The crust-core transition and the stellar matter equation of state

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Acknowledgments:

YITP YUKAWA INSTITUTE FOR THEORETICAL PHYSICS

FCT Fundação para a Ciência e a Tecnologia MINISTÉRIO DA CIÊNCIA, TECNOLOGIA E ENSINO SUPERIOR

compstar

POPH QUALIFICAR É CRESCER.

QREN QUADRO DE REFERÊNCIA ESTRATÉGICO NACIONAL PORTUGAL 2007.2013

GOVERNO DA REPÚBLICA PORTUGUESA

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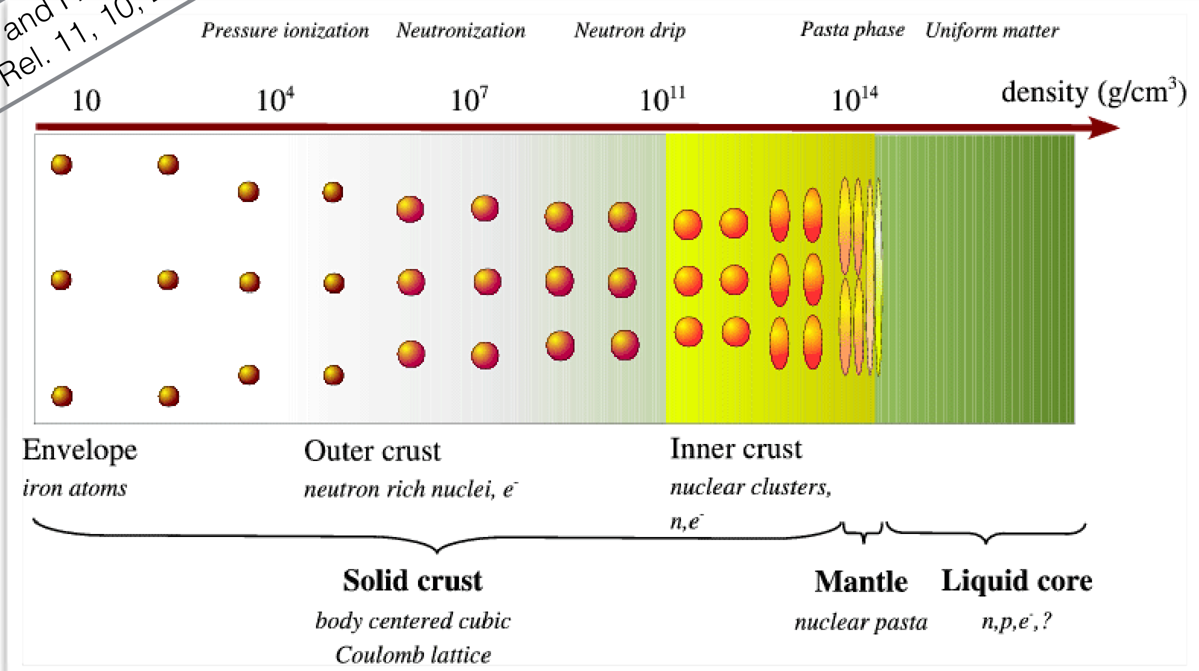
Neutron stars

- One of the most dense objects in Universe: R~10km and M~1.5 $M_{\odot}$

N. Chamel and P. Haensel. Liv. Rev. Rel. 11, 10, 2008

- Divided in 3 main layers:

  1. Outer crust
  2. Inner crust
  3. Core



Crust-core transition important:

- The choice of **inner crust EoS** and the **matching to the core EoS** can be critical :

Variations have been found of **0.5km** for a M=1.4 $M_{\odot}$ star!

- P_t plays crucial role in fraction of I in crust of star: $I_{crust} \sim \frac{16\pi}{3}\frac{R_t^6 P_t}{R_s}$

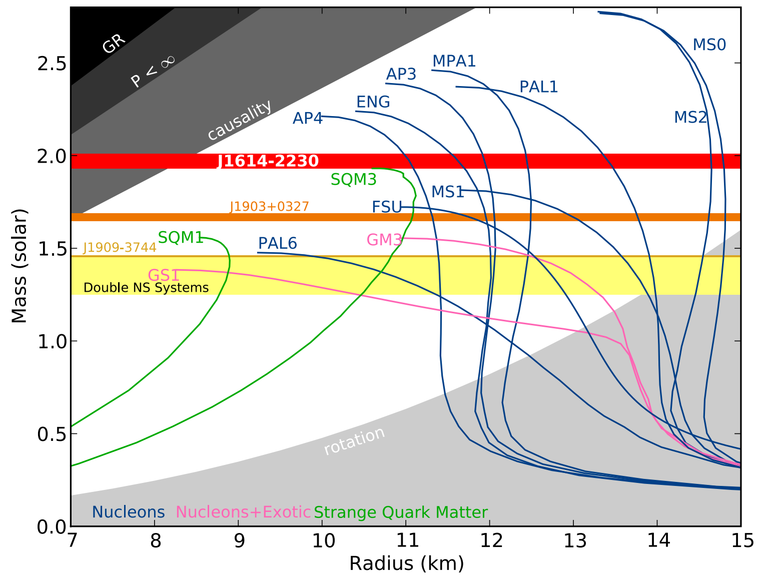
which also depends on **crust thickness**, R_t

Describing neutron stars

P.B. Demorest *et al*, Nature 467, 1081, 2010

Prescription:

1. EoS: $P(E)$ for a system at given ρ and T
2. Compute TOV equations
3. Get star M(R) relation



Problem: Which EoS to choose?

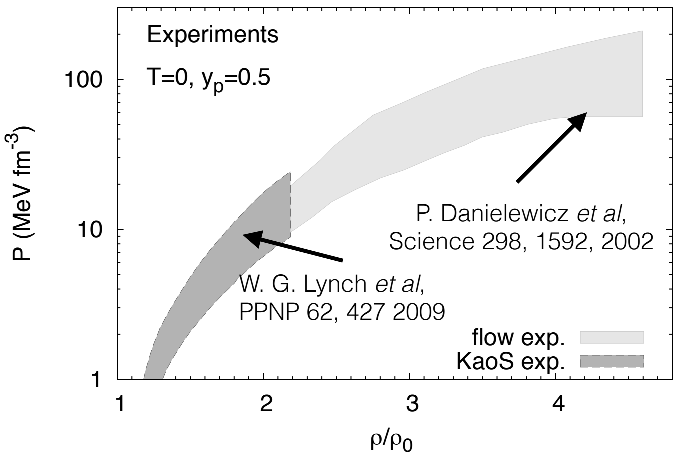
Many EoS models in literature:

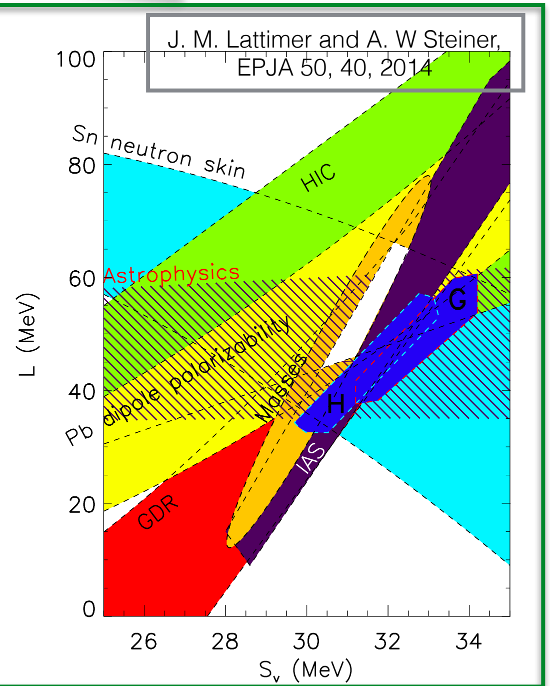
- Phenomenological models (parameters are fitted to nuclei properties): RMF, Skyrme…
- Microscopic models (starts from n-body nucleon interaction): (D)BHF, APR…

Solution: **Need Constrains!!**

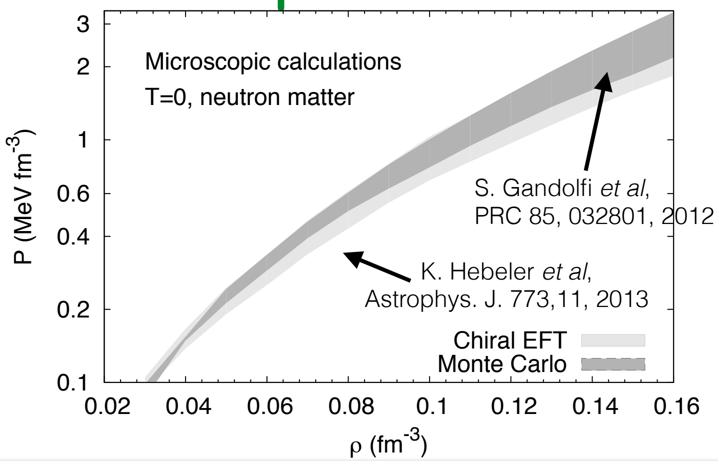
EoS Constrains

•Experiments

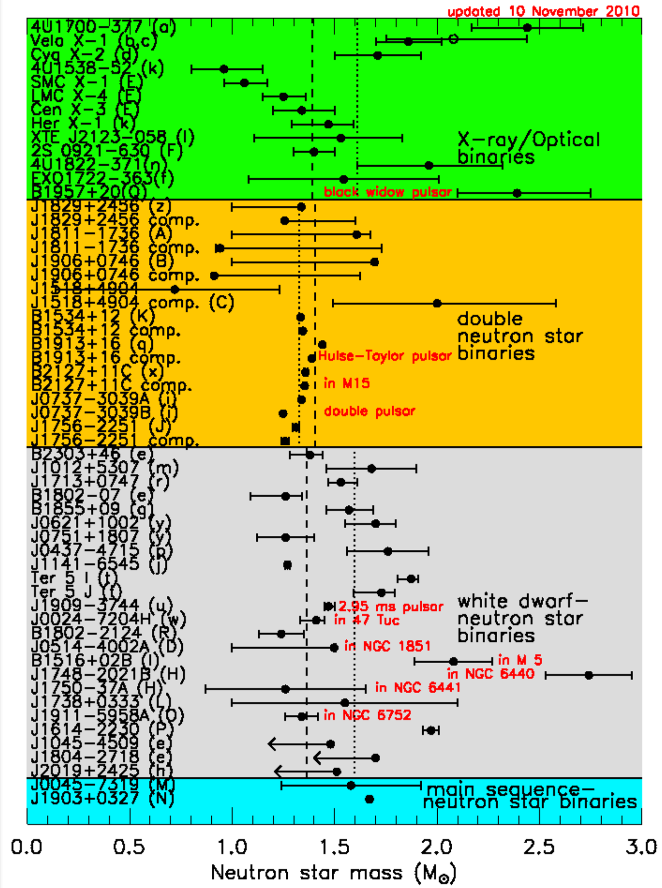




•Microscopic calculations



•Observations



J. M. Lattimer and M. Prakash,
arXiv: 1012.3208 [astro-ph.SR] 2010

Choosing the EoS(s)

We need unified EoS, but if we don't have it..

Choose 1 EoS for each NS layer:

arXiv:1604.01944 [astro-ph.SR] 2016

- Outer crust EoS (BPS or HP or RHS) → M(R) not affected
- *Inner crust EoS (1)* → *pasta phases ?, unified core EoS ?*
- Core EoS → homogeneous matter

and then

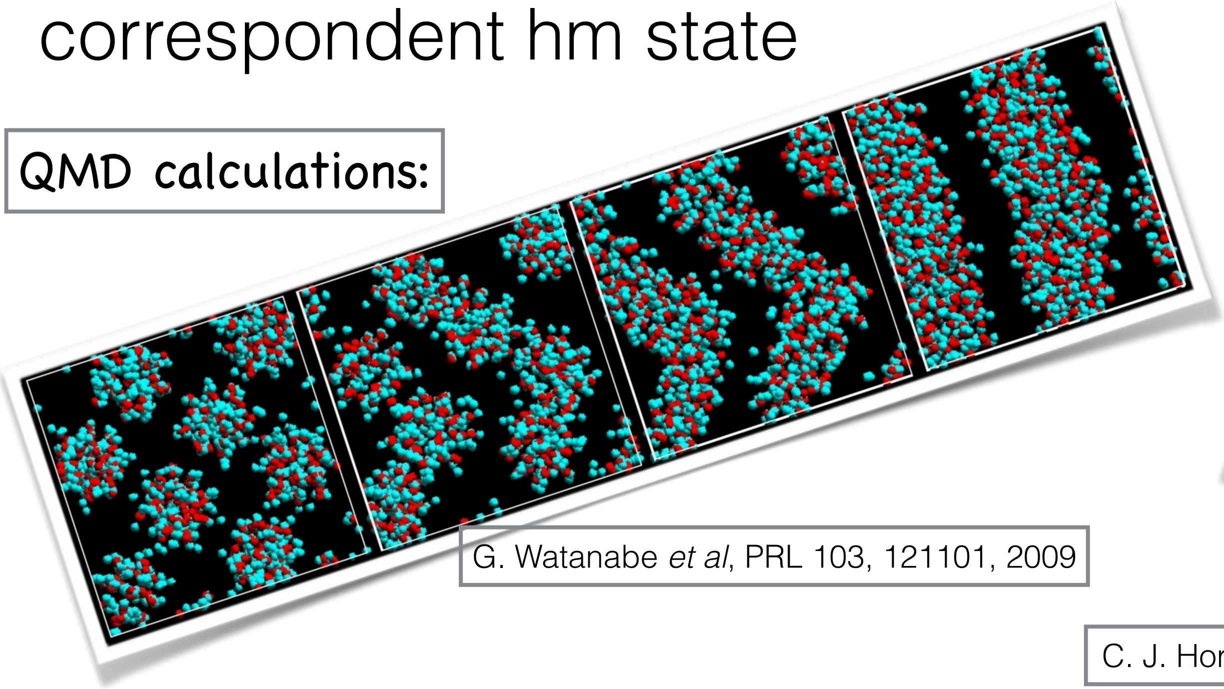
- Match OC EoS at the neutron drip with IC EoS
- Match IC EoS at *crust-core transition (2)* with Core EoS

We are going to focus on (1) and (2) to obtain the transition densities and pressures!

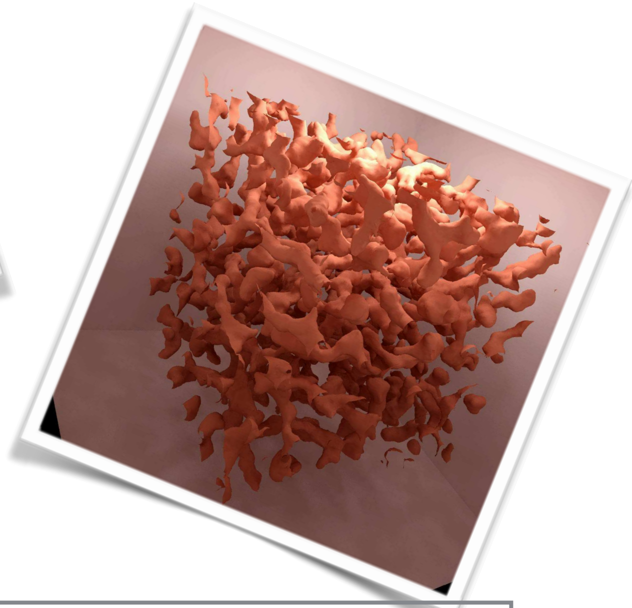
The pasta phases

- Competition between Coulomb and nuclear forces leads to frustrated system
- Geometrical structures, the **pasta phases**, evolve with density until they melt ⟶ **crust-core transition**
- Criterium: pasta free energy must be lower than the correspondent hm state

QMD calculations:



G. Watanabe *et al*, PRL 103, 121101, 2009



C. J. Horowitz *et al*, PRC 70, 065806, 2004

Pasta phases - calculation (I)

- Thomas-Fermi (TF) approximation:
  - Nonuniform npe matter system described inside **Wigner-Seitz cell**:

    Sphere, cilinder or slab in 3D (spherical symmetry), 2D (axial symmetry around z axis) and 1D (reflexion symmetry).
  - **Matter is assumed locally homogeneous** and, at each point, its density is determined by the corresponding local Fermi momenta.
  - **Fields are assumed to vary slowly** so that baryons can be treated as moving in locally constant fields at each point.
  - **Surface effects are treated self-consistently**.
  - Quantities such as the energy and entropy densities are **averaged over the cells**. The free energy density and pressure are calculated from these two thermodynamical functions.

Pasta phases - calculation (II)

check PRC 91, 055801 2015

- Coexistence Phase (CP) approximation:

  - Separated regions of higher and lower density**: pasta phases**, and a **background nucleon gas**.
  - **Gibbs equilibrium conditions**: for $T = T^I = T^{II}$:

$$\mu_p^I = \mu_p^{II} \qquad \mu_n^I = \mu_n^{II} \quad P^I = P^{II}$$

  - **Finite size effects are taken into account** by a surface and a Coulomb terms in the energy density, **after the coexisting phases are achieved**.
  - Total $\mathcal{F}$ and total ρ_p of the system:

$$\mathcal{F} = f\mathcal{F}^I + (1-f)\mathcal{F}^{II} + \mathcal{F}_e + \epsilon_{surf} + \epsilon_{coul}$$
$$\rho_p = \rho_e = y_p\rho = f\rho_p^I + (1-f)\rho_p^{II}$$

Pasta phases - calculation (III)

check PRC 91, 055801 2015

- Compressible Liquid Drop (CLD) approximation:

The total free energy density is minimized, **including the surface and Coulomb terms.**

The **equilibrium conditions** become:

$$\mu_n^I = \mu_n^{II},$$

$$\mu_p^I = \mu_p^{II} - \frac{\epsilon_{surf}}{f(1-f)(\rho_p^I - \rho_p^{II})},$$

$$P^I = P^{II} - \epsilon_{surf}\left(\frac{1}{2\alpha} + \frac{1}{2\phi}\frac{\partial \phi}{\partial f} - \frac{\rho_p^{II}}{f(1-f)(\rho_p^I - \rho_p^{II})}\right)$$

Non-linear Walecka Model

mesons: mediation of nuclear force

$$\mathcal{L} = \sum_{i=p,n} \mathcal{L}_i + \mathcal{L}_e + \mathcal{L}_\gamma + \underbrace{\mathcal{L}_\sigma + \mathcal{L}_\omega + \mathcal{L}_\rho}_{\text{mesons}} + \mathcal{L}_{\sigma\omega\rho}$$

nucleons

electrons

em

non-linear mixing couplings

$$\mathcal{L}_i = \bar{\psi}_i \left[\gamma_\mu i D^\mu - M^*\right] \psi_i$$

$$\mathcal{L}_e = \bar{\psi}_e \left[\gamma_\mu \left(i\partial^\mu + eA^\mu\right) - m_e\right] \psi_e$$

$$\mathcal{L}_\gamma = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu}$$

$$\mathcal{L}_\sigma = \frac{1}{2}\left(\partial_\mu \phi \partial^\mu \phi - m_s^2 \phi^2 - \frac{1}{3}\kappa\phi^3 - \frac{1}{12}\lambda\phi^4\right)$$

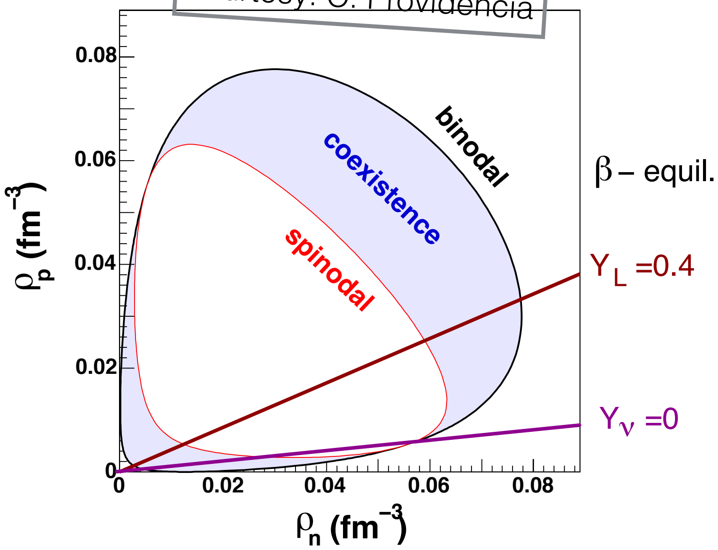
$$\mathcal{L}_\omega = -\frac{1}{4}\Omega_{\mu\nu}\Omega^{\mu\nu} + \frac{1}{2} m_v^2 V_\mu V^\mu + \frac{1}{4!}\xi g_v^4 (V_\mu V^\mu)^2$$

$$\mathcal{L}_\rho = -\frac{1}{4}\mathbf{B}_{\mu\nu} \cdot \mathbf{B}^{\mu\nu} + \frac{1}{2} m_\rho^2 \boldsymbol{b}_\mu \cdot \boldsymbol{b}^\mu$$

non-linear mixing couplings terms:
responsible for density dependence of Esym!

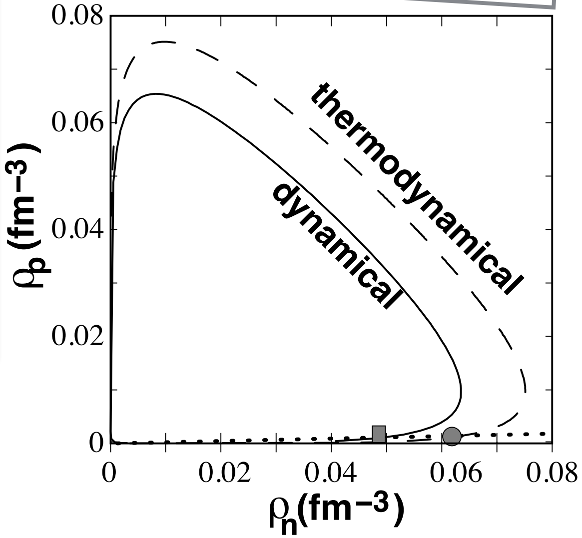
$$\begin{aligned}\mathcal{L}_{\sigma\omega\rho} = {} & \Lambda_{3\sigma} g_s g_v^2 \phi V_\mu V^\mu + \Lambda_{2\sigma} g_s^2 g_v^2 \phi^2 V_\mu V^\mu + \Lambda_{1\sigma} g_s g_\rho^2 \phi \mathbf{b}_\mu \cdot \mathbf{b}^\mu \\ & + \Lambda_\sigma g_s^2 g_\rho^2 \phi^2 \mathbf{b}_\mu \cdot \mathbf{b}^\mu + \Lambda_v g_v^2 g_\rho^2 \mathbf{b}_\mu \cdot \mathbf{b}^\mu V_\mu V^\mu\end{aligned}$$

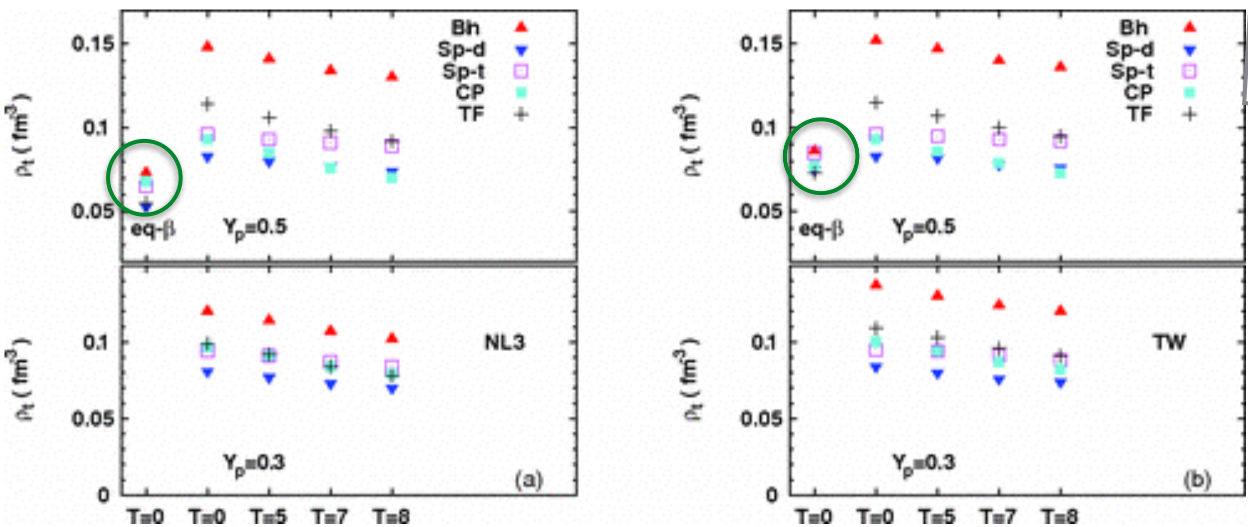
How to calculate transition density?



1) Get the instability region:
- Dynamical spinodal or
- Thermodynamical spinodal

2) Intersect EoS with that boundary to get ρ_t





PRC 82, 055807, 2010
PRC 85, 059904(E), 2012

For β-eq. matter and T=0, dyn. spinodal very coincident with TF calculation

Thermodynamical spinodal

check PRC 74, 024317 2006

- The (free) energy curvature matrix for asymmetric NM is defined by:

$$\mathcal{C} = \left(\frac{\partial^2 \mathcal{F}}{\partial \rho_i \partial \rho_j} \right)$$

- Stability conditions: $Tr(\mathcal{C}) > 0, Det(\mathcal{C}) > 0$

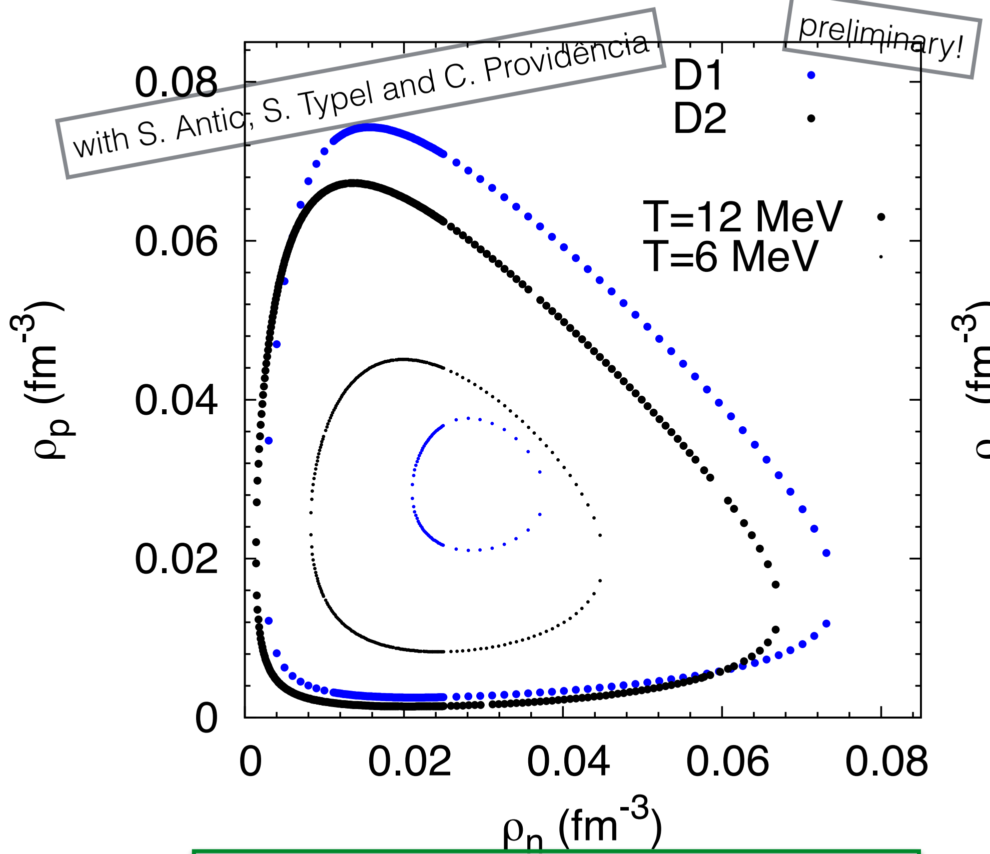
- The spinodal is given by (T, ρ_p, ρ_n) for which $Det(\mathcal{C}) = 0$

i.e., one of **eigenvalues is negative in the region of instability and goes to zero at border**:

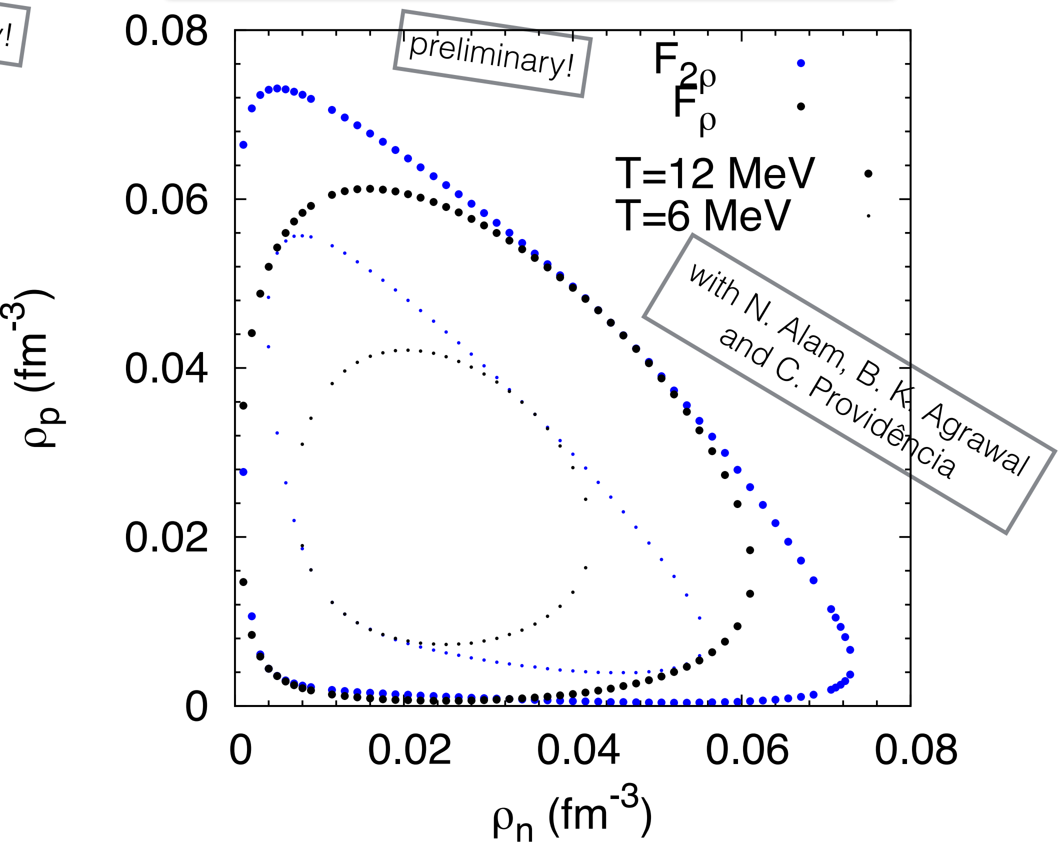
$$\lambda_- = \frac{1}{2}\left(Tr(\mathcal{C}) - \sqrt{Tr(\mathcal{C})^2 - 4Det(\mathcal{C})} \right) = 0$$

The crust-core transition - thermodynamical spinodal approach

a) density-dependent models



b) **non-linear mixing** meson couplings models



D* models: scalar and vector self-energies depend on E: the couplings are adjusted to the **optical potential** in nuclear matter.

Nucl. Phys. A 938, 92 2015

Different mixing couplings: different L: L(Fρ)=70 MeV, L(F2ρ)=46 MeV.

e.g. PRC 81, 034323 2010

Dynamical spinodal

check PRC 94, 015808 2016

- Dynamical instabilities are given by collective modes that correspond to small oscillations around equilibrium state.
- Very good tool to estimate crust-core transition in cold neutrino-free neutron stars.

check PRC 82, 055807 2010; PRC 85, 059904(E) 2012

- These small deviations are described by linearized equations of motion.
- Perturbed fields: $F_i = F_{i0} + \delta F_i$
- Perturbed distribution function: $f_i = f_{i0} + \delta f_i$

Dynamical spinodal (cont)

- The time evolution of the distribution functions is described by the Vlasov equation: $\frac{\partial f_i}{\partial t} + \{f_i, h_i\} = 0, \qquad i = p,\, n,\, e$

  semiclassical approach, that is a good approximation to t-dependent Hartree-Fock eqs at low energies

- We get a set of equations for the fields and particles, whose solutions form a complete set of eigenmodes, that lead to the following matrix:

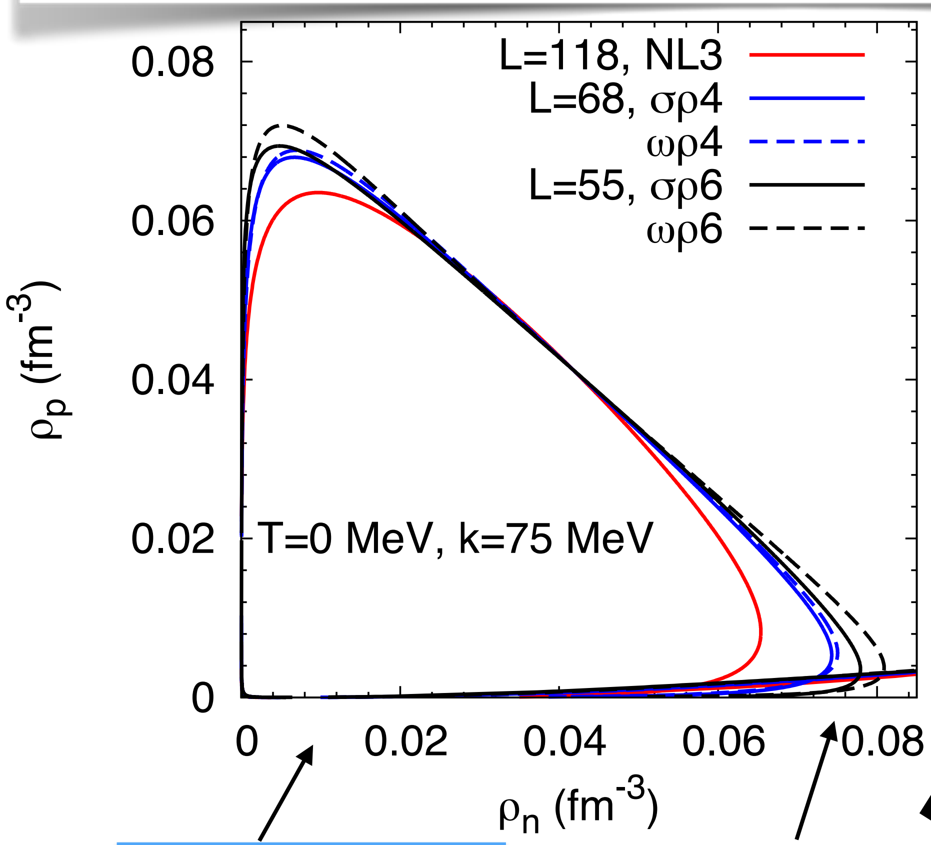
$$\begin{pmatrix} 1 + F^{pp} L_p & F^{pn} L_p & C_A^{pe} L_p \\ F^{np} L_n & 1 + F^{nn} L_n & 0 \\ C_A^{ep} L_e & 0 & 1 - C_A^{ee} L_e \end{pmatrix} \begin{pmatrix} \frac{2P_{F_p}}{3k} \frac{\delta\rho_p}{\rho_p} \\ \frac{2P_{F_n}}{3k} \frac{\delta\rho_n}{\rho_n} \\ \frac{2P_{F_e}}{3k} \frac{\delta\rho_e}{\rho_e} \end{pmatrix} = 0$$

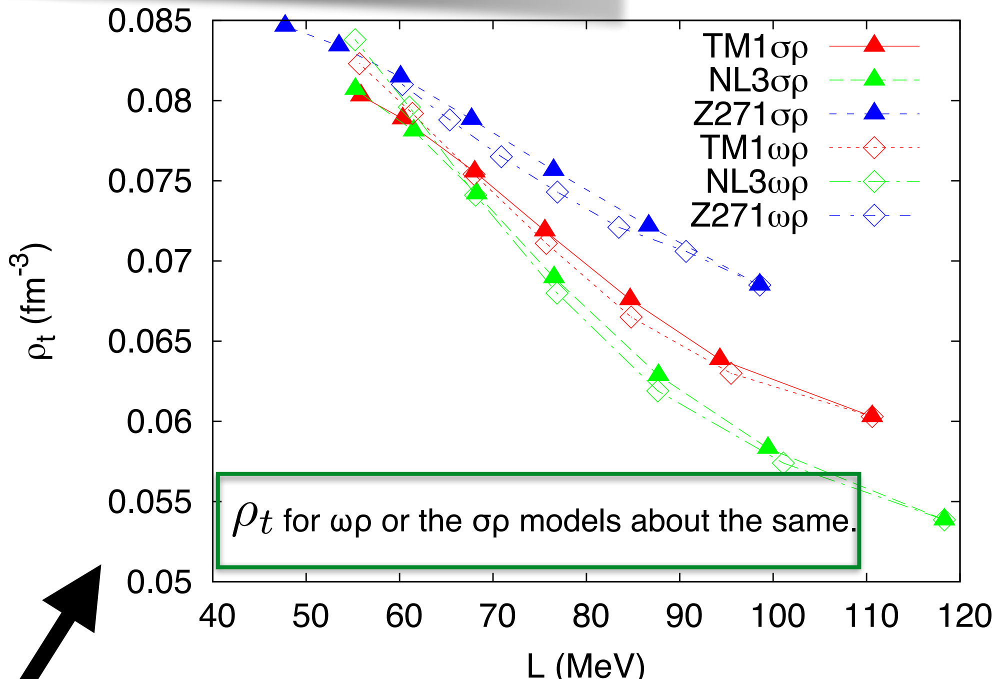
- The **dynamical spinodal** surface is defined by the region in (ρ_p, ρ_n) space, for a given wave vector **k** and temperature *T*, limited by the surface $\omega = 0$.

- In the **k**=0 MeV limit, the thermodynamic spinodal is obtained.

The crust-core transition – dynamical spinodal approach

PRC 94, 015808 2016



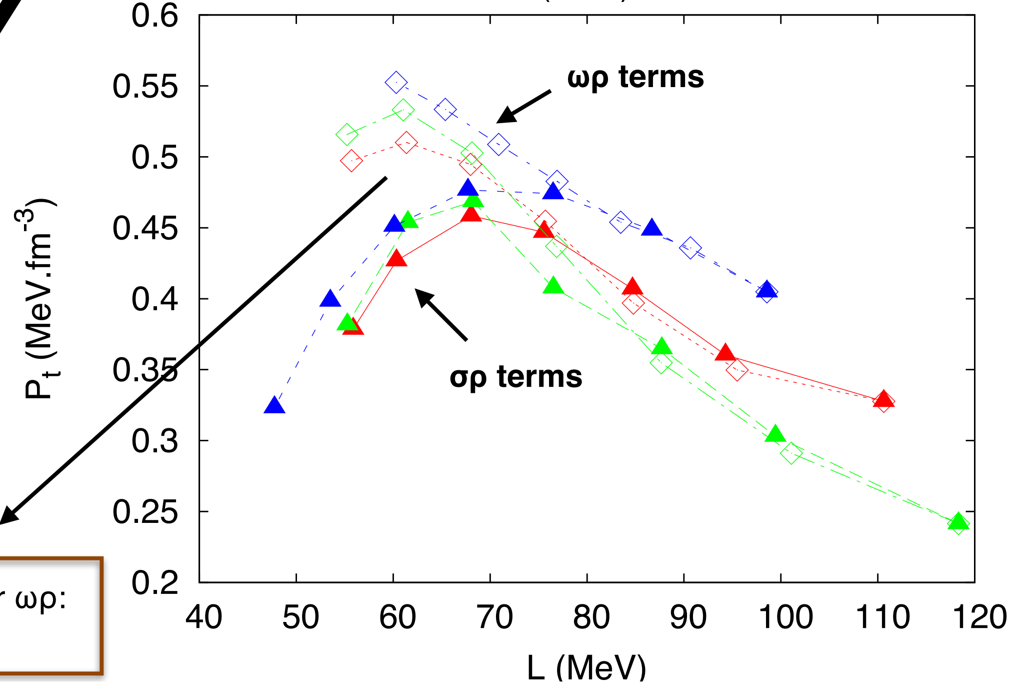


1.The larger L, the smaller the spinodal section.

2.The term ωρ makes the spinodal section larger compared to σρ.

but

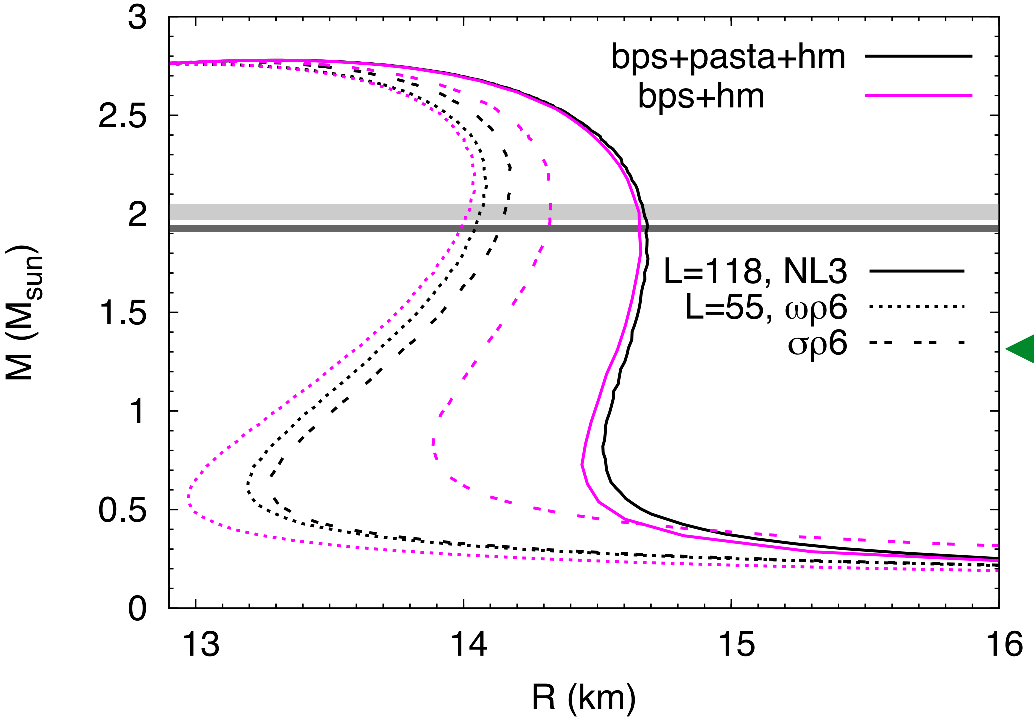
3.Crossing of the ωρ and σρ spinodals, for a given L, occurs close to the crossing of the β-eq EoS with the spinodals



L<80MeV, Pt is larger for ωρ: **direct implication in l**

M(R) relations

a) effect of pasta:



Effect on Mmax is negligible, not true for the radius!

Stars with unified inner crust-core EoS (black lines) **have larger** (smaller) **radii than configurations without inner crust** (pink lines) **for the NL3ωρ** (NL3σρ) models.

σρ give slightly larger radii than ωρ models, the differences being larger for M >~ 1.4M⊙. For 1.4M⊙ star, difference is of ~ 100 m.

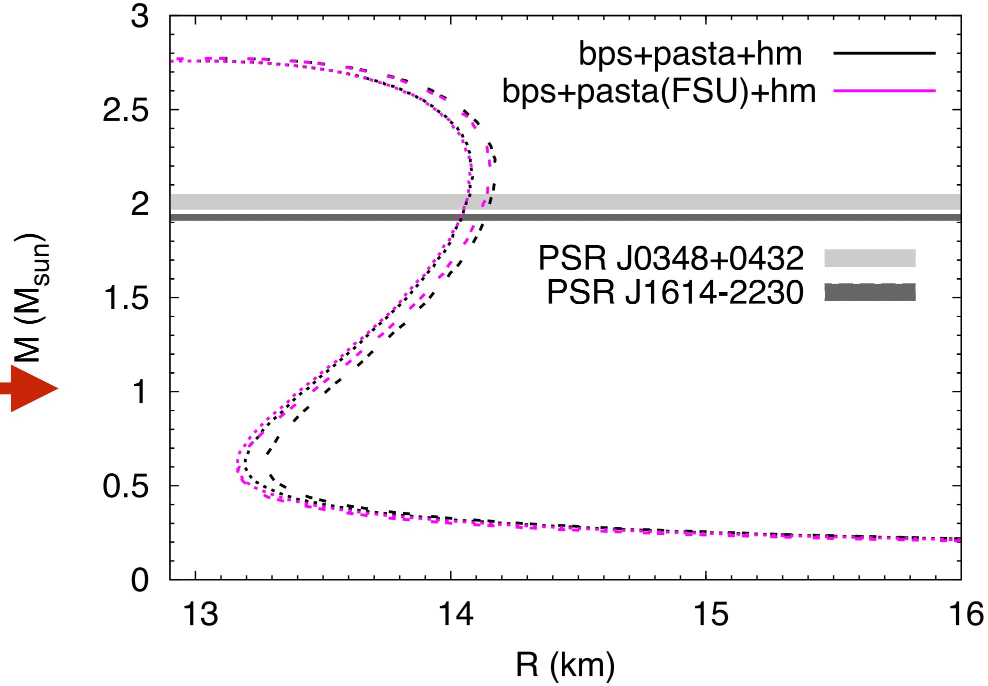
b) effect of different inner crust EoS with L close to core EoS:

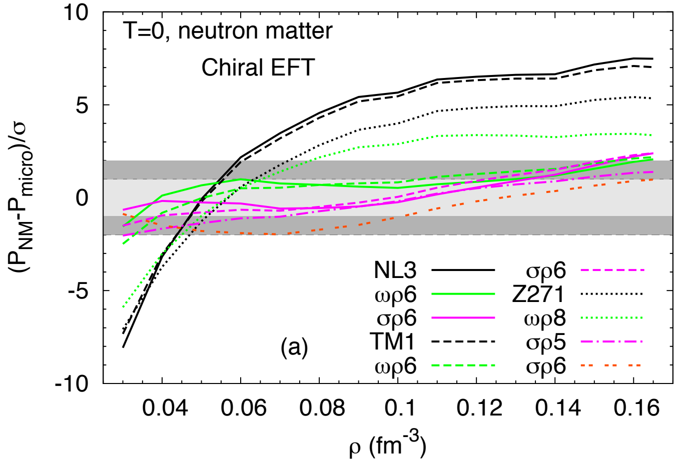
We get the transition density from a **dyn. spin.** calculation.

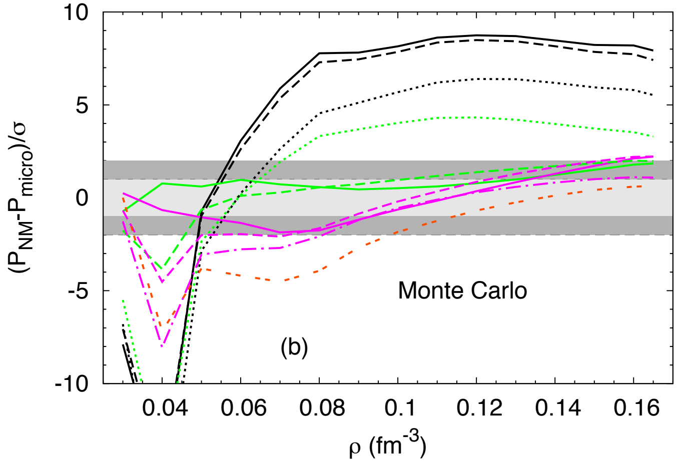
the **error on the determination of the radius is negligible** for all masses!

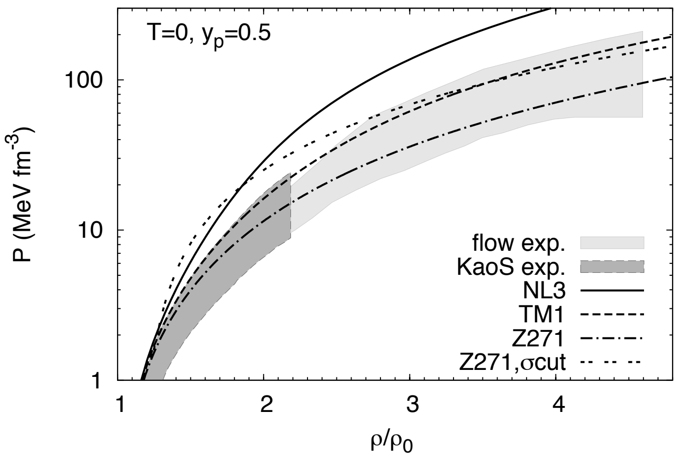
exceptions: NL3σρ6, difference of ~50 m (~ 40 m) for a 1M⊙ (1.4M⊙) star; NL3ωρ6, ~ 20 m for a 1M⊙ star

tested for **other models** (TM1 and Z271): **same result**!

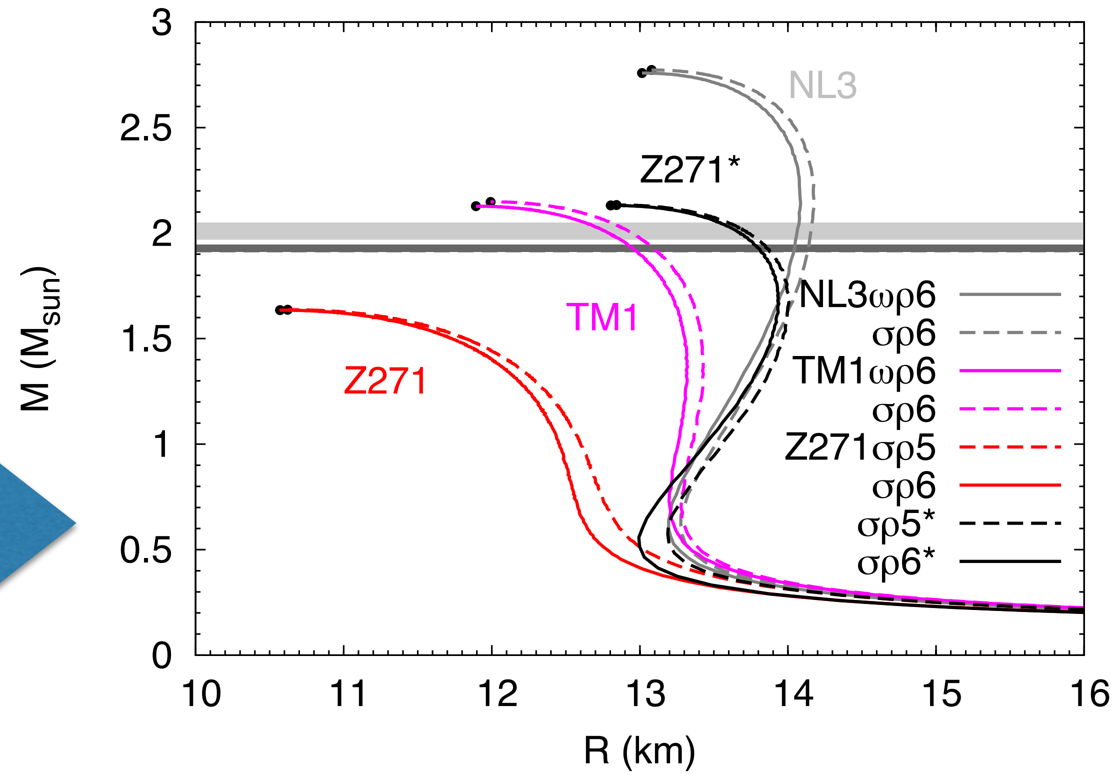








If we combine the 3 constrains, we get the following models:



For 1.4M⊙ stars, these models predict R=13.6 ± 0.3 km and a crust thickness of 1.36 ± 0.06km.

exceptions: **NL3** did not pass exp. constrain

and **Z271** did not pass obs. constrain but

Z271*: extra potential dependent on σ meson, that makes M* to stop decreasing above saturation density, as suggested in K. A. Maslov, E. E. Kolomeitsev, and D. N. Voskresensky, Phys. Rev. C 92, 052801 (2015).

ext. Nambu–Jona-Lasinio Model

PRC 93, 065805 2016

- Set of models with chiral symmetry included, unlike RMFs
- Since chiral symmetry is satisfied, EoS valid at higher densities!

$$\mathcal{L} = \bar{\psi}(i\gamma^\mu\partial_\mu - m)\psi$$

to make restoration of the chiral symmetry less abrupt

$$+G_s[(\bar{\psi}\psi)^2 + (\bar{\psi}i\gamma_5\vec{\tau}\psi)^2] - G_v(\bar{\psi}\gamma^\mu\psi)^2$$

short range attraction

short range repulsion

$$-G_{sv}[(\bar{\psi}\psi)^2 + (\bar{\psi}i\gamma_5\vec{\tau}\psi)^2](\bar{\psi}\gamma^\mu\psi)^2$$

density dependence of scalar coupling

$$-G_\rho\left[(\bar{\psi}\gamma^\mu\vec{\tau}\psi)^2 + (\bar{\psi}\gamma_5\gamma^\mu\vec{\tau}\psi)^2\right]$$

isospin asymmetric nuclear matter

$$-G_{v\rho}(\bar{\psi}\gamma^\mu\psi)^2\left[(\bar{\psi}\gamma^\mu\vec{\tau}\psi)^2 + (\bar{\psi}\gamma_5\gamma^\mu\vec{\tau}\psi)^2\right]$$

$$-G_{s\rho}\left[(\bar{\psi}\psi)^2 + (\bar{\psi}i\gamma_5\vec{\tau}\psi)^2\right]\left[(\bar{\psi}\gamma^\mu\vec{\tau}\psi)^2 + (\bar{\psi}\gamma_5\gamma^\mu\vec{\tau}\psi)^2\right]$$

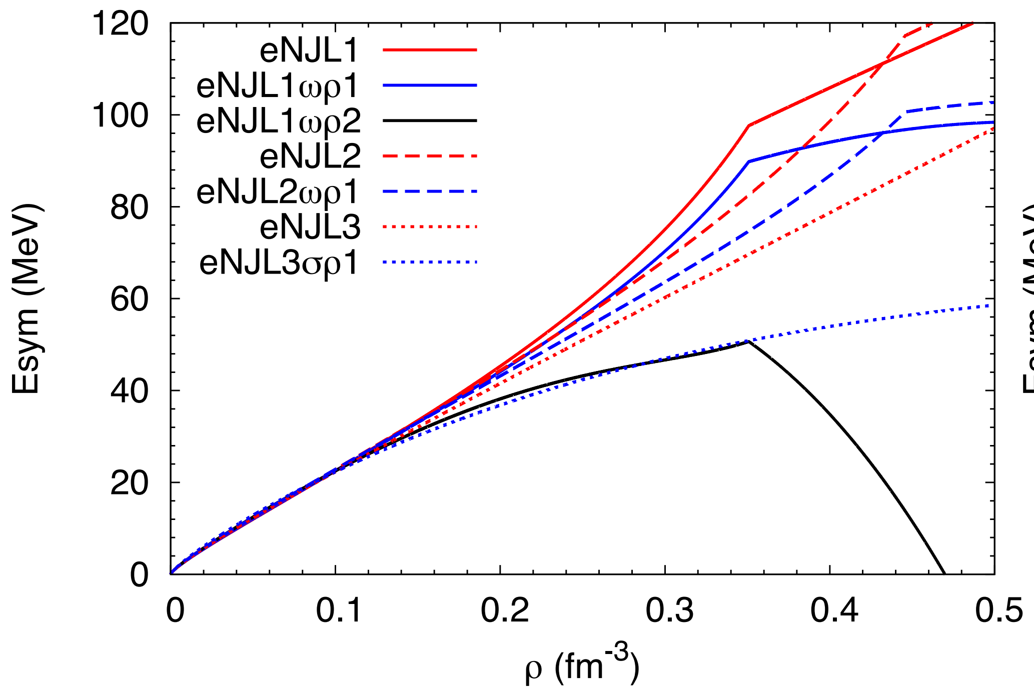
make the symmetry energy softer

$$M = m - 2G_s\rho_s + 2G_{sv}\rho_s\rho^2 + 2G_{s\rho}\rho_s\rho_3^2$$

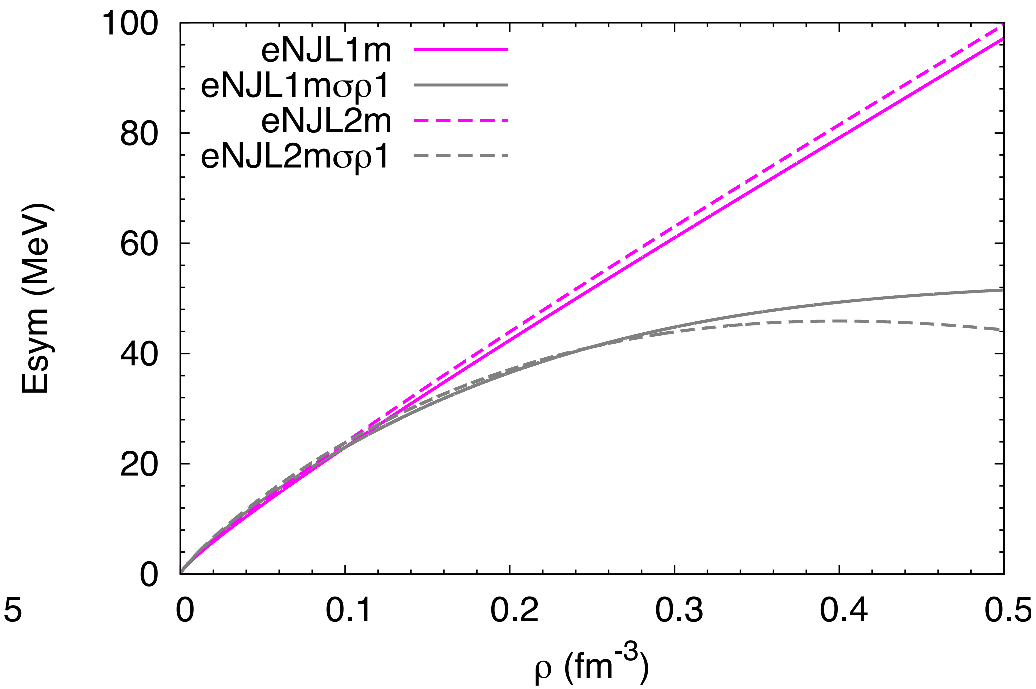
nucleon effective mass

eNJL models

- In this study, we used eNJLx, eNJLxωρy, and eNJLxσρy type of models
- We also considered models with a current mass: eNJLxm, and eNJLxmσρy.
- To make Esym softer: *ωρ* and *σρ* models, where we fixed the Esym at ρ=0.1 at the same value of eNJLx (eNJLxm), and we calculated the new Gρ, fixing the Gvρ (Gsρ) constant.



without current mass



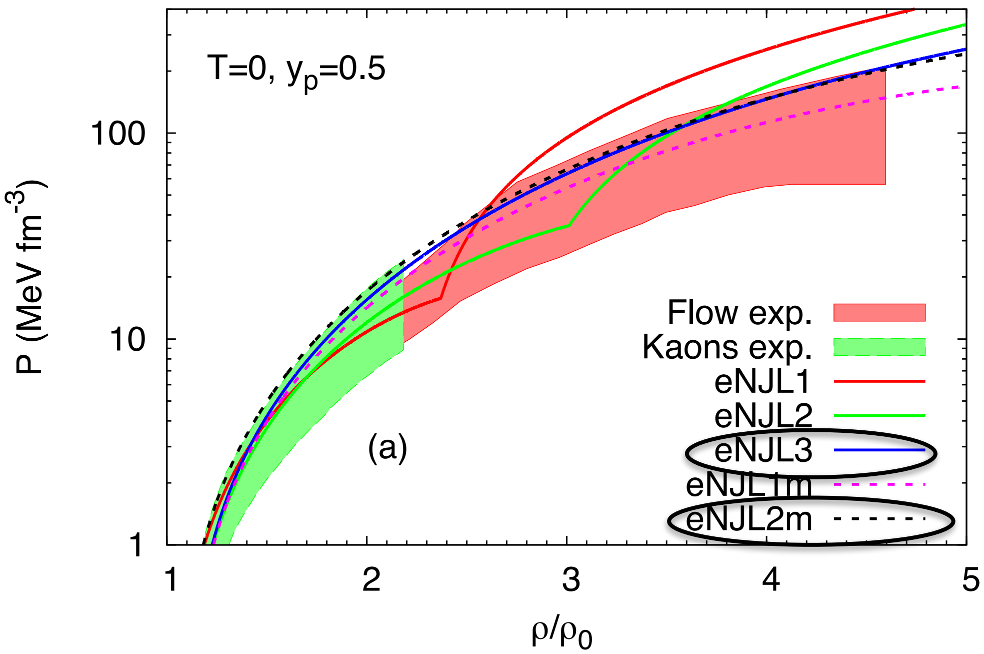
with current mass

eNJL models-Constrains

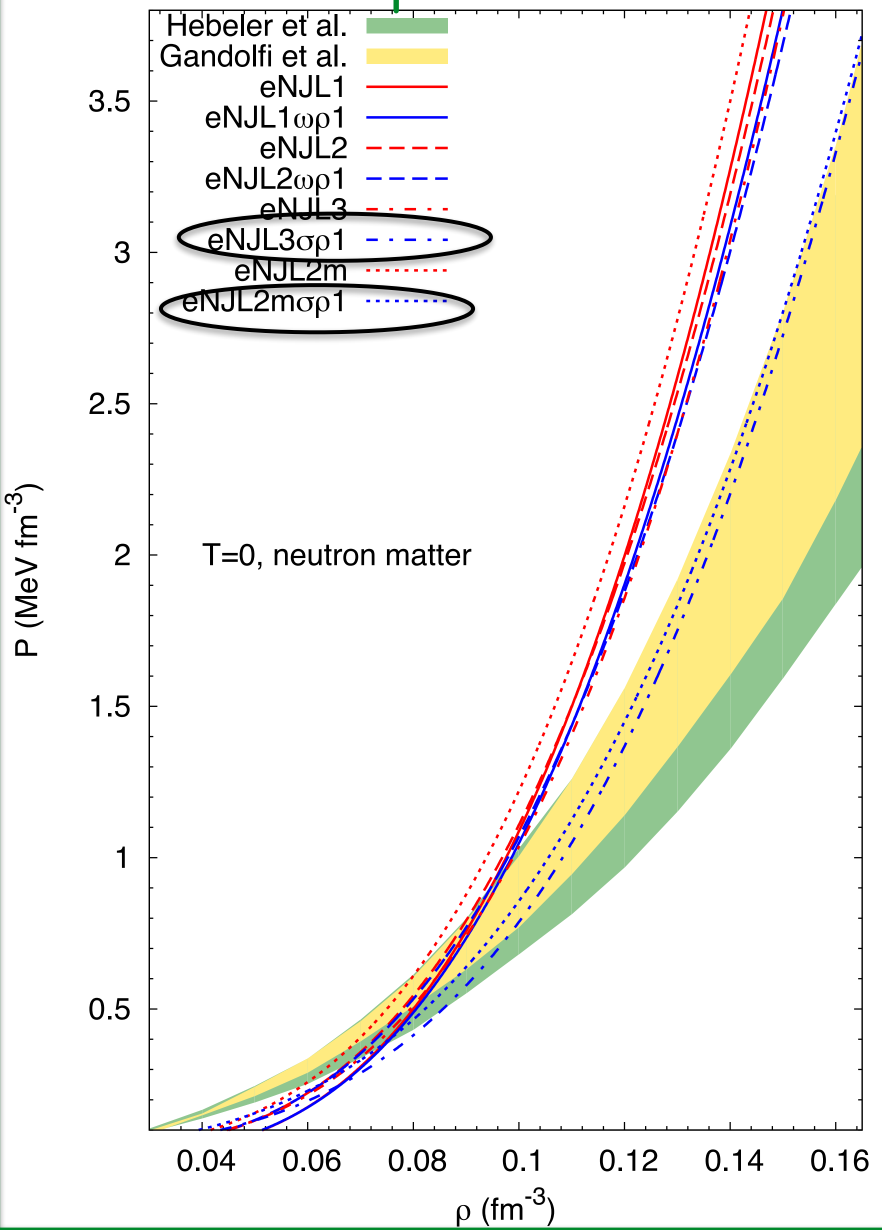
But the models need to fulfil the constrains…

only 2 models passed:
eNJL3σρ1 and **eNJL2mσρ1**

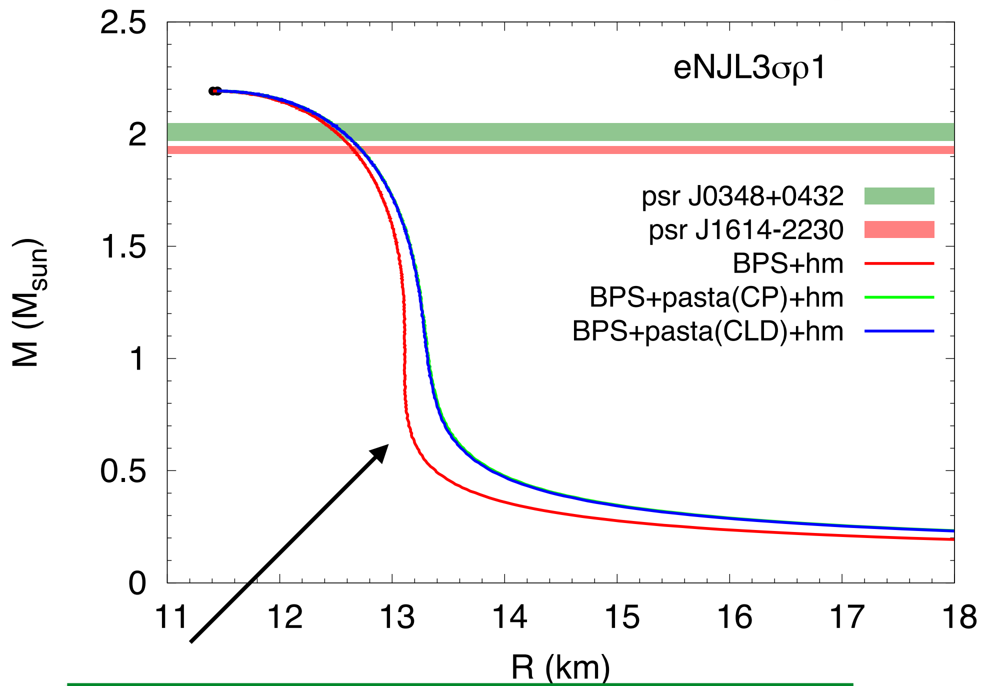
Experiments



Microscopic calculations



M(R) relations



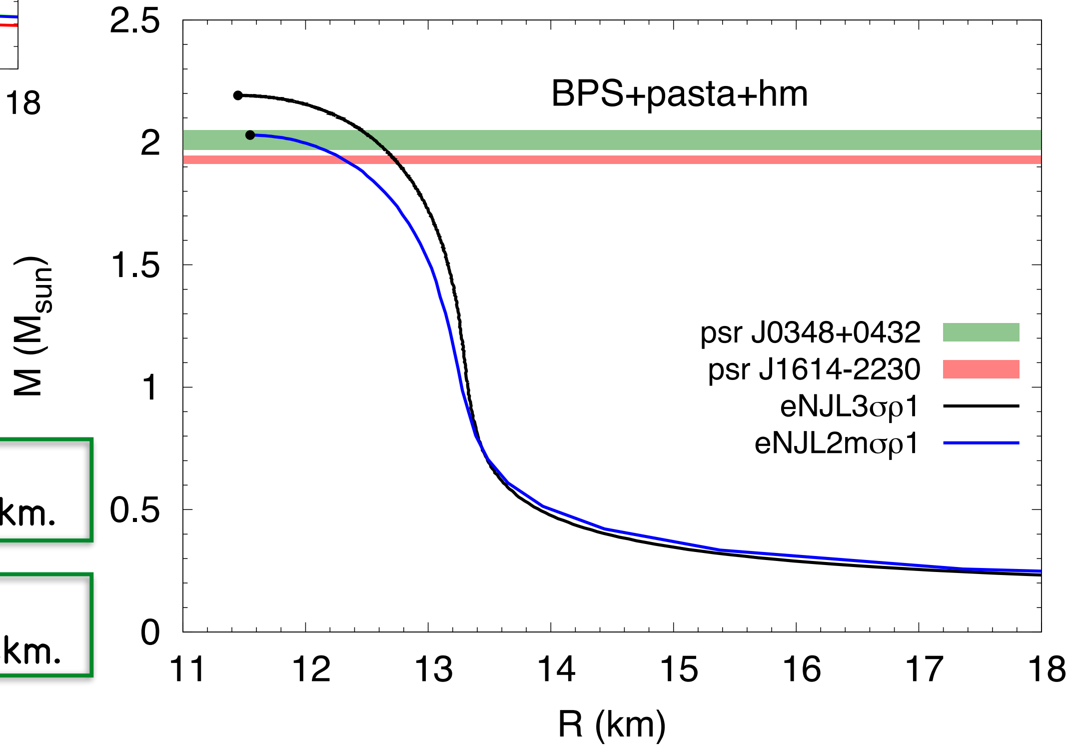
CP or CLD: no difference in M(R)

1) EoS:

1) outer crust: BPS
2) inner crust: pasta from a CP or CLD calculation
3) core: hom. nucleonic matter, same model as inner crust

2) integrate TOV

3) get M(R)



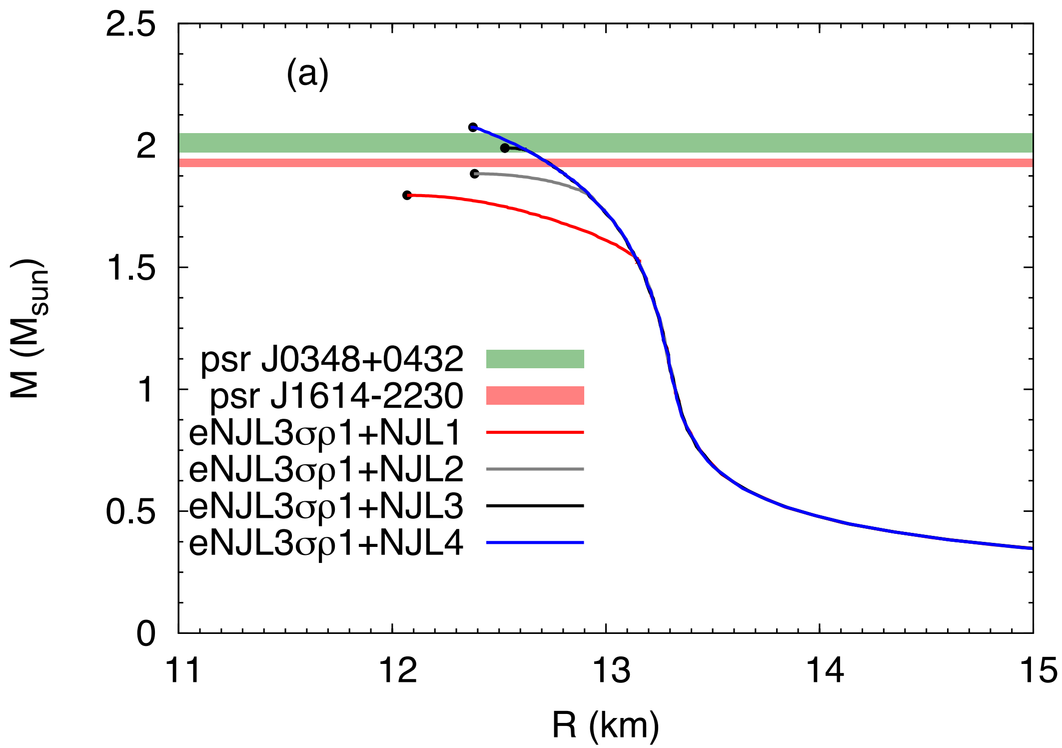
eNJL3σρ1:
R(M=1.4M⊙)=13.212 km, with ΔR(M=1.4M⊙)=1.405km.

eNJL2mσρ1:
R(M=1.4M⊙)=13.084 km, with ΔR(M=1.4M⊙)=1.408km.

M(R) relations (cont)

Considering hybrid stars:

- quark core described within SU(3) NJL model
- perform a Maxwell construction to get hadronic to quark transition



The deconfinement phase transition decreases maximum mass

though…

We are still able to describe stable 2M⊙ stars with a quark core!

Summary

- Inclusion of the inner crust EoS has strong effect on the radius of low and intermediate mass neutron stars!
- Unified EoS are needed!

but...

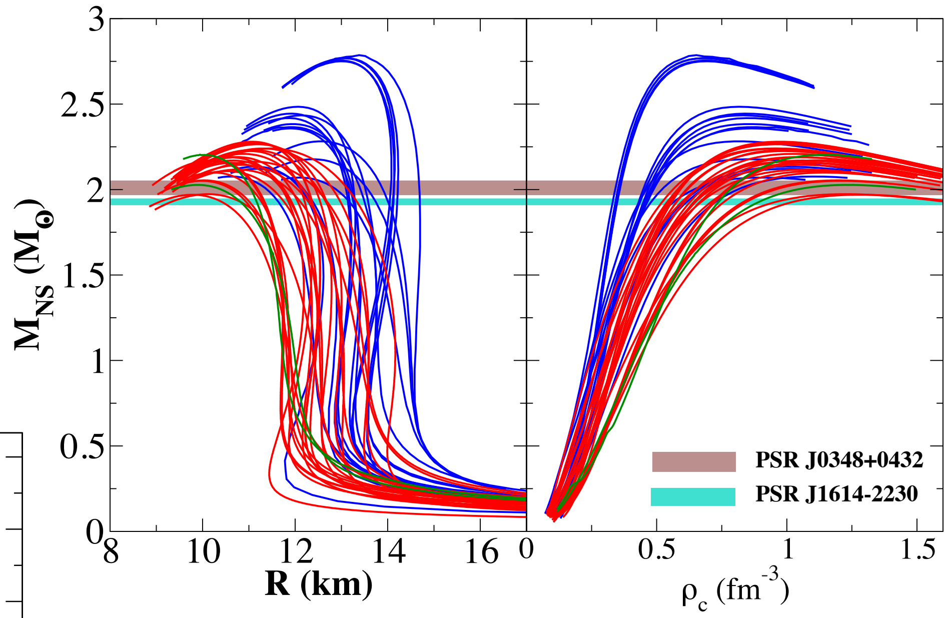
- Inner crust EoS with similar symmetry energy properties as the core EoS: effect on radii for stars with $M > 1\,M_\odot$ is negligible!
- For RMF models, $R(M=1.4M_\odot)=13.6\pm0.3$km, with $\Delta R(M=1.4M_\odot)=1.36\pm0.06$km.
- Inner-crust–core unified EoS with chiral symmetry and pasta allows the description of 2 $M_\odot$ stars, with $R(M=1.4M_\odot)=13.148\pm0.064$km.

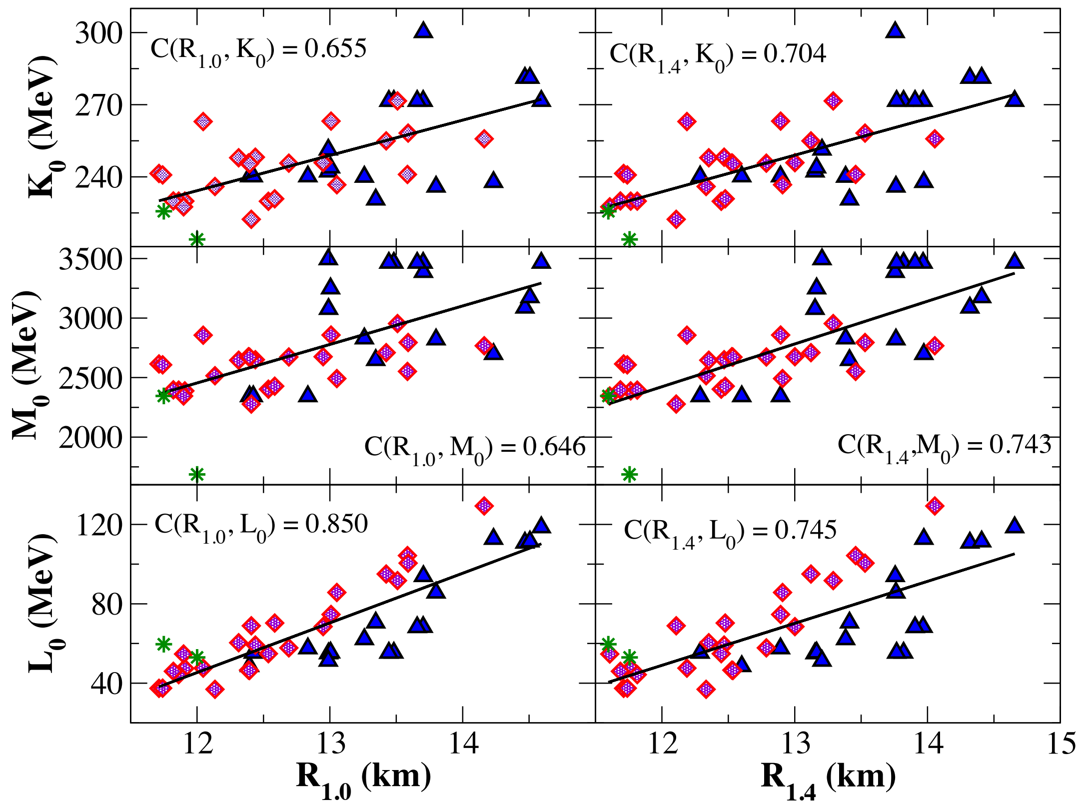
Strong correlations of neutron star radii with the slopes of nuclear matter incompressibility and symmetry energy at saturation

accep. PRC (R), arXiv: 1610.06344[nucl-th]

set of **24 Skyrme-type** effective forces and **18 RMF** models, and **2 microscopic calculations**, all describing 2M⊙ neutron stars.

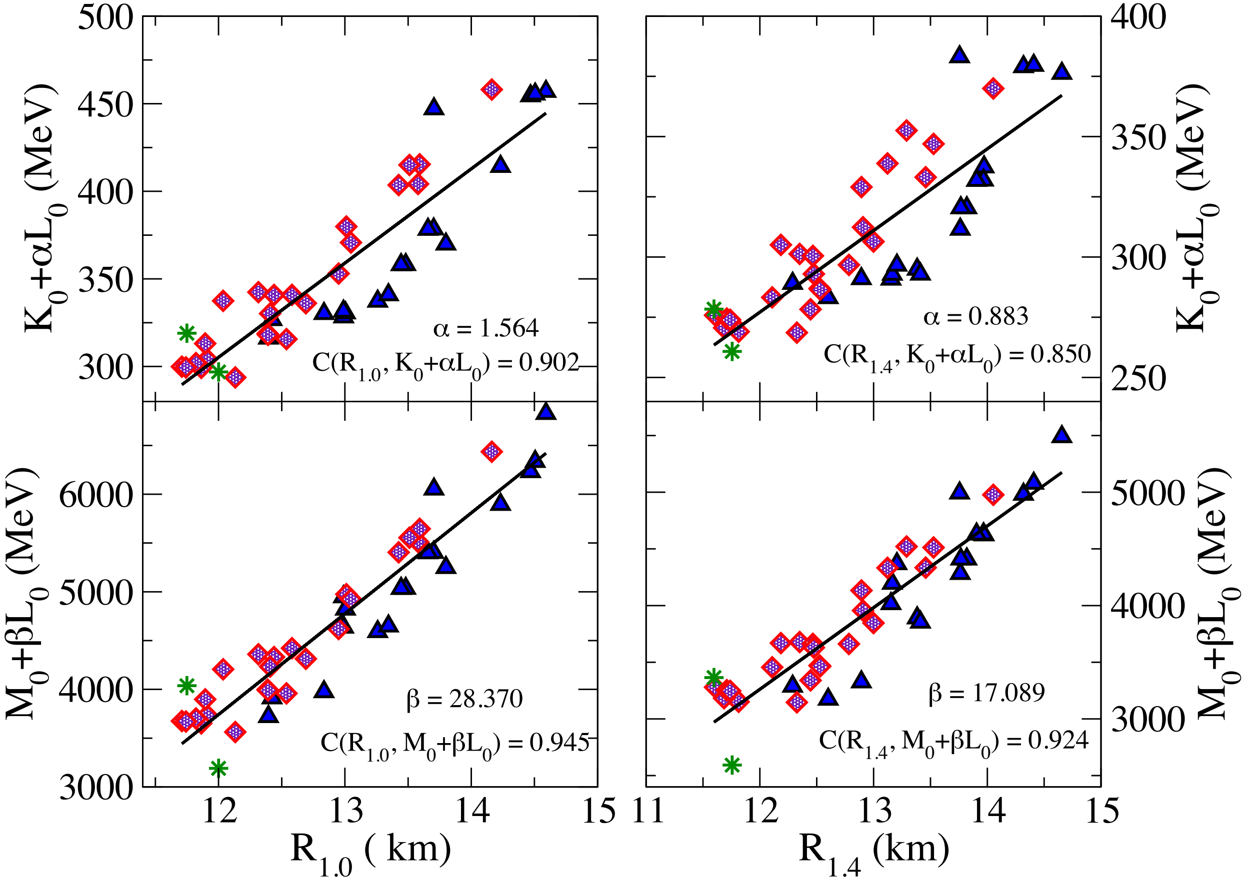
Unified EoSs for the inner-crust-core region have been built for all the phenomenological models, both relativistic and non-relativistic.



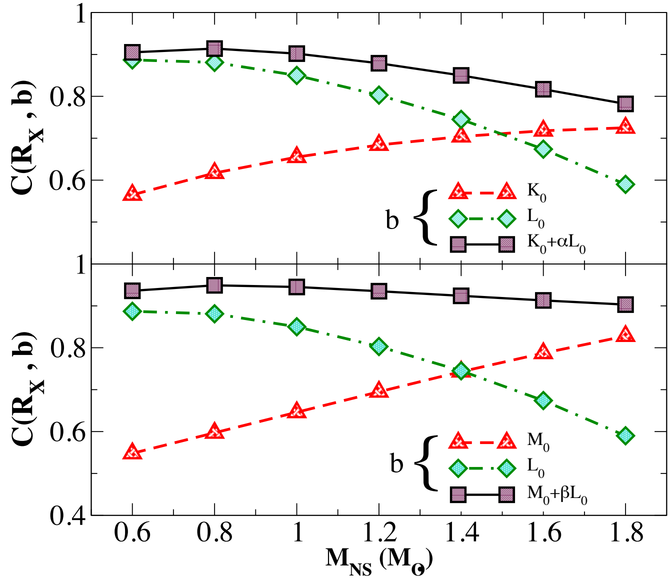


We started by calculating correlation of R with K, M and L…

and then…



and….



We found **strong correlation** of the neutron star **R** with **linear combination of M and L,** and **almost independent of the neutron star mass** in the range 0.6-1.8M⊙.

This correlation can be linked to the empirical relation between R and P at a nucleonic density between 1-2 saturation density, and the dependence of P on K, M and L.

Thank you!