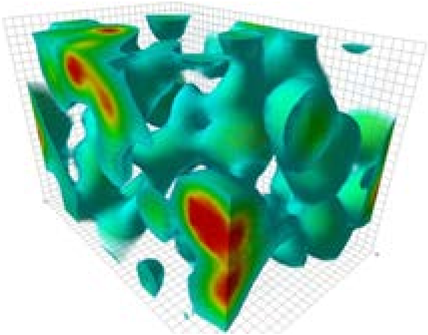
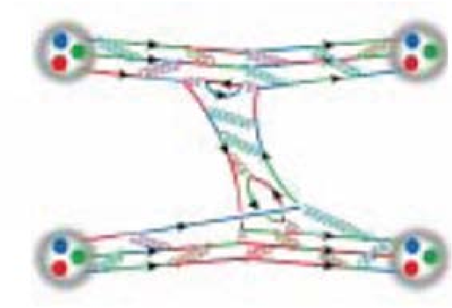
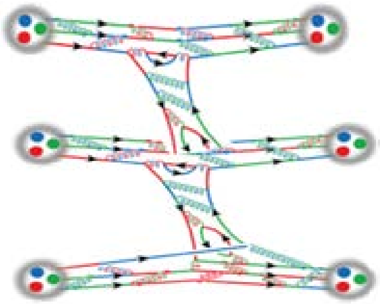
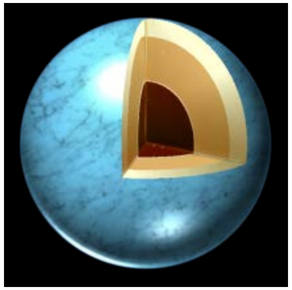
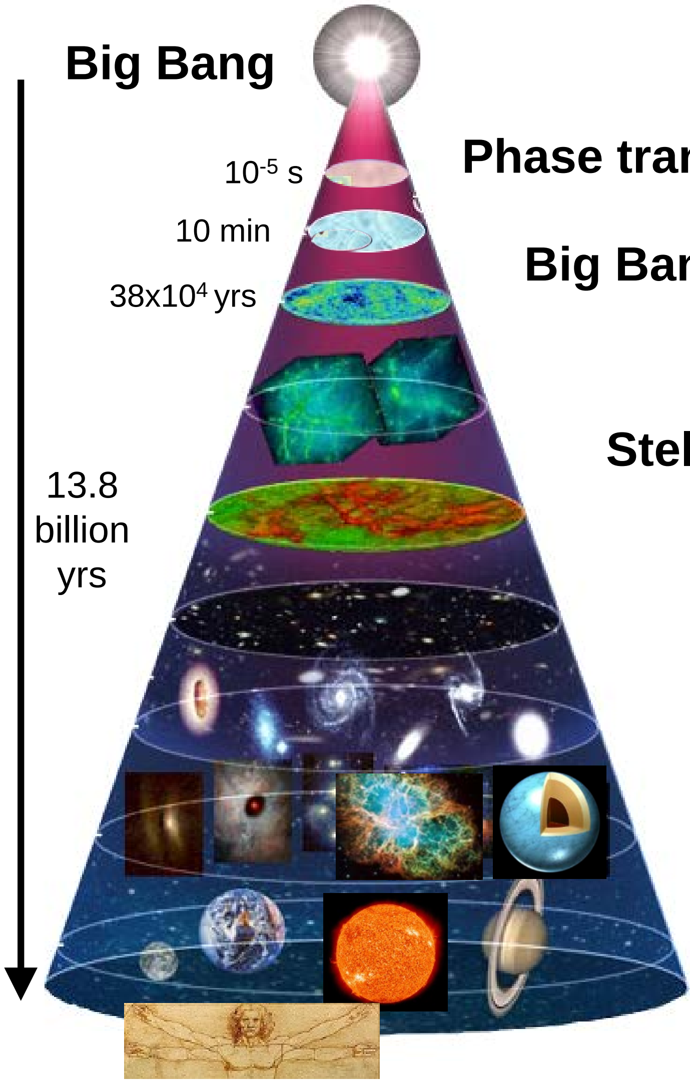
Two- and Three-Baryon Forces from Lattice QCD

Takumi Doi
(Nishina Center, RIKEN)

Where do we come from ? Where are we going ?



Phase transition: Birth of (visible) matter

Big Bang Nucleosynthesis for light elements
(H, ^{2}H (deuteron), ^{3}He, ^{4}He, ^{7}Li, …)

Stellar Nucleosynthesis for medium elements
(^{4}He, …, ^{12}C, ^{14}N, ^{16}O, …, ^{56}Fe)

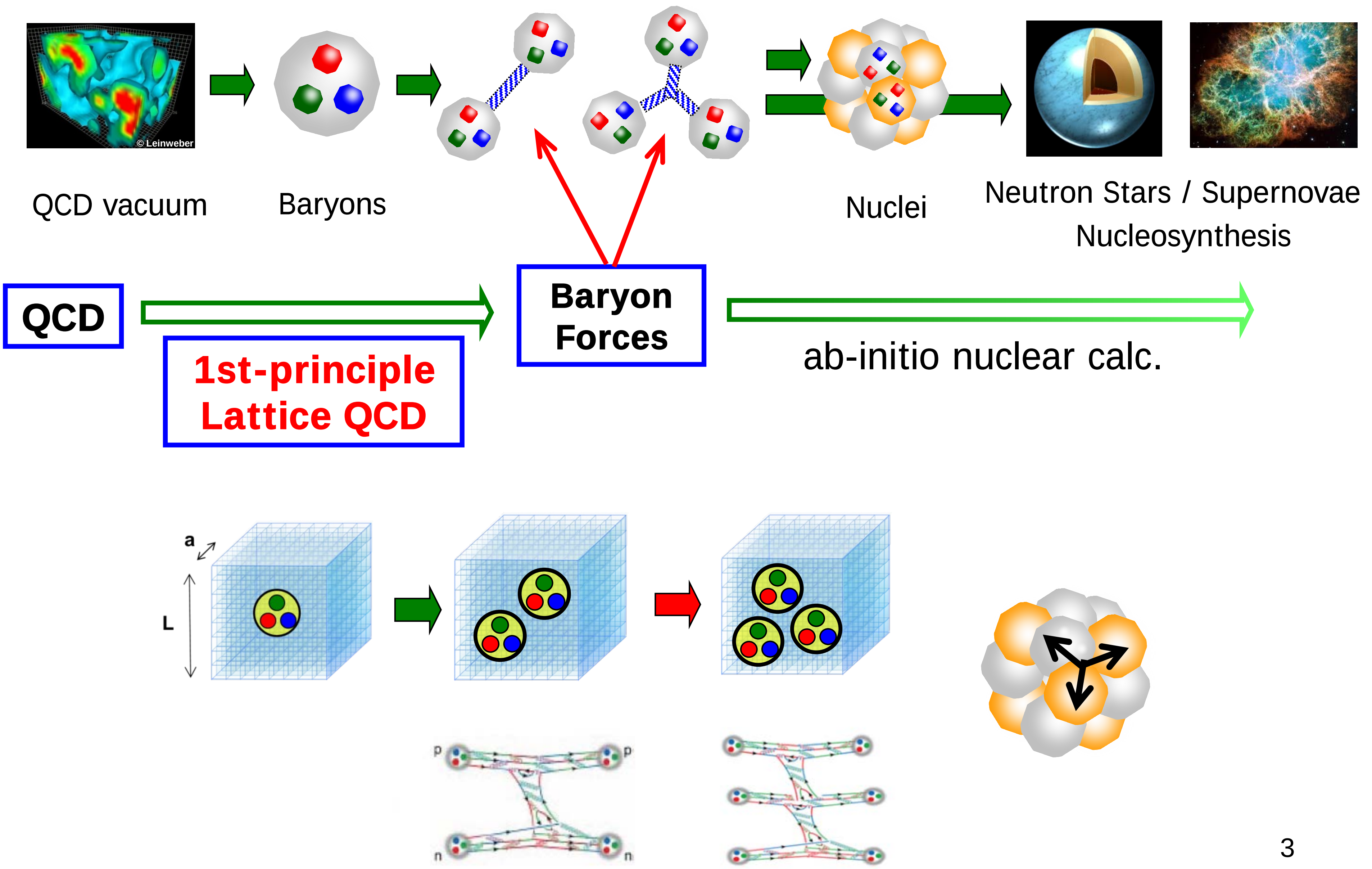
Fate of the matter:
Supernova ➔ Neutron Star vs. Black Hole

Nucleosynthesis for heavy elements
(Fe < : e.g., ^{197}Au, ^{238}U)

Nucleosynthesis by human

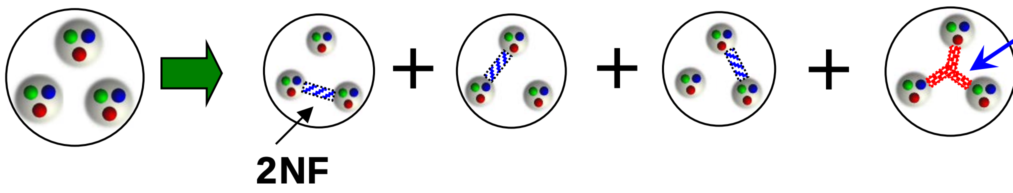
$^{278}_{113}$Nh

The Odyssey from Quarks to Universe



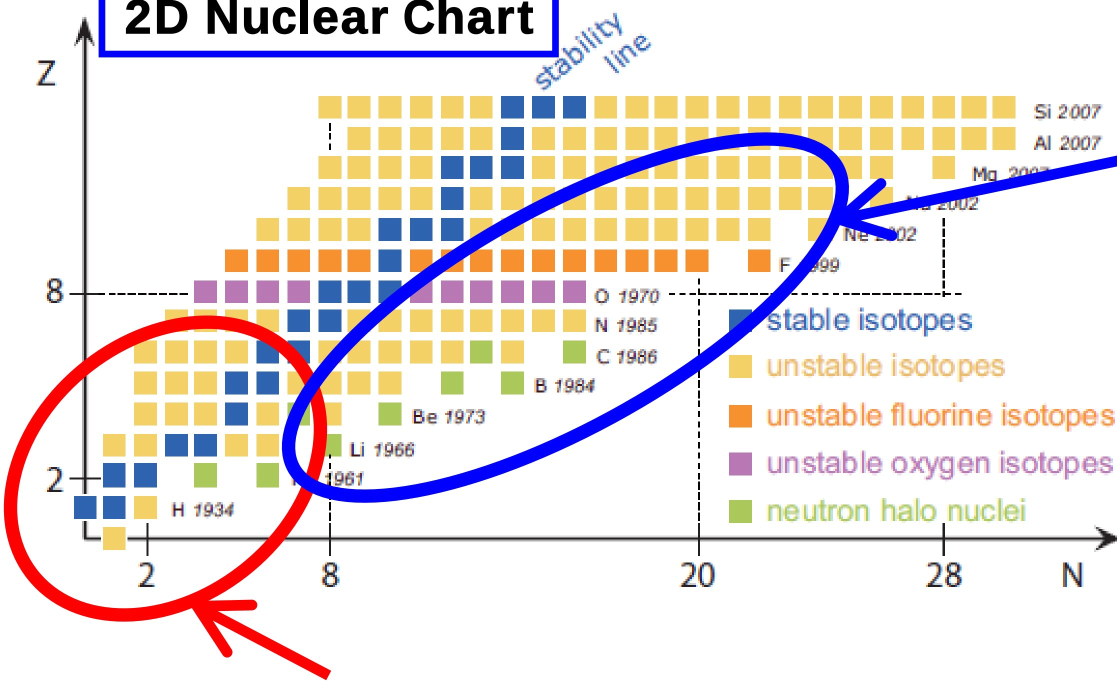
Three-nucleon forces (3NF)

What is 3NF ?



3NF: Forces which cannot be explained by pair-wise 2NF

2D Nuclear Chart



Precise ab initio calculations show 3NF is indispensable

Paradigm Shift in
Unstable Nuclei
(New Magic Numbers !)

🡐 **Important role of 3NF**

T.Otsuka et al., PRL105(2010)032501

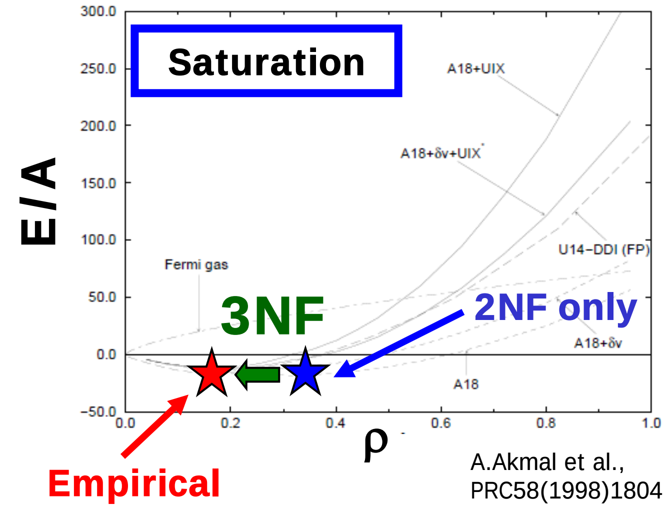
➔ **r-process Nucleosynthesis**

RIBF/FRIB

New Horizons w/ **Three-Nucleon/Baryon Forces**

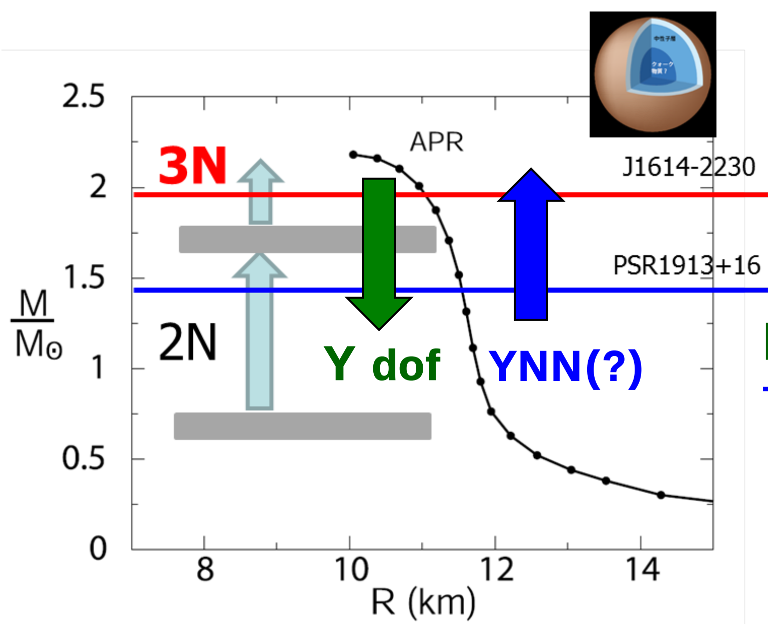
- 3NF is crucial to understand EoS of high density matter

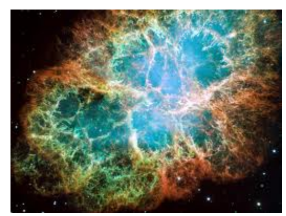
Short-range repulsive 3NF
is phenomenologically introduced

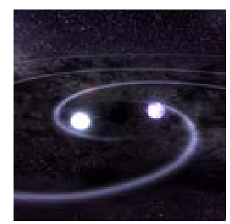


A.Akmal et al., PRC58(1998)1804

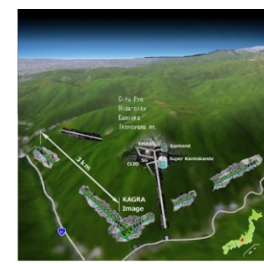
Neutron Star / Supernova / Nucleosynthesis







NS-NS merger



aLIGO/KAGRA

Hyperon Forces (YN/YY) and
Three-Baryon Forces (YNN etc.) important

Nishizaki et al. ('02), Takatsuka et al. ('08)

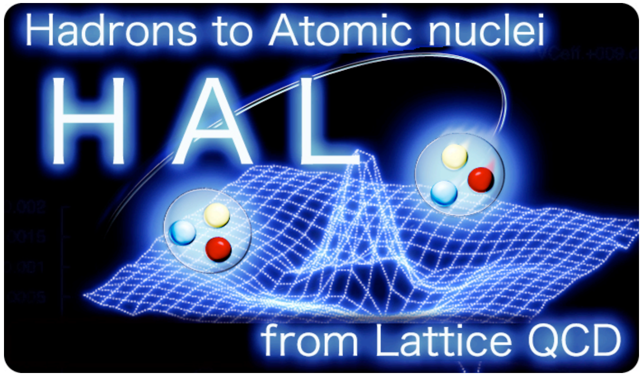
A. Ohnishi's talk (Fri.)

Quark matter ?

Masuda et al. ('12)

- **Outline**
  - ~~Introduction~~
  - Theoretical framework
  - Three-Nucleon Forces at heavy quark masses
  - Two-Baryon Forces at physical quark masses
  - Summary / Prospects

Hadrons to **A**tomic nuclei from **L**attice QCD
(**HAL** QCD Collaboration)



S. Aoki, D. Kawai, T. Miyamato, K. Sasaki (YITP)
T. Doi, T. Hatsuda, T. Iritani (RIKEN)
F. Etminan (Univ. of Birjand)
S. Gongyo (Univ. of Tours)
Y. Ikeda, N. Ishii, K. Murano (RCNP)
T. Inoue (Nihon Univ.)
H. Nemura (Univ. of Tsukuba)

Interactions on the Lattice

- Direct method (Luscher's method)
  - Phase shift & B.E. from temporal correlation in finite V

M.Luscher, CMP104(1986)177
CMP105(1986)153
NPB354(1991)531

➔ Pursued by Yamazaki et al. / NPL Coll. / CalLat Coll.

- HAL QCD method
  - "Potential" from spacial (& temporal) correlation
  - Phase shift & B.E. by solving Schrodinger eq in infinite V

Ishii-Aoki-Hatsuda, PRL99(2007)022001, PTP123(2010)89
HAL QCD Coll., PTEP2012(2012)01A105

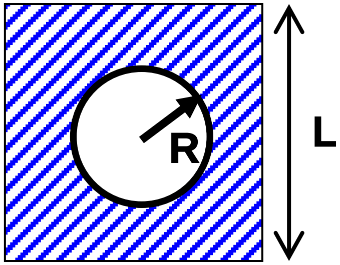
➔ Pursued by HAL QCD Coll.

“Potential” as a representation of S-matrix [HAL QCD method]

- Nambu-Bethe-Salpeter (NBS) wave function

$$\psi(\vec{r}) = \langle 0 |\ N(\vec{r}) N(\vec{0})\ | N(\vec{k}) N(-\vec{k}); in \rangle$$

$$(\nabla^2 + k^2)\psi(\vec{r}) = 0, \quad r > R$$



  - phase shift at asymptotic region

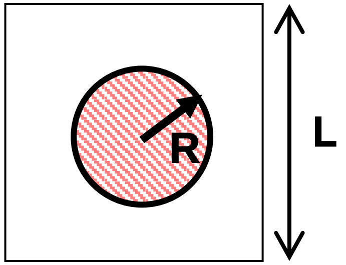
$$\psi(r) \simeq A \frac{\sin(kr - l\pi/2 + \delta(k))}{kr}$$

M.Luscher, NPB354(1991)531
C.-J.Lin et al., NPB619(2001)467
N.Ishizuka, PoS LAT2009 (2009) 119
CP-PACS Coll., PRD71(2005)094504

- Consider the wave function at “interacting region”

$$(\nabla^2 + k^2)\psi(\boldsymbol{r}) = m \int d\boldsymbol{r}' U(\boldsymbol{r}, \boldsymbol{r}')\psi(\boldsymbol{r}'), \quad r < R$$

  - U(r,r’): faithful to the phase shift by construction
    - U(r,r’): E-independent, while non-local in general
      - Non-locality ➔ derivative expansion



Aoki-Hatsuda-Ishii PTP123(2010)89

Extension to multi-particle systems (n>=3)

S.Aoki et al. (HAL Coll.), PRD88(2013)014036

- Unitarity of S-matrix

$$T_{[L]}(Q) = -\frac{2n^{3/2}}{mQ^{3n-5}} \boxed{e^{i\delta_{[L]}(Q)} \ \sin\delta_{[L]}(Q)}$$

c.f. R.B. Newton (1974) for n = 3

Similar formula to 2-body system
(w/ diagonalization matrix U which includes dynamics)

- NBS wave function

$$\psi_\alpha([\boldsymbol{x}]) = _{\text{in}}\langle 0|\phi([\boldsymbol{x}])|\alpha\rangle_{\text{in}} = _{\text{in}}\langle 0|N(\vec{x}_1)N(\vec{x}_2)\cdots N(\vec{x}_n)|\alpha\rangle_{\text{in}}$$

$$\psi_{[L],[K]}(R,Q_A) \propto \sum_{[N]} U_{[L][N]}(Q_A) \boxed{e^{i\delta_{[N]}(Q_A)} \frac{\sin(Q_A R - \Delta_L + \delta_{[N]}(Q_A))}{(Q_A R)^{(D-1)/2}}} U^\dagger_{[N][K]}(Q_A)$$

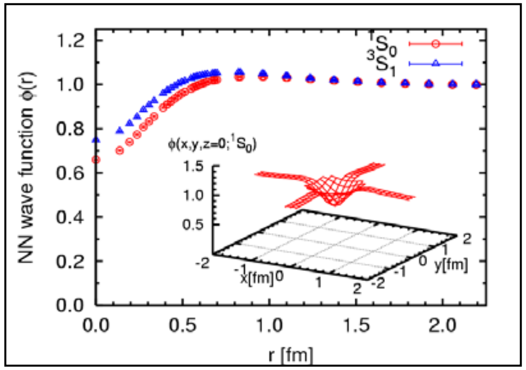
Similar asymptotic behavior to 2-body system (non-rela approx.)

c.f. Finite V spectrum, n=3 only, relativistic: Hansen-Sharpe ('14, '15, '16)

HAL QCD method

Lattice QCD

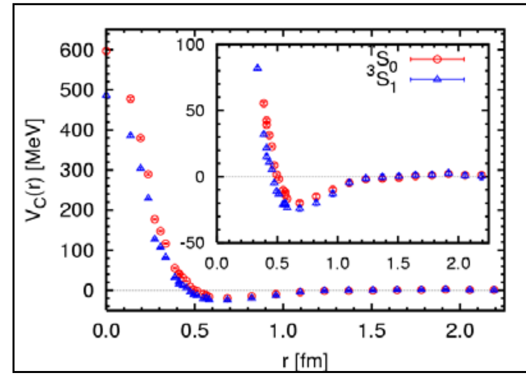
NBS wave func.



$$\psi_{NBS}(\vec{r}) = \langle 0|N(\vec{r})N(\vec{0})|N(\vec{k})N(-\vec{k}), in\rangle$$
$$\simeq e^{i\delta_l(k)} \sin(kr - l\pi/2 + \delta_l(k))/(kr)$$

(at asymptotic region)

Lat Baryon Force



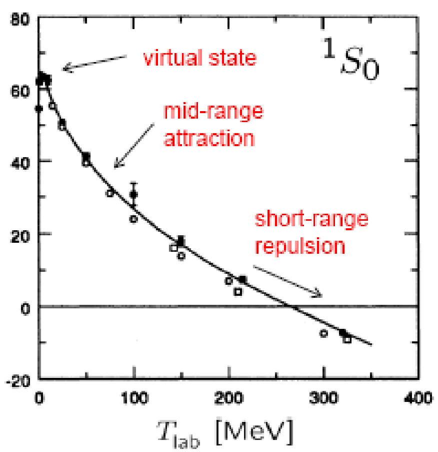
$$\left(k^2/m_N - H_0\right)\psi(\vec{r}) = \int d\vec{r}\,' U(\vec{r},\vec{r}\,')\psi(\vec{r}\,')$$

Lat potential is faithful to phase shift by construction

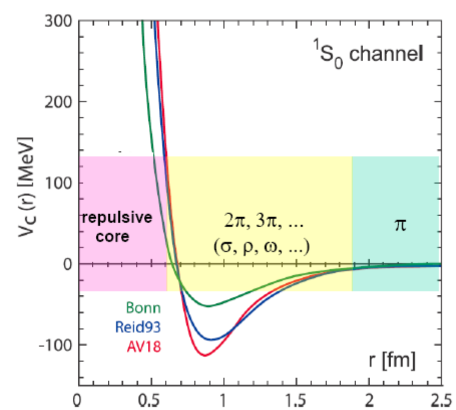
Analog to …

Scattering Exp.

Phase shifts



Phen. Potential



- **Outline**
  - – Introduction
  - – Theoretical framework
    - Challenges for multi-body systems on the lattice
  - – Three-Nucleon Forces at heavy quark masses
  - – Two-Baryon Forces at physical quark masses
  - – Summary / Prospects

Signal/Noise Issue

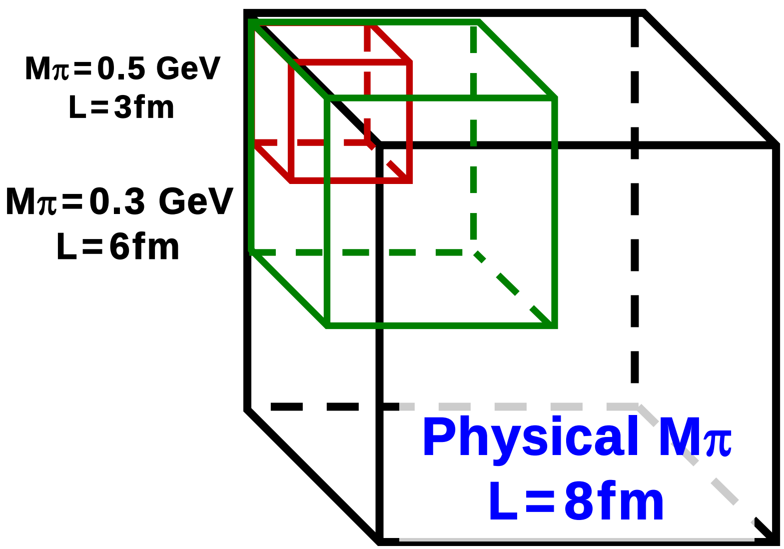
- Challenge in traditional LQCD method : G.S. saturation

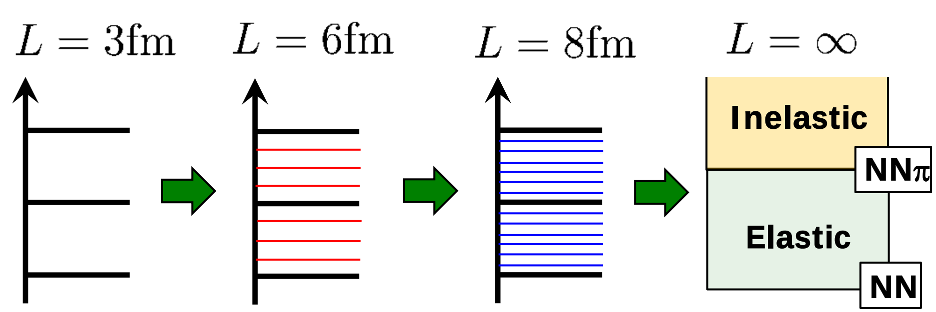
$$G(r,t) = \langle 0|\mathcal{O}(r,t)\overline{\mathcal{O}}(0)|0|\rangle = \sum_n \alpha_n \psi_n(r) e^{-E_n t} \xrightarrow[t\to\infty]{} \alpha_0 \psi_0(r) e^{-E_0 t}$$

$$S/N \sim \exp[-\mathbf{A} \times (\mathbf{m_N} - \mathbf{3/2m_\pi}) \times \mathbf{t}]$$

Parisi, Lepage(1989)

$$1/t \ll E_1 - E_0 \simeq \frac{1}{m_N}\frac{(2\pi)^2}{L^2}$$





(simple)

$S/N \propto$ 10^{-4} 10^{-13} 10^{-25}

System w/o Gap

New Challenge for multi-body systems

(For both of Direct method / (old) HAL method)

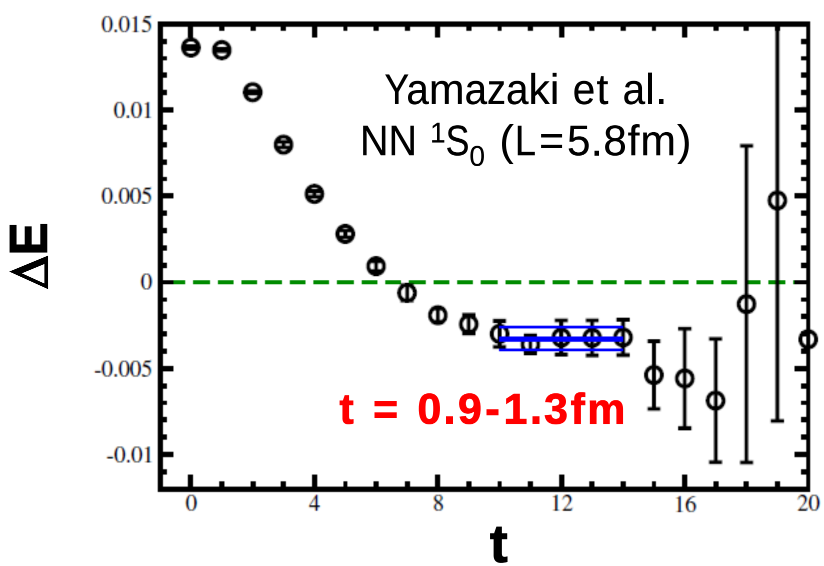
How serious is this issue ?
~ manifestation in the Direct method ~

T. Iritani et al. arXiv:1607.06371; in prep.

- LQCD @ m(pi)=0.51GeV, L <= 5.8 fm
  - Excitation energy E_1-E_0 >= 35 MeV ➔ G.S. saturation requires t ~ 10fm
- People tried to bypass the issue by, e.g., tuning the operators
  - ➔ t ~ 1 fm by observing "plateau-like" structure as a sign of G.S. saturation
  - Bound NN states are claimed at heavy quark masses

T. Yamazaki et al. PRD86(2012)074514

"Sanity Check"



ΔE = E_{BB}-$2m_B$ (➔ phase shift δ_E)

$$\Delta E(t) = \ln\left[\frac{R(t)}{R(t+1)}\right] \xrightarrow[t\to\infty]{} \Delta E\Big|_{\mathrm{G.S.}}$$

$$R(t) \sim e^{-\Delta E t}$$

"plateau-like" structure

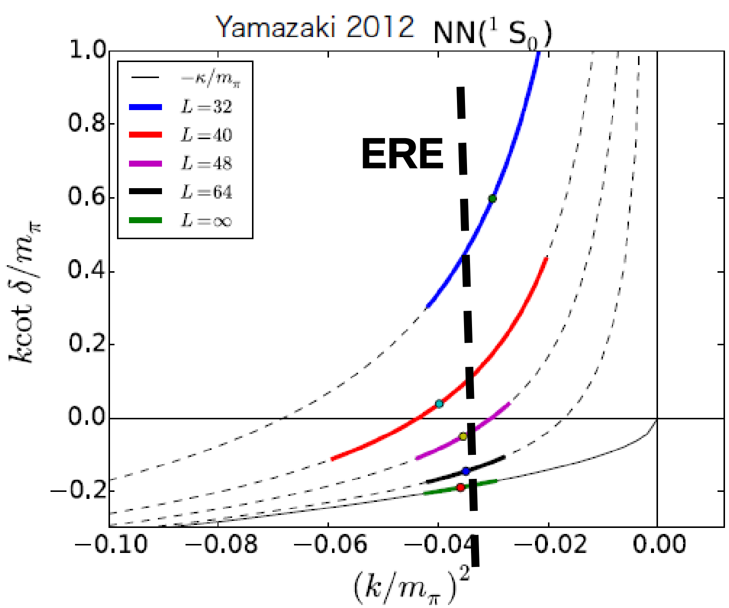
How serious is this issue ?
~ manifestation in the Direct method ~

T. Iritani et al. arXiv:1607.06371; in prep.

- LQCD @ m(pi)=0.51GeV, L <= 5.8 fm
  - Excitation energy E_1-E_0 >= 35 MeV ➔ G.S. saturation requires t ~ 10fm
- People tried to bypass the issue by, e.g., tuning the operators
  - ➔ t ~ 1 fm by observing "plateau-like" structure as a sign of G.S. saturation
  - Bound NN states are claimed at heavy quark masses

T. Yamazaki et al. PRD86(2012)074514

"Sanity Check"



Effective Range Expansion (ERE)
w/ Luscher's finite V formula

$$k\cot\delta = \frac{1}{a} + \frac{1}{2} r k^2 + \cdots$$

⟷ 1/a ~= −∞, r ~=−∞

(very unrealistic !)

Lesson: **We cannot bypass this issue**
We have to confront it !

Our solution

Time-dependent HAL method

N.Ishii et al. (HAL QCD Coll.) PLB712(2012)437

E-indep of potential U(r,r') ➔ *(excited) scatt states share the same U(r,r')*

They are ***not contaminations, but signals***

Original (t-indep) HAL method

$$G_{NN}(\vec{r},t) = \langle 0|N(\vec{r},t)N(\vec{0},t)\overline{\mathcal{J}_{\rm src}(t_0)}|0\rangle$$

$$R(\boldsymbol{r},t) \equiv G_{NN}(\boldsymbol{r},t)/G_N(t)^2 = \sum_i A_{W_i}\psi_{W_i}(\boldsymbol{r})e^{-(W_i-2m)t}$$

🡐 Many states contribute

$$\int d\boldsymbol{r}' U(\boldsymbol{r},\boldsymbol{r}')\psi_{W_0}(\boldsymbol{r}') = (E_{W_0} - H_0)\psi_{W_0}(\boldsymbol{r})$$

$$\int d\boldsymbol{r}' U(\boldsymbol{r},\boldsymbol{r}')\psi_{W_1}(\boldsymbol{r}') = (E_{W_1} - H_0)\psi_{W_1}(\boldsymbol{r})$$

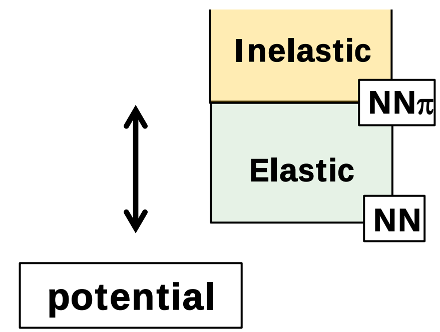
. . .

New t-dep HAL method

All equations can be combined as

$$\int d\boldsymbol{r}' U(\boldsymbol{r},\boldsymbol{r}')R(\boldsymbol{r}',t) = \left(-\frac{\partial}{\partial t} + \frac{1}{4m}\frac{\partial^2}{\partial t^2} - H_0\right)R(\boldsymbol{r},t)$$

System w/ Gap



~~G.S. saturation~~ ➔ "Elastic state" saturation

[Exponential Improvement]

Reliability Test for LQCD methods

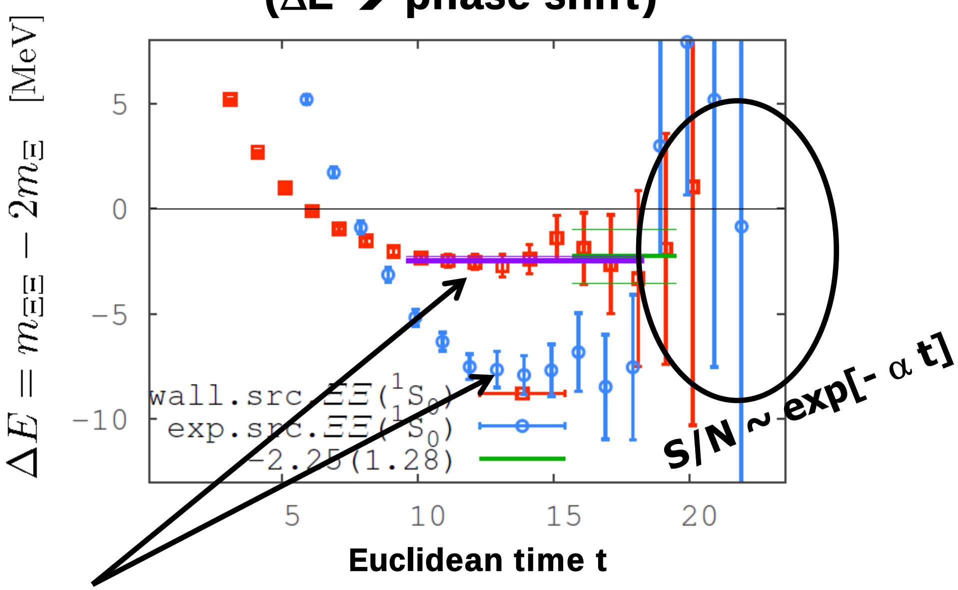
- High-stat study for BB-system (@m(pi)=0.5GeV)
  - Benchmark w/ two LQCD setup (wall & smeared src)

T. Iritani et al. (HAL Coll.)
arXiv:1607.06371

← **Physical outputs should NOT depend on these setup**

Direct method (traditional)

(ΔE → phase shift)

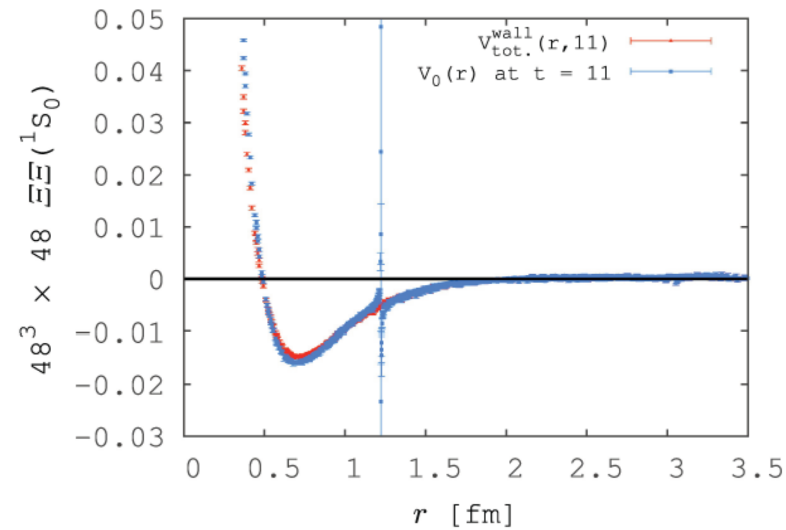


Inconsistent "signal" (red (wall) vs blue (smeared))
→ cannot judge which (or neither) is reliable

FAILED

t-dep HAL method (new)

(V(r) → phase shift)

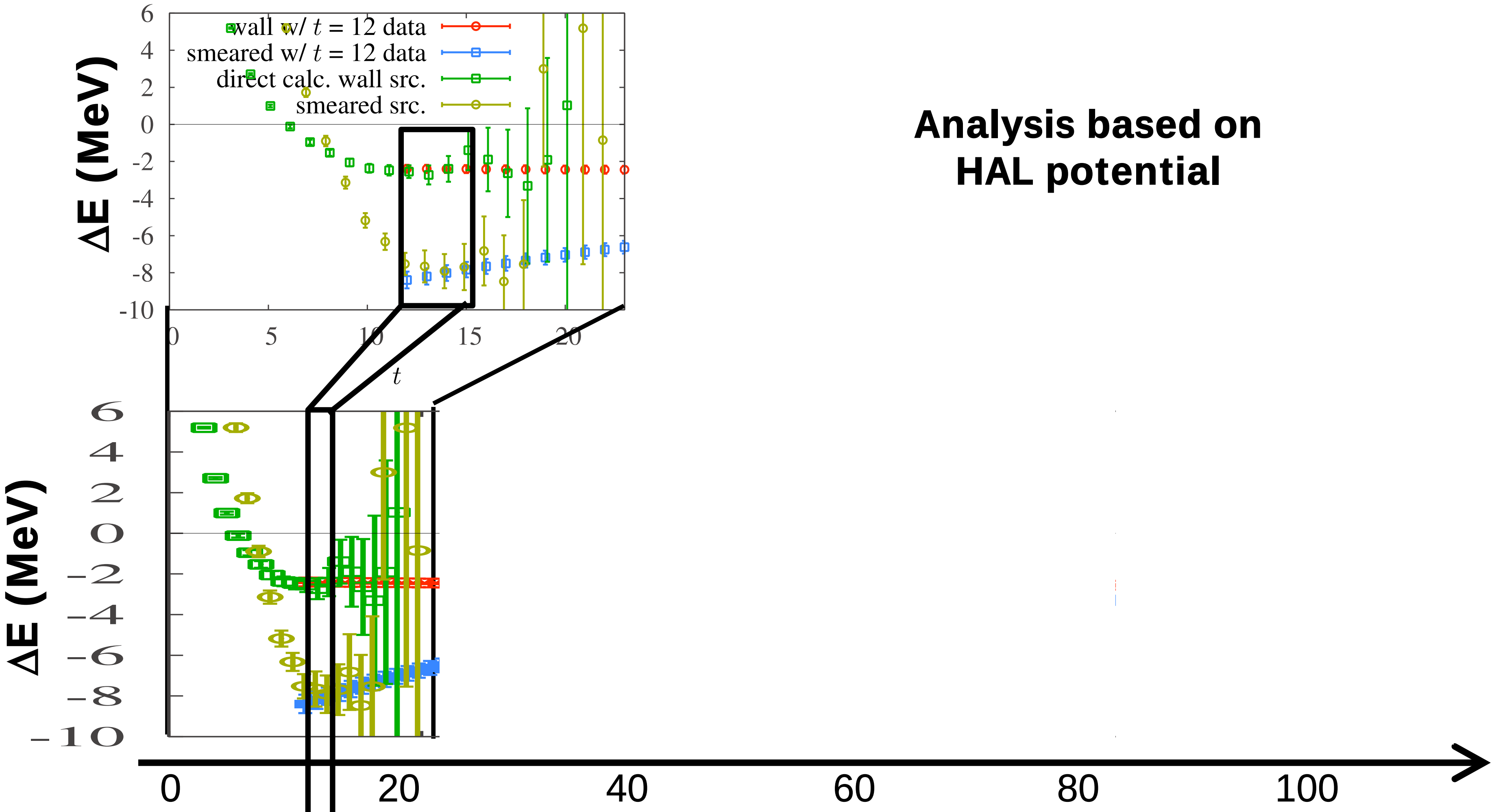


V^{eff}(r) from wall &
V^{LO}(r) from wall+smeared
are consistent

PASSED

The origin of "fake plateaux" in Direct method

➔ Excited State Contaminations



Analysis based on HAL potential

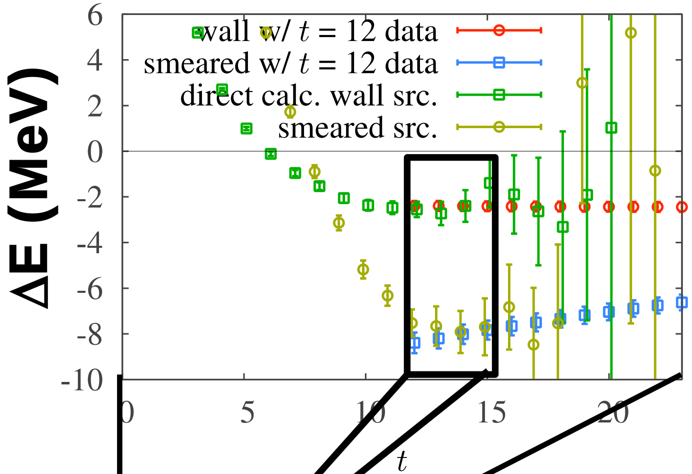
"fake plateaux" at t ~ 1fm

"real plateau" at t ~ 10fm

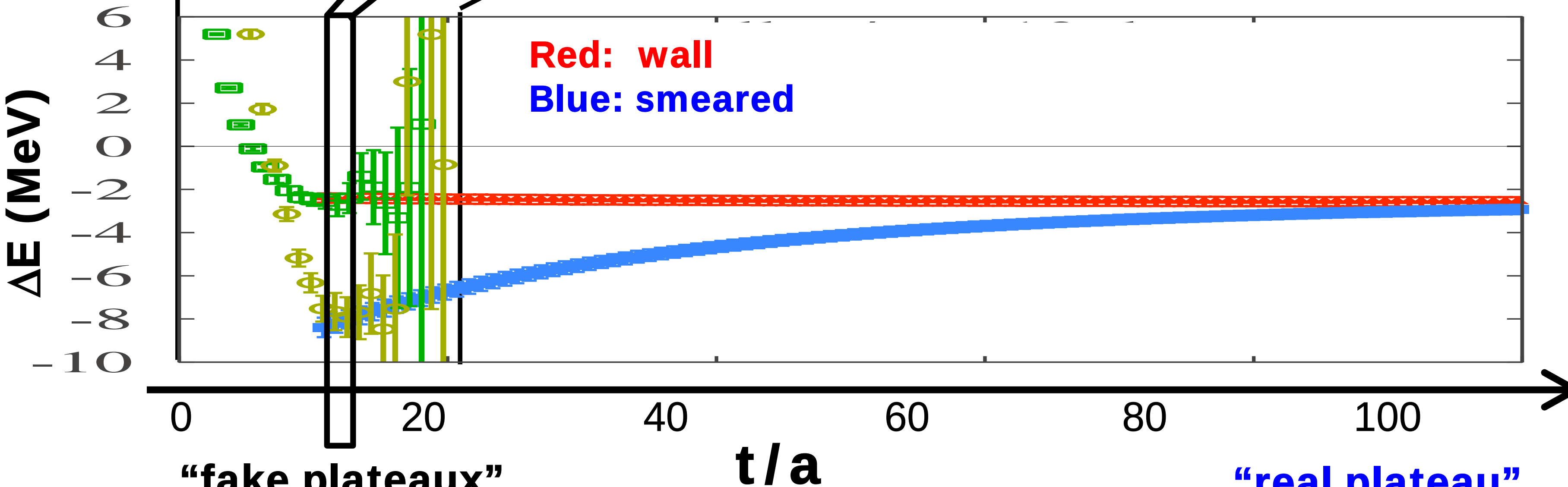
HAL method is crucial !

The origin of “fake plateaux” in Direct method

➔ **Excited State Contaminations**



Analysis based on HAL potential



“fake plateaux” at t ~ 1fm

“real plateau” at t ~ 10fm

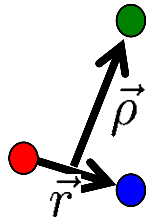
HAL method is crucial !

- **Outline**
  - Introduction
  - Theoretical framework
  - Three-Nucleon Forces at heavy quark masses in HAL method
    - Identification of genuine Three-Nucleon Forces
    - Computational Cost Issue
    - Results
  - Two-Baryon Forces at physical quark masses
  - Summary / Prospects

3NF from NBS wave function [HAL QCD method]

- Nambu-Bethe-Salpeter (NBS) wave function

$$\psi(\vec{r},\vec{\rho}) = \langle 0|N(\vec{x}+\vec{r})N(\vec{x})N(\vec{x}+\vec{r}/2+\vec{\rho})|3N\rangle$$



- Obtain 3NF through

$$(E-H_0^r-H_0^\rho)\psi(\vec{r},\vec{\rho}) = \left[\sum_{i<j} V_{ij}(\vec{r}_{ij}) + V_{3NF}(\vec{r},\vec{\rho})\right]\psi(\vec{r},\vec{\rho})$$

by 2N calc

- NBS is obtained by 6pt. correlator

$$G(\vec{r},\vec{\rho},t-t_0) = \sum_{\vec{x}}\langle 0|N(\vec{x}+\vec{r},t)N(\vec{x},t)N(\vec{x}+\vec{r}/2+\vec{\rho},t)\overline{NNN}(t_0)|0\rangle$$

➔ **time-dependent HAL QCD method**

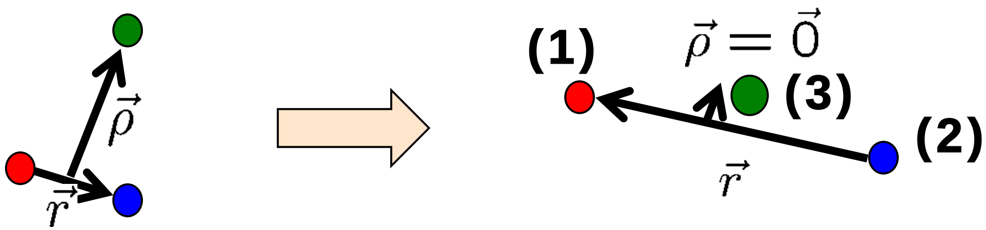
$$\left(-H_0 - \frac{\partial}{\partial t}\right)R(\boldsymbol{r},t) = V(\boldsymbol{r})R(\boldsymbol{r},t)$$

➔ Ground state saturation is NOT necessary !

3NF calculation in Lat QCD

- We fix the geometry of 3N (🡐 this is not an approximation)

- We study **linear setup**



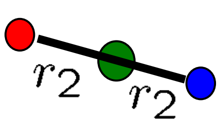
We consider Triton channel

$(\vec{r}_2 \equiv \vec{r}/2)$

  - → $L^{(1,2)\text{-pair}} = L^{\text{total}}$ = 0 or 2 only
  - → **Bases are only three**, labeled by **$^1S_0$, $^3S_1$, $^3D_1$ for (1,2)-pair**

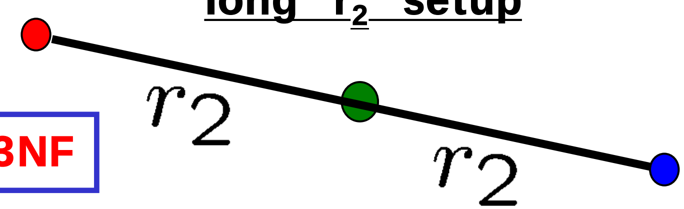
- **Linear setup** with various distance **"r_2"**

short "r_2" setup



Study r_2-dependence of 3NF

long "r_2" setup



(1) Identification of Genuine 3NF

- **Genuine 3NF** can be extracted from **3x3 coupled channel**
  - Both of parity-even 2NF and parity-odd potential required

$$\hat{H}_0 \begin{pmatrix} \psi(^1S_0) \\ \psi(^3S_1) \\ \psi(^3D_1) \end{pmatrix} + \Big(V \Big) \begin{pmatrix} \psi(^1S_0) \\ \psi(^3S_1) \\ \psi(^3D_1) \end{pmatrix} = E \begin{pmatrix} \psi(^1S_0) \\ \psi(^3S_1) \\ \psi(^3D_1) \end{pmatrix}$$

$$V = (V_{2N} + V_{3NF})$$

V_{2N}:

$V_C^{I,S=1,0}, V_C^{I,S=0,1}, V_T^{I,S=0,1}$: (P = even)

$V_C^{I,S=0,0}, V_C^{I,S=1,1}, V_T^{I,S=1,1}$: (P = odd)

V_{3NF}: **Target to be determined**

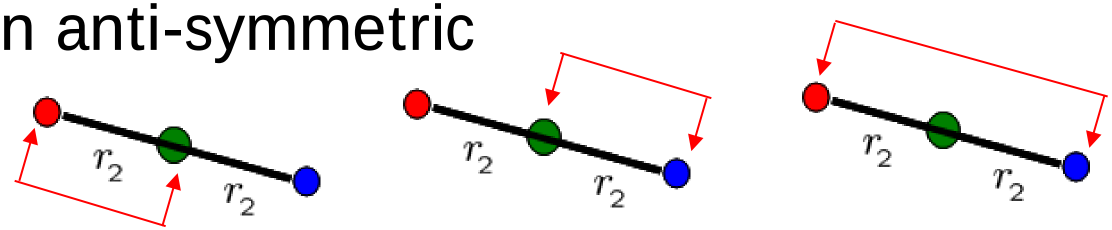
- S/N : parity-even 2NF > parity-odd 2NF in Lat QCD
  - ➔ **Desirable to extract 3NF w/ parity-even 2NF only**

c.f. K. Murano et al. (HAL Coll.) PLB735(2014)19

Solution using “symmetric” wave function

- We can construct the wave function in which <u>any 2N pair</u> is spin/isospin anti-symmetric

$$\psi_S =$$



- ➔ **<u>L=even for any 2N pair</u>** automatically guaranteed
- Bases are rotated as $|\psi_{^1S_0}\rangle, |\psi_{^3S_1}\rangle, |\psi_{^3D_1}\rangle \longrightarrow |\psi_S\rangle, |\psi_M\rangle, |\psi_{^3D_1}\rangle$

$$|\psi_S\rangle = 1/\sqrt{2}\left(-|\psi_{^1S_0}\rangle + |\psi_{^3S_1}\rangle\right) \qquad |\psi_M\rangle = 1/\sqrt{2}\left(+|\psi_{^1S_0}\rangle + |\psi_{^3S_1}\rangle\right)$$

$$\hat{H}_0 \begin{pmatrix}\psi_S\\ \psi_M\\ \psi_{^3D_1}\end{pmatrix} + \begin{pmatrix} & & \\ & V_{2N} & \\ & & \end{pmatrix}\begin{pmatrix}\psi_S\\ \psi_M\\ \psi_{^3D_1}\end{pmatrix} + \hat{V}_{3NF}\begin{pmatrix}\psi_S\\ \psi_M\\ \psi_{^3D_1}\end{pmatrix} = E\begin{pmatrix}\psi_S\\ \psi_M\\ \psi_{^3D_1}\end{pmatrix}$$

All pair P=even

No V(P=odd)

Solution using "symmetric" wave function

- We can construct the wave function in which <u>any 2N pair</u> is spin/isospin anti-symmetric
  - ➔ **<u>L=even for any 2N pair</u>** automatically guaranteed
- 3x3 coupled channel is reduced to
  - **<u>one channel with only 3NF unknown</u>**
  - two channels with $V_C^{I,S=0,0}$, $V_C^{I,S=1,1}$, $V_T^{I,S=1,1}$,(3NF) unknown

$$\left(H_0 \right) \begin{pmatrix} \psi_S \\ \psi_M \\ \psi_{^3D_1} \end{pmatrix} + \left(V_{2N} \right) \begin{pmatrix} \psi_S \\ \psi_M \\ \psi_{^3D_1} \end{pmatrix} + \left(V_{3NF} \right) \begin{pmatrix} \psi_S \\ \psi_M \\ \psi_{^3D_1} \end{pmatrix} = E \begin{pmatrix} \psi_S \\ \psi_M \\ \psi_{^3D_1} \end{pmatrix}$$

No V(P=odd)

Target to be determined

- ➔ Even without parity-odd V, we can determine one 3NF
  - This method works for any fixed 3D-geometry other than linear

(2) Computational Challenge

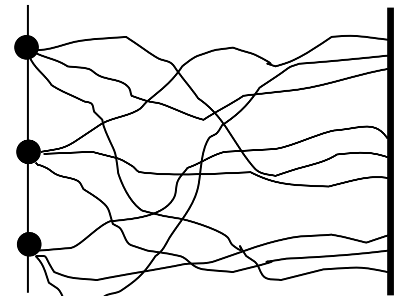
- **Enormous comput. cost for multi-baryon correlators**
  - Wick contraction (permutations)

    $\sim [\,(\frac{3}{2}A)!\,]^2$ (A: mass number)

  - color/spinor contractions

    $\sim 6^A \cdot 4^A \quad \text{or} \quad 6^A \cdot 2^A$

    See also T. Yamazaki et al., PRD81(2010)111504



  - **Unified Contraction Algorithm (UCA)** TD, M.Endres, CPC184(2013)117
    - A novel method which unifies two contractions

Permuted Sum

$$\Pi^{2N} \simeq \langle qqqqqq(t)\bar{q}(\xi_1')\bar{q}(\xi_2')\bar{q}(\xi_3')\bar{q}(\xi_4')\bar{q}(\xi_5')\bar{q}(\xi_6')(t_0)\rangle \times \text{Coeff}^{2N}(\xi_1',\cdots,\xi_6')$$

Sum over color/spinor unified list

Drastic Speedup

$\times 192$ for ^{3}H/^{3}He, $\times 20736$ for ^{4}He, $\times 10^{11}$ for ^{8}Be (x add'l. speedup)

See also subsequent works: Detmold et al., PRD87(2013)114512
Gunther et al., PRD87(2013)094513

Lattice simulation setup

- Nf=2 dynamical clover fermion + RG improved gauge action
  - a^{-1}=1.269GeV, a=0.1555fm (beta=1.95)
  - 16^3 X 32 lattice, L=2.5fm

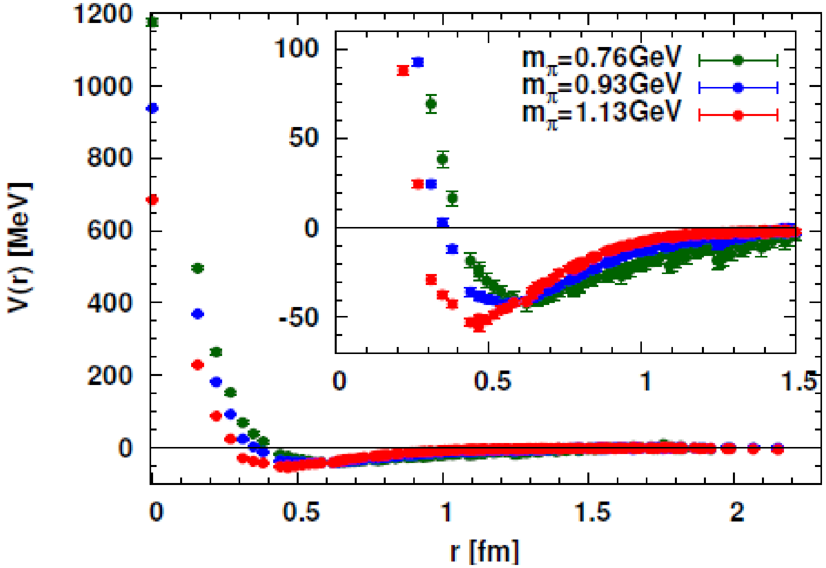
CP-PACS Coll. S. Aoki et al., Phys. Rev. D65 (2002) 054505

- Masses: (π, N, Δ) = (1.13, 2.15, 2.31) GeV
  - Kappa(ud)=0.13750
  - 599 configs x 32 measurements, t+1=[5,12]
- Masses: (π, N, Δ) = (0.925, 1.85, 2.02) GeV
  - Kappa(ud)=0.13900
  - 686 configs x 32 measurements, t+1=[5,12]
- Masses: (π, N, Δ) = (0.757, 1.61, 1.81) GeV
  - Kappa(ud)=0.14000
  - 686 configs x 32 measurements, t+1=[5,12]

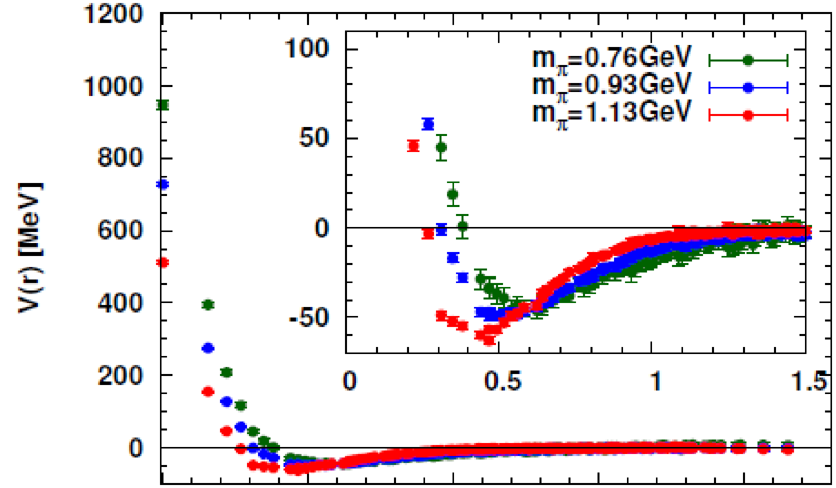
Wall source w/ Coulomb gauge

2NF on the lattice

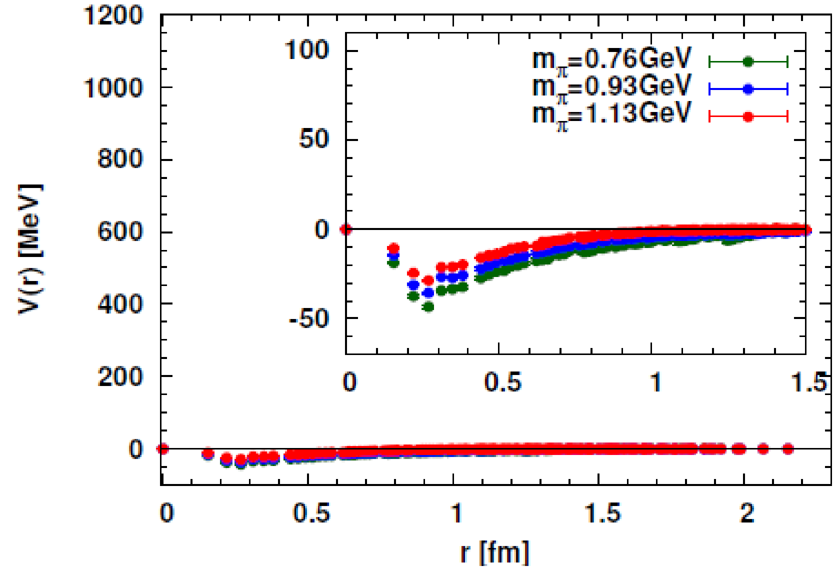
Central in $^1S_0$



$^3S_1$-$^3D_1$ channel



Central



Tensor

Lighter mass corresponds to…

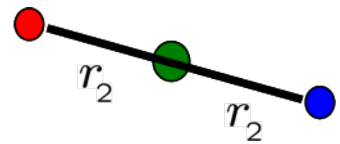
- **Longer interaction range**
- **Larger Repulsive Core**
- **Stronger Tensor Force**

(t-t_0=7.5)

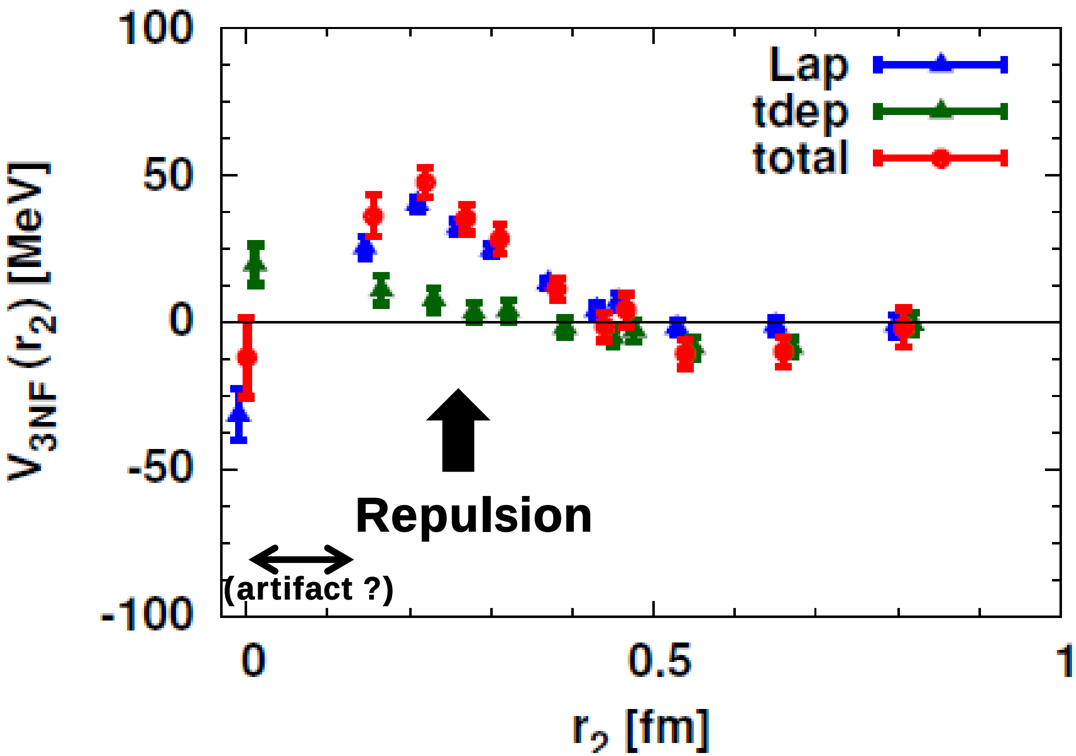
3N-forces (3NF) on the lattice

T.D. et al. (HAL QCD Coll.) PTP127(2012)723 + t-dep method updates etc.

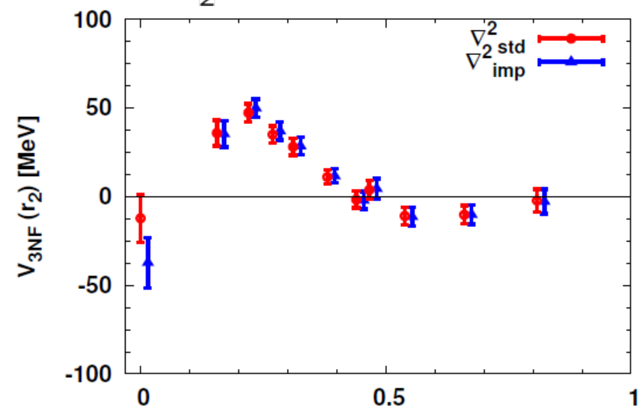
m_π=1.1GeV



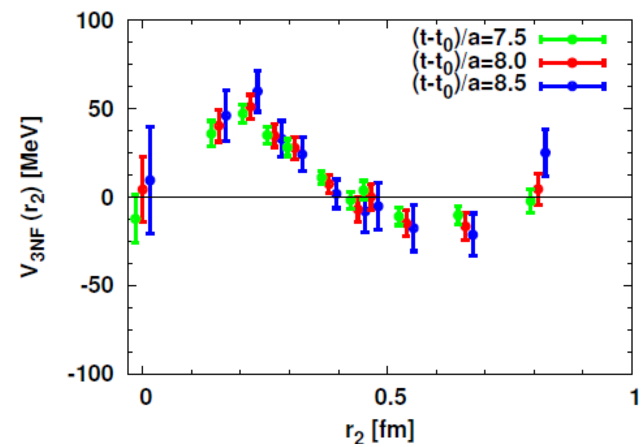
Triton channel



(breakup at $t-t_0$=7.5)



Discretization artifact ~ r=0



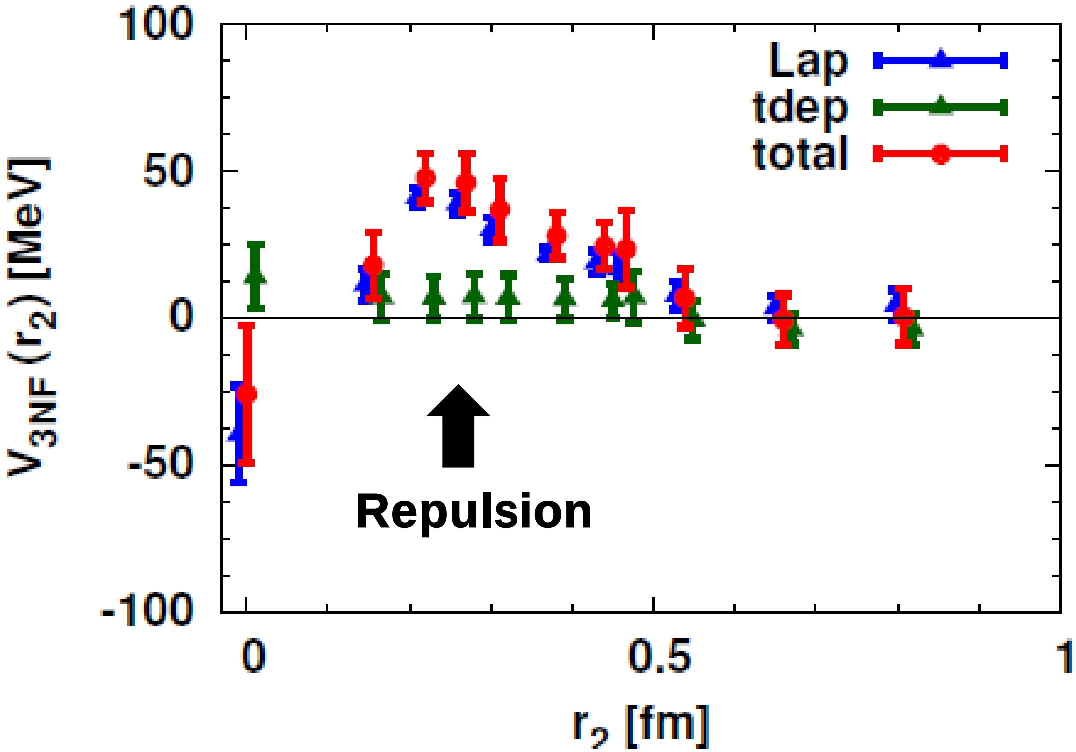
Sink time dependence is small

Lap tdep

$$\left(-H_0 - \frac{\partial}{\partial t}\right) R(\boldsymbol{r}, t) = V(r) R(\boldsymbol{r}, t)$$

3N-forces (3NF) on the lattice

$m_\pi = 0.93$GeV

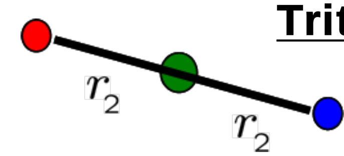


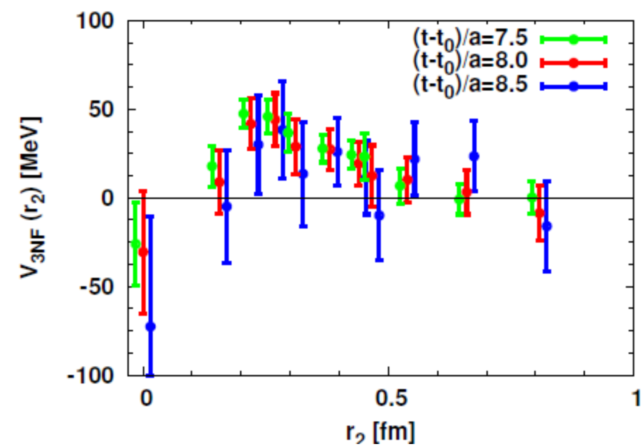
(breakup at $t-t_0=7.5$)

Lap tdep

$$\left(-H_0 - \frac{\partial}{\partial t}\right) R(\boldsymbol{r}, t) = V(r) R(\boldsymbol{r}, t)$$

Triton channel

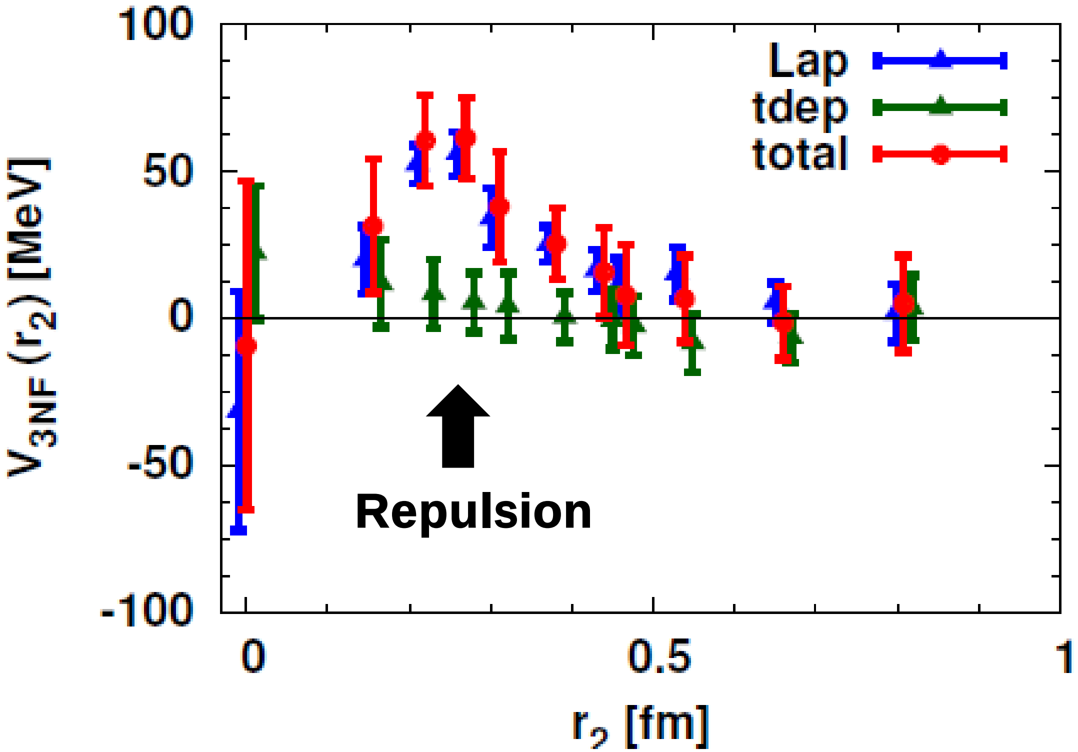




Sink time dependence is small

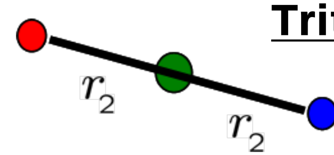
3N-forces (3NF) on the lattice

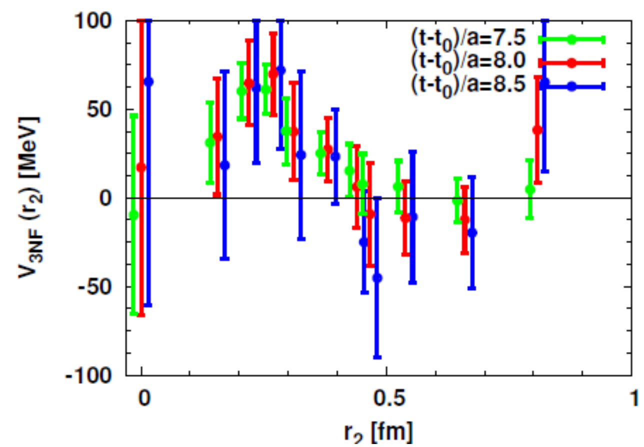
$m_\pi = 0.76$GeV



(breakup at $t\text{-}t_0=7.5$)

Triton channel





(Sink time dependence is small ?)

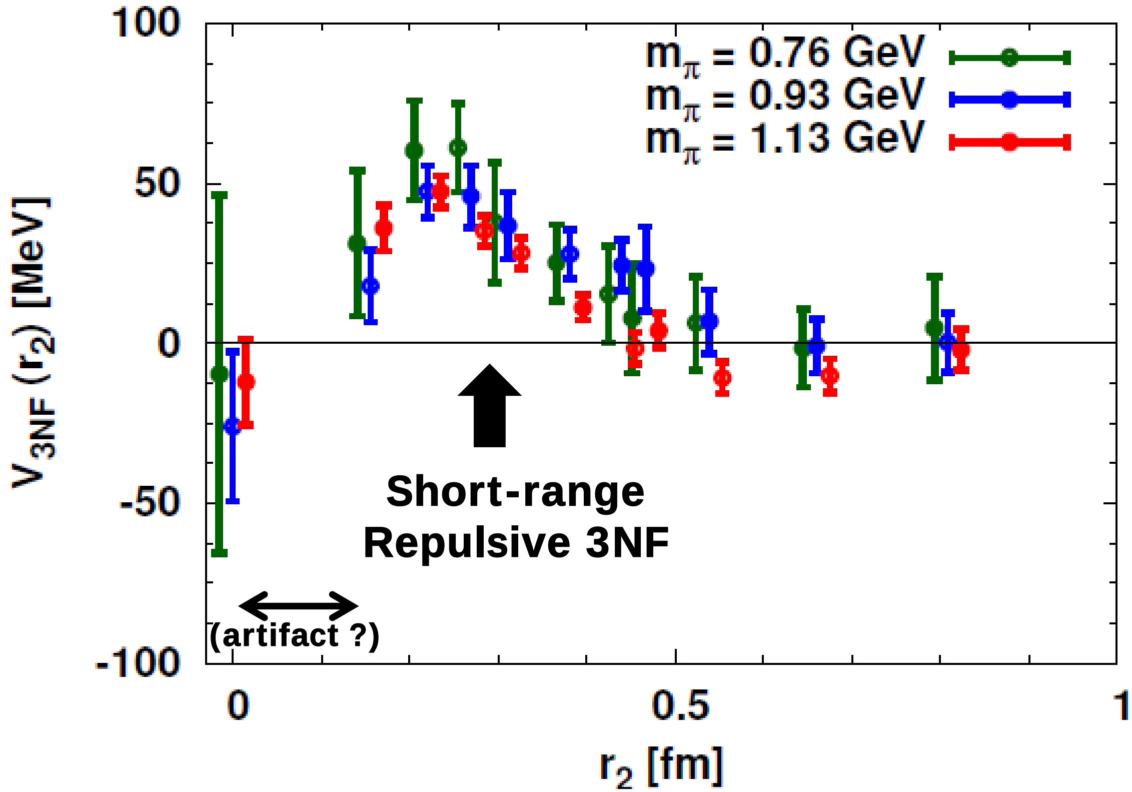
Lap tdep

$$\left(-H_0 - \frac{\partial}{\partial t}\right) R(\boldsymbol{r}, t) = V(r) R(\boldsymbol{r}, t)$$

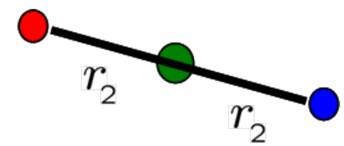
3N-forces (3NF) in mπ >=0.76 GeV

T.D. et al. (HAL QCD Coll.) PTP127(2012)723 + t-dep method updates etc.

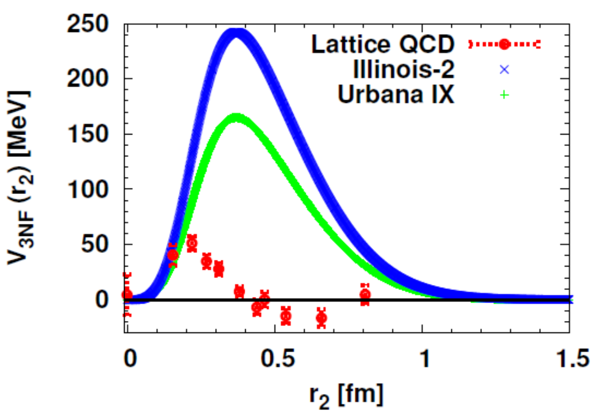
Nf=2 clover (CP-PACS), 1/a=1.27GeV,
L=2.5fm, m_π=0.76-1.1GeV, m_N=1.6-2.1GeV



Triton channel



Naïve comparison to phenomenological 3NF



(t-t0 = 7.5)

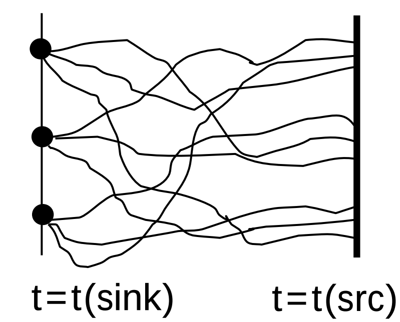
➔ Lighter quark mass important ?

Toward lighter mass: LQCD Setup

- **Nf = 2 + 1 gauge configs**
  - clover fermion + Iwasaki gauge action
  - m(pi) = 0.51 GeV, m(N) = 1.32 GeV
  - V=(64a)^4 = (5.8fm)^4, 1/a=2.19GeV, a=0.090fm

T. Yamazaki et al., PRD86(2012)074514

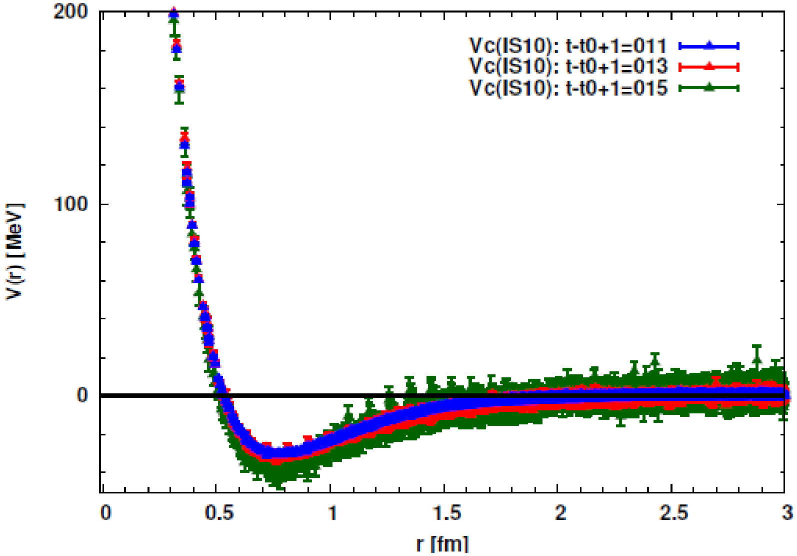
- **Measurement**
  - Triton channel, linear (& triangle) geometries
  - #stat = 327 confs x 4 rot x 32 wall src
  - **Unified Contraction Algorithm (UCA)**
  - **K-computer is used**
  - Special code tuning for K-computer
    ➔ **Kernel ~50%, total ~20$^+$ % efficiency achieved**
    - Kernel requires memory bandwidth B/F ~=4
    - K-computer: B/F(L2 cache)=2, B/F(memory)=0.5
    - ⇔ ~ perfect L2 cache hit is achieved



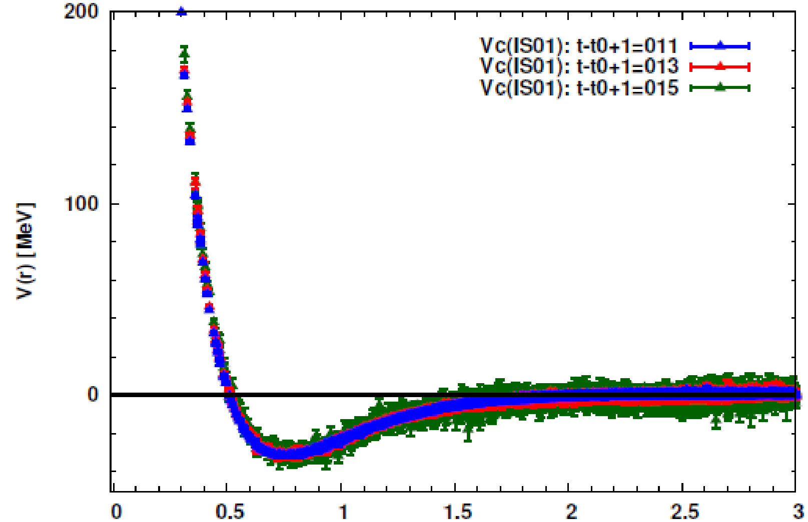
K-computer
[10PFlops]

2N-forces (2NF) @ m(pi)=0.51GeV

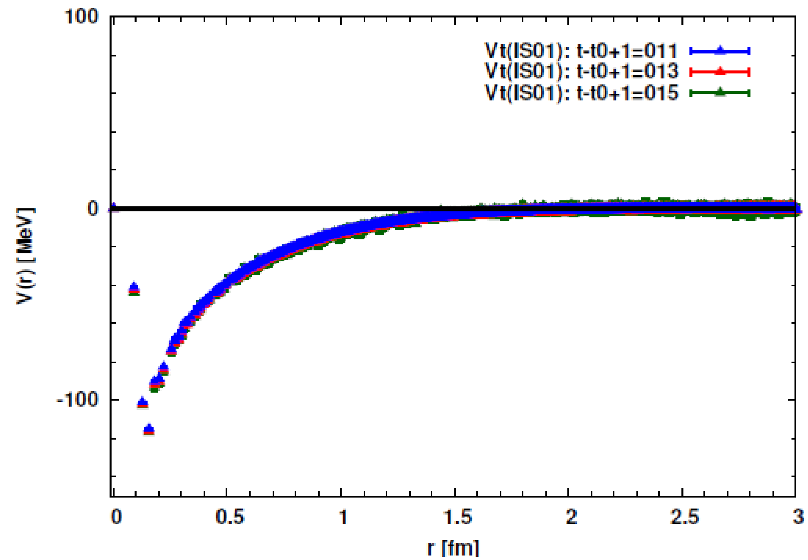
$^1S_0$ channel



$^3S_1$-$^3D_1$ channel



Central

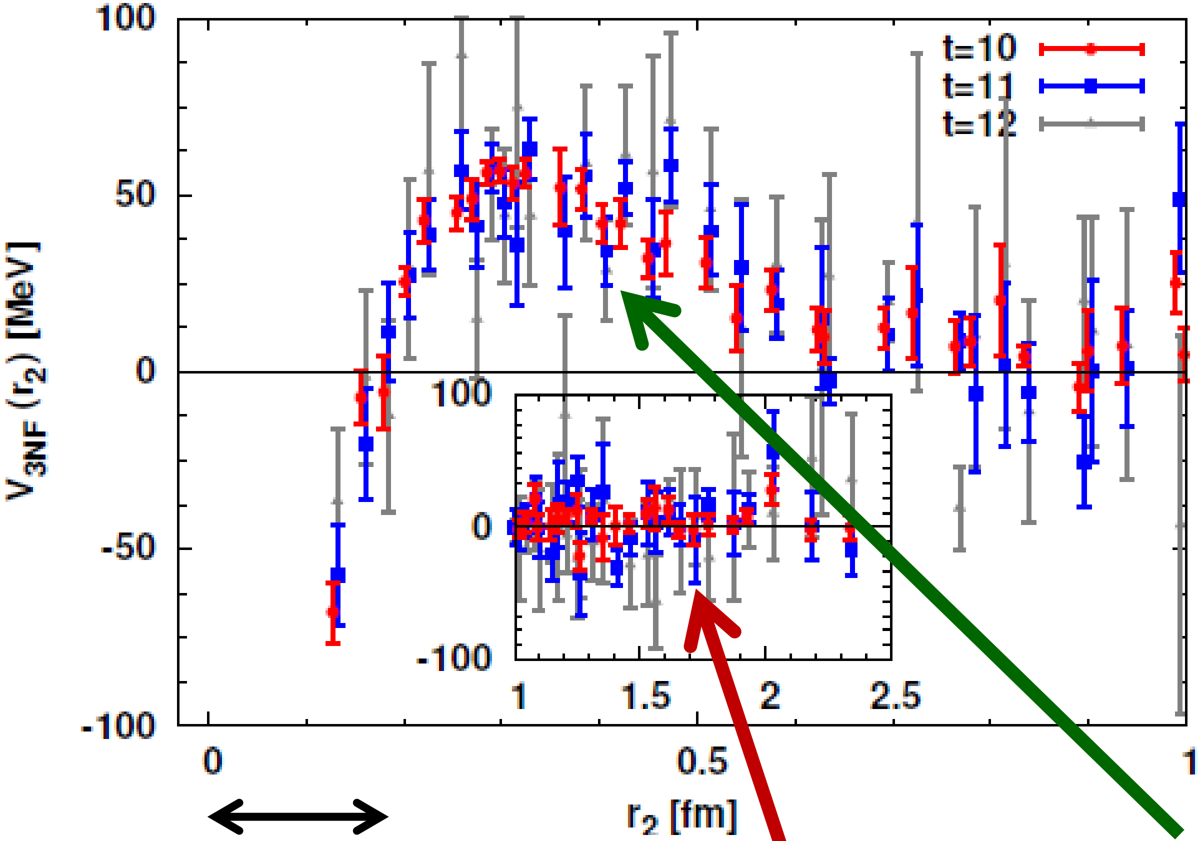


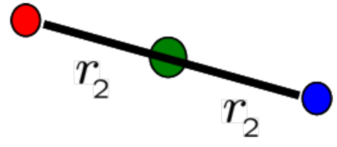
Tensor

t-dependence is small

for $t-t_0 >= 10$

3N-forces (3NF) @ m(pi)=0.51GeV





(very-short-range attractive 3NF ?)

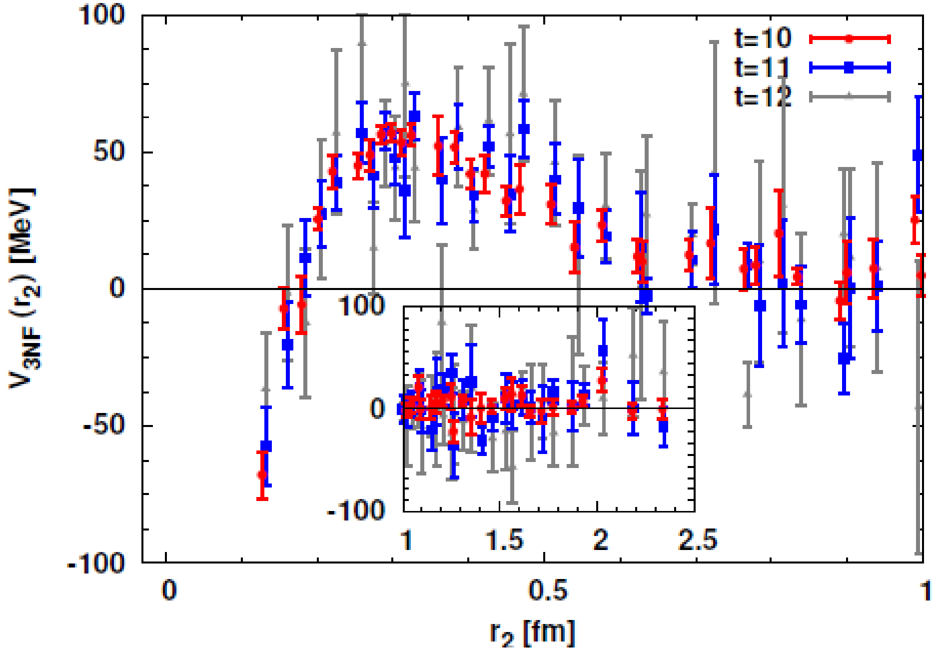
(Discretization Artifact ?)

No visible
mid-range 3NF

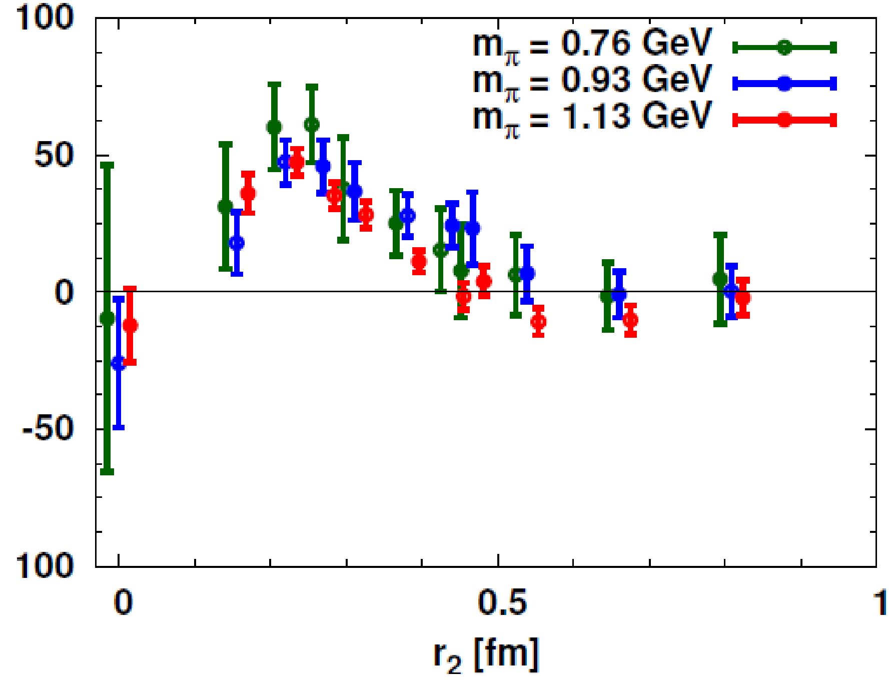
Short-range
Repulsive 3NF

Quark mass dependence of 3NF

Nf=2+1, mπ=0.51 GeV



Nf=2, mπ=0.76-1.1 GeV

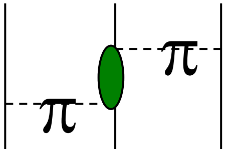


Magnitude of 3NF is similar for all masses

Range of 3NF tend to get longer (?) for m(pi)=0.5GeV

What is the origin of Lat 3NF ?

- 2πE-type 3NF (Fujita-Miyazawa) is unlikely
  - Strongly suppressed by mπ >= 0.5GeV



- It may be attributed to quark/gluon dynamics directly
  - Recall generalized 2BF in SU(3)f …
    - ➔ Pauli principle works well
  - c.f. Recent work in Quark Model
    - Pauli effect in norm kernel

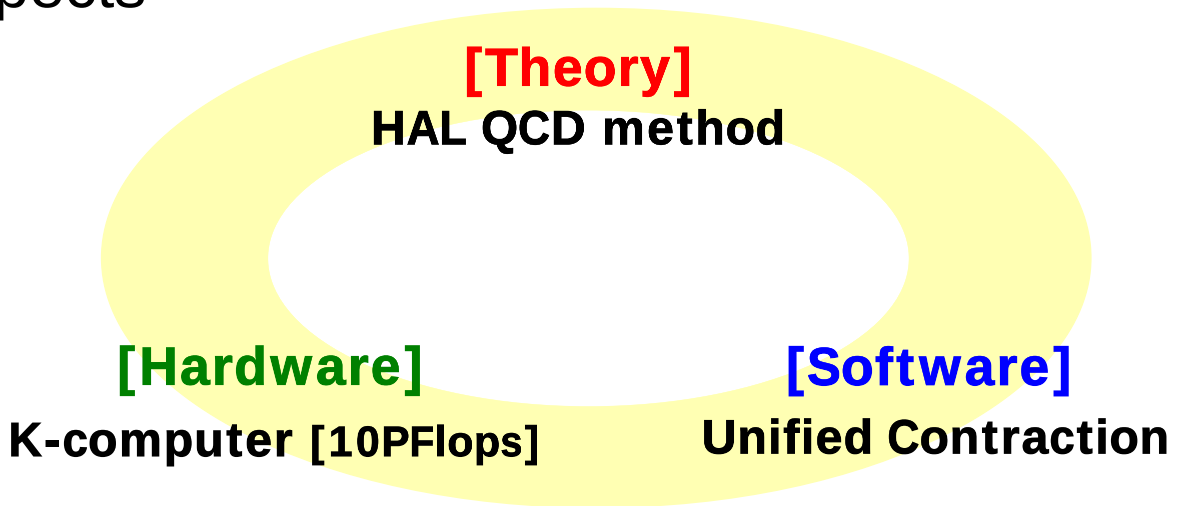
Nakamoto-Suzuki, arXiv:1606.07225

  - c.f. OPE (pert. QCD) predicts repulsive 3NF at short distance

S.Aoki et al., New J. Phys ('12) arXiv:1112.2053

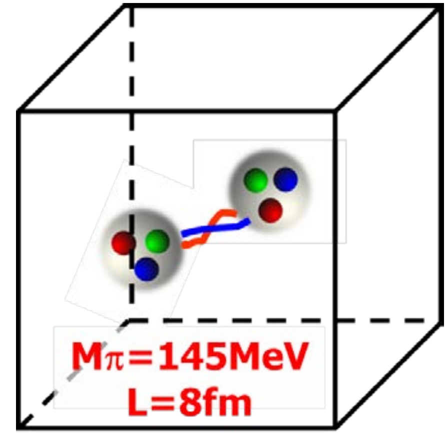
Repulsive 3NF from AdS/CFT (Hashimoto-Iizuka)

- **<u>Outline</u>**
  - Introduction
  - Theoretical framework
  - Three-Nucleon Forces at heavy quark masses
  - Two-Baryon Forces at (almost) physical quark masses
    - Hyperon Forces
    - Impact on dense matter
  - Summary / Prospects



Lattice QCD Setup

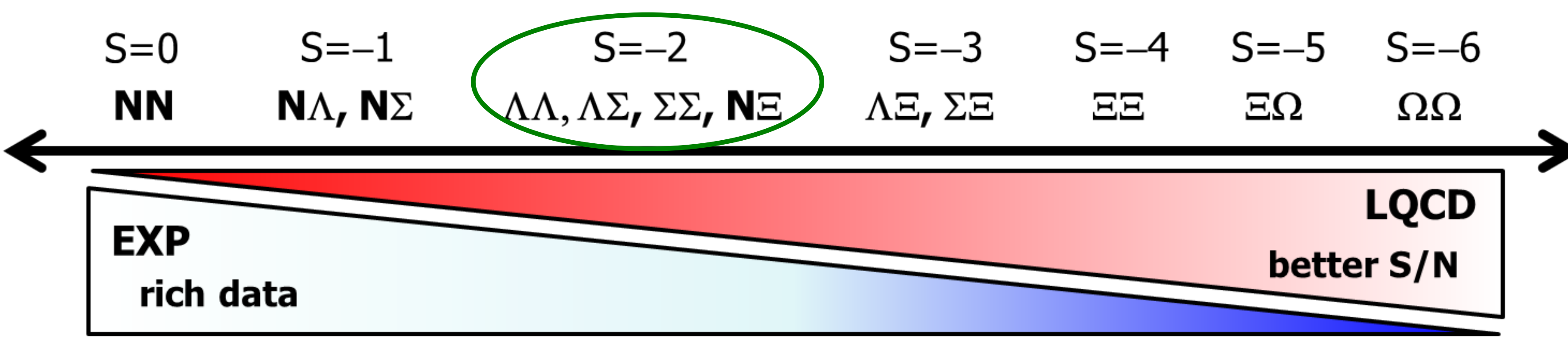
- **Nf = 2 + 1 gauge configs**
  - clover fermion + Iwasaki gauge w/ stout smearing
  - V=$(8.1\text{fm})^4$, a=0.085fm (1/a = 2.3 GeV)
  - m(pi) ~= 145 MeV, m(K) ~= 525 MeV
  - #traj ~= 2000 generated



K.I. Ishikawa et al., PoS LAT2015, 075

- **Measurement**
  - #stat = 414 confs x 4 rot x 28 wall src (calc in progress)
  - All of NN/YN/YY for central/tensor forces in P=(+) (S, D-waves)

Hyperon forces provide precious predictions



Baryon-baryon system with S=-2

Spin singlet states

Isospin	BB channels		
I=0	$\Lambda\Lambda$	$N\Xi$	$\Sigma\Sigma$
I=1	$N\Xi$	$\Lambda\Sigma$	—
I=2	$\Sigma\Sigma$	—	—

Spin triplet states

Isospin	BB channels		
I=0	$N\Xi$	—	—
I=1	$N\Xi$	$\Lambda\Sigma$	$\Sigma\Sigma$

Relations between BB channels and SU(3) irreducible representations

$$8 \times 8 = 27 + 8_s + 1 + \underline{10} + 10 + 8_A$$

J^P=0⁺, I=0

$$\begin{pmatrix} \Lambda\Lambda \\ N\Xi \\ \Sigma\Sigma \end{pmatrix} = \frac{1}{\sqrt{40}} \begin{pmatrix} -\sqrt{5} & -\sqrt{8} & \sqrt{27} \\ \sqrt{20} & \sqrt{8} & \sqrt{12} \\ \sqrt{15} & -\sqrt{24} & -1 \end{pmatrix} \begin{pmatrix} 1 \\ 8 \\ 27 \end{pmatrix}$$

J^P=1⁺, I=0

$$N\Xi \Leftrightarrow 8$$

J^P=1⁺, I=1

$$\begin{pmatrix} N\Xi \\ \Sigma\Lambda \\ \Sigma\Sigma \end{pmatrix} = \frac{1}{\sqrt{6}} \begin{pmatrix} -\sqrt{2} & -\sqrt{2} & \sqrt{2} \\ \sqrt{3} & -\sqrt{3} & 0 \\ 1 & 1 & \sqrt{4} \end{pmatrix} \begin{pmatrix} 8 \\ 10 \\ 10 \end{pmatrix}$$

J^P=0⁺, I=1

$$\begin{pmatrix} N\Xi \\ \Sigma\Lambda \end{pmatrix} = \frac{1}{5} \begin{pmatrix} \sqrt{2} & -\sqrt{3} \\ \sqrt{3} & \sqrt{2} \end{pmatrix} \begin{pmatrix} 27 \\ 8 \end{pmatrix}$$

J^P=0⁺, I=2

$$\Sigma\Sigma \Leftrightarrow 8$$

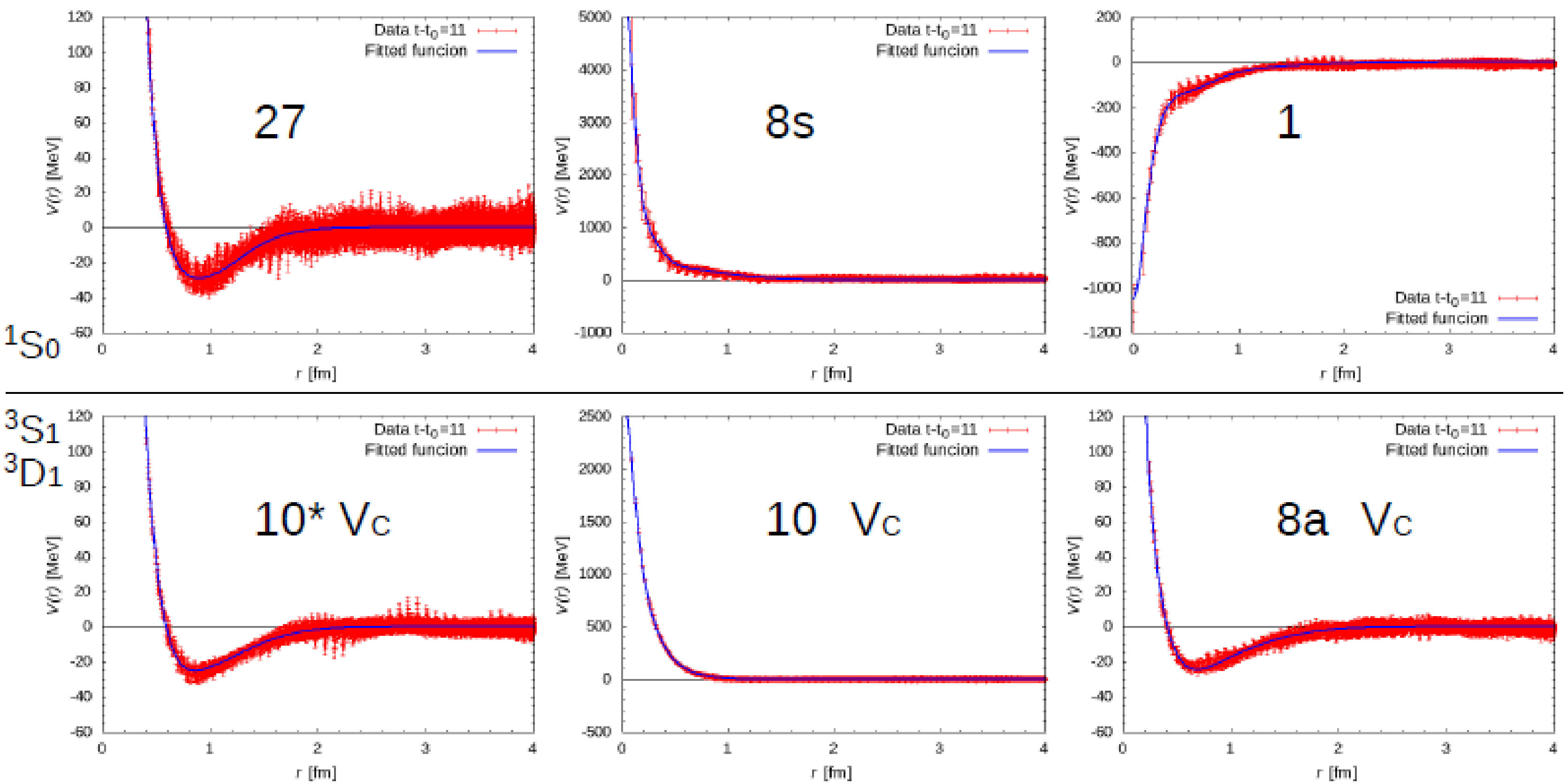
Features of flavor singlet interaction is integrated into the S=-2 J^P=0⁺, I=0 system.

[K. Sasaki]

S=-2 interactions suitable to grasp whole NN/YN/YY interactions

Central Force in Irrep-base (diagonal)

$$8 \times 8 = \underbrace{27 + 8_s + 1}_{^1S_0} + \underbrace{10^* + 10 + 8_a}_{^3S_1,\ ^3D_1}$$



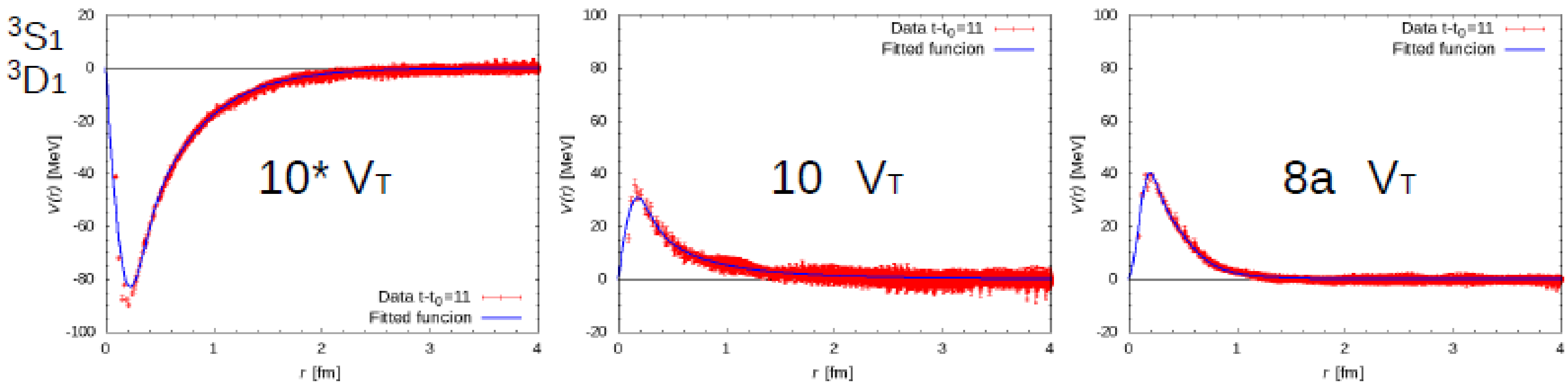
(off-diagonal component is small)

[K. Sasaki]

S=-2 interactions suitable to grasp whole NN/YN/YY interactions

Tensor Force in Irrep-base (diagonal)

$$8 \times 8 = \underbrace{27 + 8s + 1}_{^1S_0} + \underbrace{10^* + 10 + 8a}_{^3S_1,\ ^3D_1}$$



➔ **We calculate single-particle energy of hyperon in nuclear matter w/ LQCD baryon forces**

(off-diagonal component neglected)

We fit by

$$V(r) = a_1 e^{-a_2 r^2} + a_3 e^{-a_4 r^2} + a_5 \left[\left(1 - e^{-a_6 r^2}\right) \frac{e^{-a_7 r}}{r} \right]^2 \quad \text{(central)}$$

$$V(r) = a_1 \left(1 - e^{-a_2 r^2}\right)^2 \left(1 + \frac{3}{a_3 r} + \frac{3}{(a_3 r)^2}\right) \frac{e^{-a_3 r}}{r} + a_4 \left(1 - e^{-a_5 r^2}\right)^2 \left(1 + \frac{3}{a_6 r} + \frac{3}{(a_6 r)^2}\right) \frac{e^{-a_6 r}}{r} \quad \text{(tensor)}$$

Brueckner-Hartree-Fock LOBT

- Hyperon single-particle potential

M. Baldo, G.F. Burgio, H.-J. Schulze, Phys. Rev. C58, 3688 (1998)

$$U_Y(k) = \sum_{N=n,p} \sum_{SLJ} \sum_{k' \le k_F} \langle k k' | G^{SLJ}_{(YN)(YN)}\left(e_Y(k)+e_N(k')\right) | k k' \rangle$$

$$^{2S+1}L_J = {}^1S_0,\ {}^3S_1,\ {}^3D_1,\ \Big|\ {}^1P_1,\ {}^3P_J\ \cdots$$

in our study | limitation

- YN G-matrix using $V^{\mathrm{LQCD}}_{S=-1}$, $M^{\mathrm{Phys}}_{N,Y}$, $U^{\mathrm{AV18,BHF}}_{n,p}$ and, U^{LQCD}_{Y}

Q=0
$$\begin{pmatrix} G^{SLJ}_{(\Lambda n)(\Lambda n)} & G_{(\Lambda n)(\Sigma^0 n)} & G_{(\Lambda n)(\Sigma^- p)} \\ G_{(\Sigma^0 n)(\Lambda n)} & G_{(\Sigma^0 n)(\Sigma^0 n)} & G_{(\Sigma^0 n)(\Sigma^- p)} \\ G_{(\Sigma^- p)(\Lambda n)} & G_{(\Sigma^- p)(\Sigma^0 n)} & G_{(\Sigma^- p)(\Sigma^- p)} \end{pmatrix}$$

Q=+1
$$\begin{pmatrix} G^{SLJ}_{(\Lambda p)(\Lambda p)} & G_{(\Lambda p)(\Sigma^0 p)} & G_{(\Lambda p)(\Sigma^+ n)} \\ G_{(\Sigma^0 p)(\Lambda p)} & G_{(\Sigma^0 p)(\Sigma^0 p)} & G_{(\Sigma^0 p)(\Sigma^+ n)} \\ G_{(\Sigma^+ n)(\Lambda p)} & G_{(\Sigma^+ n)(\Sigma^0 p)} & G_{(\Sigma^+ n)(\Sigma^+ n)} \end{pmatrix}$$

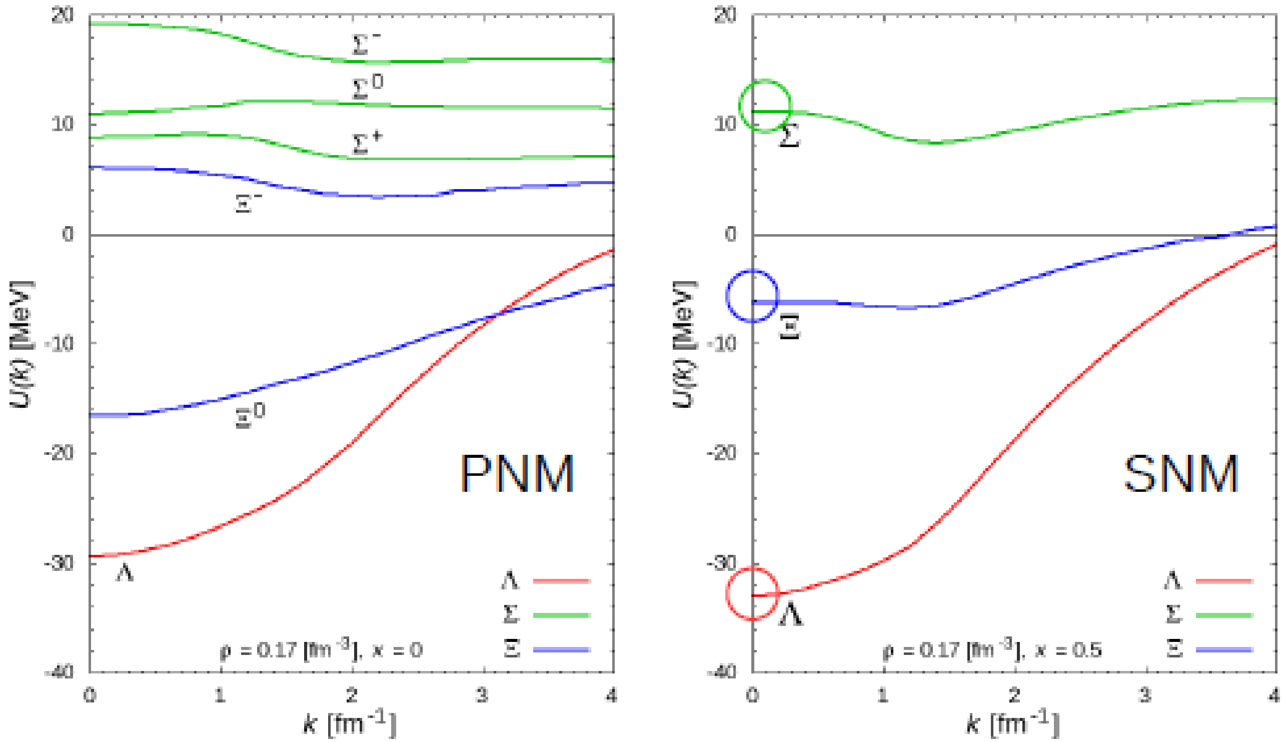
Q=−1 $G^{SLJ}_{(\Sigma^- n)(\Sigma^- n)}$

Q = +2 $G^{SLJ}_{(\Sigma^+ p)(\Sigma^+ p)}$

17

[T. Inoue]

Hyperon single-particle potentials



@ $\rho = 0.17\,[\mathrm{fm}^{-3}]$

Preliminary

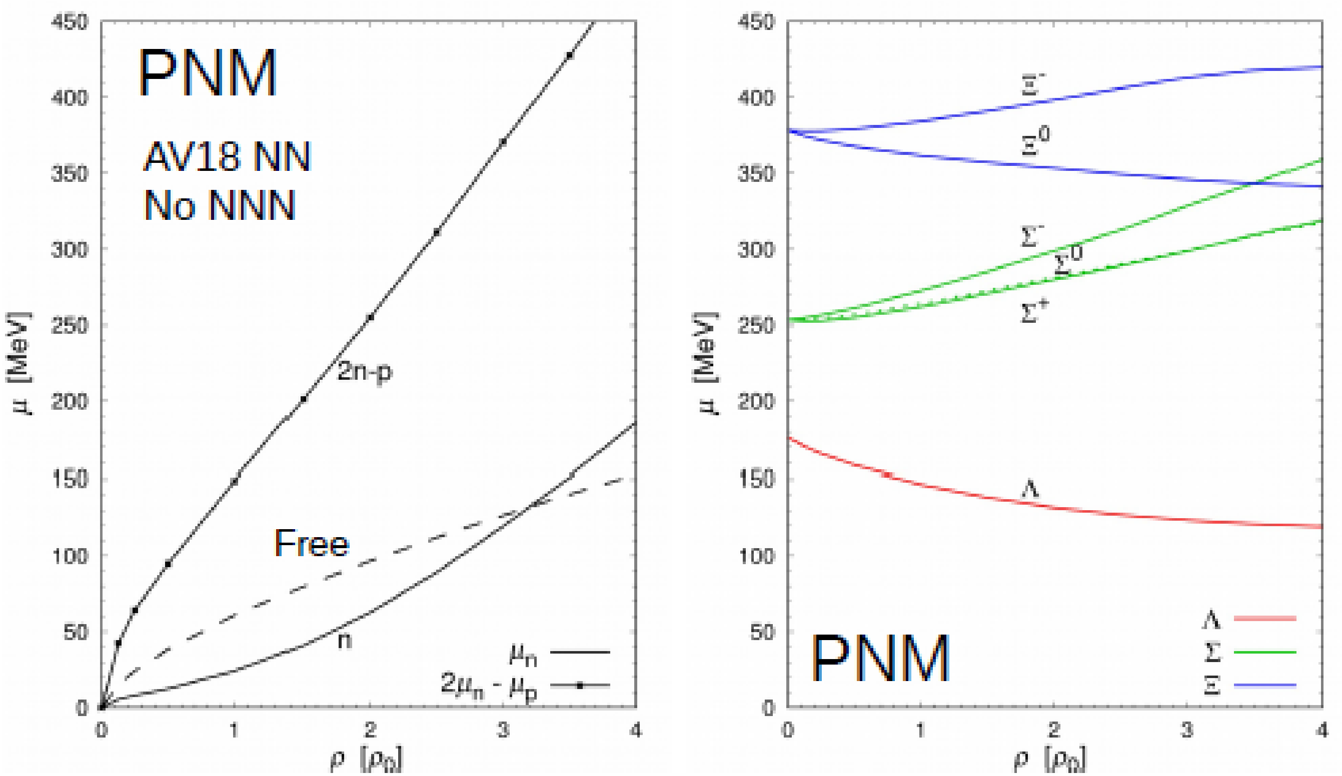
- obtained by using YN,YY forces form QCD.
- Results agree with experimental data!

$U_\Lambda^{\mathrm{Exp}}(0) \simeq -30,\quad U_\Xi(0)^{\mathrm{Exp}} \simeq -10,\quad U_\Sigma^{\mathrm{Exp}}(0) \simeq +10$ [MeV] 22

attraction — attraction small — repulsion small

[T. Inoue]

Chemical potentials



Preliminary

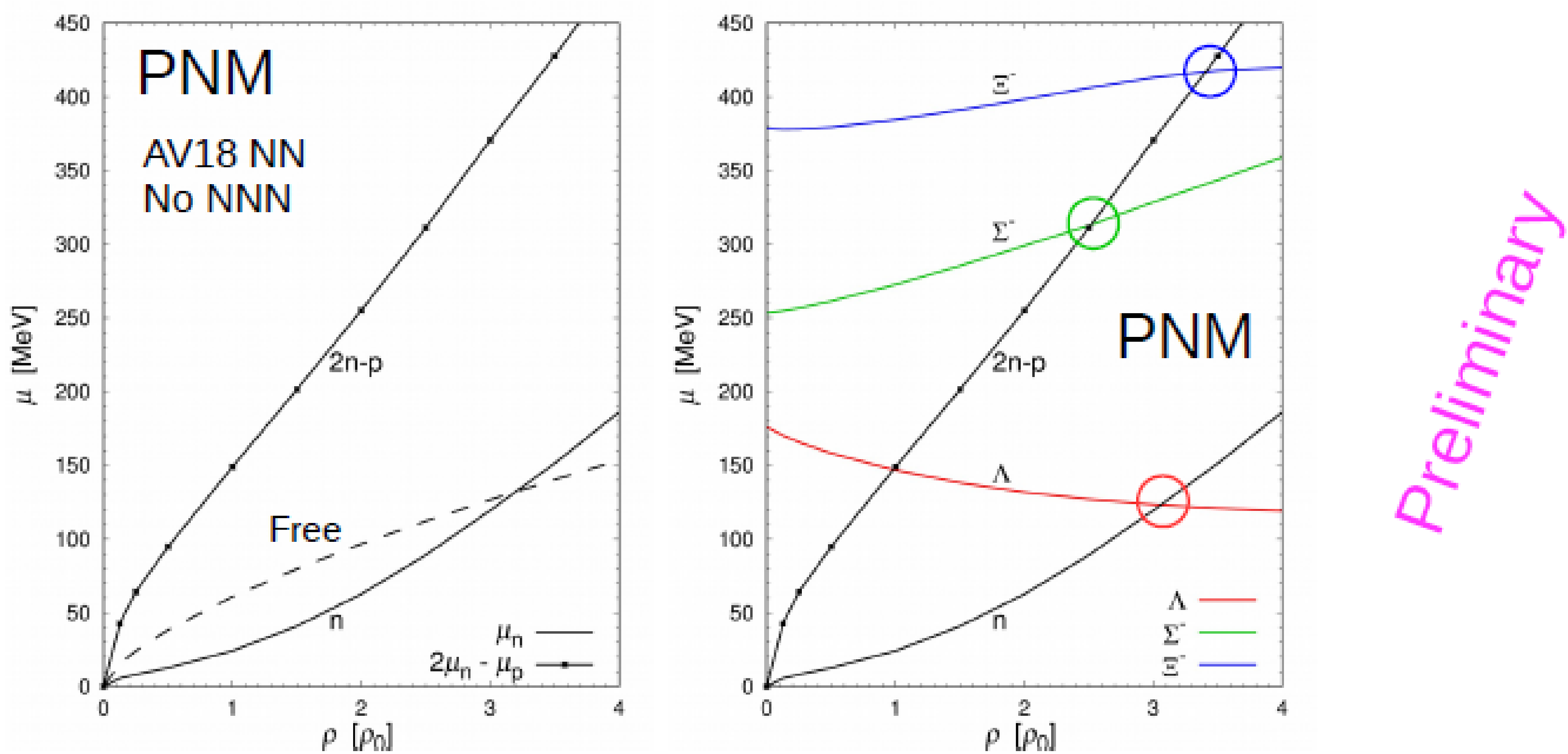
- Density dependence of chemical pot. of n & Y in PNM.
- Hyperon appear $n \rightarrow Y^0$ if $\mu_n > \mu_{Y^0}$

$nn \rightarrow pY^-$ if $2\mu_n > \mu_p + \mu_{Y^-}$

25

[T. Inoue]

Hyperon onset (just for a demonstration)



- First, Σ^- appear at 2.5 ρ_0. Next, Λ appear at 3.0 ρ_0.
  - NS matter is not PNM especially at high density.
  - We should compare with more sophisticated μ_n and μ_p.
  - P-wave YN force may be important at high density.

26

[T. Inoue]

Summary and Prospects

- Baryon forces: bridge between particle/nuclear/astro-physics
  - HAL QCD method is crucial for a reliable calculation

- Three-nucleon forces from LQCD
  - Nf=2, m_π=0.76-1.1GeV & Nf=2+1, m_π=0.51 GeV
  - Repulsive 3NF at short distance observed
  - Quark mass dependence small but tends to show up

- Two-baryon forces (NN/YN/YY) from LQCD
  - The first simulations w/ ~physical masses
  - Hyperon forces ➔ Properties of dense matter
  - Much more coming

- **Outlook**
  - Three-baryon forces (NNN/YNN/YYN/YYY) w/ lighter ud masses
  - Two-baryon forces in parity-odd channel, LS-forces
  - New Era is dawning w/ LQCD Baryon Forces !

Post-K computer
(Exascale)