Production of clusters and pions in heavy-ion collisions

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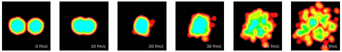
- Clusters (d, t, ^{3}He, α) in heavy-ion collisions and in AMD
- Pion production based on AMD calculation (+JAM)
- Recent progress in the comparison of many transport codes

Various densities in heavy-ion collisions

Nuclear EOS (at $T=0$)

$$(E/A)(\rho_p,\rho_n) = (E/A)_0(\rho) + S(\rho)\delta^2 + \cdots$$

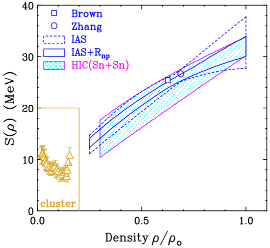
$$\rho = \rho_p + \rho_n, \quad \delta = (\rho_n - \rho_p)/(\rho_n + \rho_p).$$



^{132}Sn + ^{124}Sn, 300 MeV/u, $b \sim 0$

Constrains on $S(\rho)$ at ICNT2013 @FRIB

Horowitz et al., J. Phys. G: Nucl. Part. Phys. 41 (2014) 093001.



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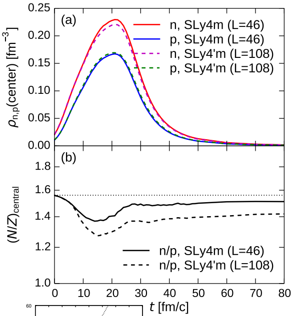
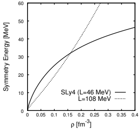
$\sim \frac{1}{10}\rho_0$ $\quad$ $\rho > \rho_0$





Probing high-density dynamics by pions

Bao-An Li, PRL88 (2002) 192701.

Pion ratio π^-/π^+ has been proposed as a good probe of

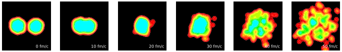
- symmetry energy at high densities
- ρ_n/ρ_p ratio in the high density region

Production/absorption of Δ and π: ($N = n$ or p)

$$
\begin{aligned}
N+N &\to N+\Delta \\
N+N &\leftarrow N+\Delta \\
\Delta &\to N+\pi \\
\Delta &\leftarrow N+\pi
\end{aligned}
$$

Simple expectation: $\pi^-/\pi^+ \approx (N/Z)^2$

- First-chance $NN \to N\Delta \to NN\pi$
- Chemical equilibrium



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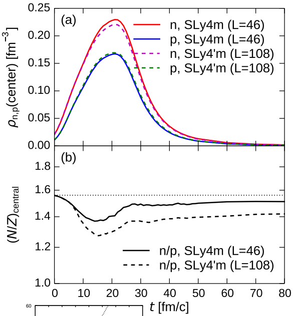
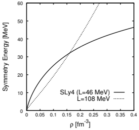
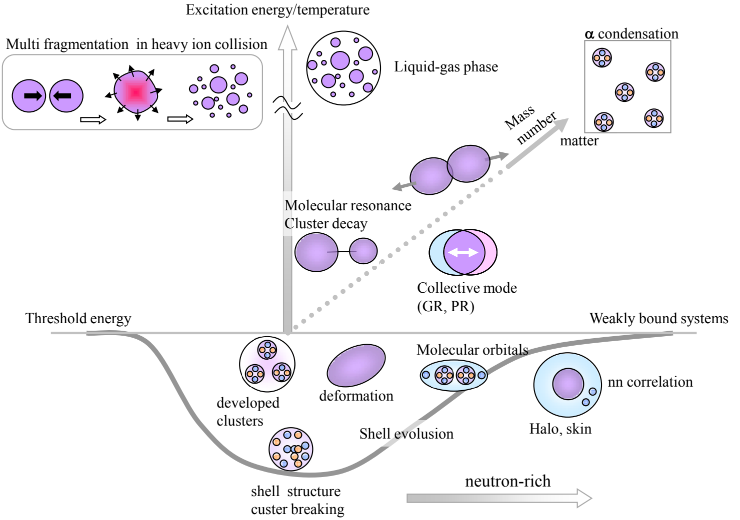




Clustering phenomena in excited states of nuclear systems



Kanada-En'yo, Kimura, Ono, Prog. Theor. Exp. Phys. 2012 01A202 (2012)

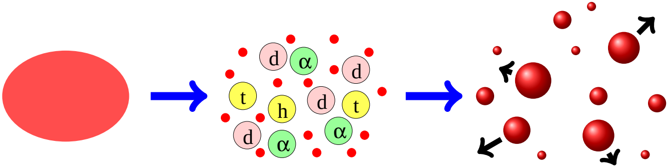
Interacting and reacting clusters in heavy-ion collisions

INDRA data, Hudan et al., PRC67 (2003) 064613.

FOPI data, Reisdorf et al., NPA 848 (2010) 366.

Partitioning of protons

	Xe + Sn 50 MeV/u	Au + Au 250 MeV/u
p	≈10%	21%
α	≈20%	20%
d, t, ^{3}He	≈10%	40%
$A > 4$	≈60%	18%



- The system may be composed of many clusters.
- Clusters are not only created but also broken by reactions.

- $n+p+X \leftrightarrow d+X'$
- $d+n+X \leftrightarrow t+X'$
- $d+p+X \leftrightarrow h+X'$
- $t+p+X \leftrightarrow \alpha+X'$
- $h+n+X \leftrightarrow \alpha+X'$
- $d+d+X \leftrightarrow \alpha+X'$
- $2n+p+X \leftrightarrow t+X'$
- $n+2p+X \leftrightarrow h+X'$
- $d+n+p+X \leftrightarrow \alpha+X'$
- $2n+2p+X \leftrightarrow \alpha+X'$
- $d+d \leftrightarrow p+t$
- $d+d \leftrightarrow n+h$
- $p+t \leftrightarrow n+h$
- $d+t \leftrightarrow n+\alpha$
- $d+h \leftrightarrow p+\alpha$
- $d+t \leftrightarrow 2n+h$
- $d+h \leftrightarrow 2p+t$
- $d+\alpha \leftrightarrow t+h$

Transport models with clusters

+decay

Transport with clusters (pBUU)

BUU with clusters

Danielewicz and Bertsch, NPA 533 (1991) 712.

Coupled equations for $f_n(\mathbf{r},\mathbf{p},t)$, $f_p(\mathbf{r},\mathbf{p},t)$, $f_d(\mathbf{r},\mathbf{p},t)$, $f_t(\mathbf{r},\mathbf{p},t)$, $f_h(\mathbf{r},\mathbf{p},t)$ are solved by the test particle method.

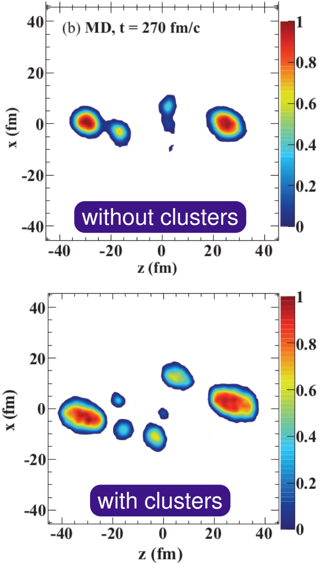
$$\frac{\partial f_n}{\partial t} + \mathbf{v}\cdot\frac{\partial f_n}{\partial \mathbf{r}} - \frac{\partial U_n}{\partial \mathbf{r}}\cdot\frac{\partial f_n}{\partial \mathbf{p}} = I_n^{\text{coll}}[f_n, f_p, f_d, f_t, f_h]$$

$$\frac{\partial f_p}{\partial t} + \mathbf{v}\cdot\frac{\partial f_p}{\partial \mathbf{r}} - \frac{\partial U_p}{\partial \mathbf{r}}\cdot\frac{\partial f_p}{\partial \mathbf{p}} = I_p^{\text{coll}}[f_n, f_p, f_d, f_t, f_h]$$

$$\frac{\partial f_d}{\partial t} + \mathbf{v}\cdot\frac{\partial f_d}{\partial \mathbf{r}} - \frac{\partial U_d}{\partial \mathbf{r}}\cdot\frac{\partial f_d}{\partial \mathbf{p}} = I_d^{\text{coll}}[f_n, f_p, f_d, f_t, f_h]$$

$$\frac{\partial f_t}{\partial t} + \mathbf{v}\cdot\frac{\partial f_t}{\partial \mathbf{r}} - \frac{\partial U_t}{\partial \mathbf{r}}\cdot\frac{\partial f_t}{\partial \mathbf{p}} = I_t^{\text{coll}}[f_n, f_p, f_d, f_t, f_h]$$

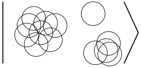
$$\frac{\partial f_h}{\partial t} + \mathbf{v}\cdot\frac{\partial f_h}{\partial \mathbf{r}} - \frac{\partial U_h}{\partial \mathbf{r}}\cdot\frac{\partial f_h}{\partial \mathbf{p}} = I_h^{\text{coll}}[f_n, f_p, f_d, f_t, f_h]$$



Coupland, Lynch, Tsang, Danielewicz, Zhang, PRC 84 (2011) 054603.

Antisymmetrized Molecular Dynamics (very basic version)

Ono, Horiuchi, Maruyama, Ohnishi, Prog. Theor. Phys. 87 (1992) 1185.



AMD wave function

$$|\Phi(Z)\rangle = \det_{ij}\left[\exp\left\{-\nu\left(\mathbf{r}_j - \frac{\mathbf{Z}_i}{\sqrt{\nu}}\right)^2\right\}\chi_{\alpha_i}(j)\right]$$

$$\mathbf{Z}_i = \sqrt{\nu}\mathbf{D}_i + \frac{i}{2\hbar\sqrt{\nu}}\mathbf{K}_i$$

ν : Width parameter = (2.5 fm)$^{-2}$

χ_{α_i} : Spin-isospin states = $p\uparrow, p\downarrow, n\uparrow, n\downarrow$

Equation of motion for the wave packet centroids Z

$$\frac{d}{dt}\mathbf{Z}_i = \{\mathbf{Z}_i, \mathcal{H}\}_{\text{PB}} \quad + \quad \text{(NN collisions)}$$

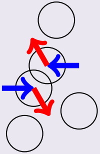
$\{\mathbf{Z}_i, \mathcal{H}\}_{\text{PB}}$: Motion in the mean field

$$\mathcal{H} = \frac{\langle\Phi(Z)|H|\Phi(Z)\rangle}{\langle\Phi(Z)|\Phi(Z)\rangle} + \text{(c.m. correction)}$$

H: Effective interaction (e.g. Skyrme force)

NN collisions

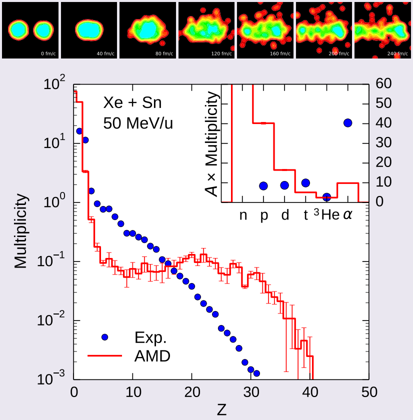
$$W_{i\to f} = \frac{2\pi}{\hbar}|\langle\Psi_f|V|\Psi_i\rangle|^2\delta(E_f - E_i)$$



- $|V|^2$ or σ_{NN} (in medium)
- Pauli blocking

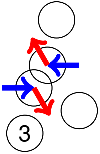
AMD with usual NN collisions (very basic version)

Central Xe + Sn at 50 MeV/u



NN collisions without or with cluster correlations

$$W_{i\to f} = \frac{2\pi}{\hbar}|\langle\Psi_f|V|\Psi_i\rangle|^2\delta(E_f - E_i)$$

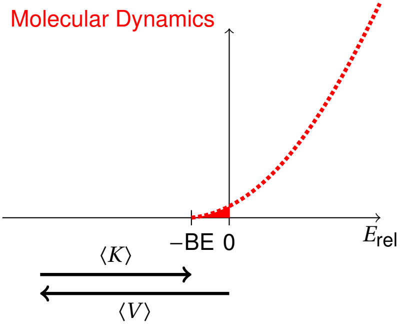


In the usual way of NN collision, only the two wave packets are changed.

$$\left\{|\Psi_f\rangle\right\} = \left\{|\varphi_{k_1}(1)\varphi_{k_2}(2)\Psi(3,4,\ldots)\rangle\right\}$$

(ignoring antisymmetrization for simplicity of presentation.)

Phase space or the density of states for two nucleon system



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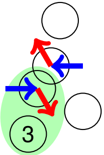

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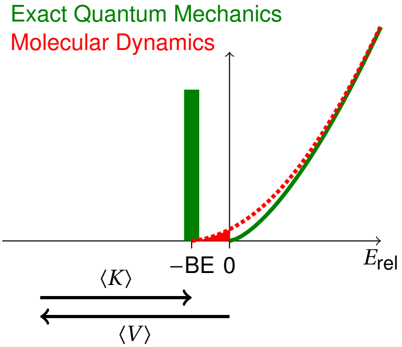
(ignoring antisymmetrization for simplicity of presentation.)

Extension for cluster correlations

Include correlated states in the set of the final states of each NN collision.

$$\{|\Psi_f\rangle\} \ni |\varphi_{k_1}(1)\psi_d(2,3)\Psi(4,\ldots)\rangle,\ \ldots$$

Phase space or the density of states for two nucleon system

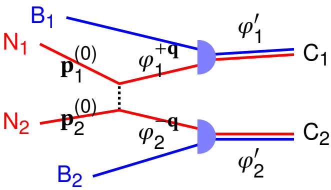


NN collisions with cluster correlations

Similar to Danielewicz et al., NPA533 (1991) 712.

$$\mathrm{N}_1 + \mathrm{B}_1 \; + \; \mathrm{N}_2 + \mathrm{B}_2 \; \to \mathrm{C}_1 + \mathrm{C}_2$$

- $\mathrm{N}_1, \mathrm{N}_2$: Colliding nucleons
- $\mathrm{B}_1, \mathrm{B}_2$: Spectator nucleons/clusters
- $\mathrm{C}_1, \mathrm{C}_2$: N, $(2N)$, $(3N)$, $(4N)$ (up to α cluster)



Transition probability

$$W(\mathrm{NBNB} \to \mathrm{CC}) = \frac{2\pi}{\hbar} |\langle \mathrm{CC}|V|\mathrm{NBNB}\rangle|^2 \delta(E_f - E_i)$$

$$v\,d\sigma \propto |\langle \varphi_1' | \varphi_1^{+\mathbf{q}} \rangle|^2 |\langle \varphi_2' | \varphi_2^{-\mathbf{q}} \rangle|^2 |M|^2 \delta(E_f - E_i) p_{\mathrm{rel}}^2 \, dp_{\mathrm{rel}} \, d\Omega$$

$|M|^2 = |\langle \mathrm{NN}|V|\mathrm{NN}\rangle|^2$: Matrix elements of NN scattering

$\Leftarrow (d\sigma/d\Omega)_{\mathrm{NN}}$ in medium (or in free space)

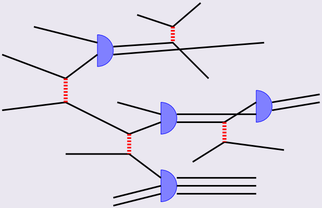
$$\mathbf{p}_{\mathrm{rel}} = \tfrac{1}{2}(\mathbf{p}_1 - \mathbf{p}_2) = p_{\mathrm{rel}} \hat{\boldsymbol{\Omega}}$$

$$\mathbf{q} = \mathbf{p}_1 - \mathbf{p}_1^{(0)} = \mathbf{p}_2^{(0)} - \mathbf{p}_2$$

$$\varphi_1^{+\mathbf{q}} = \exp(+i\mathbf{q}\cdot\mathbf{r}_{\mathrm{N}_1})\varphi_1^{(0)}$$

$$\varphi_2^{-\mathbf{q}} = \exp(-i\mathbf{q}\cdot\mathbf{r}_{\mathrm{N}_2})\varphi_2^{(0)}$$

NN collisions with cluster correlations (more explanations)



For each NN collision, cluster formation is considered.

$$\mathrm{N}_1 + \mathrm{B}_1 \ + \ \mathrm{N}_2 + \mathrm{B}_2 \ \to \mathrm{C}_1 + \mathrm{C}_2$$

$$W_{i\to f} = \frac{2\pi}{\hbar}|\langle \mathrm{CC}|V_{NN}|\mathrm{NBNB}\rangle|^2 \delta(E_f - E_i)$$

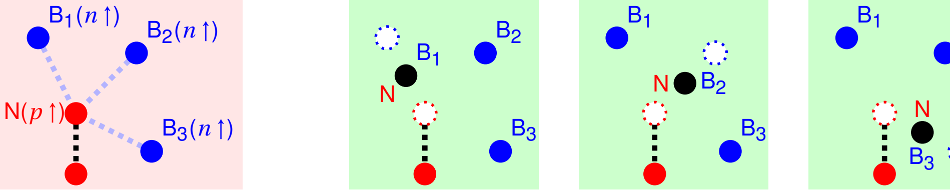
Ono, J. Phys. Conf. Ser. 420 (2013) 012103

Ikeno, Ono et al., PRC 93 (2016) 044612

- We always have a Slater determinant of nucleon wave packets. A cluster in the final states is represented by placing wave packets at the same phase space point.
- Consequently the processes such as $d + X \to n + p + X'$ and $d + X \to d + X'$ are automatically taken into account.
- No parameters have been introduced to adjust individual reactions.
- There are many possibilities to from clusters in the final states. Non-orthogonality of the final states should be carefully handled.

Construction of Final States

Clusters (in the final states) are assumed to have $(0s)^N$ configuration.



$|\Phi^{\mathbf{q}}\rangle$ After $\mathbf{p}^{(0)} \to \mathbf{p}^{(0)} + \mathbf{q}$

$|\Phi'_1\rangle$ N + B$_1$ → C$_1$

$|\Phi'_2\rangle$ N + B$_2$ → C$_2$

$|\Phi'_3\rangle$ N + B$_3$ → C

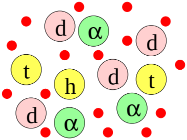
Final states are not orthogonal: $N_{ij} \equiv \langle \Phi'_i | \Phi'_j \rangle \neq \delta_{ij}$

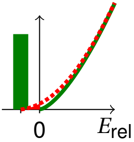
The probability of cluster formation with one of B's:

$$\hat{P} = \sum_{ij} |\Phi'_i\rangle N^{-1}_{ij} \langle \Phi'_j|, \qquad P = \langle \Phi^{\mathbf{q}} | \hat{P} | \Phi^{\mathbf{q}} \rangle \qquad \neq \sum_i |\langle \Phi'_i | \Phi^{\mathbf{q}} \rangle|^2$$

$$\begin{cases} P & \Rightarrow \text{Choose one of the candidates and make a cluster.} \\ 1 - P & \Rightarrow \text{Don't make a cluster (with any n}\uparrow\text{).} \end{cases}$$

Correlations to bind several clusters

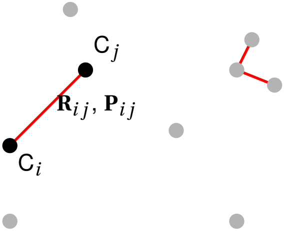




Clusters may form a loosely bound state.

e.g., $^7\mathrm{Li} = \alpha + t - 2.5$ MeV

Need more probability of $|\alpha + t\rangle \to |^7\mathrm{Li}\rangle$



Step 1 Clusters (and nucleons) C_i and C_j are *linked*,

- if C_i is one of the 3 clusters closest to C_j, and $(i \leftrightarrow j)$,
- and if the distance is 1 fm $< |\mathbf{R}_{ij}| < 7$ fm,
- and if they are slowly moving away, $\mathbf{P}_{ij}^2/2\mu_{ij} <$ 10 MeV and $\mathbf{R}_{ij} \cdot \mathbf{P}_{ij} > 0$.

Step 2 Linked clusters (CC) are identified. Following steps are taken only for CC with mass number $6 \le A \le 9$ or $19 \le A \le 23$.

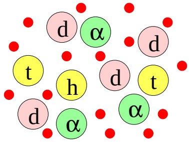
Step 3 *Transition* of the internal state of CC by eliminating the (radial component of) internal momentum

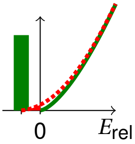
$$\mathbf{P}_i \to 0 \quad \text{for } i \in \text{CC in the c.m. of CC}$$

with some care of the momentum conservation.

Next *Energy conservation*.

Correlations to bind several clusters





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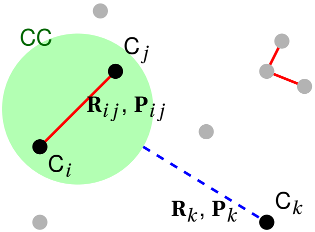


Step 4 Search a third particle for E-conservation

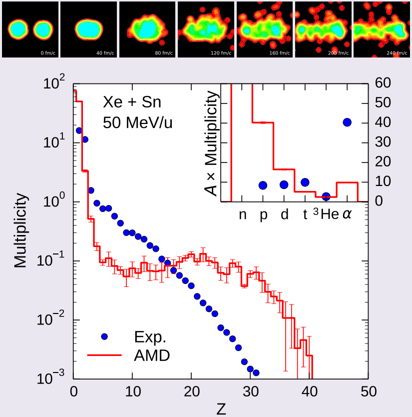
- A cluster C_k is selected, depending on the distance and momentum ($|\mathbf{R}_k|$ and $|\mathbf{P}_k|$) relative to CC.
- If the selected C_k already belongs to a CC′, this whole CC′ is treated as the third particle for E-conservation.

Step 5 Scale the radial component of the relative momentum between CC and C_k for the total energy conservation.

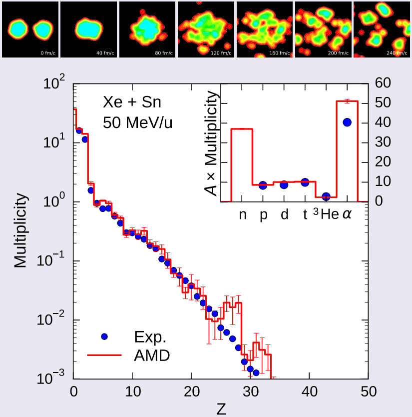
$$\mathbf{P}_k = \mathbf{P}_{k\parallel} + \mathbf{P}_{k\perp} \to \beta \mathbf{P}_{k\parallel} + \mathbf{P}_{k\perp}$$

Effect of cluster correlations: central Xe + Sn at 50 MeV/u

Without clusters

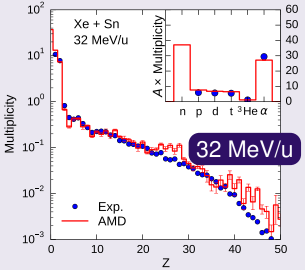


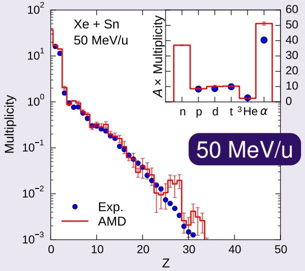
With clusters



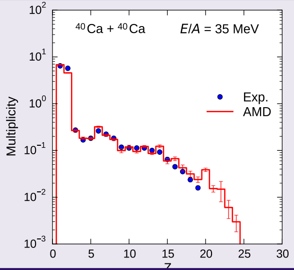
Results for multifragmentation in central collisions

Xe + Sn

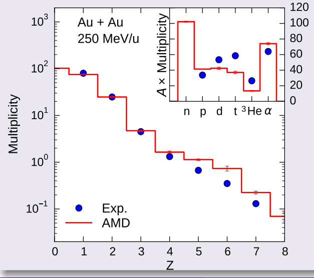




Ca + Ca at 35 MeV/u

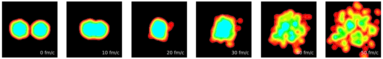


Au + Au at 250 MeV/u

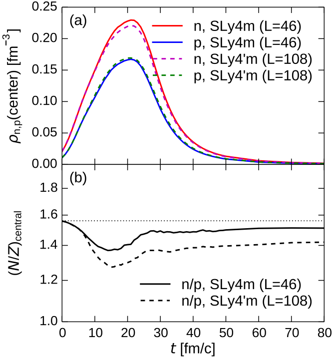


Data:
Hudan et al., PRC 67 (2003) 064613.
Hagel et al., PRC 50 (1994) 2017.
Reisdorf et al., NPA 848 (2010) 366.

Collisions at higher energies



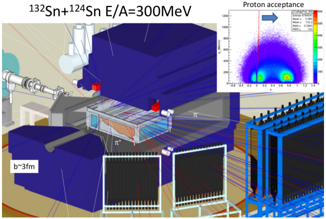
^{132}Sn + ^{124}Sn, $E/A = 300$ MeV, $b \sim 0$



Momentum dependence of Skyrme (SLy4) interaction has been corrected for high energy collisions, in a similar way to Gale, Bertsch, Das Gupta, PRC 35 (1987) 1666.

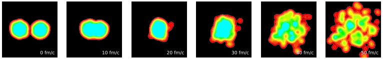
- Pions are measured to probe high-density nuclear matter by the SπRIT experment.
- Many protons are bound in clusters. In Au+Au at 250 MeV/u (FOPI data), p: 21%, α: 20%, d+t+^{3}He: 40%

The SπRIT project: TPC in SAMURAI magnet

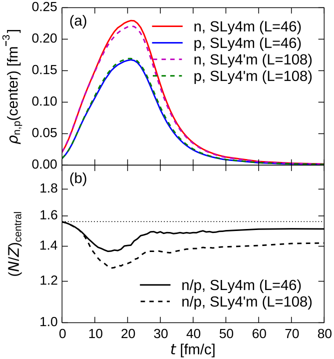


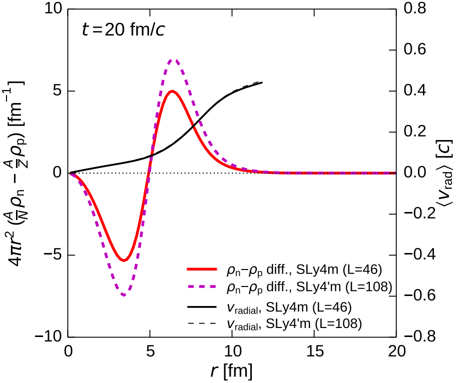
Talk by T. Isobe at NuSYM15

Collisions at higher energies



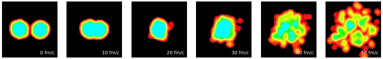
$^{132}\mathrm{Sn} + {}^{124}\mathrm{Sn}$, $E/A = 300$ MeV, $b \sim 0$



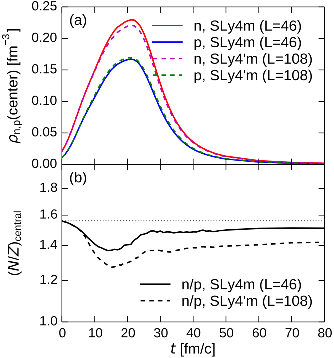


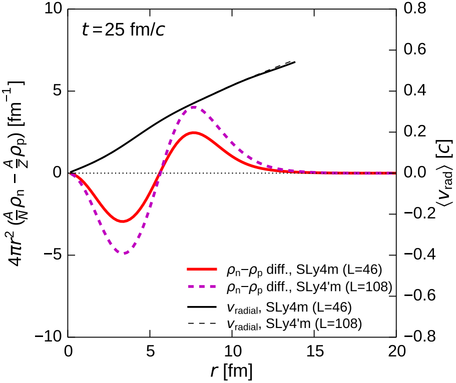
Momentum dependence of Skyrme (SLy4) interaction has been corrected for high energy collisions, in a similar way to Gale, Bertsch, Das Gupta, PRC 35 (1987) 1666.

Collisions at higher energies



$^{132}\mathrm{Sn} + {}^{124}\mathrm{Sn}$, $E/A = 300$ MeV, $b \sim 0$





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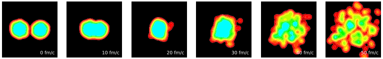
Collisions at higher energies



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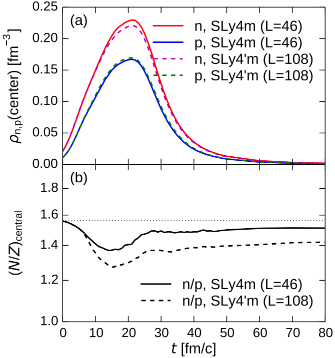

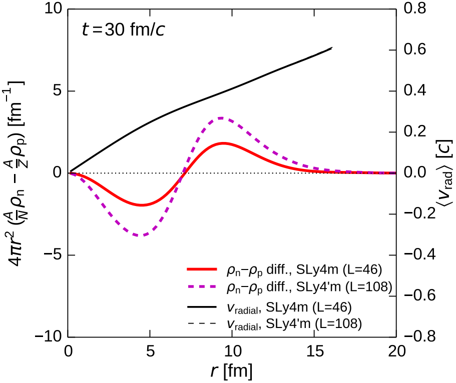

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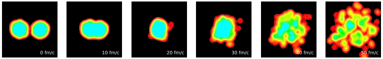
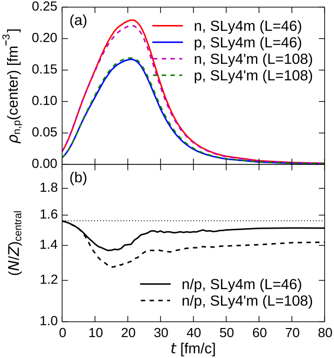
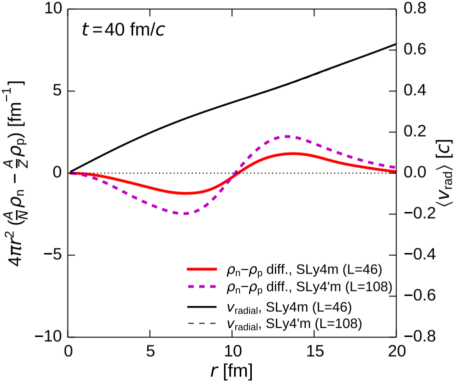


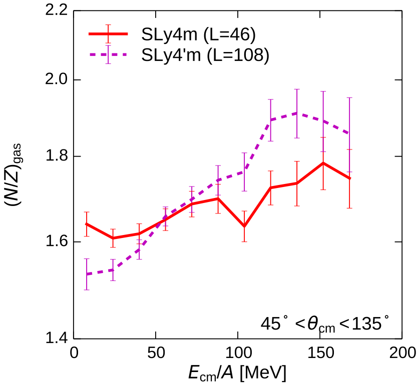
^{132}Sn + ^{124}Sn, $E/A = 300$ MeV, $b \sim 0$

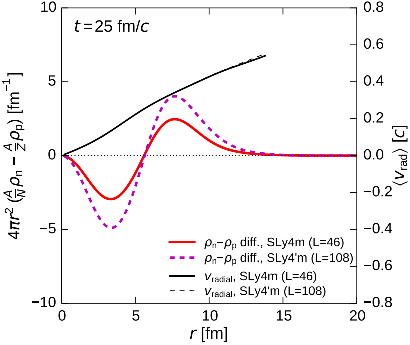




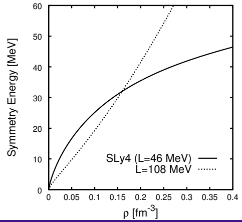
Momentum dependence of Skyrme (SLy4) interaction has been corrected for high energy collisions, in a similar way to Gale, Bertsch, Das Gupta, PRC 35 (1987) 1666.

N/Z Spectrum Ratio (AMD with clusters)

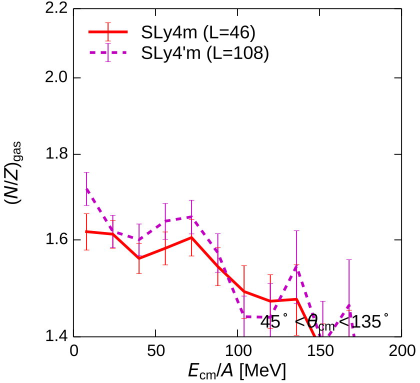


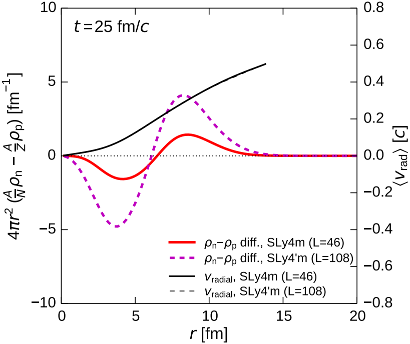


$$\left(\frac{N}{Z}\right)_{gas} = \frac{Y_n(v) + Y_d(v) + 2Y_t(v) + Y_h(v) + 2Y_\alpha(v)}{Y_p(v) + Y_d(v) + Y_t(v) + 2Y_h(v) + 2Y_\alpha(v)}$$

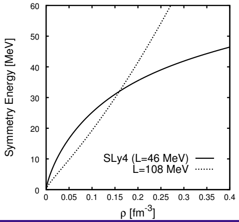


N/Z of spectrum of emitted particles is similar to the neutron-proton density difference at the compression stage.

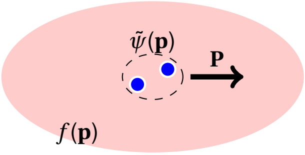




$$\left(\frac{N}{Z}\right)_{\text{gas}} = \frac{Y_n(v) + Y_d(v) + 2Y_t(v) + Y_h(v) + 2Y_\alpha(v)}{Y_p(v) + Y_d(v) + Y_t(v) + 2Y_h(v) + 2Y_\alpha(v)}$$

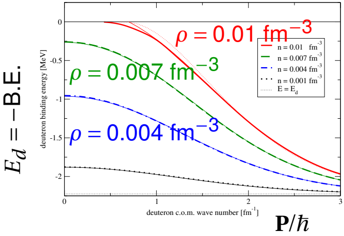


N/Z of spectrum of emitted particles is NOT similar to the neutron-proton density difference at the compression stage.



Equation for a deuteron in uncorrelated medium

$$\Big[e(\tfrac{1}{2}\mathbf{P}+\mathbf{p})+e(\tfrac{1}{2}\mathbf{P}-\mathbf{p})\Big]\tilde{\psi}(\mathbf{p}) + \Big[1-f(\tfrac{1}{2}\mathbf{P}+\mathbf{p})-f(\tfrac{1}{2}\mathbf{P}-\mathbf{p})\Big]\int\frac{d\mathbf{p}'}{(2\pi)^3}\langle\mathbf{p}|v|\mathbf{p}'\rangle\tilde{\psi}(\mathbf{p}') = E\tilde{\psi}(\mathbf{p})$$



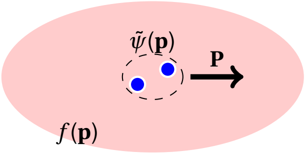
Momentum ($\mathbf{P}$) dependence of B.E.

Röpke, NPA867 (2011) 66.

- Clusters may exist even in high density matter if they have sufficient momenta so that the effect of Pauli principle is weak.
- However, clusters in high density region will be soon broken by a collisions with other particles.

Does it make sense to consider such short-lived correlations?

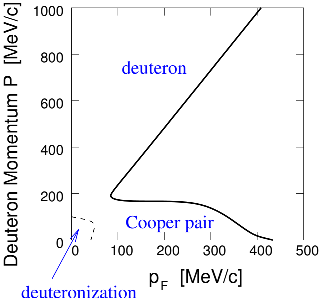
Do clusters exist in high density region?



Equation for a deuteron in uncorrelated medium

$$\left[e(\tfrac{1}{2}\mathbf{P}+\mathbf{p})+e(\tfrac{1}{2}\mathbf{P}-\mathbf{p})\right]\tilde{\psi}(\mathbf{p})$$
$$+\left[1-f(\tfrac{1}{2}\mathbf{P}+\mathbf{p})-f(\tfrac{1}{2}\mathbf{P}-\mathbf{p})\right]\int\frac{d\mathbf{p}'}{(2\pi)^3}\langle\mathbf{p}|v|\mathbf{p}'\rangle\tilde{\psi}(\mathbf{p}')$$
$$=E\tilde{\psi}(\mathbf{p})$$

Deuteron in medium (at $T=0$)



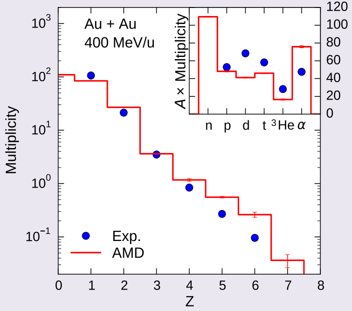
Danielewicz and Bertsch, NPA 533 (1991)

- Clusters may exist even in high density matter if they have sufficient momenta so that the effect of Pauli principle is weak.
- However, clusters in high density region will be soon broken by a collisions with other particles.

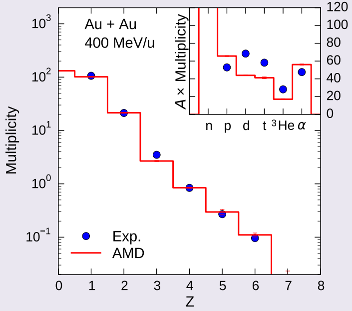
Does it make sense to consider such short-lived correlations?

Turn off clusters at high density

Always produce clusters



Produce clusters only at $\rho < \rho_0$



When the cluster production is turned off at high density $\rho > 0.16\ \mathrm{fm}^{-3}$,

- $M_p \nearrow$ (overproduction)
- $M_\alpha \searrow$
- M_d and M_t don't increase.

We need some fine tuning of the cluster production.

Transport equations for N, Δ and π

Coupled equations for $f_N(\mathbf{r},\mathbf{p},t)$, $f_\Delta(\mathbf{r},\mathbf{p},t)$, $f_\pi(\mathbf{r},\mathbf{p},t)$

$$\frac{\partial f_N}{\partial t} + \frac{\partial h_N}{\partial \mathbf{p}} \cdot \frac{\partial f_N}{\partial \mathbf{r}} - \frac{\partial h_N[f_N, f_\Delta, f_\pi]}{\partial \mathbf{r}} \cdot \frac{\partial f_N}{\partial \mathbf{p}} = I_N[f_N, f_\Delta, f_\pi] \qquad NN \to NN$$

$$\frac{\partial f_\Delta}{\partial t} + \frac{\partial h_\Delta}{\partial \mathbf{p}} \cdot \frac{\partial f_\Delta}{\partial \mathbf{r}} - \frac{\partial h_\Delta[f_N, f_\Delta, f_\pi]}{\partial \mathbf{r}} \cdot \frac{\partial f_\Delta}{\partial \mathbf{p}} = I_\Delta[f_N, f_\Delta, f_\pi] \qquad NN \leftrightarrow N\Delta$$

$$\frac{\partial f_\pi}{\partial t} + \frac{\partial h_\pi}{\partial \mathbf{p}} \cdot \frac{\partial f_\pi}{\partial \mathbf{r}} - \frac{\partial h_\pi[f_N, f_\Delta, f_\pi]}{\partial \mathbf{r}} \cdot \frac{\partial f_\pi}{\partial \mathbf{p}} = I_\pi[f_N, f_\Delta, f_\pi] \qquad \Delta \leftrightarrow N\pi$$

Assumption: Δ and pion productions are rare (in low-energy collisions), so that they can be treated as perturbation.

$$I_N[f_N, f_\Delta, f_\pi] = I_N^{\mathrm{el}}[f_N, 0, 0] + \lambda I'_N[f_N, f_\Delta, f_\pi]$$

$$f_N = f_N^{(0)} + \lambda f_N^{(1)} + \cdots, \qquad f_\Delta = O(\lambda), \quad f_\pi = O(\lambda)$$

Ikeno, Ono, Nara, Ohnishi, PRC 93 (2016) 044612.

Zeroth order equation for f_N — solved by AMD

$$\frac{\partial f_N^{(0)}}{\partial t} + \frac{\partial h_N}{\partial \mathbf{p}} \cdot \frac{\partial f_N^{(0)}}{\partial \mathbf{r}} - \frac{\partial h_N[f_N^{(0)},0,0]}{\partial \mathbf{r}} \cdot \frac{\partial f_N^{(0)}}{\partial \mathbf{p}} = I_N^{\text{el}}[f_N^{(0)},0,0]$$

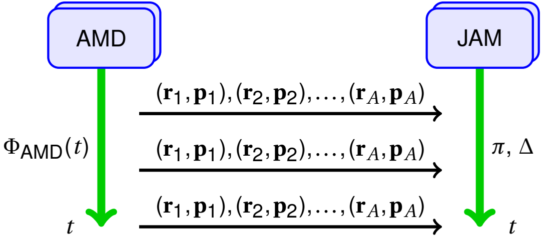
First order equations for f_Δ and f_π — solved by JAM for given $f_N^{(0)}$

$$\frac{\partial f_\Delta}{\partial t} + \frac{\partial h_\Delta}{\partial \mathbf{p}} \cdot \frac{\partial f_\Delta}{\partial \mathbf{r}} - \frac{\partial h_\Delta[f_N^{(0)}, f_\Delta, f_\pi]}{\partial \mathbf{r}} \cdot \frac{\partial f_\Delta}{\partial \mathbf{p}} = I_\Delta[f_N^{(0)}, f_\Delta, f_\pi]$$

$$\frac{\partial f_\pi}{\partial t} + \frac{\partial h_\pi}{\partial \mathbf{p}} \cdot \frac{\partial f_\pi}{\partial \mathbf{r}} - \frac{\partial h_\pi[f_N^{(0)}, f_\Delta, f_\pi]}{\partial \mathbf{r}} \cdot \frac{\partial f_\pi}{\partial \mathbf{p}} = I_\pi[f_N^{(0)}, f_\Delta, f_\pi]$$

Send nucleon test particles from AMD to JAM at every 2 fm/c, with corrections for the conservation of baryon number and charge.

Ikeno, Ono, Nara, Ohnishi, PRC 93 (2016) 044612.



About JAM

JAM: Jet AA Microscopic transport model

Y. Nara, N. Otuka, A. Ohnishi, K. Niita, S. Chiba, PRC61 (2000) 024901.

- Applied to high-energy collisions ($1 \sim 158A$ GeV)
- Hadron-Hadron reactions are based on experimental data and the detailed balance.
  e.g.,

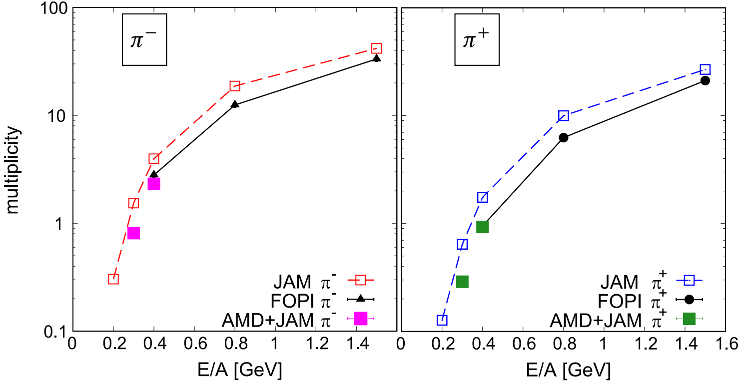
$$\frac{d\sigma_{NN\to N\Delta}}{dm} = \frac{C_I}{p_i s}\frac{|\mathscr{M}|^2}{16\pi} \times \frac{2}{\pi}\frac{m^2\Gamma(m)}{(m^2-m_\Delta^2)^2+m^2\Gamma(m)^2}p_f(m)$$

$$|\mathscr{M}|^2 = A\frac{s\Gamma_\Delta^2}{(s-m_\Delta^2)^2+s\Gamma_\Delta^2}$$

- No mean field (default).
- s-wave pion production ($NN \to NN\pi$) is turned off.

Pions from JAM and AMD+JAM calculations

Au + Au central collisions   Data: Reisdorf et al., NPA 848 (2010) 366.



The energy violation, due to the mismatch of AMD and JAM, is about 2 MeV/nucleon on average at $t \approx 20$ fm/c in collisions at 300 MeV/nucleon. This corresponds to the 10% overestimation of the pion multiplicity.

About potentials for Δ and π

$$N_{\tau_1} + N_{\tau_2} \longleftrightarrow N_{\tau_3} + \Delta_{\tau_4}$$

$$U^{(N)}_{\tau_1} + U^{(N)}_{\tau_2} \qquad U^{(N)}_{\tau_3} + U^{(\Delta)}_{\tau_4}$$

$$+q_1 U_\mathrm{C} + q_2 U_\mathrm{C} \qquad + q_3 U_\mathrm{C} + q_4 U_\mathrm{C}$$

$$\Delta_{\tau_1} \longleftrightarrow N_{\tau_3} + \pi_{\tau_4}$$

$$U^{(\Delta)}_{\tau_1} \qquad U^{(N)}_{\tau_3} + U^{(\pi)}_{\tau_4}$$

$$+q_1 U_\mathrm{C} \qquad + q_3 U_\mathrm{C} + q_4 U_\mathrm{C}$$

- $U^{(*)}_\tau$: Isospin(τ)-dependent potential due to the strong interaction
- U_C: Coulomb potential

In JAM, reaction thresholds are the same as in free space. Therefore AMD+JAM assumes

$$U^{(N)}_{\tau_1} + U^{(N)}_{\tau_2} = U^{(N)}_{\tau_3} + U^{(\Delta)}_{\tau_4}, \qquad U^{(\Delta)}_{\tau_1} = U^{(N)}_{\tau_3} + U^{(\pi)}_{\tau_4} \qquad \text{for } \tau_1(+\tau_2) = \tau_3 + \tau_4$$

This is satisfied in case

$$U^{(N,\Delta)}_\tau = U_0(\mathbf{r}) + \tau U_\mathrm{sym}(\mathbf{r}), \qquad U^{(\pi)}_\tau = \tau U_\mathrm{sym}(\mathbf{r})$$

c.f. pBUU : Hong and Danielewicz, PRC 90 (2014) 024605.

Difference choices of Δ potential

Our choice

$$V_{\text{asy}}(\Delta^-) = 3V_{\text{asy}}(n)$$
$$V_{\text{asy}}(\Delta^0) = V_{\text{asy}}(n)$$
$$V_{\text{asy}}(\Delta^+) = V_{\text{asy}}(p)$$
$$V_{\text{asy}}(\Delta^{++}) = 3V_{\text{asy}}(p)$$

Another choice

$$V_{\text{asy}}(\Delta^-) = V_{\text{asy}}(n)$$
$$V_{\text{asy}}(\Delta^0) = \tfrac{1}{3}V_{\text{asy}}(n)$$
$$V_{\text{asy}}(\Delta^+) = \tfrac{1}{3}V_{\text{asy}}(p)$$
$$V_{\text{asy}}(\Delta^{++}) = V_{\text{asy}}(p)$$

c.f. Bao-An Li, PRC92 (2015) 034603.

Different choices of Δ potential

$\Rightarrow$ Different thresholds for Δ production

$\Rightarrow$ Different π^-/π^+ ratios

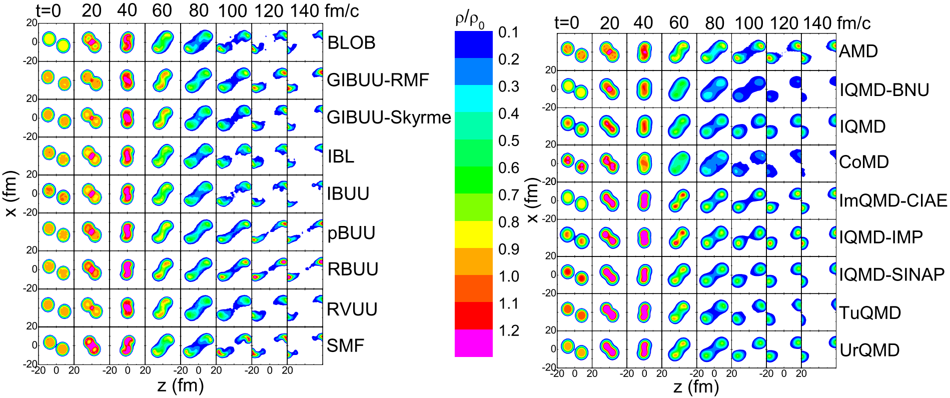
Transport code comparison

PHYSICAL REVIEW C **93**, 044609 (2016)

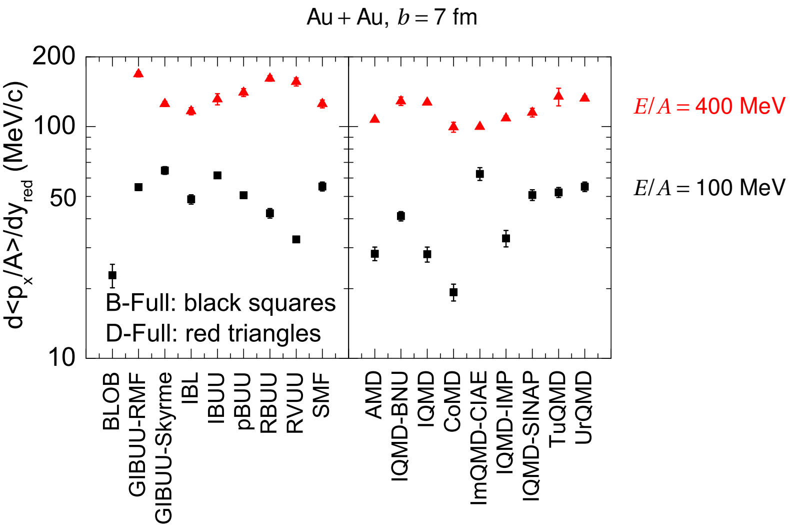
Understanding transport simulations of heavy-ion collisions at 100*A* and 400*A* MeV: Comparison of heavy-ion transport codes under controlled conditions

Jun Xu,[1,*] Lie-Wen Chen,[2,†] ManYee Betty Tsang,[3,‡] Hermann Wolter,[4,§] Ying-Xun Zhang,[5,∥] Joerg Aichelin,[6] Maria Colonna,[7] Dan Cozma,[8] Pawel Danielewicz,[3] Zhao-Qing Feng,[9] Arnaud Le Fèvre,[10] Theodoros Gaitanos,[11] Christoph Hartnack,[6] Kyungil Kim,[12] Youngman Kim,[12] Che-Ming Ko,[13] Bao-An Li,[14] Qing-Feng Li,[15] Zhu-Xia Li,[5] Paolo Napolitani,[16] Akira Ono,[17] Massimo Papa,[18] Taesoo Song,[19] Jun Su,[20] Jun-Long Tian,[21] Ning Wang,[22] Yong-Jia Wang,[15] Janus Weil,[19] Wen-Jie Xie,[23] Feng-Shou Zhang,[24] and Guo-Qiang Zhang[1]

J. Xu et al., PRC93 (2016) 044609.

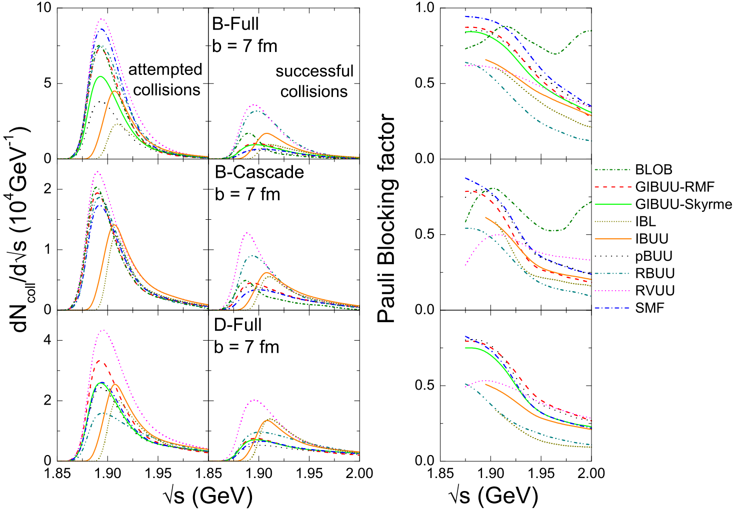


Transport code comparison: Transverse flow

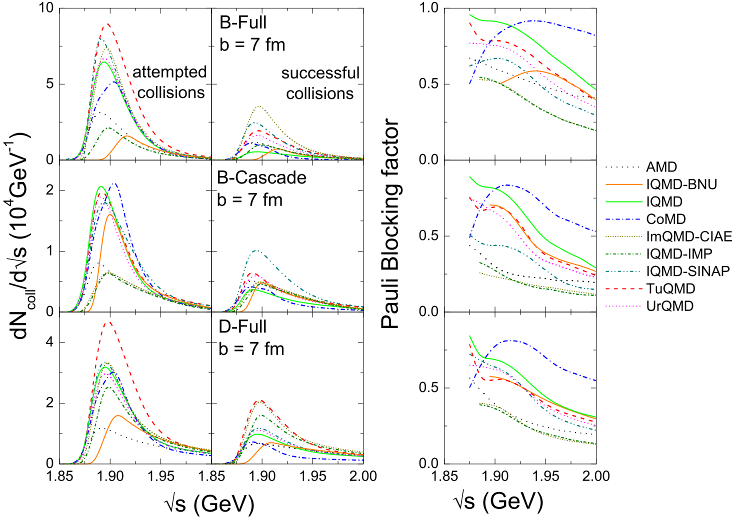


Uncertainties in the flow parameter : about 30% (100 MeV), about 13% (400 MeV)

Transport code comparison: NN collisions and Pauli blocking



Transport code comparison: NN collisions and Pauli blocking

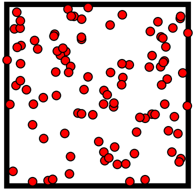


Box simulations

What is the origin of the divergence of the number of NN collisions?

- Due to the difference of the realized states, and/or
- Due to the difference of the procedure for NN collisions.

$\Rightarrow$ Let's do Box Simulations.



- Simple initialization ($T = 0$ Fermi)
- No mean field
- $\sigma_{NN} = 40$ mb
- Turn off Pauli blocking

We still had large divergence. ($\sim 30\%$)

$\Rightarrow$ Many codes fixed **an issue of the Bertsch prescription**.

$\Rightarrow$ Much better agreement! (a few %)

Next (1) Pauli blocking

Next (2) with mean field

Next (3) π and Δ in the box

The Bertsch prescription

PHYSICS REPORTS (Review Section of Physics Letters) 160, No. 4 (1988) 189–233. North-Holland, Amsterdam

A GUIDE TO MICROSCOPIC MODELS FOR INTERMEDIATE ENERGY HEAVY ION COLLISIONS

G.F. BERTSCH

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and

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Received September 1987

Bertsch and Das Gupta,

Phys. Rep. 160 (1988) 189.

Many codes use the prescription described in this article. However, there are a few points that must be corrected implicitly.

The value of t_1 is chosen from

$$\int_{t_0}^{t_1} a\,e^{bt}\,dt \Big/ \int_{t_0}^{0} a\,e^{bt}\,dt = x_1 . \tag{B.4}$$

The value of θ_s is chosen from $\cos\theta_s = 1 - t_1/t_0$; ϕ_s is chosen randomly. Thus for each scattering, the momenta and energy are conserved but not the angular momentum. The cumulative effect of angular momentum nonconservation has been found to be small.

If isotropic scattering is assumed in the inelastic channel, then θ_s can be chosen from $\cos\theta_s = 1 - 2x$.

We have given all the parameters for the calculation. We now give some details of implementation. We have $\sigma^t(\sqrt{s}) \le \sigma^t_{max} = 55$ mb which corresponds to $b_{max} = 1.32$ fm. Thus in a given time interval δt, two particles cannot collide if the distance between them is greater than some preassigned value $d_{min} = \sqrt{(1.32)^2 + c^2\,\delta t^2}$. For two particles whose separation is less, we now proceed to check if the particles pass the point of closest approach and if this distance of closest approach is less than 1.32 fm.

The center-of-mass energy $\sqrt{s} = \sqrt{(E_1+E_2)^2 - (\boldsymbol{p}_1+\boldsymbol{p}_2)^2}$. The velocity of the c.m. of the colliding particles is $\boldsymbol{\beta} = (\boldsymbol{p}_1+\boldsymbol{p}_2)/(E_1+E_2)$. The momentum of particle 1 in the frame is

$$\boldsymbol{p} = \gamma\left(\frac{\boldsymbol{p}_1\cdot\boldsymbol{\beta}}{\beta} - \beta E_1\right)\frac{\boldsymbol{\beta}}{\beta} + \left(\boldsymbol{p}_1 - \frac{\boldsymbol{p}_1\cdot\boldsymbol{\beta}}{\beta}\,\frac{\boldsymbol{\beta}}{\beta}\right).$$

The momentum of the second particle is $-\boldsymbol{p}$. The distance $\Delta\boldsymbol{r}$ in the c.m. frame is $(\gamma-1)\{(\boldsymbol{r}_1-\boldsymbol{r}_2)\cdot\boldsymbol{\beta}/\beta\}\boldsymbol{\beta}/\beta + (\boldsymbol{r}_1-\boldsymbol{r}_2)$. In the time interval $-\delta t/2$ to $\delta t/2$ the two particles become candidates for collision if

$$\left|\frac{\Delta\boldsymbol{r}\cdot\boldsymbol{p}}{p}\right| < \left(\frac{p}{\sqrt{p^2+m_1^2}} + \frac{p}{\sqrt{p^2+m_2^2}}\right)\delta t/2$$

and $b = \sqrt{(\Delta\boldsymbol{r})^2 - (\Delta\boldsymbol{r}\cdot\boldsymbol{p}/p)^2}$ is less than 1.32 fm. The following sequence of operations is now performed:

1. The elastic cross section $\sigma^e_{nn}(\sqrt{s})$ is computed from eq. (B.1). We generate a random number h. If $h < \sigma^e_{nn}(\sqrt{s})/55$, elastic scattering occurs and the angle of scattering chosen according to eq. (B.4). Then we branch to step 5.
If $h > \sigma^e_{nn}(\sqrt{s})/55$ go to step 2 which examines the possibility of inelastic scattering.

2. If $\sqrt{s} < 2.015$ both are nucleons ($m_n = 0.938$) but there is not enough energy to produce a Δ. We branch to step 4.
If m_1 and m_2 are both greater than 0.938 (both are Δ's), there is no possibility of inelastic scattering and we branch to step 4.
We now compute the inelastic cross section $\sigma^{in}_{nn\to n\Delta}$ (eq. (B.2)). If one of the colliding particles is a nucleon and the other one is a Δ, we branch to step 3. Otherwise, we compare h with $(\sigma^{in}_{nn\to n\Delta} + \sigma^{el}_{nn})/55$. If h is greater, then we branch to step 4. If h is less, then Δ production will occur. One has to choose the mass m_Δ. Different prescriptions have been used. The simplest one is $m_\Delta = 1.077 + 0.75(\sqrt{s} - 2.015)$ for $2.015 < \sqrt{s} < 2.2203$; $m_\Delta = 1.231$ for $\sqrt{s} > 2.2203$ [23]. Cugnon [16] has used

$$x = \int_{M_N+M_\pi}^{M_\Delta} f(M')\,dM' \Big/ \int_{M_N+M_\pi}^{\sqrt{s}-M_N} f(M')\,dM'$$

Problem of the Bertsch prescription

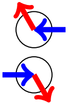
Bertsch prescription (nonrelativistic version)

Two nucleons may collide

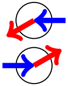
- when the relative distance is minimum during a time step, $2|\mathbf{r}\cdot\mathbf{v}| < \mathbf{v}^2\Delta t$,
- and when the minimum distance is $\mathbf{r}^2 - (\mathbf{r}\cdot\mathbf{v})^2/\mathbf{v}^2 < \sigma/\pi$,

where $\mathbf{r}$ and $\mathbf{v}$ are the relative coordinate and velocity, respectively, at the current time t.

Then the same pair of two particles can collide more than once, as in the second example.



- By the collision, the momenta are changed from the blue to the red arrows. After the collision, the particles are moving away from each other ($\mathbf{r}\cdot\mathbf{v} > 0$), so they will not collide again.



- After the collision (red arrows), the particles are again approaching to each other ($\mathbf{r}\cdot\mathbf{v} < 0$) and the distance is $|\mathbf{r}| < \sqrt{\sigma/\pi}$, so they will collide again at a later time.

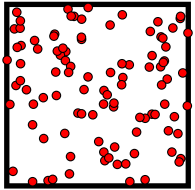
Several collisions occur **spuriously** when two nucleons come into the scattering distance.

Box simulations

What is the origin of the divergence of the number of NN collisions?

- Due to the difference of the realized states, and/or
- Due to the difference of the procedure for NN collisions.

⇒ Let's do Box Simulations.



- Simple initialization ($T = 0$ Fermi)
- No mean field
- $\sigma_{NN} = 40$ mb
- Turn off Pauli blocking

We still had large divergence. ($\sim 30\%$)

⇒ Many codes fixed **an issue of the Bertsch prescription**.

⇒ Much better agreement! (a few %)

Next (1) Pauli blocking

Next (2) with mean field

Next (3) π and Δ in the box

Summary

- Clusters and pions are produced from the same collisions at several hundred MeV/nucleon.
  - AMD with clusters
  - Combining JAM with AMD to predict pions.
- If the cluster correlation is strong, . . .
  - Strong influence on the symmetry-energy observables such as the n/p ratio and the π^-/π^+ ratio (c.f. Ikeno's talk).
  - The expansion dynamics is simple so that some information at an early time may be directly seen in the final observables.
- Transport code comparison is going on.
  - Very recent progresses in box simulations.
- TODO:
  - Threshold in medium for Δ production
  - Box pion simulation by JAM
  - Condition for cluster correlations in medium