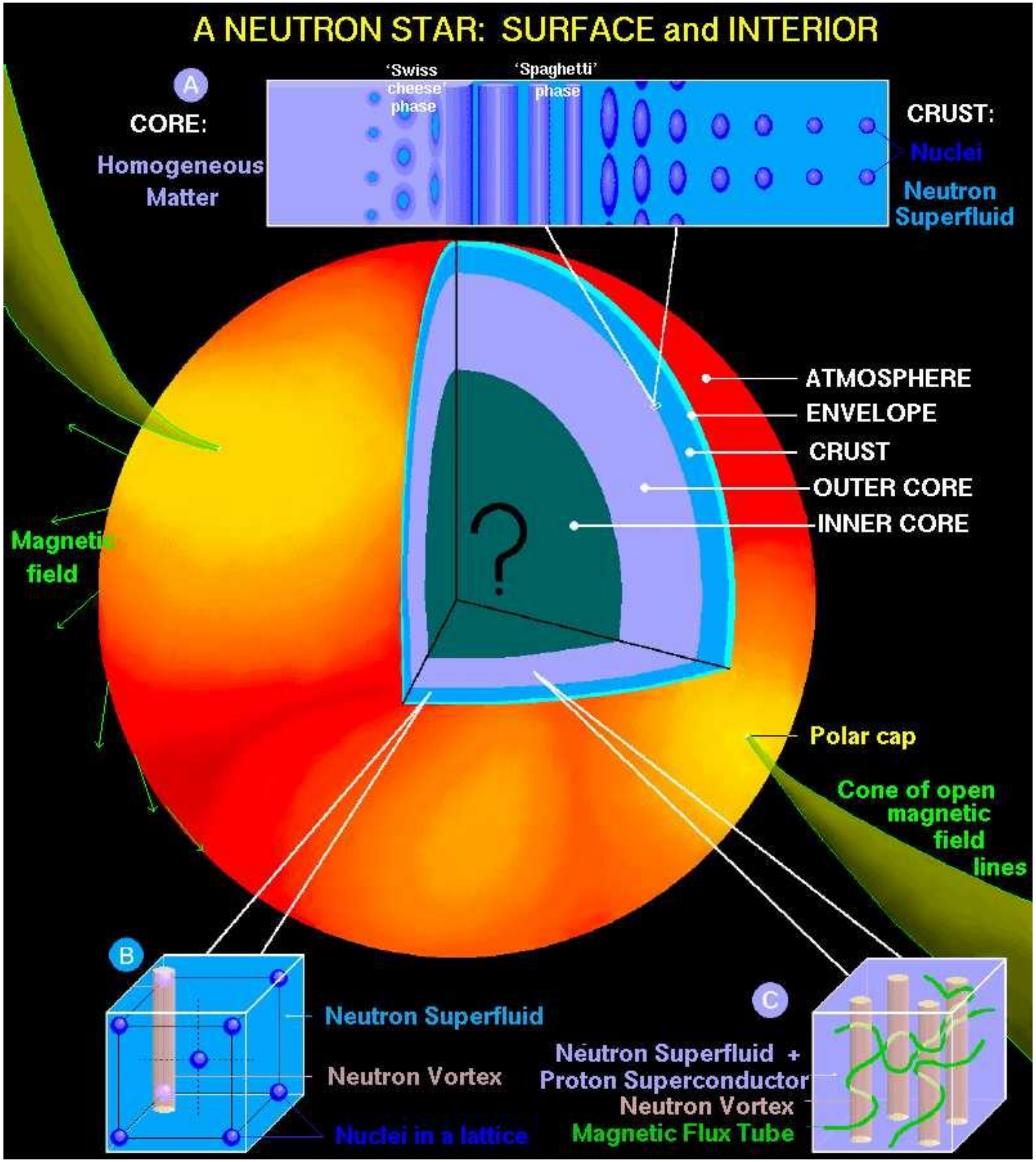


K+ : Lynch et al. , Prog. Part. Nucl. Phys. 62, 427 (2009)

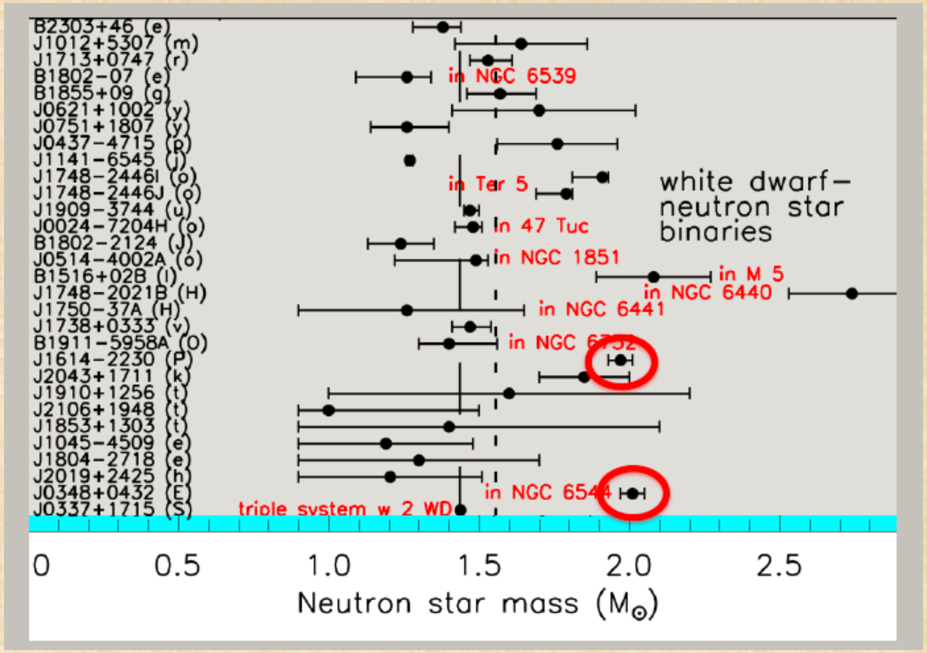
Flow : Danielewicz et al. , Science 298, 1592 (2002)

EOS from Neutron Stars

Confer to Hebeler et al., ApJ 773, 11 (2013)



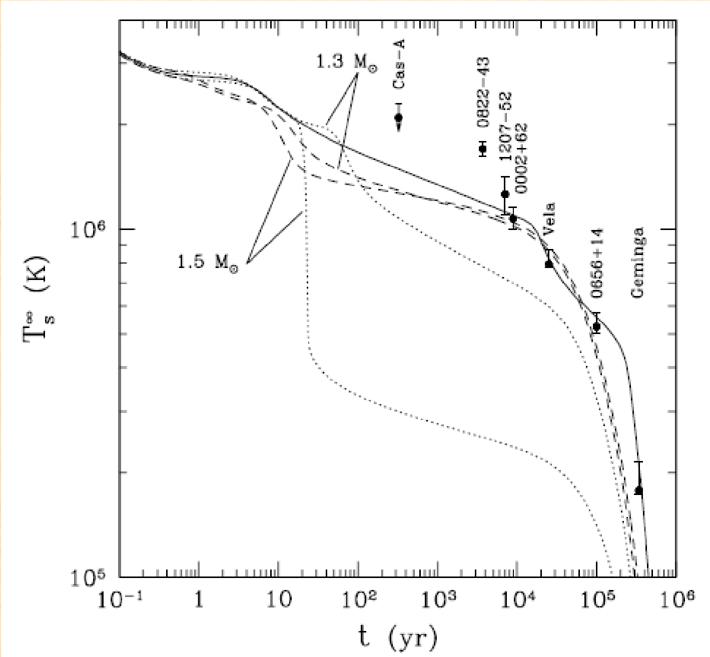
A section (schematic) of a neutron star



Credit : Jim Lattimer, David Nice

The largest NS mass

Stiffness of the EOS

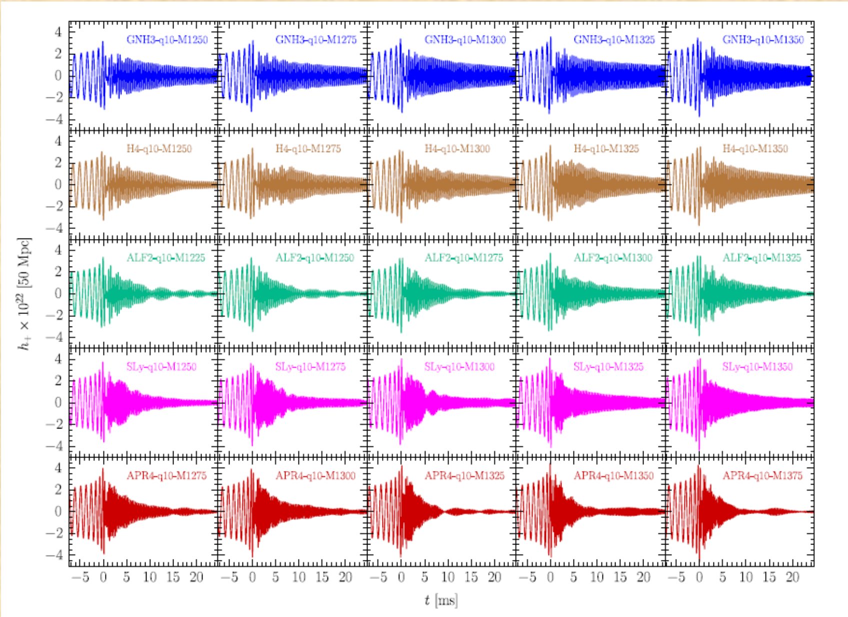


Cooling : the onset of the URCA process

Uncertainty : role of superfluidity

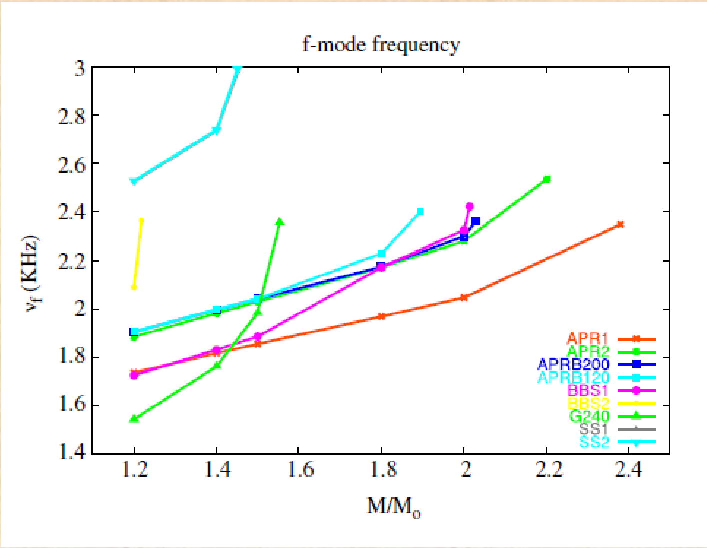
Yakovlev et al., Phys. Rep. 354, 1 (2001)

For the future (GW)



GW signal from two NS mergers depends on the EOS

Takami et al., PRD91, 064001 (2015)



The frequency of the fundamental f-mode depends on the EOS

$\omega \approx M/R^3$ $\lambda \approx M/R$

O. Benhar et al., PRD70, 124015 (2004)

Other signals from compact objects

1. Neutrino from supernovae
(L.F. Roberts et al., Phys. Rev. Lett. 108, 061103 (2012))

2. NS radii
(Lattimer & Steiner, ApJ 784, 123 (2014))

Developments in Heavy Ions

1. Improvements at intermediate energies

2. The CBM experiment at FAIR

What can we get from this overall set of constraints ?

1. The constraints restrict the properties of the EOS but surely they do not fix it. An ample family of EOS can be compatible with the phenomenological bounds.

2. The constraints are obtained generally through the use of phenomenological Energy Density Functionals, which can generate spurious correlations among physical quantities.

One can follow a different approach :
Develop a microscopic many-body theory of the EOS and compare with the phenomenological constraints. Then one gets :

1. Selection of the EOS

2. Hints on the structure of nuclear matter

There are two main basic elements in the microscopic approach.

A. The microscopic many-body approach

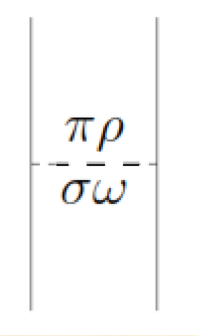
1. The Brueckner-Bethe-Goldstone expansion (BBG) and Coupled Cluster (CC) expansion
2. Self-consistent Green's function.
3. The variational method
4. The relativistic Dirac-Brueckner approach
5. The renormalization group

B. The choice of the bare nucleonic force

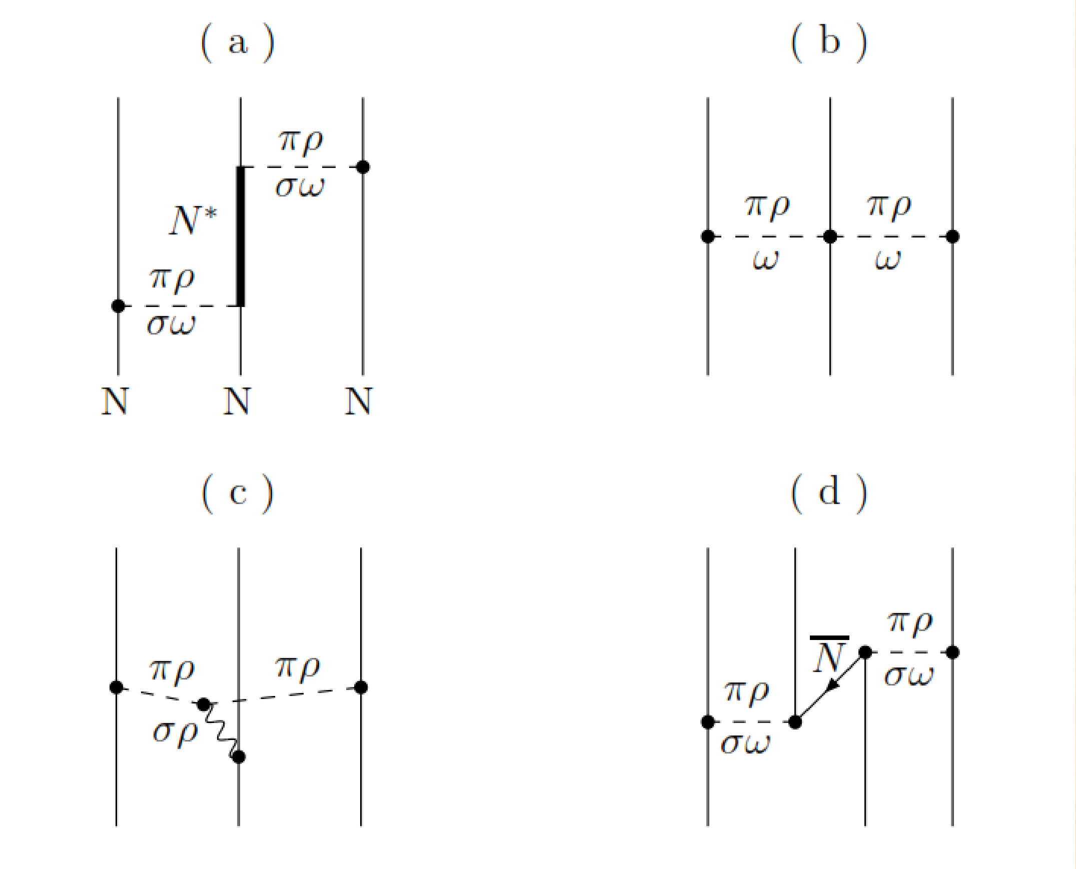
1. Meson exchange models
2. Chiral approach
3. Quark models

Meson exchange models

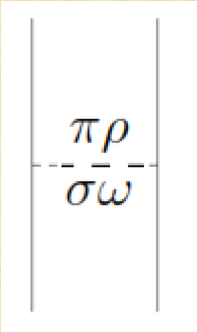
Two-body forces



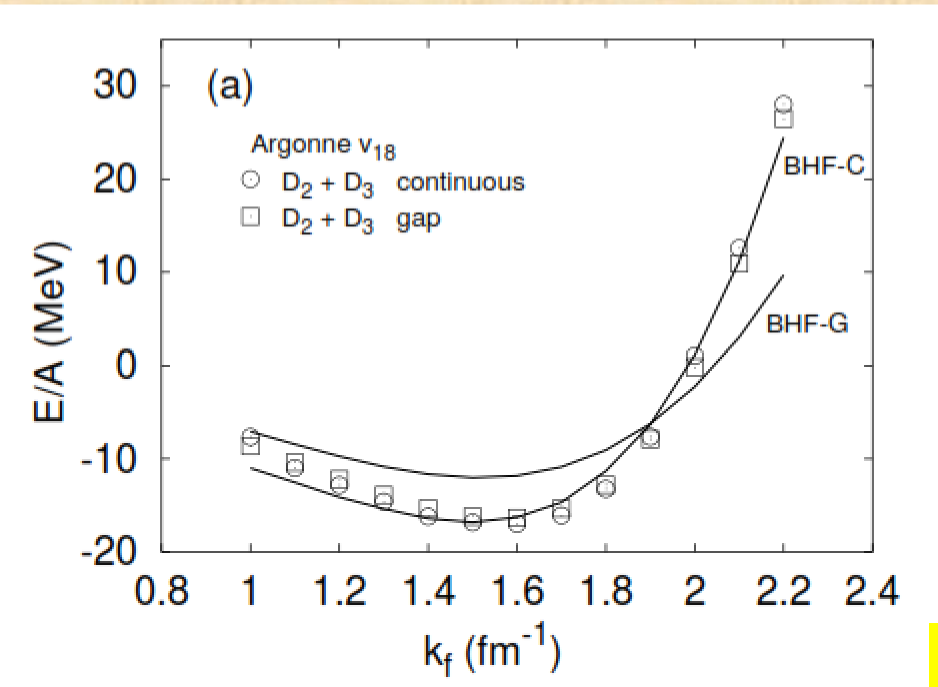
Three-body forces

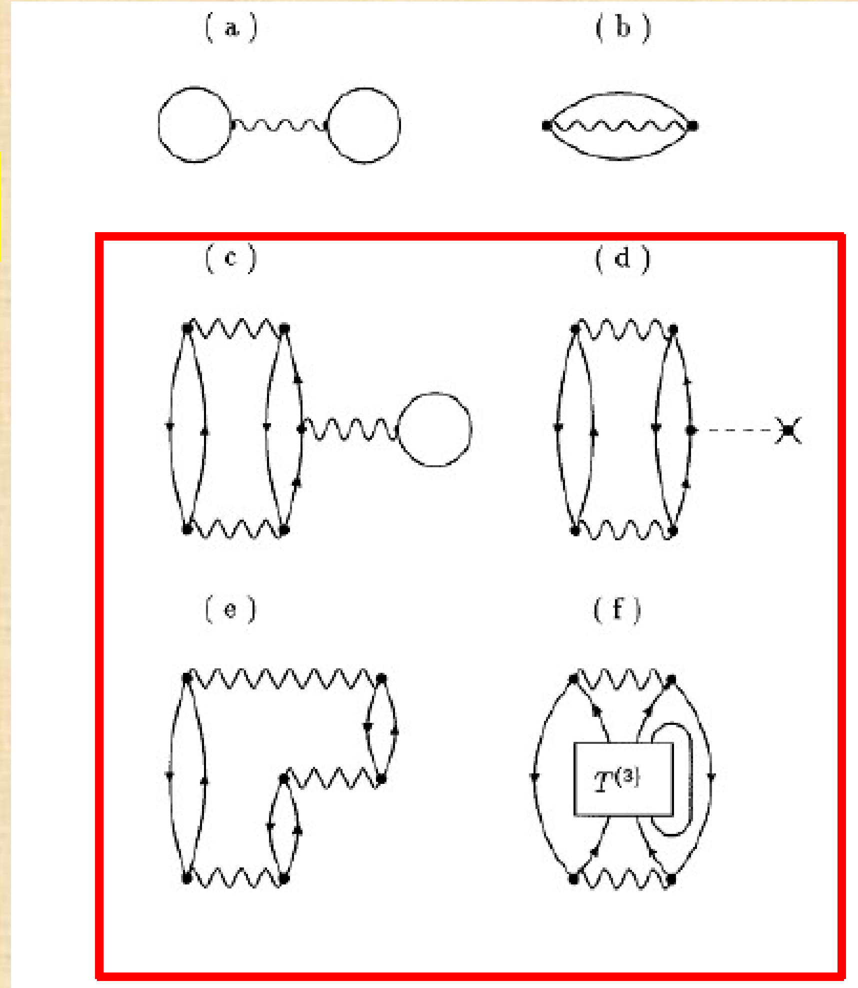


BBG



Two-body force only

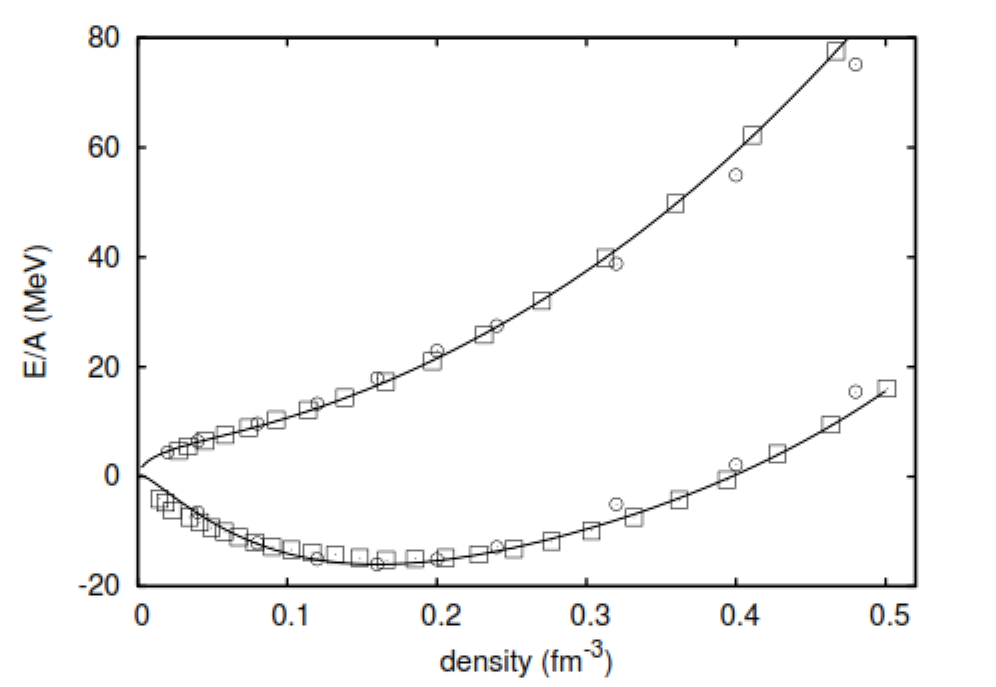




Two and three body correlations

In the continuous choice three-body correlations turn out to be small. One assumes that this is still true once the three-body forces are introduced.

Three-body forces are necessary to get the correct saturation point. However their contribution is much smaller than the two-body one (around saturation)

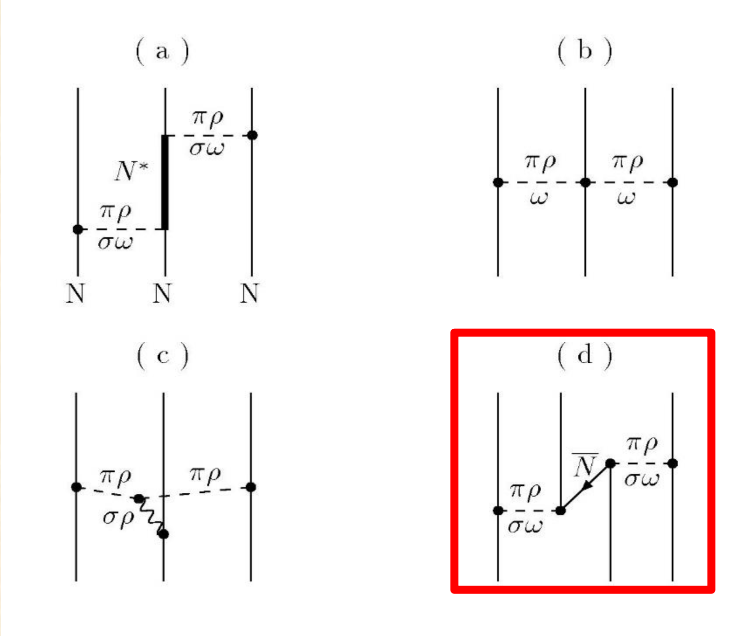


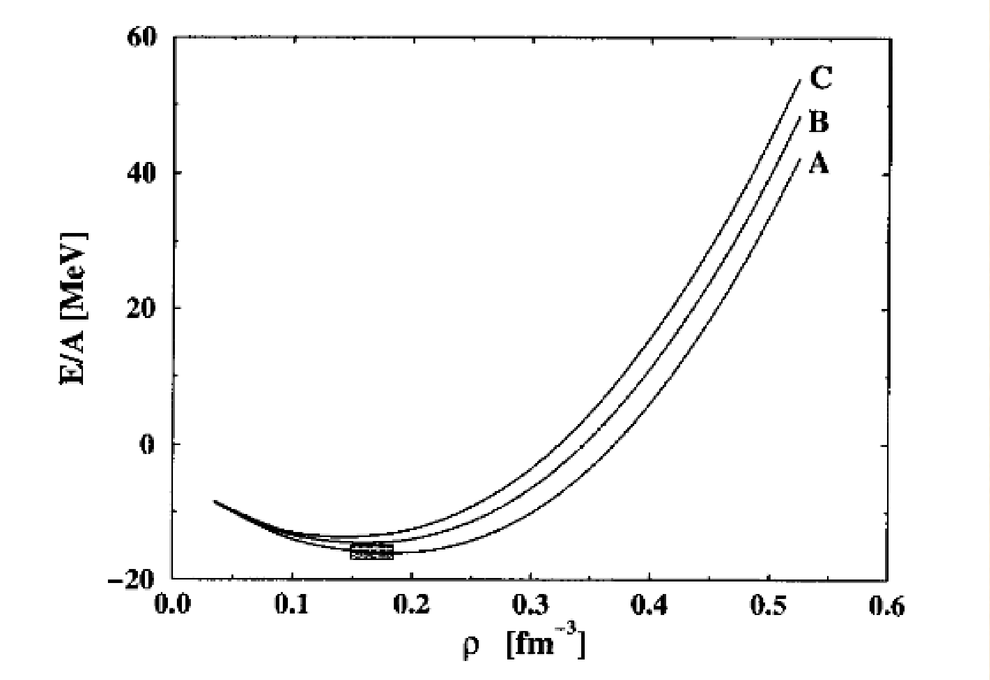
Phenomenological three-body forces

□ BHF (M.B. et al. PRC87, 064305 (2013))

○ Variational (Akmal et al. PRC58, 1804 (1998))

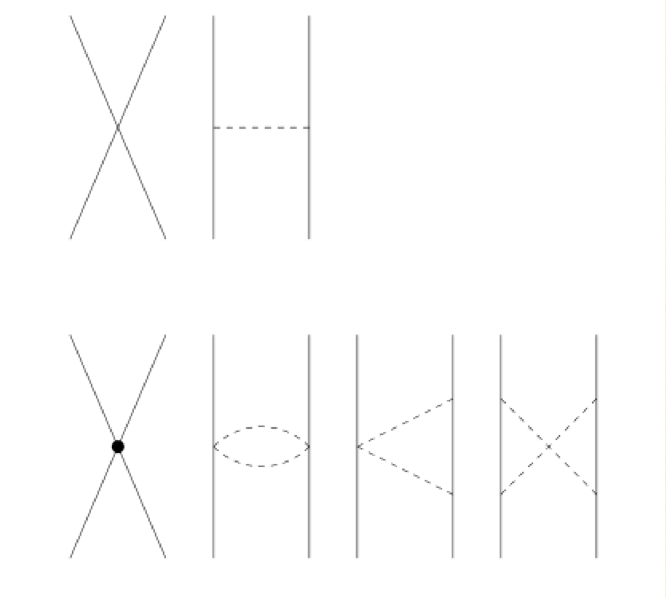
Dirac – Brueckner. Two-body forces only
NN interactions Bonn A, B, C



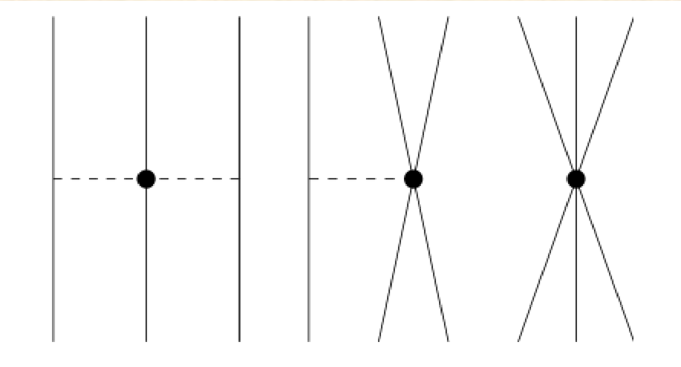


T. Gross-Boelting et al.
NPA 648, 105 (1999)

Chiral expansion approach, from QCD symmetry



Two-body forces
Pion exchange +
point interactions

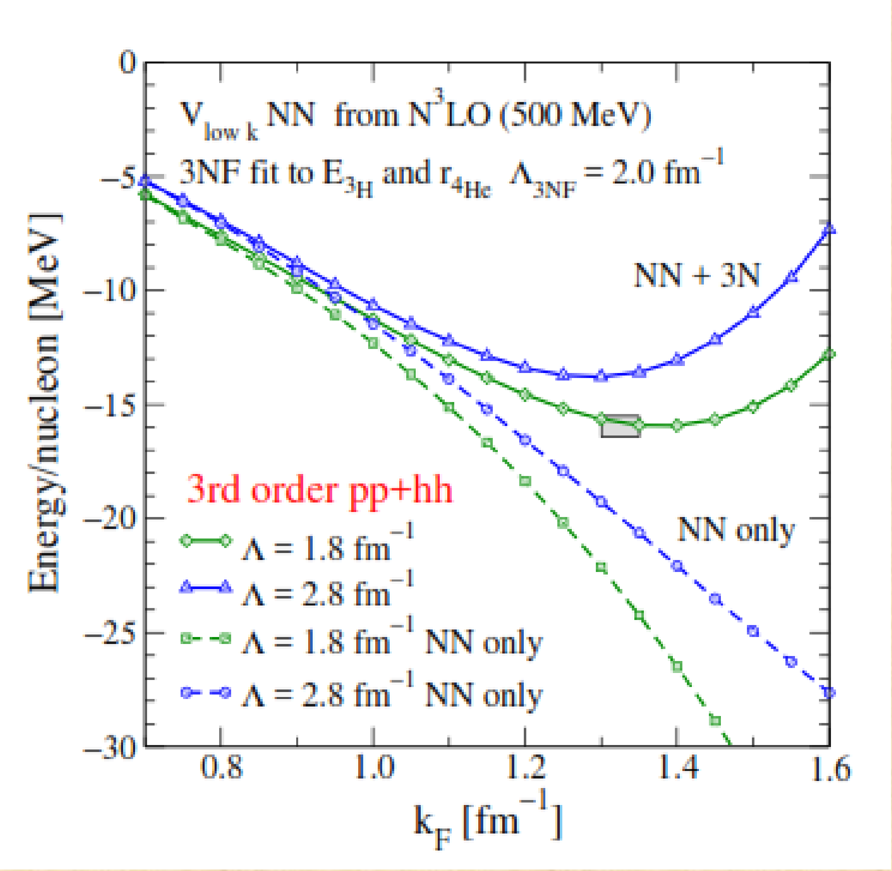


Three-body forces

Systematic hierarchy of the relevance of the forces

The quark degrees of freedom do not appear explicitly

Chiral force + RG, perturbative calculation

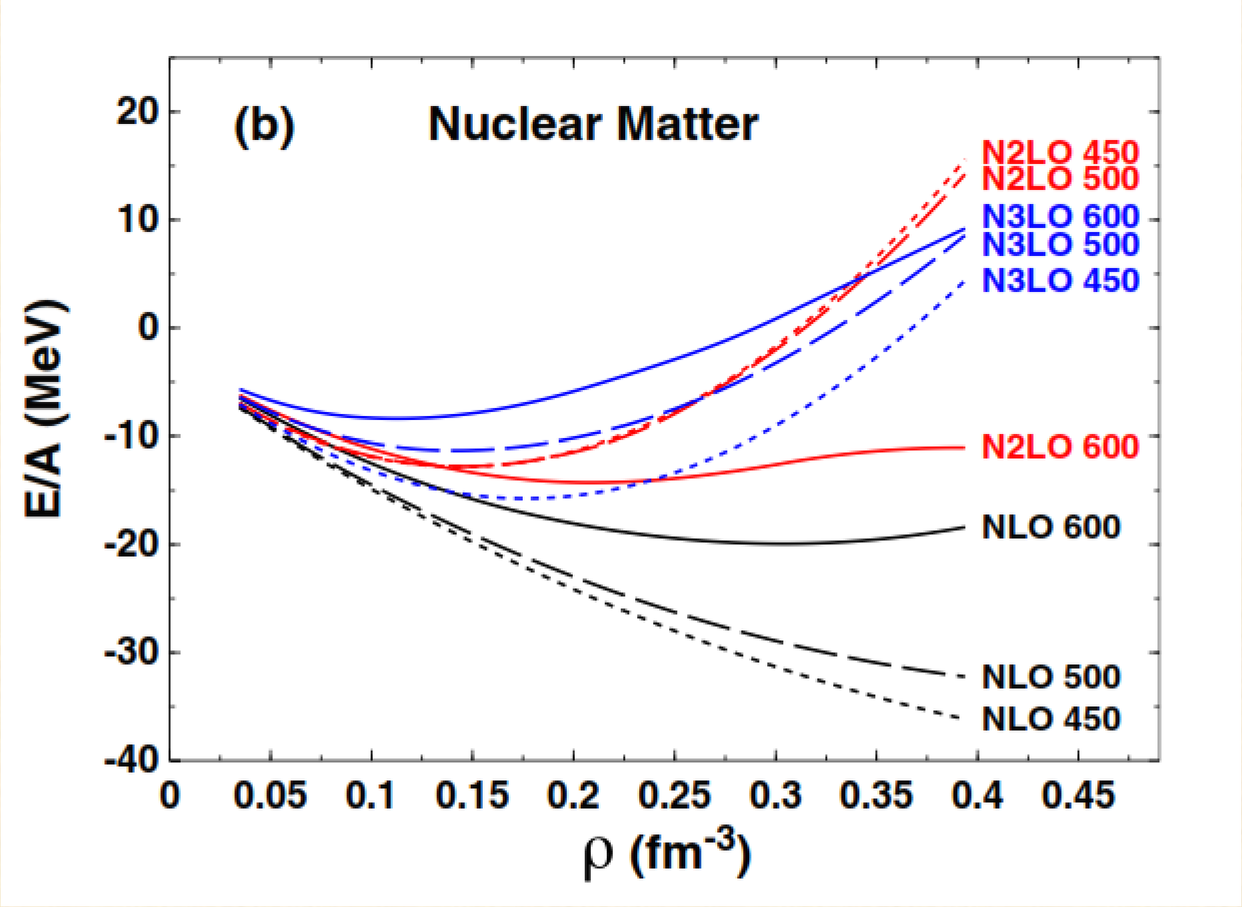


Hebeler et al.
PRC 83, 031301 (2011)

No saturation with only two-body forces
Three-body forces essential and large even at saturation

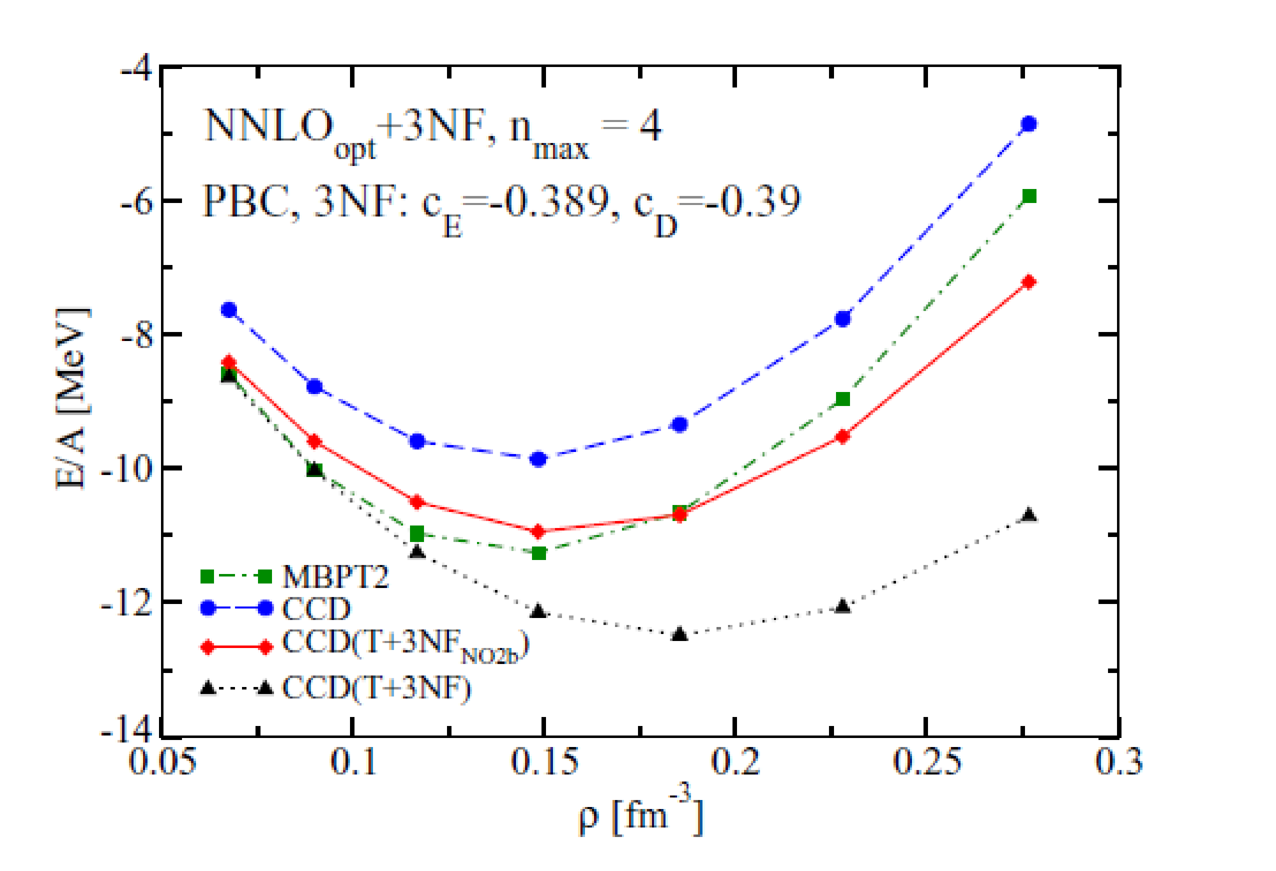
Chiral force + RG, Brueckner calculations

Proceeding order by order in the force hierarchy.
The rate of convergence is cut-off dependent



F. Sammaruca et al., PRC 91, 054311 (2015) (BHF calculations)

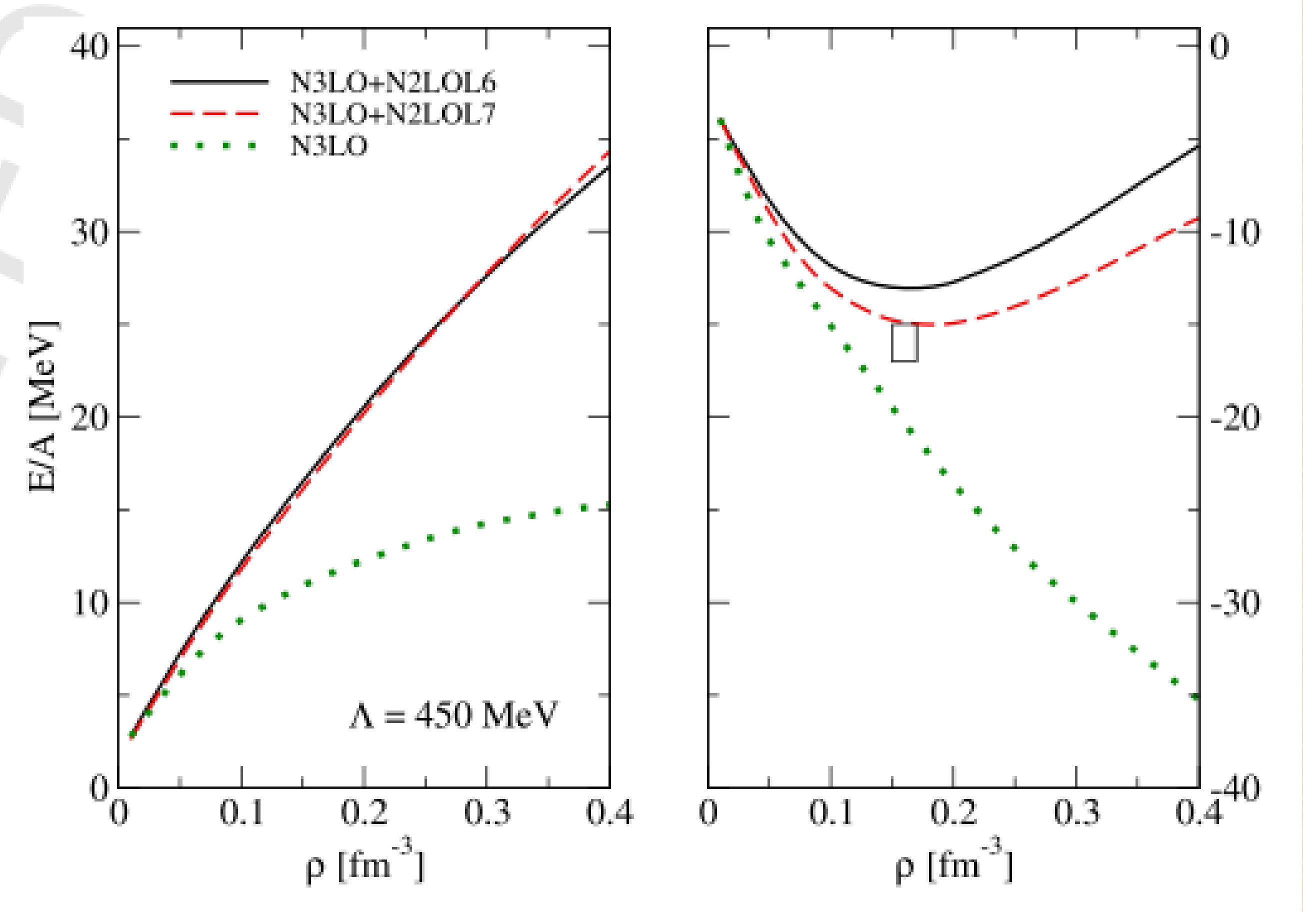
Difficulty in fitting both few-body and Nuclear Matter saturation point



Hagen et al., PRC 89, 014319 (2014)

Coupled Cluster calculations up to selected triples, chiral forces.
Situation similar to the one for meson exchange models.

Optimizing few-body and Nuclear Matter



Logoteta et al., Phys. Lett. B 758, 449 (2016)

BHF calculations, Av18 + chiral TBF

Quark-Model Baryon-Baryon (QM BB) Interactions

Structure of the NN interaction.

• Nucleons described as three-quarks clusters with confinement.

• Quark exchange interactions.

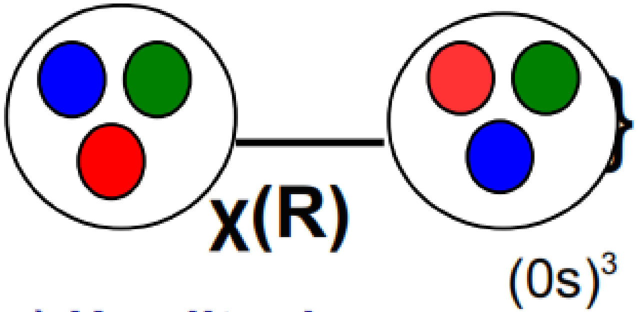
• One gluon exchange + effective meson exchange interaction between quarks.

Feature : Highly non-local with natural cut-off.

fss2 and FSS Y. Fujiwara Y. Suzuki and N. Nakamoto, Prog. Part. Nucl. Phys. 58, 439 (2007)

(3q)-(3q) resonating-group method (RGM)

$\Psi = A\{$



$\}$

solved in M. Oka, K. Yazaki
PLB, 90, 41 (1980). PTP 66, 556, 572. (1981)

RGM equation

$$\langle \phi(3q)\phi(3q) | (E-H) | A\{\phi(3q)\phi(3q)\chi(R)\}\rangle = 0$$

Origin of high nonlocality

(3q)-(3q) Hamiltonian

$$H = \sum_i \left(m_i c^2 + \frac{p_i^2}{2m} - T_G\right) + \sum_{i<j}\left(U_{ij}^{cf} + U_{ij}^{FB} + U_{ij}^{Meson}\right)$$

Confinement+Fermi-Breit (OGEP)+Effective meson exchange (EMEP)

short-range medium+ long-range

Notes:

1. The **energy-independent** version is used.

N: RGM normalization kernel

$$(H_0 + V)\chi = EN\chi \Rightarrow (H_0 + V + W)\psi = E\psi \quad \text{with} \quad \psi \equiv \sqrt{N}\chi$$

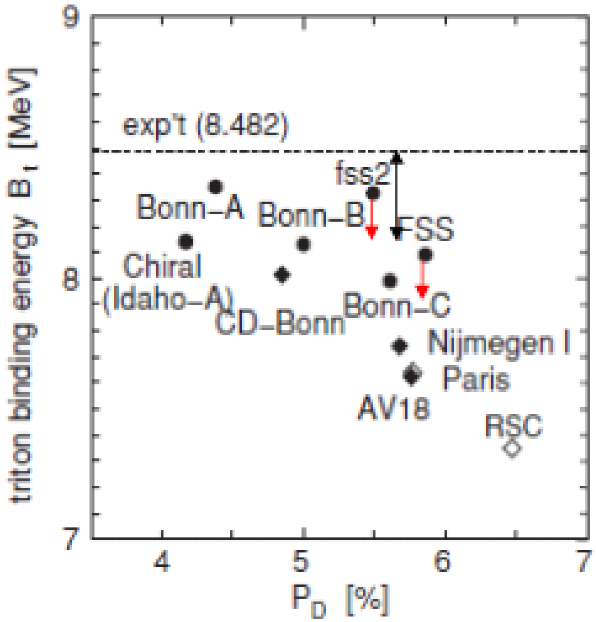
$$W = \sqrt{1/N}(H_0 + V)\sqrt{1/N} - (H_0 + V)$$

2. Gaussian representation of fss2 is used.

Y. Suzuki et al., PLB 659,160 (2008)
Y. Fujiwara and K. Fukukawa, PTP 124,433 (2010)

Bound and scattering three-body system

Triton binding energy



Deuteron *D*-state probability

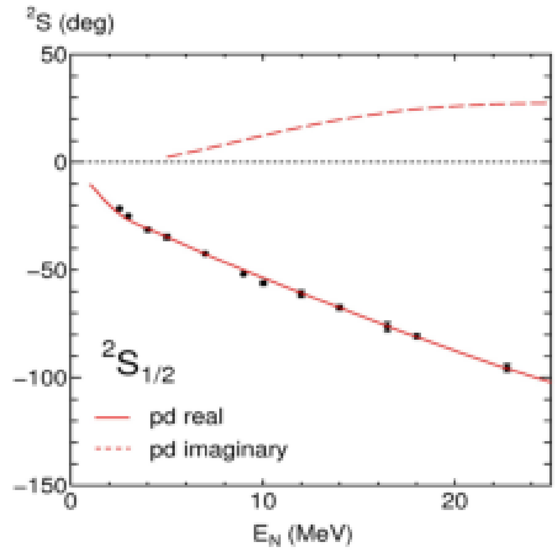
Y. Fujiwara et al. PRC77 (2008) 027001

◆ : take into account the charge dependence

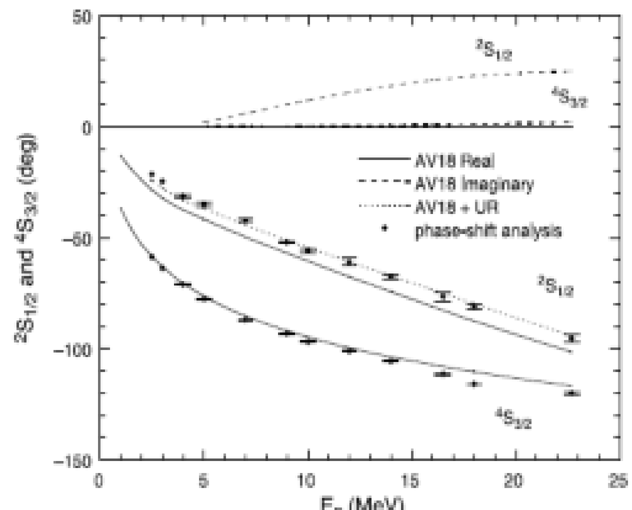
● : do not take into account charge dependence

The energy deficiency by fss2 ~350 keV

$^2S_{1/2}$ phase shifts in proton-deuteron elastic scattering

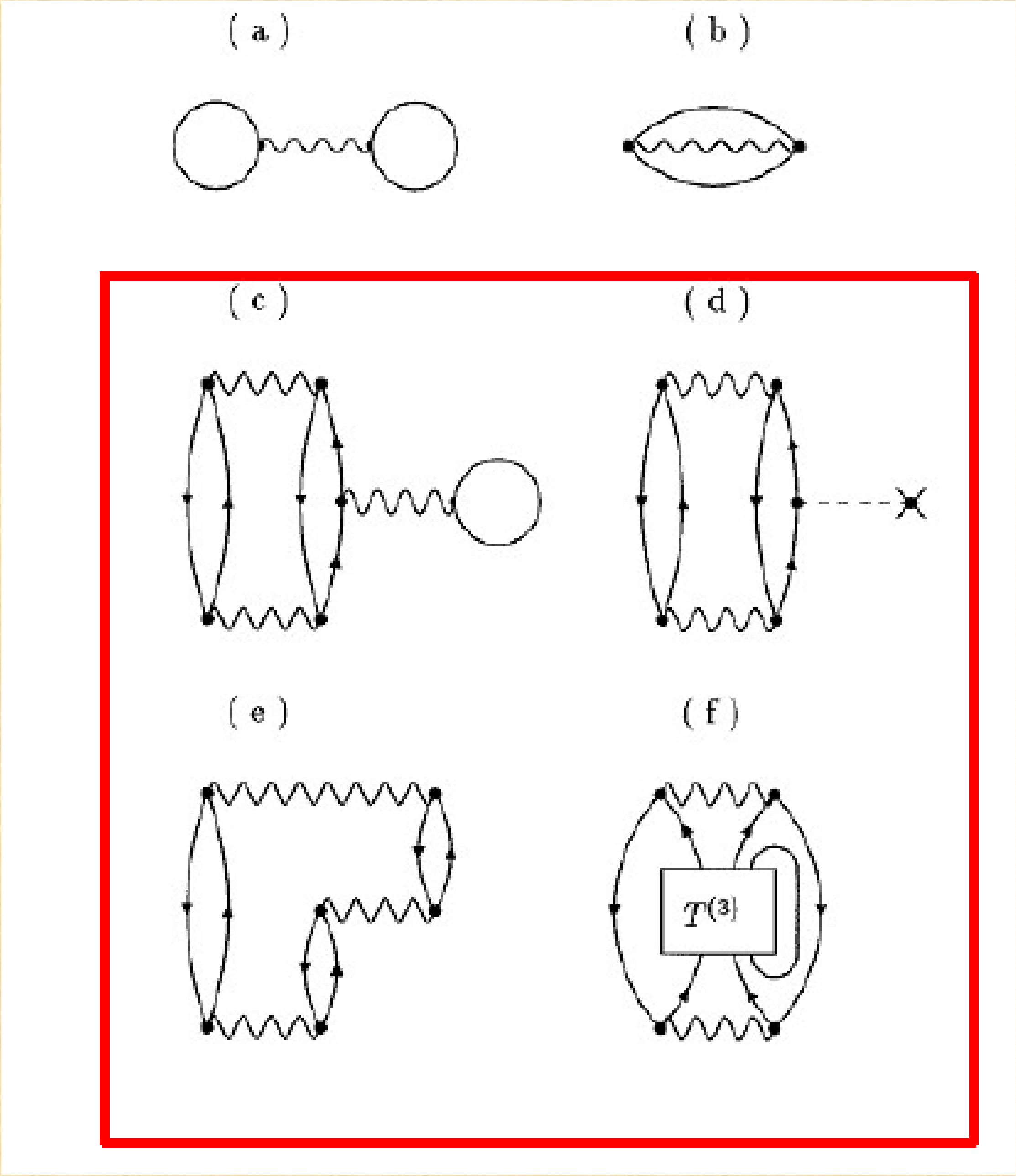


K. Fukukawa
Doctor thesis

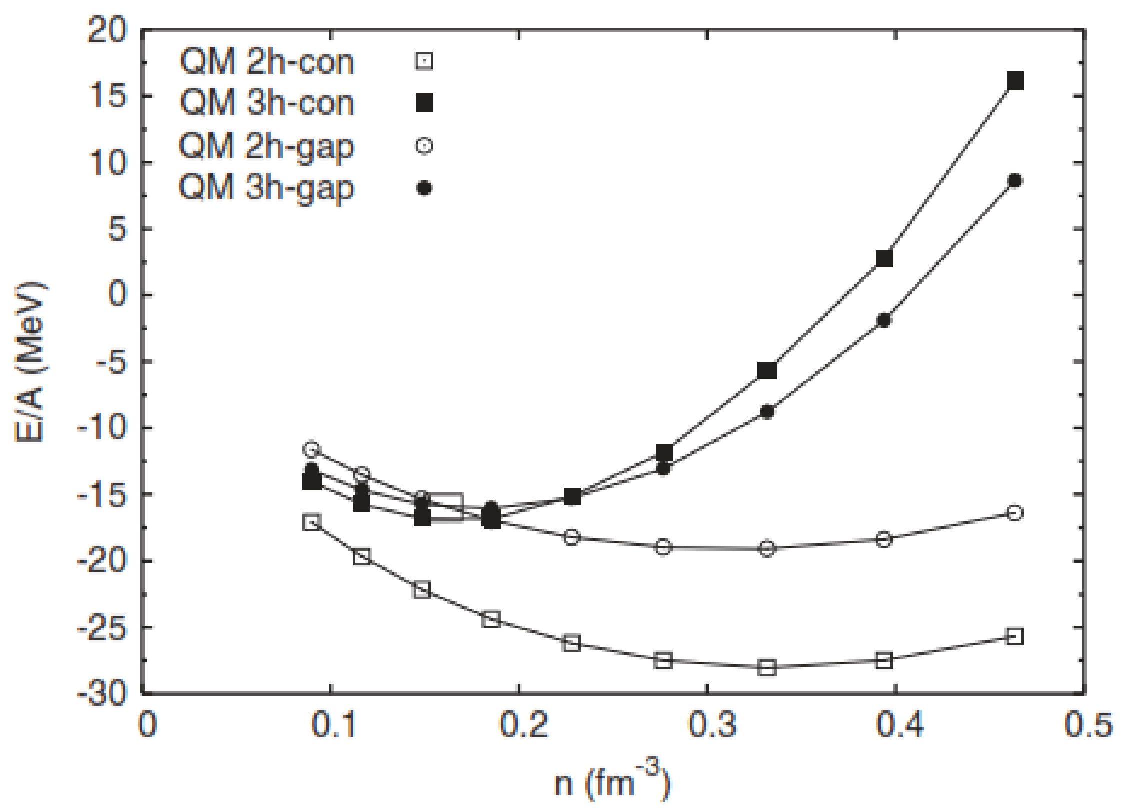


Z. M. Chen, W. Tornow and A. Kievsky, Few-Body Syst. 35, 15 (2004)

Including three-body correlations in Nuclear Matter



Relevance of three-body correlations for the QM interaction

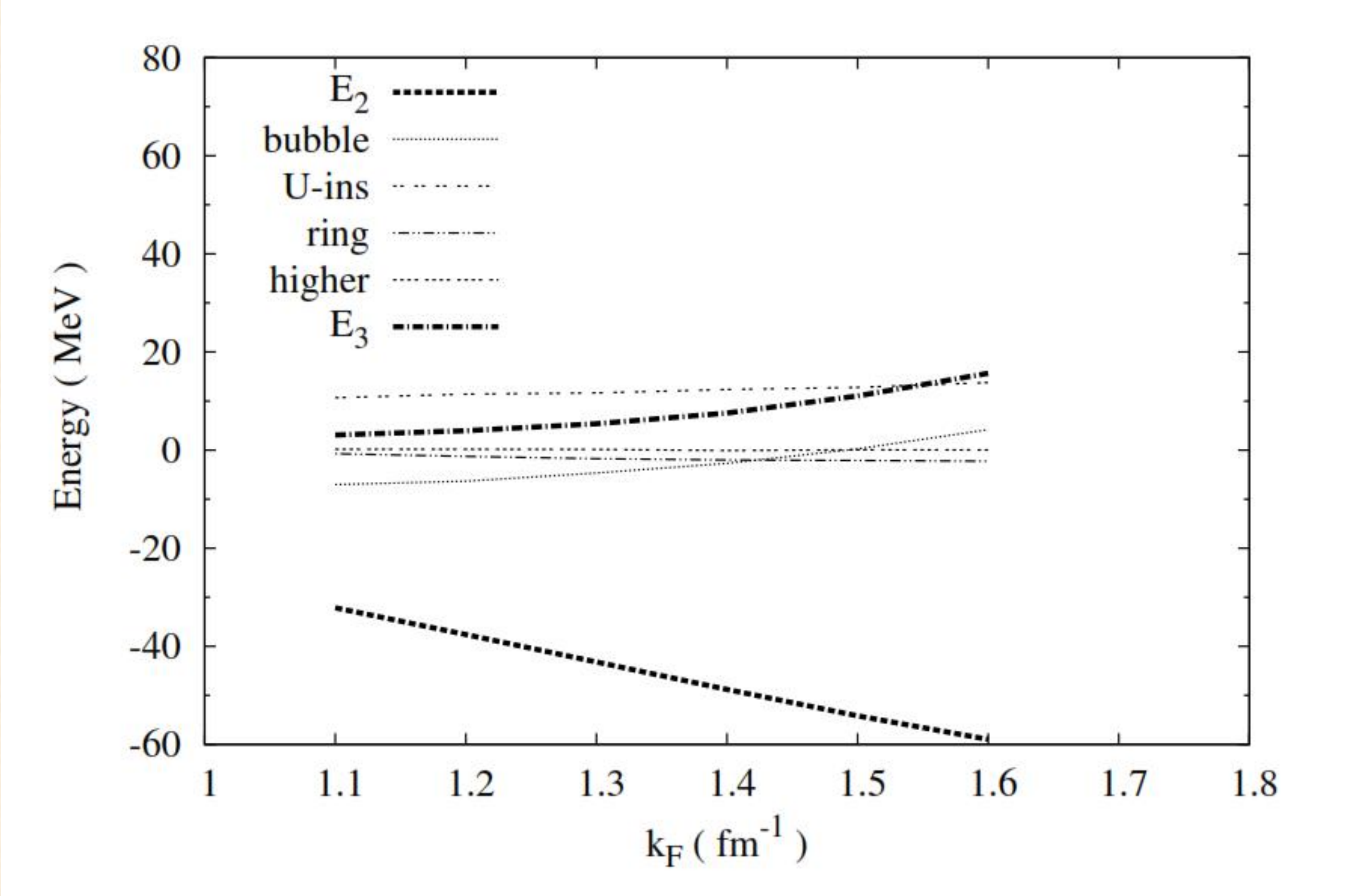


At saturation
n = 0.157 fm-3
K = 219 MeV
E/A = -16.3 MeV

This is at variance with respect to the other NN interactions that need three-boy forces (non relativistic)

M.B. and K. Fukukawa, PRL 113, 242501 (2015)

Comparing two-body and three-body CORRELATIONS

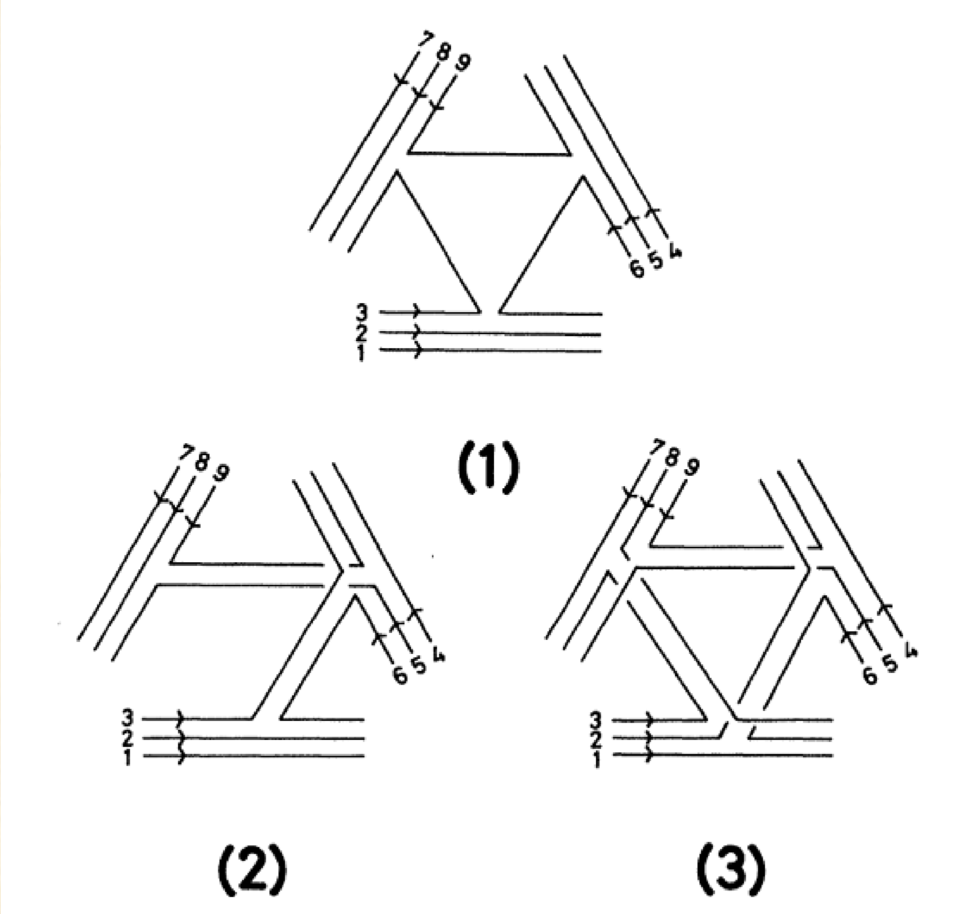


The saturation point and compressibility are well reproduced with this two-body force. The remaining physical effects not included are mainly

- Three-body forces.
- Relativistic effects.
- Four (or higher) nucleon forces.

However, the results indicate that the overall size of these effects must be marginal.

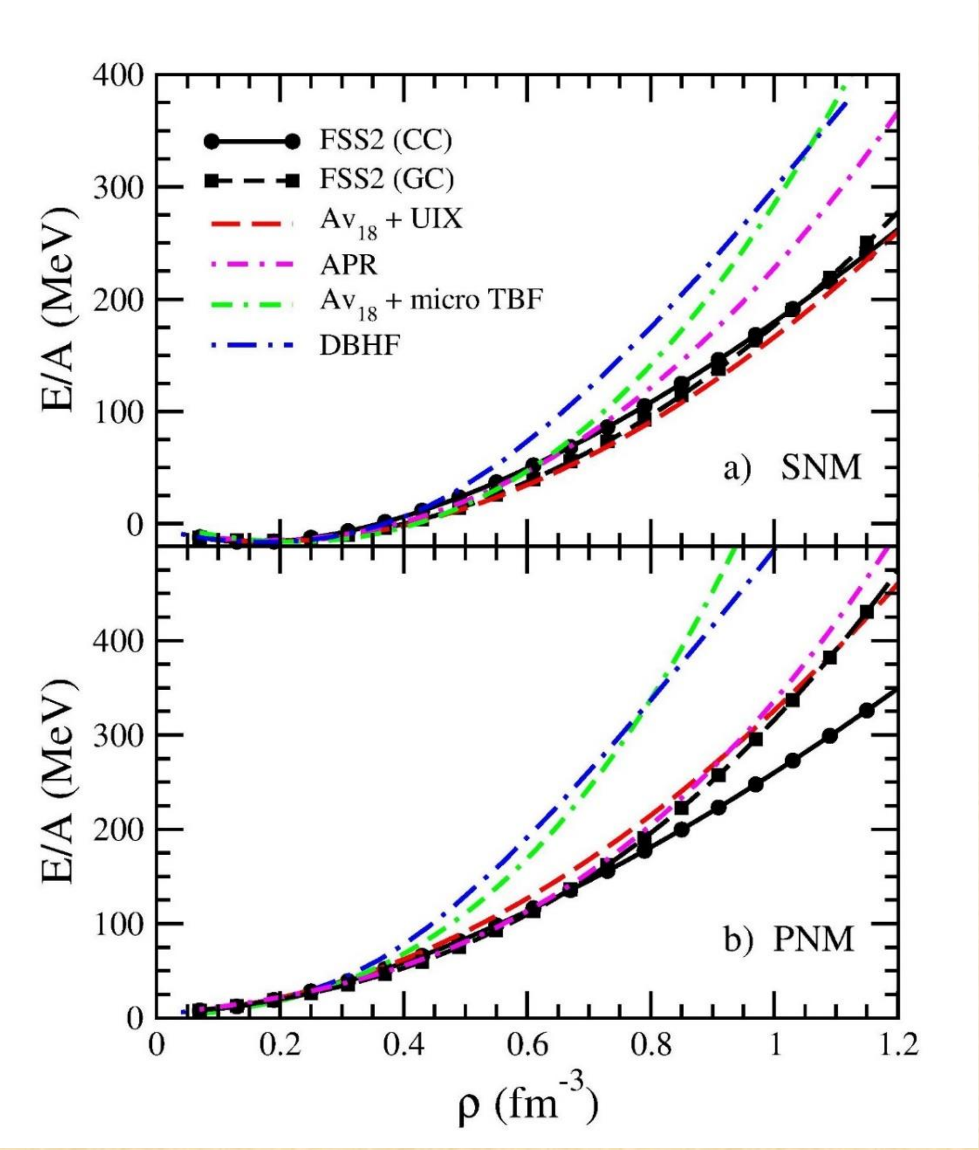
Possible three-body forces in the quark model



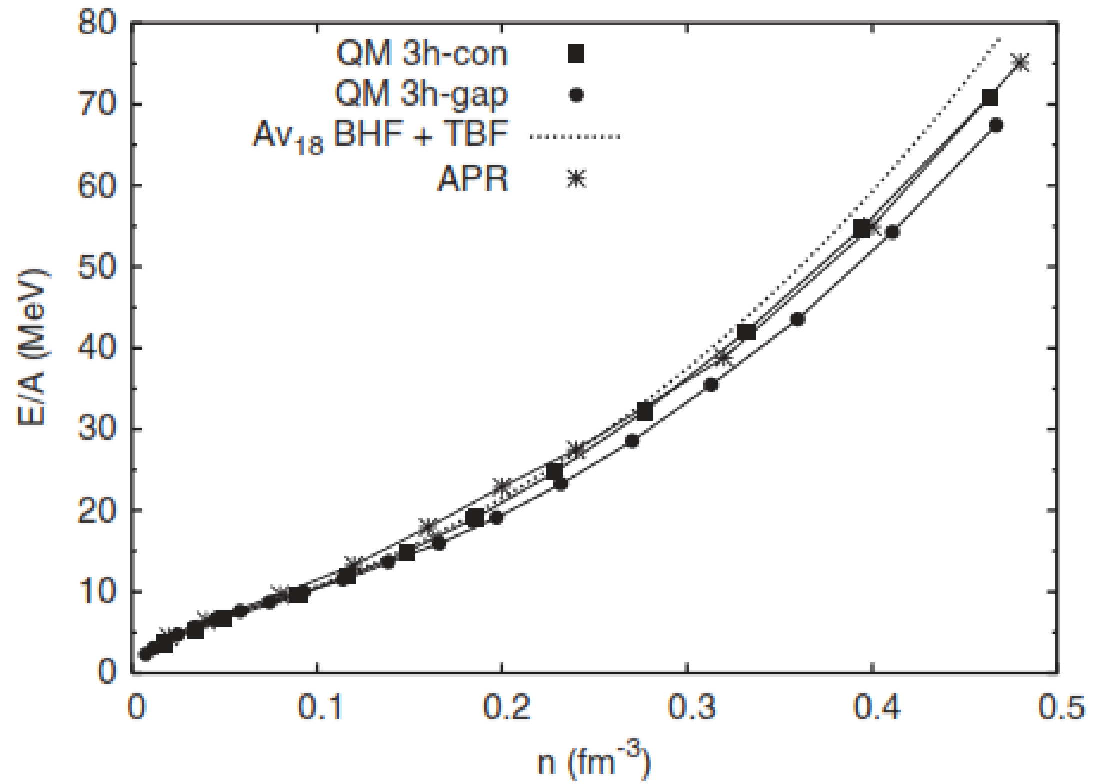
They turn out to be small

Y. Suzuki and K.T. Hecht, PRC29, 1586 (1984)

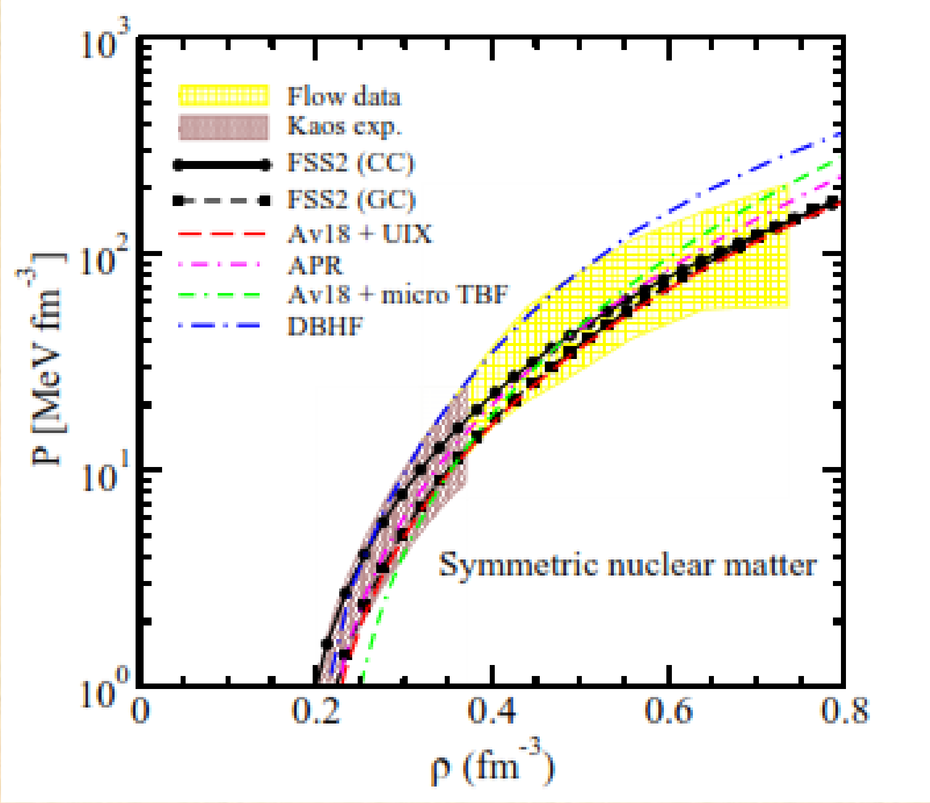
Comparing the QM EOS with other models

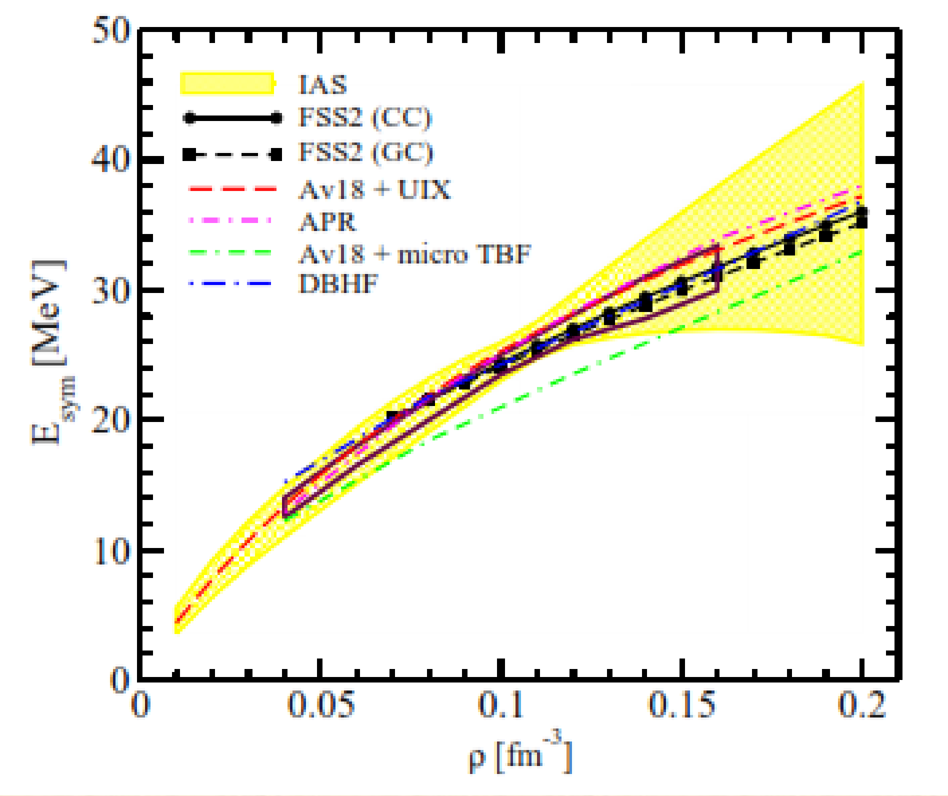


Comparison with other non relativistic models for pure Neutron Matter



Let us consider a brief survey of the comparison with the phenomenological constraints. Only the EOS that give the correct saturation point will be included

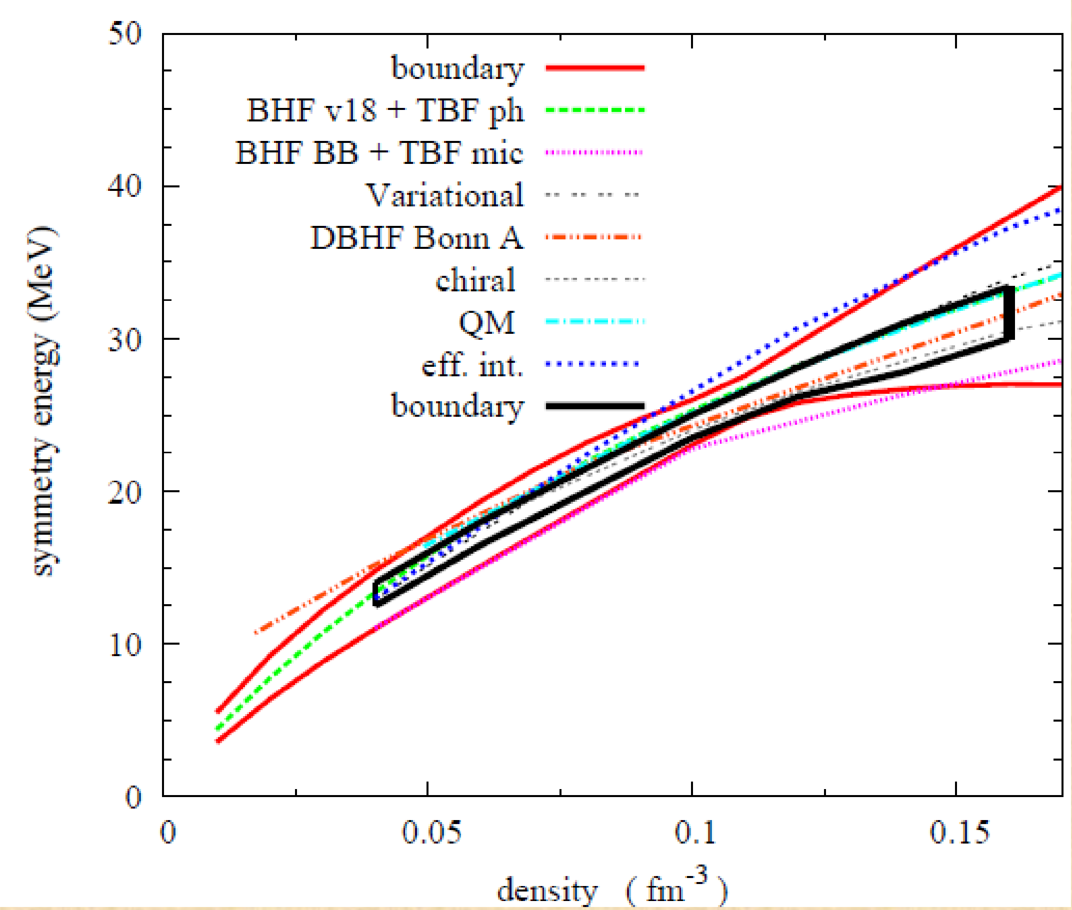




Heavy ions

Nuclear structure : Symmetry energy

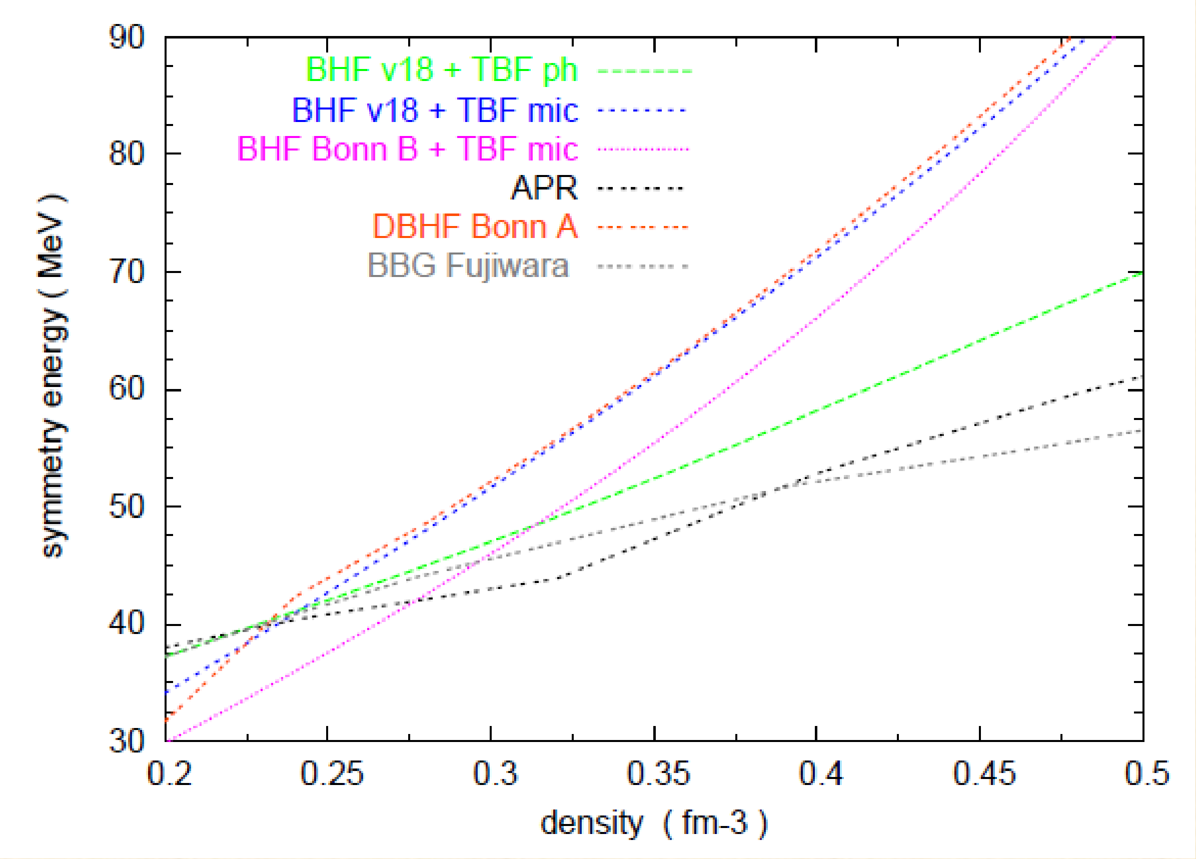
Overall comparison of the symmetry energy below saturation.
IAS + neutron skin data



Fair agreement among different EOS
Some discrepancy close to saturation

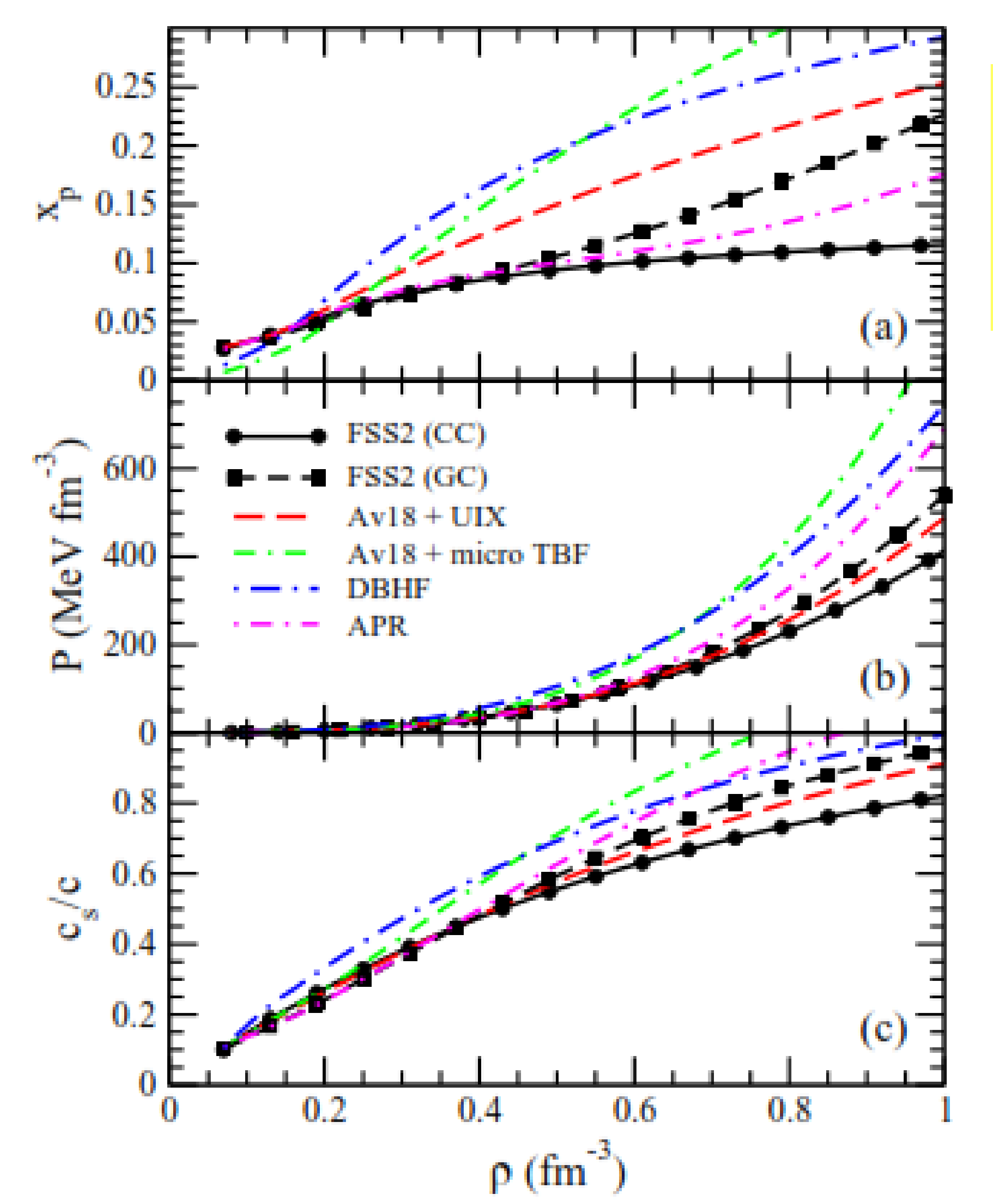
From : M.B. & G.F. Burgio, Prog. Part. Nucl. Phys. 2016

Symmetry energy above saturation



Substantial disagreement among the different EOS
No relevant constraints from Heavy Ion data (up to now)
Higher density constraints would be quite selective.

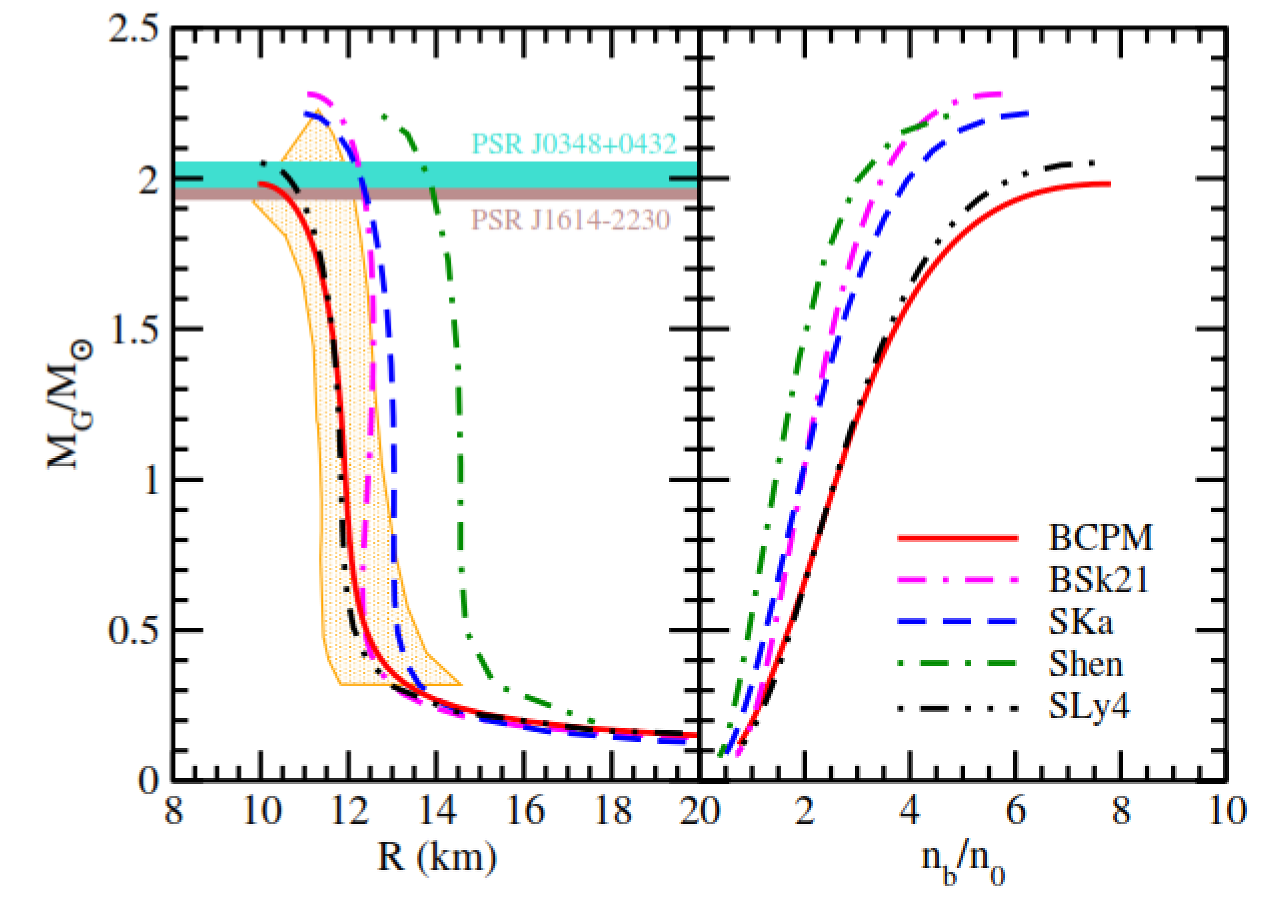
From : M.B. & G.F. Burgio, Prog. Part. Nucl. Phys. 2016



EoS for NS matter
i.e. beta-stable nuclear matter with components :

n, p, e^-, μ

The constraint from the observed maximum mass.

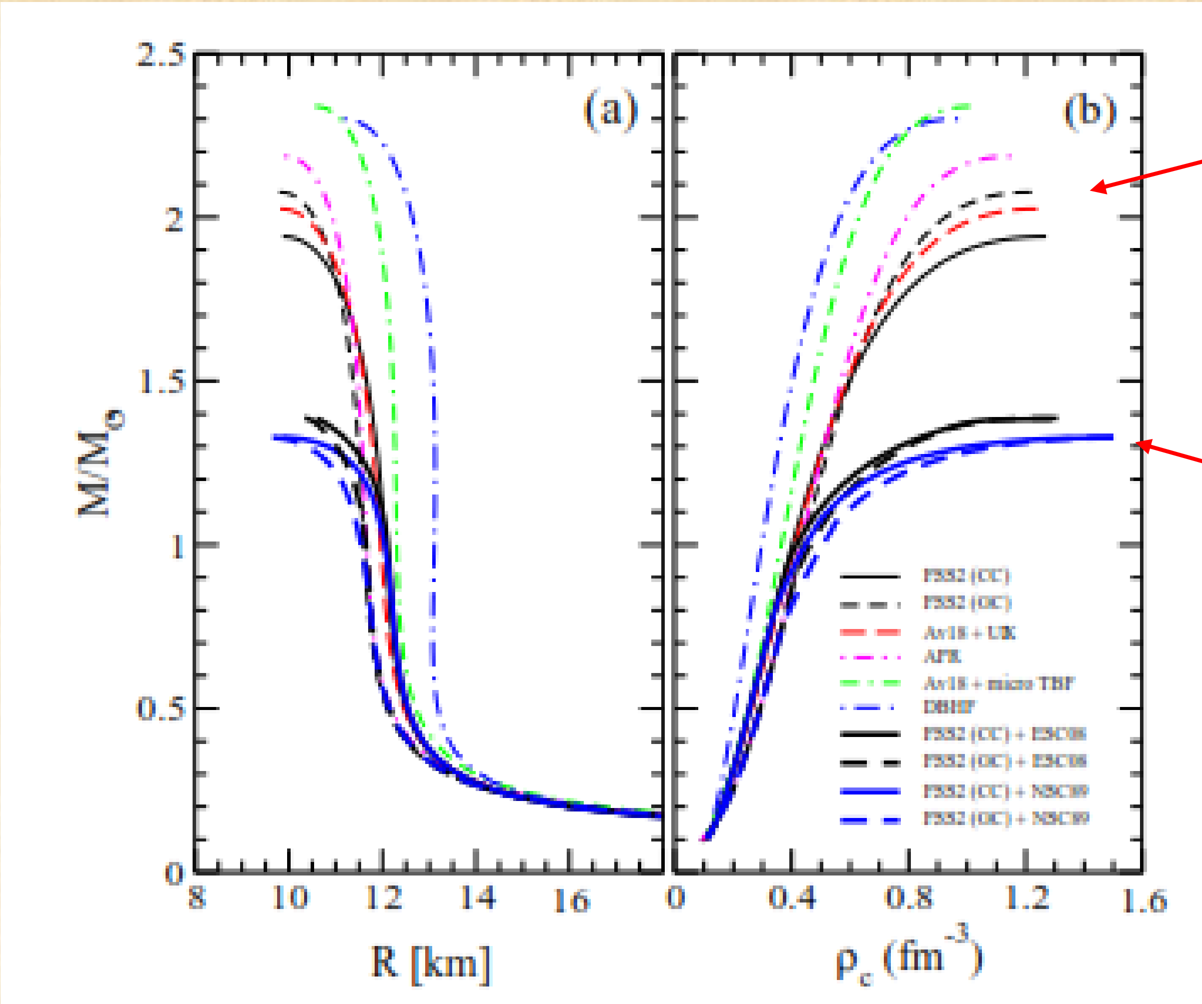


Hatched area : Bayesian analysis by Lattimer & Steiner, EPJA50, 40 (2014)

Different functionals, including Skyrme. Crust included.

Sharma et al., A&A 584, A103 (2015)

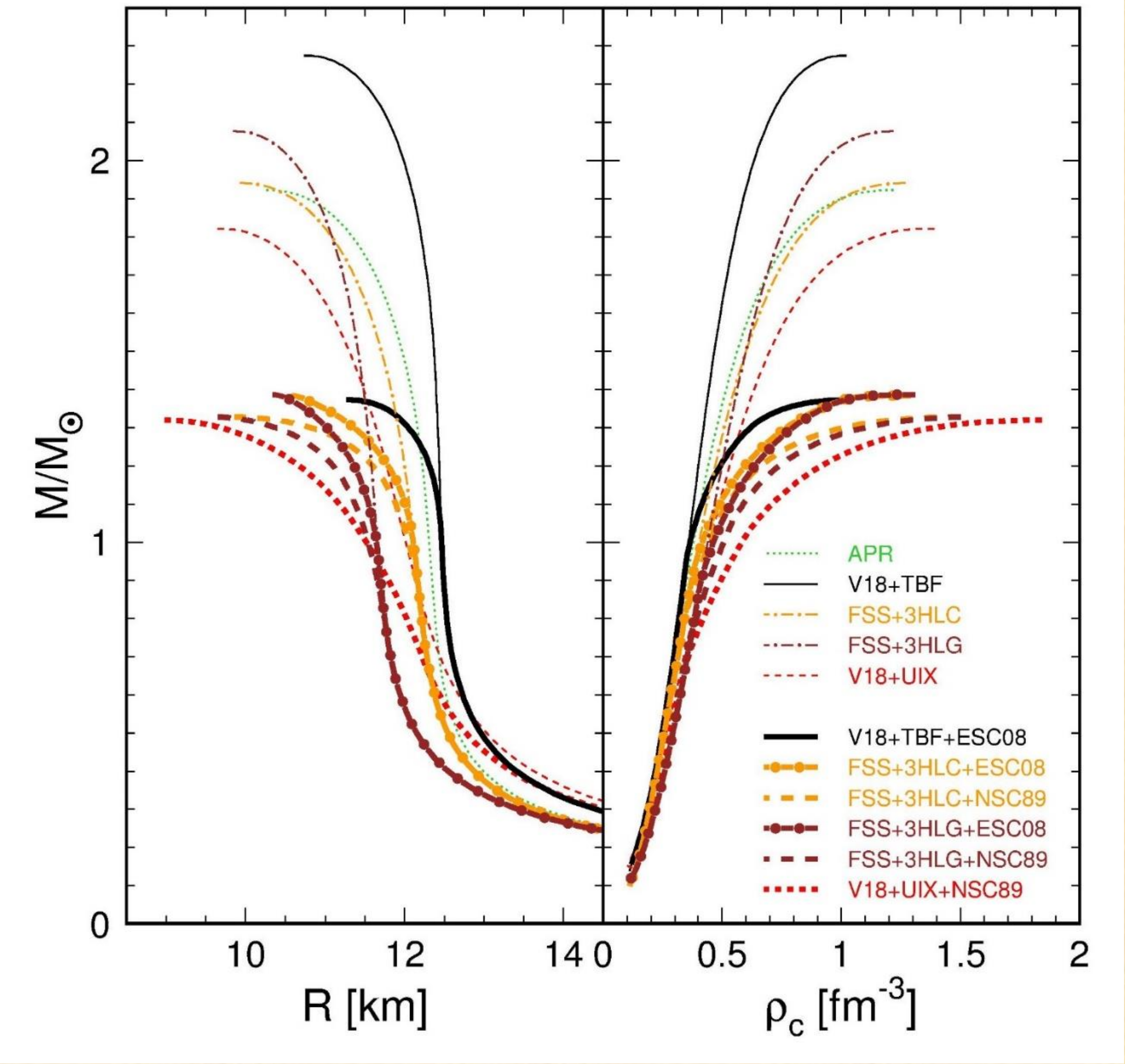
Neutron Star mass and radius



Other (microscopic) EOS.

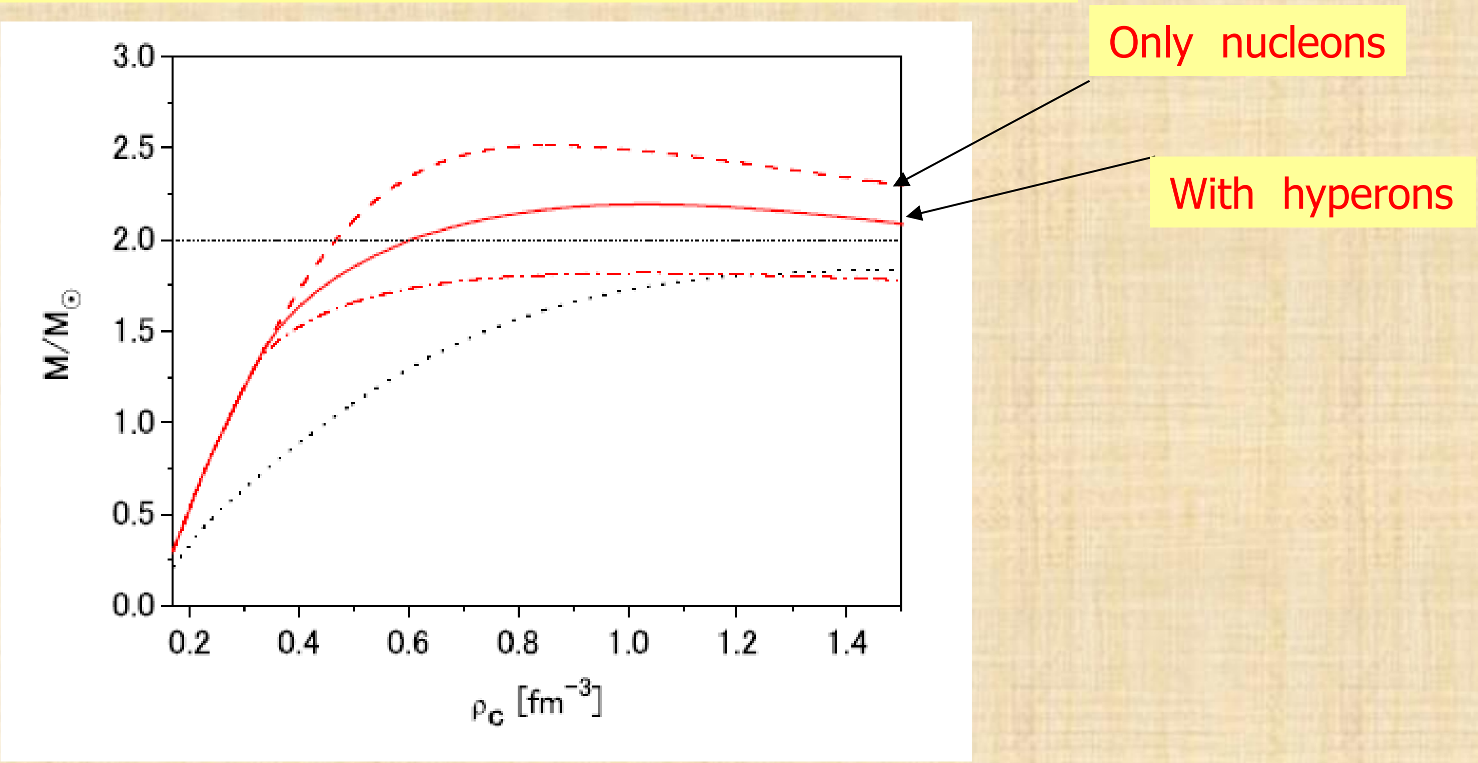
Dramatic effect of the hyperon component

Other hyperon-nucleon and hyperon-hyperon interaction models



Possible solution

Introducing multi-body forces for hyperons
Multi-pomeron exchange potential (MPP)



Universal repulsive force for all baryon sectors, including hyperons

Yamamoto et al., PRC90, 045805 (2014)

Universal repulsive baryon-baryon interaction related to three anf four-body forces.

$$V_{eff}^{(3)}(r) = g_P^{(3)}(g_P)^3 \frac{\rho}{\mathcal{M}^5} F(r) ,$$
$$V_{eff}^{(4)}(r) = g_P^{(4)}(g_P)^4 \frac{\rho^2}{\mathcal{M}^8} F(r) ,$$
$$F(r) = \frac{1}{4\pi} \frac{4}{\sqrt{\pi}} \left(\frac{m_P}{\sqrt{2}}\right)^3 \exp\left(-\frac{1}{2} m_P^2 r^2\right)$$

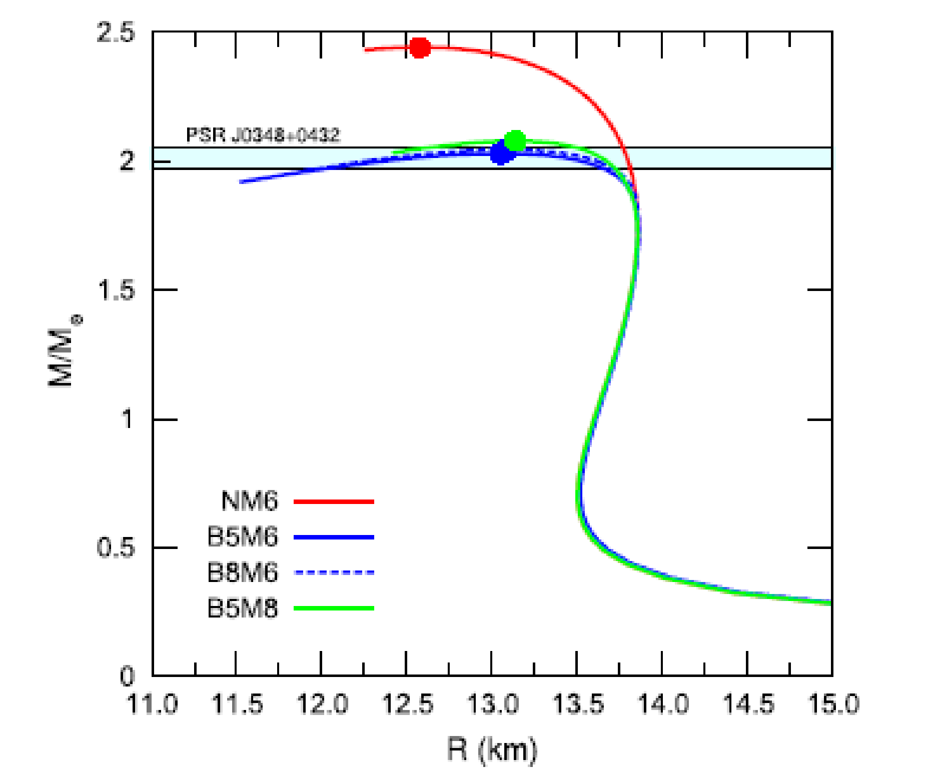
One can coclude that :

1.Extra repulsion is needed.

2.The multi-body forces in the hyperonic sector must be at least as strong as in the nucleonic sector.

Similar conclusion in D. Lonardoni et al., PRL114, 092301 (2015).

A similar conclusion is obtained also in DBHF, assuming SU(6), which is equivalent to take the same TBF in the nucleonic and hyperonic sector

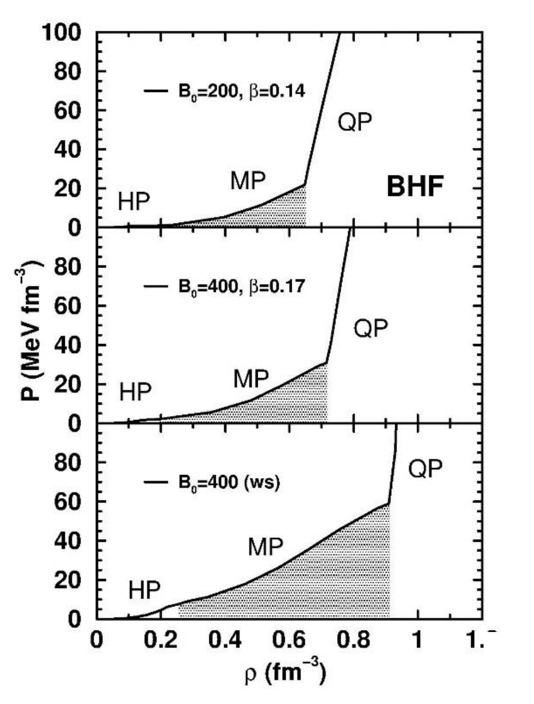


Katayama & Saito, PLB 747, 43 (2015)

Can the solution come from the quark degrees of freedom ?

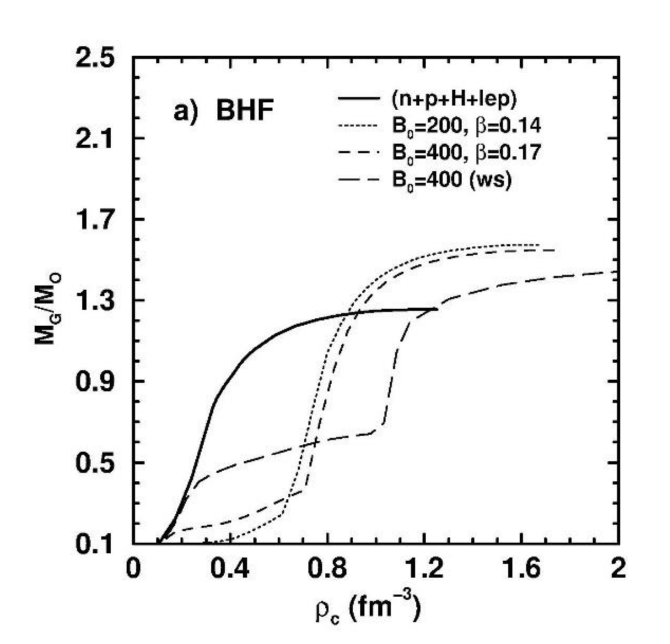
Introducing the quark degrees of freedom

Bag model with density dependent bag constant



Shaded area : mixed phase
QP : pure quark matter

Hyperons mainly disappear and the maximum mass is determined by the quark EOS, but it is still below the observational limit



G.F. Burgio et al., PLB 526, 19 (2002)

With respect to the MIT bag model there is need of additional repusion at high density.
This problem has been approached within several schemes

1. Color dielectric model

2. Nambu – Jona Lasinio model + additional interactions

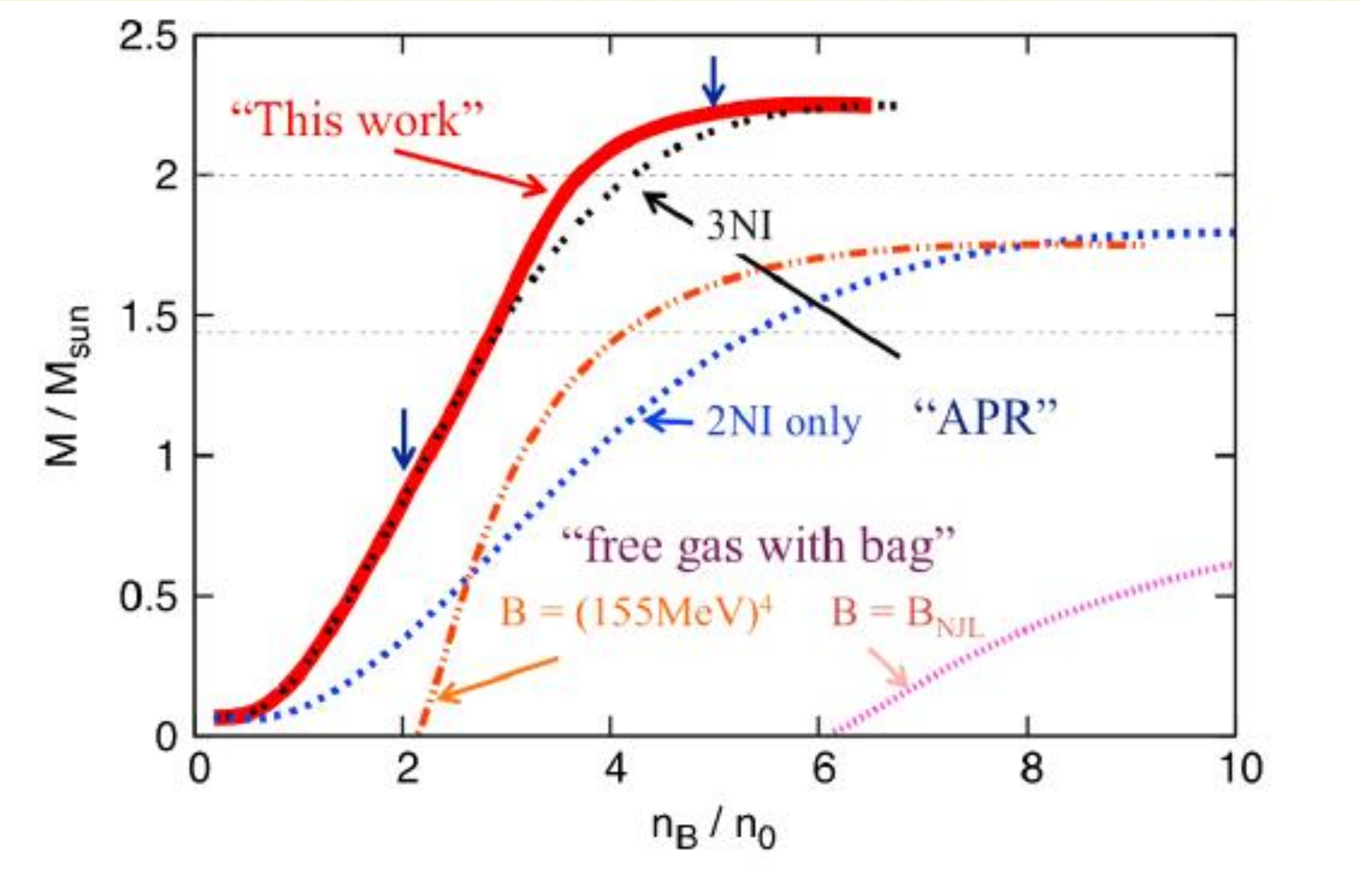
3. Dyson – Schwinger equation

4. Field correlator method

5. Freedman & McLerran model of QCD

With a suitable choice of the parameters they are able to reach the two solar mass limit (but one must check that hyperons are prevented to appear or they have little effect)

The quark matter EOS can be as stiff as the nucleonic EOS at high density



T. Koyo et al., PRD91, 045003 (2015), extended NJL model
Vector + diquark interaction

CONCLUDING REMARKS

1. There is a set of microscopic nucleonic EOS that are compatible with the phenomenological constraints. More constraints are expected from GW, heay ions and astrophysical data

2. They substantially agree up to density just above saturation, in particular on the symmetry energy

3. Disagreements appear at higher density, which means that **constraints in this density region would be very effective in selecting the microscopic EOS**

3. If hyperonic and quark degrees of freedom are introduced, the observed masses of NS require a substantial additional repulsion with respect to the simplest models, **either to stiffen the EOS or to hinder the appearence of these ‘exotic’ components.** A sound QCD theoretical basis for this repulsion is still lacking

5. A systematic application of these microscopic many-body theories to the calculations of other NS properties (e.g. MURCA, transport, GW, ……) is still missing. Hopefully this could provide further selection.

Major uncertainties :

1. Three-body forces unknown at high density.
   Their relevance is model dependent.
   If quark degrees of freedom are introduced their relevance seems to be reduced to a minimum.

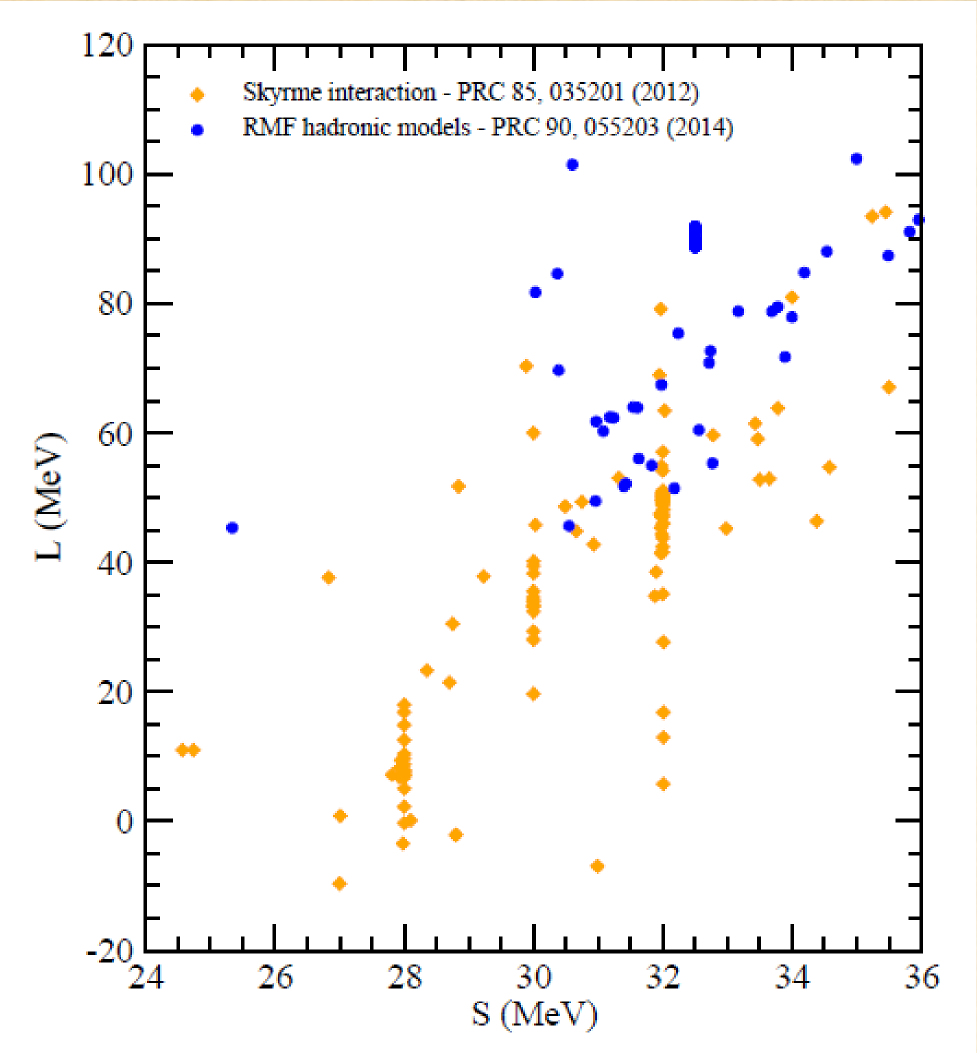
2. The effect of ‘exotic’ components (mainly hyperon and quark) has not a sound theoretical framework

The phenomelogical constraints are selective on the acceptable EOS. They can give, especially the astrophysical ones, hints on the direction where to move**, e.g. the additional repulsion at high density in the ‛exotic‛ sector.**

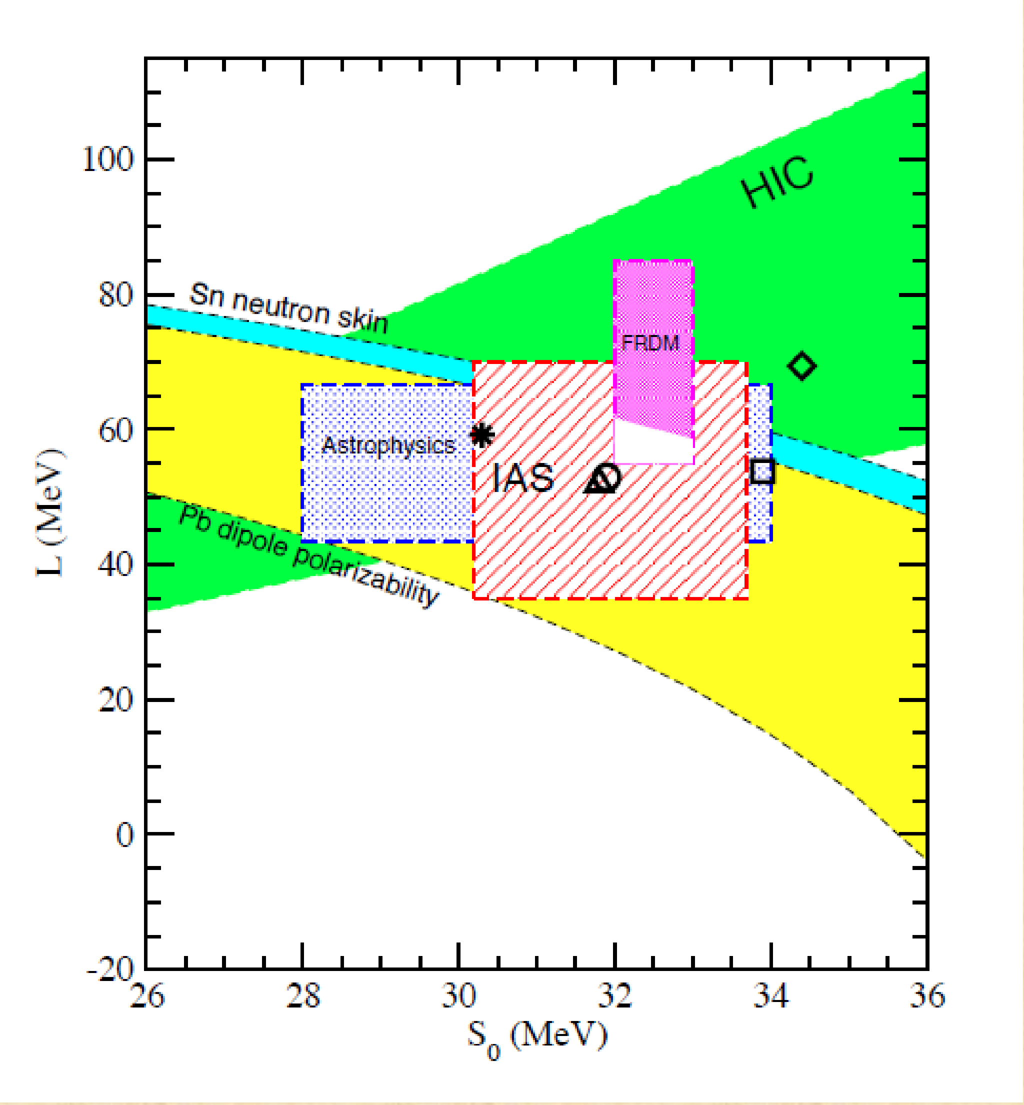
It is indeed the mutual interaction between phenomenology and theory that can support additional progresses in the field

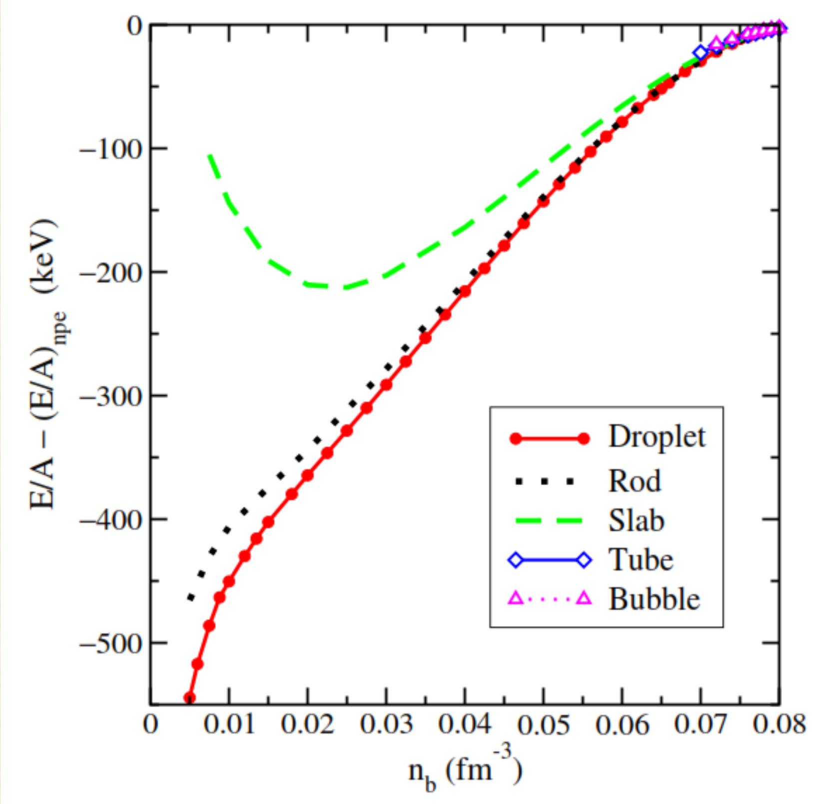
MANY THANKS !

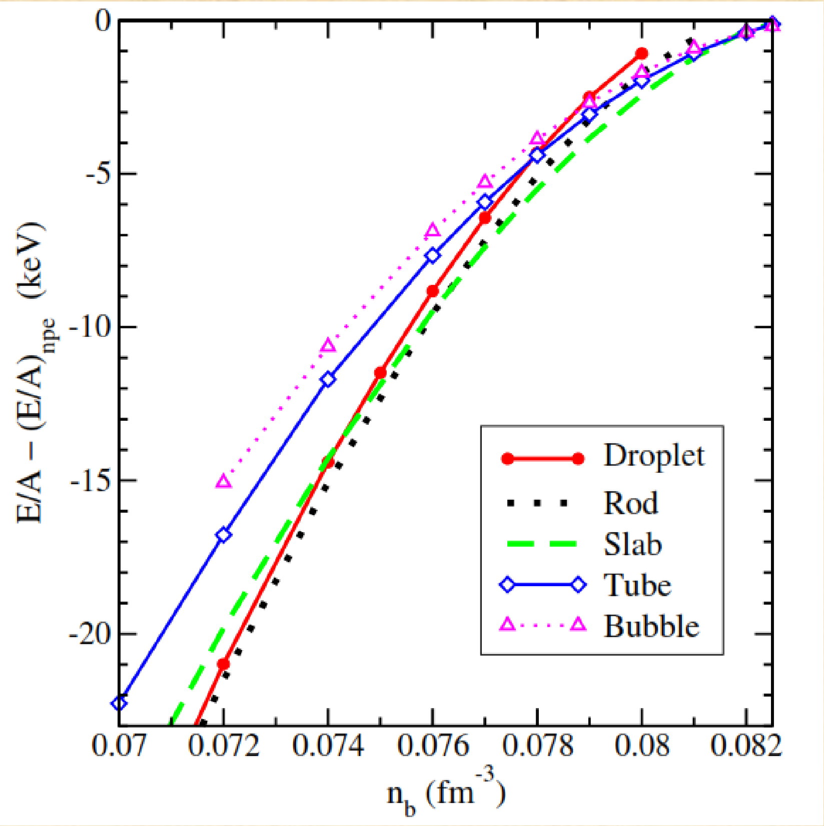
Skyrme and RMF functionals with saturation energy and compressibility compatible with phenomenology



Comparing microscopic theories with phenomenology







Structure of the Neutron Star crust

The densities involved are typical nuclear densities
Physical conditions quite different

B.K. Sharma, M. Centelles, X. Vinas, G.F. Burgio, M.B.
Astronomy & Astophysics 584, A103 (2015).