Clustering 

Clusters (d, t, $^3$He, $\alpha$) in heavy-ion collisions and in AMD

Pion production based on AMD calculation (+JAM)

Recent progress in the comparison of many transport codes
Various densities in heavy-ion collisions

**Nuclear EOS (at $T = 0$)**

$$(E/A)(\rho_p, \rho_n) = (E/A)_0(\rho) + S(\rho)\delta^2 + \cdots$$

$$\rho = \rho_p + \rho_n, \quad \delta = (\rho_n - \rho_p)/(\rho_n + \rho_p).$$

**Constrains on $S(\rho)$ at ICNT2013 @FRIB**


**$^{132}$Sn + $^{124}$Sn, 300 MeV/u, $b \sim 0$**

Akira Ono (Department of Physics, Tohoku University)
Probing high-density dynamics by pions


Pion ratio $\pi^-/\pi^+$ has been proposed as a good probe of

- symmetry energy at high densities
- $\rho_n/\rho_p$ ratio in the high density region

Production/absorption of $\Delta$ and $\pi$: ($N = n$ or $p$)

$$N + N \rightarrow N + \Delta$$
$$N + N \leftarrow N + \Delta$$

$$\Delta \rightarrow N + \pi$$
$$\Delta \leftarrow N + \pi$$

Simple expectation: $\pi^-/\pi^+ \approx (N/Z)^2$

- First-chance $NN \rightarrow N\Delta \rightarrow NN\pi$
- Chemical equilibrium
Clustering phenomena in excited states of nuclear systems

- Molecular resonance
- Cluster decay
- Excitation energy/temperature
- Liquid-gas phase
- Multi fragmentation in heavy ion collision
- Mass number
- Weakly bound systems
- Shell evolution
- Halo, skin
- nn correlation
- Shell structure
- Cluster breaking
- Deformation
- Collective mode (GR, PR)
- Molecular orbitals
- Molecular resonance
- Cluster decay
- Excitation energy/temperature
- Liquid-gas phase
- Developed clusters
- Threshold energy
The system may be composed of many clusters.

Clusters are not only created but also broken by reactions.

Transport models with clusters + decay
Transport with clusters (pBUU)

**BUU with clusters**


Coupled equations for $f_n(r, p, t)$, $f_p(r, p, t)$, $f_d(r, p, t)$, $f_t(r, p, t)$, $f_h(r, p, t)$ are solved by the test particle method.

\[
\begin{align*}
\frac{\partial f_n}{\partial t} + \mathbf{v} \cdot \frac{\partial f_n}{\partial \mathbf{r}} - \frac{\partial U_n}{\partial \mathbf{r}} \cdot \frac{\partial f_n}{\partial \mathbf{p}} &= I_n^{\text{coll}}[f_n, f_p, f_d, f_t, f_h] \\
\frac{\partial f_p}{\partial t} + \mathbf{v} \cdot \frac{\partial f_p}{\partial \mathbf{r}} - \frac{\partial U_p}{\partial \mathbf{r}} \cdot \frac{\partial f_p}{\partial \mathbf{p}} &= I_p^{\text{coll}}[f_n, f_p, f_d, f_t, f_h] \\
\frac{\partial f_d}{\partial t} + \mathbf{v} \cdot \frac{\partial f_d}{\partial \mathbf{r}} - \frac{\partial U_d}{\partial \mathbf{r}} \cdot \frac{\partial f_d}{\partial \mathbf{p}} &= I_d^{\text{coll}}[f_n, f_p, f_d, f_t, f_h] \\
\frac{\partial f_t}{\partial t} + \mathbf{v} \cdot \frac{\partial f_t}{\partial \mathbf{r}} - \frac{\partial U_t}{\partial \mathbf{r}} \cdot \frac{\partial f_t}{\partial \mathbf{p}} &= I_t^{\text{coll}}[f_n, f_p, f_d, f_t, f_h] \\
\frac{\partial f_h}{\partial t} + \mathbf{v} \cdot \frac{\partial f_h}{\partial \mathbf{r}} - \frac{\partial U_h}{\partial \mathbf{r}} \cdot \frac{\partial f_h}{\partial \mathbf{p}} &= I_h^{\text{coll}}[f_n, f_p, f_d, f_t, f_h]
\end{align*}
\]

**without clusters**

**with clusters**

Antisymmetrized Molecular Dynamics (very basic version)


**AMD wave function**

\[ |\Phi(Z)\rangle = \text{det}_{i,j}\left[ \exp\left\{ -\nu \left( r_j - \frac{Z_i}{\sqrt{\nu}} \right)^2 \right\} \chi_{\alpha_i}(j) \right] \]

**Equation of motion for the wave packet centroids** \( Z \)

\[ \frac{d}{dt} Z_i = \{Z_i, H\}_{\text{PB}} + (\text{NN collisions}) \]

**\{Z_i, H\}_{\text{PB}}**: Motion in the mean field

\[ H = \frac{\langle \Phi(Z)|H|\Phi(Z)\rangle}{\langle \Phi(Z)|\Phi(Z)\rangle} + (\text{c.m. correction}) \]

\( H \): Effective interaction (e.g. Skyrme force)

**NN collisions**

\[ W_{i\rightarrow f} = \frac{2\pi}{\hbar} |\langle \Psi_f |V|\Psi_i \rangle|^2 \delta(E_f - E_i) \]

- \( |V|^2 \) or \( \sigma_{NN} \) (in medium)
- Pauli blocking
Failure of fragmentation

AMD with usual NN collisions (very basic version)

Central Xe + Sn at 50 MeV/u

Production of clusters and pions in heavy-ion collisions
NN collisions without or with cluster correlations

\[ W_{i \to f} = \frac{2\pi}{\hbar} |\langle \Psi_f | V | \Psi_i \rangle|^2 \delta(E_f - E_i) \]

In the usual way of NN collision, only the two wave packets are changed.

\[ \left\{ |\Psi_f \rangle \right\} = \left\{ |\varphi_{k_1}(1)\varphi_{k_2}(2)\Psi(3,4,\ldots)\rangle \right\} \]

(ignoring antisymmetrization for simplicity of presentation.)

Phase space or the density of states for two nucleon system

Molecular Dynamics

Phase space or the density of states for two nucleon system

Akira Ono (Department of Physics, Tohoku University) Production of clusters and pions in heavy-ion collisions NPCSM 2016 9 / 34
\[ W_{i \rightarrow f} = \frac{2\pi}{\hbar} |\langle \Psi_f | V | \Psi_i \rangle|^2 \delta (E_f - E_i) \]

In the usual way of NN collision, only the two wave packets are changed.

\[ \{ |\Psi_f \rangle \} = \{ |\varphi_{k_1}(1)\varphi_{k_2}(2)\Psi(3,4,\ldots)\rangle \} \]

(ignoring antisymmetrization for simplicity of presentation.)

**Extension for cluster correlations**

Include correlated states in the set of the final states of each NN collision.

\[ \{ |\Psi_f \rangle \} \ni |\varphi_{k_1}(1)\psi_d(2,3)\Psi(4,\ldots)\rangle, \ldots \]
NN collisions with cluster correlations

\[ N_1 + B_1 + N_2 + B_2 \rightarrow C_1 + C_2 \]

- \( N_1, N_2 \): Colliding nucleons
- \( B_1, B_2 \): Spectator nucleons/clusters
- \( C_1, C_2 : N, (2N), (3N), (4N) \) (up to \( \alpha \) cluster)

**Transition probability**

\[
W(\text{NBNB} \rightarrow \text{CC}) = \frac{2\pi}{\hbar} |\langle \text{CC} | V | \text{NBNB} \rangle|^2 \delta (E_f - E_i)
\]

\[
v d\sigma \propto |\langle \varphi'_1 | \varphi_1^+ q \rangle|^2 |\langle \varphi'_2 | \varphi_2^- q \rangle|^2 |M|^2 \delta (E_f - E_i) p_{\text{rel}}^2 dp_{\text{rel}} d\Omega
\]

\[ |M|^2 = |\langle \text{NN} | V | \text{NN} \rangle|^2: \text{Matrix elements of NN scattering} \]

\[ \left\langle d\sigma/d\Omega \right\rangle_{\text{NN}} \text{ in medium (or in free space)} \]

Similar to Danielewicz et al.,

\[ p_{\text{rel}} = \frac{1}{2} (p_1 - p_2) = p_{\text{rel}} \hat{\Omega} \]

\[ q = p_1^{(0)} - p_2^{(0)} = p_{2}^{(0)} - p_2 \]

\[ \varphi_1^{+ q} = \exp (+ i q \cdot r_{N_1}) \varphi_1^{(0)} \]

\[ \varphi_2^{- q} = \exp (- i q \cdot r_{N_2}) \varphi_2^{(0)} \]
For each NN collision, cluster formation is considered.

\[
N_1 + B_1 + N_2 + B_2 \rightarrow C_1 + C_2
\]

\[
W_{i \rightarrow f} = \frac{2\pi}{\hbar} |\langle CC | V_{NN} | NBNB \rangle| \delta(E_f - E_i)
\]

We always have a Slater determinant of nucleon wave packets. A cluster in the final states is represented by placing wave packets at the same phase space point.

Consequently the processes such as 
\[
d + X \rightarrow n + p + X'\]
\[
d + X \rightarrow d + X'
\]
are automatically taken into account.

No parameters have been introduced to adjust individual reactions.

There are many possibilities to from clusters in the final states. Non-orthogonality of the final states should be carefully handled.
Construction of Final States

Clusters (in the final states) are assumed to have \((0s)^N\) configuration.

\[
|\Phi^q\rangle
\]

After \(p^{(0)} \rightarrow p^{(0)} + q\)

\[
|\Phi'_1\rangle
\]

\[
|\Phi'_2\rangle
\]

\[
|\Phi'_3\rangle
\]

Final states are not orthogonal: 

\[ N_{ij} \equiv \langle \Phi'_i | \Phi'_j \rangle \neq \delta_{ij} \]

The probability of cluster formation with one of \(B\)'s:

\[
\hat{P} = \sum_{ij} |\Phi'_i\rangle N_{ij}^{-1} \langle \Phi'_j|, \quad P = \langle \Phi^q | \hat{P} | \Phi^q \rangle \neq \sum_i |\langle \Phi'_i | \Phi^q \rangle|^2
\]

\[
\begin{cases} 
P & \Rightarrow \text{Choose one of the candidates and make a cluster.} \\
1 - P & \Rightarrow \text{Don’t make a cluster (with any } n\uparrow). 
\end{cases}
\]
Correlations to bind several clusters

Clusters may form a loosely bound state.

\[ ^7\text{Li} = \alpha + t - 2.5 \text{ MeV} \]

Need more probability of \(|\alpha + t\rangle \rightarrow |^7\text{Li}\rangle\)

---

**Step 1** Clusters (and nucleons) \(C_i\) and \(C_j\) are linked,

- if \(C_i\) is one of the 3 clusters closest to \(C_j\), and \((i \leftrightarrow j)\),
- and if the distance is \(1 \text{ fm} < |R_{ij}| < 7 \text{ fm}\),
- and if they are slowly moving away,
  \[ P_{ij}^2/2\mu_{ij} < 10 \text{ MeV} \text{ and } R_{ij} \cdot P_{ij} > 0. \]

**Step 2** Linked clusters (CC) are identified.
Following steps are taken only for CC with mass number \(6 \leq A \leq 9\) or \(19 \leq A \leq 23\).

**Step 3** Transition of the internal state of CC by eliminating the (radial component of) internal momentum

\[ P_i \rightarrow 0 \quad \text{for } i \in \text{CC in the c.m. of CC} \]

with some care of the momentum conservation.

**Next** Energy conservation.
Correlations to bind several clusters

Clusters may form a loosely bound state.

e.g., $^7\text{Li} = \alpha + t - 2.5 \text{ MeV}$

Need more probability of $|\alpha + t\rangle \rightarrow |^7\text{Li}\rangle$

**Step 4** Search a third particle for E-conservation

- A cluster $C_k$ is selected, depending on the distance and momentum ($|R_k|$ and $|P_k|$) relative to CC.
- If the selected $C_k$ already belongs to a CC', this whole CC' is treated as the third particle for E-conservation.

**Step 5** Scale the radial component of the relative momentum between CC and $C_k$ for the total energy conservation.

$$P_k = P_{k\parallel} + P_{k\perp} \rightarrow \beta P_{k\parallel} + P_{k\perp}$$
Effect of cluster correlations: central Xe + Sn at 50 MeV/u
Results for multifragmentation in central collisions

**Xe + Sn**


**Ca + Ca at 35 MeV/u**

**Au + Au at 250 MeV/u**
Collisions at higher energies

$^{132}\text{Sn} + ^{124}\text{Sn}$, $E/A = 300$ MeV, $b \sim 0$

- Pions are measured to probe high-density nuclear matter by the $S\pi$RIT experiment.
- Many protons are bound in clusters.
  - In Au+Au at 250 MeV/u (FOPI data),
    - $p$: 21%, $\alpha$: 20%, $d+t+^3\text{He}$: 40%

The $S\pi$RIT project: TPC in SAMURAI magnet

Momentum dependence of Skyrme (SLy4) interaction has been corrected for high energy collisions, in a similar way to Gale, Bertsch, Das Gupta, PRC 35 (1987) 1666.
Collisions at higher energies

\[ ^{132}\text{Sn} + ^{124}\text{Sn}, \ E/A = 300 \text{ MeV}, \ b \sim 0 \]

Momentum dependence of Skyrme (SLy4) interaction has been corrected for high energy collisions, in a similar way to Gale, Bertsch, Das Gupta, PRC 35 (1987) 1666.
Collisions at higher energies

$^{132}\text{Sn} + ^{124}\text{Sn}$, $E/A = 300$ MeV, $b \sim 0$

Momentum dependence of Skyrme (SLy4) interaction has been corrected for high energy collisions, in a similar way to Gale, Bertsch, Das Gupta, PRC 35 (1987) 1666.
$^{132}\text{Sn} + ^{124}\text{Sn}$, $E/A = 300$ MeV, $b \sim 0$

Momentum dependence of Skyrme (SLy4) interaction has been corrected for high energy collisions, in a similar way to Gale, Bertsch, Das Gupta, PRC 35 (1987) 1666.
Collisions at higher energies

$^{132}\text{Sn} + ^{124}\text{Sn}$, $E/A = 300$ MeV, $b \sim 0$

Momentum dependence of Skyrme (SLy4) interaction has been corrected for high energy collisions, in a similar way to Gale, Bertsch, Das Gupta, PRC 35 (1987) 1666.
\[
\frac{N}{Z} \text{ of spectrum of emitted particles is similar to the neutron-proton density difference at the compression stage.}
\]
$\frac{N}{Z}$ Spectrum Ratio (AMD without clusters)

\[ \left( \frac{N}{Z} \right)_{\text{gas}} = \frac{Y_n(v) + Y_d(v) + 2Y_t(v) + Y_h(v) + 2Y_\alpha(v)}{Y_p(v) + Y_d(v) + Y_t(v) + 2Y_h(v) + 2Y_\alpha(v)} \]

$N/Z$ of spectrum of emitted particles is NOT similar to the neutron-proton density difference at the compression stage.
Do clusters exist in high density region?

Clusters may exist even in high density matter if they have sufficient momenta so that the effect of Pauli principle is weak.

However, clusters in high density region will be soon broken by collisions with other particles.

Does it make sense to consider such short-lived correlations?

Equation for a deuteron in uncorrelated medium

\[
\begin{align*}
\left[ e\left(\frac{1}{2}P + p\right) + e\left(\frac{1}{2}P - p\right) \right] \tilde{\psi}(p) \\
+ \left[ 1 - f\left(\frac{1}{2}P + p\right) - f\left(\frac{1}{2}P - p\right) \right] \int \frac{d{p}'}{(2\pi)^3} \langle p|\nu|p'\rangle \tilde{\psi}(p') \\
= E\tilde{\psi}(p)
\end{align*}
\]
Do clusters exist in high density region?

Clusters may exist even in high density matter if they have sufficient momenta so that the effect of Pauli principle is weak.

However, clusters in high density region will be soon broken by collisions with other particles.

Does it make sense to consider such short-lived correlations?

\[ \left[ e\left(\frac{1}{2}P + p\right) + e\left(\frac{1}{2}P - p\right) \right] \tilde{\psi}(p) \]
\[ + \left[ 1 - f\left(\frac{1}{2}P + p\right) - f\left(\frac{1}{2}P - p\right) \right] \int \frac{dp'}{(2\pi)^3} \langle p | \nu | p' \rangle \tilde{\psi}(p') \]
\[ = E\tilde{\psi}(p) \]

Danielewicz and Bertsch, NPA 533 (1991)
Turn off clusters at high density

Always produce clusters

Produce clusters only at $\rho < \rho_0$

When the cluster production is turned off at high density $\rho > 0.16 \text{ fm}^{-3}$,

- $M_p \uparrow$ (overproduction)
- $M_\alpha \downarrow$
- $M_d$ and $M_t$ don’t increase.

We need some fine tuning of the cluster production.
Transport equations for $N$, $\Delta$ and $\pi$

**Coupled equations for $f_N(r, p, t)$, $f_\Delta(r, p, t)$, $f_\pi(r, p, t)$**

\[
\begin{align*}
\frac{\partial f_N}{\partial t} + \frac{\partial h_N}{\partial p} \cdot \frac{\partial f_N}{\partial r} - \frac{\partial h_N[f_N, f_\Delta, f_\pi]}{\partial r} \cdot \frac{\partial f_N}{\partial p} &= I_N[f_N, f_\Delta, f_\pi] \\
\frac{\partial f_\Delta}{\partial t} + \frac{\partial h_\Delta}{\partial p} \cdot \frac{\partial f_\Delta}{\partial r} - \frac{\partial h_\Delta[f_N, f_\Delta, f_\pi]}{\partial r} \cdot \frac{\partial f_\Delta}{\partial p} &= I_\Delta[f_N, f_\Delta, f_\pi] \\
\frac{\partial f_\pi}{\partial t} + \frac{\partial h_\pi}{\partial p} \cdot \frac{\partial f_\pi}{\partial r} - \frac{\partial h_\pi[f_N, f_\Delta, f_\pi]}{\partial r} \cdot \frac{\partial f_\pi}{\partial p} &= I_\pi[f_N, f_\Delta, f_\pi]
\end{align*}
\]

**Assumption:** $\Delta$ and pion productions are rare (in low-energy collisions), so that they can be treated as perturbation.

\[
I_N[f_N, f_\Delta, f_\pi] = I_N^{el}[f_N, 0, 0] + \lambda I_N'[f_N, f_\Delta, f_\pi]
\]

\[
f_N = f_N^{(0)} + \lambda f_N^{(1)} + \cdots, \quad f_\Delta = O(\lambda), \quad f_\pi = O(\lambda)
\]

Ikeno, Ono, Nara, Ohnishi, PRC 93 (2016) 044612.
Zeroth order equation for $f_N$
solved by AMD

$$\frac{\partial f_N^{(0)}}{\partial t} + \frac{\partial h_N}{\partial p} \cdot \frac{\partial f_N^{(0)}}{\partial r} - \frac{\partial h_N[f_N^{(0)},0,0]}{\partial r} \cdot \frac{\partial f_N^{(0)}}{\partial p} = i_N^{el}[f_N^{(0)},0,0]$$

First order equations for $f_\Delta$ and $f_\pi$
solved by JAM for given $f_N^{(0)}$

$$\frac{\partial f_\Delta}{\partial t} + \frac{\partial h_\Delta}{\partial p} \cdot \frac{\partial f_\Delta}{\partial r} - \frac{\partial h_\Delta[f_N^{(0)},f_\Delta,f_\pi]}{\partial r} \cdot \frac{\partial f_\Delta}{\partial p} = I_\Delta[f_N^{(0)},f_\Delta,f_\pi]$$

$$\frac{\partial f_\pi}{\partial t} + \frac{\partial h_\pi}{\partial p} \cdot \frac{\partial f_\pi}{\partial r} - \frac{\partial h_\pi[f_N^{(0)},f_\Delta,f_\pi]}{\partial r} \cdot \frac{\partial f_\pi}{\partial p} = I_\pi[f_N^{(0)},f_\Delta,f_\pi]$$

Send nucleon test particles from AMD to JAM at every 2 fm/c, with corrections for the conservation of baryon number and charge.

Ikeno, Ono, Nara, Ohnishi,
PRC 93 (2016) 044612.
JAM: Jet AA Microscopic transport model


- Applied to high-energy collisions (1 ∼ 158 A GeV)
- Hadron-Hadron reactions are based on experimental data and the detailed balance.

\[
\frac{d\sigma_{NN\rightarrow N\Delta}}{dm} = \frac{C_I}{p_i s} \frac{|\mathcal{M}|^2}{16\pi} \times \frac{2}{\pi} \frac{m^2 \Gamma(m)}{(m^2 - m_{\Delta}^2)^2 + m^2 \Gamma(m)^2} p_f(m)
\]

\[
|\mathcal{M}|^2 = A \frac{s \Gamma_{\Delta}^2}{(s - m_{\Delta}^2)^2 + s \Gamma_{\Delta}^2}
\]

- No mean field (default).
- s-wave pion production ($NN \rightarrow NN\pi$) is turned off.
The energy violation, due to the mismatch of AMD and JAM, is about 2 MeV/nucleon on average at $t \approx 20 \text{ fm}/c$ in collisions at 300 MeV/nucleon. This corresponds to the 10% overestimation of the pion multiplicity.
About potentials for $\Delta$ and $\pi$

\[
N_{\tau_1} + N_{\tau_2} \leftrightarrow N_{\tau_3} + \Delta_{\tau_4}
\]
\[
U^{(N)}_{\tau_1} + U^{(N)}_{\tau_2} \leftrightarrow U^{(N)}_{\tau_3} + U^{(\Delta)}_{\tau_4} + q_1 U_C + q_2 U_C + q_3 U_C + q_4 U_C
\]

\[
\Delta_{\tau_1} \leftrightarrow N_{\tau_3} + \pi_{\tau_4}
\]
\[
U^{(\Delta)}_{\tau_1} \leftrightarrow U^{(N)}_{\tau_3} + U^{(\pi)}_{\tau_4} + q_1 U_C + q_3 U_C + q_4 U_C
\]

- $U^{(*)}_\tau$: Isospin($\tau$)-dependent potential due to the strong interaction
- $U_C$: Coulomb potential

In JAM, reaction thresholds are the same as in free space. Therefore AMD+JAM assumes

\[
U^{(N)}_{\tau_1} + U^{(N)}_{\tau_2} = U^{(N)}_{\tau_3} + U^{(\Delta)}_{\tau_4}, \quad U^{(\Delta)}_{\tau_1} = U^{(N)}_{\tau_3} + U^{(\pi)}_{\tau_4} \quad \text{for} \quad \tau_1 (+ \tau_2) = \tau_3 + \tau_4
\]

This is satisfied in case

\[
U^{(N,\Delta)}_\tau = U_0(r) + \tau U_{\text{sym}}(r), \quad U^{(\pi)}_\tau = \tau U_{\text{sym}}(r)
\]

Difference choices of $\Delta$ potential

**Our choice**

- $V_{\text{asy}}(\Delta^-) = 3V_{\text{asy}}(n)$
- $V_{\text{asy}}(\Delta^0) = V_{\text{asy}}(n)$
- $V_{\text{asy}}(\Delta^+) = V_{\text{asy}}(p)$
- $V_{\text{asy}}(\Delta^{++}) = 3V_{\text{asy}}(p)$

**Another choice**

- $V_{\text{asy}}(\Delta^-) = V_{\text{asy}}(n)$
- $V_{\text{asy}}(\Delta^0) = \frac{1}{3}V_{\text{asy}}(n)$
- $V_{\text{asy}}(\Delta^+) = \frac{1}{3}V_{\text{asy}}(p)$
- $V_{\text{asy}}(\Delta^{++}) = V_{\text{asy}}(p)$

different choices of $\Delta$ potential

⇒ Different thresholds for $\Delta$ production

⇒ Different $\pi^-/\pi^+$ ratios

---

Transport code comparison

Understanding transport simulations of heavy-ion collisions at 100A and 400A MeV:
Comparison of heavy-ion transport codes under controlled conditions

Jun Xu,1,* Lie-Wen Chen,2,4 ManYee Betty Tsang,3,4 Hermann Wolter,4,5 Ying-Xun Zhang,5,6 Joerg Aichelin,6
Maria Colonna,7 Dan Cozma,8 Pawel Danielewicz,9 Zhao-Qing Feng,9 Arnaud Le Fèvre,10 Theodoros Gaitanos,11
Christoph Hartnack,6 Kyungil Kim,12  Youngman Kim,12 Che-Ming Ko,13 Bao-An Li,14 Qing-Feng Li,15 Zhu-Xia Li,5
Paolo Napolitani,16 Akira Ono,17 Massimo Papa,18 Taesoo Song,19 Jun Su,20 Jun-Long Tian,21 Ning Wang,22 Yong-Jia Wang,15
Janus Weil,19 Wen-Jie Xie,23 Feng-Shou Zhang,24 and Guo-Qiang Zhang

Akira Ono (Department of Physics, Tohoku University)
Production of clusters and pions in heavy-ion collisions
Transport code comparison: Transverse flow

Au + Au, $b = 7$ fm

$E/A = 400$ MeV

$E/A = 100$ MeV

Uncertainties in the flow parameter: about 30% (100 MeV), about 13% (400 MeV)
Transport code comparison: NN collisions and Pauli blocking

Akira Ono (Department of Physics, Tohoku University) Production of clusters and pions in heavy-ion collisions NPCSM 2016 29 / 34
Transport code comparison: NN collisions and Pauli blocking

\[ \frac{dN_{\text{coll}}}{d\sqrt{s}} \left( 10^4 \text{GeV}^{-1} \right) \]

\[ \frac{dN_{\text{coll}}}{d\sqrt{s}} \left( 10^4 \text{GeV}^{-1} \right) \]

<table>
<thead>
<tr>
<th>Code</th>
<th>Successful Collisions</th>
<th>Attempted Collisions</th>
<th>( b = 7 \text{ fm} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>B-Full</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>B-Cascade</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>D-Full</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\[ \text{Pauli Blocking factor} \]

\[ \text{Pauli Blocking factor} \]

\[ \text{\( \sqrt{s} \ (\text{GeV}) \)} \]

\[ \text{\( \sqrt{s} \ (\text{GeV}) \)} \]

- AMD
- IQMD-BNU
- IQMD
- CoMD
- ImQMD-CIAE
- IQMD-IMP
- IQMD-SINAP
- TuQMD
- UrQMD
Box simulations

What is the origin of the divergence of the number of NN collisions?

- Due to the difference of the realized states, and/or
- Due to the difference of the procedure for NN collisions.

⇒ Let’s do Box Simulations.

We still had large divergence. (~ 30%)

⇒ Many codes fixed an issue of the Bertsch prescription.
⇒ Much better agreement! (a few %)

Next (1) Pauli blocking
Next (2) with mean field
Next (3) $\pi$ and $\Delta$ in the box

Simple initialization ($T = 0$ Fermi)
No mean field
$\sigma_{NN} = 40$ mb
Turn off Pauli blocking
The value of $t_1$ is chosen from

$$ t_1 = \frac{1}{\beta} \int_{t_0}^{t} a e^{\beta t} dt = x_1. \quad (B.4) $$

The value of $\theta_1$ is chosen from $\cos \theta_1 = 1 - t_1/\beta$, $\phi_1$ is randomly chosen. Thus for each scattering, the momentum and energy are conserved but not the angular momentum. The cumulative effect of angular momentum nonconservation has been found to be small.

If isotropic scattering is assumed in the inelastic channel, then $\theta_1$ can be chosen from $\cos \theta_1 = 1 - 2x$. We have given all the parameters for the calculation. We now give some details of implementation. We have $\sigma^R(\sqrt{s}) < \alpha_1 = 55$ mb which corresponds to $b_{\max} = 1.32$ fm. Thus in a given time interval $\delta t$, two particles cannot collide if the distance between them is greater than some preassigned sign $d_{\min} = \sqrt{(1.32)^2 + c^2 \delta t}$. For two particles whose separation is less, we now proceed to check if the particles pass the point of closest approach and if this distance of closest approach is less than 1.32 fm.

The center-of-mass energy $\sqrt{s} = \sqrt{(E_1 + E_2)^2 - (p_1 + p_2)^2}$. The velocity of the c.m. of the colliding particles is $\beta = (p_1 + p_2)/(E_1 + E_2)$. The momentum of particle 1 in the frame is

$$ p = \gamma \left( p_1 \cdot \beta + \beta E_1 \right) \beta + \left( p_1 - p_1 \cdot \beta \beta \right). $$

The momentum of the second particle is $-p$. The distance $\Delta r$ in the c.m. frame is $\left( \gamma - 1 \right) ((r_1 - r_2) \cdot \beta/\beta) + (r_1 - r_2)$. In the time interval $-\delta t/2$ to $\delta t/2$ the two particles become candidates for collision if

$$ \frac{\Delta r \cdot p}{p} < \left( \frac{p}{\sqrt{p^2 + m_1^2}} + \frac{p}{\sqrt{p^2 + m_2^2}} \right) \delta t/2 $$

and $b = \sqrt{(\Delta r)^2 - (\Delta r \cdot p/p)^2}$ is less than 1.32 fm. The following sequence of operations is now performed:

1. The elastic cross section $\sigma_{ee}(\sqrt{s})$ is computed from eq. (B.1). We generate a random number $h$. If $h < \alpha_1(\sqrt{s})/55$, elastic scattering occurs and the angle of scattering chosen according to eq. (B.4). Then we branch to step 5.

2. If $h > \alpha_1(\sqrt{s})/55$ go to step 2 which examines the possibility of inelastic scattering.

3. If $s < 2.015$ both are nucleons ($m_a = 0.938$) but there is not enough energy to produce a $\Delta$. We branch to step 4.

4. If $m_1$ and $m_2$ are both greater than 0.938 (both are $\Delta$s), there is no possibility of inelastic scattering and we branch to step 4.

We now compute the inelastic cross section $\sigma_{in}^{m-a}(\sqrt{s})$ (eq. (B.2)). If one of the colliding particles is a nucleon and the other one is a $\Delta$, we branch to step 3. Otherwise, we compare $h$ with $(\sigma_{in}^{m-a} + \sigma_{in}^{a-m})/55$. If $h$ is greater, then we branch to step 4. If $h$ is less, then $\Delta$ production will occur. One has to choose the mass $m_a$. Different prescriptions have been used. The simplest one is $m_a = 1.077 + 0.75(\sqrt{s} - 2.015)$ for $2.015 < \sqrt{s} < 2.2005, m_a = 1.231$ for $\sqrt{s} > 2.2005$ [23]. Cugnon [16] has used

$$ x = \int_{m_a + m_1}^{m_{a+2}} f(M') dM' \int_{m_a + m_2}^{m_{a+2}} f(M') dM' $$

Many codes use the prescription described in this article. However, there are a few points that must be corrected implicitly.
Problem of the Bertsch prescription

Bertsch prescription (nonrelativistic version)

Two nucleons may collide

- when the relative distance is minimum during a time step, \(2|\mathbf{r} \cdot \mathbf{v}| < v^2 \Delta t\),
- and when the minimum distance is \(r^2 - (\mathbf{r} \cdot \mathbf{v})^2/v^2 < \sigma/\pi\),

where \(r\) and \(v\) are the relative coordinate and velocity, respectively, at the current time \(t\).

Then the same pair of two particles can collide more than once, as in the second example.

- By the collision, the momenta are changed from the blue to the red arrows. After the collision, the particles are moving away from each other \((\mathbf{r} \cdot \mathbf{v} > 0)\), so they will not collide again.
- After the collision (red arrows), the particles are again approaching to each other \((\mathbf{r} \cdot \mathbf{v} < 0)\) and the distance is \(|\mathbf{r}| < \sqrt{\sigma/\pi}\), so they will collide again at a later time.

Several collisions occur spuriously when two nucleons come into the scattering distance.
Box simulations

What is the origin of the divergence of the number of NN collisions?

- Due to the difference of the realized states, and/or
- Due to the difference of the procedure for NN collisions.

⇒ Let’s do Box Simulations.

![Box Simulation Diagram]({"width":200,"height":200})

We still had large divergence. (~30%)

⇒ Many codes fixed **an issue of the Bertsch prescription**.

⇒ Much better agreement! (a few %)

Next (1) Pauli blocking

Next (2) with mean field

Next (3) $\pi$ and $\Delta$ in the box

- Simple initialization ($T = 0$ Fermi)
- No mean field
- $\sigma_{NN} = 40$ mb
- Turn off Pauli blocking
Clusters and pions are produced from the same collisions at several hundred MeV/nucleon.

- AMD with clusters
- Combining JAM with AMD to predict pions.

If the cluster correlation is strong, . . .

- Strong influence on the symmetry-energy observables such as the $n/p$ ratio and the $\pi^-/\pi^+$ ratio (c.f. Ikeno's talk).
- The expansion dynamics is simple so that some information at an early time may be directly seen in the final observables.

Transport code comparison is going on.

- Very recent progresses in box simulations.

TODO:

- Threshold in medium for $\Delta$ production
- Box pion simulation by JAM
- Condition for cluster correlations in medium