Nuclear matter EOS in the leading order Brueckner theory with the three-nucleon interaction from chiral EFT

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- LOBT calculations of the nuclear matter saturation curve, using NN and 3N interactions in chiral effective field theory (Ch-EFT).
  - Role of the 3NF to reproduce correct saturation properties, and a physical picture behind.
- Results for neutron matter.
- Hyperon properties in nuclear matter: implication of the present NLO YN interactions by the Bonn-Julich group.
  - Possible contributions from the NLO ΛNN force.
Salient features of atomic nuclei

- Atomic nuclei: basic components in our ordinary hadronic world, in the intermediate-period between the big-bang and black-holes or neutron stars.

- Salient features of nuclei: fundamental for various quantum mechanical phenomena of nuclei.
  - Saturation
  - Shell-structure (single-particle aspect)

- The basic problem is how to understand these properties on the basis of underlying NN interactions and/or QCD.

- Quantitative understanding is not complete: e.g., discussions about EoS, $E_{sym}$, slope parameter $L$, and so on are still ongoing.
Saturation

- inner densities of nuclei and B.E./A are almost constant from light to heavy nuclei: infinite matter limit, $\rho_0 = 0.17$ fm$^{-3}$ and B.E./A= 16 MeV
  - a liquid-drop model was thought in the early stage.

Shell-structure

- Experiments established single-particle aspects of nuclei, in spite of singularly repulsive NN interactions.

Bruckner theory in the 1950’s paved the way for a (semi-) quantitative explanation.

Recent progresses:

- developments of the description of NN and 3N interactions on the basis of chiral symmetry of QCD and effective interaction theory such as $V_{\text{low}k}$ and SRG.
- advances of quantum many-body calculations, theoretically and computationally: CCM, GFMC, NCSM, • • •
How the saturation and s.-p. structure of nuclei appear, in spite of the NN interaction singular at short distance?

- Brueckner theory (1950s) provided a semi-quantitative explanation.
  - The “effective” interaction namely G-matrix, after short-range correlations are taken into account by the matrix equation
    \[ G(\omega) = v + v \frac{Q}{\omega - H} G(\omega), \]
    is weak enough to produce mean field.
  - Importance of the Pauli and dispersion effects.
  - Saturation mechanism: the attraction from the tensor correlation becomes weaker at larger densities due to the Pauli blocking.

- However, no realistic NN potential with high accuracy can reproduce correct saturation properties in nuclear matter.
  (Energies and radii of nuclei are not simultaneously reproduced in \textit{ab initio} calculations.)
Understanding nuclear saturation properties

- Microscopic studies based on bare NN forces
  - Brueckner theory (1950's, LOBT)
- Standard explanation
  - Strong short-range repulsion
  - Pauli effects suppress tensor correlations.

- Quantitative insufficient.
  - Saturation point is located at higher densities.

![Graph showing partial wave contributions to the E/A with angle-average Q operators and a graph showing symmetric nuclear matter E/A [MeV] vs. k_F [fm^{-1}].]
The unitary tf. $e^{S_{12}}$ should satisfy a decoupling condition

$$\langle Q | \tilde{H} | P \rangle = \langle P | \tilde{H} | Q \rangle = 0$$

in two-body space (block-diagonal).

- Singular high-momentum components are eliminated.

- Eigenvalues, namely on-shell properties, in the restricted (P) space do not change.
  - Off-shell properties naturally change.

- Induced many-body forces appear in many-body space.

Apply a unitary transformation $e^{S_{12}}$ to $H$ to obtain an equivalent Hamiltonian $\tilde{H}$ in a restricted (P) space

[Suzuki and Lee, PTP64 (1980)]
Nuclear matter saturation curves with various modern NN forces

- LOBT calculations do not reproduce the empirical saturation point. (Higher order contributions are believed to be small.)
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Missing effects or contributions?

- Possible effects to be taken care of:
  - Higher orders, relativistic effects, 3NF, medium modification, ...

- Since the 1950s, 3NF effects have been expected.
  - It has been known that the $\Delta$-excitation, which provides attraction in the $^1S_0$ channel in free space, is Pauli-blocked in the medium.
  - This effect can be understood as the contribution of the Fujita-Miyazawa 3NFs.

- Phenomenological terms and adjustments were included in all studies of 3NF effects in the literature.
  - Variational calculations by Pandharipande et al. (1980-2000)
Progress in the description of NN interaction: Ch-EFT

At N^3LO level, comparable accuracy for reproducing NN scattering data with other modern NN interactions is achieved.

3NFs are introduced (defined) systematically and consistently with the NN sector.

These 3NFs can, as shown later, reproduce quantitatively saturation properties.

2π-exchange is a lower-energy process to be considered before including anti-nucleons, for example.
Brief introduction of baryon-baryon interactions in Ch-EFT

- Low energy effective theory: starting from a general Lagrangian, which satisfies chiral symmetry of QCD, for low-energy elements of nucleons and pions (a Goldstone boson of the symmetry breaking), NN potential is perturbatively constructed in power counting scheme, by calculating, e.g., Feynman diagrams.

\[ \cdots \]

\((\pi, K, \eta \text{ exchange in the case of SU}(3))\)

- Coupling constants are determined by \(\pi N\) and NN data.
  - NN scattering and bound states are not treated by perturbation.
  - Lippmann-Schwinger eq.

- Renormalization for the divergence of Feynman diagrams and regularization of Lippmann-Schwinger equation at the cutoff scale of the order of \(\Lambda = 500 - 600\) MeV.
NNLO 3NFs $\nu_{123}$ in Ch-EFT

\[ V_{123}^{(2\pi)} = \sum_{i\neq j\neq k} \frac{g_A^2}{8f_\pi^4} \frac{(\sigma_i \cdot q_i)(\sigma_j \cdot q_j)}{(q_i^2 + m_\pi^2)(q_j^2 + m_\pi^2)} F_{ijk}^{\alpha\beta} \tau_i^\alpha \tau_j^\beta \]

\[ F_{ijk}^{\alpha\beta} = \delta^{\alpha\beta} \left[ -4c_1 m_\pi^2 + 2c_3 q_i \cdot q_j \right] + c_4 \epsilon^{\alpha\beta\gamma\tau_k} \tau_k \sigma_k \cdot (q_i \times q_j) \]

\[ V_{123}^{(1\pi)} = -\sum_{i\neq j\neq k} \frac{g_A c_D}{8f_\pi^4 \Lambda_\chi} \frac{\sigma_j \cdot q_j}{(q_j^2 + m_\pi^2)} \sigma_i \cdot q_j \tau_i \cdot \tau_j \]

\[ V_{123}^{(ct)} = \sum_{i\neq j\neq k} \frac{c_E}{2f_\pi^4 \Lambda_\chi} \tau_i \cdot \tau_j \]

\[ q_i = p_i - p_i, \Lambda_\chi = 700 \text{ MeV} \]

$c_1 = -0.81 \text{ GeV}^{-1}, c_3 = -3.4 \text{ GeV}^{-1},$ and $c_4 = 3.4 \text{ GeV}^{-1}$ are fixed in NN sector. $c_D$ and $c_E$ are to be determined in many-body systems.
Reduction of 3NF $\nu_{123}$ to density dependent NN $\nu_{12(3)}$

$$\langle ab|\nu_{12(3)}|cd\rangle_A \equiv \sum_h \langle abh|\nu_{123}|cdh\rangle_A$$


- Expand them into partial waves, add them to NN and carry out G matrix calculation.
  - A factor of $\frac{1}{3}$ is needed for the calculation of energy.
- This diagram partly corresponds to the Pauli blocking of the isobar $\Delta$ excitation in a conventional picture.
Statistical factors for 3NF contributions on the HF level

Some caution in evaluating 3NF contributions to the total energy $E$ and s.p. energy $e_h$.

- **Total energy (in the HF approximation)**

  $$
  E = \sum_h \langle h|t|h \rangle + \frac{1}{2!} \sum_{hh'} \langle hh'|v_{12}|hh' \rangle_A + \frac{1}{3!} \sum_{hh'h''} \langle hh'h''|v_{123}|hh'h'' \rangle_A
  $$

  Define $\langle ab|v_{12(3)}|cd \rangle_A \equiv \sum_h \langle abh|v_{123}|cdh \rangle_A$

  $$
  = \sum_h \langle h|t|h \rangle + \frac{1}{2!} \sum_{hh'} \langle hh'|v_{12} + \frac{1}{3} v_{12(3)}|hh' \rangle_A
  $$

- **Single-particle energy (potential)**

  $$
  e_h = \langle h|t|h \rangle + \sum_{h'} \langle hh'|v_{12}|hh' \rangle_A + \frac{1}{2!} \sum_{h'h''} \langle hh'h''|v_{123}|hh'h'' \rangle_A
  $$

  $$
  = \langle h|t|h \rangle + \sum_{h'} \langle hh'|v_{12} + \frac{1}{2} v_{12(3)}|hh' \rangle_A
  $$

  $$
  = \langle h|t|h \rangle + \sum_{h'} \langle hh'|v_{12} + \frac{1}{3} v_{12(3)}|hh' \rangle_A + \sum_{h'} \langle hh'|\frac{1}{6} v_{12(3)}|hh' \rangle_A
  $$
Prescription for the G matrix equation including "3NF" term

\[ G_{12} = \left( v_{12} + \frac{1}{3} v_{12(3)} \right) + \left( v_{12} + \frac{1}{3} v_{12(3)} \right) \frac{Q}{\omega - H} G_{12} \]

- total energy is given by \( E = \sum_k \left\{ \langle k | t | k \rangle + \frac{1}{2} U_E(k) \right\} \),
  where \( U_E(k) \equiv \sum_{k'} \langle kk' | G_{12} | kk' \rangle_A \)
- s.p. energy \( e_k = \langle k | t | k \rangle + U_G(k) \), and the potential in the propagator \( \omega - H = e_{k_1} + e_{k_2} - (t_1 + U_G(k'_1) + t_2 + U_G(k'_2)) \)
- The factor in front of \( v_{12(3)} \) is different in \( U_G(k) \):
  \[ U_G(k) \equiv \sum_{k'} \left\langle kk' \left| G_{12} + \frac{1}{6} v_{12(3)} \left( 1 + \frac{Q}{\omega - H} \right) G_{12} \right| kk' \right\rangle_A \]
- Note that the difference between \( U_E(k) \) and \( U_G(k) \) is typically 5 MeV, and this difference does not much affect \( G \), because of the cancellation in the denominator \( \omega - H \).
- Note that OMP potential corresponds to \( U_G(k) \).
Reducing 3NFs to effective NN interactions in infinite matter

- Reduction to effective two-body forces by folding the third nucleon in infinite matter.

  Contributions of left two-diagrams (wave-function-renormalization and vertex-correction types) tend to cancel.

- In free space:
  - Attractive in $^1S_0$.
  - Suppress 1-$\pi$ exchange tensor.

- In the medium:
  - Reduce attraction in $^1S_0$.
  - Reduce the suppression of tensor force.

- $c_1, c_3$, and $c_4$ are fixed in the NN sector.
- $c_D$ and $c_E$ are determined in many-body systems.

Diagram:

- $V_{3N}^{(2\pi)}$,
- $V_{3N}^{(1\pi)}$,
- $V_{3N}^{(ct)}$.
LOBT calculations with NN+”3NF” of Ch-EFT

- Calculated saturation curves with three choices of cutoff $\Lambda$.
- Results of $c_D = 0$ and $c_E = 0$.
- Pauli effects are sizable.

Tune $c_D$ and $c_E$.  

Pauli effects are sizable.
Born energies from the $c_D$ and $c_E$ terms in nuclear matter

- $c_D$ and $c_E$ terms provide very similar contributions to the nuclear matter HF-level energy, if $c_D \approx -4c_E$.
- When $c_D \approx 4c_E$ is satisfied, contributions of the $c_D$ and $c_E$ terms cancel.
- There are continuous uncertainties for $c_D$ and $c_E$ values as far as NM energies are concerned.

Parametrization in the quadratic form of the density $\rho$:

$$E_{c_D}(\rho) = c_D \times (-0.1902 + 2.952\rho + 37.16\rho^2)$$

$$E_{c_E}(\rho) = c_E \times (0.8695 - 17.52\rho - 128.3\rho^2).$$
each spin- and isospin-channel contribution in LOBT E/A

thin curves: $\Lambda = 450$ MeV, thick curves: $\Lambda = 550$ MeV

Repulsive contributions of 3NF in $^3\text{O}$ and $^1\text{S}$ channels.

- Repulsive effects of the 3NFs are smaller in $^3\text{E}$ state than in $^1\text{E}$ state, because of the attraction from the enhanced ($\sim 15\%$) tensor component.
- Sizable repulsive contribution in $^3\text{O}$ state.
Physics behind the repulsive effect in the $^1S_0$ and $^3O$ states

- It has been known that the $\Delta$-excitation, which produces attraction in the $^1S_0$ state in free space, is Pauli-blocked in the medium.

- This effect can be viewed as the contribution of the Fujita-Miyazawa 3NFs.

- The corresponding effects appear through a $\pi\pi NN$ vertex in Ch-EFT.
Physics behind the tensor force enhancement in the $^3S_1$ state

- Strong tensor force from one-pion exchange
  - In a OBEP model, $\rho$-meson exchange provides tensor force in an opposite sign: $\rightarrow$ natural cut necessary to explain scattering data.
  - In Ch-EFT
    - $\rho$-meson is not present (out of scale).
    - Tensor component from the $2\pi$-exchange plays the role of the $\rho$-meson. The $2\pi$-exchange is Pauli blocked in the medium, which reduces the suppression of the tensor force: $\rightarrow$ enhancement.
Tensor force including the 3NF effects at normal density

- Bare diagonal matrix elements with and without 3NF effects.
- Diagonal matrix elements of low-momentum tensor interaction.

Tensor force is enhanced by about 15% by the 3NF.
Cutoff-scale dependence is small in low-momentum space.
Neutron matter

- EoS of neutron matter: basic to theoretical studies of neutron star.
  - EoS of APR, including phenomenological 3NFs, has been standard.
    - Necessity of the repulsive contributions from 3NF.

- Dependence on different two-body NN interactions is small, because of the absence of tensor effects in the $^3E$ state.
- The contribution of Ch-EFT 3NFs (no $c_D$ and $c_E$ terms) is similar to the standard phenomenological one by APR.
  - Ch-EFT is not to be applied to the high-density region of $\rho > 2\rho_0$.

Neutron matter as a function of $\rho/\rho_0$

- $E/A$ profile of Gogny D1 force (and some of Skyrme int.) is unrealistic from the point of view of microscopic calculations.

- The density-dependent repulsion is solely taken care of by the $^3E$ state when $x_0 = 1$ in $t_0(1 + x_0 P_\sigma) \rho^\alpha (r_1) \delta(r_1 - r_2)$.

- 3NFs of Ch-EFT imply that repulsion is dominant in $^1E$ and $^3O$ states.

![Graph showing neutron and nuclear matter energies](image-url)

Neutron and nuclear matter energies

- Neutron matter and symmetric nuclear matter energies for various forces.
- Renormalized and bare $G_0$ interactions.
- Friedman–Pandharipande and Bonn–B forces.

Graph illustrates the renormalized and bare $G_0$ interactions, as well as neutron and symmetric nuclear matter energies for different forces.
When the s.p. energy of the $\Lambda$ hyperon becomes smaller than that of the neutron in neutron matter, the $\Lambda$ hyperon starts to appear through weak processes.

$$m_\Lambda + \frac{\hbar^2}{2m_\Lambda} \times 0^2 + U_\Lambda(0) < m_n + \frac{\hbar^2}{2m_n} \times k_{F_n}^2 + U_n(k_{F_n})$$

$$U_\Lambda(0) < \frac{\hbar^2}{2m_n} \times k_{F_n}^2 - (m_\Lambda - m_n) + U_n(k_{F_n})$$

Calculations in the literature, using realistic NN and YN interactions, indicates that the $\Lambda$ hyperon should appear at the density of around $3\rho_0$.

The appearance of the $\Lambda$ hyperon softens the EoS at high densities.

Recent observation of $2m_\odot$ neutron stars seems not to support this scenario. **Hyperon puzzle.**
YN interactions in Ch-EFT

- Parameterization by the Bonn-Julich group
  - Lowest order:
    - Polinder et al., Nucl. Phys. A779, 244 (2006)
    - Parameters: $f_{NN\pi} = \frac{2f_\pi}{g_A}$, $\alpha = \frac{F}{F+D}$, and 5 low-energy constants $C_{1S0}^{\Lambda\Lambda}, C_{3S1}^{\Lambda\Lambda}, C_{1S0}^{\Sigma\Sigma}, C_{3S1}^{\Sigma\Sigma}, C_{3S1}^{\Lambda\Sigma}$
  - Next-to-Leading order
  - Leading three-baryon forces
Coupling constants in the NLO level are almost determined by the present YN and hyper-nuclear data.

The depth of the $\Lambda$ s.p. potential is shallower than that of other YN potential models.
Lambda and Sigma s.p. potentials in symmetric nuclear matter

- The attraction of the Lambda s.p. potential comes from strong Lambda-N-Sigma N coupling.
- The Sigma s.p. potential is repulsive.
- The depth of the Lambda s.p. potential becomes shallower at higher densities, which has not been seen in other potential models.
Comparison of the $\Lambda N-\Sigma N$ coupling: NSC97 and Ch-EFT

The attraction from the $\Lambda N-\Sigma N$ coupling to the $\Lambda$ s.p. potential is of the order of 60 MeV at the normal density.
Large uncertainties for coupling constants in the $S = -2$ sector.

- One constraint: absence of the $H$ particle state.

Several-baryon-channels coupling ($\Xi N$-$\Lambda\Lambda$-$\Sigma\Sigma$ ($T=0$), $\Xi N$-$\Lambda\Sigma$-$\Sigma\Sigma$ ($T=1$)) leads to strong state dependence and density dependence.

Scarce of experimental data.

Recent experimental progresses:

- Kiso event: $\Xi^- - ^{14}\text{N}$ (Nakazawa)
- $\Xi$ bound state in the $^{12}\text{C}(K^- , K^+)X$ reaction at 1.8 GeV/c (Nagae)

2-π exchange ΛNN interaction (a) is first reduced to effective two-body interaction in infinite matter, diagrams (c, d, e), then the contribution to the Λ s.p. potential (c) is estimated. ((d) and (e) are absent for Λ.)

- Contributions from (d) and (e) diagrams almost cancel each other.
- Pauli blocking type (c) suppress attraction in free space, which means repulsive effect.
Estimation of 2-π exchange ΛNN contributions to Λ s.p. energies

\[ V_{\Lambda NN} = \frac{g_A^2}{3f_0^4} \left( \tau_2 \cdot \tau_3 \right) \frac{(\sigma_3 \cdot q_{63})(\sigma_2 \cdot q_{52})}{(q_{63}^2 + m_\pi^2)(q_{63}^2 + m_\pi^2)} \times \left\{ -(3b_0 + b_D)m_\pi^2 + (2b_2 + 3b_4)q_{63} \cdot q_{52} \right\} \]

\[ U_\Lambda(k_\Lambda) = \frac{1}{2} \sum_{2,3} \langle k_\Lambda k_2 k_3 | V_{\Lambda NN} | k_\Lambda k_2 k_3 \rangle_A \]

\[ = \frac{g_A^2}{3f_0^4} \frac{1}{(2\pi)^6} \int_0^{k_F} q^2 \, dq \, \frac{64\pi^2}{3} \frac{4q^2}{(4q^2 + m_\pi^2)^2} \times (2k_F + q) \times \left\{ -(3b_0 + b_D)m_\pi^2 + (2b_2 + 3b_4)q^2 \right\} \]

Contributions from contact terms have also to be considered.
Appearance of $\Lambda$ in neutron matter?

A naive condition for the $\Lambda$ hyperon to appear in pure neutron matter.

\[ U_\Lambda(0) < \frac{\hbar^2}{2m_n} \times k_{F_n}^2 + U_n(k_{F_n}) \]

- The NLO YN interactions predict a shallow $\Lambda$ s.p. potential.
  - $U_\Lambda(0) \cong -20 \sim -30$ MeV
- Non-phenomenological YNN interactions have to be considered seriously.
  - Medium modification for the YN interaction, such as Pauli-blocking effects, should exist.
- $\Lambda NN$-$\Sigma NN$ is to be included.
Appearance of $\Lambda$ in neutron matter?

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- $\Lambda$NN-$\Sigma$NN is to be included.
Saturation point of $N^3$LO NN potential in Ch-EFT (as accurate as other modern NN potentials) in nuclear matter is in nuclear matter is naturally on the Coester band.

Saturation properties are much improved by including 3NFs, which are introduced consistently to the NN sector.

In the present calculations, 3NFs are reduced to effective NN forces by folding them over the third nucleon.

3NF effects, in addition to the repulsive contribution:

- Tensor component is enhanced, which brings about some attraction in the $^3S_1$ channel. (and enhance the imaginary potential for the scattering state).
- Spin-orbit force is enhanced, which is important to account for an empirical strength for shell-structure. (not shown in this talk)
- Using recent NLO YN and YNN interactions, $\Lambda$ hyperon properties in nuclear matter are calculated.