Variational Method with Explicit Energy Functionals for Nuclear Matter

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1. Introduction
2. Fermion systems with central forces and ^{3}He systems.
3. Nuclear matter with central and tensor forces.
4. Extension of the explicit energy functional
5. Summary

2016.10.21. Nuclear Physics, Compact Stars, and Compact Star Mergers 2016@YITP

0. Nuclear EOS for Core-Collapse Supernovae with the Variational Method

H. Togashi,[1)] K. Nakazato,[2)] Y. Takehara,[3)] S. Yamamuro,[3)] H. Suzuki,[3)] and M. Takano

[1)] Riken, [2)] Kyushu University, [3)] Tokyo University of Science

An EOS table for supernova numerical simulations constructed with the cluster variational method based on the Argonne v18 two-body potential and the Urbana IX three-body potential

APR-EOS

Grid point

Parameter	Minimum	Maximum	Mesh	Number
$\log_{10}(T)$ [MeV]	-1.00	2.60	0.04	$91 + 1$
Y_p	0.00	0.65	0.01	66
$\log_{10}(\rho_B)$ [g/cm^3]	5.1	16.0	0.10	110

For Uniform Nuclear Matter

Jastrow wave function

$$\Psi(x_1,\cdots,x_N) = Sym\left[\prod_{i<j} f_{ij}\right]\Phi_F(x_1,\cdots,x_N)$$

Φ_F: Fermi-gas wave function

Sym[ ]: symmetrizer

f_{ij}: correlation function (variational function)

The expectation value of the Hamiltonian is cluster-expanded.

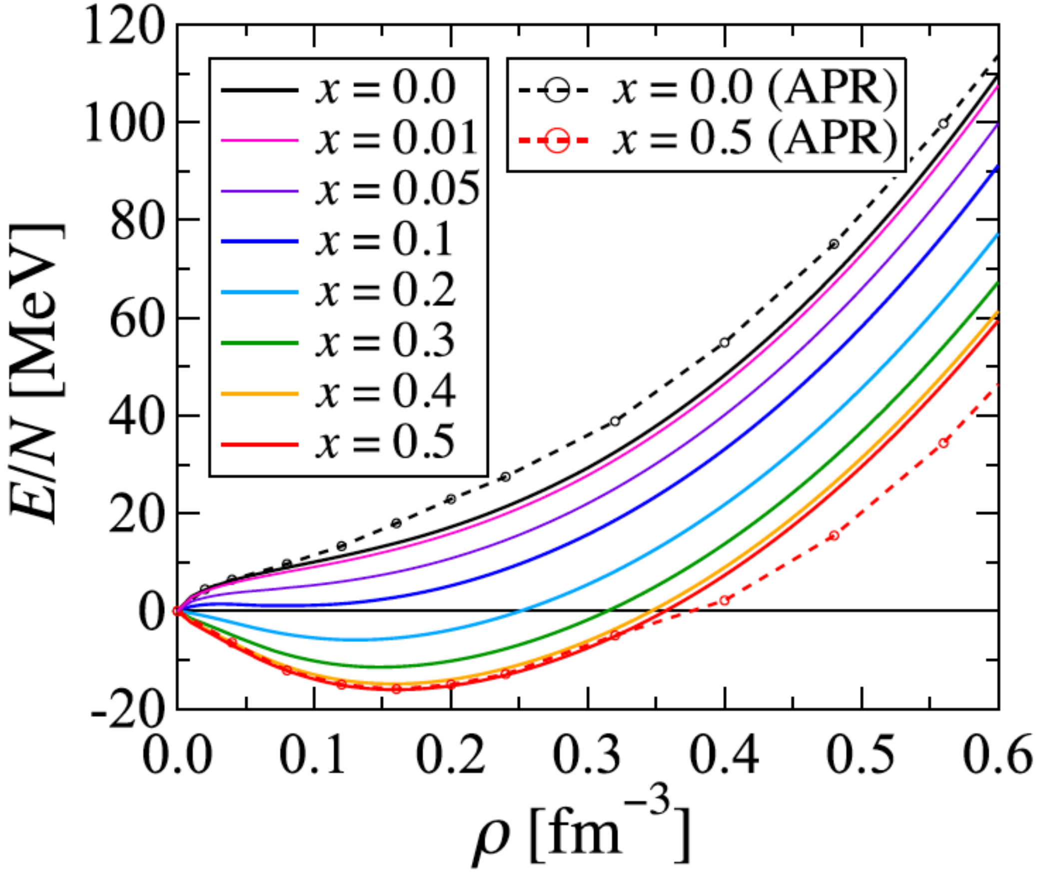
Two-body cluster approximation is employed.

At Finite Temperatures

The prescription by Schmidt and Pandharipande (SP) is empoyed

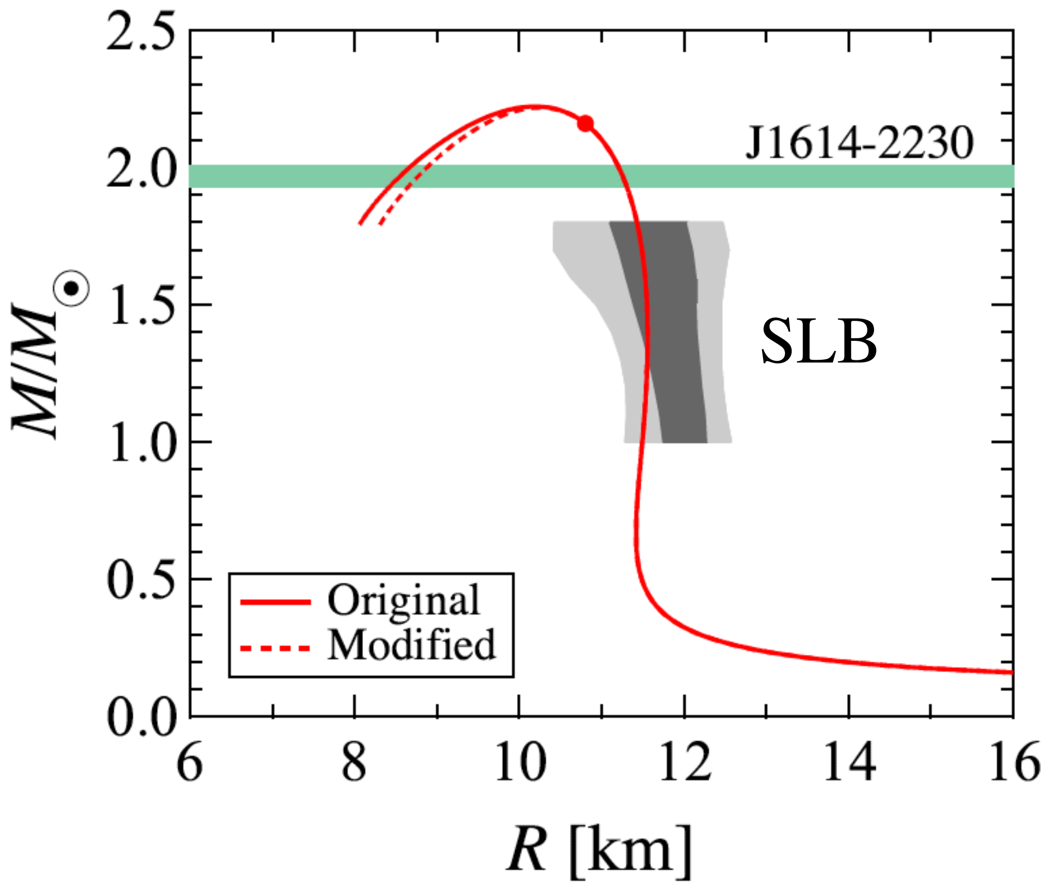
SP: K. E. Schmidt and V. R. Pandharipande, Phys. Lett. B87 (1979) 89.

Energy per nucleon of nuclear matter



x: proton fraction

Neutron Stars with our EOS

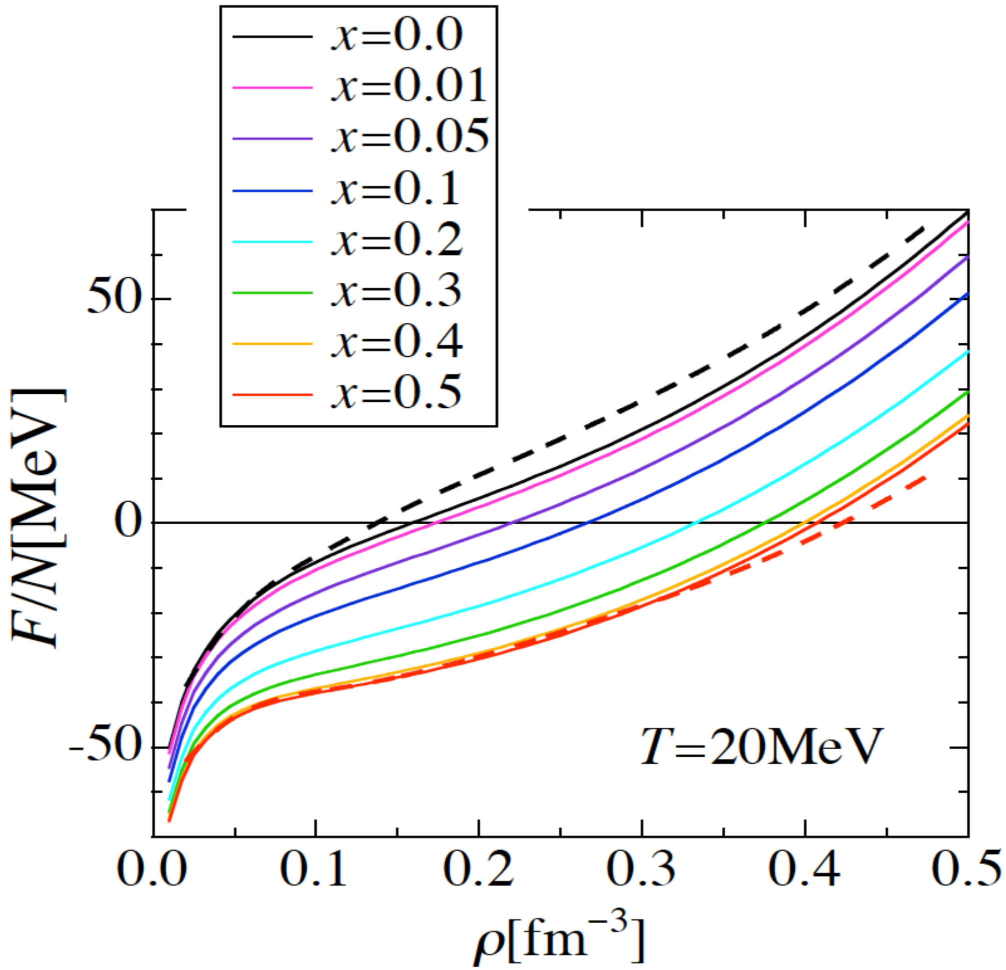


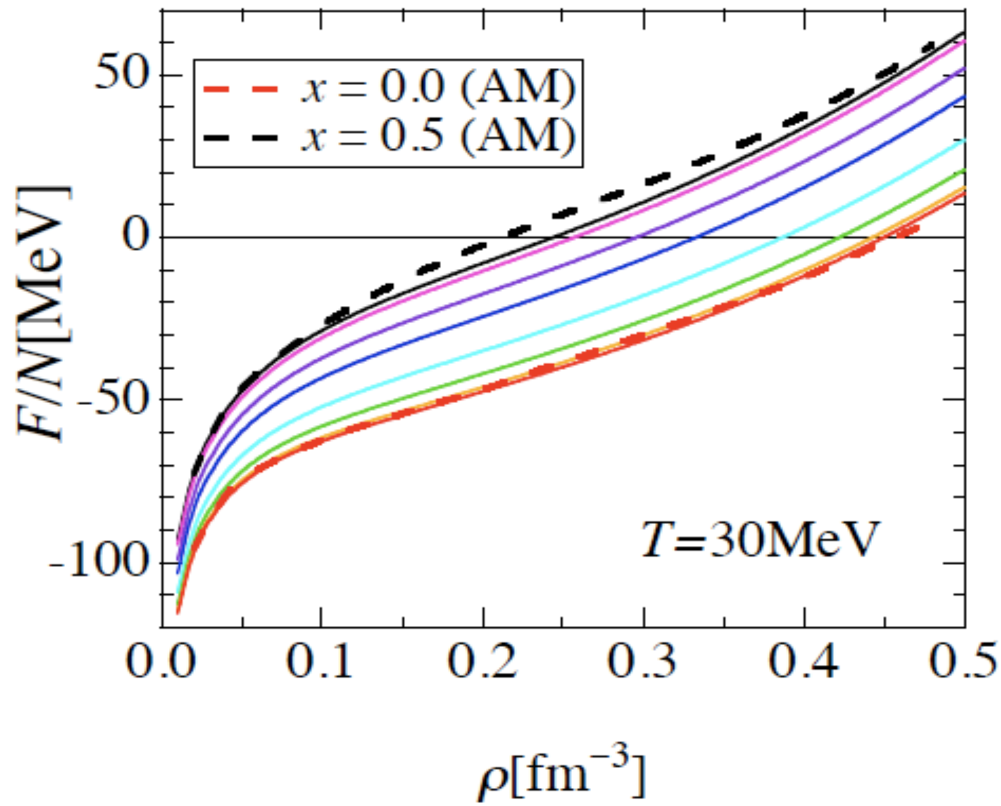
H. Togashi and M. T., Nucl. Phys. A902 (2013) 53.

APR: Akmal, Pandharipande, Ravenhall, Phys. Rev. C58 (1998) 1804.

SLB: A. W. Steiner, J. M. Lattimer, E. F. Brown, Astrophys. J. 722 (2010) 33.

Free energy per nucleon at finite temperature





H. Togashi and M. T. Nucl. Phys. A 902 (2013) 53.

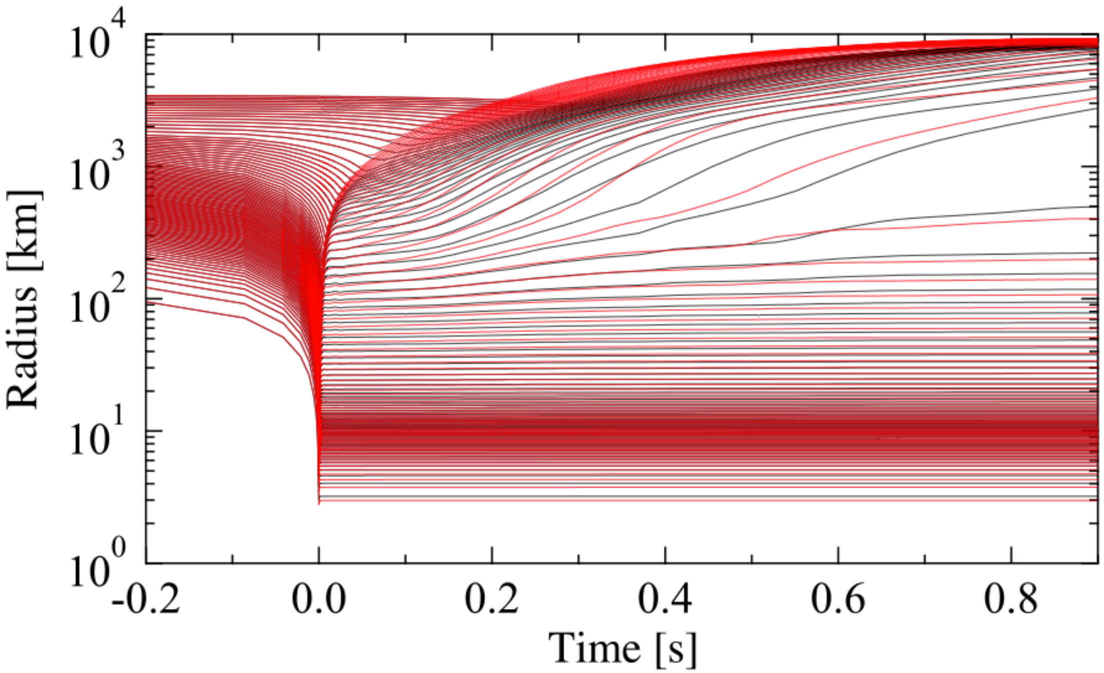
AM: A. Mukherjee, PRC79(2009)045811.

Application to Spherical Core-collapse Supernovae

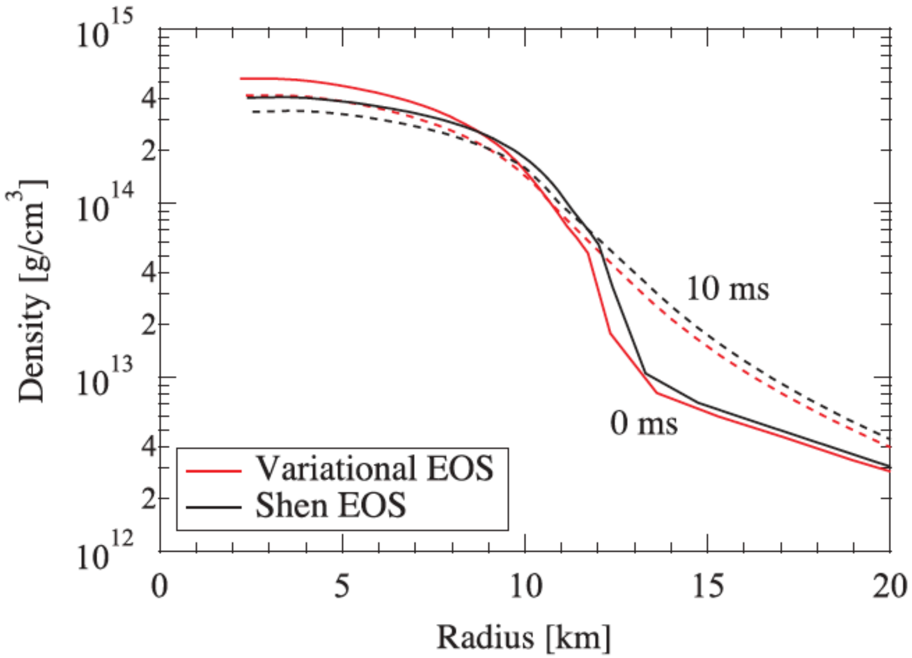
H. Togashi, M. T., K. Sumiyoshi and K. Nakazato, Prog. Theor. Exp. Phys. 2014, 023D05.

The Shen EOS is employed for non-uniform phase

Adiabatic calculation (1D)



Density profiles



The Shen EOS: H. Shen, H. Toki, K. Oyamatsu and K. Sumiyoshi, Astrophys. J. Suppl. 197 (2011) 20.

For Non-uniform Nuclear Matter

The Thomas-Fermi calculation is performed

Free energy of a Wigner Seitz cell $F_{\rm cell}(n_{\rm B}, Y_{\rm p}, T) = F_{\rm bulk} + E_{\rm grad} + E_{\rm C},$

Bulk energy

$$F_{\rm bulk} = \int_{\rm cell} d\boldsymbol{r} f(n_{\rm p}(r), n_{\rm n}(r), n_{\alpha}(r), T),$$

Gradient energy

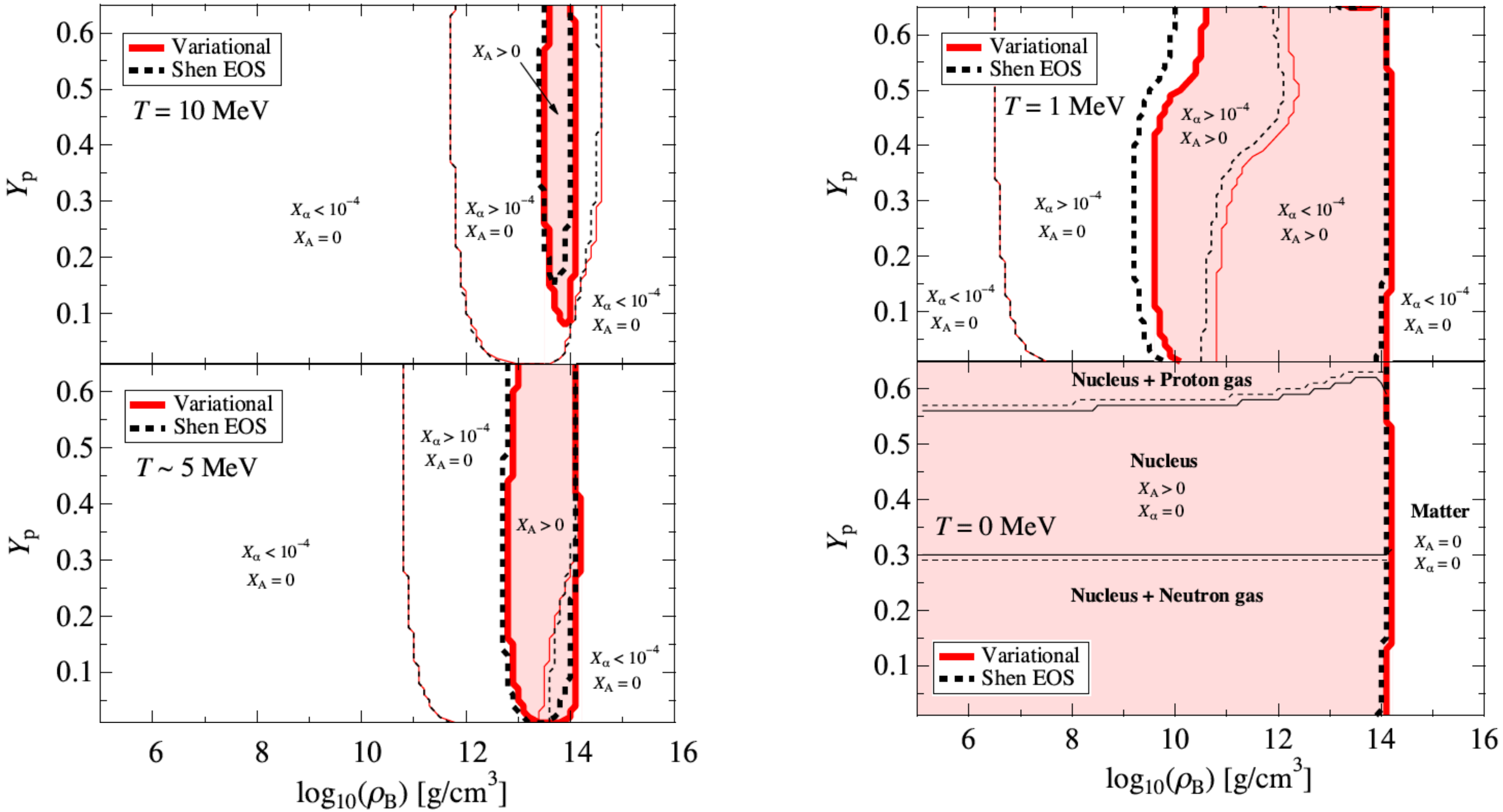
$$E_{\rm grad} = F_0 \int_{\rm cell} d\boldsymbol{r} |\nabla(n_{\rm p}(r) + n_{\rm n}(r))|^2,$$

Coulomb energy

$$E_{\rm C} = \frac{e^2}{2} \int_{\rm cell} d\boldsymbol{r} \int_{\rm cell} d\boldsymbol{r'} \frac{[n_{\rm p}(r) + 2n_{\alpha}(r) - n_{\rm e}][n_{\rm p}(r') + 2n_{\alpha}(r') - n_{\rm e}]}{|\boldsymbol{r} - \boldsymbol{r'}|}$$
$$+ \; c_{\rm bcc} \frac{(Z_{\rm non} e)^2}{a},$$

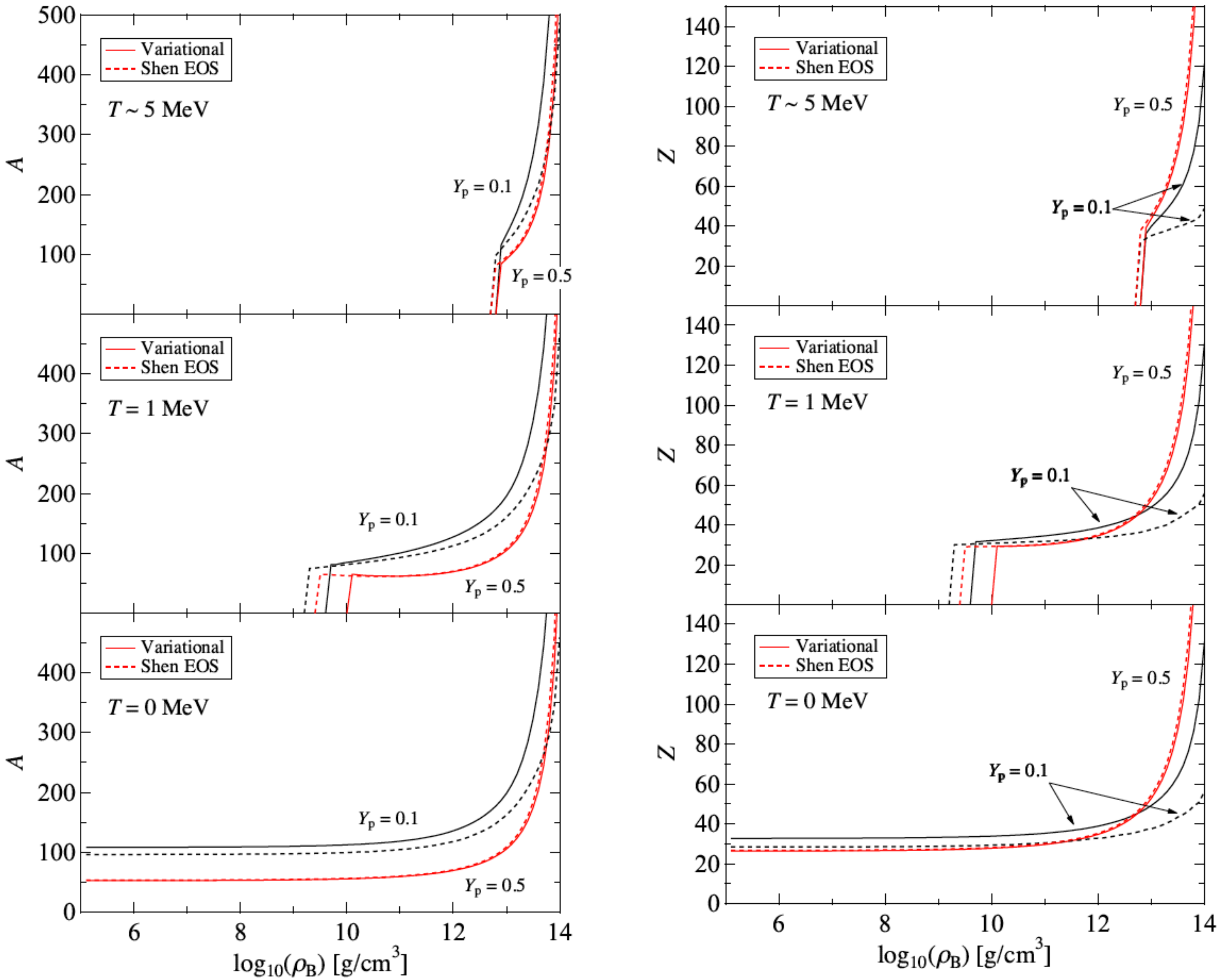
$n_{\rm p}(r)$: Proton number density in the WS cell,
$n_{\rm n}(r)$: Neutron number density in the WS cell,
$n_{\alpha}(r)$: Alpha-particle number density in the WS cell

Phase Diagram of Nuclear Matter



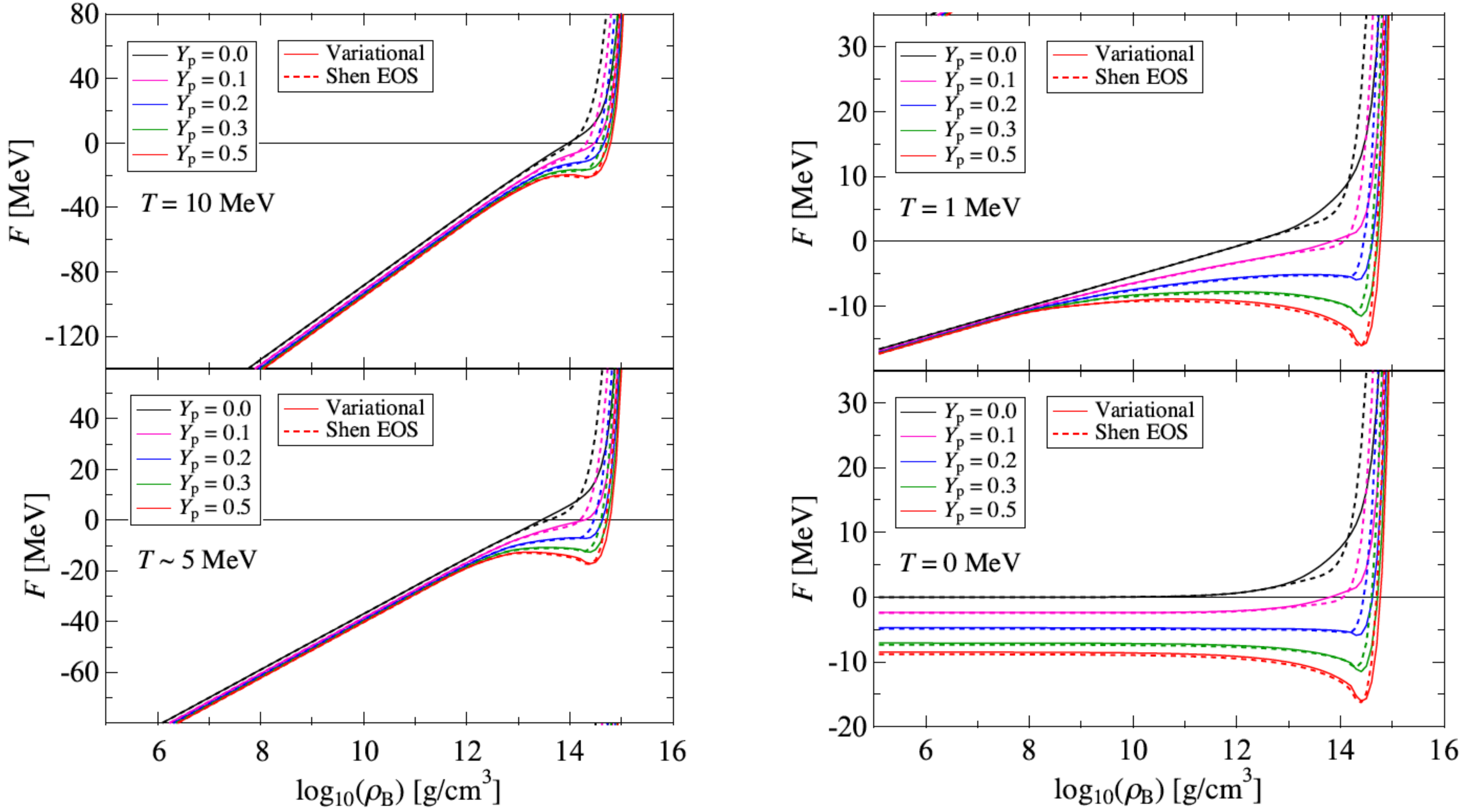
H. Togashi et al., in preperation

Heavy Nuclei in Supernova Matter (Single Nucleus Approximation)



H. Togashi et al., in preperation

Free energy



For details, please come to the poster by H. Togashi

H. Togashi et al., in preperation

1. Variational Method with Explicit Energy Functional

We calculate the energy per particle of
strongly correlated uniform fermion systems
starting from bare interactions between fermions

The energy per particle of a uniform fermion system
is expressed explicitly
with two-body distribution functions.

Explicit Energy Functional

⇩

The Euler-Lagrange equations are solved numerically

Fully minimized energy per particle is obtained

Explicit Energy Functional of Fermion Systems with a Two-body Central force

- Spin-dependent radial distribution function

$$F_s(r_{12}) = \Omega^2 \sum_{\text{spin}} \int \Psi^\dagger(x_1,\cdots,x_N) P_{s12} \Psi(x_1,\cdots,x_N)\, d\boldsymbol{r}_3 \cdots d\boldsymbol{r}_N \qquad (s = 0, 1)$$

P_{sij} : Spin projection operator

- Structure function

$$S_{c1}(k) \equiv \frac{1}{N} \left\langle \left| \sum_{i=1}^{N} \exp(i\boldsymbol{k}\cdot\boldsymbol{r}_i) \right|^2 \right\rangle = 1 + S_1(k) + S_0(k) \geq 0$$

$$S_{c2}(k) \equiv \frac{1}{3N} \left\langle \left| \sum_{i=1}^{N} \boldsymbol{\sigma}_i \exp(i\boldsymbol{k}\cdot\boldsymbol{r}_i) \right|^2 \right\rangle = 1 + \frac{1}{3} S_1(k) - S_0(k) \geq 0$$

$$S_s(k) = \rho \int \left[F_s(r) - F_s(\infty) \right] \exp(i\boldsymbol{k}\cdot\boldsymbol{r}) d\boldsymbol{r} \qquad (s = 0, 1)$$

Explicit Energy Functional for Fermion Systems with Central Forces

$$\frac{E}{N}(\rho) = \frac{3}{5}E_{\mathrm{F}} + 2\pi\rho\sum_{s=0}^{1}\int_0^\infty F_s(r)V_s(r)r^2dr$$
$$+\frac{\pi\hbar^2\rho}{2m}\sum_{s=0}^{1}\int_0^\infty\left[\frac{1}{F_s(r)}\frac{dF_s(r)}{dr}-\frac{1}{F_{\mathrm{F}s}(r)}\frac{dF_{\mathrm{F}s}(r)}{dr}\right]^2F_s(r)\,r^2dr$$
$$-\frac{\hbar^2}{16\pi^2m\rho}\sum_{n=1}^{2}\int_0^\infty(2n-1)\frac{\left[S_{\mathrm{c}n}(k)-1\right]\left[S_{\mathrm{c}n}(k)-S_{\mathrm{cF}}(k)\right]^2}{S_{\mathrm{c}n}(k)/S_{\mathrm{cF}}(k)}k^4dk$$

E_{F} : Fermi energy

ρ: particle number density

$V_s(r)$: Spin-dependent two-body central potential

$F_{\mathrm{F}s}(r)$: $F_s(r)$ for the degenerate Fermi gas

$S_{\mathrm{cF}}(k)$: $S_{\mathrm{c}n}(k)$ for the degenerate Fermi gas

M. T. and M. Yamada, Prog. Theor. Phys. **91** (1994) 1149

Explicit Energy Functional for Fermion Systems with Central Forces

1. The potential energy expectation value: **the exact expression**

2. The kinetic energy expectation value: approximate expression

   When **the Jastrow wave function** is assumed

$$\Psi(x_1,\cdots,x_N) = Sym\left[\prod_{i<j} f_{ij}\right]\Phi_{\mathrm{F}}(x_1,\cdots,x_N)$$

The **kinetic energy expectation value $<T>/N$** is cluster-expanded.

The two-body cluster term of $<T>/N$

The main part of the three-body cluster direct term of $<T>/N$

The main part of the four-body cluster direct term of $<T>/N$

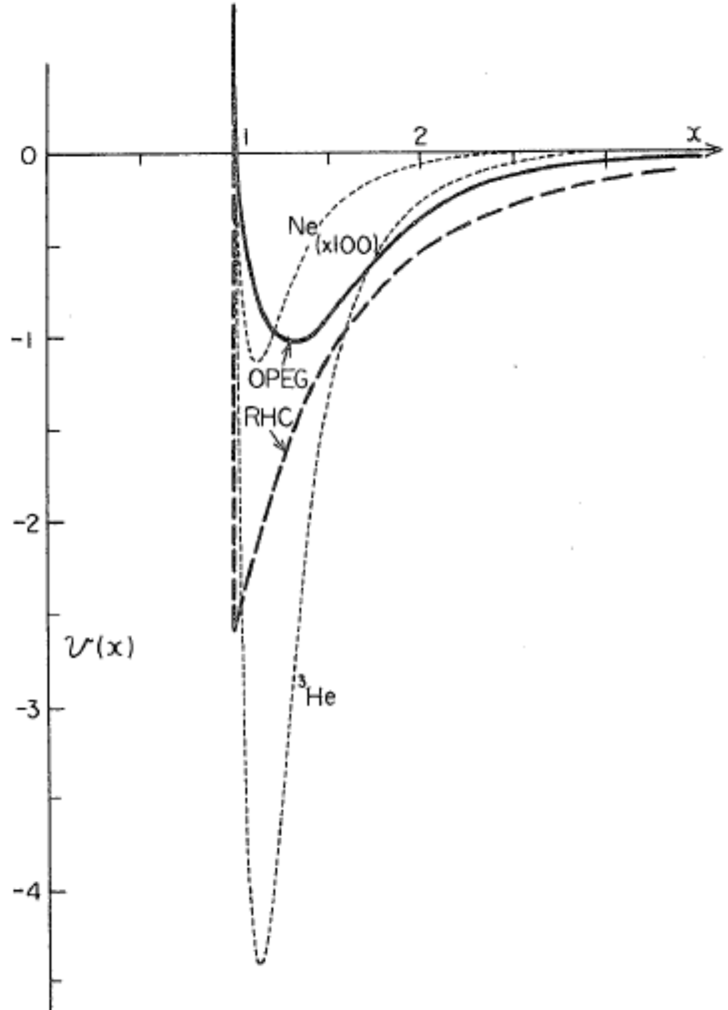
are included in the explicit energy functional.

3. Necessary conditions on the structure functions $S_{cn}(k) \geq 0$ are guaranteed.

2. Nuclear Matter and Liquid ^{3}He

Strongly correlated Fermion systems

Nucleon	⟷	^{3}He atom (spin 1/2)
Two-body nuclear force	⟷	Interatomic force e.g. HFDHE2 pot.



Nuclear force and Interatomic force

Strong short-range attraction

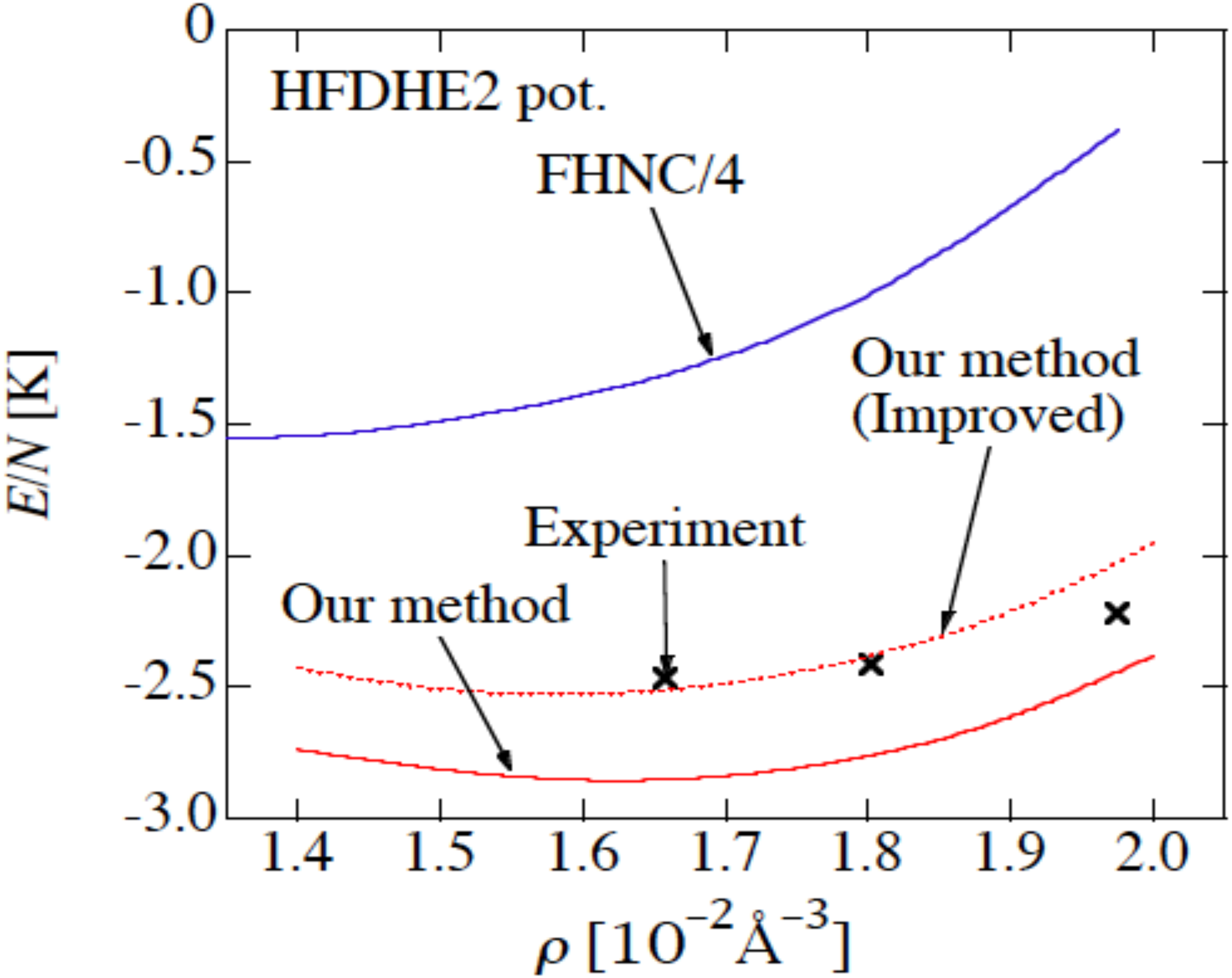
+ repulsive core

Interatomic force
- Central Force
- State Independent Force
- Two-body Force only

Liquid ^{3}He : similar to nuclear matter, but much denser

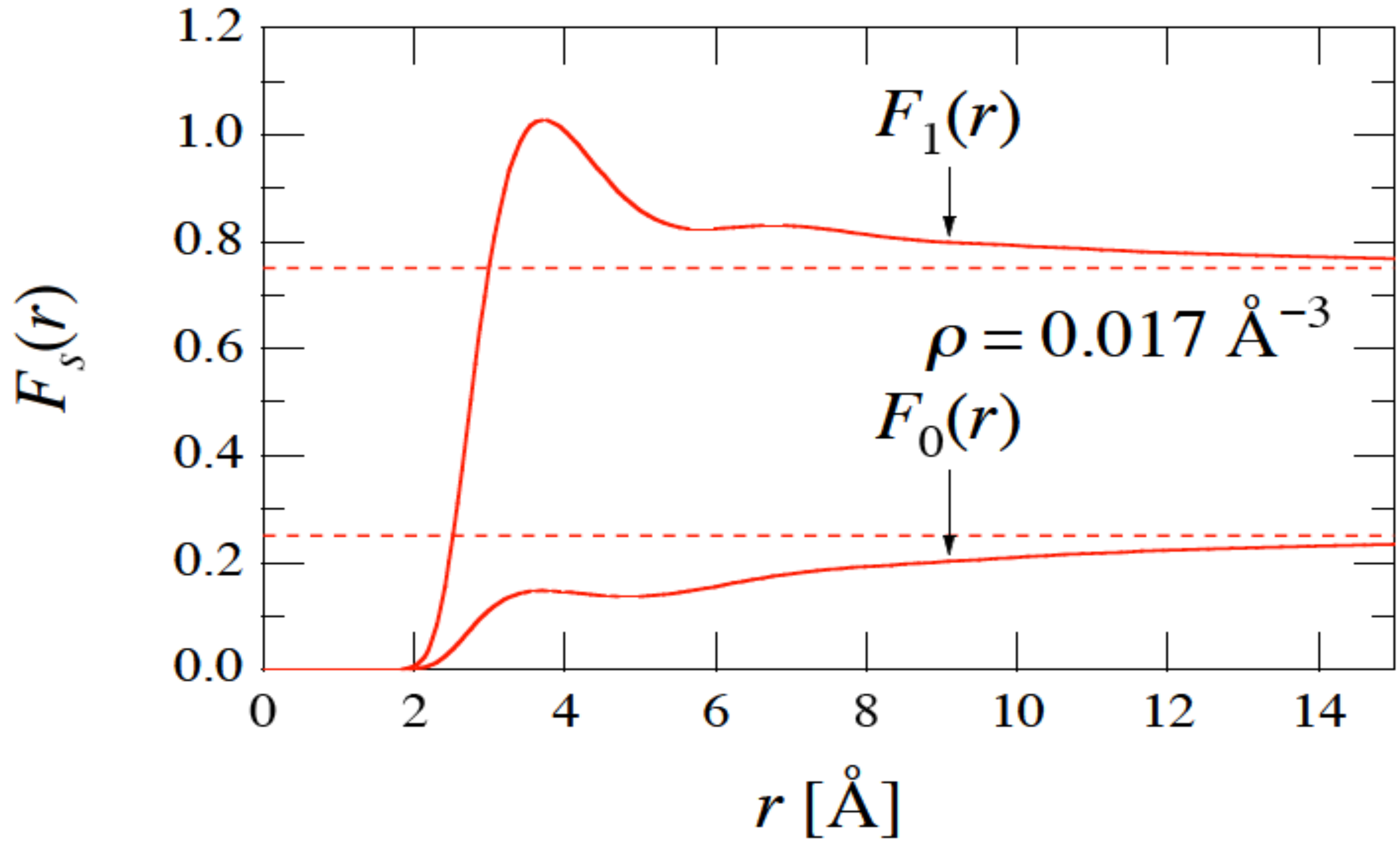
($\rho \sim$ several ρ_0)

R. Tamagaki, Prog. Theor. Phys. Suppl. 112 (1993) 2



Energy per particle of Liquid ^{3}He

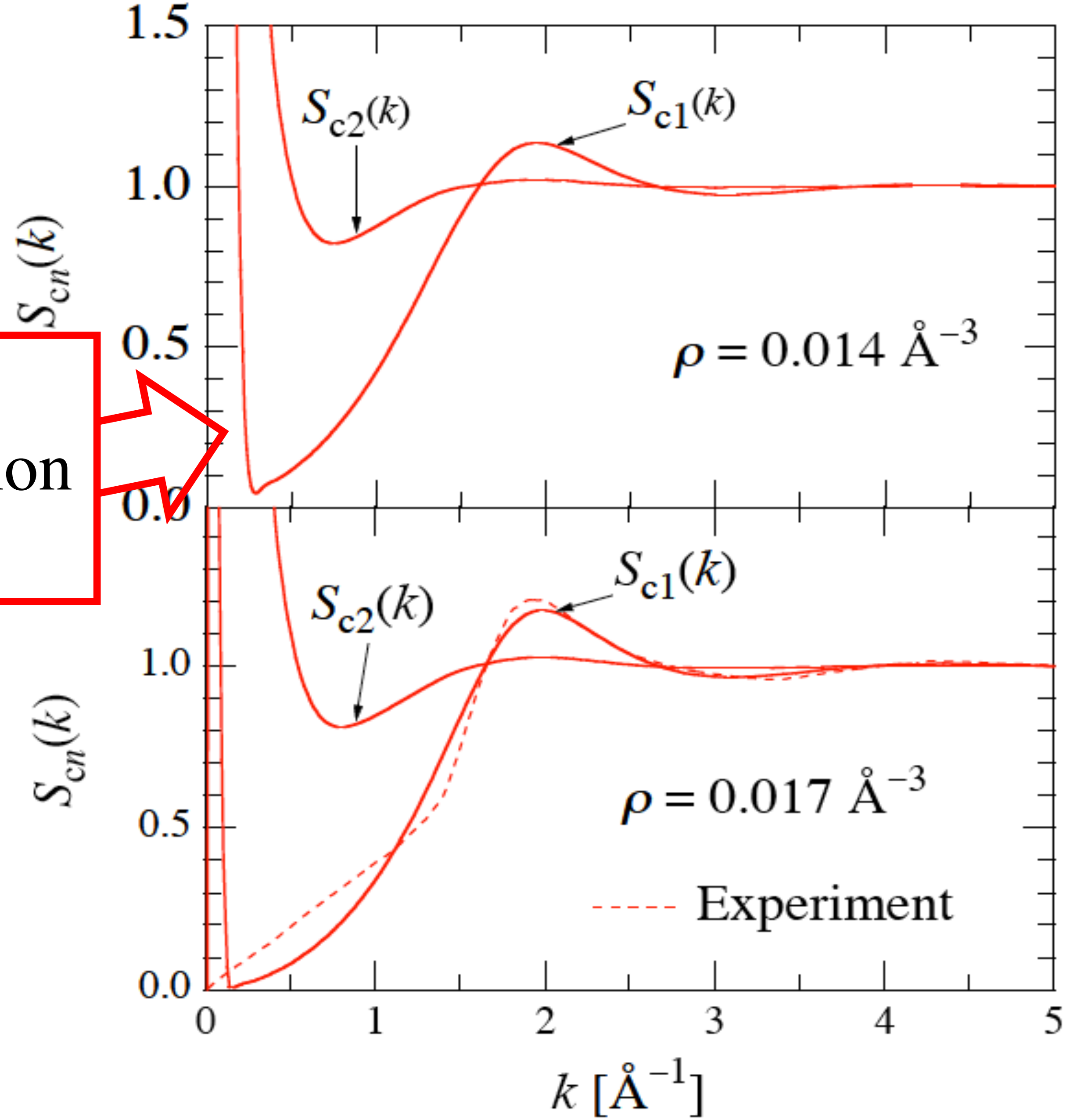
M. T. and M. Yamada, Prog. Theor. Phys. **91** (1994) 1149



Radial Distribution Functions for Liquid ^{3}He

The HFDHE2 pot.

Violation of
the Mayer condition
$S_{c1}(0) = 0$



Structure functions for liquid ^{3}He

The HFDHE2 pot.

Two-dimensional ^{3}He systems

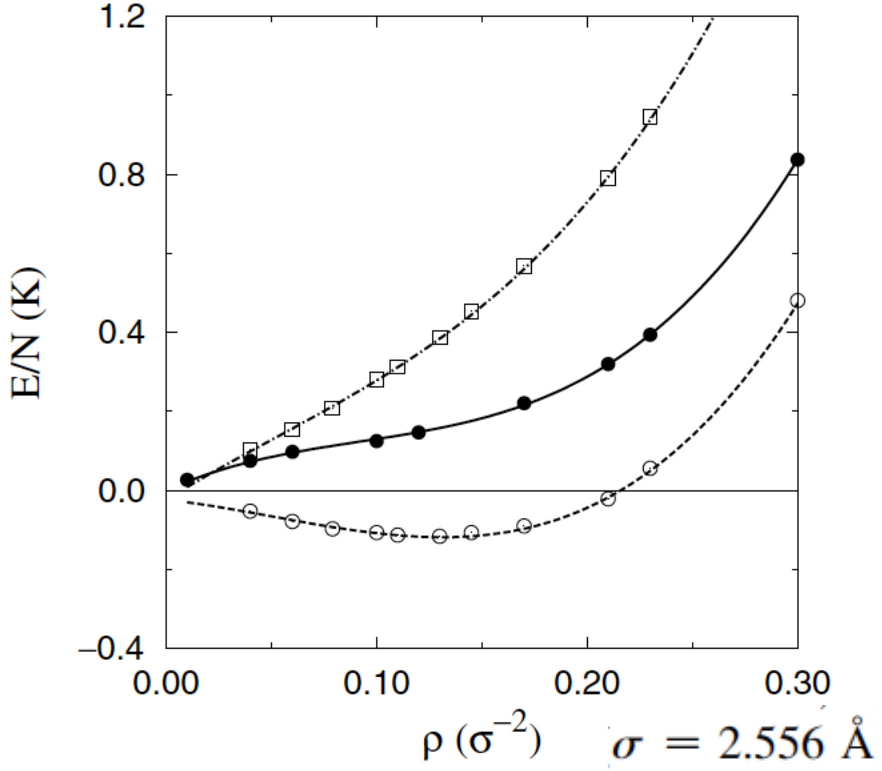
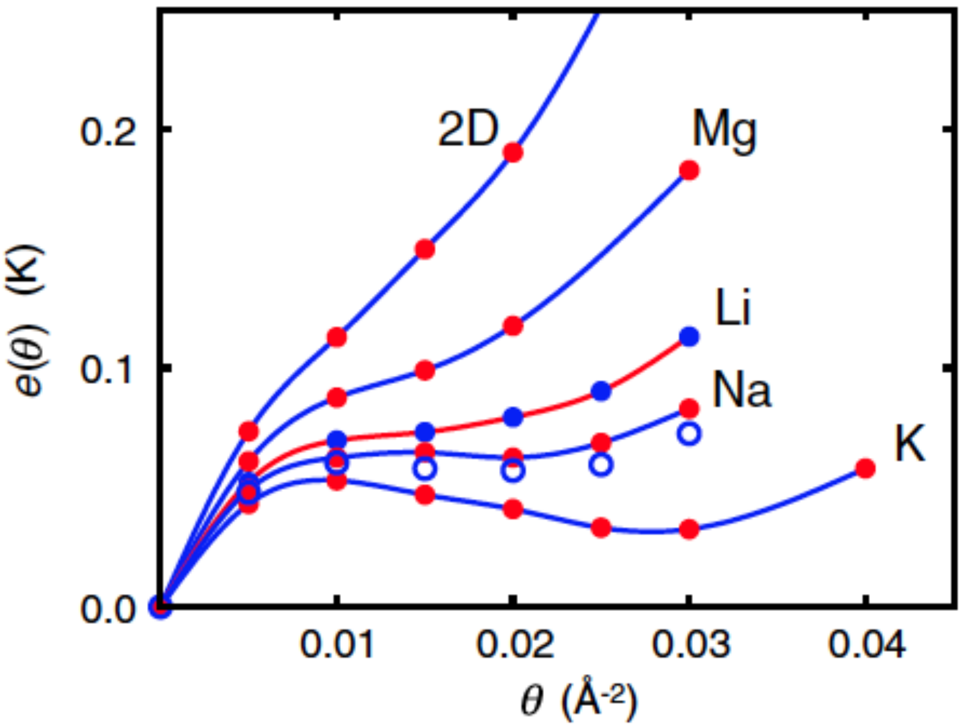


FIG. 3. Influence of the Fermi statistics on the energy of 2D ^{3}He. Filled and empty circles correspond to Fermi and Bose ^{3}He, respectively. Squares represent the sum of the boson energy and the Fermi gas kinetic energy. The lines are polynomial fits to the data.

V. Grau, J. Boronat, and J. Casulleras, Phys. Rev. Lett. 89 (2002) 045301

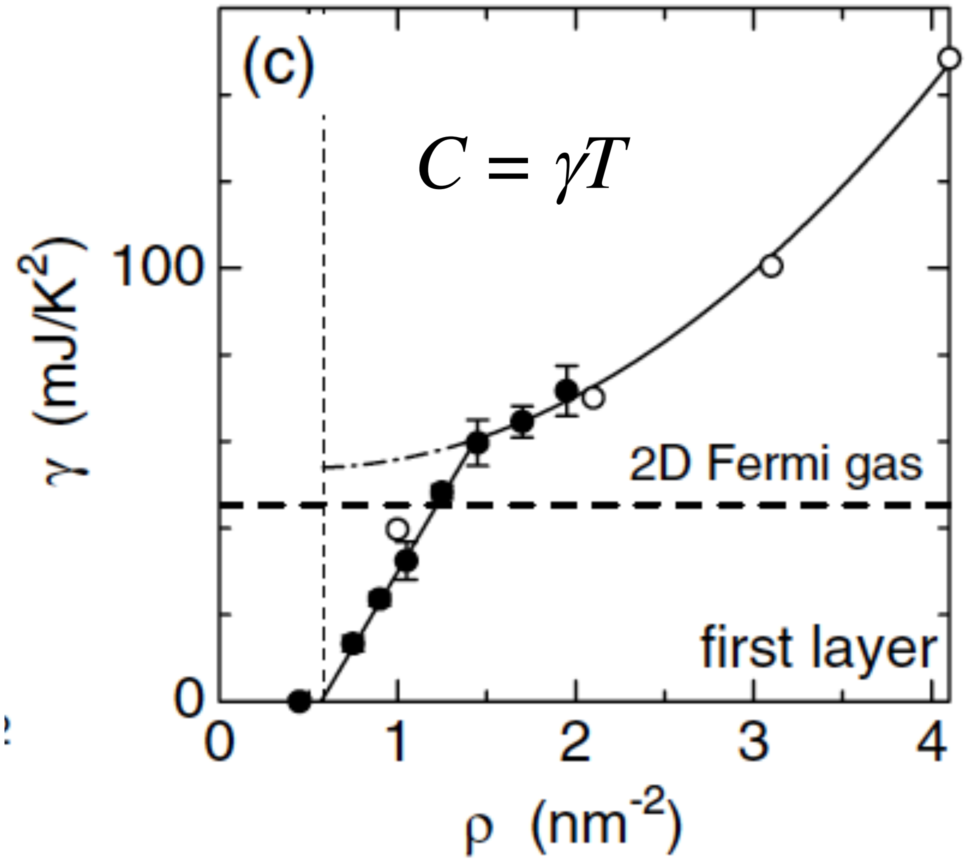


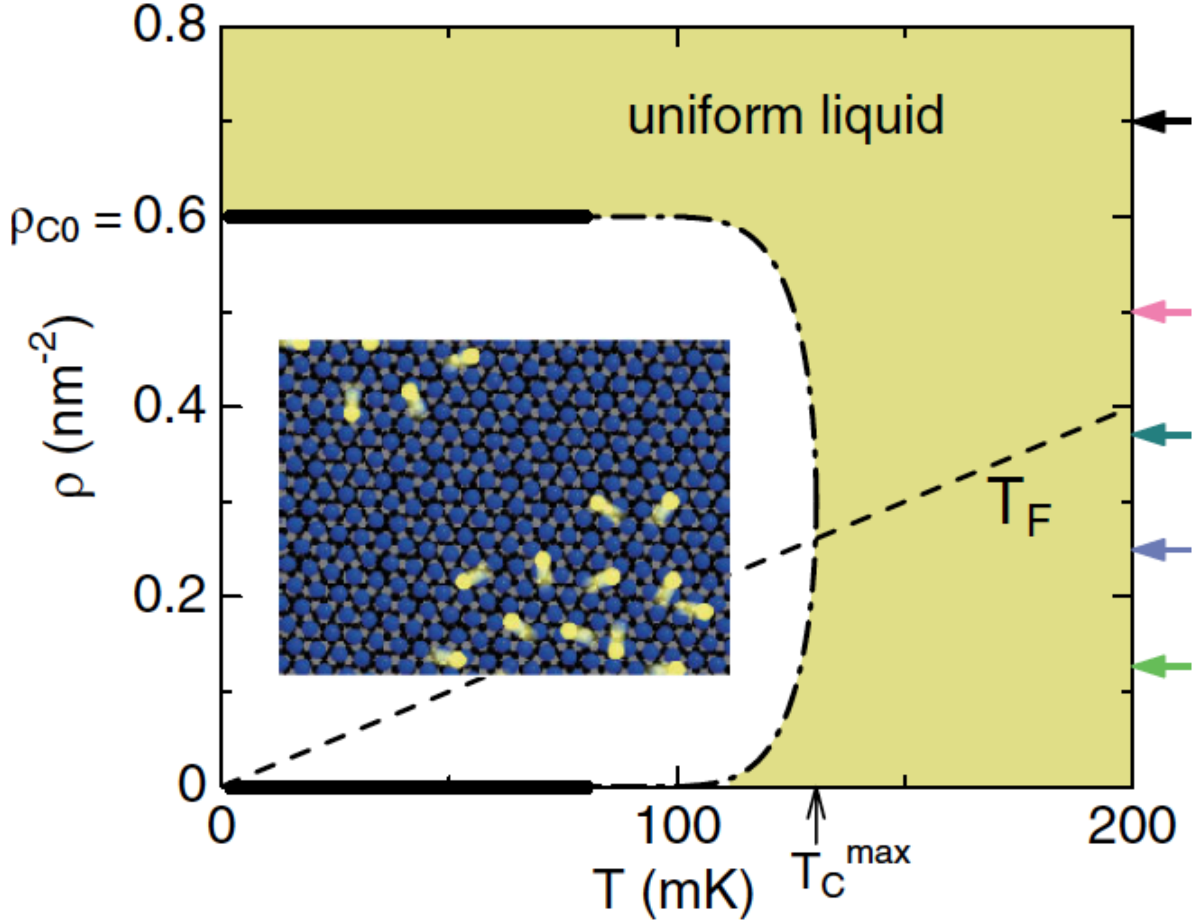
^{3}He atoms on various substrates

M. Ruggeri, S. Morini and M. Boninsegni, Phys. Rev. Lett. 111 (2013) 045303.

No liquid state predicted

2D self-bound ^{3}He absorbed on graphite ?



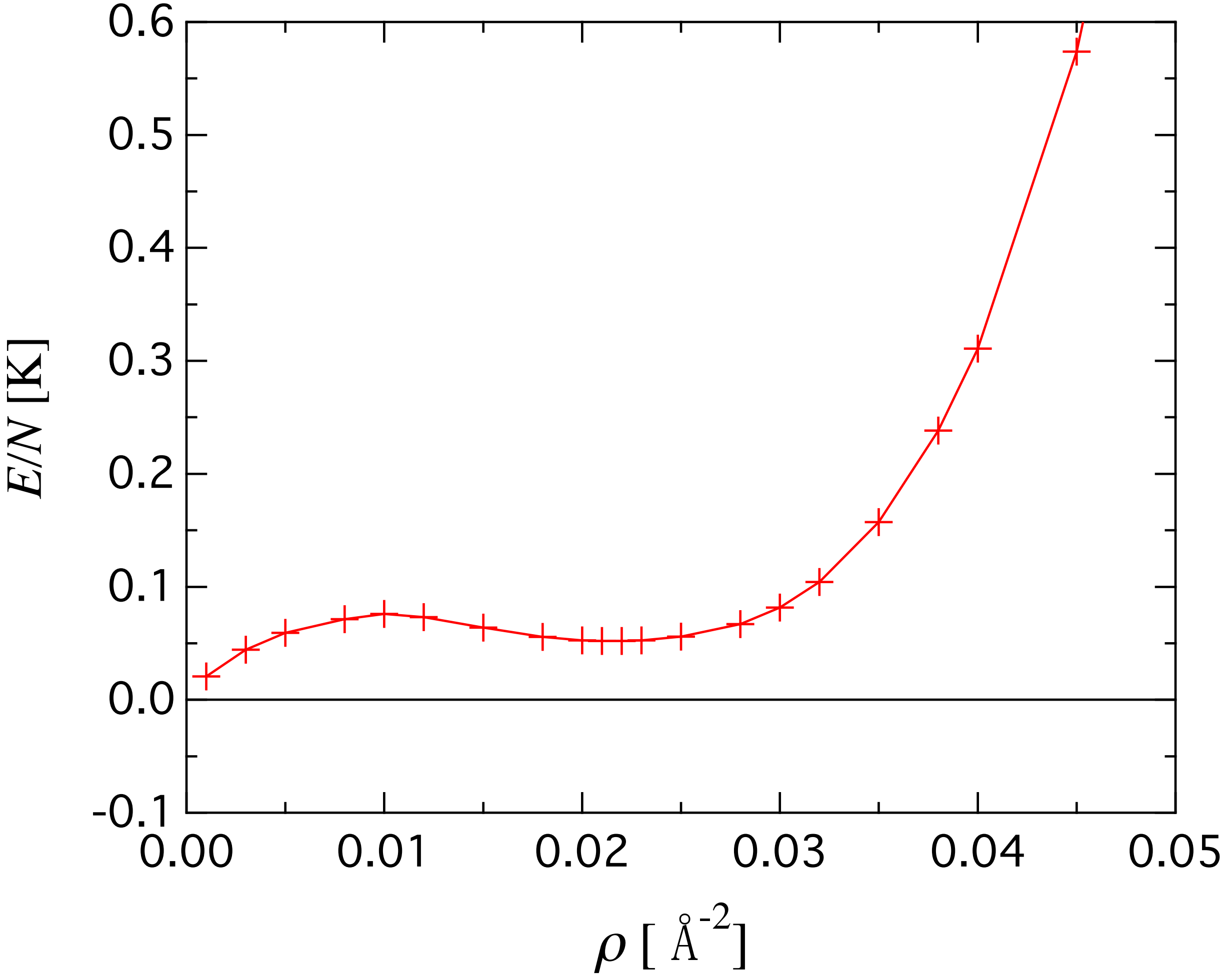


D. Sato, K. Naruse, T. Matsui, and H. Fukuyama, Phys. Rev. Lett. 109 (2012) 235306.

The Explicit Energy Functional for Two-dimentional ^3He system

T. Suzuki, N. Sakumichi and M. T.

(in preperation)



Quasi-stable state is seen

3. Nuclear Matter with Central and Tensor Forces

The Nuclear Hamiltonian

$$H = -\frac{\hbar^2}{2m}\sum_{i=1}^{N}\nabla_i^2 + \sum_{i>j}^{N} V_{ij} + \sum_{i>j>k}^{N} V_{ijk}$$

m: Neutron mass, N: The number of neutrons

The two-body potential: AV6’

$$V_{ij} = \sum_{t=0}^{1}\sum_{s=0}^{1}\left\{V_{\mathrm{C}ts}(r_{ij}) + V_{\mathrm{T}t}(r_{ij})S_{\mathrm{T}ij}\right\}P_{tsij}$$

The three-body potential: The repulsive part of the UIX.

$$V^{\mathrm{R}}(\boldsymbol{r}_i, \boldsymbol{r}_j, \boldsymbol{r}_k) = U\sum_{\mathrm{cyc}} T(r_{ij})\, T(r_{jk})$$

Energy Functional with Tensor Forces

For Neutron Matter

Radial distribution function

$$F_s(r_{12}) = \Omega^2 \sum_{spin} \int \Psi^* (x_1, x_2, \ldots, x_N) P_{s12} \Psi(x_1, x_2, \ldots, x_N) d\boldsymbol{r}_3 \ldots d\boldsymbol{r}_N$$

Tensor distribution function

$$F_{\mathrm{T}}(r_{12}) = \Omega^2 \sum_{spin} \int \Psi^* (x_1, x_2, \ldots, x_N) S_{\mathrm{T}12} \Psi(x_1, x_2, \ldots, x_N) d\boldsymbol{r}_3 \ldots d\boldsymbol{r}_N$$

Ψ: Wave function Ω: Volume of the system

Auxiliary Functions: $F_{\mathrm{C}s}(r)$ and $g_{\mathrm{T}}(r)$

$$F_s(r) = F_{\mathrm{C}s}(r) + 8s\left[g_{\mathrm{T}}(r)\right]^2 F_{\mathrm{F}s}(r)$$

$$F_{\mathrm{T}}(r) = 16\left\{\sqrt{F_{\mathrm{C}1}(r) F_{\mathrm{F}1}(r)}\, g_{\mathrm{T}}(r) - \left[g_{\mathrm{T}}(r)\right]^2 F_{\mathrm{F}1}(r)\right\}$$

Spin-dependent structure functions

$$S_{\mathrm{c1}}(k) = \frac{1}{N}\left\langle \left| \sum_{i=1}^{N} \exp\left[i\boldsymbol{k}_i \cdot \boldsymbol{r} \right] \right|^2 \right\rangle = 1 + S_1(k) + S_0(k) \geq 0$$

$$S_{\mathrm{c2}}(k) = \frac{1}{3N}\left\langle \left| \sum_{i=1}^{N} \boldsymbol{\sigma}_i \exp\left[i\boldsymbol{k}_i \cdot \boldsymbol{r} \right] \right|^2 \right\rangle = 1 + \frac{1}{3} S_1(k) - S_0(k) \geq 0$$

$$S_{\mathrm{cT1}}(k) = \frac{1}{N}\left\langle \left| \sum_{i=1}^{N} \frac{(\boldsymbol{\sigma}_i \cdot \boldsymbol{k}_i)}{k_i} \exp\left[i\boldsymbol{k}_i \cdot \boldsymbol{r} \right] \right|^2 \right\rangle = S_{\mathrm{c2}}(k) - \frac{1}{3} S_{\mathrm{T}}(k) \geq 0$$

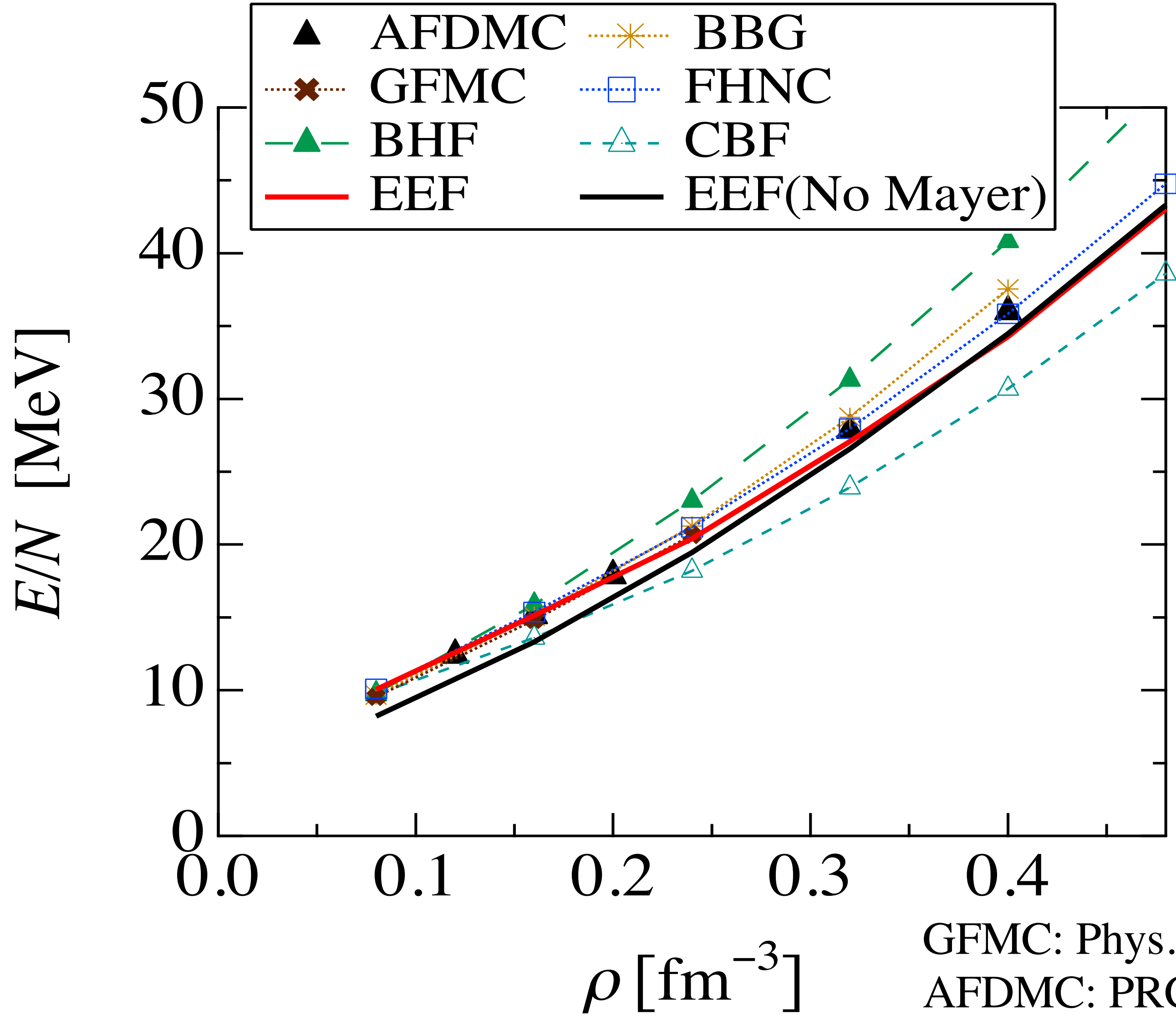
$$S_{\mathrm{cT2}}(k) = \frac{1}{N}\left\langle \left| \sum_{i=1}^{N} \frac{(\boldsymbol{\sigma}_i \times \boldsymbol{k}_i)}{k_i} \exp\left[i\boldsymbol{k}_i \cdot \boldsymbol{r} \right] \right|^2 \right\rangle = S_{\mathrm{c2}}(k) + \frac{1}{6} S_{\mathrm{T}}(k) \geq 0$$

$$S_s(k) = \rho \int \left[F_s(r) - F_s(\infty) \right] \exp\left[i\boldsymbol{k} \cdot \boldsymbol{r} \right] d\boldsymbol{r} \qquad S_{\mathrm{T}}(k) = \rho \int F_{\mathrm{T}}(r)\, j_2(kr)\, d\boldsymbol{r}$$

Explicit Energy Functional for Neutron Matter with AV6' Pot.

$$
\begin{aligned}
\frac{E_2}{N} = \frac{3}{5}E_{\mathrm{F}} &+ 2\pi\rho \int \left\{ \left[\sum_{S=0}^{1} F_s(r) V_{\mathrm{C}s}(r) \right] + F_{\mathrm{T}}(r) V_{\mathrm{T}}(r) \right\} r^2 dr \\
&+ \frac{\pi\hbar^2\rho}{2m} \int \sum_{s=0}^{1} F_{\mathrm{C}s}(r) \left[\frac{1}{F_{\mathrm{C}s}(r)} \frac{dF_{\mathrm{C}s}(r)}{dr} - \frac{1}{F_{\mathrm{F}s}(r)} \frac{dF_{\mathrm{F}s}(r)}{dr} \right]^2 r^2 dr \\
&+ \frac{2\pi\hbar^2\rho}{m} \int \left[8 \left\{ \left[\frac{dg_{\mathrm{T}}(r)}{dr} \right]^2 + \frac{6}{r^2} \left[g_{\mathrm{T}}(r) \right]^2 \right\} F_{\mathrm{F1}}(r) \right] r^2 dr \\
&- \frac{\hbar^2}{16\pi^2 m\rho} \int \frac{\left[S_{\mathrm{c1}}(k) - 3 + 2S_{\mathrm{cF}}(k) \right] \left\{ \left[S_{\mathrm{c1}}(k) - S_{\mathrm{cF}}(k) \right]^2 \right\}}{S_{\mathrm{c1}}(k) / S_{\mathrm{cF}}(k)} k^4 dk \\
&- \frac{\hbar^2}{16\pi^2 m\rho} \int \sum_{n=1}^{2} \frac{\left[S_{\mathrm{cT}n}(k) - 3 + 2S_{\mathrm{cF}}(k) \right] \left[S_{\mathrm{cT}n}(k) - S_{\mathrm{cF}}(k) \right]^2}{S_{\mathrm{cT}n}(k) / S_{\mathrm{cF}}(k)} k^4 dk + \frac{E_{\mathrm{nod}}}{N}
\end{aligned}
$$

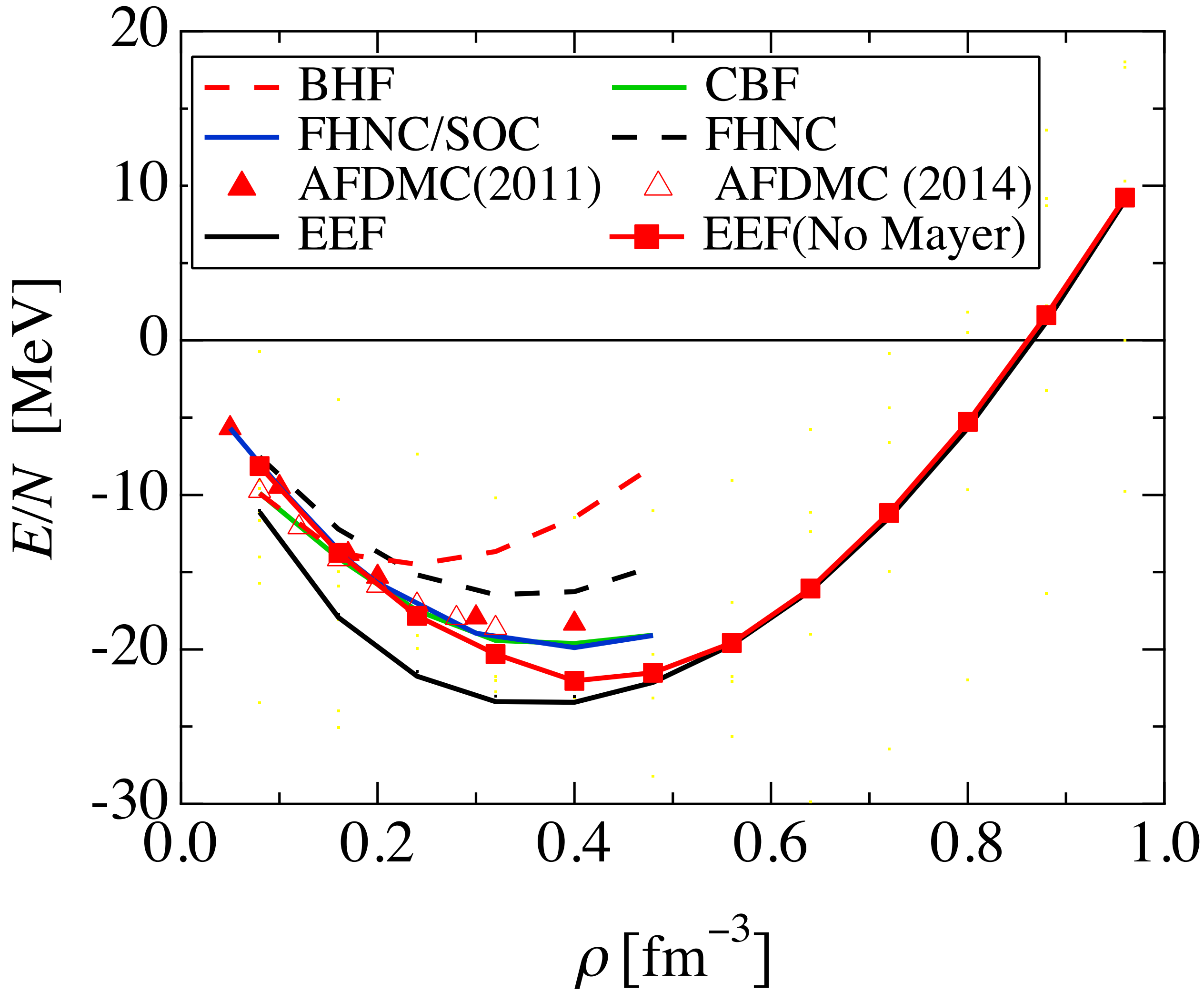
Energies of Neutron Matter with the v6’ pot.



GFMC: Phys. Rev. C 68 (2003) 025802.
AFDMC: PRC68 (2003) 024308,

EEF: The present result with the Mayer’s condition.
EEF(No Mayer): The present result without constraints.

Energies of Symmetric Nuclear Matter with the v6' pot.



AFDMC(2011), FHNC/SOC: PRC83 (2011) 054003, AFDMC(2014) PRC90 (2014) 061306(R)
BHF, FHNC, CBF: Phys. Lett. B609 (2005) 232.

EEF: The present result with the Mayer's condition.

EEF(No Mayer): The present result without constraints.

Three-body Nuclear Force:
The UIX Repulsive Part for Neutron Matter

The three-body Hamiltonian $H_3 = \sum_{i>j>k}^{N} V_3(r_{ij}, r_{jk}, r_{ki})$

$$\frac{E_3^{\mathrm{R}}}{N} = \frac{\langle H_3 \rangle}{N} = \frac{\rho^2}{6} \int F_3(\boldsymbol{r}_1, \boldsymbol{r}_2, \boldsymbol{r}_3) V_3(r_{12}, r_{23}, r_{31}) d\boldsymbol{r}_{12} d\boldsymbol{r}_{23}$$

$F_3(\boldsymbol{r}_i, \boldsymbol{r}_j, \boldsymbol{r}_k)$: The three-body distribution function

We employ an extended Kirkwood approximation for the three-body distribution function.

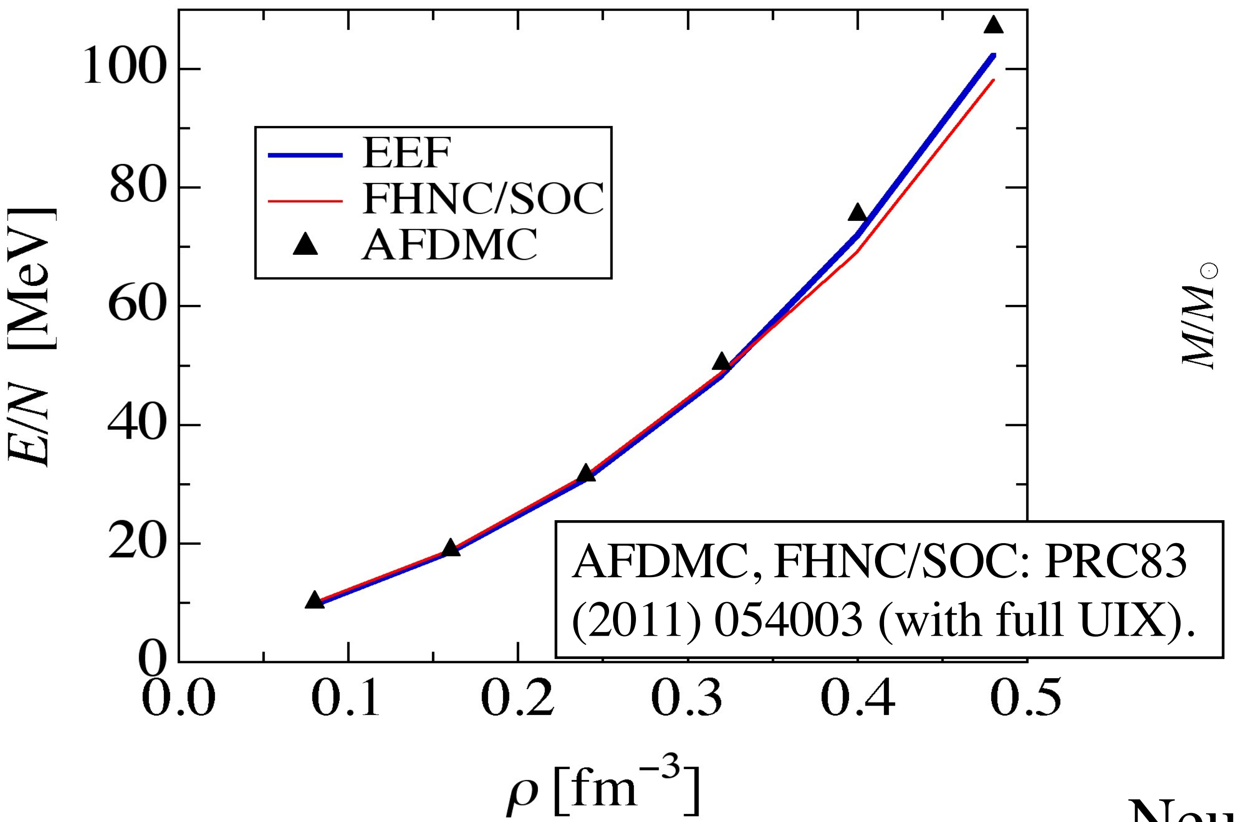
$$F_3(\boldsymbol{r}_1, \boldsymbol{r}_2, \boldsymbol{r}_3) = \frac{F_{3\mathrm{F}}(\boldsymbol{r}_1, \boldsymbol{r}_2, \boldsymbol{r}_3)}{F_{\mathrm{F}}(r_{12}) F_{\mathrm{F}}(r_{23}) F_{\mathrm{F}}(r_{31})} F(r_{12}) F(r_{23}) F(r_{31})$$

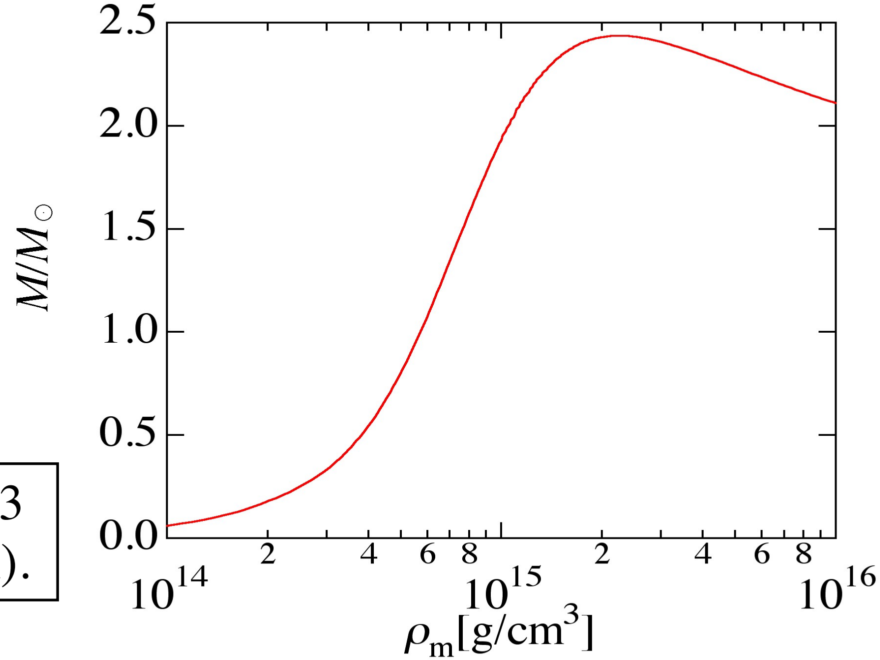
$$F_{\mathrm{F}}(r) = \sum_{s=0}^{1} F_{\mathrm{F}s}(r) \qquad F(r) = \sum_{s=0}^{1} F_s(r)$$

The total energy (v6' + repulsive UIX) $\dfrac{E_{\mathrm{tot}}^{\mathrm{R}}}{N} = \dfrac{E_2}{N} + \dfrac{E_3^{\mathrm{R}}}{N}$

is minimized with respect to $F_{\mathrm{C}s}(r)$ and $g_{\mathrm{T}}(r)$

Energy of neutron matter with v6'+UIX(repulsive)





Neutron stars with the EOS of pure neutron matter (v6'+UIX(repulsive))

4-1. Three-body Nuclear Force:
The 2π exchange component of the UIX potential

Fujita-Miyazawa three-nucleon potential

$$V_{ijk}^{2\pi} = A \sum_{cyc} \left[\{\boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j, \boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_k\}\{x_{ij}, x_{ik}\} + \frac{1}{4}[\boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j, \boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_k][x_{ij}, x_{ik}] \right]$$

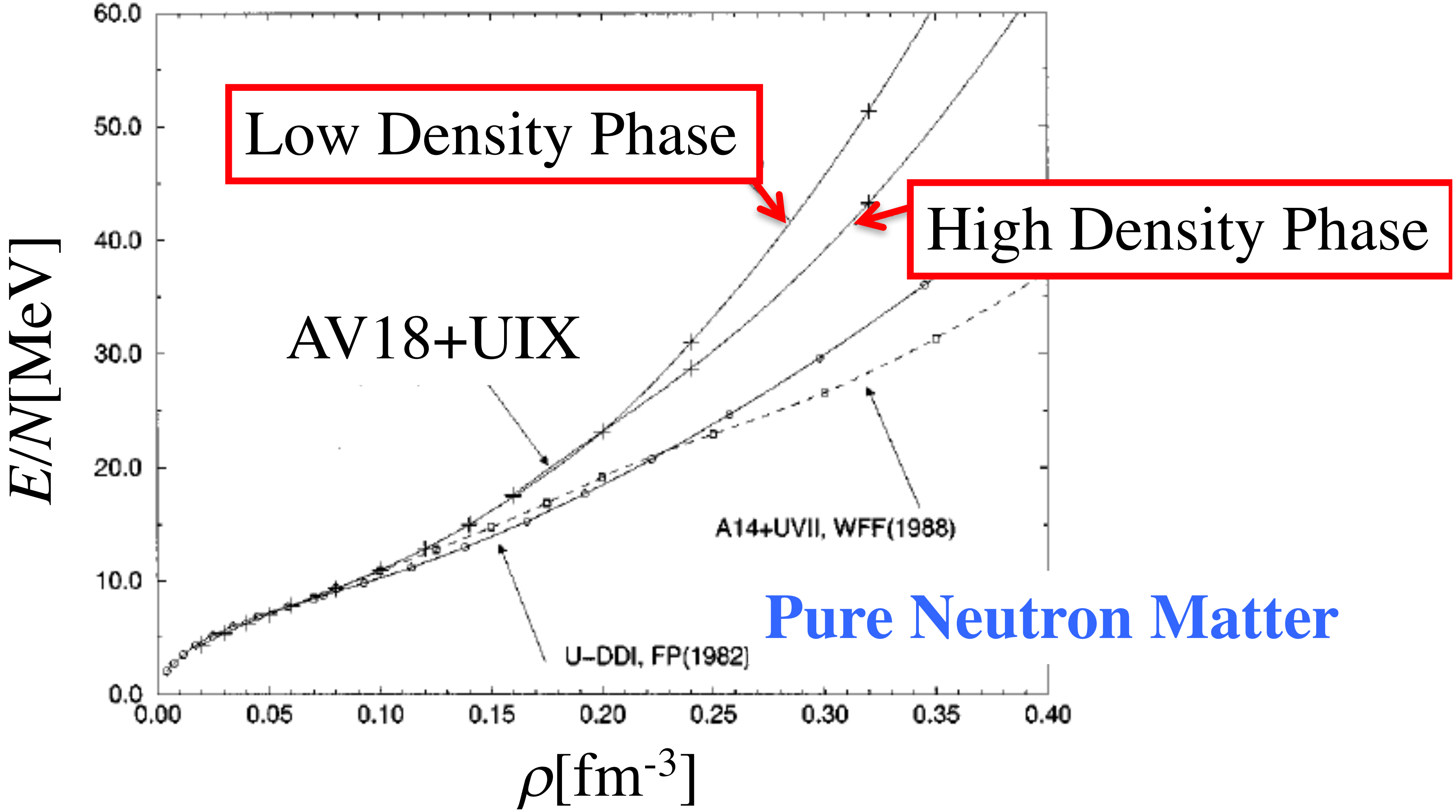
$$x_{ij} = T(r_{ij}) S_{\mathrm{T}ij} + Y(r_{ij})\, \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j$$

Tensor force **Central force**

$$Y(r) = \frac{e^{-\mu r}}{\mu r}\left(1 - \mathrm{e}^{-c_t r^2}\right)$$

$$T(r) = \left(1 + \frac{3}{\mu r} + \frac{3}{\mu^2 r^2}\right) \frac{e^{-\mu r}}{\mu r}\left(1 - e^{-c_t r^2}\right)^2$$

π-condensation calculated with the FHNC method



2π-exchange three nucleon force induces π-condensation

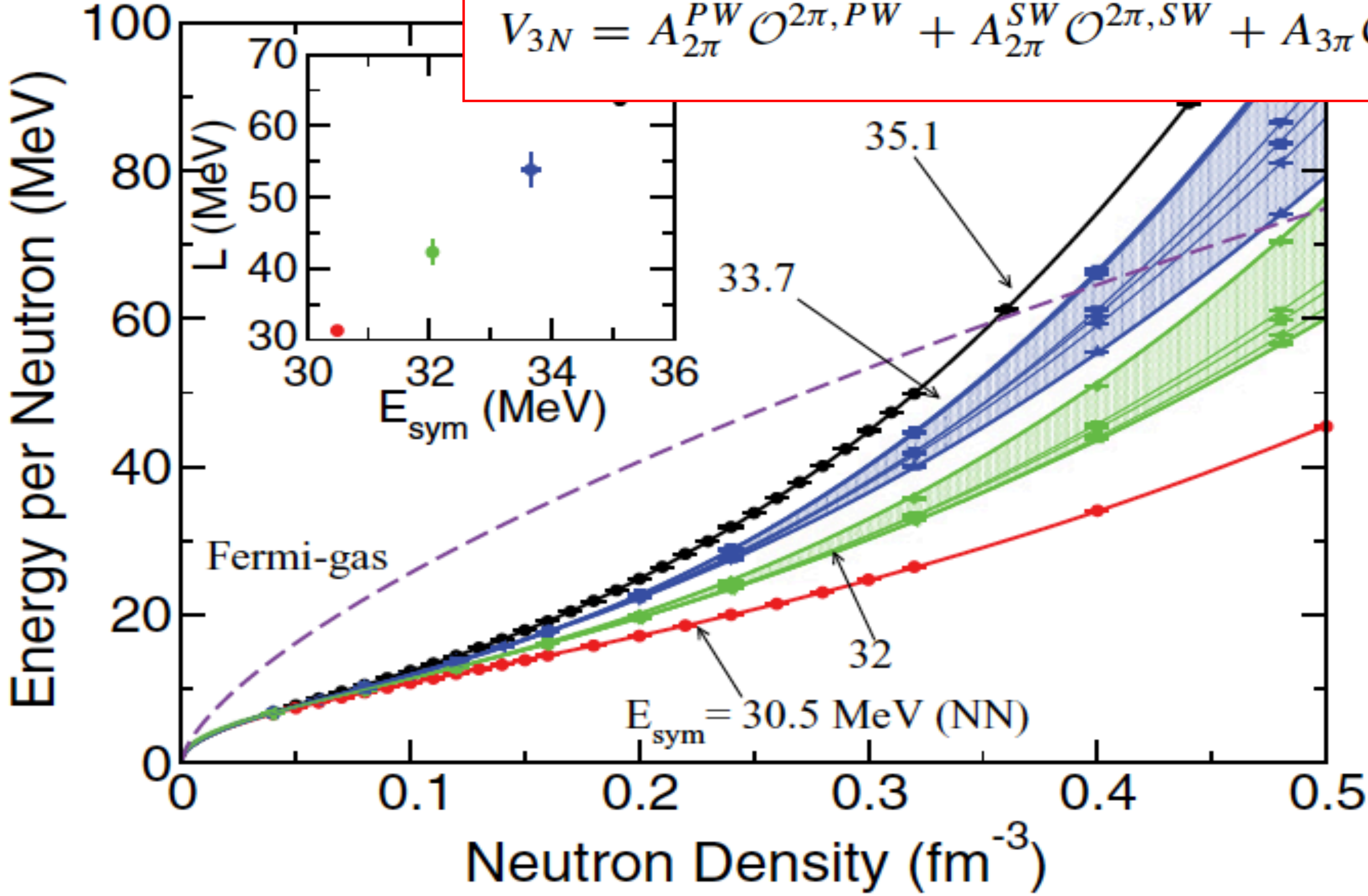
A. Akmal and V. A. Pandharipande, Phys. Rev. C56(1997)2262.

Monte Carlo calculation: No π-condensation ?

Two-body force：v8’（Central ＋ Tensor ＋ Spin-orbit ）

Three-body force：IL models（Extensions of the UIX）

$$V_{3N} = A_{2\pi}^{PW}\mathcal{O}^{2\pi,PW} + A_{2\pi}^{SW}\mathcal{O}^{2\pi,SW} + A_{3\pi}\mathcal{O}^{3\pi} + A_R\mathcal{O}^R$$



Gandolfi et al., PRC85(2012)032801(R)

Explicit Energy Expressions for the 2π exchange three-nucleon force

R. Yokota and M. T.

1. **Kirkwood's assumption** is extended.

Three-body distribution functions are expressed as **products** of **state dependent two-body distribution functions**

2. **When the Jastrow wave function is assumed**

$$\Psi(x_1,\cdots,x_N) = Sym\left[\prod_{i<j} f_{ij}\right]\Phi_{\mathrm{F}}(x_1,\cdots,x_N)$$

$$f_{ij} = \sum_{s=0}^{1} f_s(r_{ij})P_{sij} + f_{\mathrm{T}}(r_{ij})S_{\mathrm{T}ij},$$

The main part of the three-body cluster terms of the potential energy is included exactly.

Explicit Energy Expressions for the 2π exchange three-nucleon force

$$\frac{E_3^{2\pi}}{N} = \frac{E_{30\mathrm{c}}^{2\pi}}{N} + \frac{E_{31\mathrm{T}}^{2\pi}}{N} + \frac{E_{32\mathrm{c}}^{2\pi}}{N} + \frac{E_{32\mathrm{T}}^{2\pi}}{N}$$

$$\frac{E_{30\mathrm{c}}^{2\pi}}{N} = -\frac{9A}{4\pi^2\rho}\int_0^\infty H_{\mathrm{G}}(k)\,[H_{\mathrm{Y}}(k)]^2 k^2 dk \qquad \frac{E_{31\mathrm{T}}^{2\pi}}{N} = \frac{6A}{\pi^2\rho}\int_0^\infty H_{\mathrm{F}}(k) H_{\mathrm{T}}(k) H_{\mathrm{Y}}(k)\, k^2 dk$$

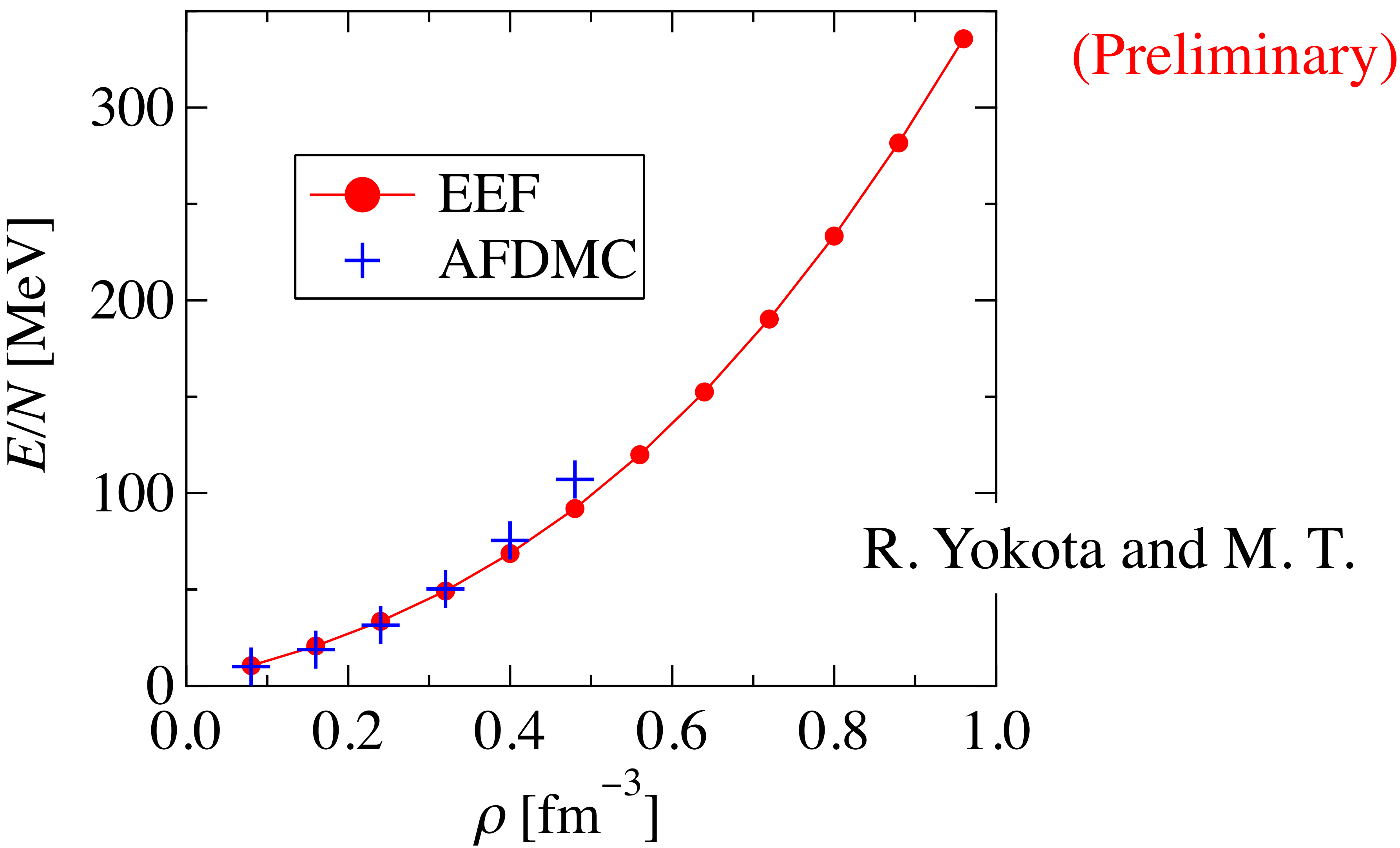
$$\frac{E_{32\mathrm{c}}^{2\pi}}{N} = \frac{18A}{\pi^2\rho}\int_0^\infty H_{\mathrm{G}}(k)\,[H_{\mathrm{T}}(k)]^2 k^2 dk \qquad \frac{E_{32\mathrm{T}}^{2\pi}}{N} = -\frac{3A}{\pi^2\rho}\int_0^\infty H_{\mathrm{F}}(k)\,[H_{\mathrm{T}}(k)]^2 k^2 dk$$

$$H_{\mathrm{G}}(k) = 4\pi\rho\int_0^\infty \left[\frac{1}{3}F_1(r) - F_0(r)\right] j_0(kr) r^2 dr \qquad H_{\mathrm{Y}}(k) = 4\pi\rho\int_0^\infty F(r)Y(r)\, j_0(kr) r^2 dr$$

$$H_{\mathrm{F}}(k) = 4\pi\rho\int_0^\infty F_{\mathrm{T}}(r)\, j_2(kr) r^2 dr \qquad H_{\mathrm{T}}(k) = 4\pi\rho\int_0^\infty F(r)T(r)\, j_2(kr) r^2 dr$$

The total energy $\dfrac{E_{\mathrm{tot}}}{N} = \dfrac{E_2}{N} + \dfrac{E_3^{\mathrm{R}}}{N} + \dfrac{E_3^{2\pi}}{N}$ is minimized.

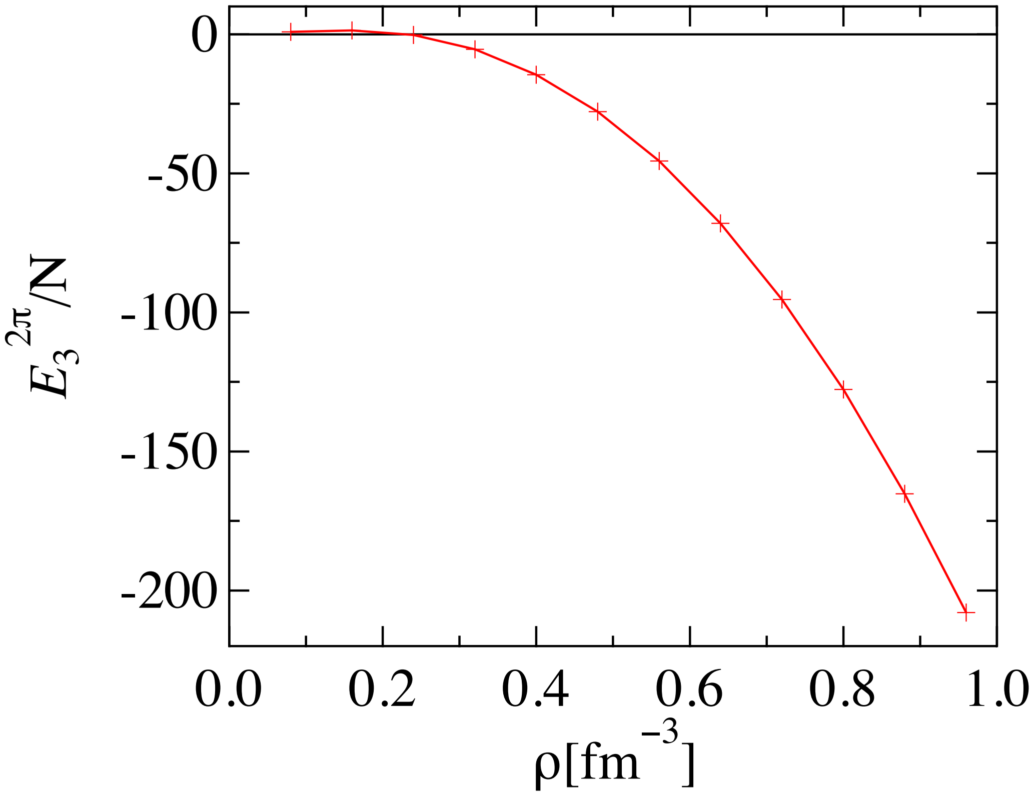
Energy per Neutron of Neutron Matter with the AV6'+UIX potentials

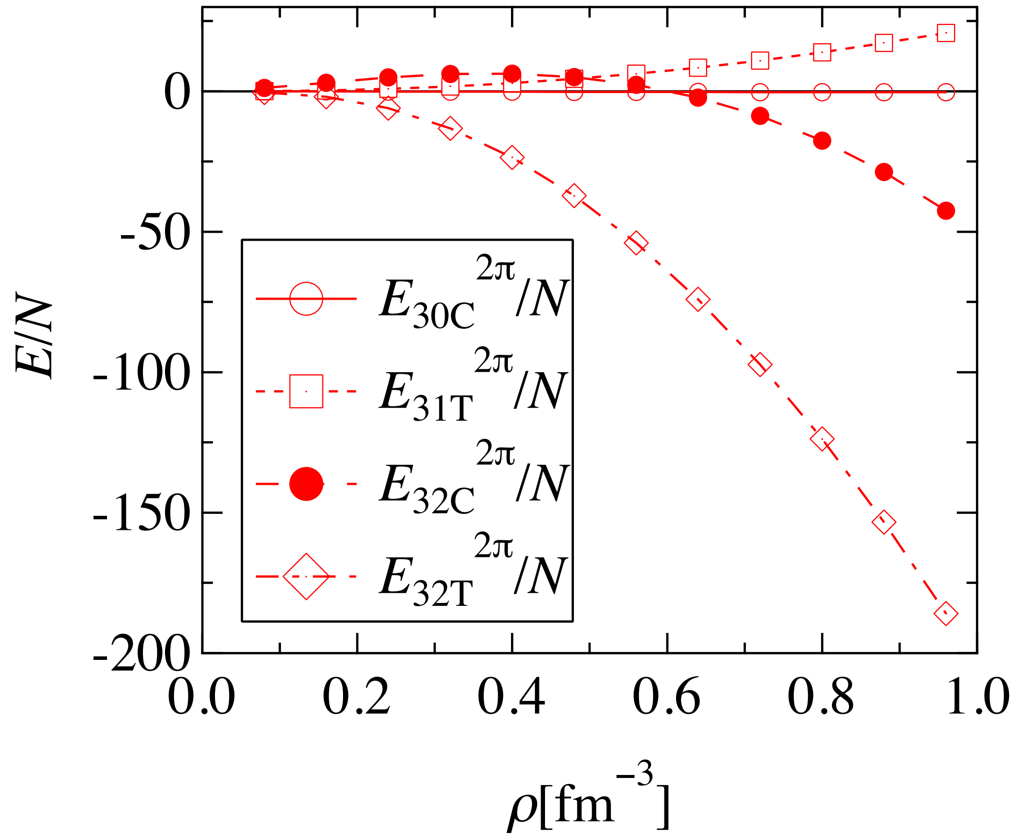


AFDMC: A. Lovato, O. Benhar, S. Fantoni, A. Yu. Illarioniv, and K. E. Schmidt, Phys. Rev. C83 (2011) 054003.

Two π Exchange Potential Energy

(Preliminary)





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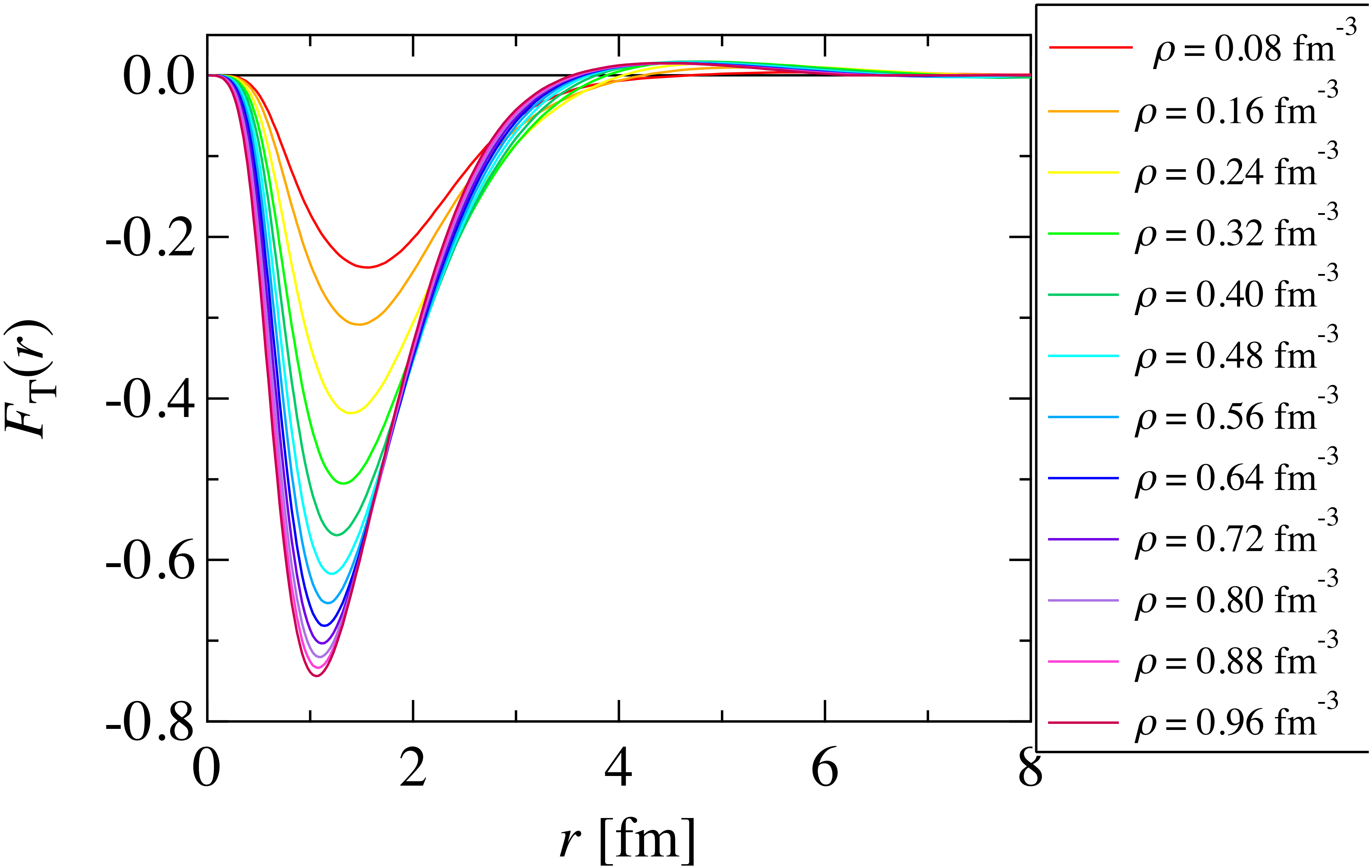
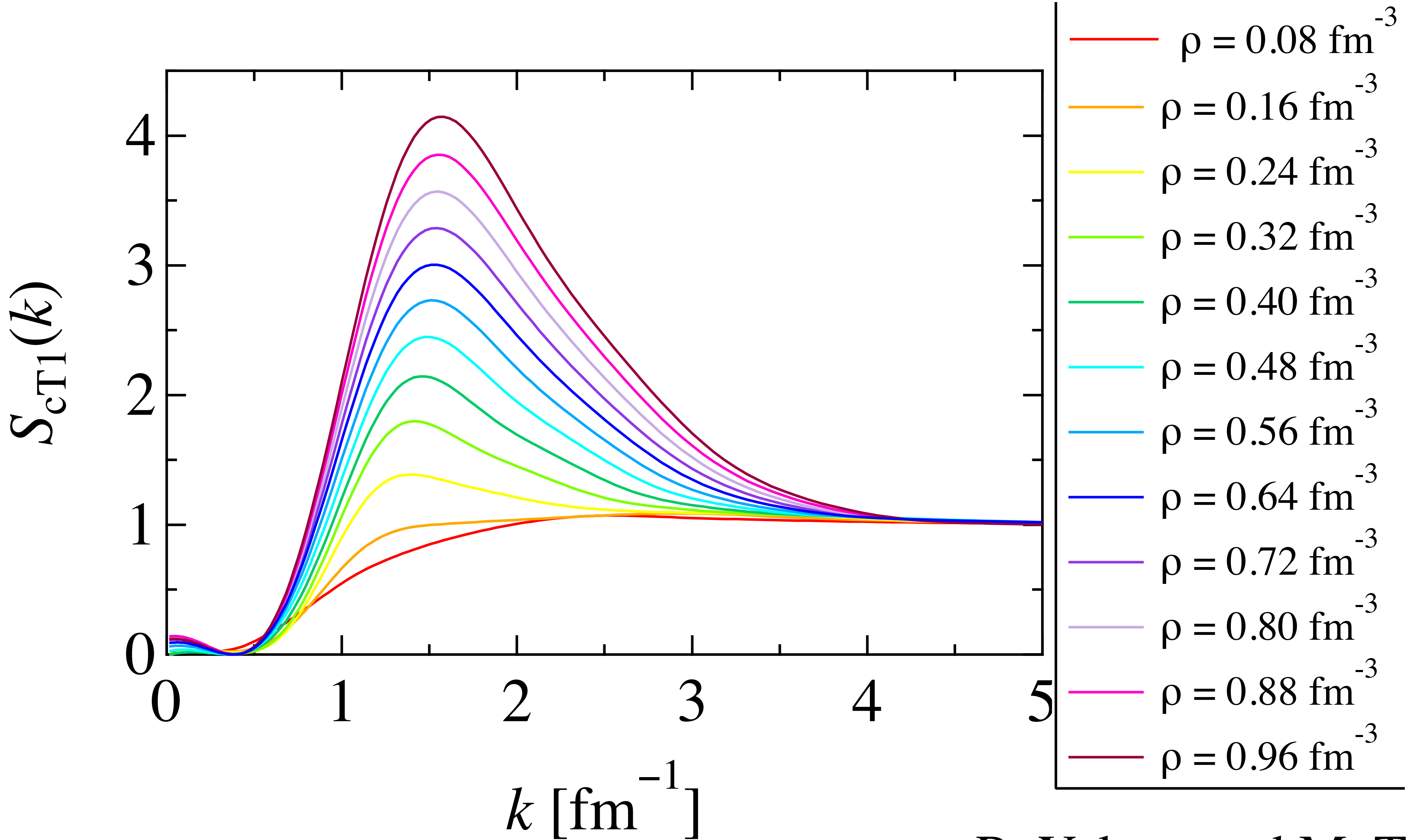
Tensor Distribution Function (Preliminary)



R. Yokota and M. T.

Tensor Structure Function (Spin-Longitudinal Response)

(Preliminary)



R. Yokota and M. T.

4-2. Extension to the Spin-Orbit Force

Neutron Matter

In addition to $F_s(r)$ and $F_{\mathrm{T}}(r)$

Spin-orbit distribution function

$$F_{\mathrm{SO}}(r_{12}) = \Omega^2 \sum_{spin} \int \Psi\ (x_1, x_2, \ \ldots\ , x_N)\left[s \bullet L_{12}\right]\Psi(x_1, x_2, \ \ldots\ , x_N)\, d\boldsymbol{r}_3 \ldots d\boldsymbol{r}_N$$

Auxiliary Functions: $F_{\mathrm{C}s}(r)$, $g_{\mathrm{T}}(r)$ and $g_{\mathrm{SO}}(r)$

$$F_s(r) = F_{\mathrm{C}s}(r) + 8s\left[g_{\mathrm{T}}(r)\right]^2 F_{\mathrm{F}s}(r) + \frac{2}{3}s\left[g_{\mathrm{SO}}(r)\right]^2 F_{\mathrm{qF}s}(r)$$

$$F_{\mathrm{T}}(r) = 16\left\{\sqrt{F_{\mathrm{C1}}(r)F_{\mathrm{F1}}(r)}\,g_{\mathrm{T}}(r) - \left[g_{\mathrm{T}}(r)\right]^2 F_{\mathrm{F1}}(r)\right\} - \frac{2}{3}\left[g_{\mathrm{SO}}(r)\right]^2 F_{\mathrm{qF1}}(r)$$

$$F_{\mathrm{SO}}(r) = -24\left[g_{\mathrm{T}}(r)\right]^2 F_{\mathrm{F1}}(r) + \frac{4}{3}\left\{\sqrt{\frac{F_{\mathrm{C1}}(r)}{F_{\mathrm{F1}}(r)}}\,g_{\mathrm{T}}(r) - \frac{\left[g_{\mathrm{T}}(r)\right]^2}{4} - g_{\mathrm{T}}(r)g_{\mathrm{SO}}(r)\right\}F_{\mathrm{qF1}}(r)$$

Explicit Energy Functional for Neutron Matter with AV8' Pot.

$$\frac{E_2}{N} = \frac{3}{5}E_{\mathrm{F}} + 2\pi\rho\int\left\{\left[\sum_{s=0}^{1}F_s(r)V_{\mathrm{C}s}(r)\right] + F_{\mathrm{T}}(r)V_{\mathrm{T}}(r) + F_{\mathrm{SO}}(r)V_{\mathrm{SO}}(r)\right\}r^2dr$$

$$+\frac{\pi\hbar^2\rho}{2m}\int\sum_{s=0}^{1}F_{\mathrm{C}s}(r)\left[\frac{1}{F_{\mathrm{C}s}(r)}\frac{dF_{\mathrm{C}s}(r)}{dr} - \frac{1}{F_{\mathrm{C}s}(r)}\frac{dF_{\mathrm{C}s}(r)}{dr}\right]^2 r^2dr$$

$$+\frac{2\pi\hbar^2\rho}{m}\int\left[8\left\{\left[\frac{dg_{\mathrm{T}}(r)}{dr}\right]^2 + \frac{6}{r^2}\left[g_{\mathrm{T}}(r)\right]^2\right\}F_{\mathrm{F}t1}(r) + \frac{2}{3}\left[\frac{dg_{\mathrm{SO}}(r)}{dr}\right]^2 F_{\mathrm{qF1}}(r)\right]r^2dr$$

$$-\frac{\hbar^2}{16\pi^2 m\rho}\int\frac{\left[S_{\mathrm{c1}}(k) - 3 + 2S_{\mathrm{cF}}(k)\right]\left\{\left[S_{\mathrm{c1}}(k) - S_{\mathrm{cF}}(k)\right]^2 + \frac{15}{2}\left[S_{\mathrm{SO}}(k)\right]^2\right\}}{S_{\mathrm{c1}}(k)/S_{\mathrm{cF}}(k)}k^4dk$$

$$-\frac{\hbar^2}{16\pi^2 m\rho}\int\frac{\left[S_{\mathrm{cT1}}(k) - 3 + 2S_{\mathrm{cF}}(k)\right]\left[S_{\mathrm{cT1}}(k) - S_{\mathrm{cF}}(k)\right]^2}{S_{\mathrm{cT1}}(k)/S_{\mathrm{cF}}(k)}k^4dk$$

$$-\frac{\hbar^2}{16\pi^2 m\rho}\int 2\frac{\left[S_{\mathrm{cT2}}(k) - 3 + 2S_{\mathrm{cF}}(k)\right]\left\{\left[S_{\mathrm{cT2}}(k) - S_{\mathrm{cF}}(k)\right]^2 + \frac{15}{4}\left[S_{\mathrm{SO}}(k)\right]^2\right\}}{S_{\mathrm{cT2}}(k)/S_{\mathrm{cF}}(k)}k^4dk + \frac{E_{\mathrm{nod}}}{N}$$

Spin-Orbit Structure Functions

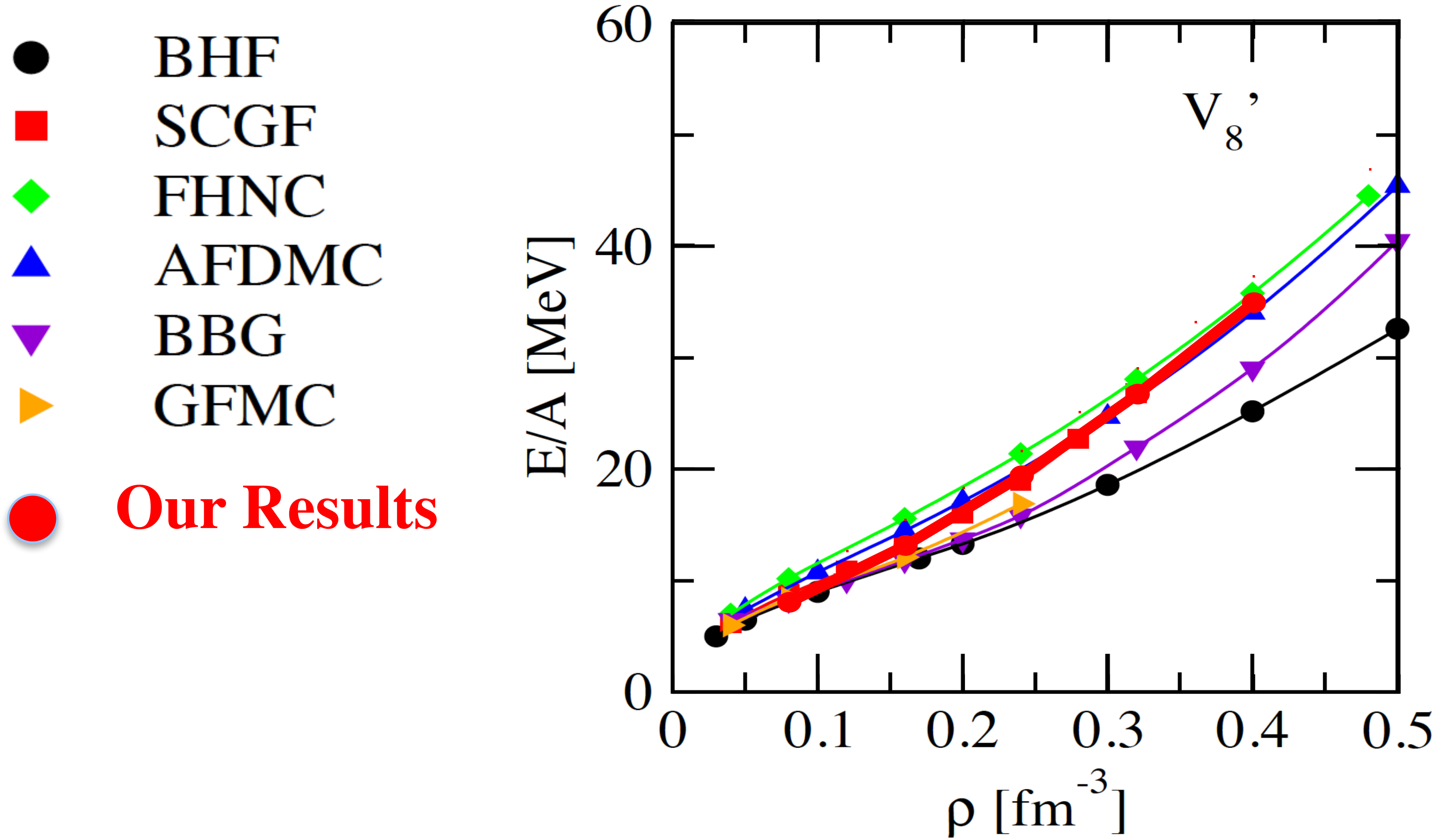
$$S_{\mathrm{SO}}(k) = \rho\int F_{\mathrm{SO}}(r)\frac{j_1(kr)}{k_{\mathrm{F}}r}dr$$

E_{nod}/N

Nodal-diagram part

Energies of Neutron Matter with the v8' pot.

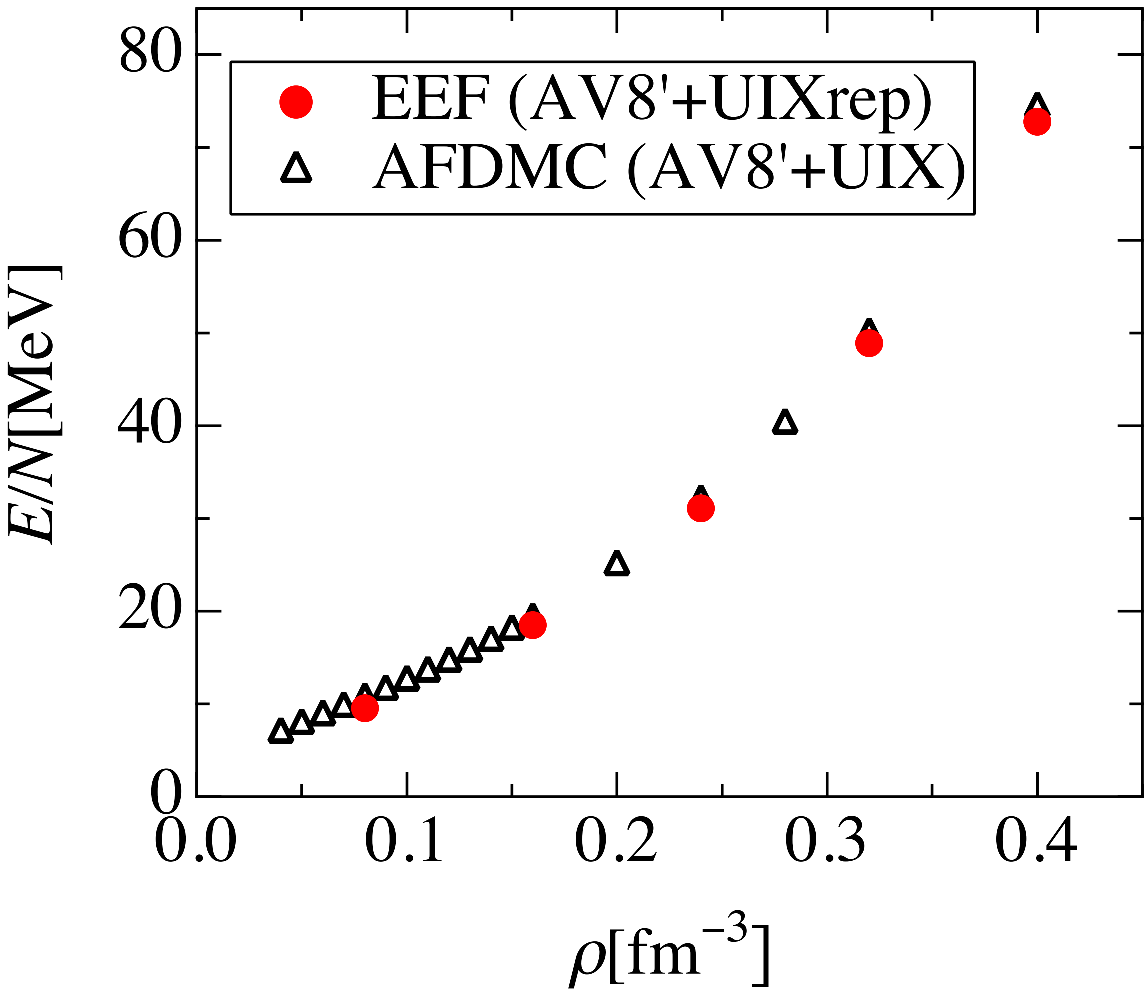
(Preliminary)



M. Baldo et al., Phys. Rev. C86 (2012) 064001

Energy of neutron matter with the v8’+UIX (Repulsive)

Preliminary



AFDMC, FHNC: PRC83 (2011) 054003 (with full UIX).

4-3. Explicit Energy Expression at Finite Temperature

The variational method by Schmidt and Pandharipande

The free energy per nucleon $$\frac{F}{N} = \frac{E_{T0}}{N} - T\frac{S_0}{N}$$

Entropy per nucleon Based on the Landau's Fermi Liquid Theory

$$\frac{S_0}{N} = -\frac{2k_B}{\pi^2\rho}\int_0^\infty \{[1-n(k)]\ln[1-n(k)] + n(k)\ln n(k)\}k^2\,dk.$$

Average occupation probability

$$n(k) = \left\{1 + \exp\left[\frac{\varepsilon(k)-\mu_0}{k_B T}\right]\right\}^{-1}. \quad \varepsilon(k) = \frac{\hbar^2 k^2}{2m^*}$$

m^*: Effective mass

The Internal energy per nucleon E_{T0}/N

At zero temperature: $E_0[n_0(k)]/N$

The expectation value of the Hamiltonian with the Jastrow wave function

$$\Psi = \mathrm{Sym}\left[\prod_{i>j} f_{ij}\right]\Phi_F\left[n_0(k)\right]$$

Φ_F: The Fermi-gas wave function

$n_0(k) = \Theta(k_F - k)$

Occupation probability at $T = 0$

At finite temperature: $E_{T0}[n(k)]/N$

The correlation function f_{ij} is chosen as at zero temperature:
Frozen correlation approximation

$$\frac{F}{N} = \frac{E_{T0}}{N} - T\frac{S_0}{N}$$ is minimized with respect to m^*

Explicit Energy Expression at Finite Temperature

Entropy per Nucleon

$$\frac{S_0}{N} = -\frac{2k_B}{\pi^2\rho}\int_0^\infty \{[1-n(k)]\ln[1-n(k)] + n(k)\ln n(k)\}k^2\,dk.$$

Internal Energy per Nucleon

Energy per Nucleon at Zero Temperature $E_0\,[F_s(r), n_0(k)]/N$

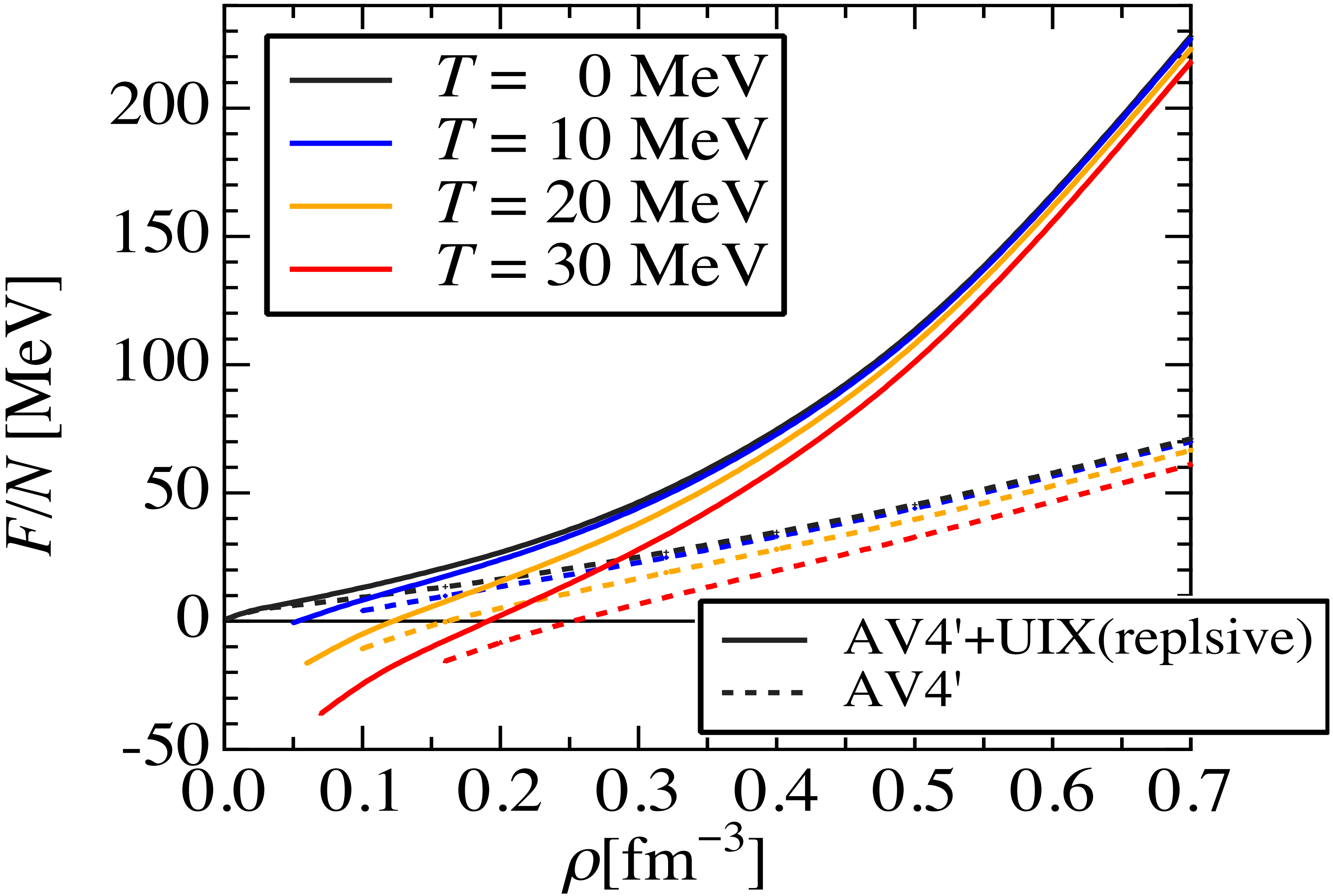
⇩

Internal Energy per Nucleon at Finite Temperature

$E_{T0}\,[F_s(r), n(k)]/N$

$$\boxed{\frac{F}{N} = \frac{E_{T0}}{N} - T\frac{S_0}{N}}$$ is minimized with respect to $F_s(r)$ and m^*

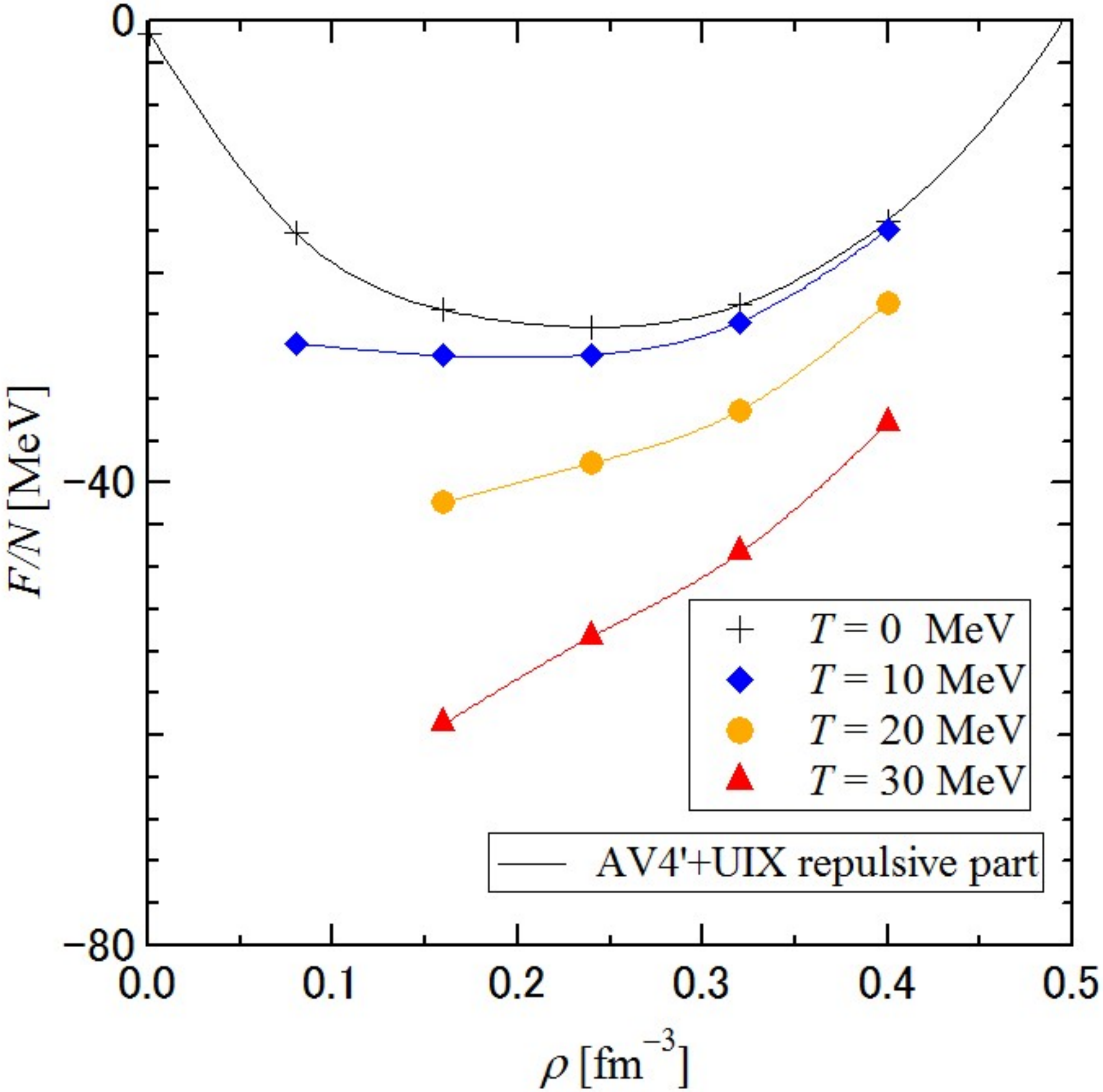
Free energy of neutron matter with v4'+UIX(Repulsive)



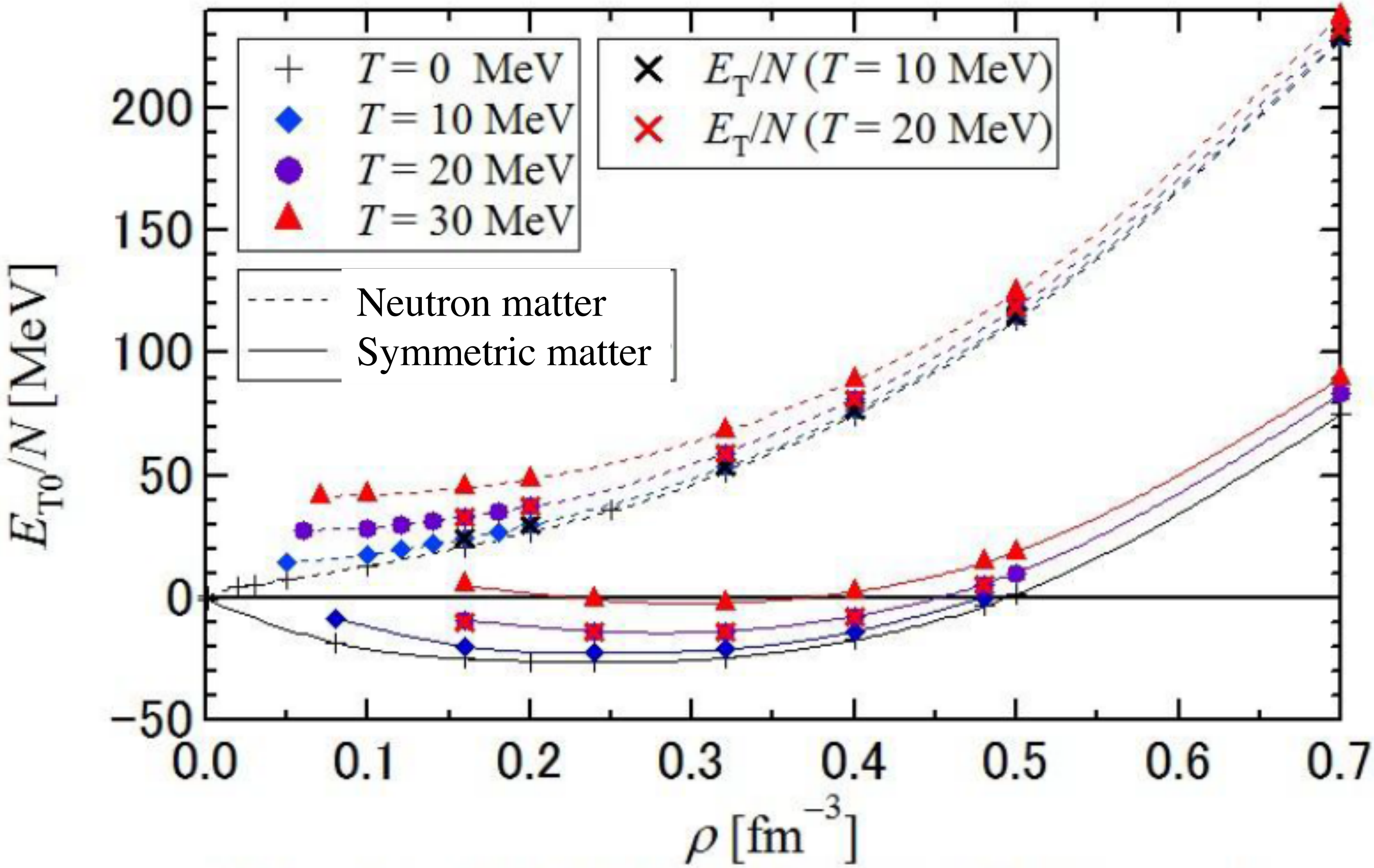
M. T., K. Kato and M. Yamada, J. Phys. Conf. Seri. 529 (2014) 012025

Free energy of symmetric nuclear matter with v4'+UIX(Repulsive)

Preliminary



Internal Energy per Nucleon (Preliminary)



The thermodynamic quantities are self consistent

Summary

Variational Method with Explicit Energy Functional

Liquid ^{3}He: Reasonable results + 2D quasistable state ?

Neutron Matter: AV6' (central + tensor) + UIX (repulsive)

Extension of the theory

2π-exchange three-body force: π condensation ?

1st order phase transition is not found

Two-body spin-orbit force (AV8')

Free energy at finite temperature:

Thermodynamic quantities are self consistent

⇩

Systematic Extentions and Application to Astrophysics