Three Baryon Interaction in the Quark Cluster Model --- 3B Interaction from Determinant Interaction of Quarks as an example

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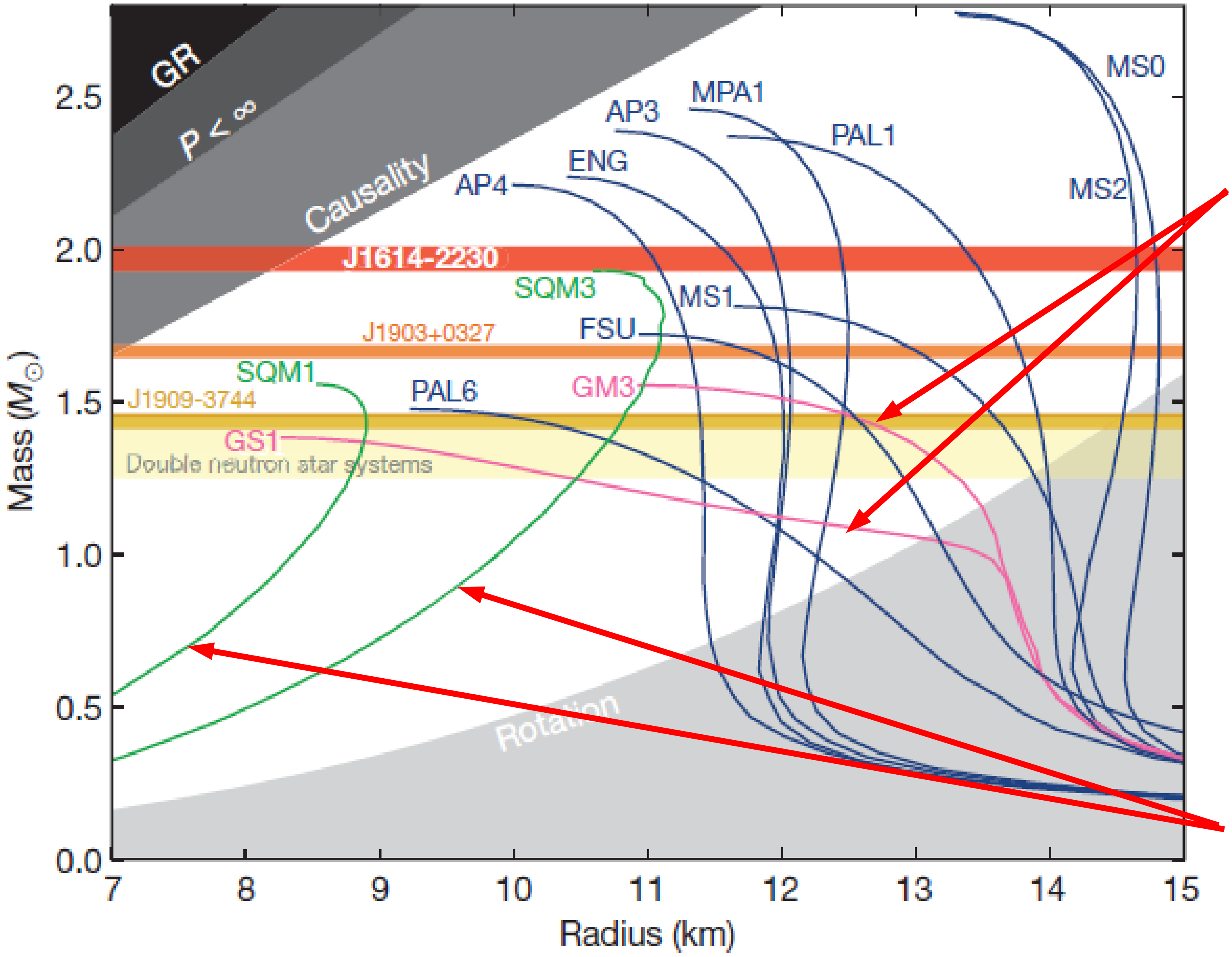
AO, K. Kashiwa, K. Morita, arXiv:1610.06306

Will be announced TODAY !

Contents

Hyperon Puzzle (or Hyperon Crisis)

Demorest et al., Nature 467 (2010) 1081 (Oct.28, 2010).



EOS with hyperons or Kaons

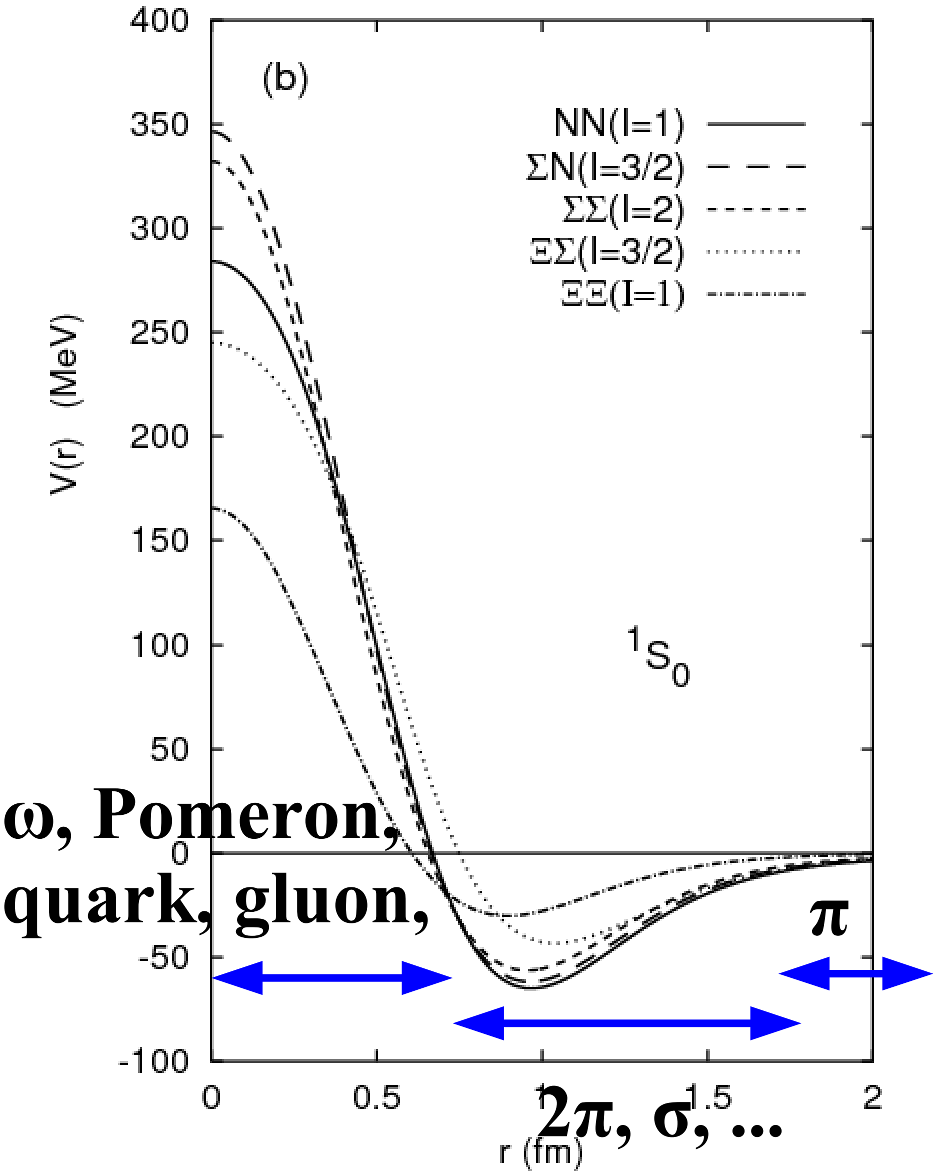
Quark matter EOS

Proposed Solutions

- **Hyperonic EOS cannot support massive NS (M ~ 2 $M_\odot$).**
  Demorest et al. (2010), Antoniadis et al. (2013)
- **Proposed Solutions**
  - **Hyperonic Three-Body Force (or density dep. coupling)**
    Bednarek et al. ('12), Jiang et al. ('12); Long et al. ('12); Yamamoto et al. ('14); Lonardoni et al. ('15); Tsubakihara et al. ('13), T. Miyatsu et al. ('13), …
  - **Crossover Transition to Quark Matter**
    Bonanno et al.('12); Masuda et al. ('13); Bejger et al.('16), …
  - **Modified Gravity**
    Astashenok et al. ('14)
- **Three-nucleon interaction is known to be necessary.**
  How can we determine YNN (+YYN, YYY) potential ?

Baryon-Baryon Force

- **Long-range (r>2 fm): π exch.**
- **Intermediate (r ~ 1 fm):**
  multi π exch., boson exch., …
- **Short range (r < 0.6 fm):**
  vector boson exch.,
  Pomeron exch.,
  quark exclusion + one gluon exch.,
  …



ω, Pomeron,
quark, gluon,

π

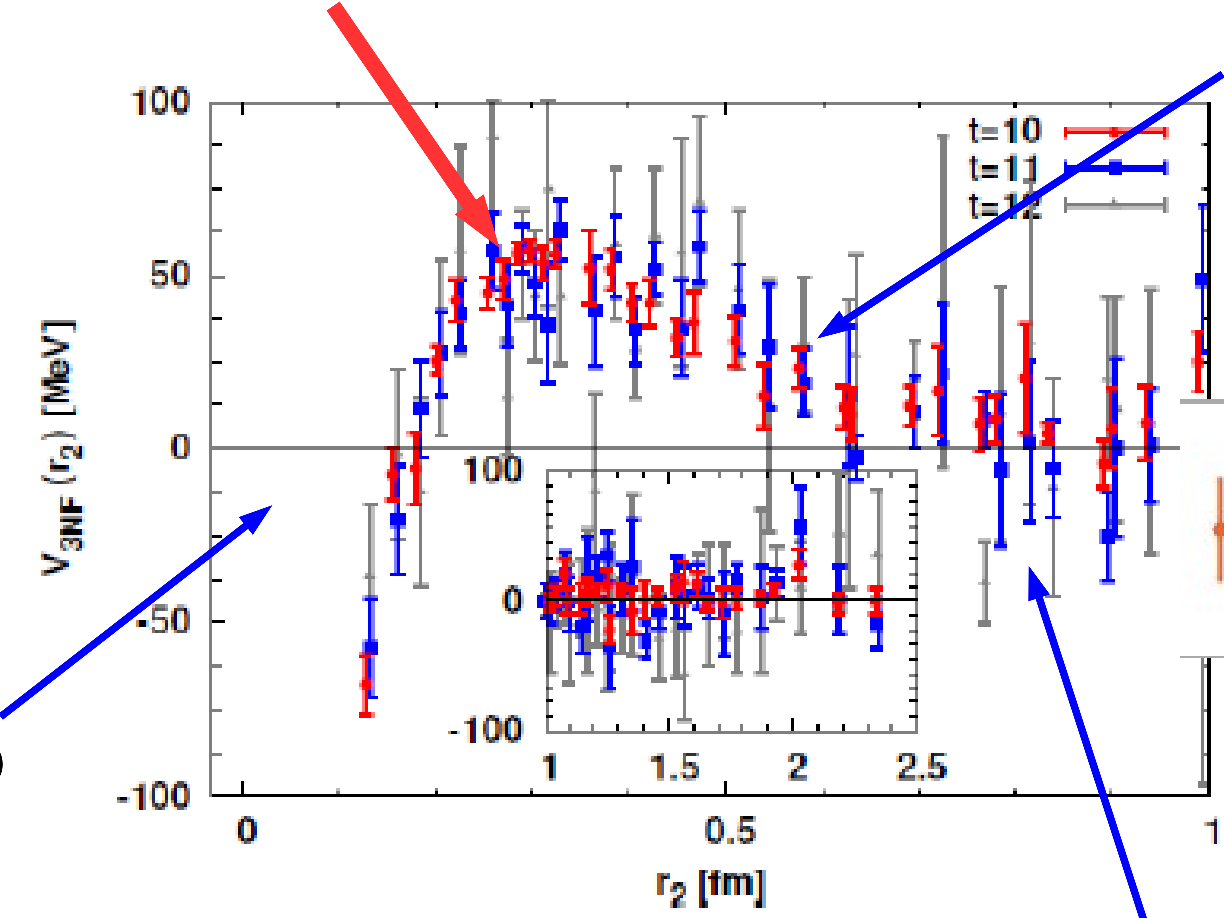
2π, σ, …

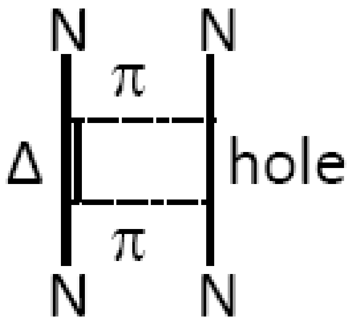
Fujiwara, Suzuki, Nakamoto ('07)

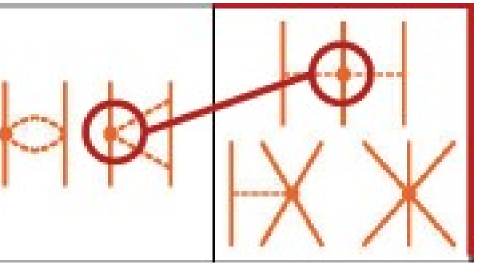
> ***Quark model description of 3B repulsion should be a promising approach !***

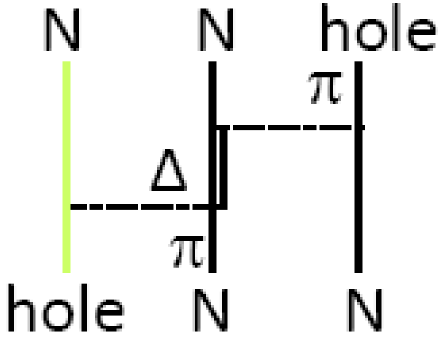
Three-Baryon force

What makes 3B repulsion at r ~ 0.5 fm ?









pQCD

Doi (Wed), Kohno (Thu), Tews (Thu)

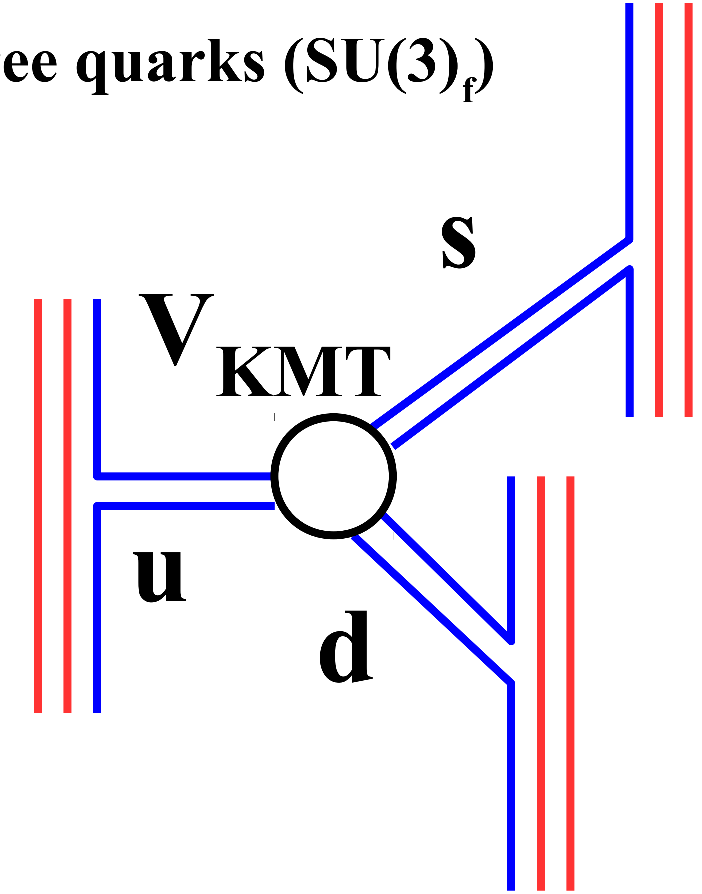
Kobayashi-Maskawa-'t Hooft (KMT) interaction

- **KMT interaction**

Kobayashi, Maskawa ('70), 't Hooft ('76)

$$\mathcal{L}=g_D\ (\det\Phi+\text{h.c.})\ ,\qquad \Phi_{ij}=\bar{q}_j(1-\gamma_5)q_i$$

- **Determinant interaction in flavor for three quarks (SU(3)$_\mathrm{f}$)**
- **Responsible for U(1)$_\mathrm{A}$ anomaly**
  η-η' mass diff.
  → g$_\mathrm{D}$= – 9.29 *Hatsuda, Kunihiro ('94)*
  – 12.36 *Rehberg, Klevanski, Hufner ('96)*
- **KMT interaction should generate**
  2B and 3B interaction
  when hyperons are involved.
- **Repulsive in ΛΛ system**
  → Pushes up H particle energy.
  Takeuchi, Oka ('91)



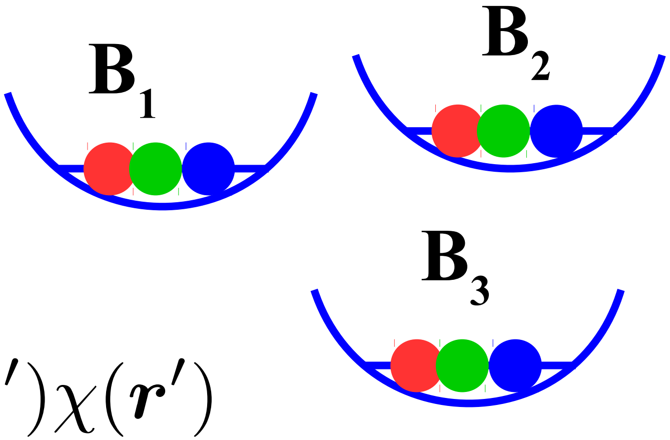
Does anomaly support massive NS ?

Quark Cluster model

- **Totally anti-symmetrized wave function of baryons**

$$|\Psi\rangle = \mathcal{A}|\chi_{12}B_1B_2\rangle$$

$$|\Psi\rangle = \mathcal{A}|\chi_{123}B_1B_2B_3\rangle$$



- **Resonating Group Method**

$$\int d\boldsymbol{r}' H(\boldsymbol{r},\boldsymbol{r}')\chi(\boldsymbol{r}') = E\int d\boldsymbol{r}' N(\boldsymbol{r},\boldsymbol{r}')\chi(\boldsymbol{r}')$$

$$\to -\frac{\hbar^2}{2\mu}\nabla^2\chi^{(N)} + (V\chi^{(N)}) = E\chi^{(N)} \quad (\chi^{(N)} = \mathcal{N}^{1/2}\chi)$$

$$H(\boldsymbol{r},\boldsymbol{r}') = \langle \boldsymbol{r}B_1B_2\ldots|H|\mathcal{A}(\boldsymbol{r}'B_1B_2\ldots)\rangle$$

$$N(\boldsymbol{r},\boldsymbol{r}') = \langle \boldsymbol{r}B_1B_2\ldots|\mathcal{A}(\boldsymbol{r}'B_1B_2\ldots)\rangle$$

- **When (wave length of χ) >> (baryon size),**

$$V(\boldsymbol{r}) \simeq \Delta K + \langle V\mathcal{A}\rangle/\langle\mathcal{A}\rangle$$

Norm Kernel

- **Antisymmetrizer makes the calculation complicated !**

$$\begin{aligned}\mathcal{A} = &[1 - 9(P_{36} + P_{39} + P_{69}) + 27(P_{369} + P_{396})\\ &+54(P_{25}\,P_{39} + P_{35}\,P_{69} + P_{38}\,P_{69})]\,\mathcal{A}_B\\ &- 216 P_{25}\,P_{38}\,P_{69}\ ,\end{aligned}$$

$$\mathcal{A}_B = \sum_{\mathcal{P}} (-1)^{\pi(\mathcal{P})}\,\mathcal{P}$$

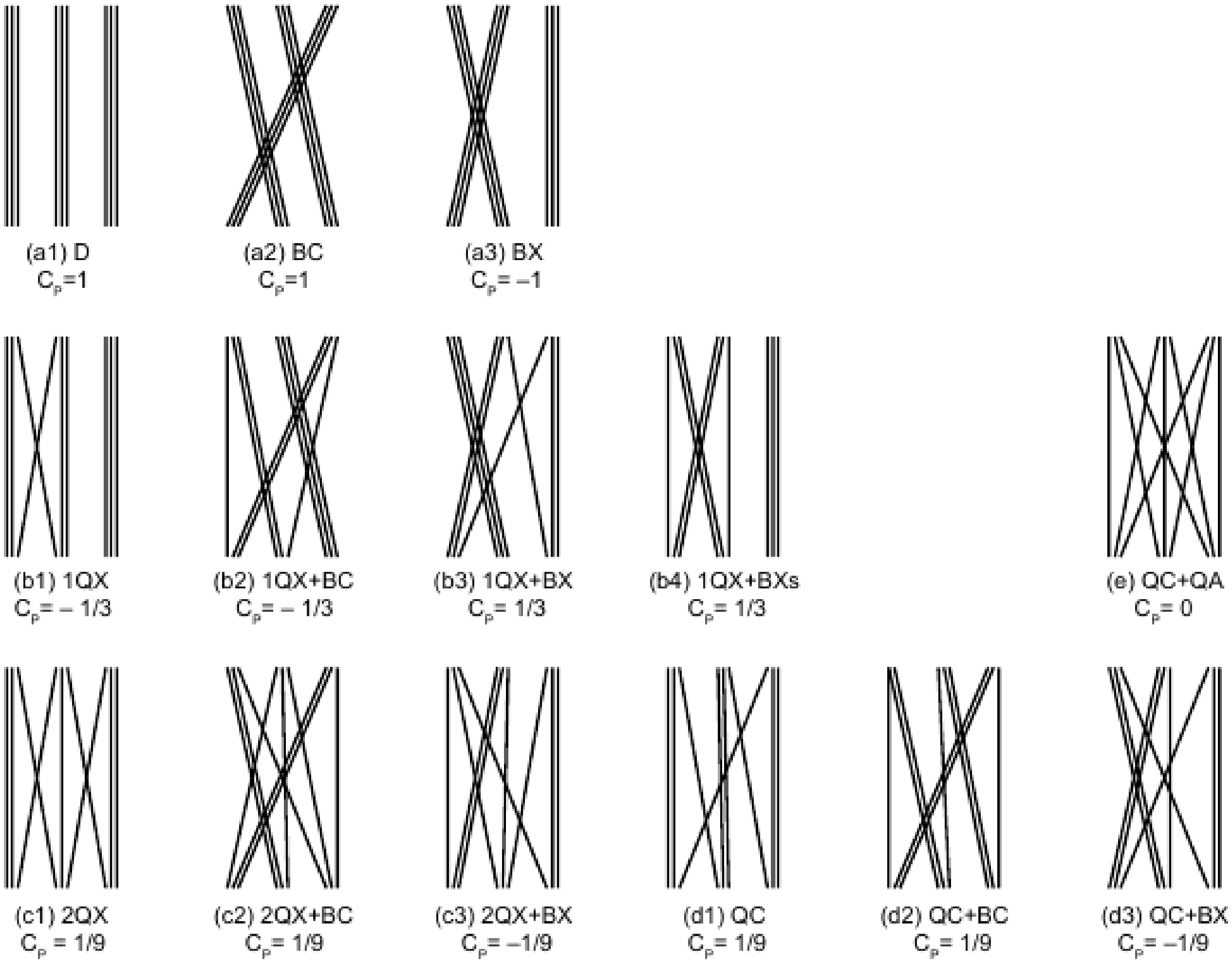
Toki, Suzuki, Hecht ('82)

- **Recent work**

Nakamoto, Suzuki ('16)

→ Norm kernel

of 3 octet B

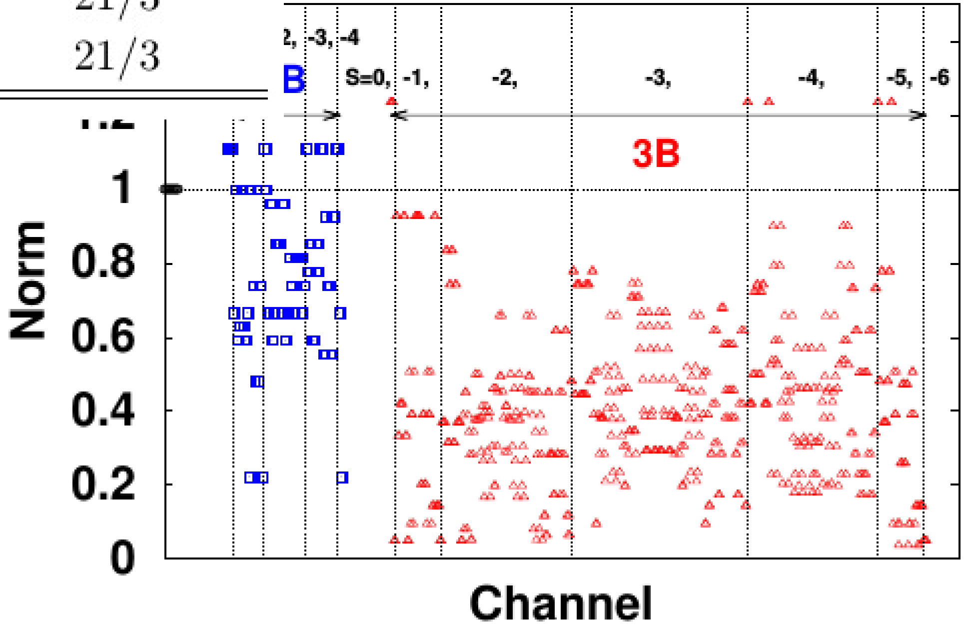


(a1) D $C_P=1$
(a2) BC $C_P=1$
(a3) BX $C_P=-1$
(b1) 1QX $C_P=-1/3$
(b2) 1QX+BC $C_P=-1/3$
(b3) 1QX+BX $C_P=1/3$
(b4) 1QX+BXs $C_P=1/3$
(e) QC+QA $C_P=0$
(c1) 2QX $C_P=1/9$
(c2) 2QX+BC $C_P=1/9$
(c3) 2QX+BX $C_P=-1/9$
(d1) QC $C_P=1/9$
(d2) QC+BC $C_P=1/9$
(d3) QC+BX $C_P=-1/9$

Norm Kernel

Baryon(s)	$\mathcal{N}_\mathcal{A}$	$\mathcal{T}_\mathcal{A}$	$\mathcal{T}$	$\mathcal{T}_{nB}(n=2,3)$
$(NN)_{(S,T)=(0,1),(1,0)}$	$10/9$	0	0	0
$N_\uparrow\Lambda_\uparrow, N_\downarrow\Lambda_\downarrow$	1	$20/3$	$20/3$	$20/3$
$N_\uparrow\Lambda_\downarrow, N_\downarrow\Lambda_\uparrow$	1	$10/3$	$10/3$	$10/3$
$(\Lambda\Lambda)_{S=0}$	1	$18/3$	$18/3$	$18/3$
$(NNN)_{(S,T)=(1/2,1/2)}$	$100/81$	0	0	0
$n_\uparrow n_\downarrow\Lambda, p_\uparrow p_\downarrow\Lambda$	$25/27$	$350/27$	14	$12/3$
$n_\uparrow p_\uparrow\Lambda_\uparrow, n_\downarrow p_\downarrow\Lambda_\downarrow$	$25/27$	$750/27$	30	$50/3$
$n_\uparrow p_\uparrow\Lambda_\downarrow, n_\downarrow p_\downarrow\Lambda_\uparrow$	$25/27$	$250/27$	10	$10/3$
$n_\uparrow p_\downarrow\Lambda, n_\downarrow p_\uparrow\Lambda$	$25/27$	$425/27$	17	$21/3$
$N\Lambda_\uparrow\Lambda_\downarrow$	$45/54$	$1035/54$	23	$21/3$

Not very small



AO, Kashiwa, Morita ('16)

KMT matrix element

- **Reduction of KMT interaction to 3 quark pot.**

$$V_{\mathrm{KMT}} \simeq -2g_{\mathrm{D}} \int d^3x\, \varepsilon_{ijk}\, u^\dagger(\boldsymbol{x}) q_i(\boldsymbol{x})\, d^\dagger(\boldsymbol{x}) q_j(\boldsymbol{x})\, s^\dagger(\boldsymbol{x}) q_k(\boldsymbol{x})$$

$$= -2g_{\mathrm{D}} \boxed{\varepsilon_{ijk} \sum_{\{\alpha,\beta,\gamma\}} \hat{T}^{u,i}_{\alpha}\, \hat{T}^{d,j}_{\beta}\, \hat{T}^{s,k}_{\gamma}} \delta(\boldsymbol{x}_\alpha - \boldsymbol{x}_\beta)\delta(\boldsymbol{x}_\beta - \boldsymbol{x}_\gamma)$$

- **Flavor exchanging operator**

$$\hat{\mathcal{T}}^{\mathrm{KMT}} = \sum_{\{\alpha,\beta,\gamma\}} \varepsilon_{ijk}\, \hat{T}^{u,i}_{\alpha}\, \hat{T}^{d,j}_{\beta}\, \hat{T}^{s,k}_{\gamma}$$

$$\mathcal{T}_A \equiv \langle \psi_A \,|\, \hat{\mathcal{T}}^{\mathrm{KMT}} \,|\, \psi_A \rangle \quad \mathcal{T} = \mathcal{T}_A / \mathcal{N}_A$$

- **Subtract the two-body part**

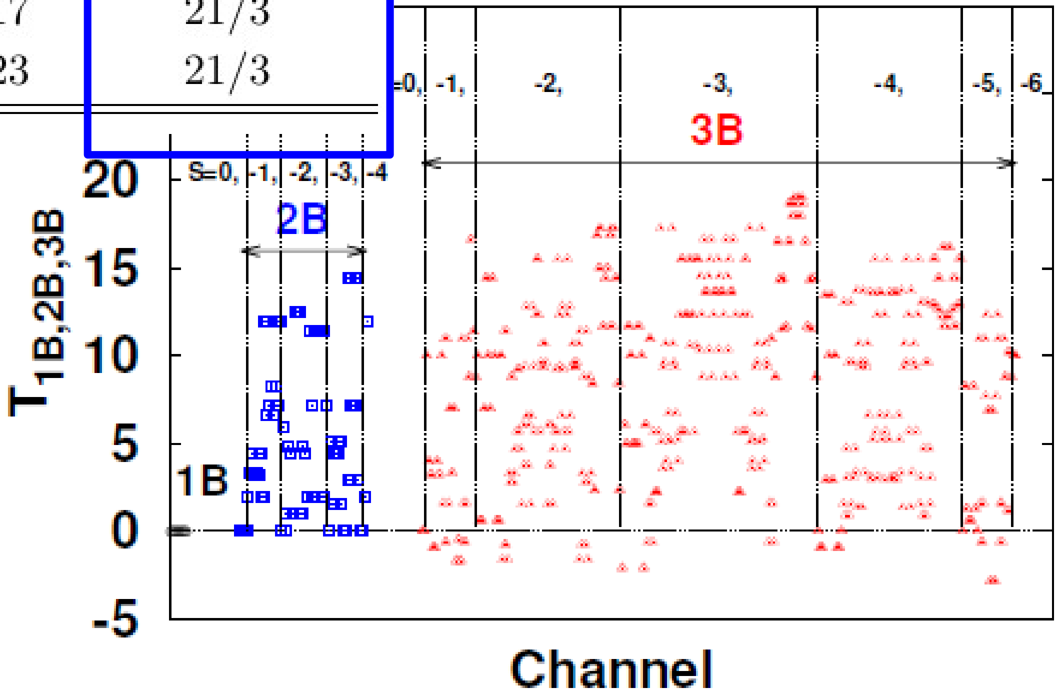
$$\mathcal{T}_{3B}(n_\uparrow n_\downarrow \Lambda_\uparrow) = \mathcal{T}(n_\uparrow n_\downarrow \Lambda_\uparrow) - \mathcal{T}(n_\uparrow \Lambda_\uparrow) - \mathcal{T}(n_\downarrow \Lambda_\uparrow)$$

KMT matrix element

Baryon(s)	$\mathcal{N}_\mathcal{A}$	$\mathcal{T}_\mathcal{A}$	$\mathcal{T}$	$\mathcal{T}_{nB}(n=2,3)$
$(NN)_{(S,T)=(0,1),(1,0)}$	10/9	0	0	0
$N_\uparrow\Lambda_\uparrow, N_\downarrow\Lambda_\downarrow$	1	20/3	20/3	20/3
$N_\uparrow\Lambda_\downarrow, N_\downarrow\Lambda_\uparrow$	1	10/3	10/3	10/3
$(\Lambda\Lambda)_{S=0}$	1	18/3	18/3	18/3
$(NNN)_{(S,T)=(1/2,1/2)}$	100/81	0	0	0
$n_\uparrow n_\downarrow\Lambda, p_\uparrow p_\downarrow\Lambda$	25/27	350/27	14	12/3
$n_\uparrow p_\uparrow\Lambda_\uparrow, n_\downarrow p_\downarrow\Lambda_\downarrow$	25/27	750/27	30	50/3
$n_\uparrow p_\uparrow\Lambda_\downarrow, n_\downarrow p_\downarrow\Lambda_\uparrow$	25/27	250/27	10	10/3
$n_\uparrow p_\downarrow\Lambda, n_\downarrow p_\uparrow\Lambda$	25/27	425/27	17	21/3
$N\Lambda_\uparrow\Lambda_\downarrow$	45/54	1035/54	23	21/3

Big for npΛ (S=3/2)

KMT matrix elements strongly depend on the channel



3B potential from KMT interaction

- **Reduction of KMT interaction to 3 quark pot.**

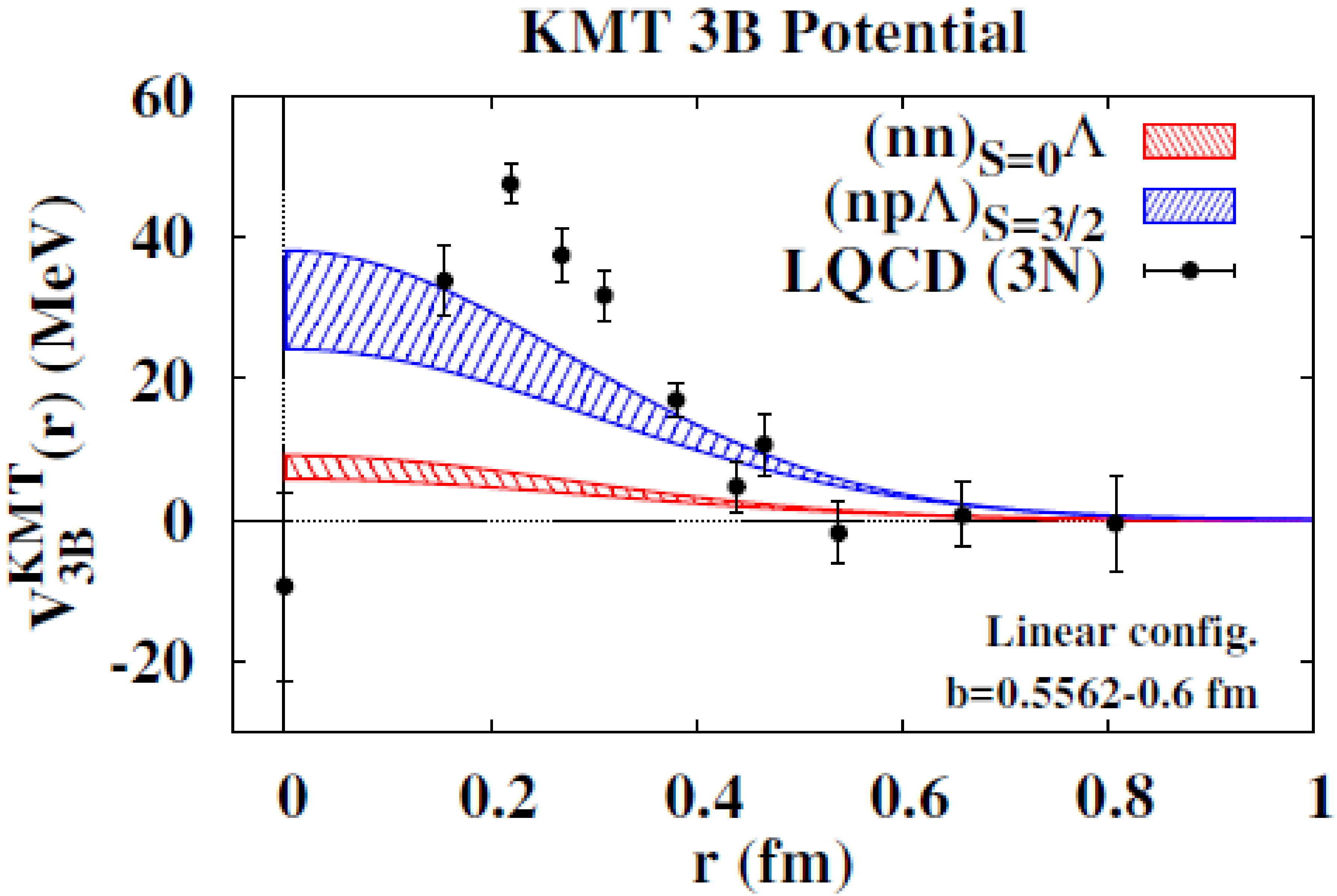
$$V_{3B}^{KMT} = -2 g_D T_{3B} \int d^3x\, \varphi^*_{R_a}(x)\varphi^*_{R_b}(x)\varphi^*_{R_c}(x)\varphi_{R_d}(x)\varphi_{R_e}(x)\varphi_{R_f}(x)$$

$$V_{3B}^{\mathrm{KMT}}(\boldsymbol{R}_1,\boldsymbol{R}_2,\boldsymbol{R}_3) \simeq V_0 T_{3B} \exp\left[-\frac{2\nu}{3}(\boldsymbol{R}_{12}^2+\boldsymbol{R}_{23}^2+\boldsymbol{R}_{31}^2)\right]$$

$$V_0 \equiv \frac{-2g_{\mathrm{D}}}{(\sqrt{3}\pi b^2)^3} = \frac{-2g_{\mathrm{D}}\Lambda^5}{(\sqrt{3}\pi b^2\Lambda^2)^3}\Lambda = \begin{cases} 1.45\ \mathrm{MeV} & (b = 0.6\ \mathrm{fm})\ , \\ 2.29\ \mathrm{MeV} & (b = 0.5562\ \mathrm{fm})\ . \end{cases}$$

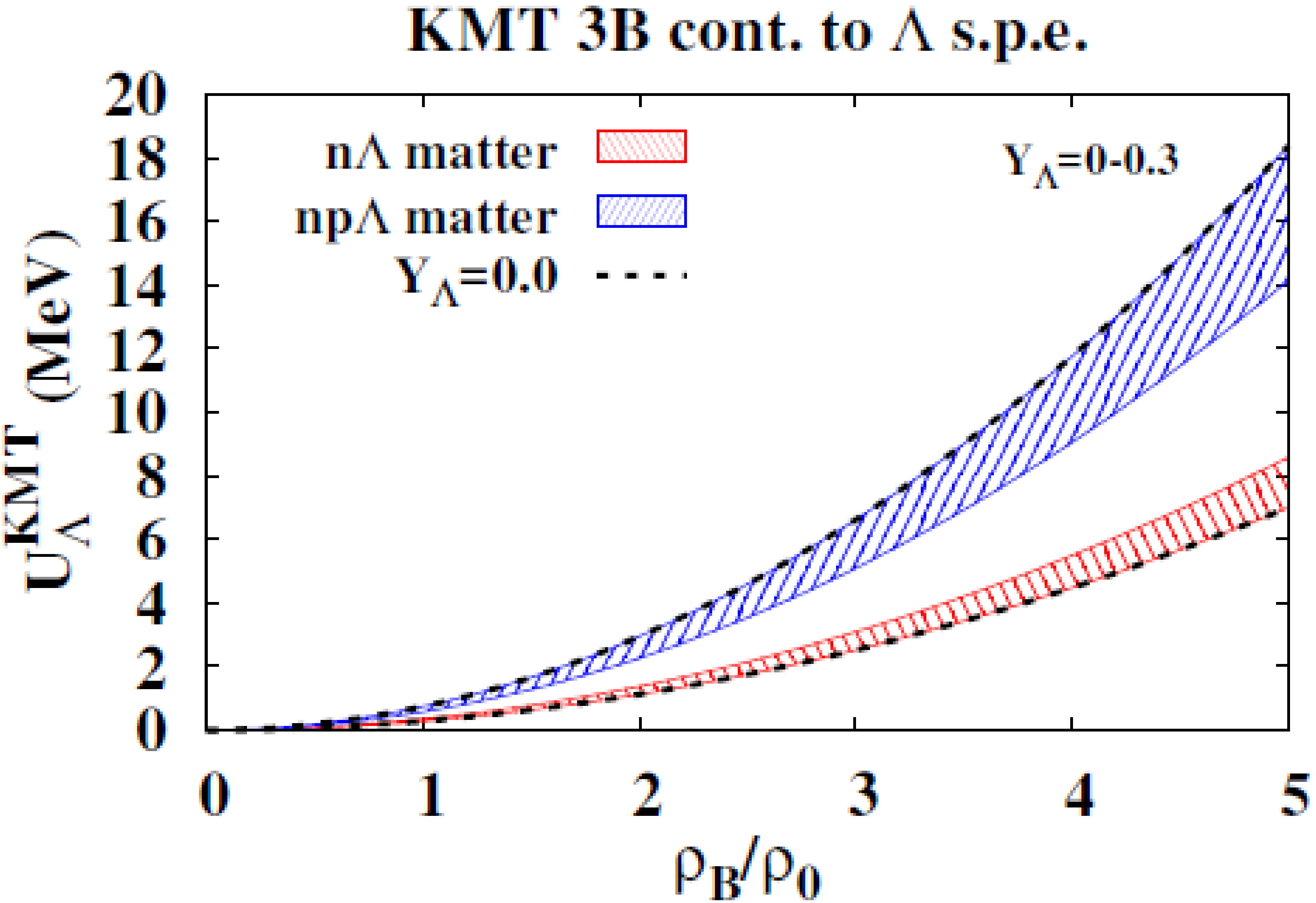
Parameters are taken from
Hatsuda, Kunihiro ('94), Rehberg, Klevanski, Hufner ('96), Fujiwara, Suzuki, Nakamoto ('07), Oka, Yazaki ('81)

3B potential from KMT interaction



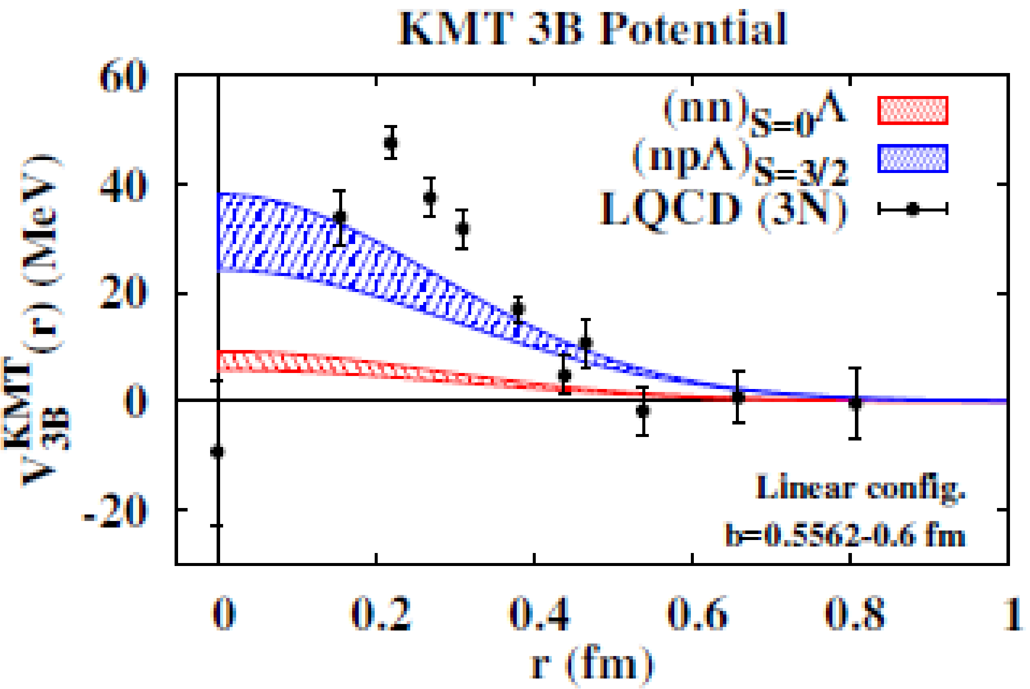
Lattice data: Doi et al. (HAL QCD) ('07)

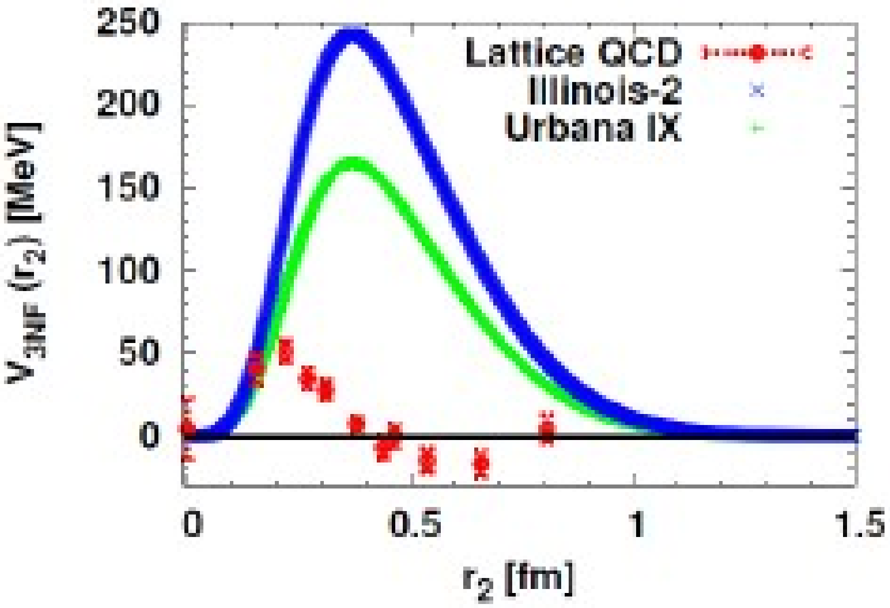
KMT-3B Contribution to Λ potential



Density is assumed to be uniform. No correlation effects.

3B potential from KMT: Repulsive enough ?





Summary

- Quark model three-baryon (3B) potential may be a promising method to evaluate the 3B potential at short distances.
- Kobayashi-Maskawa-'t Hooft (KMT) interaction generates 3q potential among u,d,s quarks, and generates 3B potential only when hyperons are involved.
- Expectation value of the KMT interaction is evaluated in the cases where 3B are located at the same spatial point.
  Matrix elements strongly depend on the baryon trio.
- 3B potential from KMT interaction is obtained.
  - It is comparable in strength to the lattice 3N potential.
  - More repulsive in npΛ than in nnΛ
    (Negative contribution to symmetry energy.)
- 3B pot. from KMT is not strong enough to solve the hyperon puzzle, but contributes to hyperon suppression.

Thank you for your attention !