Lattice QCD approach to baryon interactions

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Compact stars and gravitational waves
Oct. 31 - November 4, 2016
Yukawa Institute for Theoretical Physics, Kyoto University

For
HAL QCD Collaboration

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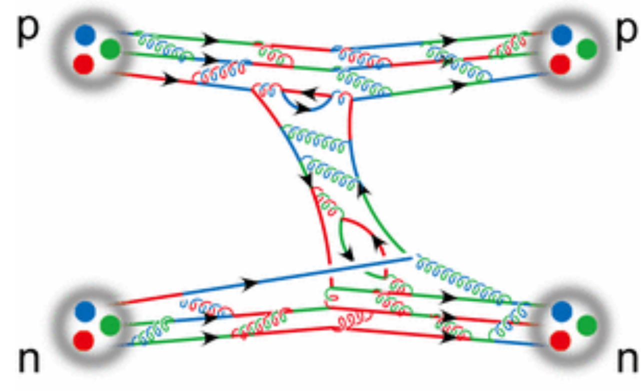
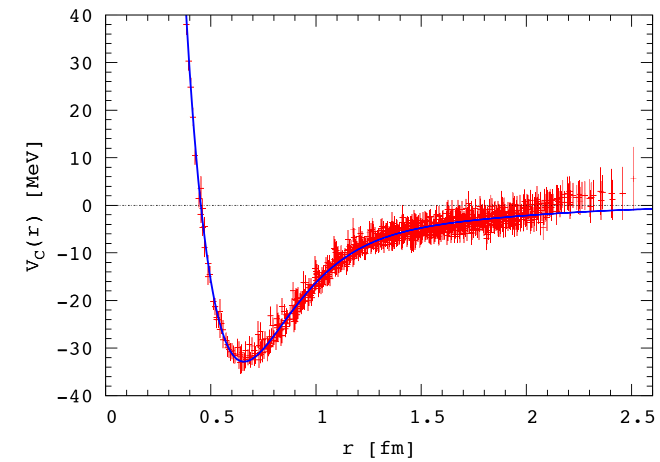
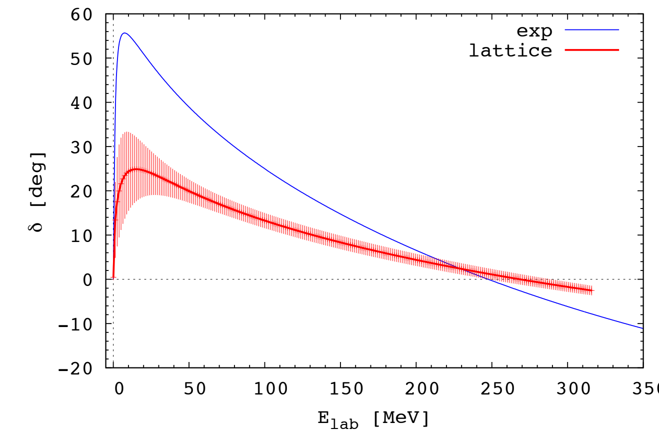
HAL QCD strategy to Nuclear/Astro physics

Potentials from
lattice QCD

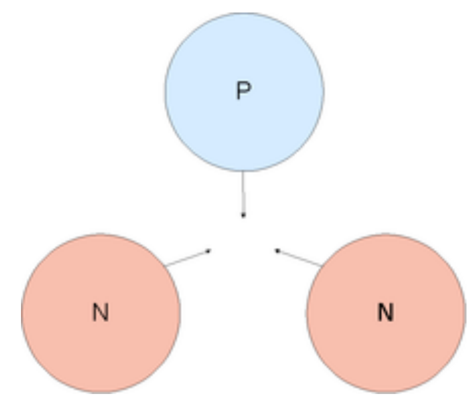




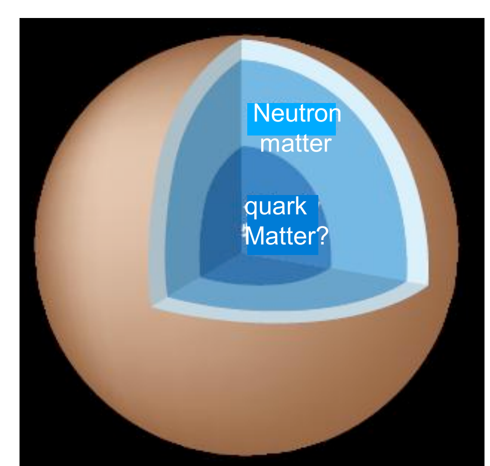


Nuclear Physics
with these potentials

parameters of EFT



Neutron stars
Supernova explosion



Before giving our results, however, there is an issue for baryon interactions in lattice QCD, which must be understood.

Some results: talk by Hatsuda on Wed.

In this talk, I will explain the issue and give our understanding.

As my talk will be more or less logical (though not difficult), please interrupt me if you get lost.

Introduction
What is an issue ?

Difficulties of two(multi)-baryon systems

Two-nucleon propagator

$$G_{NN}(t) = \langle N(t)N(t)\bar{N}(0)\bar{N}(0)\rangle = \boxed{Z_0 e^{-E_0 t}} + Z_1 e^{-E_1 t} + \cdots \to \boxed{Z_0 e^{-E_0 t},} \quad t \to \infty$$

- (systematic errors) t can not be infinite. Effects of $E_1, E_2, \cdots$.
- (statistical errors) $G_{NN}(t)$ is calculated by the Monte-Carlo average.

$N = qqq$ (3 quarks)

	Signal	Noise
Single-nucleon	$\langle N(t)\bar{N}(0)\rangle \simeq e^{-m_N t}$	$\sqrt{\langle \lvert N(t)\bar{N}(0)\rvert^2\rangle} \simeq \sqrt{e^{-3m_\pi t}} = e^{-\frac{3}{2}m_\pi t}$
A-nucleons	$\langle N^A(t)\bar{N}^A(0)\rangle \simeq e^{-Am_N t}$	$\sqrt{\langle \lvert N^A(t)\bar{N}^A(0)\rvert^2\rangle} \simeq e^{-A\frac{3}{2}m_\pi t}$

Signal-to-Noise ratio

$$\frac{S_A(t)}{N_A(t)} = \frac{\langle N^A(t)\bar{N}^A(0)\rangle}{\sqrt{\langle |N^A(t)\bar{N}^A(0)|^2\rangle}} \sim \exp\left[-A\left(m_N - \frac{3m_\pi}{2}\right)t\right]$$

becomes worse

more baryons

lighter pions

larger time

A (kind of) sign problem for fermion systems.

A single baryon is well understood. ⇨ **Baryon masses**

Only a few groups are working on two-baryon systems.
Thus still premature.

Lattice QCD methods for two-baryons

Details will be given later.

Direct method

$$G_{NN}(t) \sim e^{-E_{NN}t} \qquad t \to \infty$$

$$\Delta E = E_{NN} - 2m_N$$ → binding energy

+ finite volume formula → phase shift

Lüscher, NPB354(1991)531

Potential method **(HALQCD method)**

$$G_{NN}(r,t) = \langle N(r,t)N(0,t)\bar{N}(0)\bar{N}(0)\rangle$$ → "potential"

+ Schrödinger equation → phase shift

binding energy

Both are theoretically equivalent, but

Two nucleaon systems at heavy pions

		$^1\mathrm{S}_0$ "di-neutron"	$^3\mathrm{S}_1$ "deuteron"
Direct method		bound	bound
		interactions become stronger at heavier pions	
Potential method		unbound	unbound
		interactions become weaker at heavier pions	
Nature	$m_\pi \simeq 140\ \mathrm{MeV}$	unbound	bound

Both must agree.
We therefore have to identify sources of this discrepancy.

In this talk, I will show several evidences that some systematic uncertainties are not under control in the direct method while they are well controlled in the potential method.

Part 1. Direct method

Extraction of energy shift

Energy shift $$\Delta E \equiv E_{NN} - 2m_N$$

$O(10\ \mathrm{MeV})$ $O(2\ \mathrm{GeV})$ $O(2\ \mathrm{GeV})$

large cancellation

0.5 % accuracy required

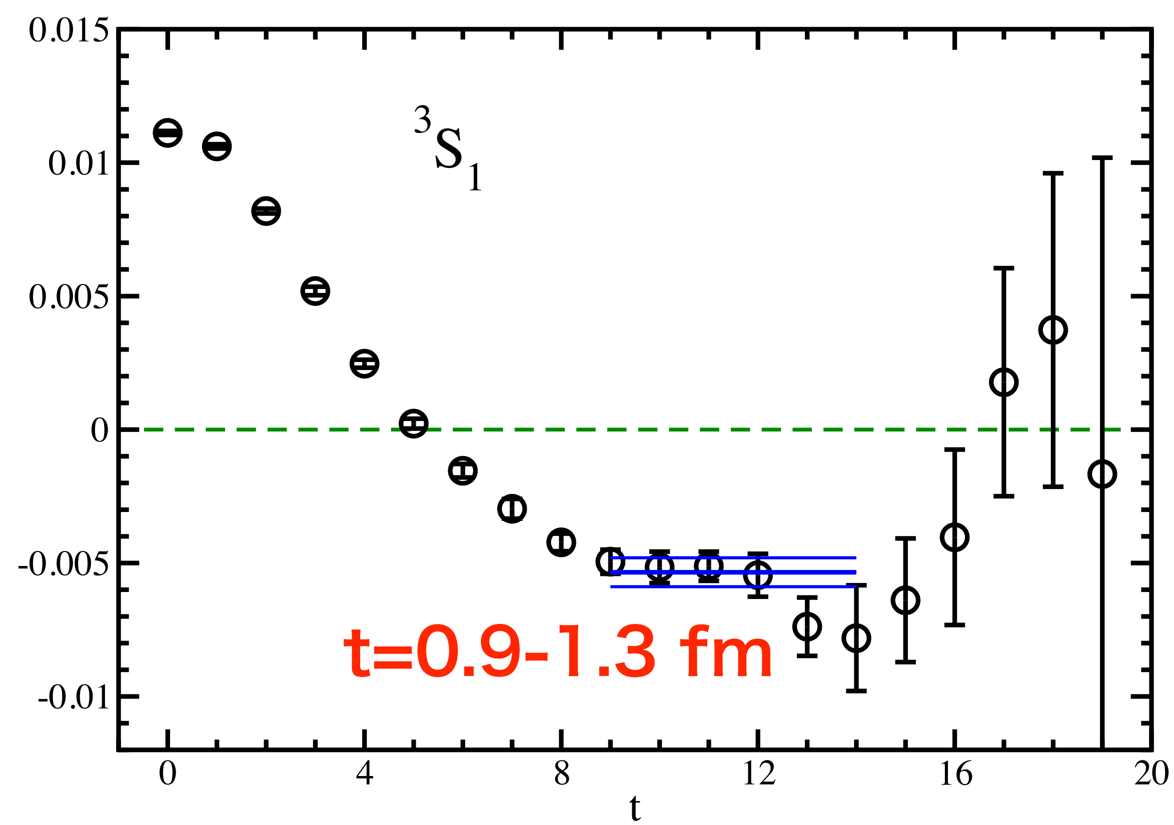
Ratio $$R(t) = \frac{G_{NN}(t)}{G_N(t)^2} \sim e^{-\Delta E t}$$

expect cancellation of both statistical and systematic errors

Effective energy shift

$$\Delta E(t) = \frac{1}{a} \log \frac{R(t)}{R(t+a)} \longrightarrow \Delta E, \qquad t \to \infty$$

YIKU 2012: PRD86(2012)074514



Plateau method

We identify $\Delta E(t)$ as ΔE, if it becomes constant.

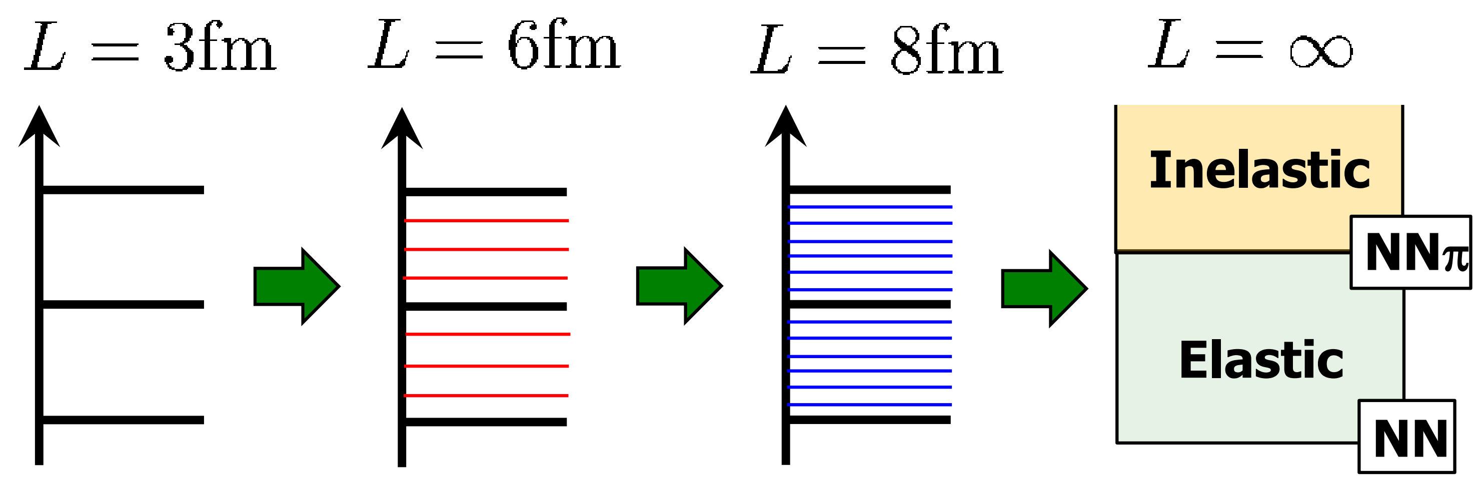
Is the plateau method reliable ?

Excitation energy $E_1 - E_0$

binding energy: very small

finite volume effect for scattering state $\simeq \frac{1}{m_N}\frac{(2\pi)^2}{L^2}$

$E_1 - E_0 \simeq 50$ MeV at $L = 4$ fm



$t \gg 1/(E_1 - E_0) \simeq 4$ fm is needed to suppress excited states.

Observing the plateau guarantees the ground state saturation even when $t \gg 1/(E_1 - E_0)$ is NOT satisfied.

claimed by Y(I)KU('11,'12,'15), NPL('12,'13,'15), CalLat('15)

Examination of the statement

Mock-up data @ $m_\pi = 0.5$ GeV, $L = 4$ fm (setup of YIKU2012)

$$R(t) = e^{-\Delta E t}\left(1 + b\, e^{-\delta E_{\rm el} t} + c\, e^{-\delta E_{\rm inel} t}\right)$$

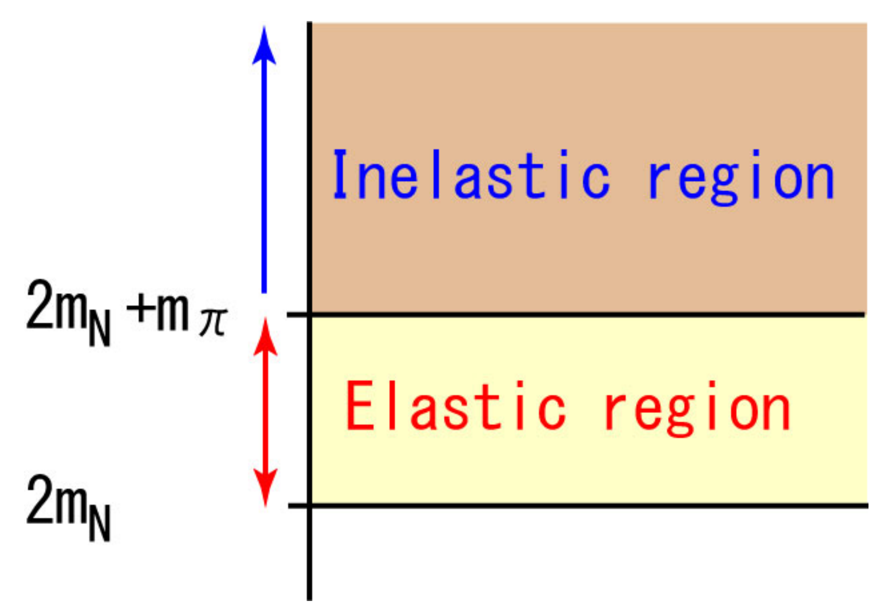
$\delta E_{\rm el} \propto \frac{1}{L^2}$ the lowest excitation energy of elastic scattering state

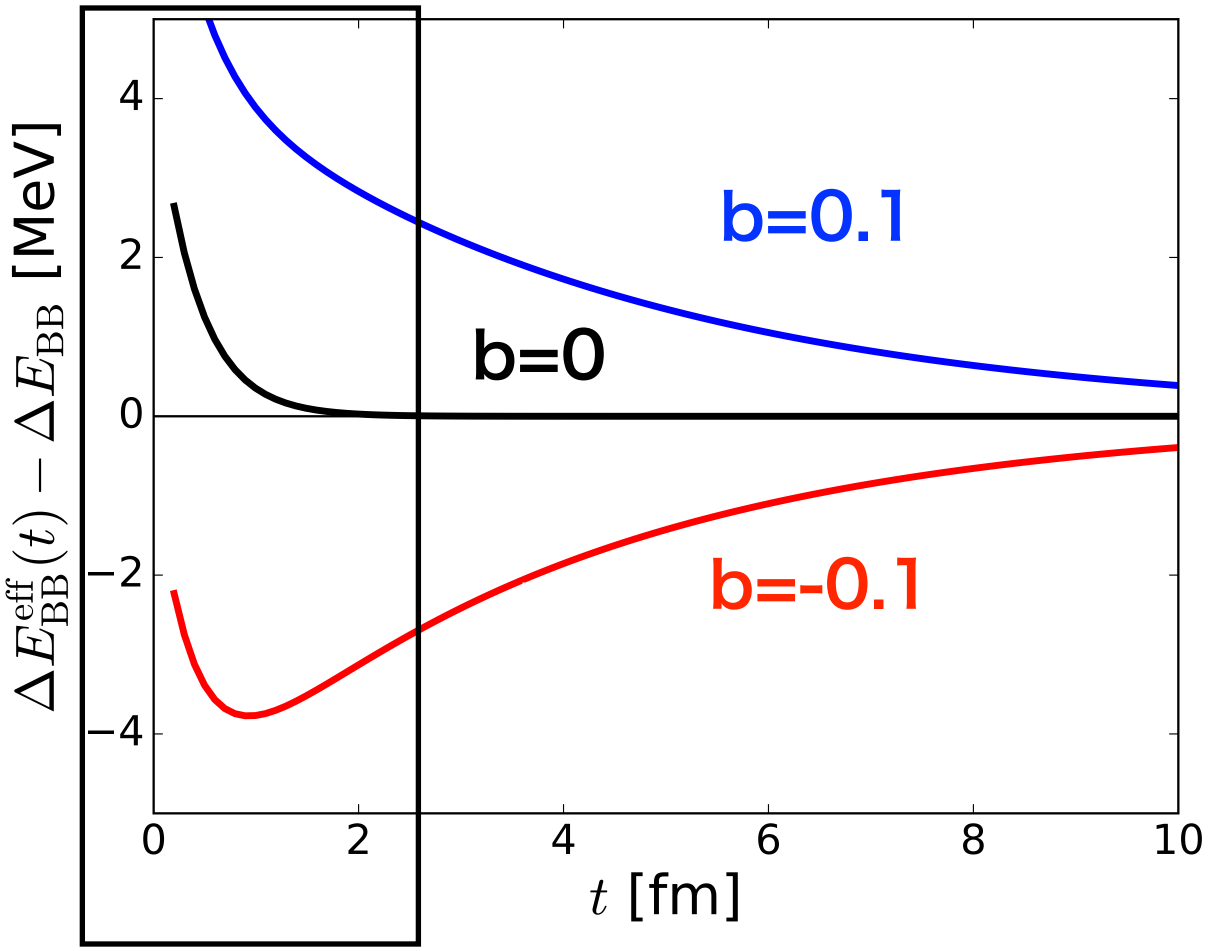
$\delta E_{\rm el} = 50$ MeV at $L \simeq 4$ fm

$b = \pm 0.1$ 10 % contamination $b = 0$ for a comparison

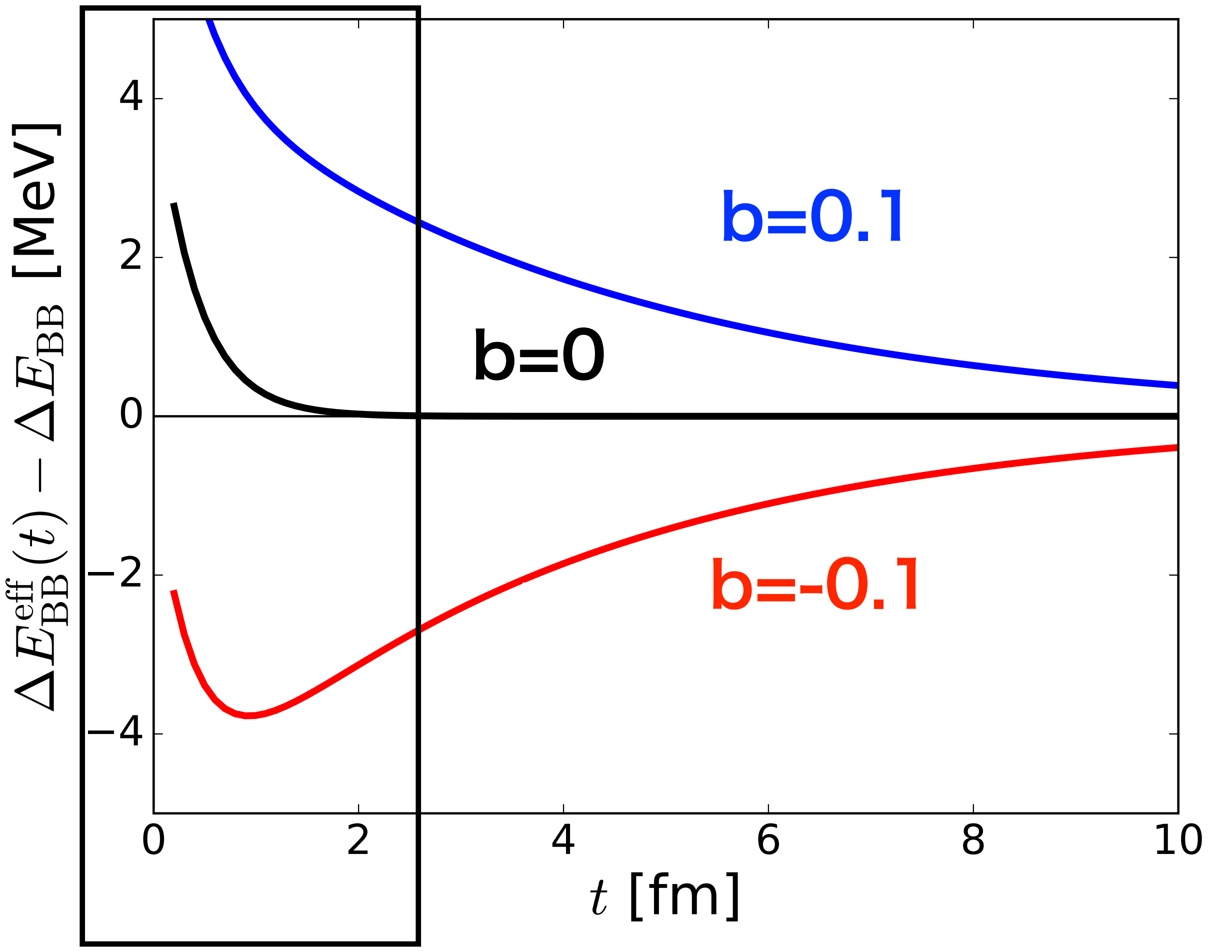
$\delta E_{\rm inel} = 500$ MeV the inelastic energy from heavy pions

$c = 0.01$ 1% contamination

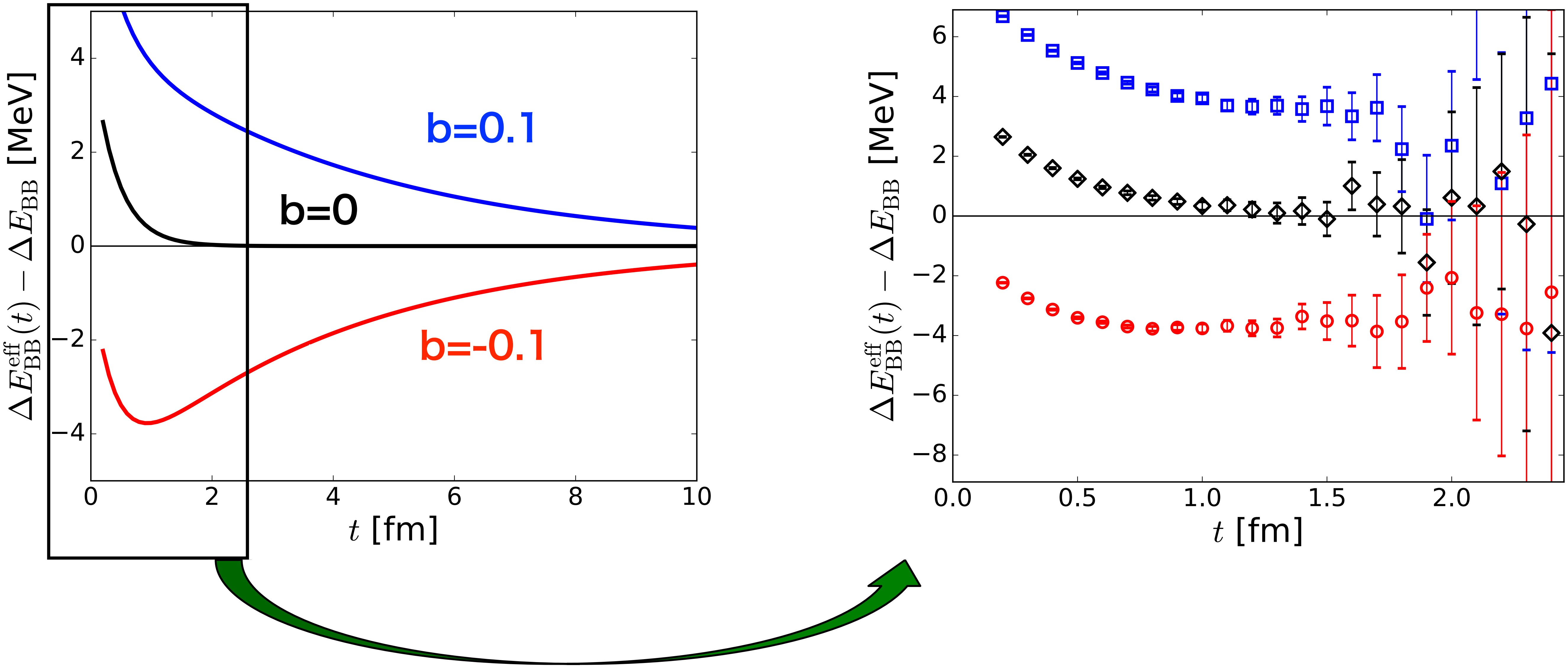




b=0.1
b=0
b=-0.1
$\Delta E_{\rm BB}^{\rm eff}(t) - \Delta E_{\rm BB}$ [MeV]
t [fm]



Zoom + increasing errors and fluctuations



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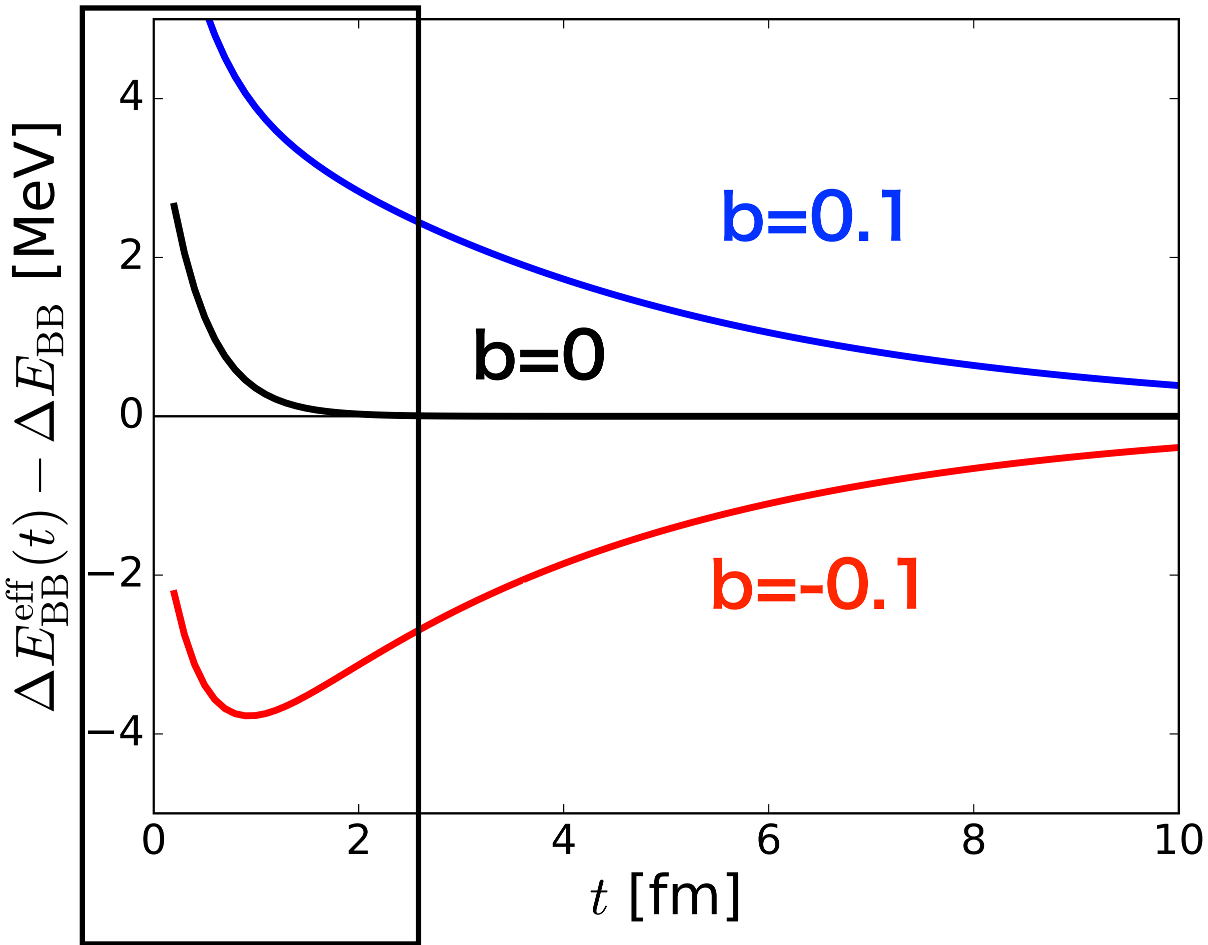
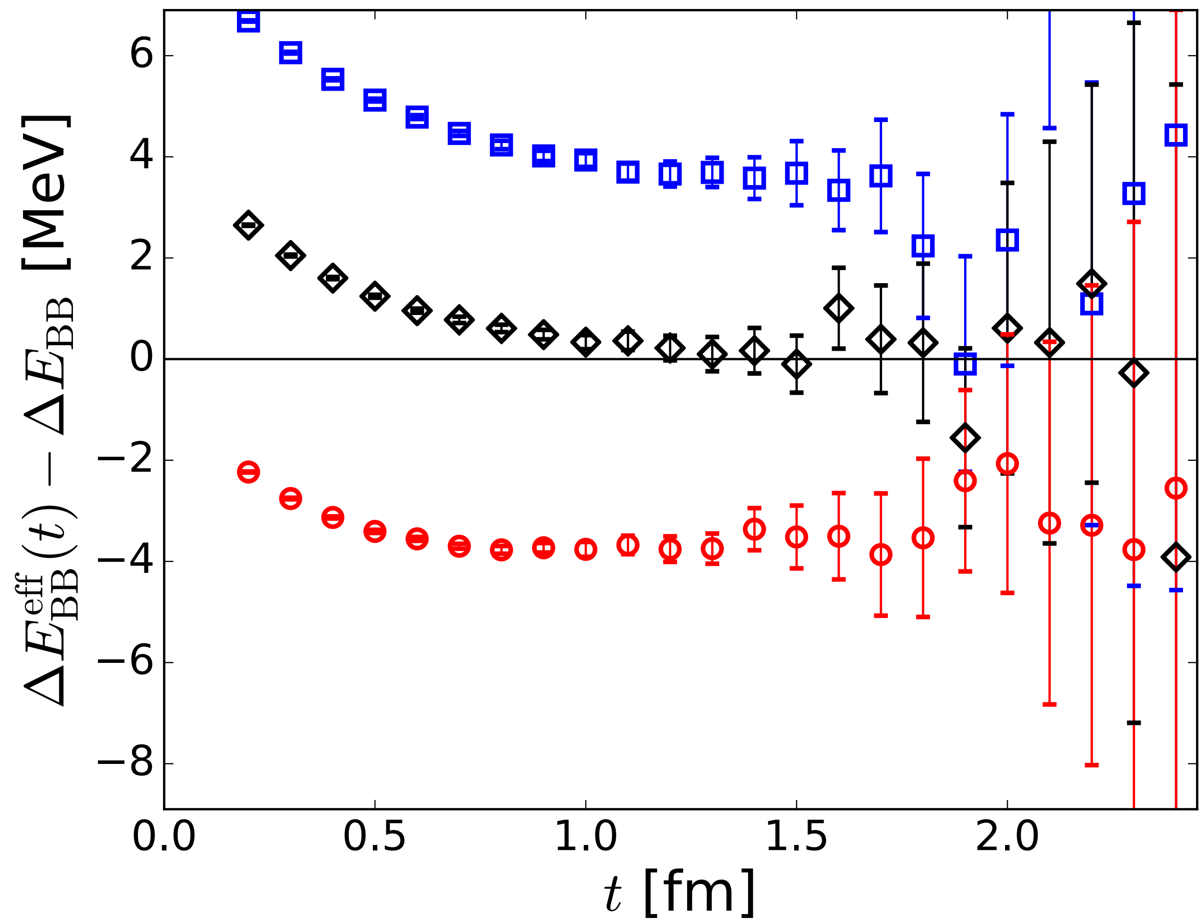
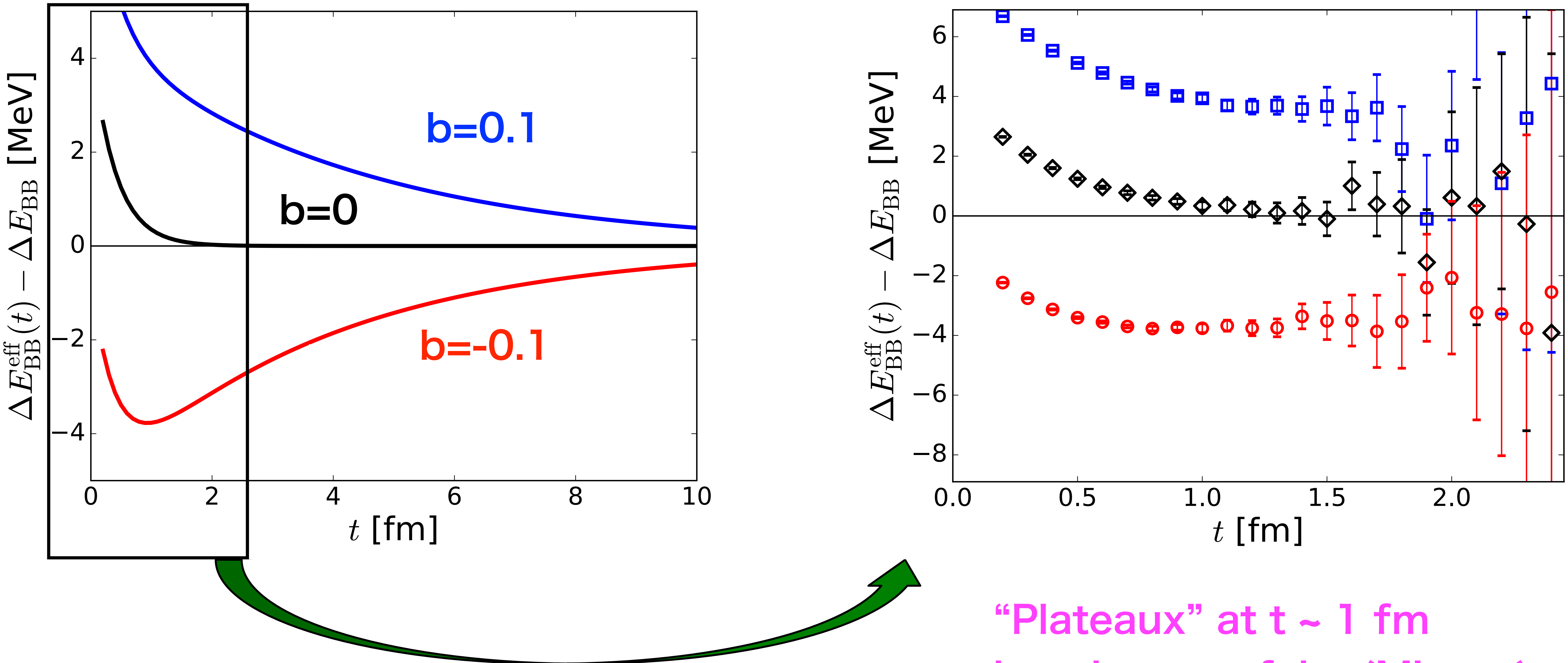




Zoom + increasing errors and fluctuations

"Plateaux" at t ~ 1 fm
but they are fake (Mirage)



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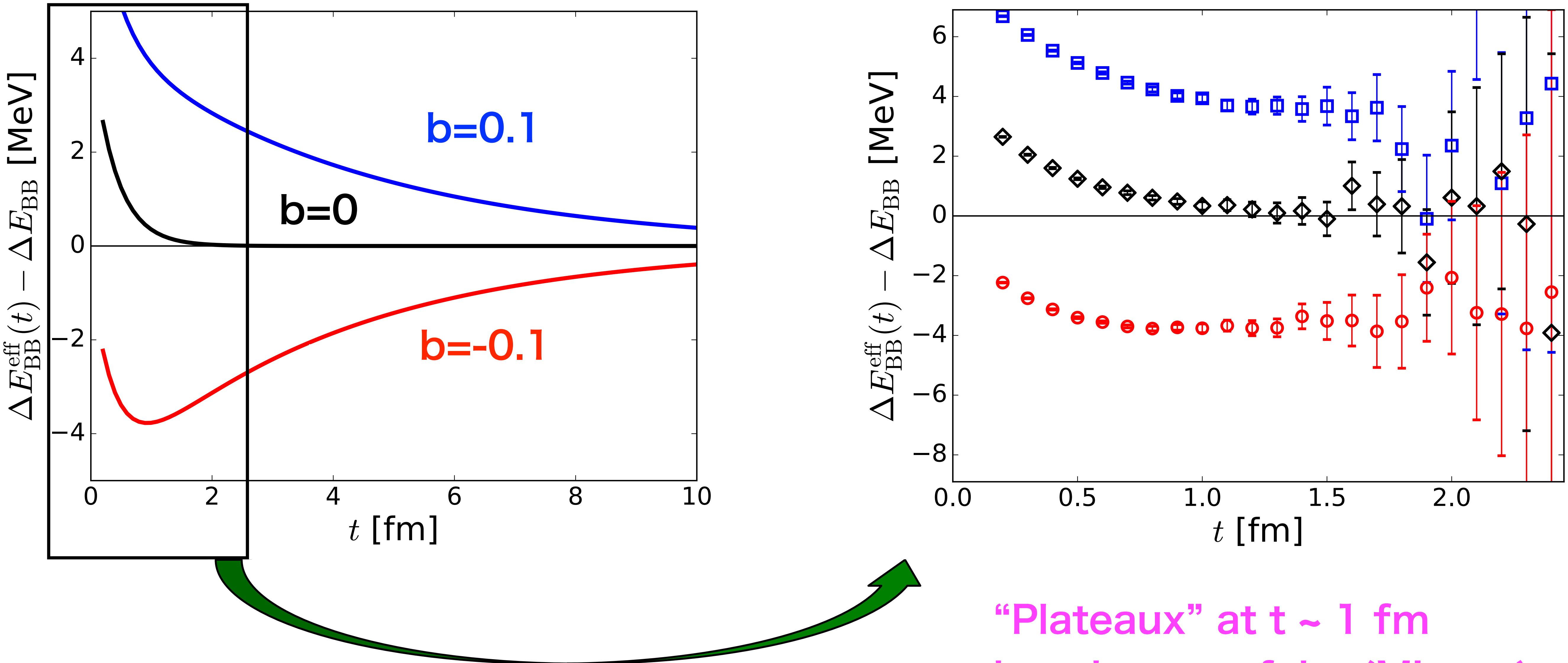
Observing the plateau guarantees the ground state saturation even when $t \gg 1/(E_1 - E_0)$ is NOT satisfied. claimed by Y(I)KU(‘11,’12,’15), NPL(’12,’13,’15), CalLat(’15)

It’s a Myth !



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It’s a Myth !

The “looking for a plateau at small t” method does not work.

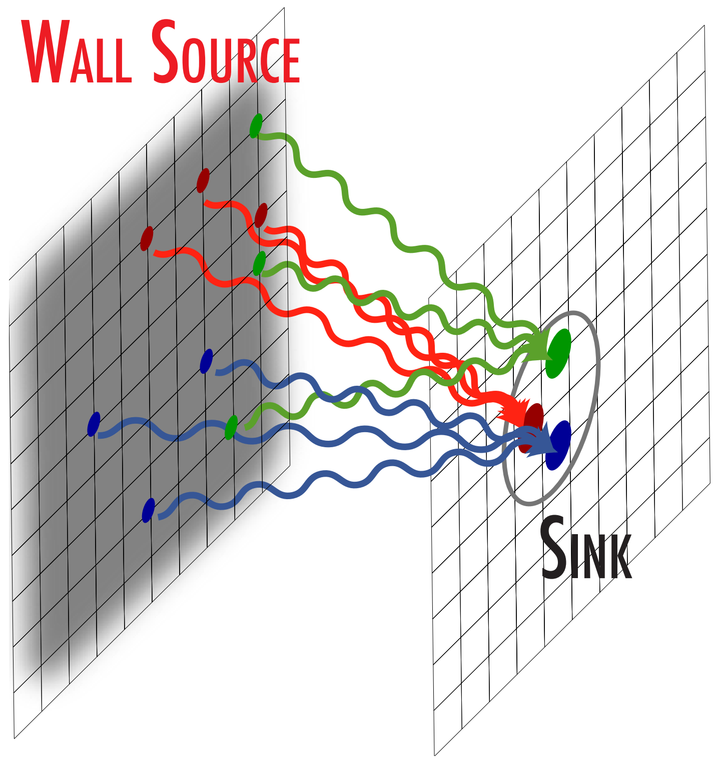
I. Mirage problem
(Operator dependence)

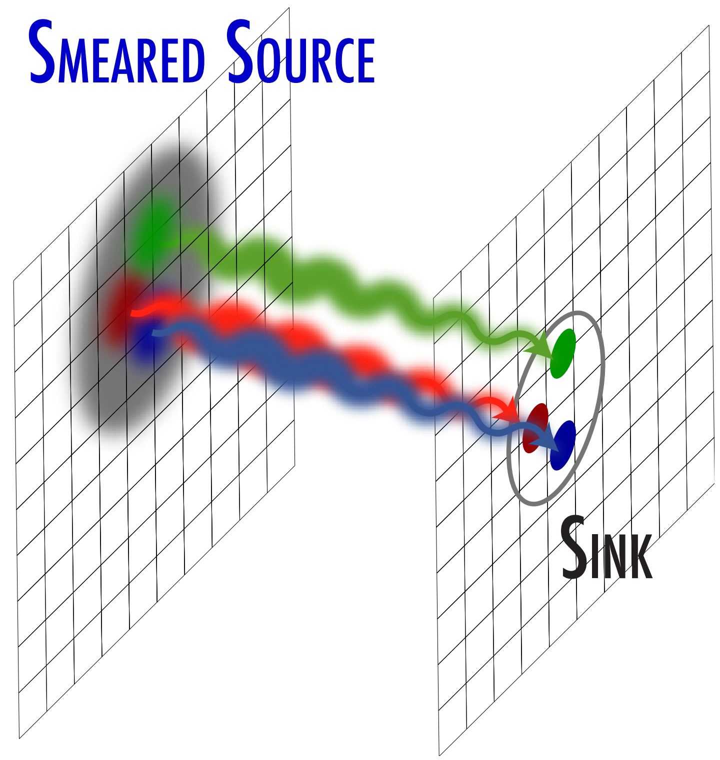
- Manifestation of the problem I -

T. Iritani et al. (HAL QCD), JHEP1610(2016)101 (arXiv:1607.06371)

Source operator dependence of plateaux

quark wall source **vs** quark smeared source





$$\sum_{\mathbf{y}} q(\mathbf{y}, t_0)$$

$$\sum_{\mathbf{y}} e^{-B|\mathbf{x}_0-\mathbf{y}|} q(\mathbf{y}, t_0)$$

b are different between the two.

Lattice setup 2+1 flavor QCD

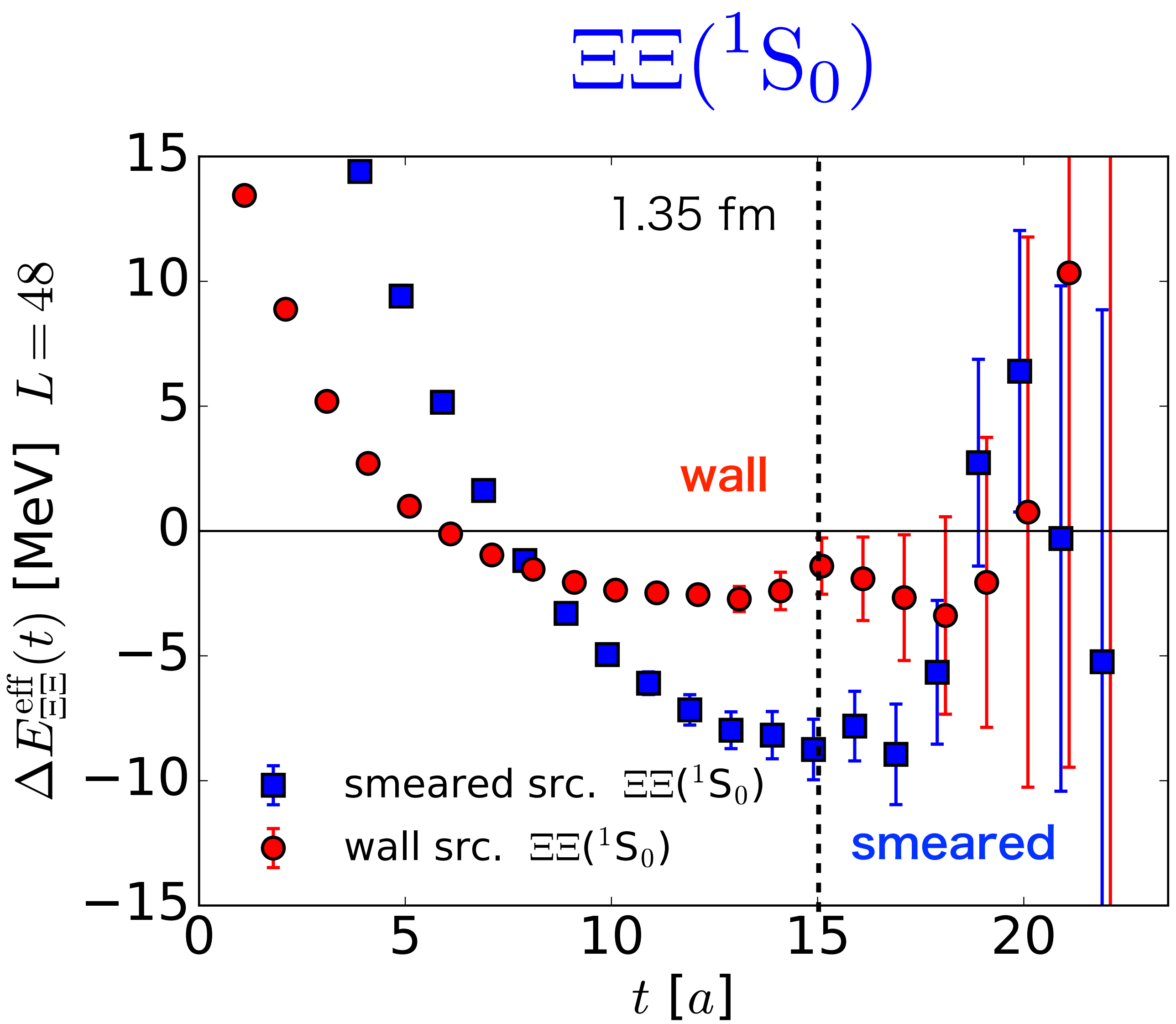
same gauge configurations of YIKU 2012

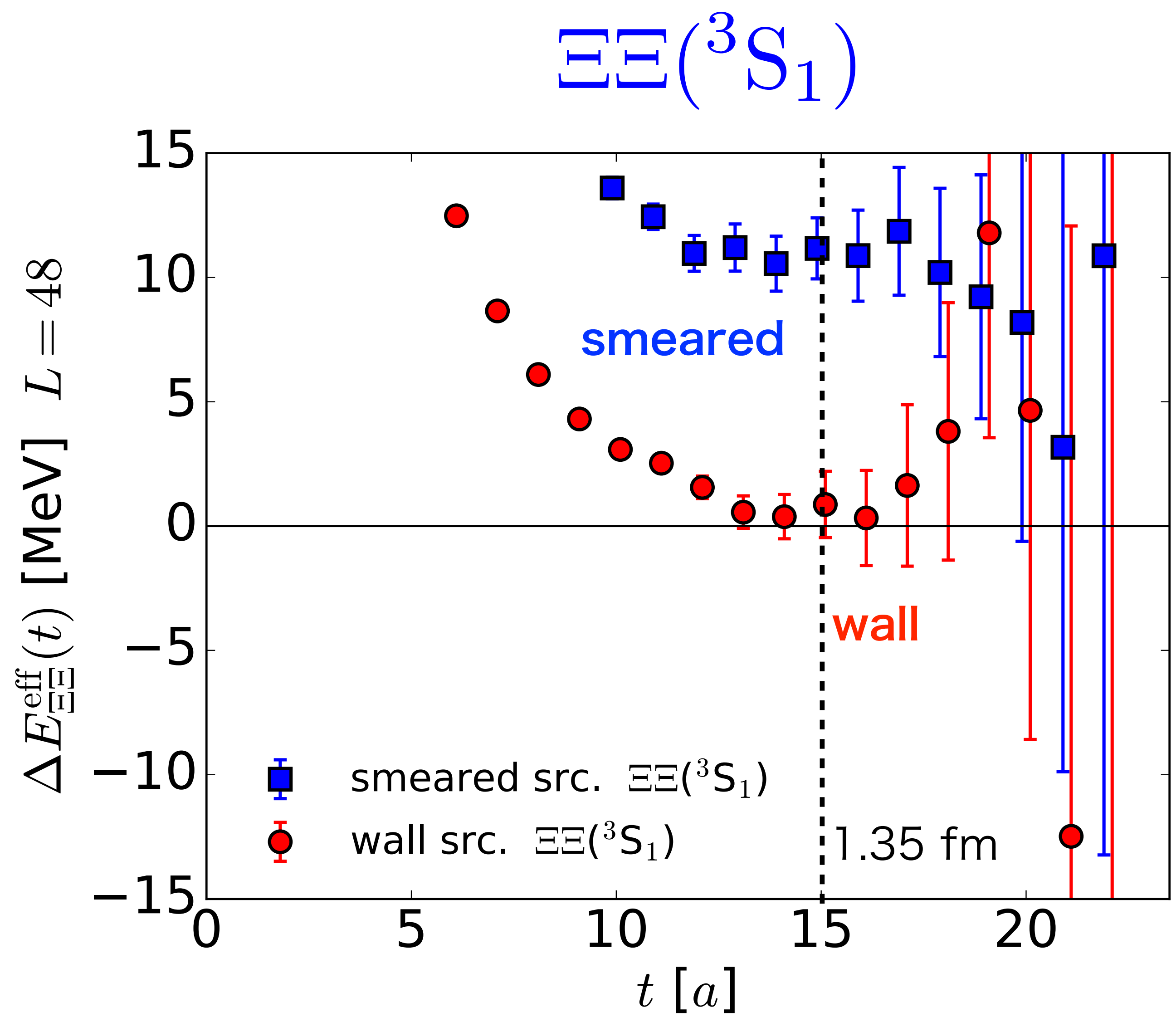
$$a = 0.09 \text{ fm } (a^{-1} = 2.2 \text{ GeV})$$

$$m_\pi = 0.51 \text{ GeV}, m_N = 1.32 \text{ GeV}, m_K = 0.62 \text{ GeV}, m_\Xi = 1.46 \text{ GeV}$$

Energy shift of ΞΞ

smaller statistical errors

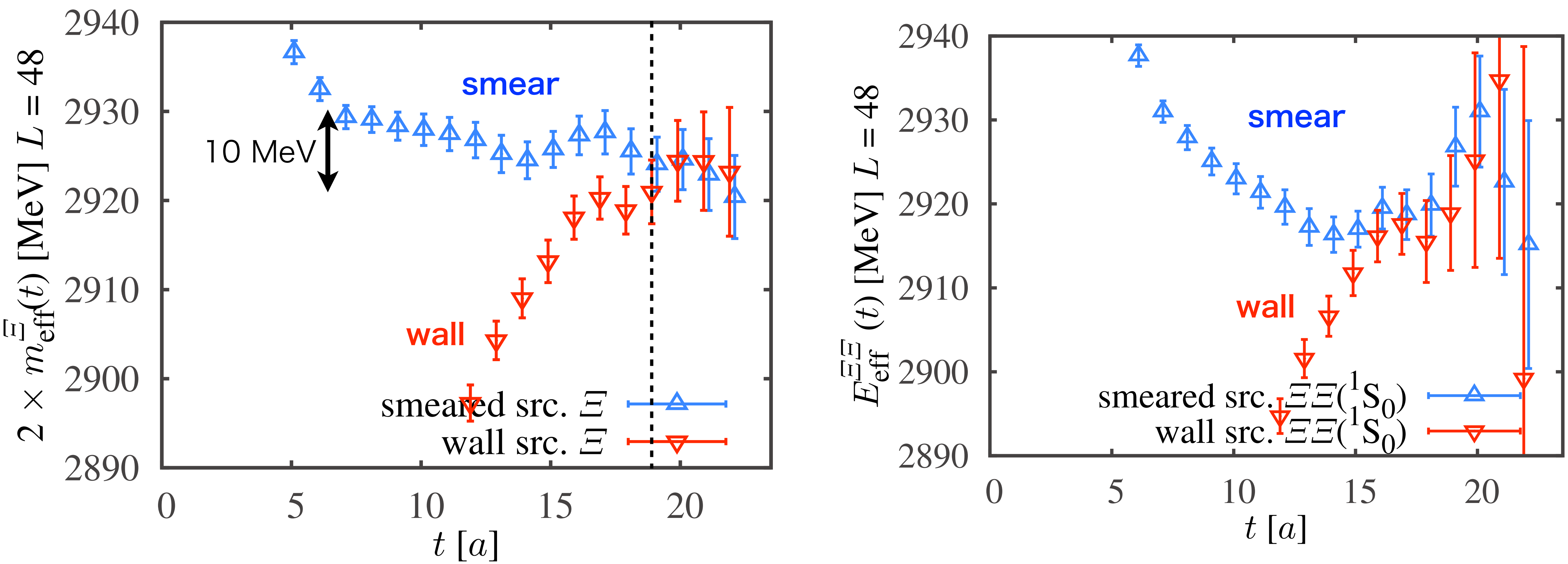




- Not surprisingly, two sources disagree.
- The potential danger becomes reality.
- Plateau-like structures around t=1-1.5 fm are by no means trustable.
- Both might agree at t > 18a, but errors are too large.

Numerator and denominator

$2m_\Xi$ $E_{\Xi\Xi}(^3\mathrm{S}_1)$

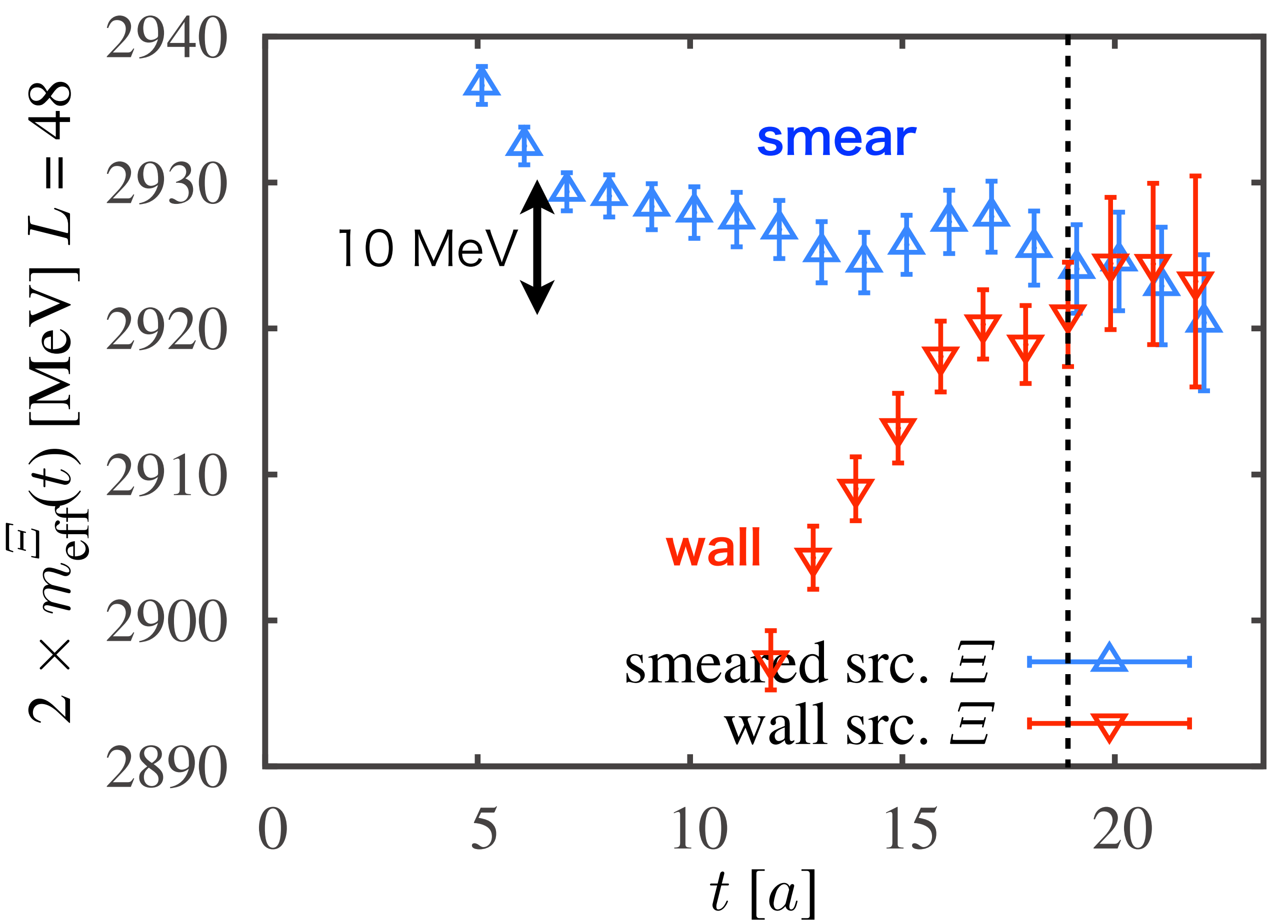


Smeared source looks better for the single baryon,
but it still keeps changing in the fine scale.

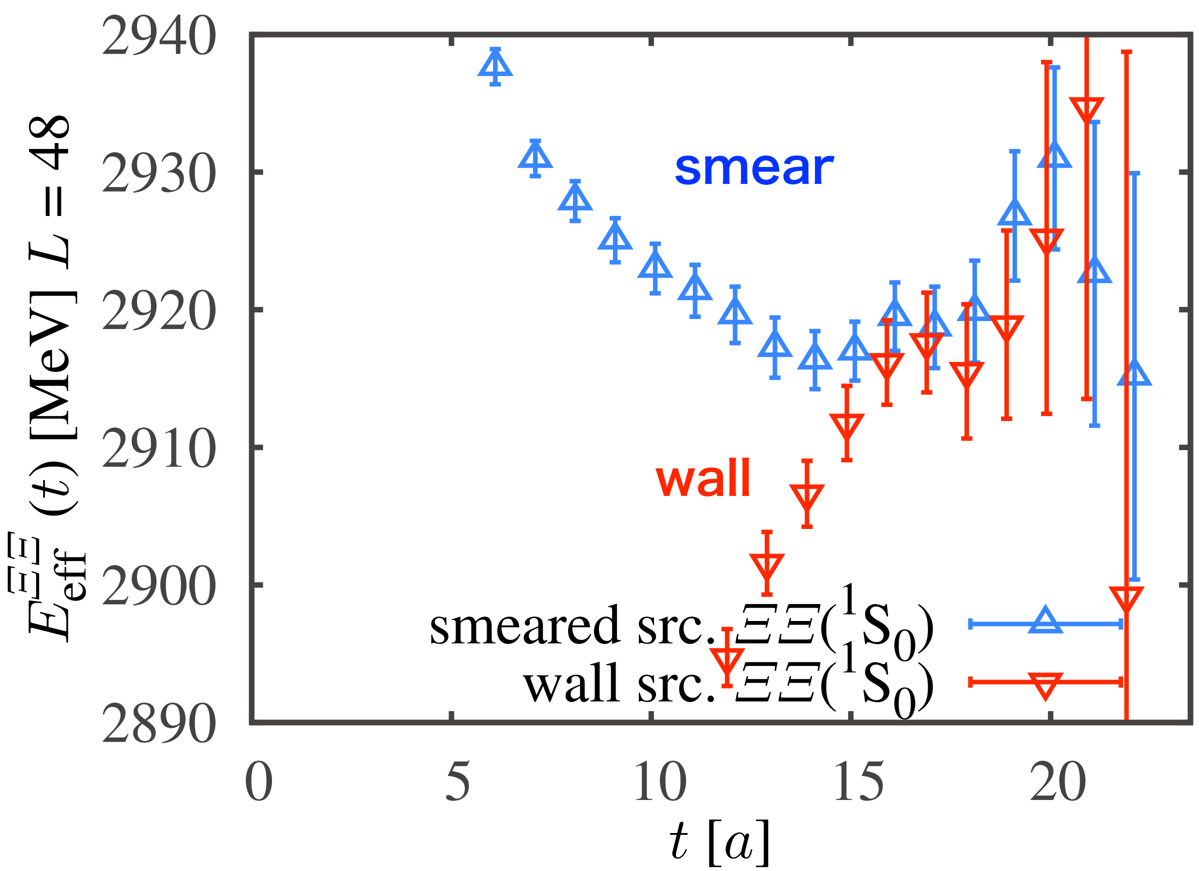
Method relies on cancellation of systematics

Numerator and denominator

$2m_{\Xi}$



$E_{\Xi\Xi}(^3\mathrm{S}_1)$



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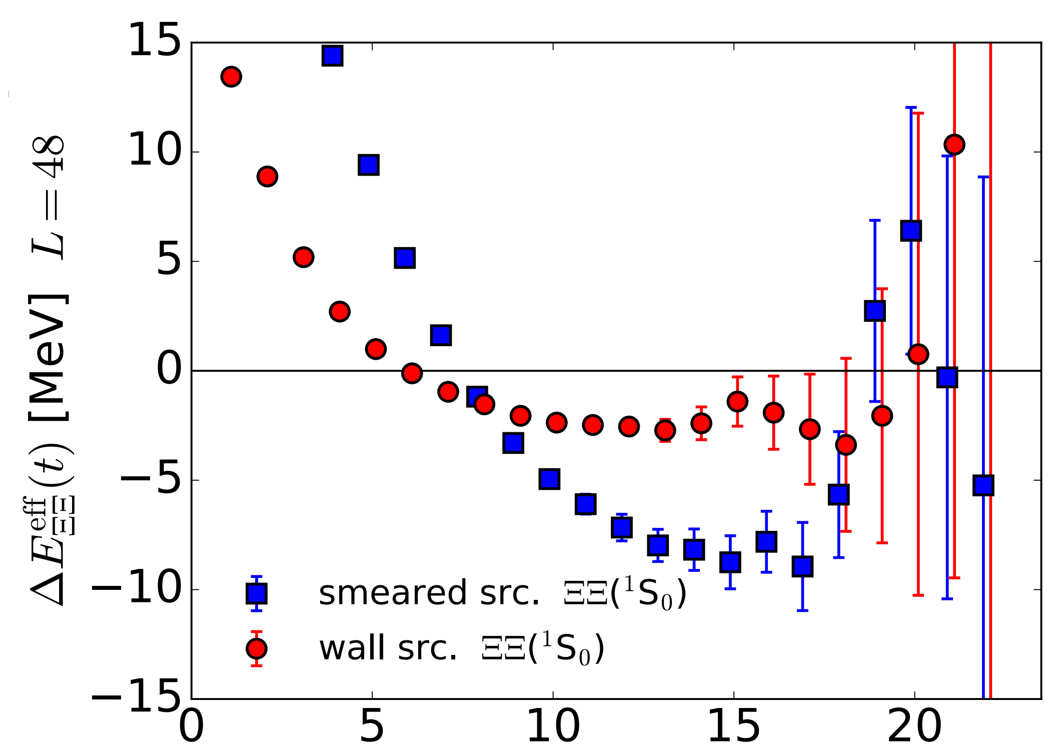
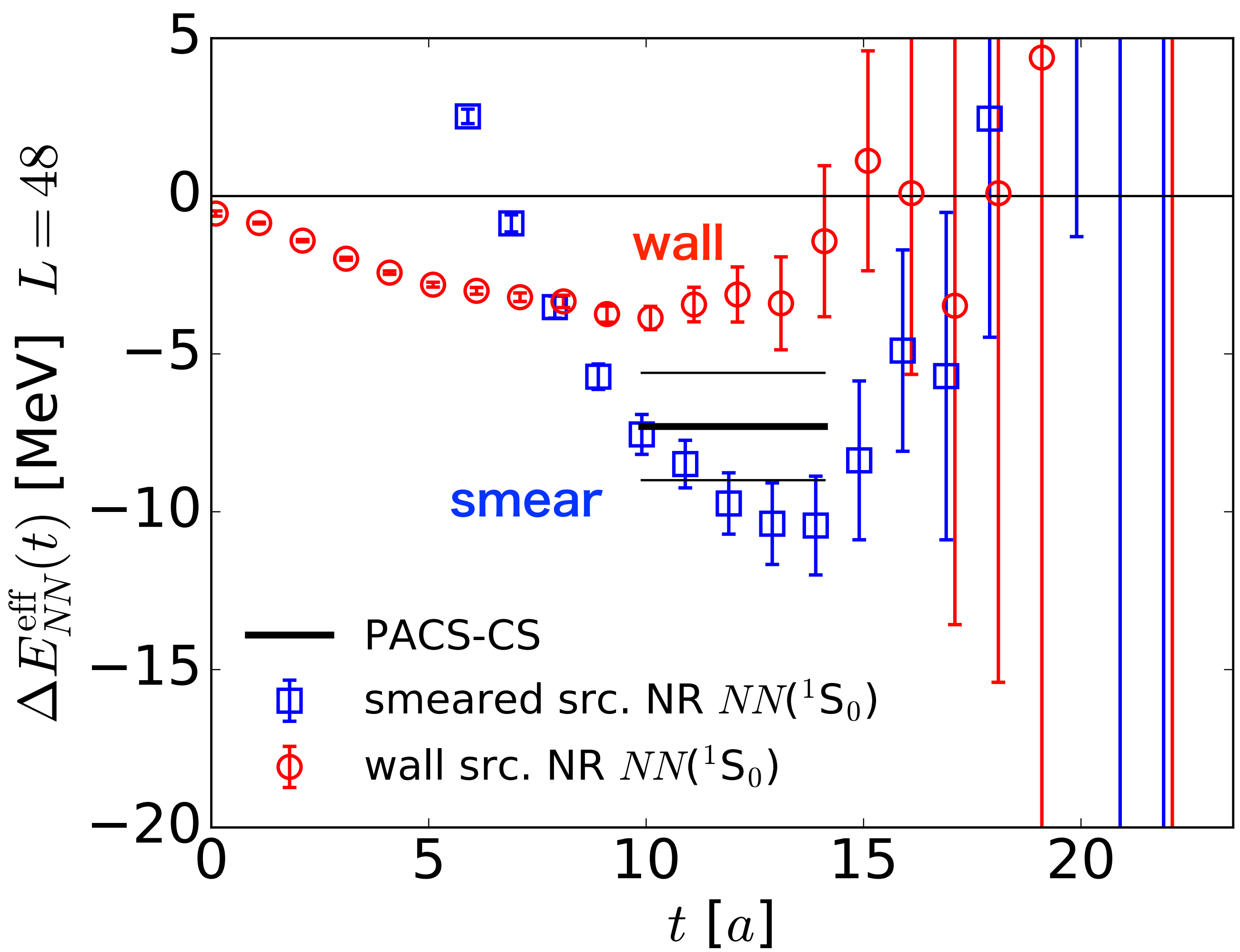
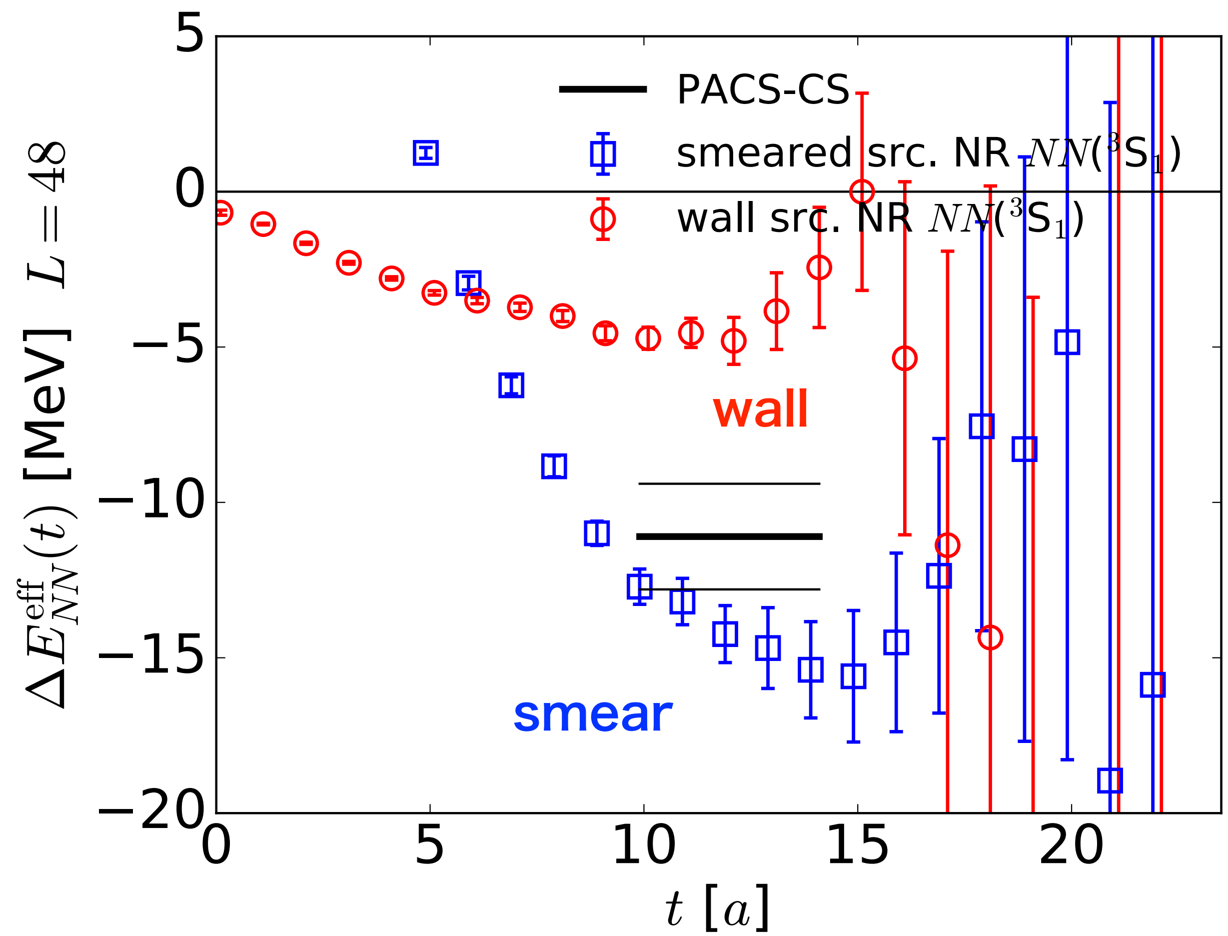


Same problem also appears for NN

$NN(^1\mathrm{S}_0)$



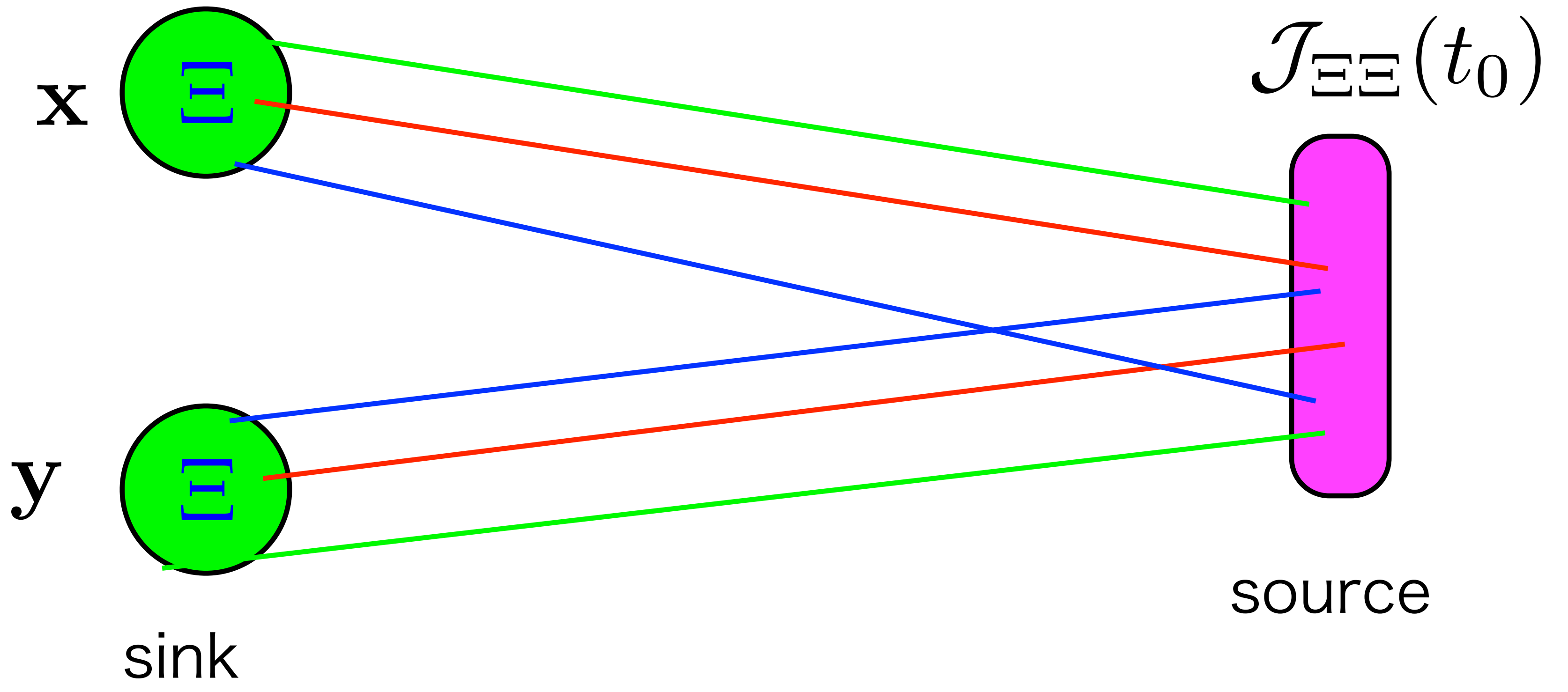
$NN(^3\mathrm{S}_1)$



With larger errors, disagreement also exists.

In addition, we may have

Sink 2-baryon operator dependence of plateaux

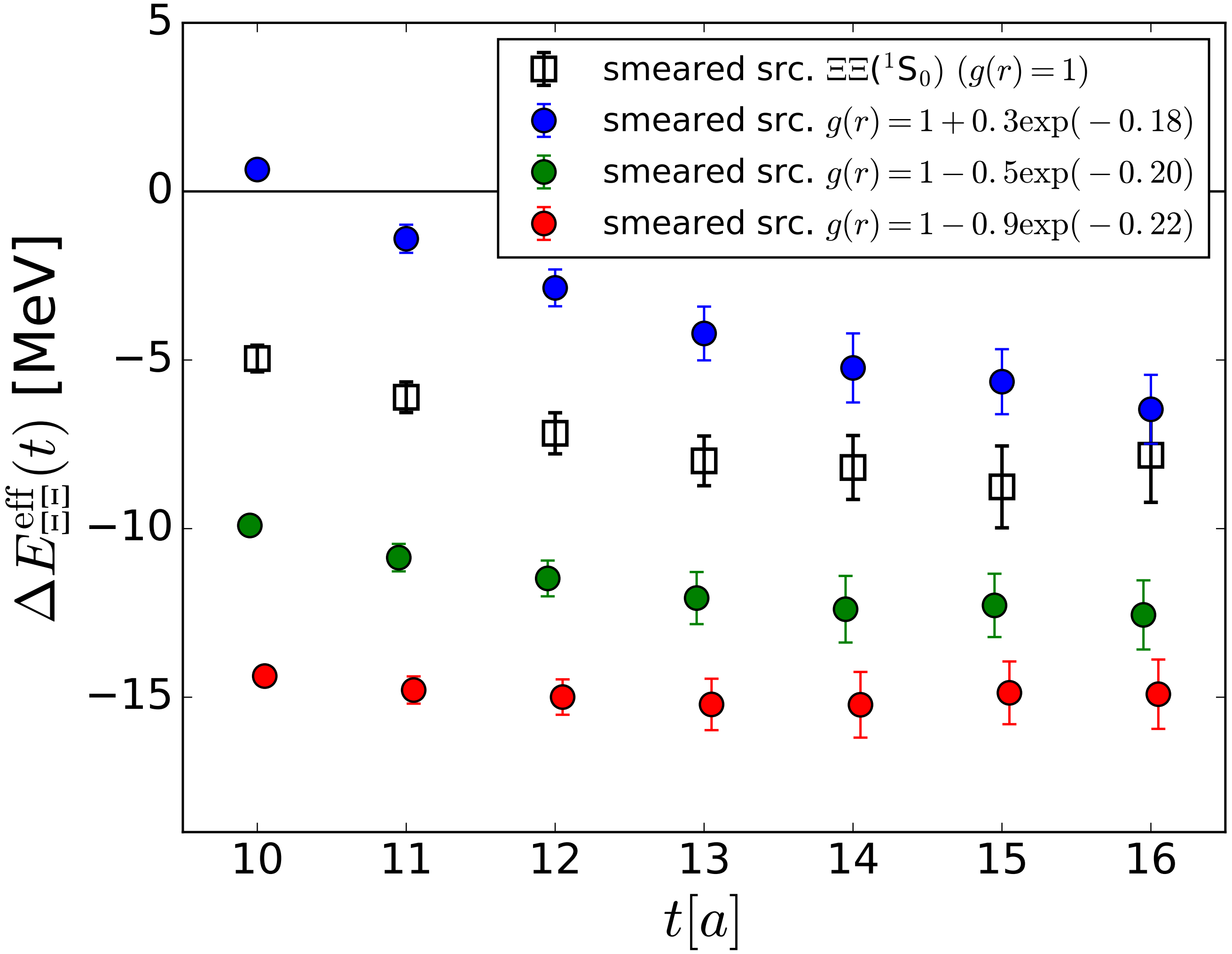


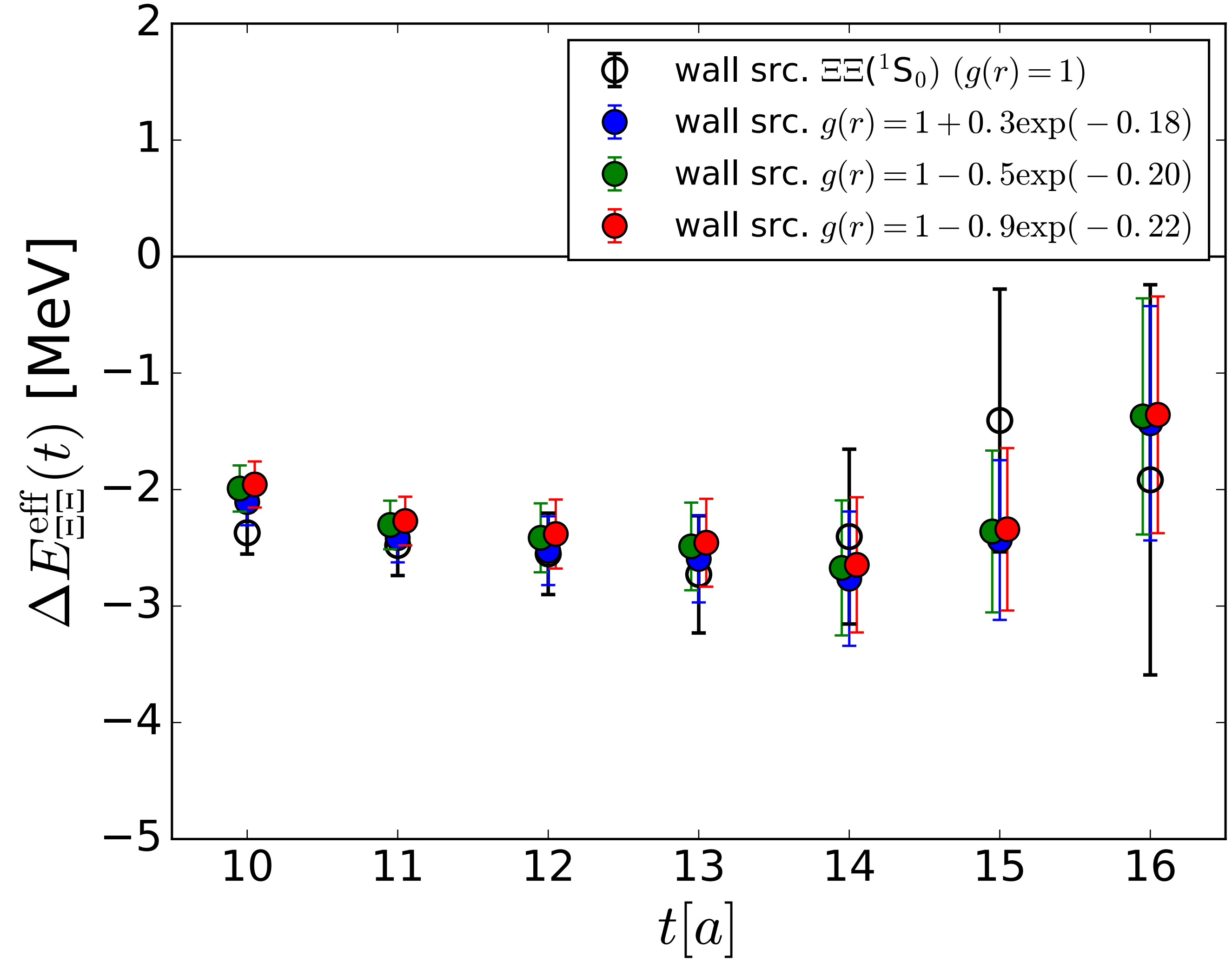
$$G_{\Xi\Xi}(t) = \sum_{\mathbf{x},\mathbf{y}} g(|\mathbf{x}-\mathbf{y}|)\langle \Xi(\mathbf{x},t)\Xi(\mathbf{y},t)\mathcal{J}_{\Xi\Xi}(t_0)\rangle$$

$$g(r) = 1 : \text{ standrad sink operator}$$

$$g(r) = 1 + A\exp(-Br) : \text{ generalized sink operator}$$

The true plateau must NOT dependent on g(r).





- smeared source is very sensitive to g(r).
  - Sometimes deeper and more stable.
  - one can produce an arbitrary value (within a certain range) by g(r).
- Wall source is insensitive to g(r).

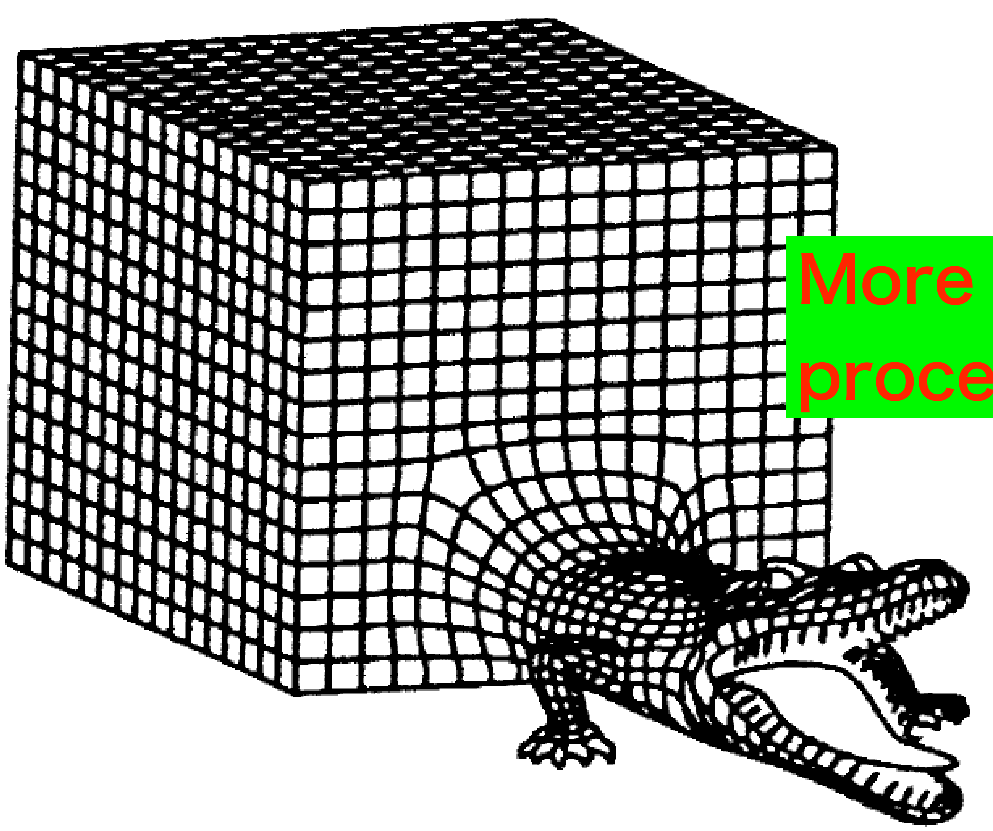
- Dangers of fake plateaux exit in principle for the direct method.

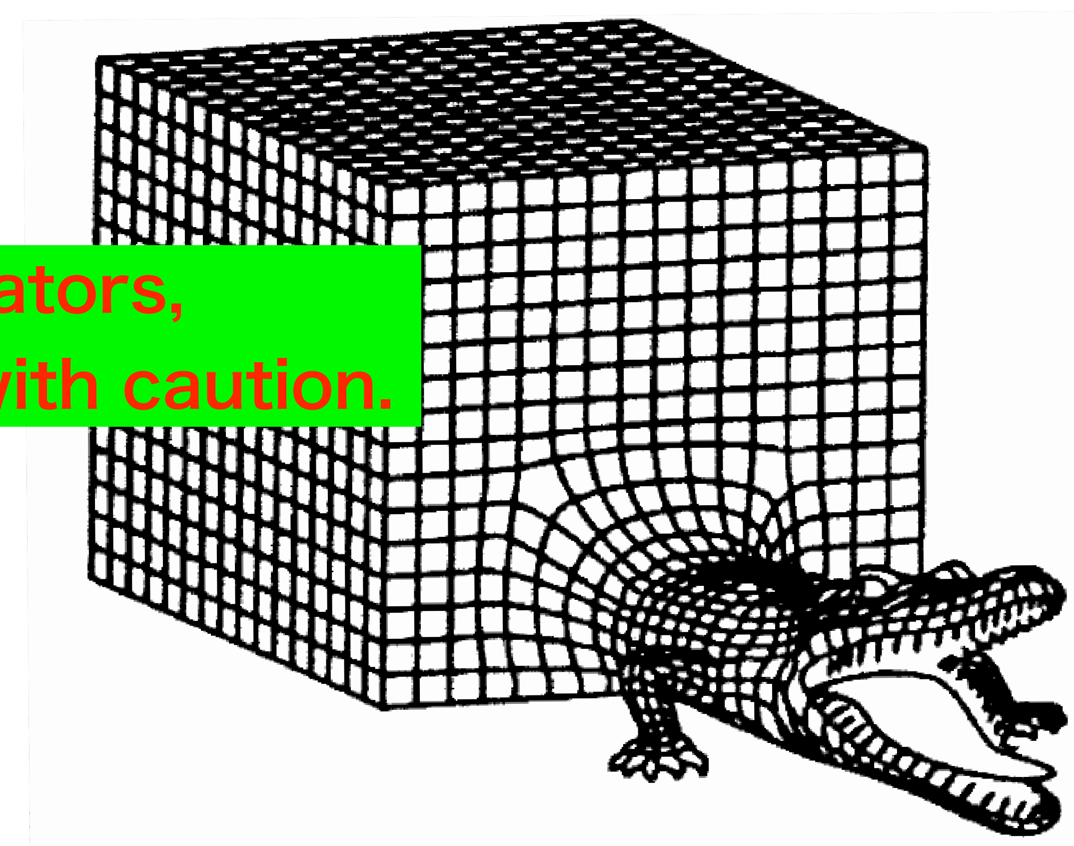
- Problem becomes manifest in the strong source/sink operator dependences of plateau values in YIKU 2012.

- Are there any symptoms in other results ?

    - Study of source dependences requires additional simulations.

    - need simpler and easier check





More alligators,
proceed with caution.

II. Sanity check

- Manifestation of the problem II -

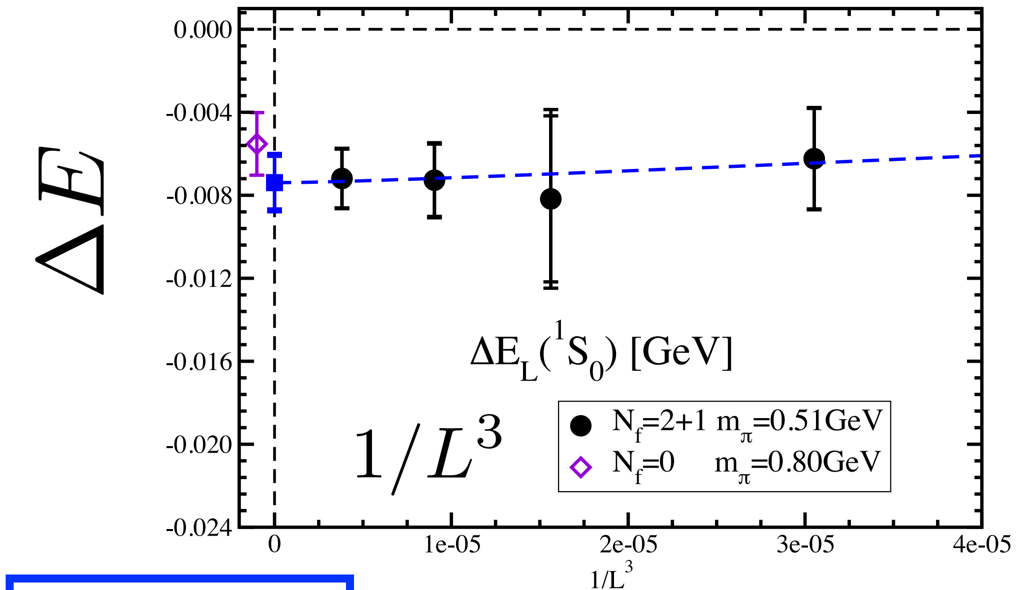
S. Aoki, T. Doi, T. Iritani, PoS(Lattice2016) 109 (aiXiv:1610:09763)

Finite volume formula

Lüscher, NPB354(1991)531

Direct method

YIKU2012

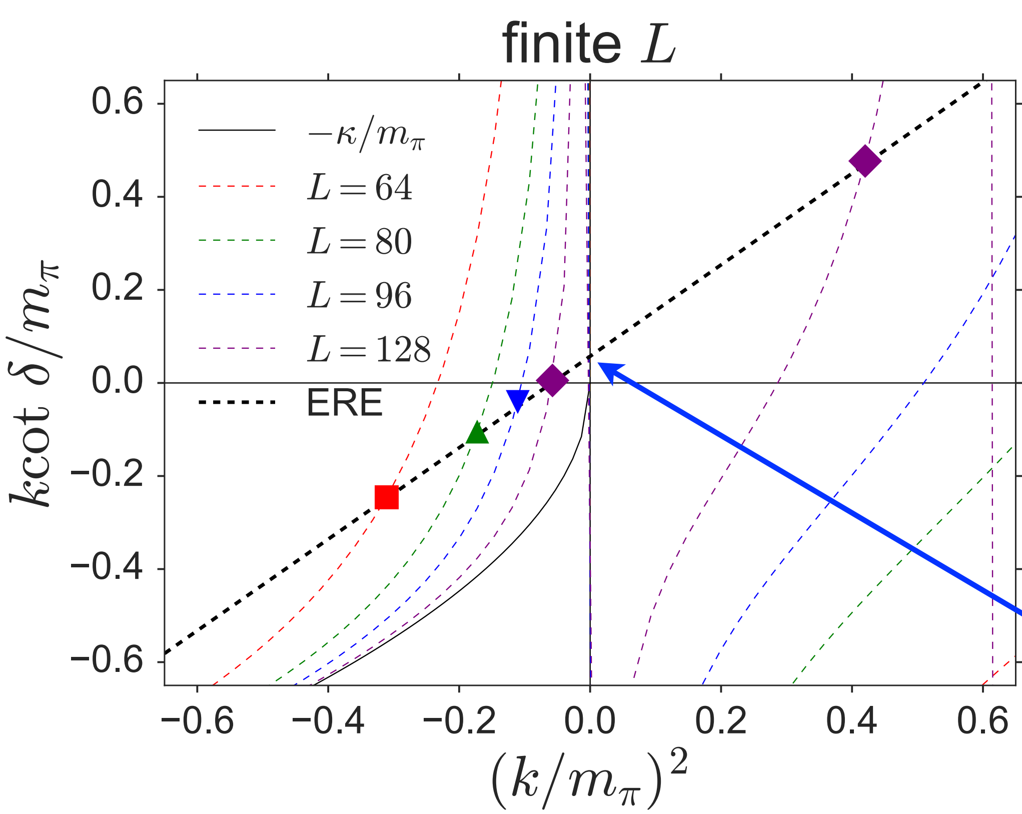


$$\Delta E = 2\sqrt{k^2 + m_N^2} - 2m_N, \quad q = \frac{kL}{2\pi}$$

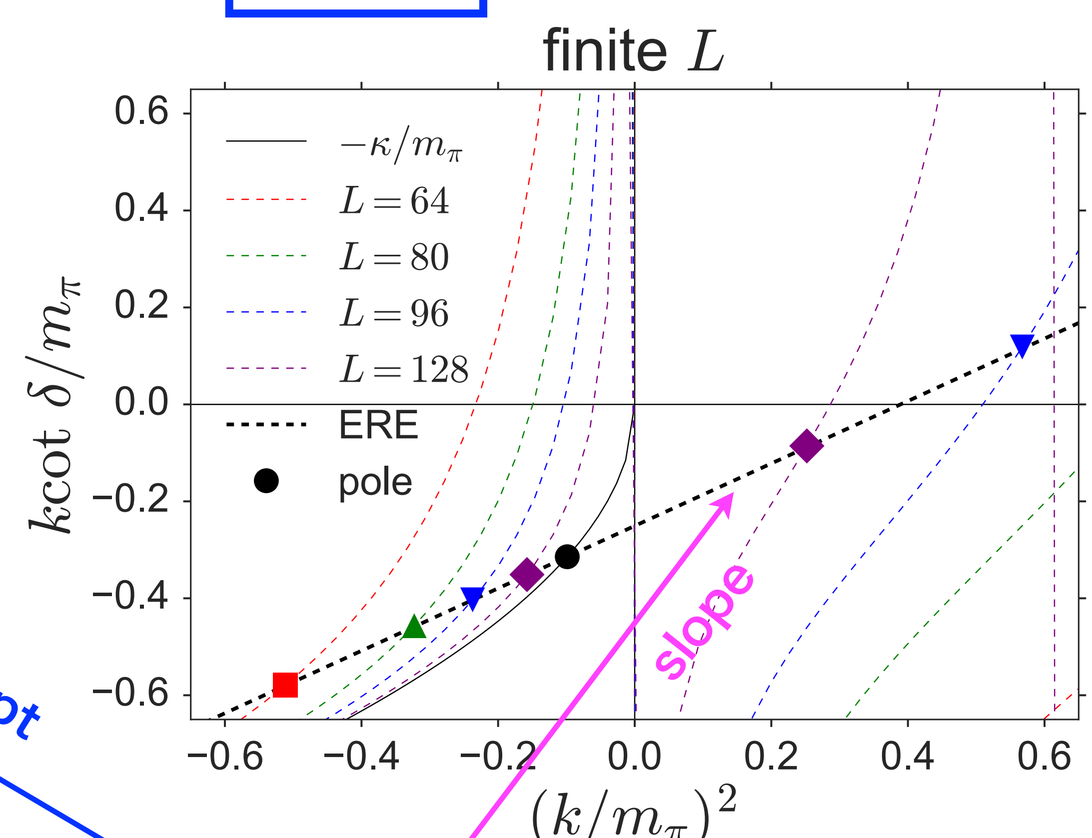
$$k \cot \delta(k) = \frac{1}{\pi L} \sum_{\vec{n} \in \mathbb{Z}^3} \frac{1}{\vec{n}^2 - q^2}$$

$\delta(k)$:scattering phase shift

unbound



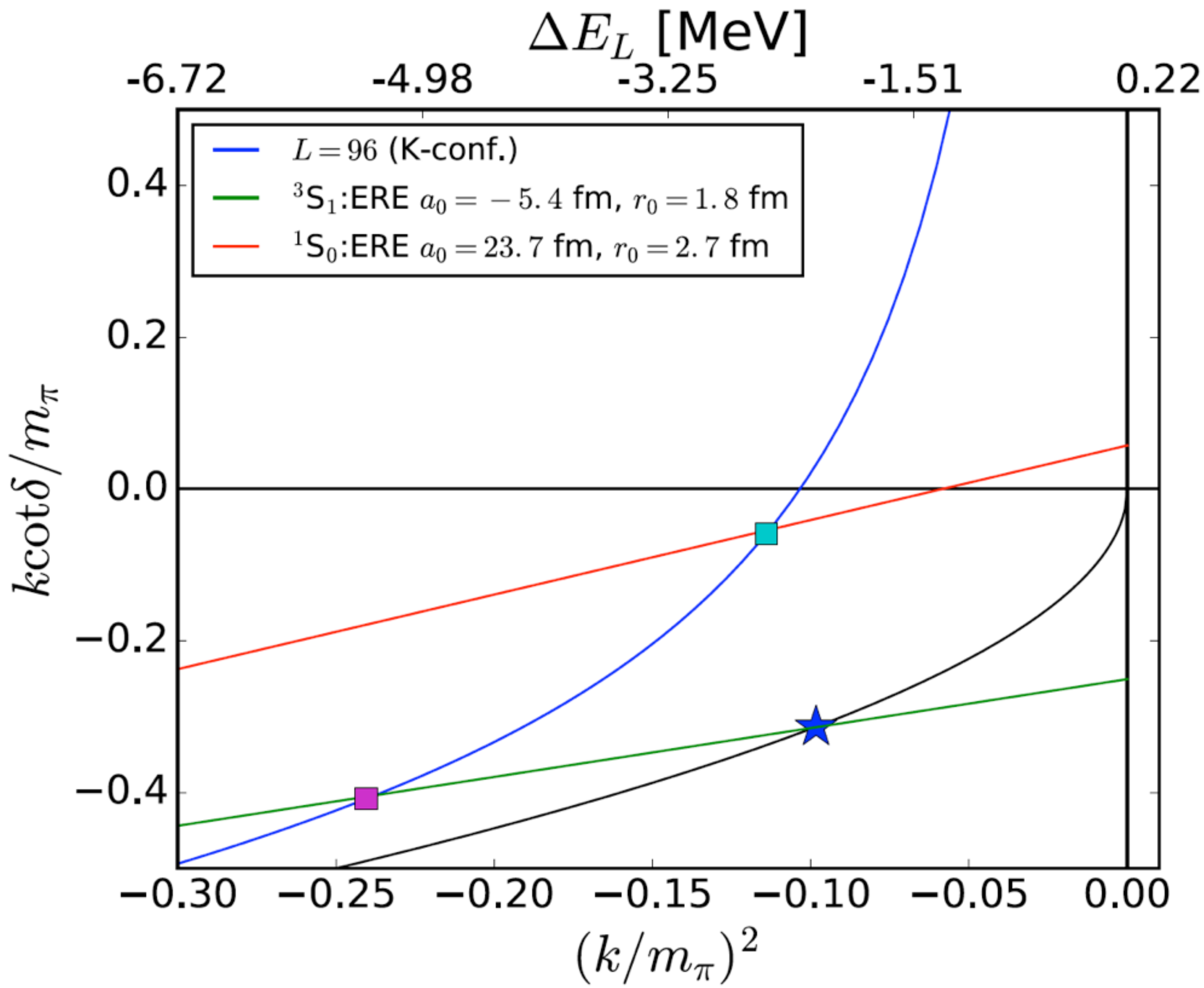
bound



Effective Range Expansion (ERE)

$$k \cot \delta(k) = \frac{1}{a} + \frac{1}{2} r k^2 + \cdots$$

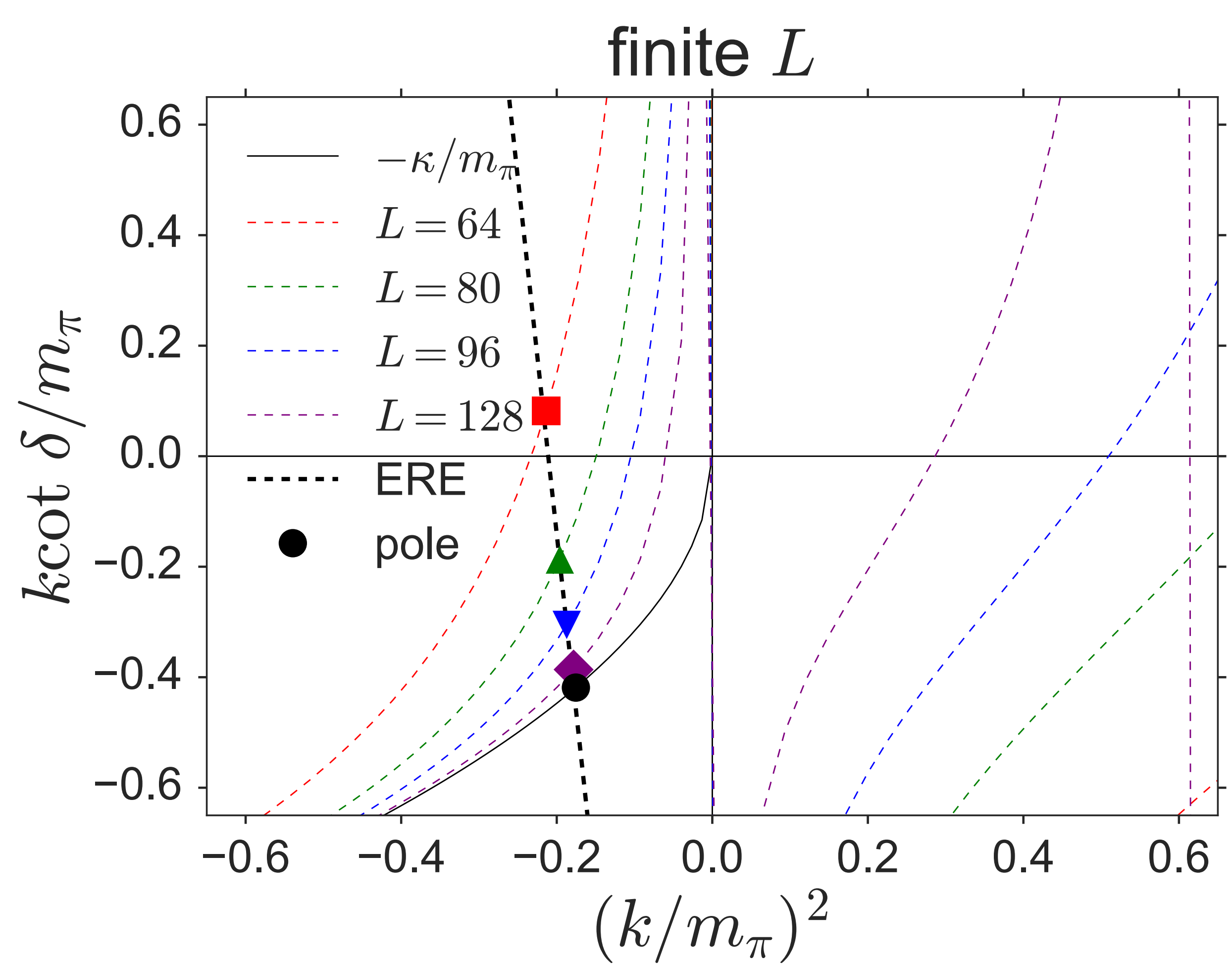
ERE at physical pion mass



Instead, a behavior shown below indicates the problem in lattice QCD data.

$$1/a \simeq -\infty, \quad r \simeq -\infty$$

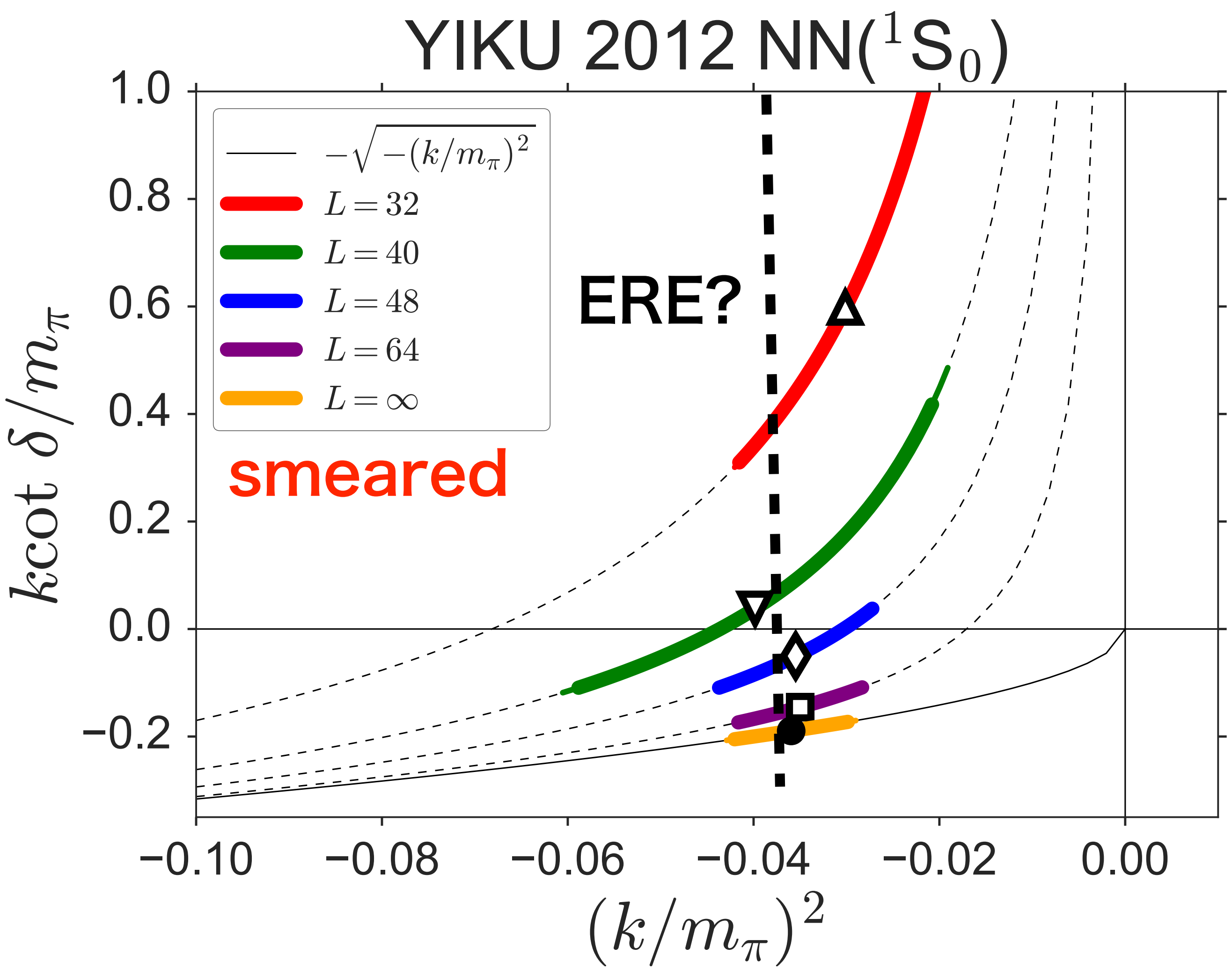
"Sanity Check"



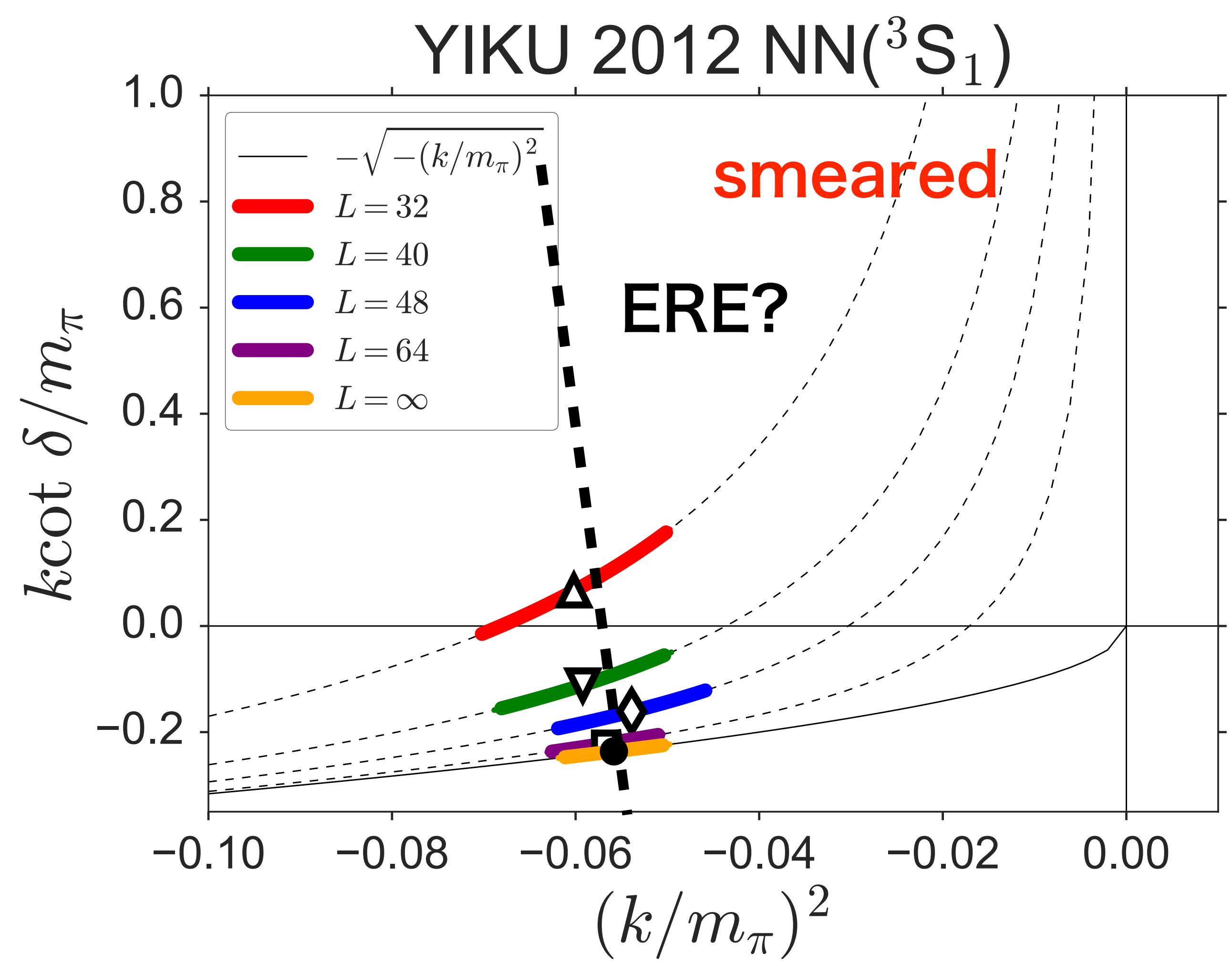
YIKU2012 Yamazaki et al. PRD86(2012)074514

$m_\pi = 0.51$ GeV, $L = 2.9 - 5.8$ fm

$\Delta E_{NN}(^1\mathrm{S}_0) = -7.4(1.3)(0.6)$ MeV



$\Delta E_{NN}(^3\mathrm{S}_1) = -11.5(1.1)(0.6)$ MeV



singular behaviors

ΔE is almost independent on L, while it is shallow bound state.

"Not Sanity"

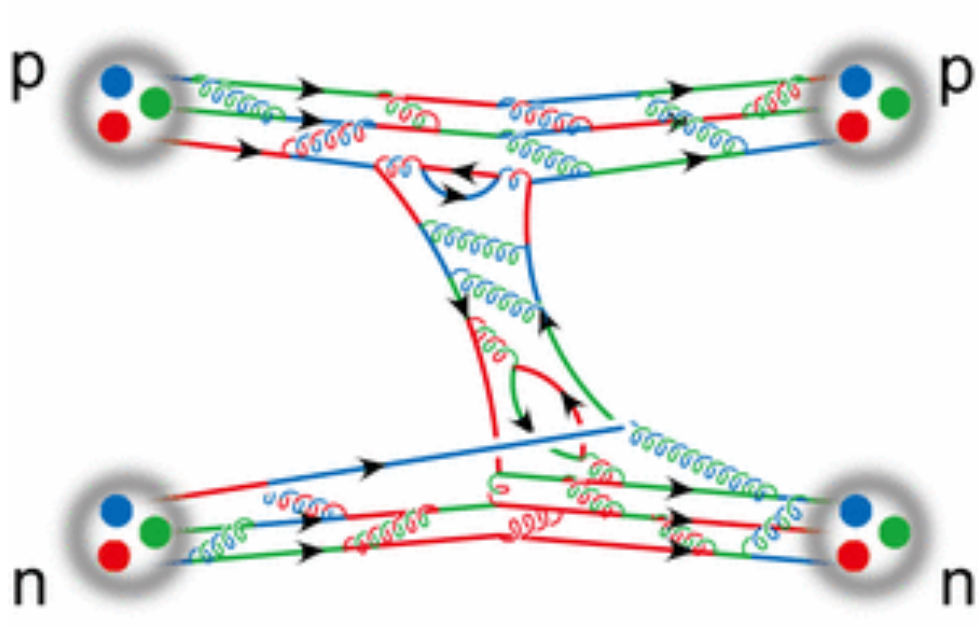
Conclusion of part 1

The direct method gives no reliable result for two(or more)-baryon systems so far, since systematic errors due to contaminations from excited (elastic) states are not under control.

Check Table for NN

	single baryon	double baryon				
	plateau check	mirage plateau	src-dep check	sink-dep check	Effective Range expansion check	**Overall Verdict**
YKU 2011	○	×	△	**Not checked**	×	**False**
YIKU 2012	○	×	×	×	×	**False**
YIKU 2015	○	×	**Not checked**	**Not checked**	×	**False**
NPL 2012	○	×	**Not checked**	**Not checked**	×	**False**
NPL 2013	○	×	**Not checked**	**Not checked**	△	**False**
NPL 2015	△	×	**Not checked**	**Not checked**	×	**False**

Part 2. HALQCD potential method



III. Strategy

Aoki, Hatsuda & Ishii, PTP123(2010)89.

Elastic scattering

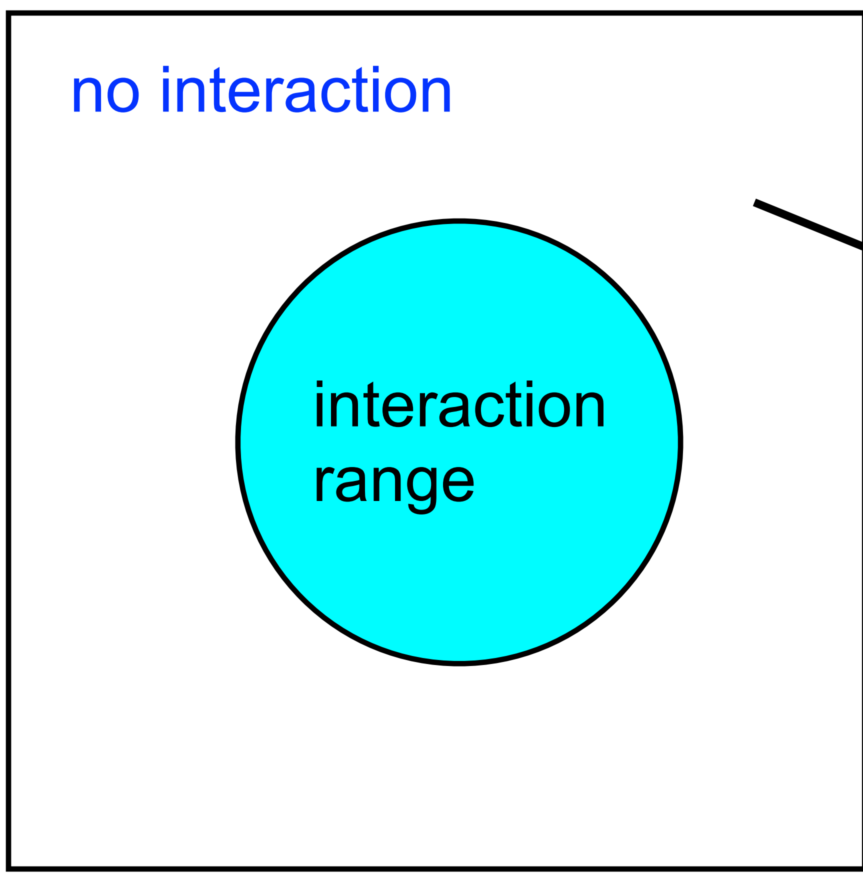
$NN \to NN$ ~~$NN \to NN +$ others~~

Nambu-Bethe-Salpeter (NBS) wave function

$$\varphi_{\mathbf{k}}(\mathbf{r}) = \langle 0 | N(\mathbf{x}+\mathbf{r},0) N(\mathbf{x},0) | NN, W_k \rangle$$

QCD eigenstate

energy $W_k = 2\sqrt{\mathbf{k}^2 + m_N^2}$



$$r = |\mathbf{r}| \to \infty$$

$$\varphi_{\mathbf{k}}(\mathbf{r}) \simeq \sum_{l,m} C_l \frac{\sin(kr - l\pi/2 + \delta_l(k))}{kr} Y_{ml}(\Omega_{\mathbf{r}})$$

$\delta_l(k)$ scattering phase shift =

phase of the S-matrix by unitarity in QCD.

Potential

Non-local but energy-independent, defined from the NBS wave function

$$\left[\epsilon_k - H_0\right] \varphi_{\mathbf{k}}(\mathbf{x}) = \int d^3y\, \underline{U(\mathbf{x}, \mathbf{y})} \varphi_{\mathbf{k}}(\mathbf{y})$$

$$\epsilon_k = \frac{\mathbf{k}^2}{2\mu} \qquad H_0 = \frac{-\nabla^2}{2\mu}$$

$$U(\mathbf{x}, \mathbf{y}) \Longleftrightarrow V_{\mathbf{k}}(\mathbf{x}) = \frac{[\epsilon_k - H_0]\varphi_{\mathbf{k}}(\mathbf{x})}{\varphi_{\mathbf{k}}(\mathbf{x})}$$

By construction

potential $U(\mathbf{x}, \mathbf{y})$ is faithful to QCD phase shift $\delta_l(k)$.

Note however that $U(\mathbf{x}, \mathbf{y})$ is not unique.

Derivative (velocity) expansion

$$U(\mathbf{x},\mathbf{y}) = V(\mathbf{x},\nabla)\delta^3(\mathbf{x}-\mathbf{y})$$

$$V(\mathbf{x},\nabla) = V_0(r) + V_\sigma(r)(\sigma_{\mathbf{1}}\cdot\sigma_{\mathbf{2}}) + V_T(r)S_{12} + V_{\mathrm{LS}}(r)\mathbf{L}\cdot\mathbf{S} + O(\nabla^2)$$

LO LO LO NLO NNLO

tensor operator $S_{12} = \frac{3}{r^2}(\sigma_{\mathbf{1}}\cdot\mathbf{x})(\sigma_{\mathbf{2}}\cdot\mathbf{x}) - (\sigma_{\mathbf{1}}\cdot\sigma_{\mathbf{2}})$

spins

At LO we simply obtain

$$V_{\mathrm{LO}}(\mathbf{x}) = \frac{[\epsilon_k - H_0]\varphi_{\mathbf{k}}(\mathbf{x})}{\varphi_{\mathbf{k}}(\mathbf{x})}$$

→ phase shifts and binding energy below inelastic threshold

Several $\varphi_k(\mathbf{x})$ are available. → We can determine $V(\mathbf{x},\nabla)$ order by order.

Note truncation of the derivative expansion introduces some systematics.

Extraction of potential

4-pt Correlation function

can be calculated in lattice QCD

source for NN

$$F(\mathbf{r}, t-t_0) = \langle 0|T\{N(\mathbf{x}+\mathbf{r},t)N(\mathbf{x},t)\}\underline{\overline{\mathcal{J}}(t_0)}|0\rangle$$

complete set for NN

$$\begin{aligned} F(\mathbf{r}, t-t_0) &= \langle 0|T\{N(\mathbf{x}+\mathbf{r},t)N(\mathbf{x},t)\} \sum_{n,s_1,s_2} \underline{|2N, W_n, s_1, s_2\rangle\langle 2N, W_n, s_1, s_2|}\overline{\mathcal{J}}(t_0)|0\rangle + \cdots \\ &= \sum_{n,s_1,s_2} A_{n,s_1,s_2} \underline{\varphi^{W_n}(\mathbf{r})} e^{-W_n(t-t_0)}, \quad A_{n,s_1,s_2} = \langle 2N, W_n, s_1, s_2|\overline{\mathcal{J}}(0)|0\rangle. \end{aligned}$$

NBS wave function

$$\varphi^{W}_{s_1,s_2}(\mathbf{r}) = \langle 0|T\{N(\mathbf{x}+\mathbf{r},0)N(\mathbf{x},0)\}|2N, W, s_1, s_2\rangle$$

Normalized 4-pt function

$$R(\mathbf{r}, t) \equiv F(\mathbf{r}, t)/G_N^2(t) = \sum_n A_n \varphi^{W_n} e^{-\Delta W_n t}$$

a sum of many NBS wave functions

$$\int d\mathbf{y}\, U(\mathbf{x},\mathbf{y})\varphi^{W_0}(\mathbf{y}) = (E_{W_0} - H_0)\varphi^{W_0}(\mathbf{x})$$

$$\int d\mathbf{y}\, U(\mathbf{x},\mathbf{y})\varphi^{W_1}(\mathbf{y}) = (E_{W_1} - H_0)\varphi^{W_1}(\mathbf{x})$$

$$\cdots$$

controlled by the same U

Time-dependent method

Ishii et al. (HALQCD), PLB712(2012) 437

$$R(\mathbf{r},t) = \sum_n A_n \varphi^{W_n}(\mathbf{r}) e^{-\Delta W_n t}$$

$$\Delta W_n = W_n - 2m_N = \frac{\mathbf{k}_n^2}{m_N} - \frac{(\Delta W_n)^2}{4m_N}$$

$$\left[\frac{\mathbf{k}_n^2}{m_N} - H_0\right]\varphi^{W_n}(\mathbf{r}) = U \cdot \varphi^{W_n}(\mathbf{r})$$

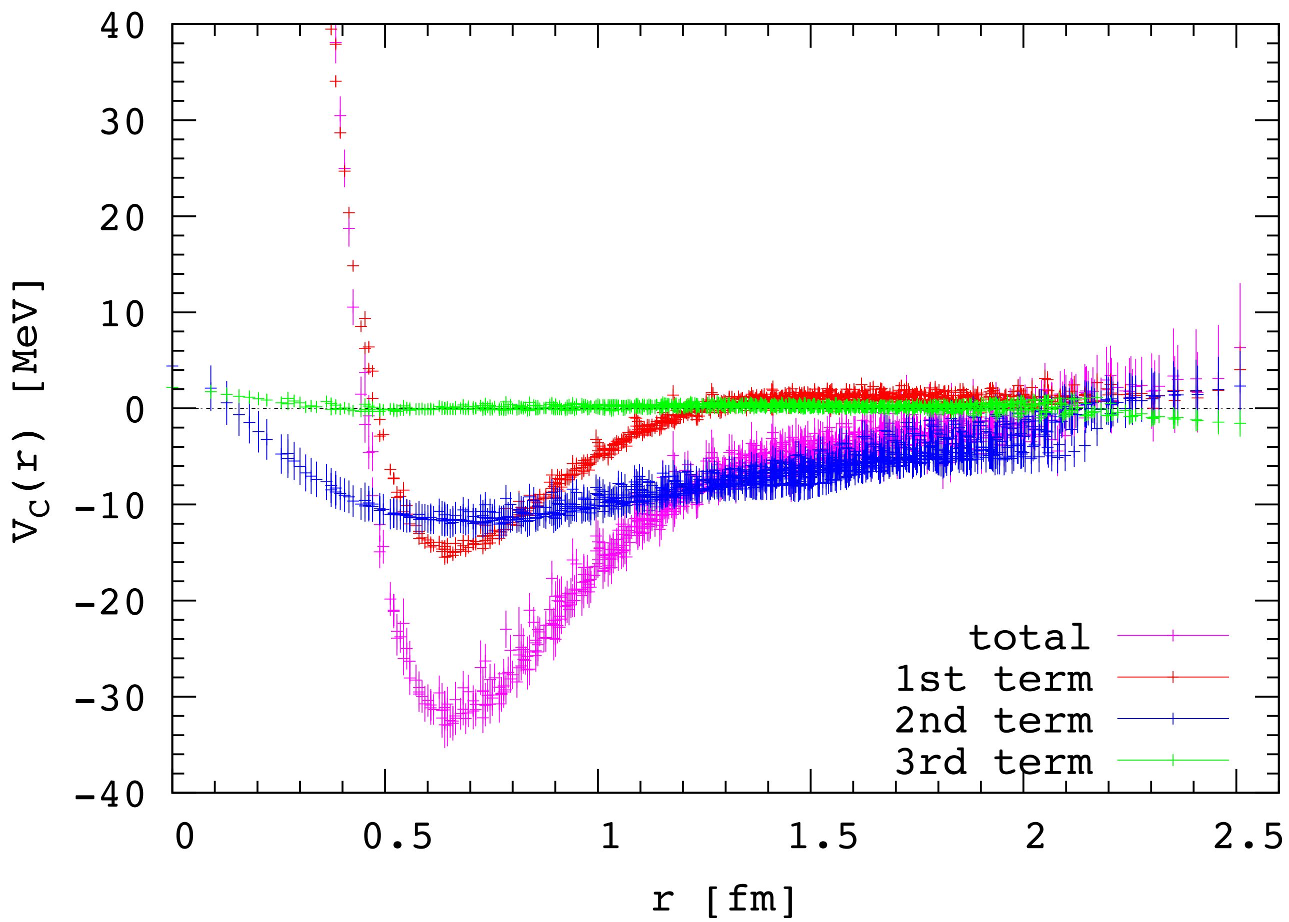
$$-\frac{\partial}{\partial t}R(\mathbf{r},t) = \left\{H_0 + U - \frac{1}{4m_N}\frac{\partial^2}{\partial t^2}\right\} R(\mathbf{r},t)$$

time corr. space corr. time corr.

Potential

$$\left\{-H_0 - \frac{\partial}{\partial t} + \frac{1}{4m_N}\frac{\partial^2}{\partial t^2}\right\} R(\mathbf{r},t) = \int d^3r'\, U(\mathbf{r},\mathbf{r}')R(\mathbf{r}',t) = V_C(\mathbf{r})R(\mathbf{r},t) + \cdots$$

1st 2nd 3rd



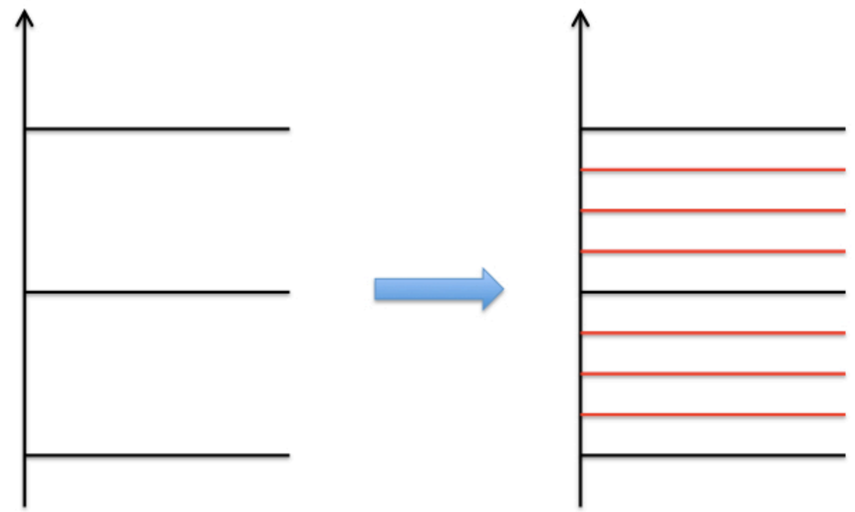
3rd term(relativistic correction) is negligible.

This method overcomes the previous difficulties in the direct method, using both space and time correlations.

Remarks

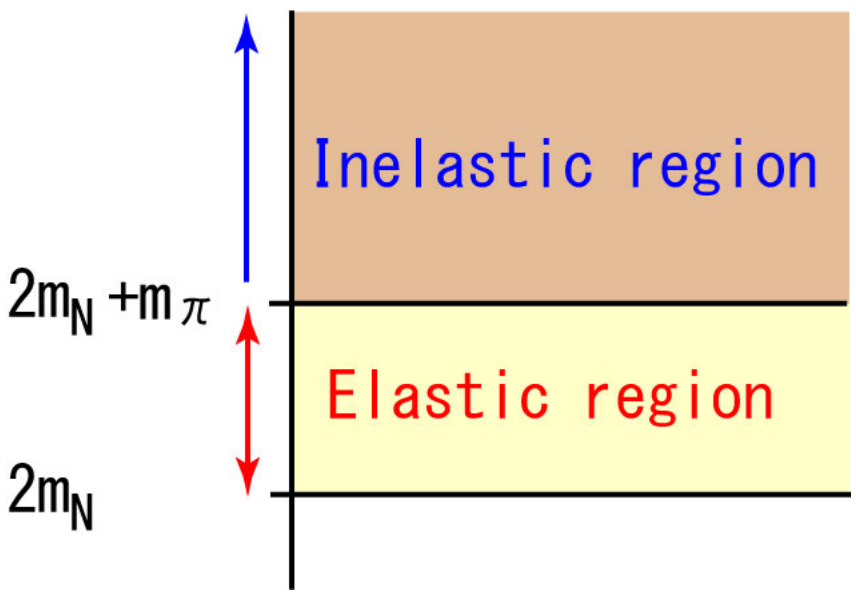
excited state contributions become bigger in the larger volume

$$\Delta E \propto \frac{1}{L^2}$$



time-dependent HAL QCD method makes this difficulty milder

$$\Delta E \simeq m_\pi$$



remaining t-dependence of the potential

1. Inelastic contributions (including excited states of one baryon)

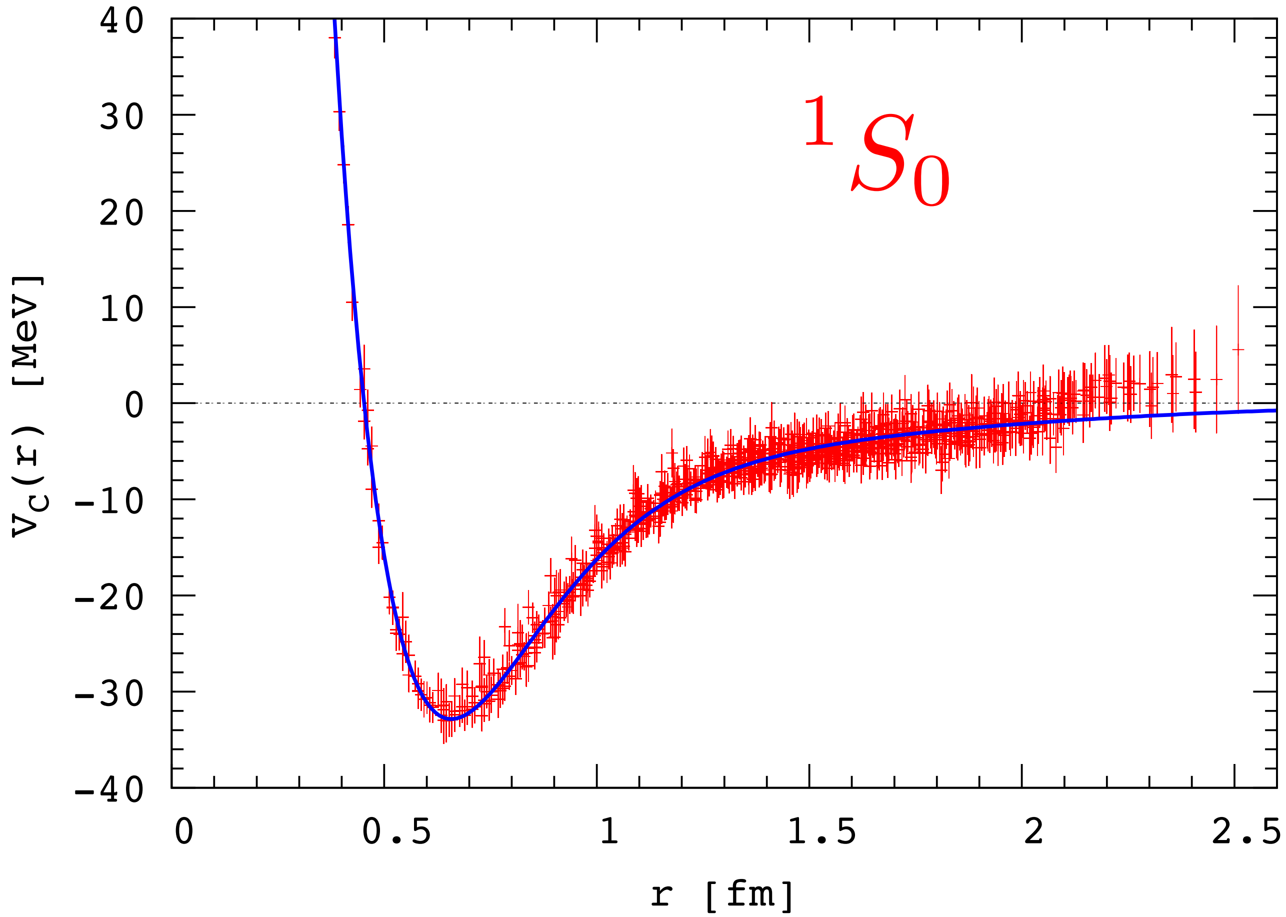
$$R(\mathbf{r}, t) = F(\mathbf{r}, t)/G_N(t)^2$$

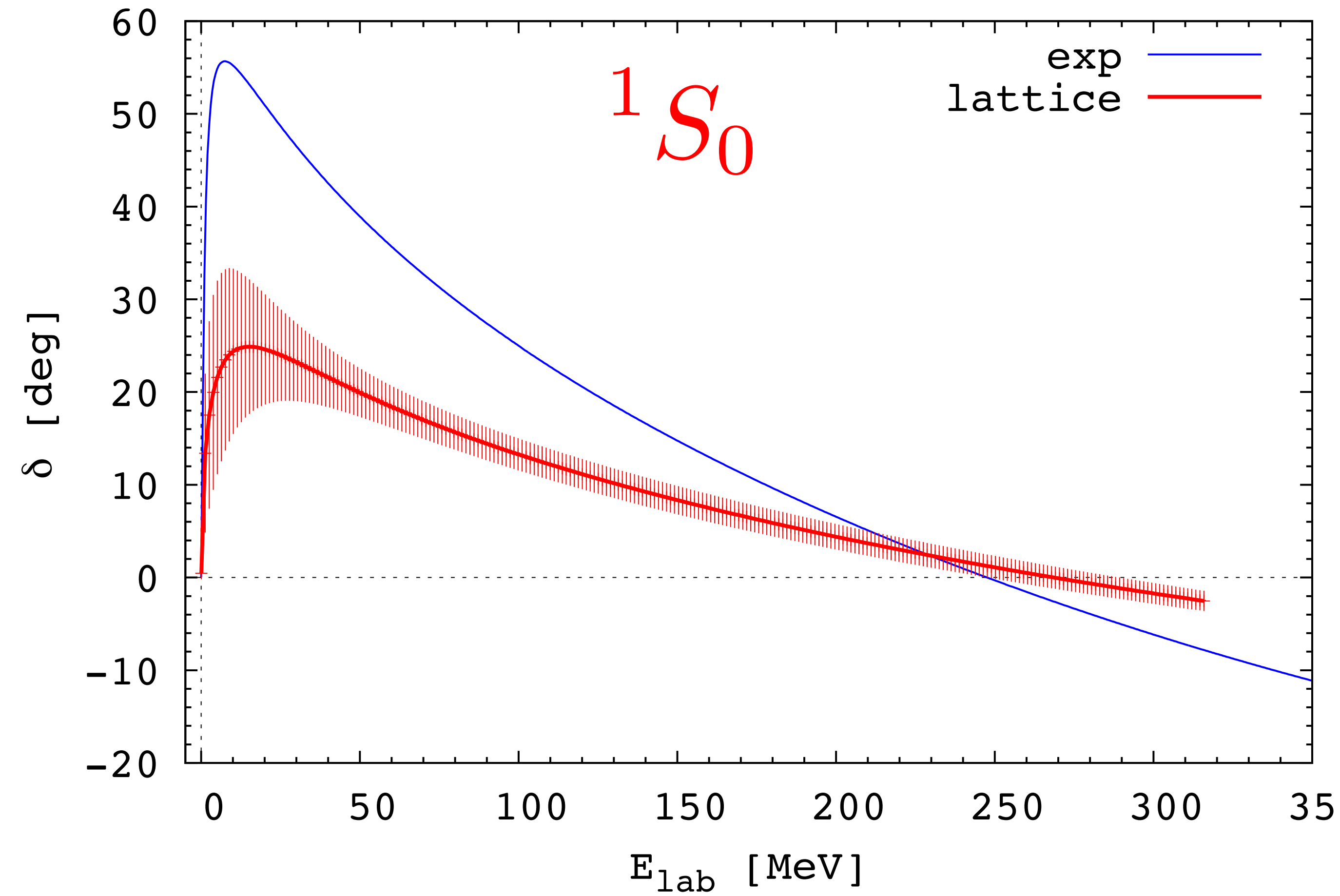
2. Higher order terms in the derivative expansion

2+1 flavor QCD a=0.09fm, L=2.9fm Ishii et al. (HALQCD), PLB712(2012) 437.

$m_\pi \simeq 700$ MeV

NN potential → phase shift





Qualitative features of NN potential are reproduced. → It has a reasonable shape. The strength is weaker due to the heavier quark mass.

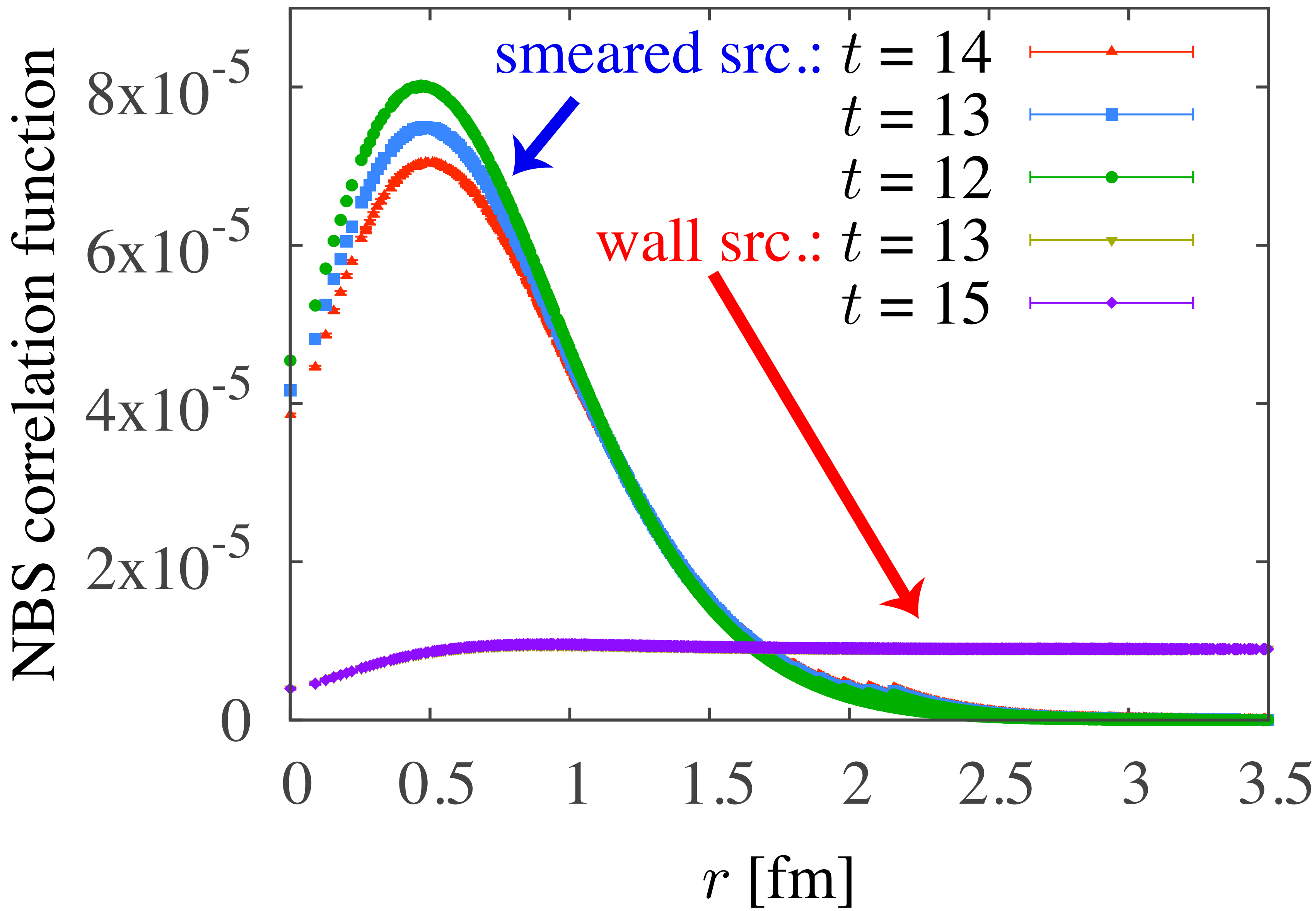
No dineutron at heavier pion mass.

IV. Source dependence of potentials

T. Iritani, Talk at Lat2016, arXiv1610.09779[hep-lat]

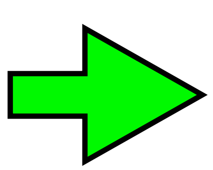
NBS wave function

$\Xi\Xi(^1S_0)$



wall source very weak t-dependence

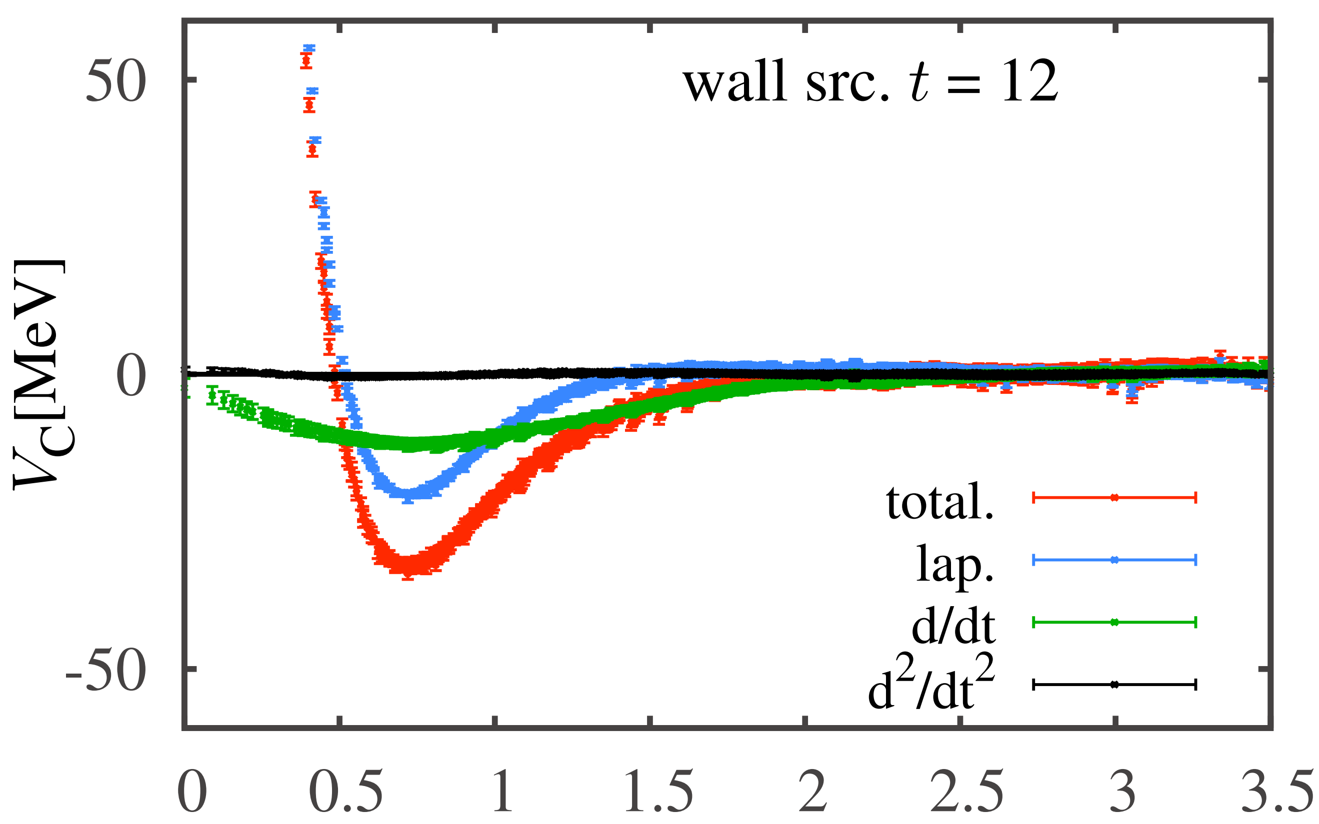
smeared source strong t-dependence

 contributions from excited states

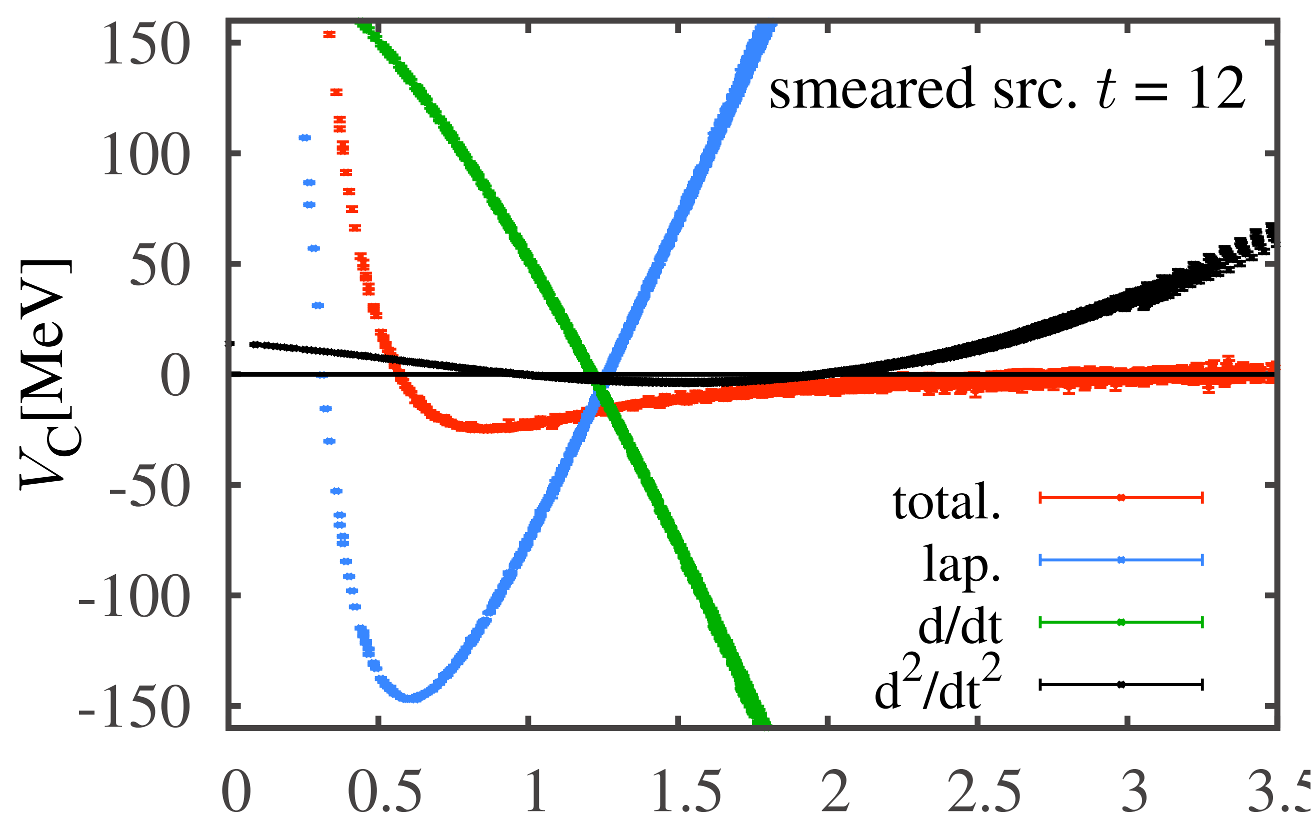
Potential

$$V_c(\mathbf{r}) = -\frac{H_0 R}{R} - \frac{(\partial/\partial t)R}{R} + \frac{(\partial/\partial t)^2 R}{4mR}$$

wall

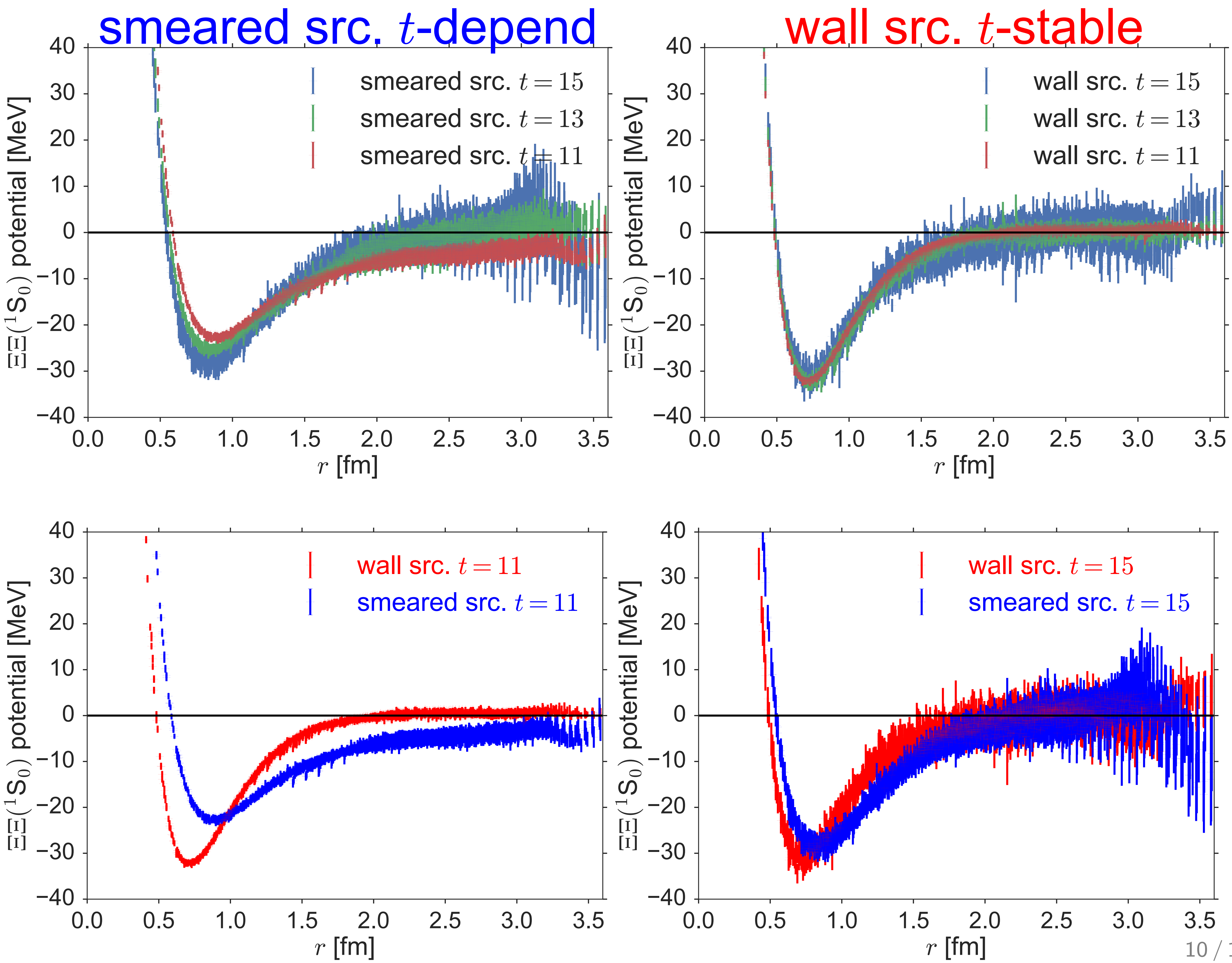


smeared



O(100) MeV cancellation

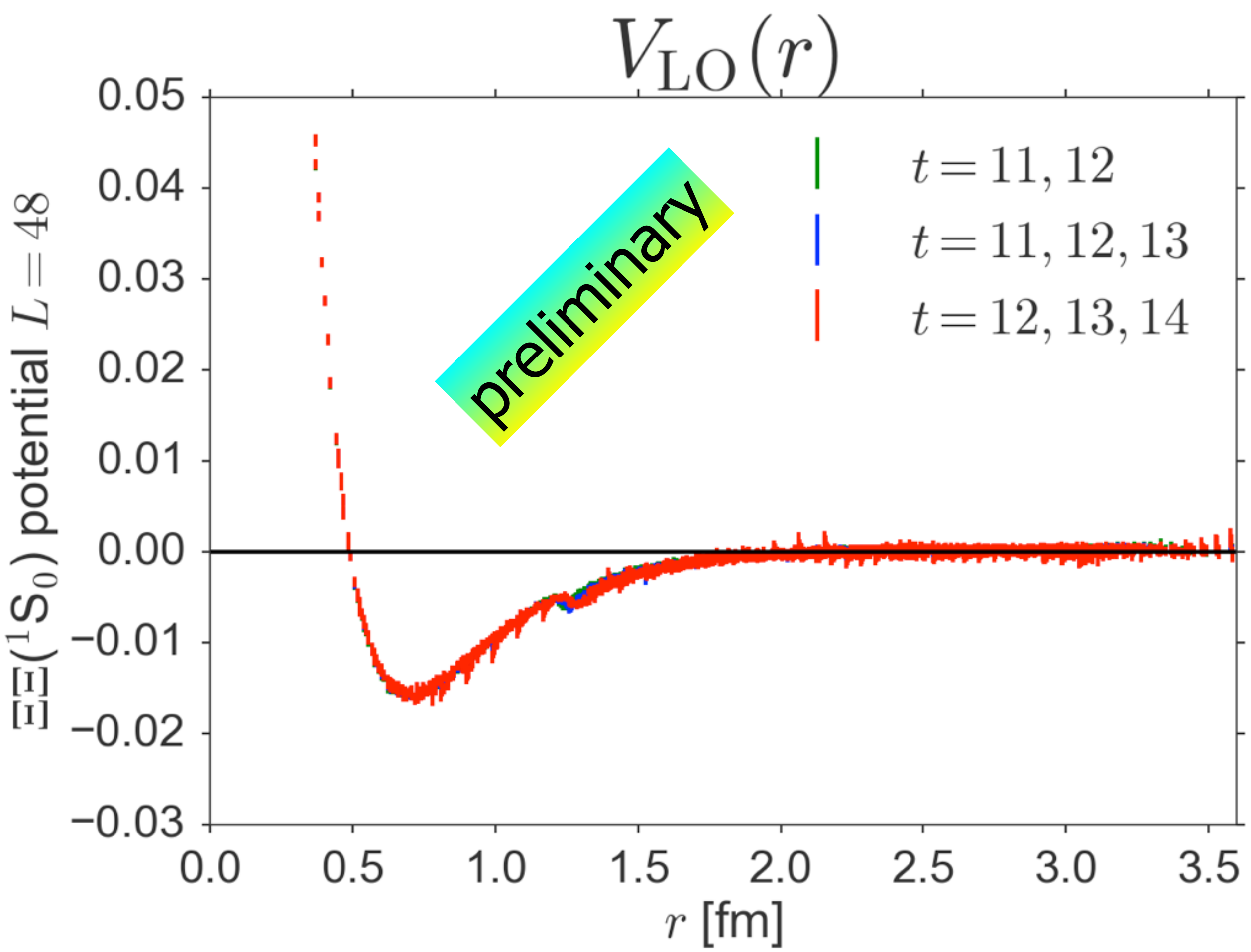
time-dependent HAL method works well

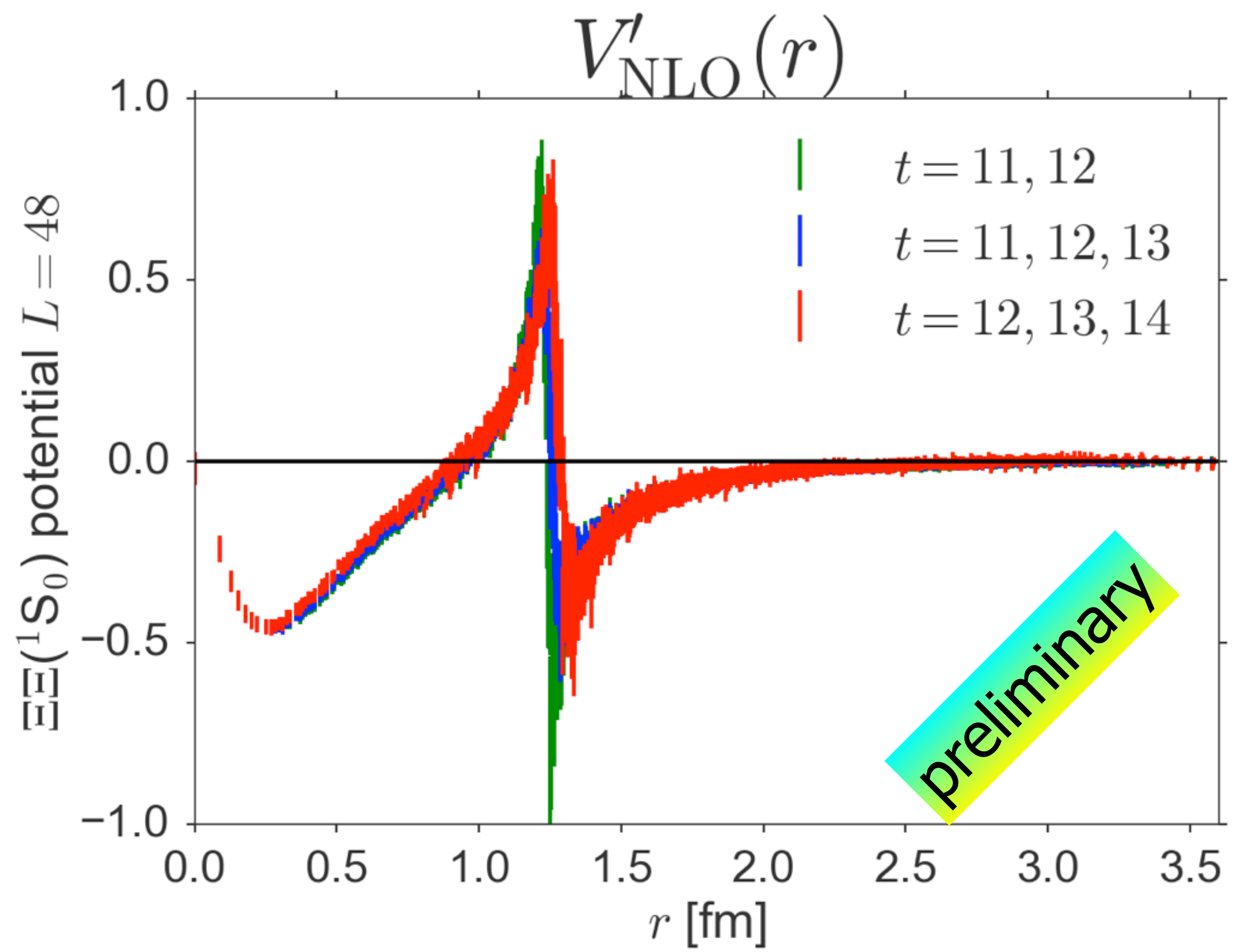


Wall src. is stable. Smeared src. -> wall src. for large t.

Analysis w/LO+ NLO potential

$R_{\mathrm{wall}}, R_{\mathrm{smeared}}$ ⇒ $U(\vec{r}, \vec{r}') = [V_{\mathrm{LO}}(\vec{r}) + V'_{\mathrm{NLO}}(\vec{r}) \nabla^2] \, \delta(\vec{r} - \vec{r}')$





The difference between wall/smeared reflects physics.

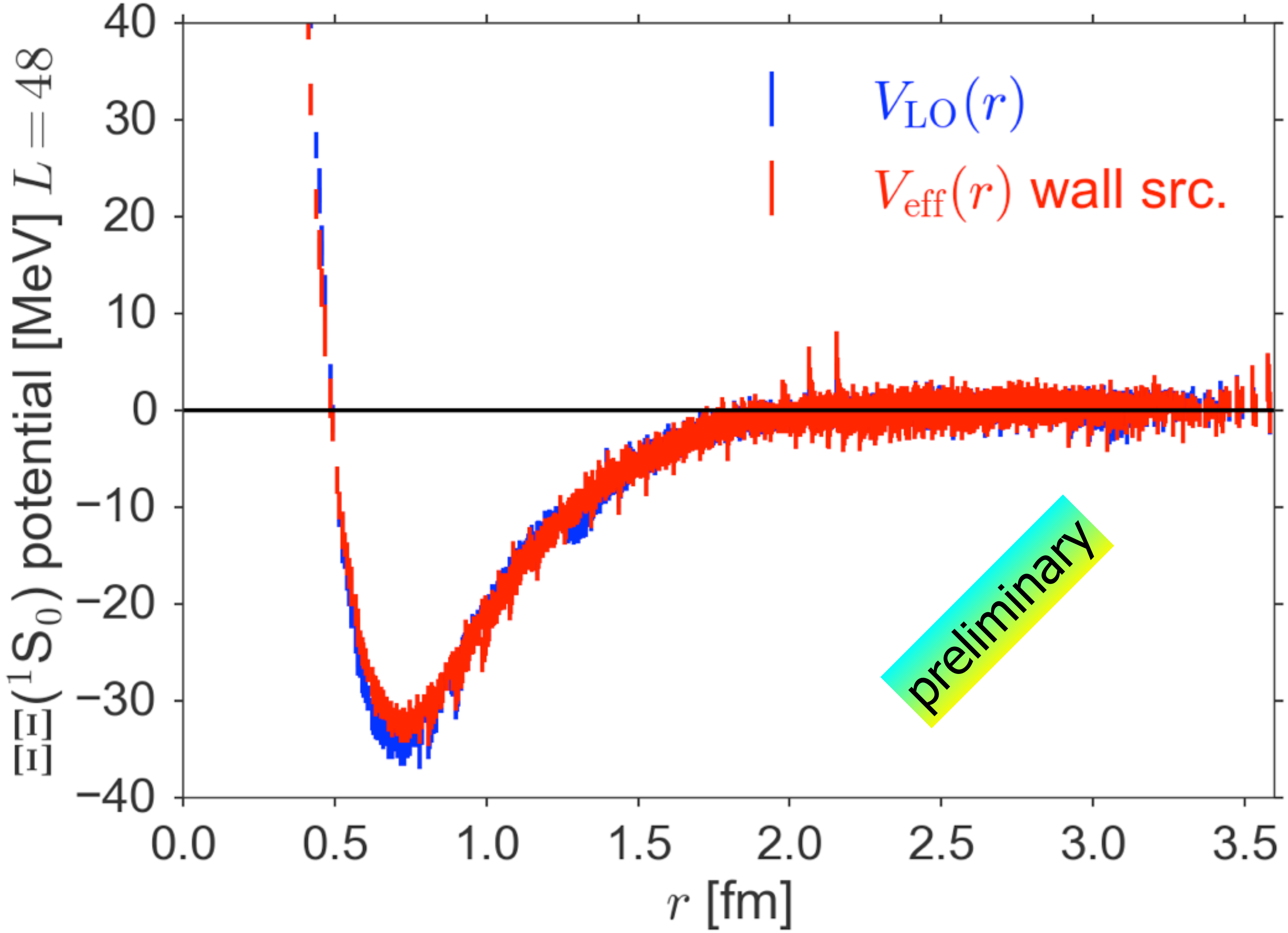
Smeared data contain much more excited states. ⇒ more sensitive to NLO

New method to extract NLO potential !

Potentials relevant at low energy

$$U(\mathbf{r},\mathbf{r}') = V_{\rm eff}(\mathbf{r})\delta(\mathbf{r}-\mathbf{r}')$$ (wall)

$$U(\mathbf{r},\mathbf{r}') = \left[V_{\rm LO}(\mathbf{r}) + V'_{\rm NLO}\nabla^2\right]\delta(\mathbf{r}-\mathbf{r}')$$ (wall & smeared)



$V_{\rm eff}(\mathbf{r}) \simeq V_{\rm LO}(\mathbf{r})$

Potential from wall src. is reliable at low energy.

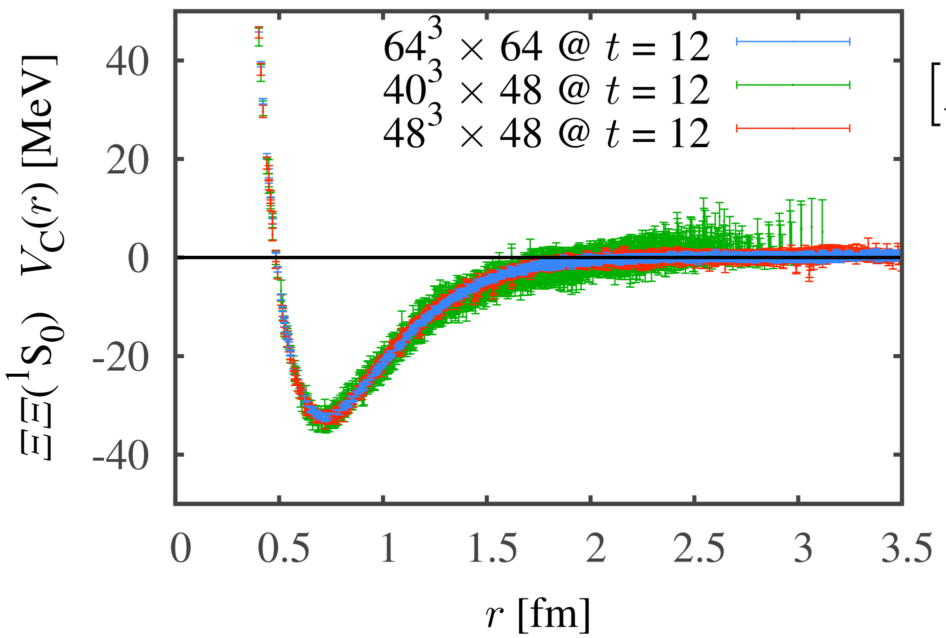
$V'_{\rm NLO}$ is relevant for high energy states.

Good convergence of the derivative expansion at low energy.

V. Anatomy of the direct method by the potential

T. Iritani, Talk at Lat2016, arXiv1610.09779[hep-lat]

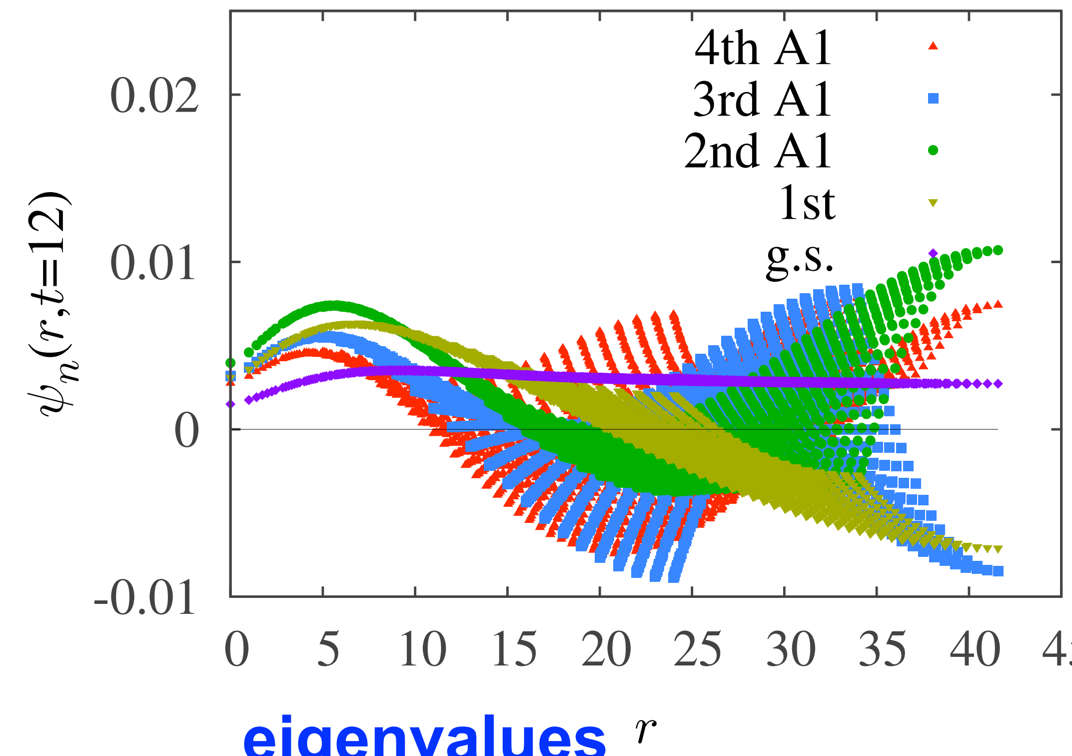
potential



$$[H_0 + V]\Psi_n = \Delta E_n \Psi_n$$

on finite Volume

eigenfunctions



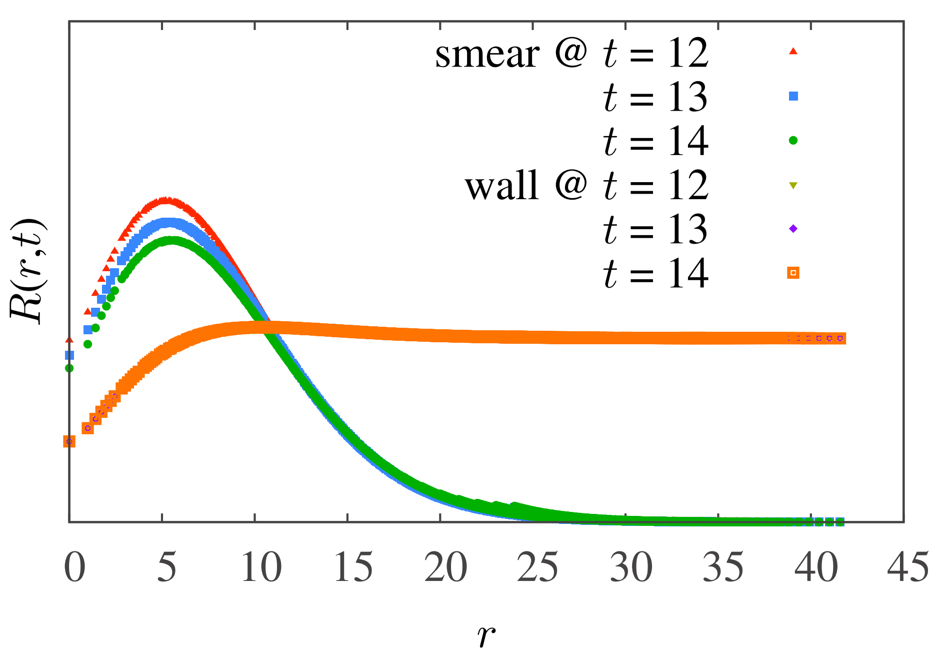
eigenvalues

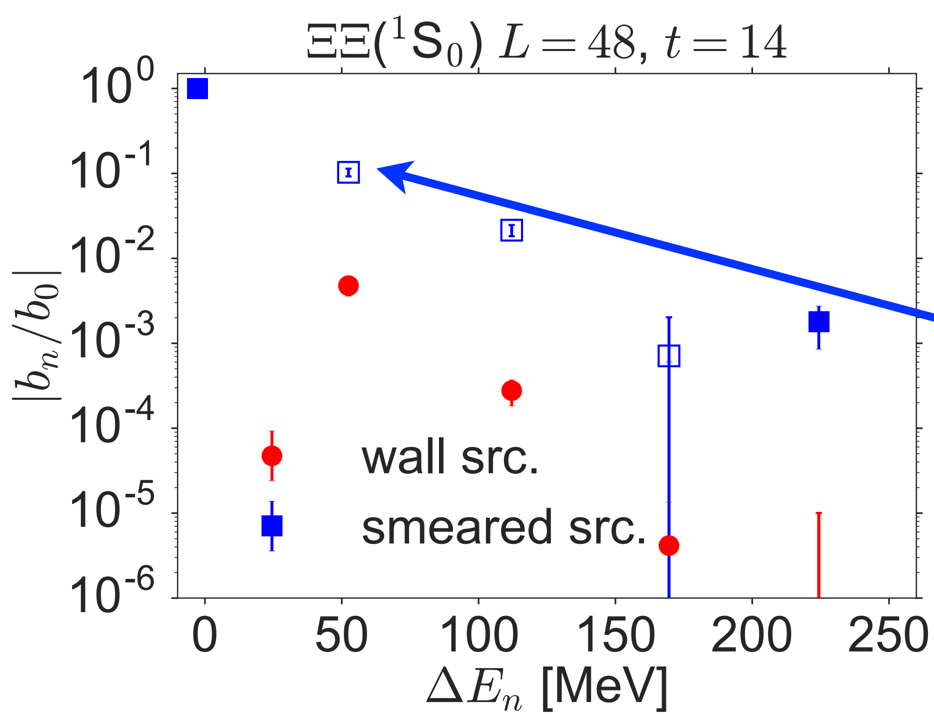
n-th A1	ΔE_n [MeV]
0	-2.58(1)
1	52.49(2)
2	112.08(2)
3	169.78(2)
4	224.73(1)

$$R^{\text{wall/smear}}(\mathbf{r}, t) \simeq \sum_n c_n^{\text{wall/smear}} \Psi_n(\mathbf{r}) \exp[-\Delta E_n t]$$

$$R^{\text{approx.}}(t) \simeq \sum_n b_n e^{-\Delta E_n t}$$

NBS wave function



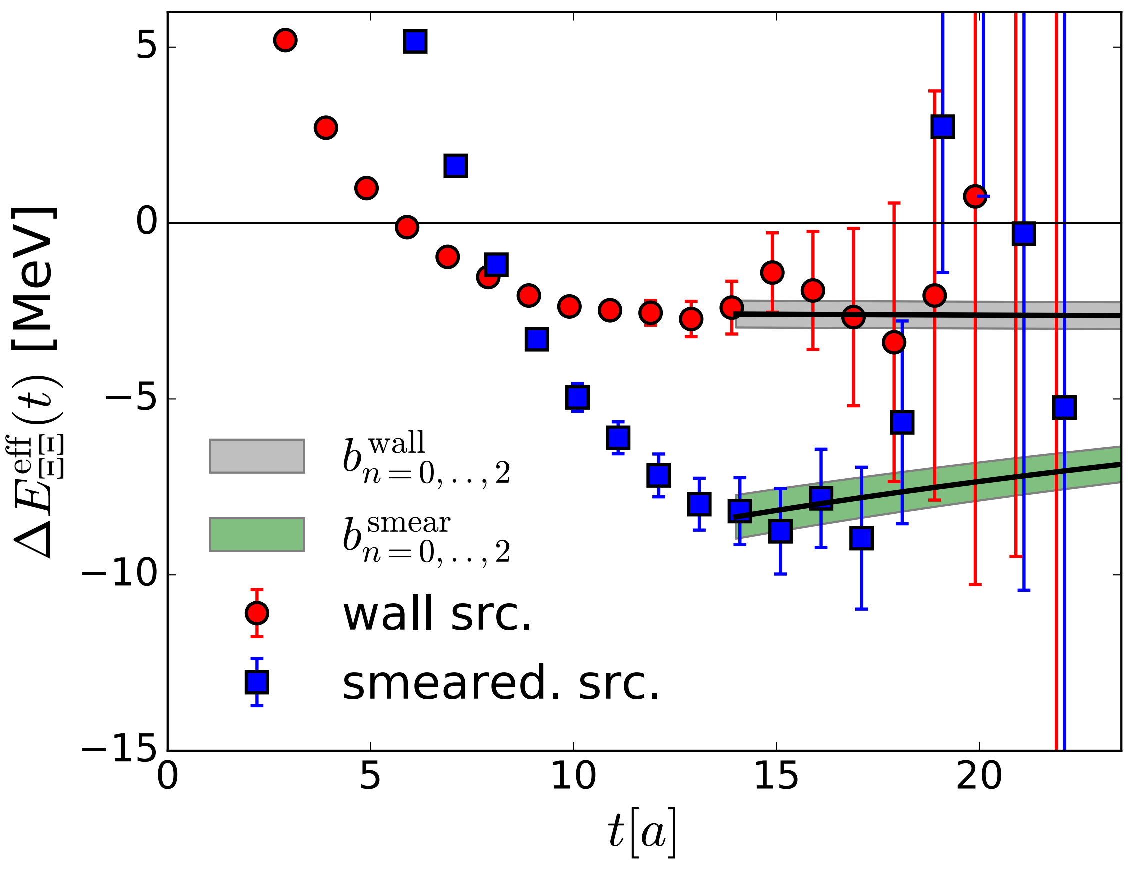


10% contamination of 1st excited states

Reconstruction of fake plateaux

$$\Delta E_{\text{eff}}^{\text{approx.}}(t) = \frac{1}{a}\log\left(\frac{R^{\text{approx.}}(t)}{R^{\text{approx.}}(t+a)}\right)$$

This explains "two plateaux"



Reconstruction of fake plateaux

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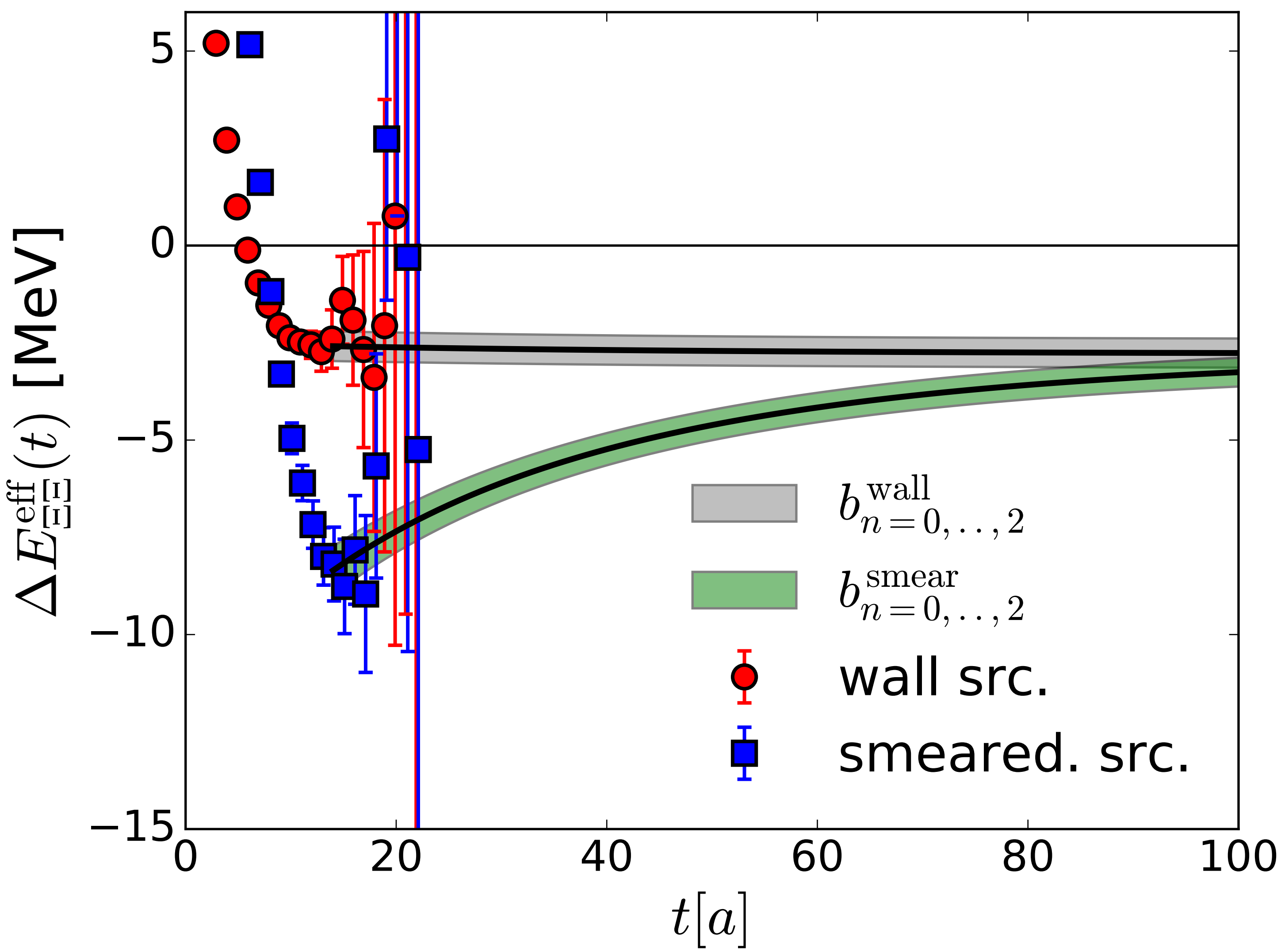

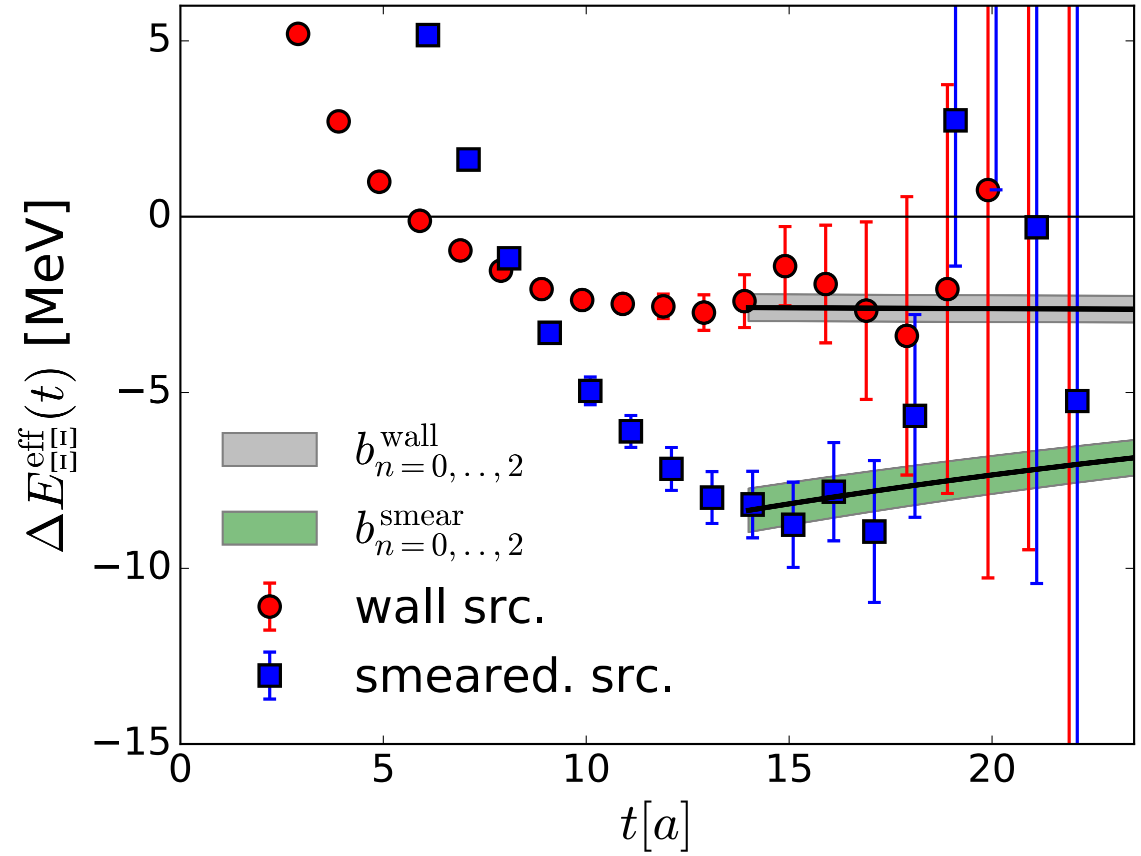

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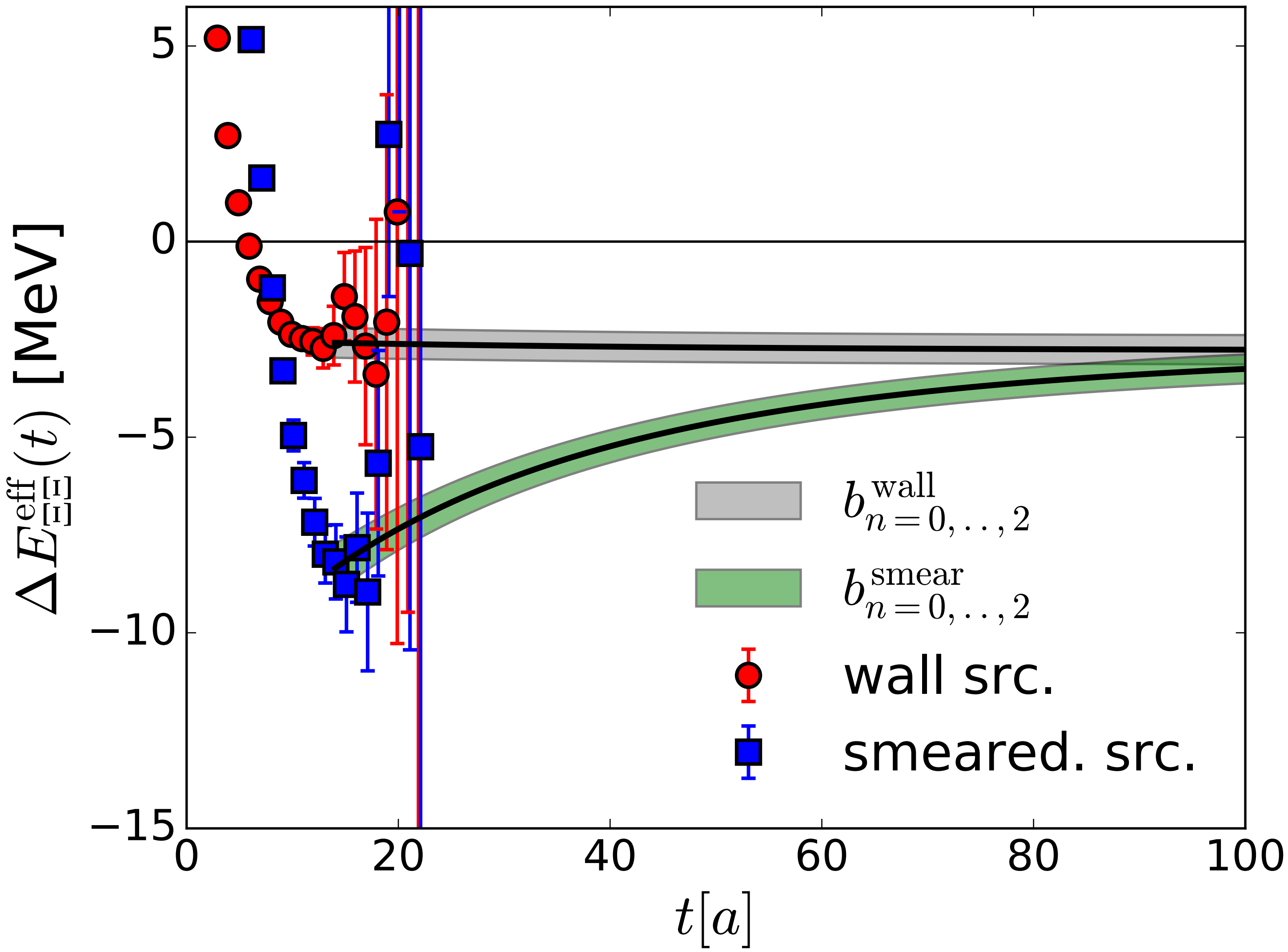

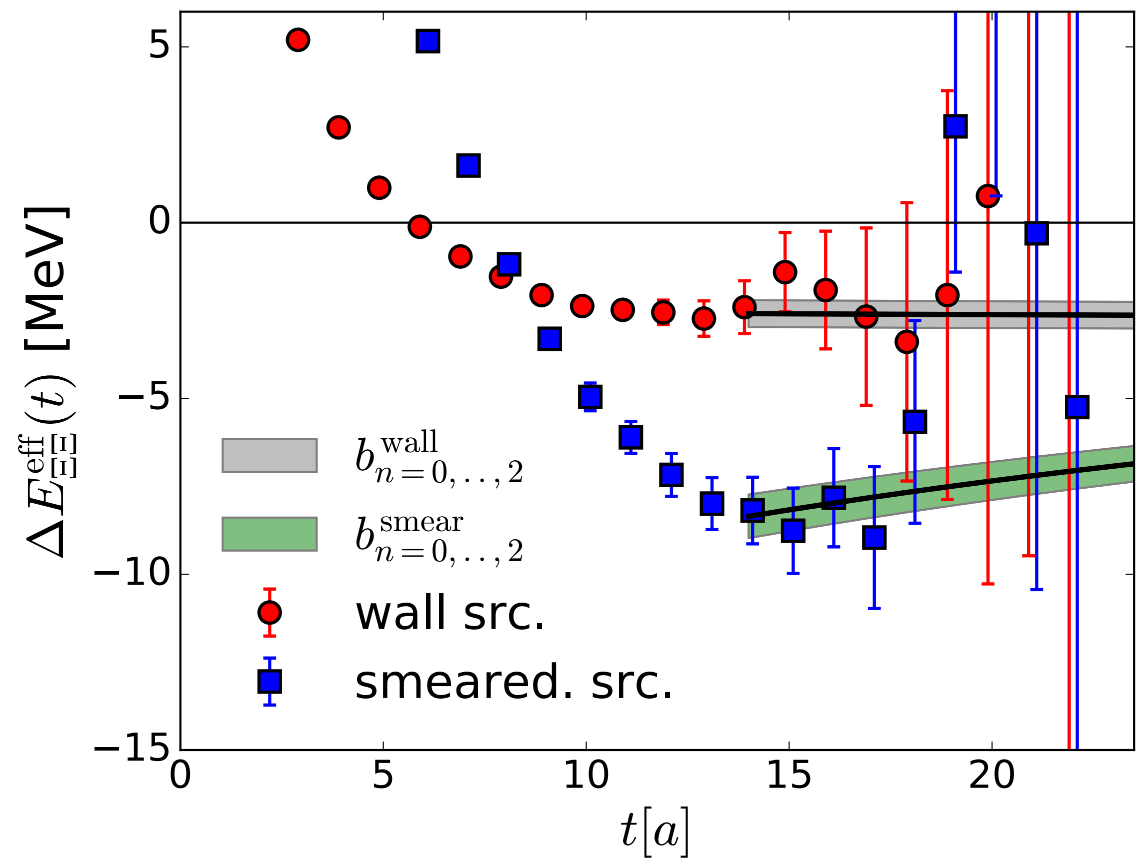

We need $t \simeq 10$ fm ($t/a \simeq 100$)
to see an agreement btw two sources

Reconstruction of fake plateaux

$$\Delta E_{\rm eff}^{\rm approx.}(t) = \frac{1}{a}\log\left(\frac{R^{\rm approx.}(t)}{R^{\rm approx.}(t+a)}\right)$$

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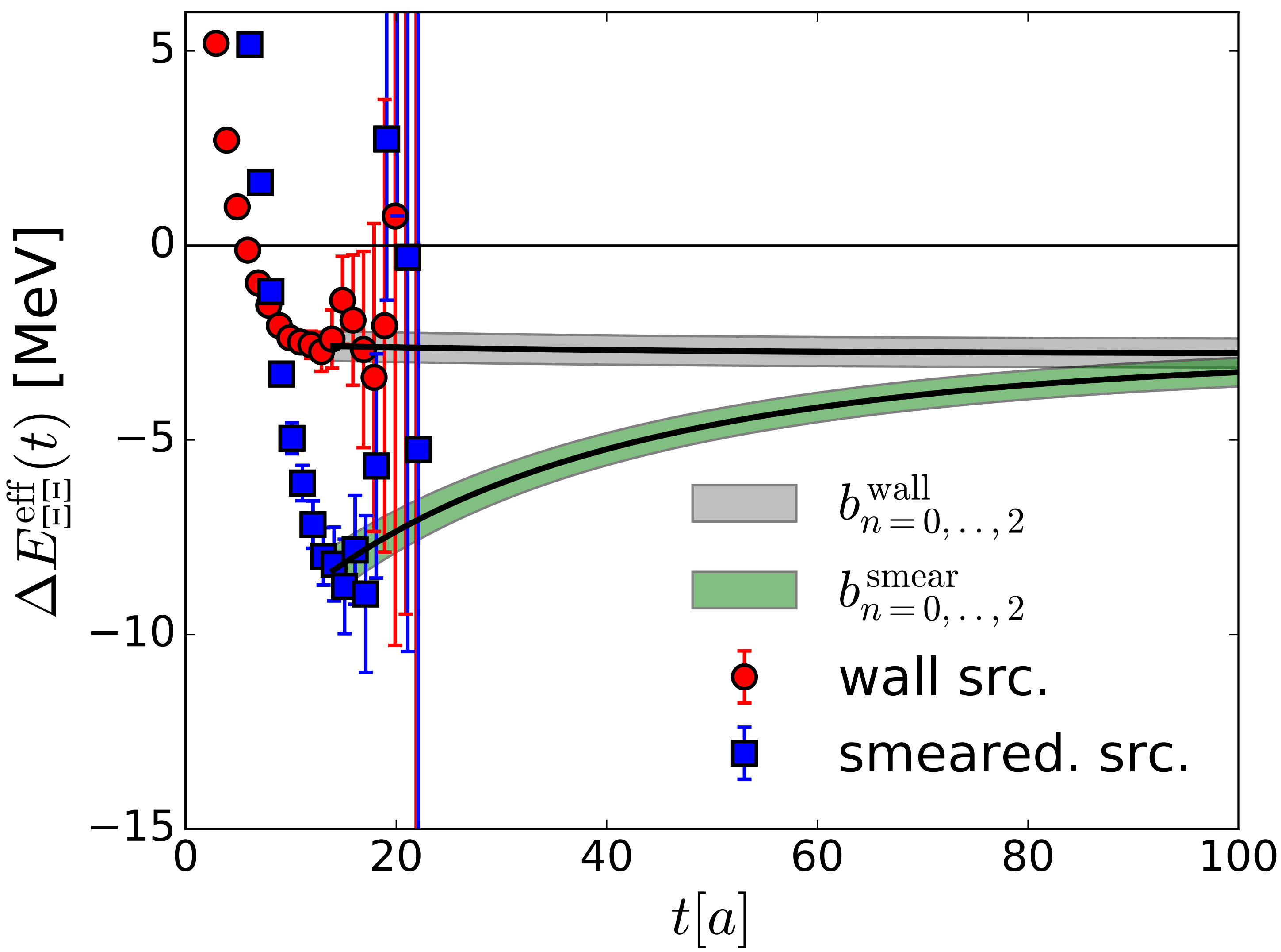

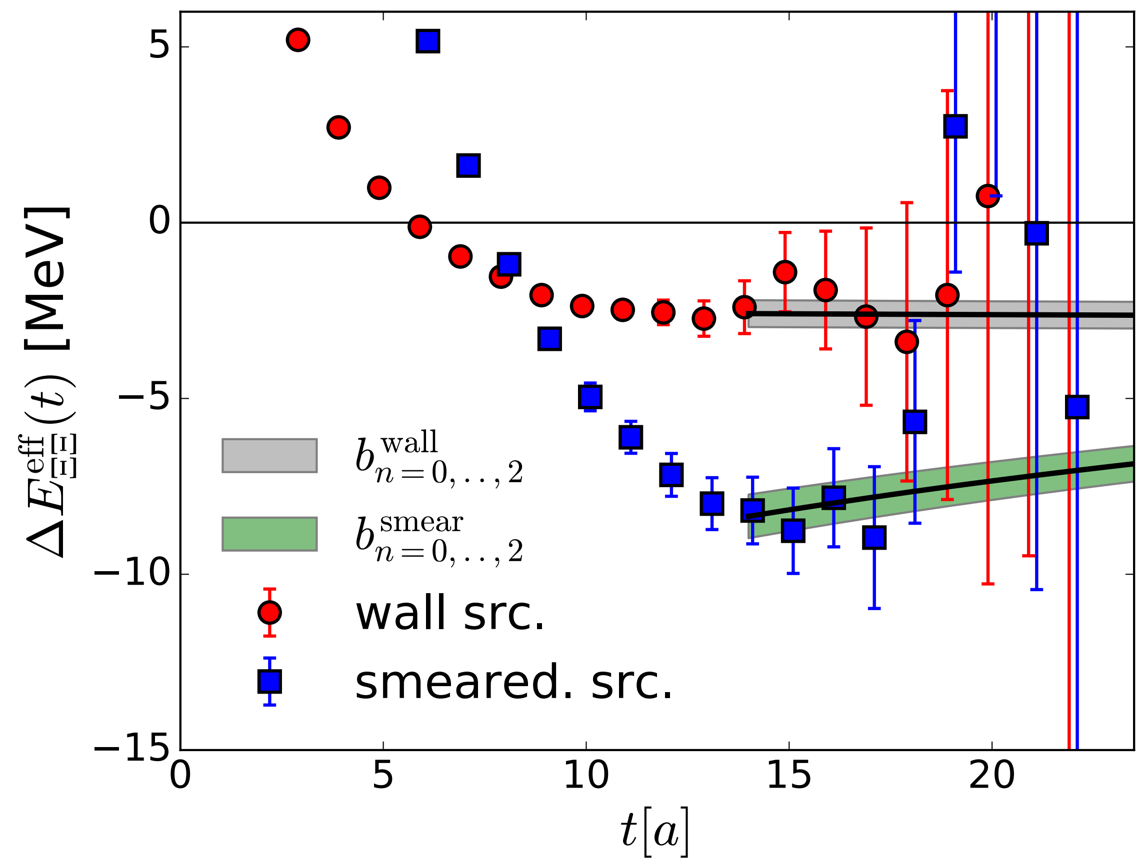

This agrees with the naive estimation.

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Reconstruction of fake plateaux

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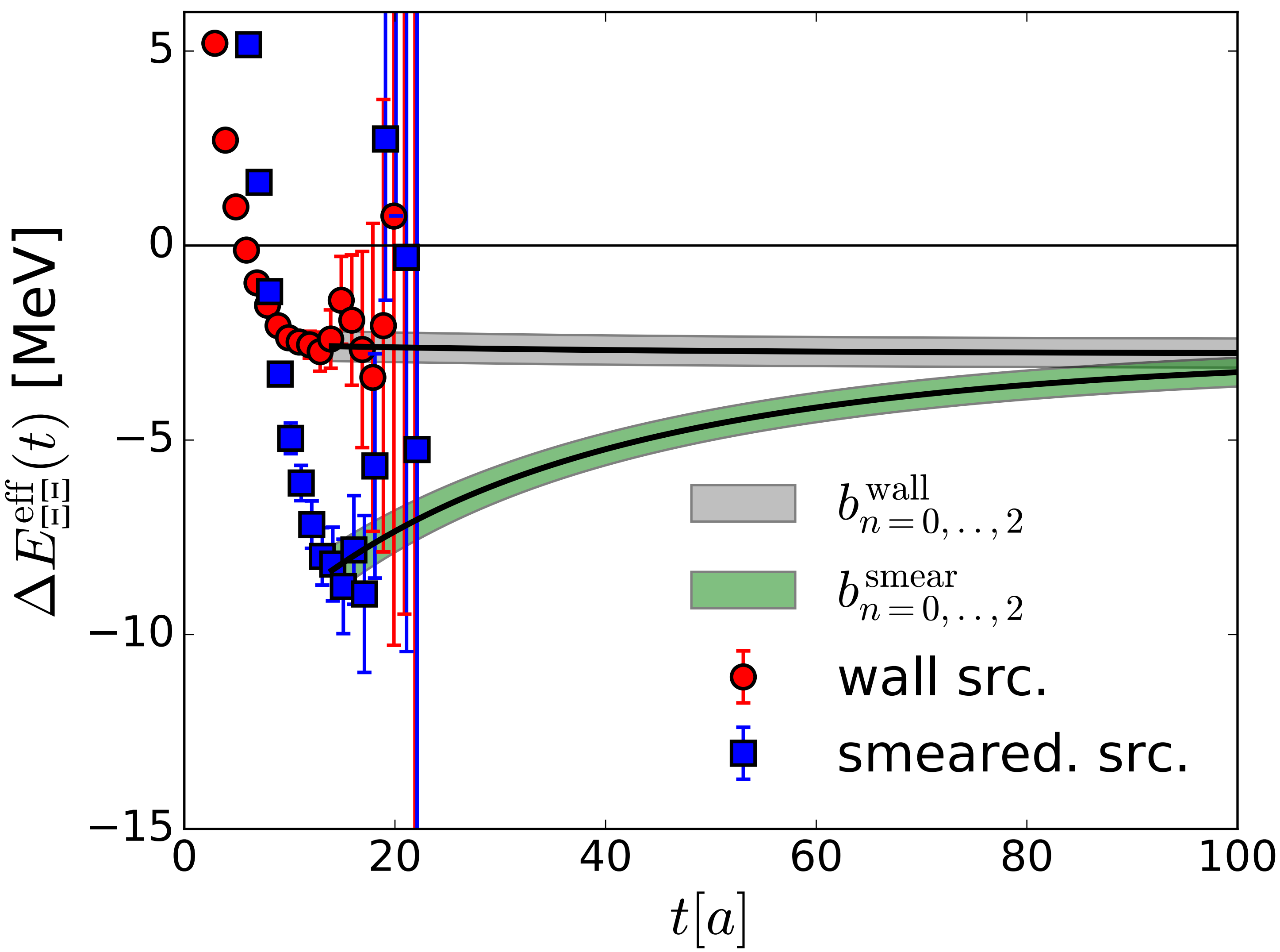

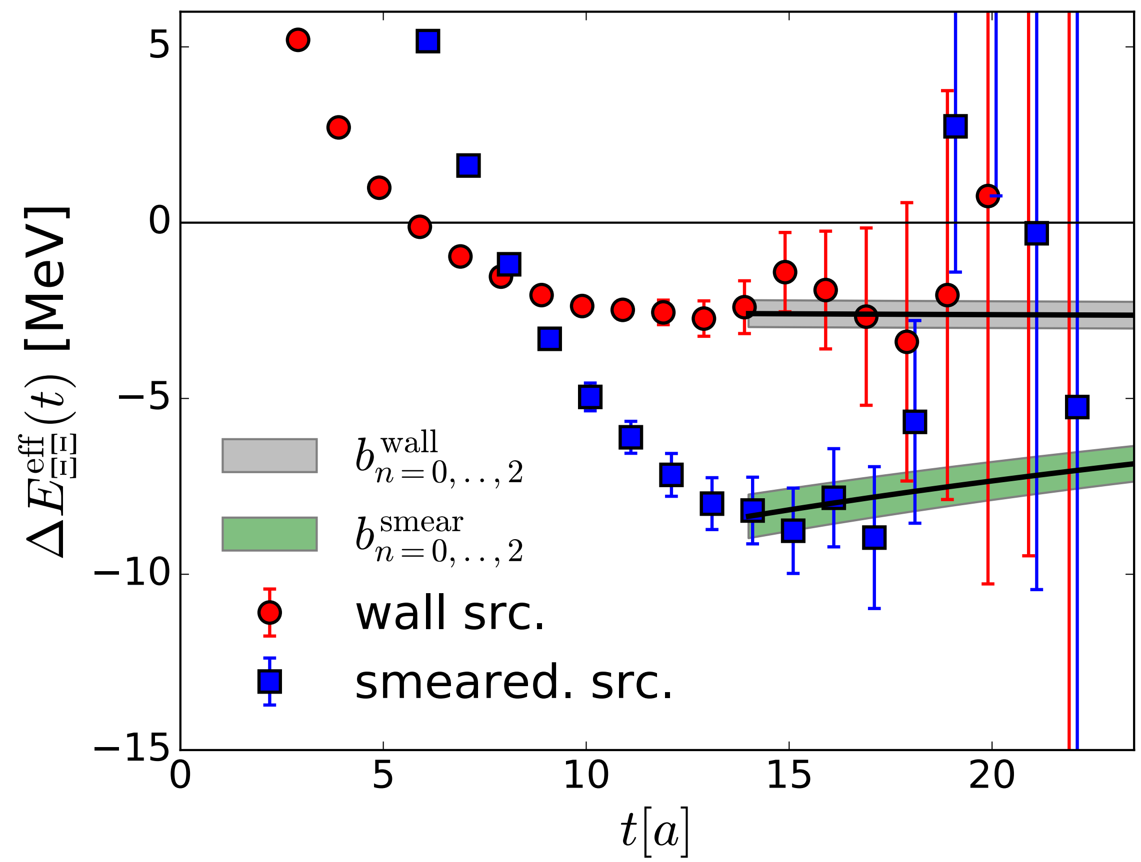

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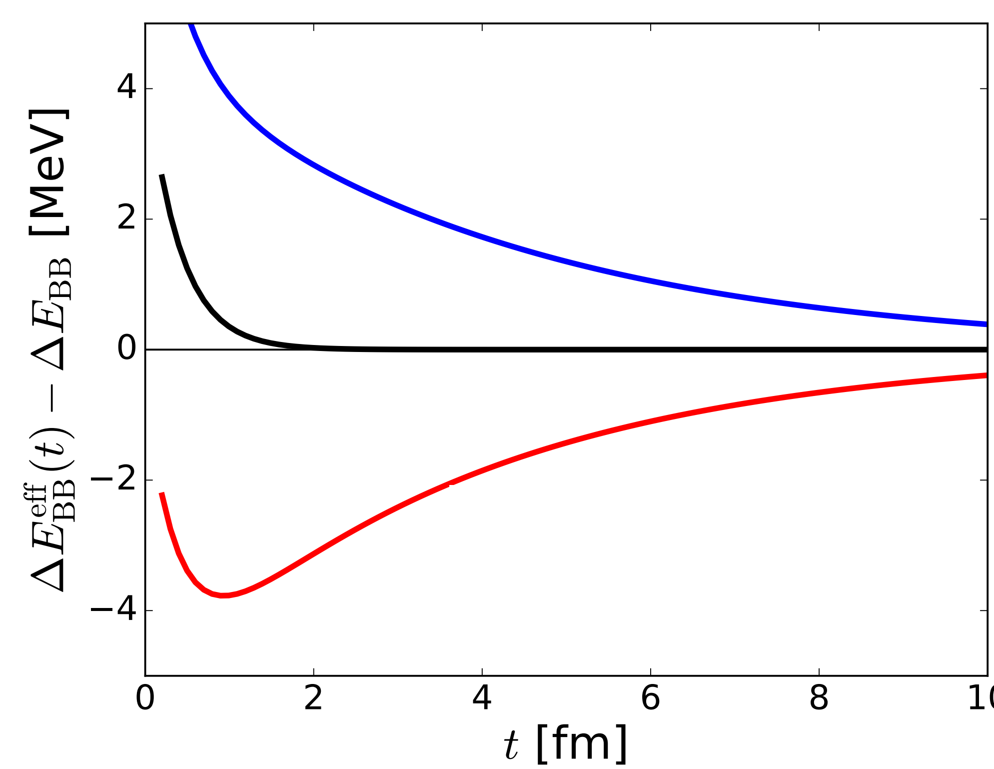


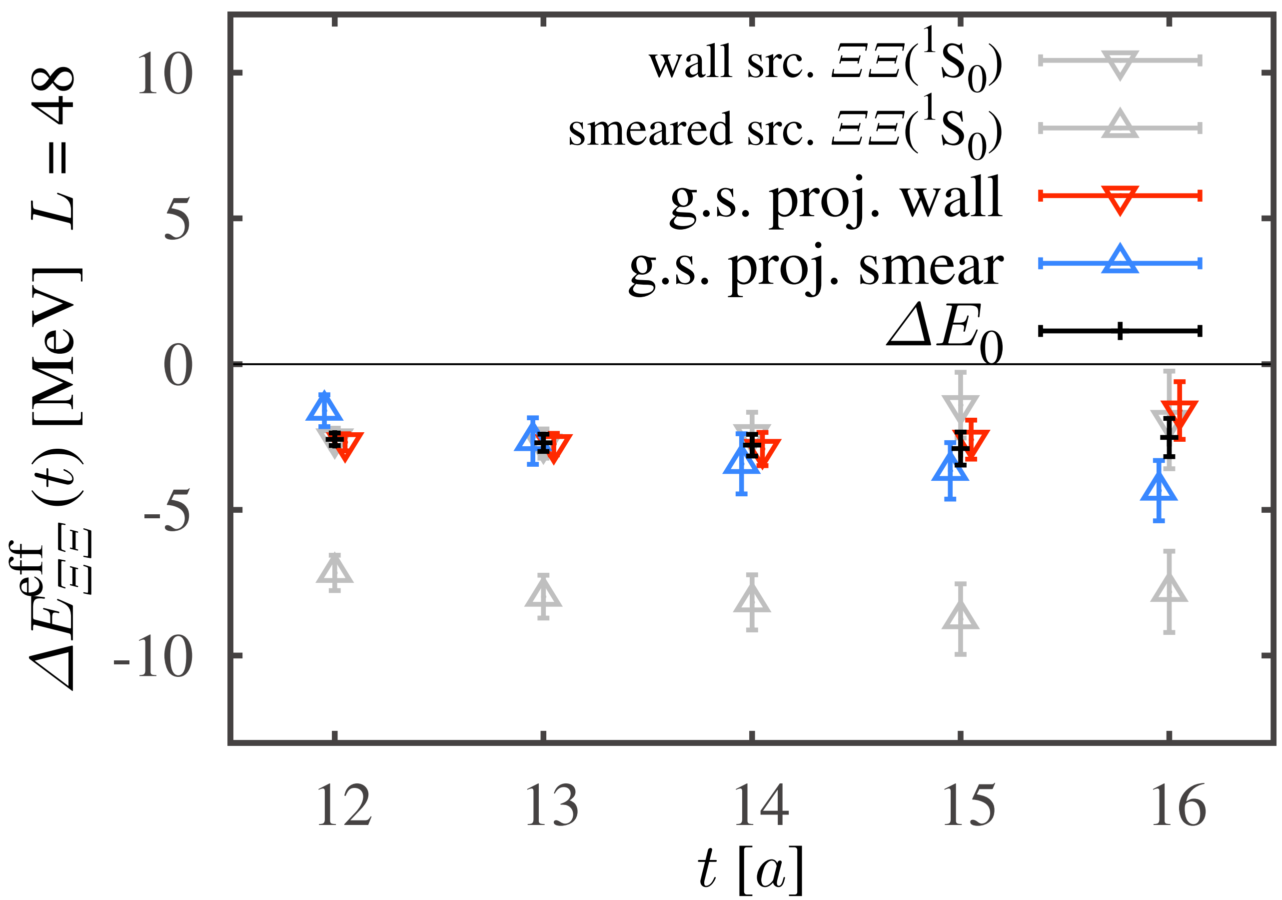
We need $t \simeq 10$ fm ($t/a \simeq 100$)
to see an agreement btw two sources

Direct method projected to an eigenstate

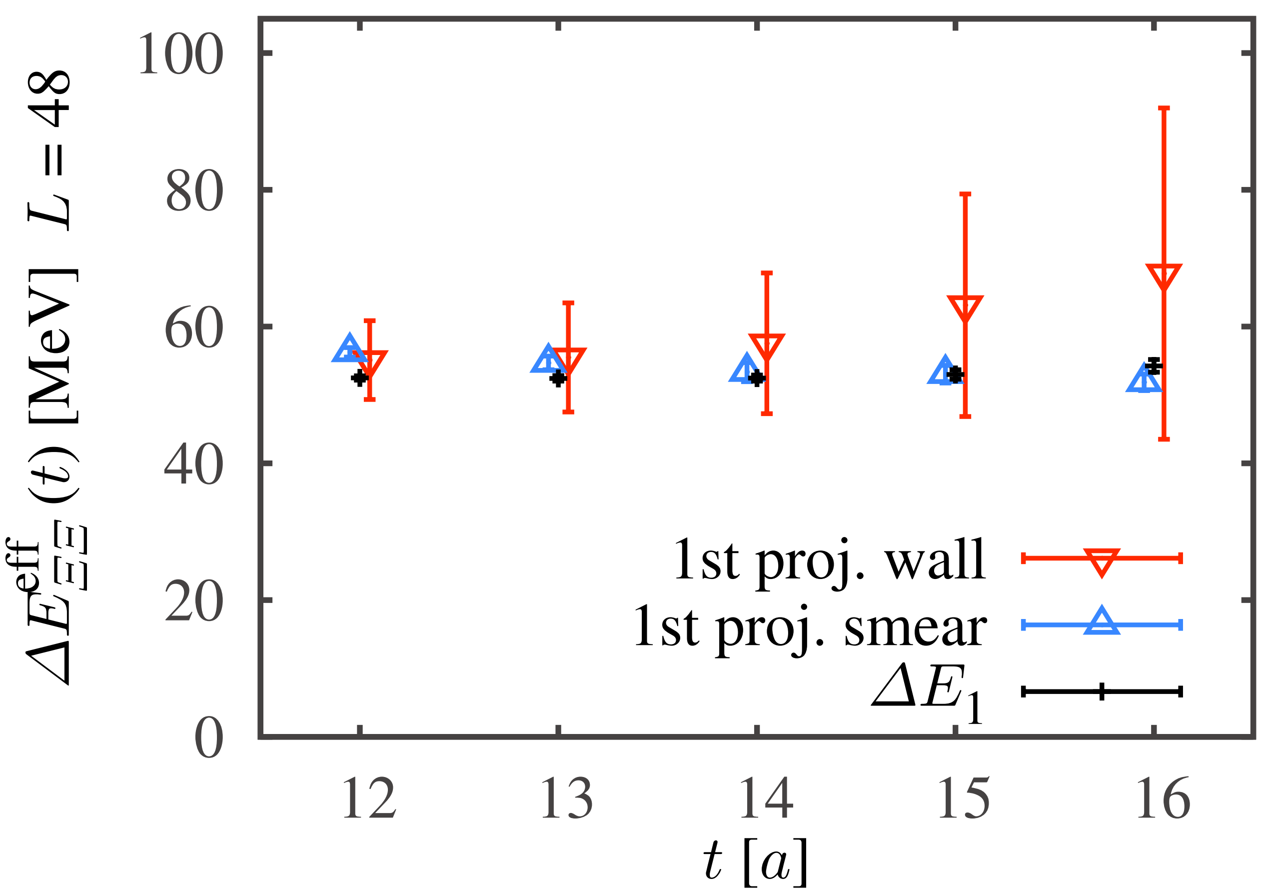
$$R_n^{\mathrm{wall/smear}}(t) = \sum_{\mathbf{r}} \underline{\Psi_n(\mathbf{r})} R^{\mathrm{wall/smear}}(\mathbf{r}, t) \quad \Rightarrow \quad \Delta E_{\mathrm{eff}}(t) = \log \frac{R_n(t)}{R_n(t+1)}$$

eigenstate

ground state



1st excited state



With the projection, even smeared src. gives the correct energy shift for the ground state at relatively short time.

We can also get the energy shift for the 1st excited state !

Errors are larger for the wall src., which has less contamination of the 1st excited state.

All analyses are consistent !

Summary

- The direct method suffers difficulties from the contamination of excited elastic states for two(or more)-baryon systems.
  - No trustable results so far.
  - Need new ideas.

Do not be misled.

- The HALQCD potential method overcomes these difficulties.
- by the time-dependent method
- gives reliable results

Please encourage your lattice colleagues to work on the potential method.

NN interactions become weaker at heavier pion masses.
No dineutron and deuteron exist there.

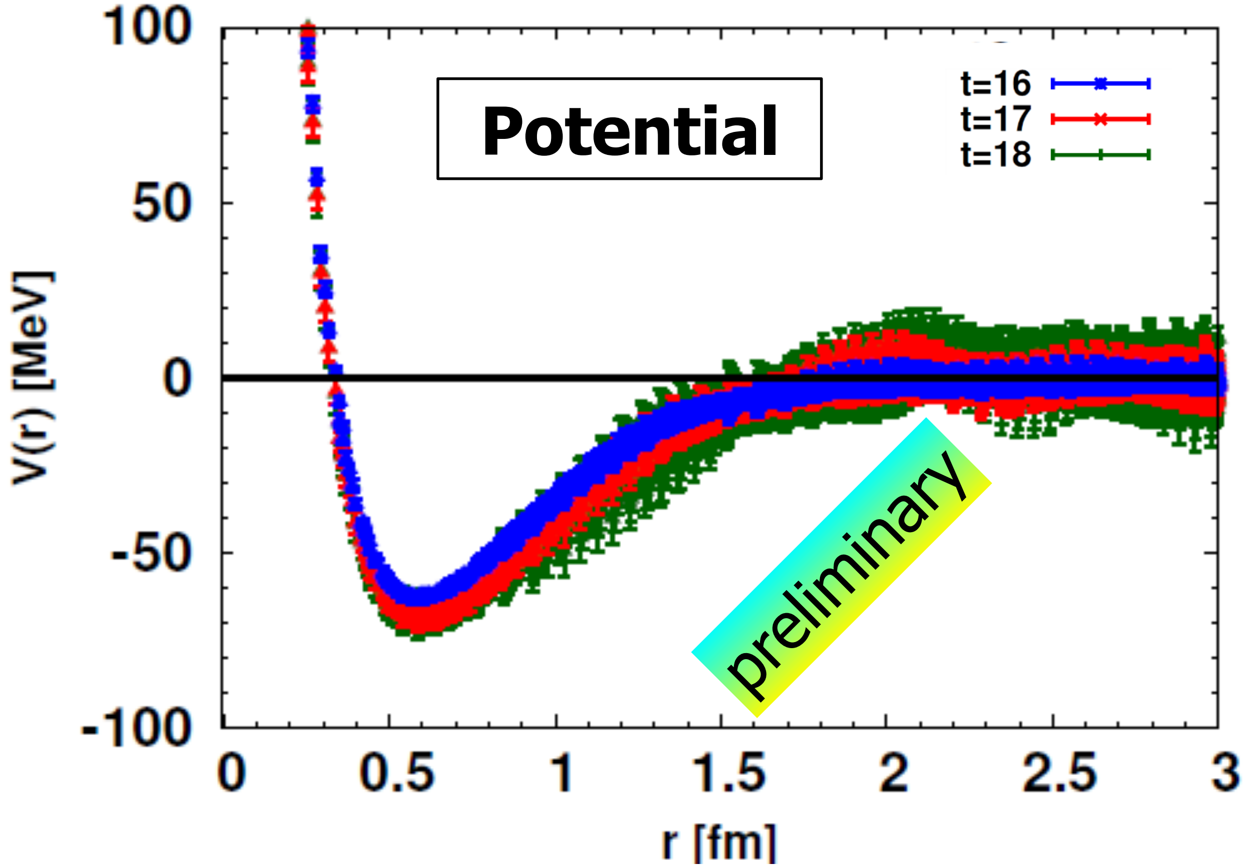
Potentials at physical pion

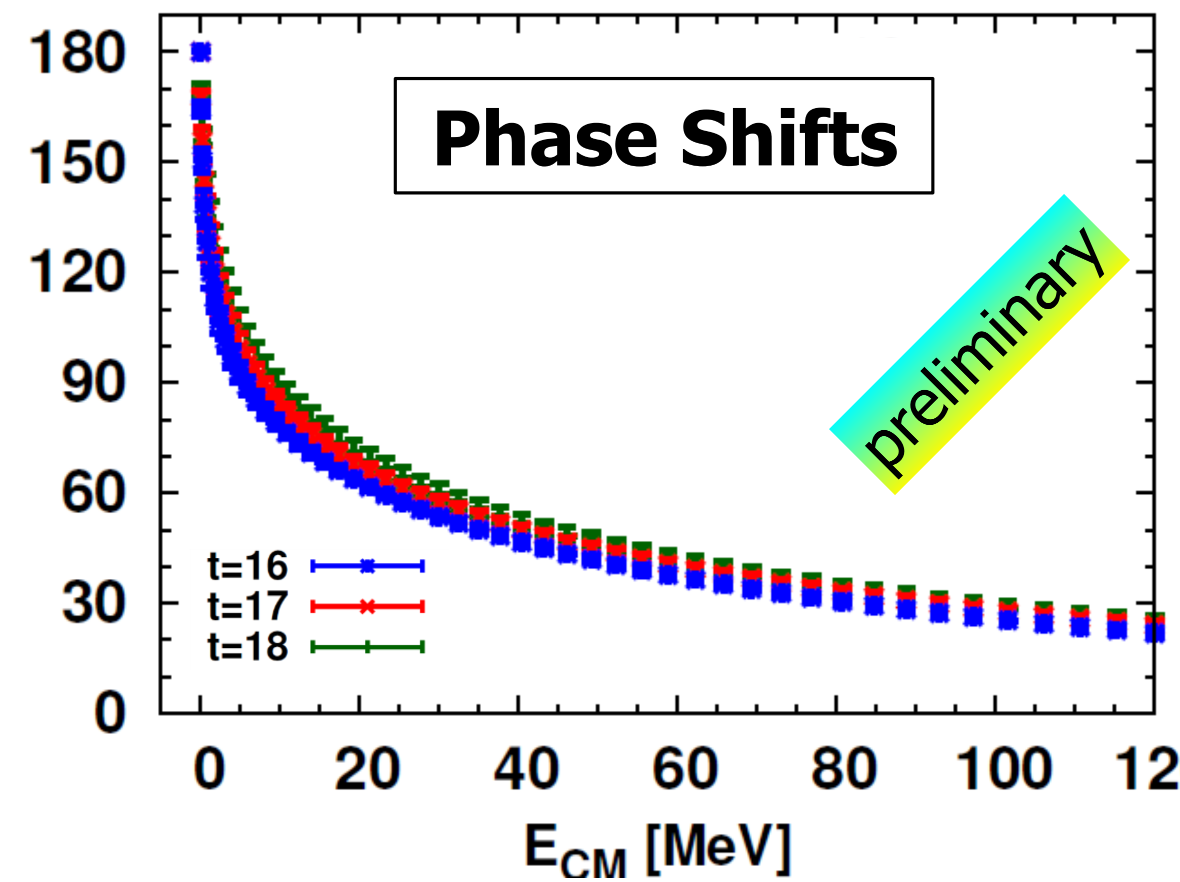
S. Gongyo

K-computer [10PFlops]

2+1 flavor QCD, $m_\pi \simeq 145$ MeV, $a \simeq 0.085$ fm, $L \simeq 8$ fm

$\Omega\Omega$ potential



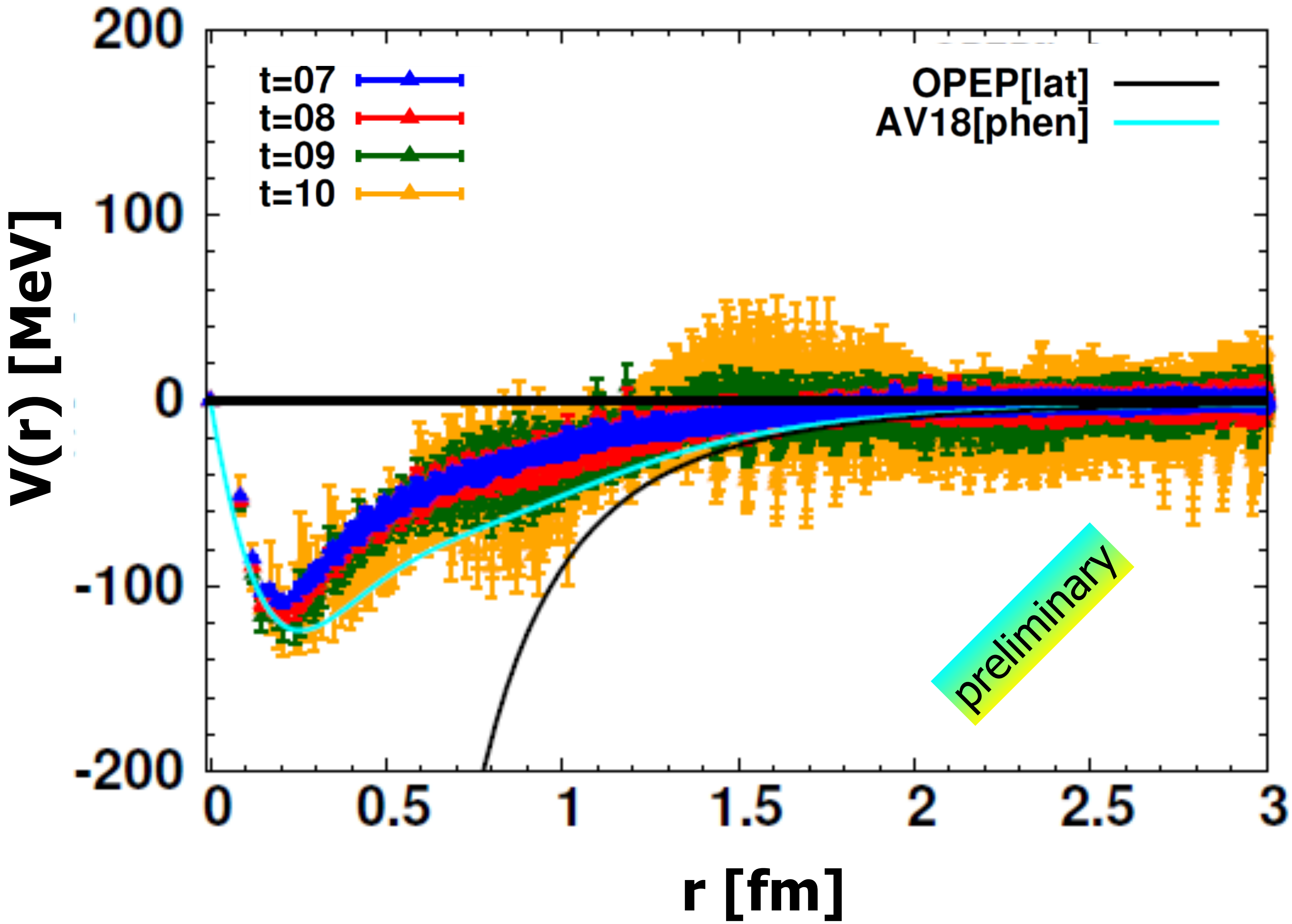


Strong attraction ➡ Vicinity of bound/unbound (~ unitary limit)

The most strange dibaryon ?

$NN(^3S_1)$ tensor potential

T. Doi



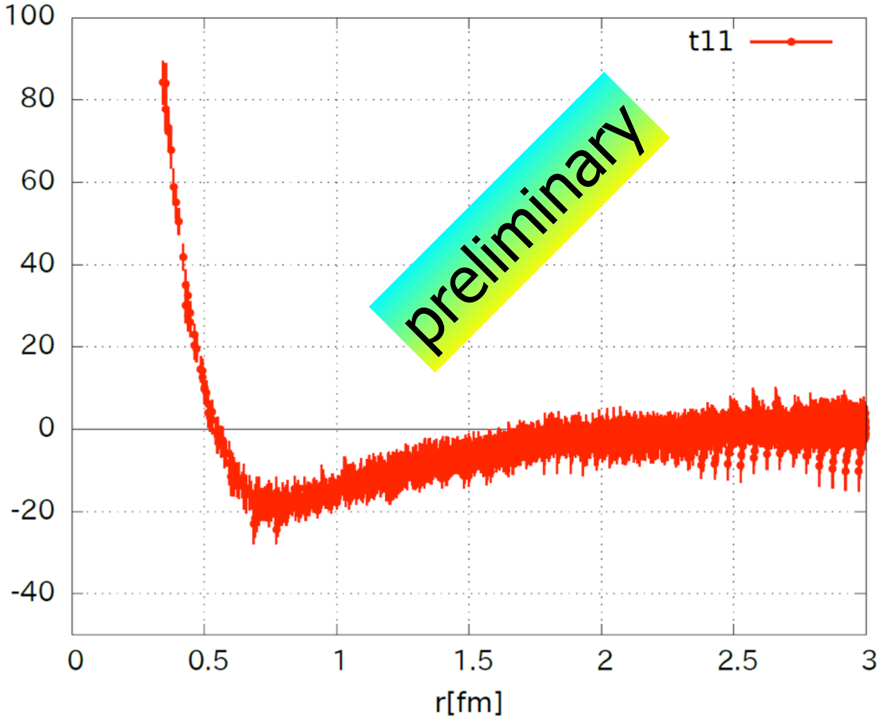
Qualitatively similar tail to one pion exchange potential (OPEP)

reduction of errors is definitely needed.

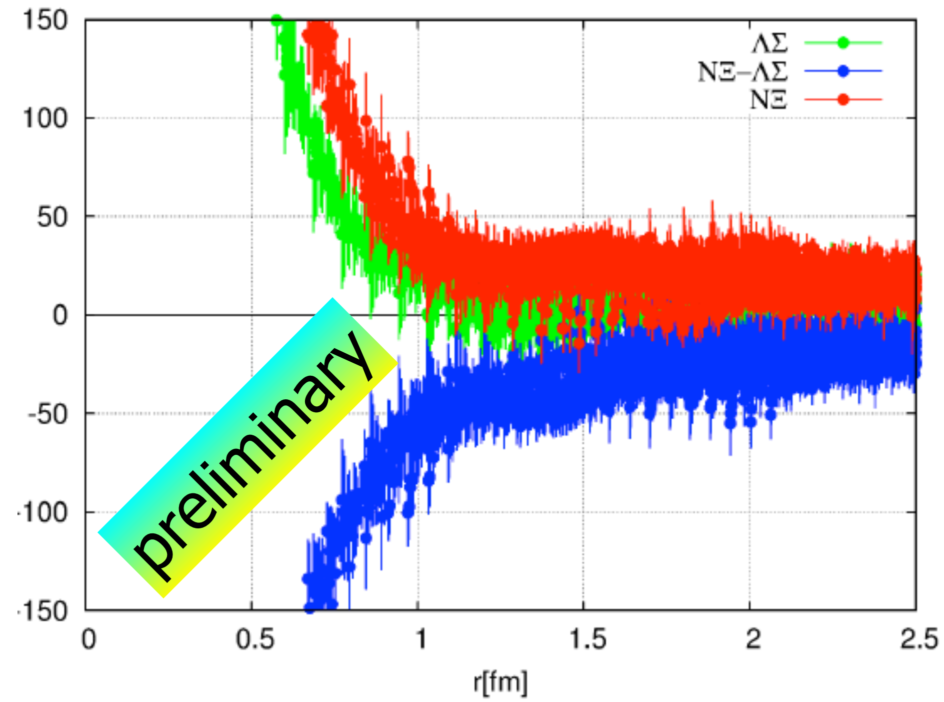
$N\Xi$ potentials

K. Sasaki

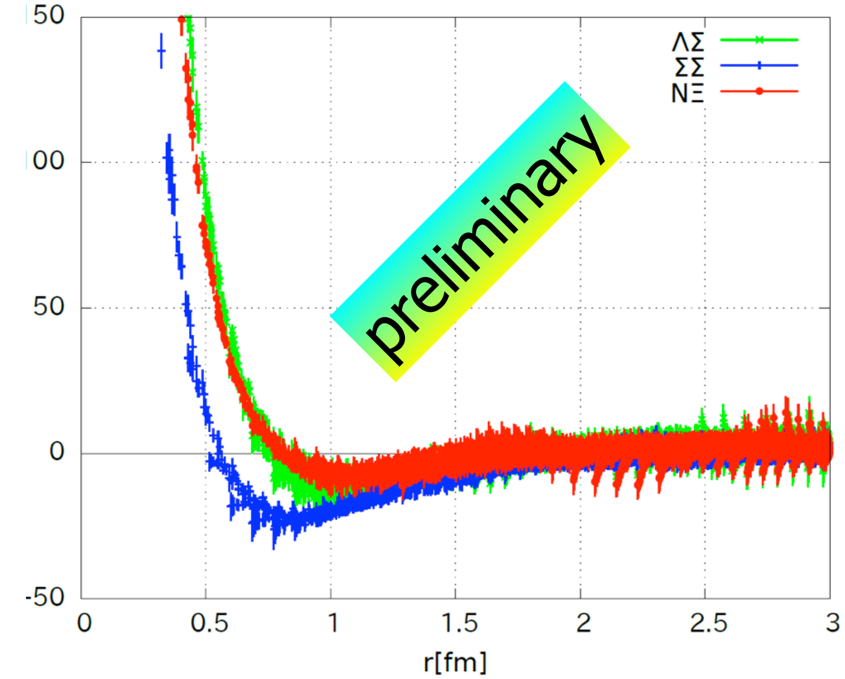
$N\Xi(I=0,^3\mathrm{S}_1)$

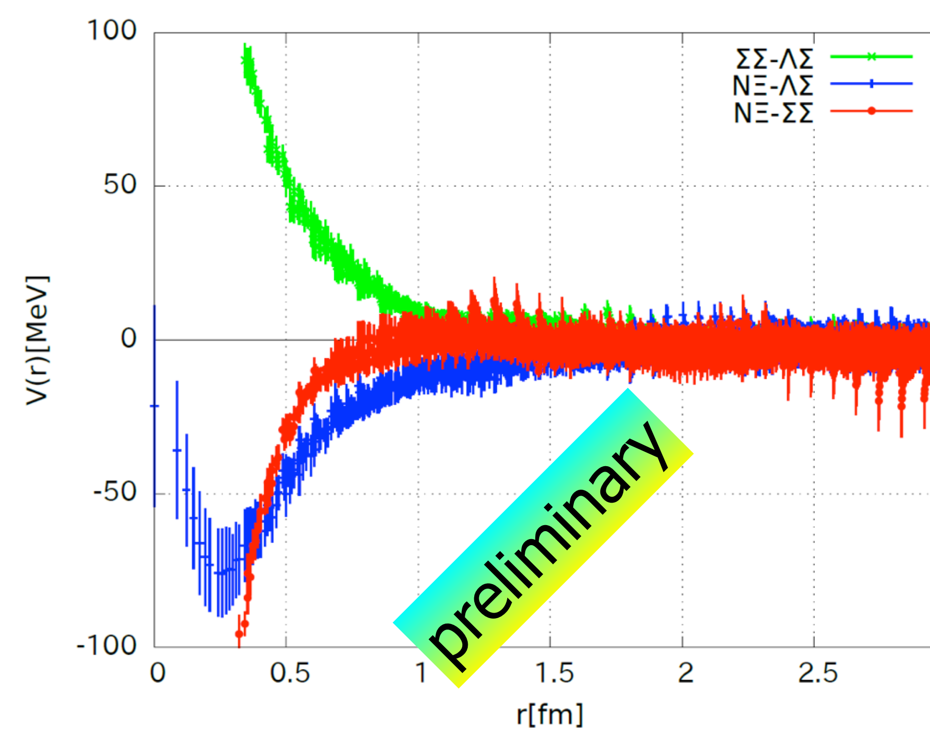


$N\Xi-\Lambda\Sigma(I=1,^1\mathrm{S}_0)$



$N\Xi-\Lambda\Sigma-\Sigma\Sigma(I=1,^3\mathrm{S}_1)$





Is the interaction net attractive ?

Stay tuned !

cf. Talk by Tamura on Wed.