Quantum Monte Carlo calculations of neutron matter with Chiral Effective Field Theory interactions

Ingo Tews,

In collaboration with J. Carlson, S. Gandolfi, A. Gezerlis, K. Hebeler, T. Krüger, J. Lynn, A. Schwenk, …

Talk, YITP program: "Nuclear Physics, Compact Stars, and Compact Star Mergers”, Oct.20, 2016, Kyoto

JINA–CEE

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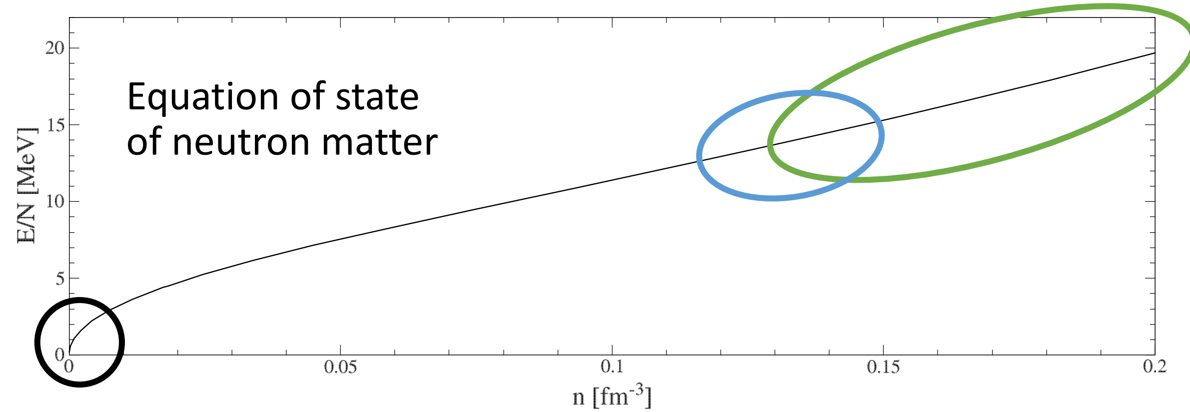
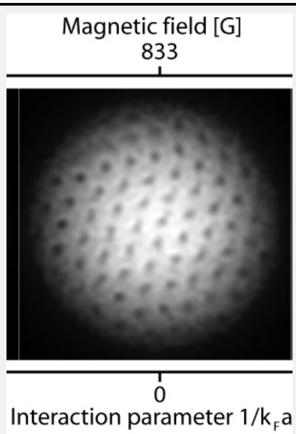
Motivation

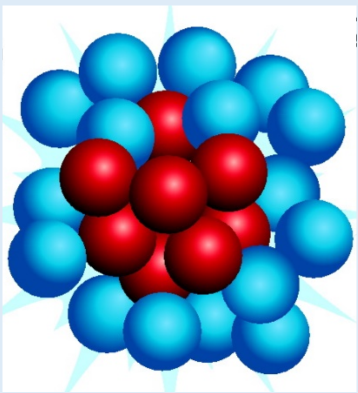
- The neutron-matter equation of state at T=0 connects several physical systems over a wide density range.
- An accurate description of the neutron-matter equation of state is therefore crucial.





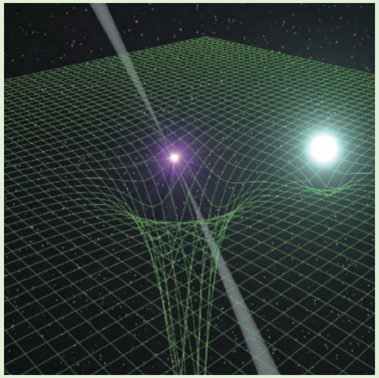
Zwierlein et al., Nature (2005)

Ultracold atoms



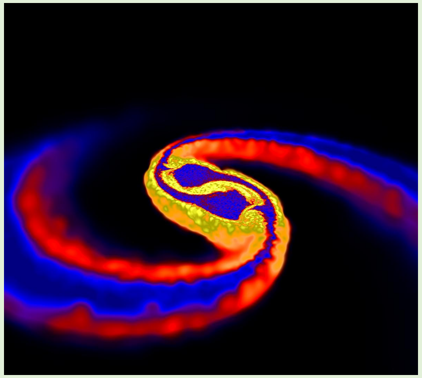
Credit: B.A. Brown

Neutron-rich nuclei



Antoniadis et al., Science (2013)

Neutron stars



Credit: S. Rosswog

Gravitational waves from neutron star mergers

Motivation

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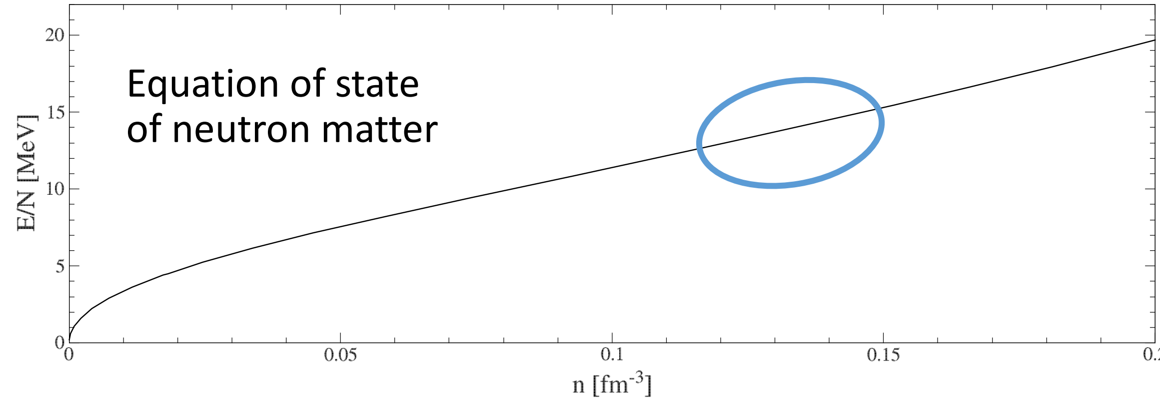
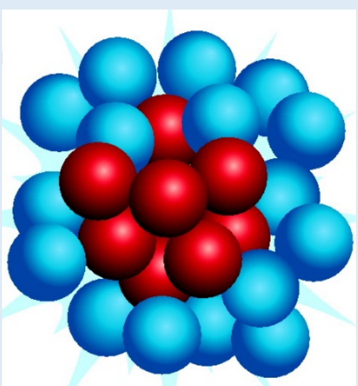
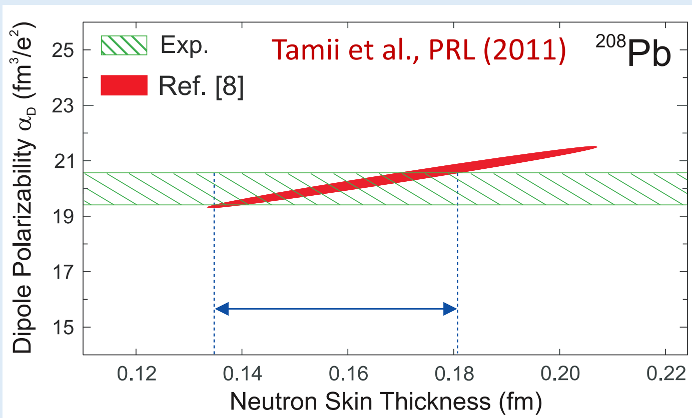
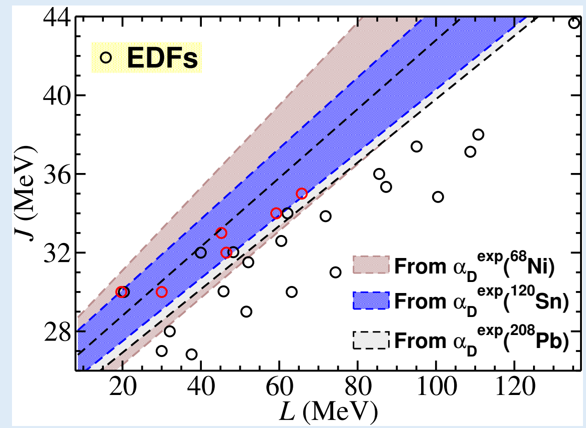




Credit: B.A. Brown

Neutron-rich nuclei



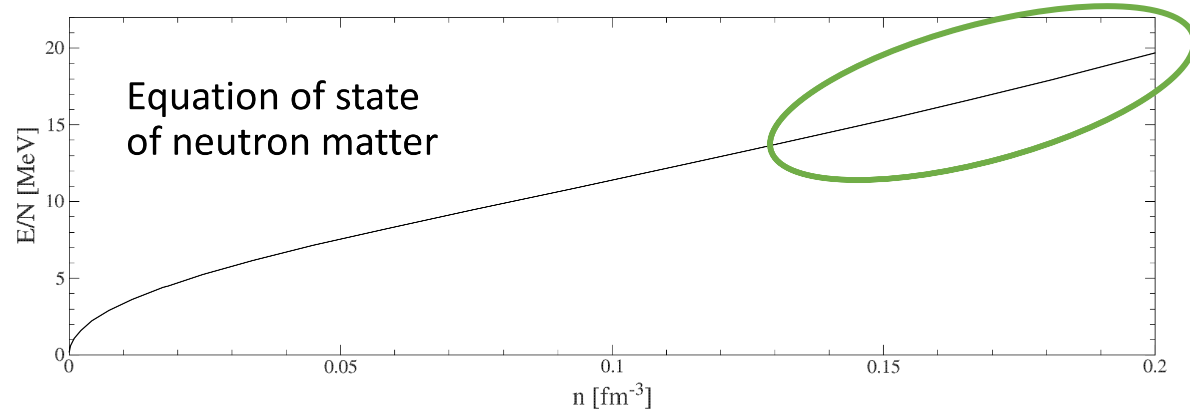


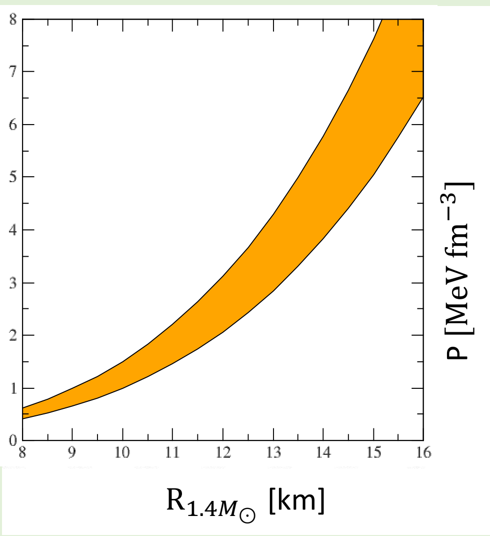
Roca-Maza et al., PRC (2013)

- Neutron matter at saturation density constrains neutron-skin thickness of neutron-rich nuclei
- Experiments at RCNP, GSI, …

Motivation

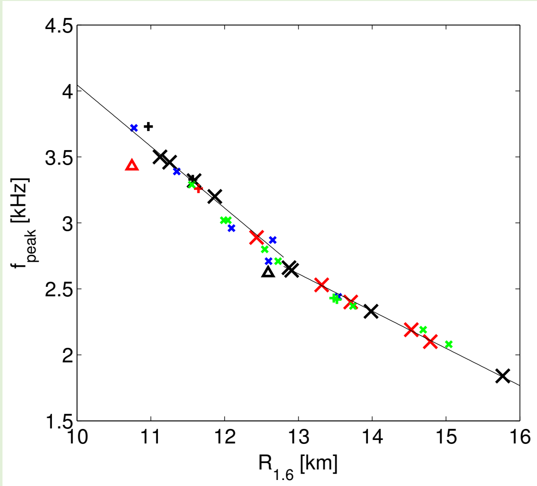
- The neutron-matter equation of state at T=0 connects several physical systems over a wide density range.
- An accurate description of the neutron-matter equation of state is therefore crucial.





Lattimer, Lim, ApJ (2013).

- Neutron matter equation of state at saturation density and above determines mass-radius relation of neutron stars and gravitational-wave signal of neutron-star mergers
- EOS properties at saturation density are correlated with neutron-star radii and gravitational wave peak frequency



Bauswein et al., PRD (2012)

Motivation

How to obtain the EOS in an ab initio approach?

Quantum Chromodynamics

Nuclear Forces

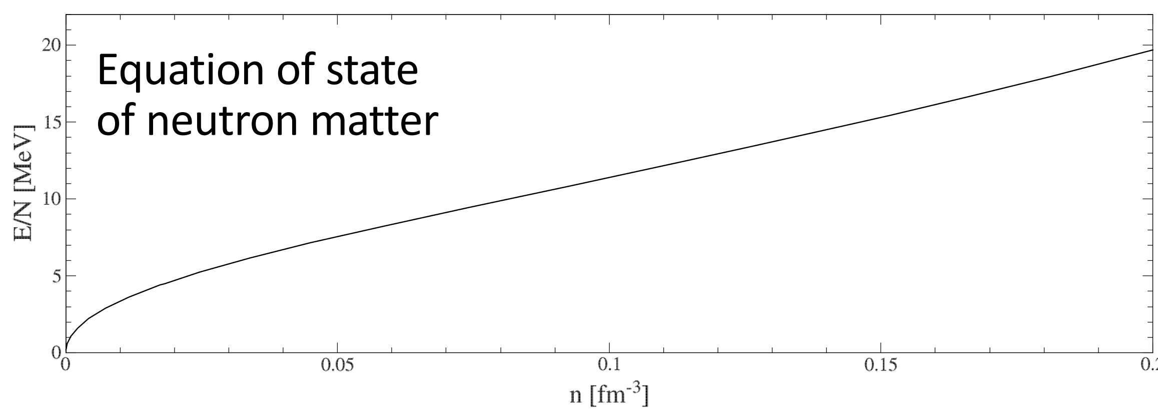
Phenomenological forces (e.g. AV18 + UIX)

Chiral effective field theory

Many-body methods

Quantum Monte Carlo: Very reliable

Broad range of methods



Outline

- Chiral effective field theory: Epelbaum *et al.*, PPNP (2006) and RMP (2009)
  - Systematic basis for low-energy nuclear forces, connected to QCD
  - naturally includes many-body forces
  - Very successful in calculations of nuclei and nuclear matter

- Ab-initio calculations using chiral EFT can be used to constrain equation of state of neutron matter

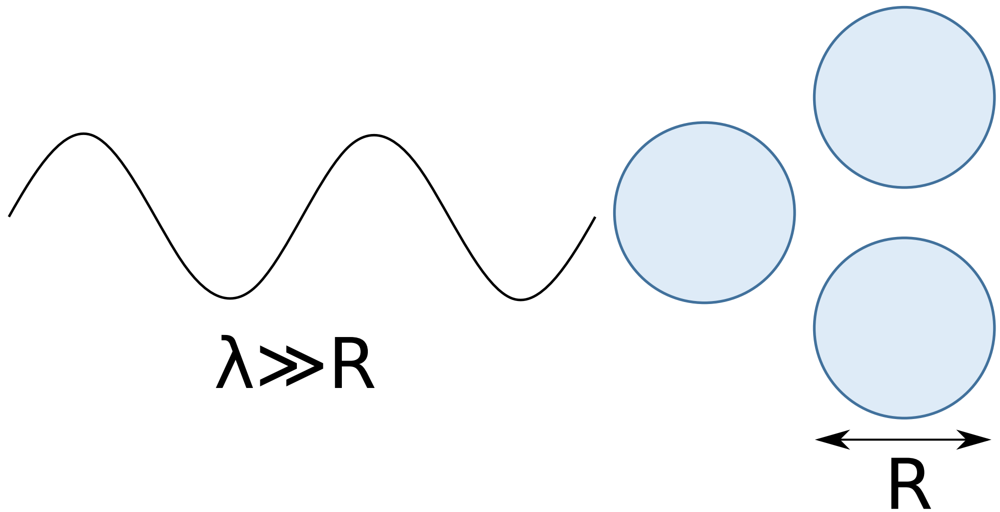
- Neutron-matter applications: IT, Krüger, Hebeler, Schwenk, PRL & PRC & PLB (2013)
  - Symmetry energy
  - Neutron-star mass-radius relation

- Improving neutron-matter results using Quantum Monte Carlo methods
  Gezerlis, IT, et al., PRL & PRC (2013, 2014, 2016)

- Summary

Chiral effective field theory for nuclear forces

Basic principle of effective field theory:

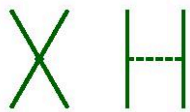
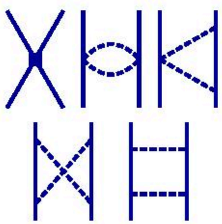
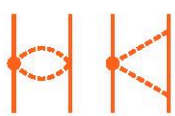
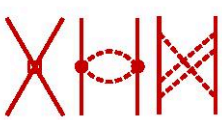


Effective field theory for nuclear forces

At low energies (long wavelength) details not resolved!

- Choose relevant degrees of freedom for low-energy processes
- Systematic expansion of interactions constrained by symmetries

Chiral effective field theory for nuclear forces

	NN
LO $\mathcal{O}\left(\frac{Q^0}{\Lambda^0}\right)$	
NLO $\mathcal{O}\left(\frac{Q^2}{\Lambda^2}\right)$	
N²LO $\mathcal{O}\left(\frac{Q^3}{\Lambda^3}\right)$	
N³LO $\mathcal{O}\left(\frac{Q^4}{\Lambda^4}\right)$	 + ...

Explicit degrees of freedom:

- Pions and nucleons

Write most general Lagrangian consistent with the symmetries of QCD

Separation of scales:

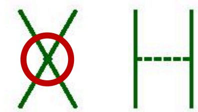
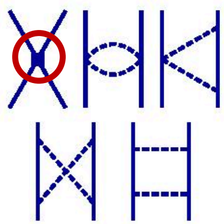
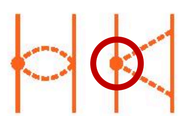
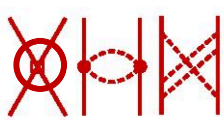
- Low momenta $Q \ll$ breakdown scale Λ_b
- Expand in powers of $\left(\frac{Q}{\Lambda_b}\right)^\nu \sim \left(\frac{1}{3}\right)^\nu$

Power counting:

- $\nu = 0$: leading order (LO),
- $\nu = 2$: next-to-leading order (NLO), ...

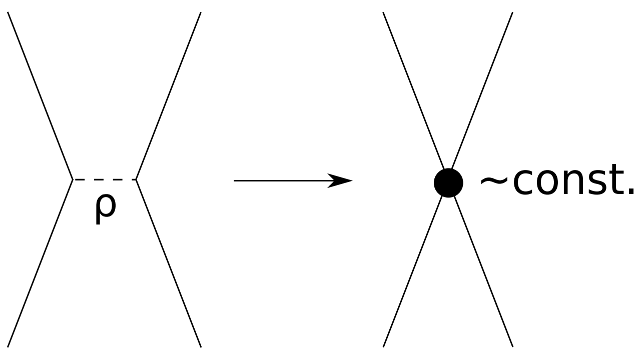
Weinberg, van Kolck, Kaplan, Savage, Wise, Epelbaum, Kaiser, Machleidt, Meißner, Hammer ...

Chiral effective field theory for nuclear forces

	NN	
LO $\mathcal{O}\left(\frac{Q^0}{\Lambda^0}\right)$		2 LECs
NLO $\mathcal{O}\left(\frac{Q^2}{\Lambda^2}\right)$		7 LECs
N²LO $\mathcal{O}\left(\frac{Q^3}{\Lambda^3}\right)$		
N³LO $\mathcal{O}\left(\frac{Q^4}{\Lambda^4}\right)$	 + ...	15 LECs

Explicit degrees of freedom:

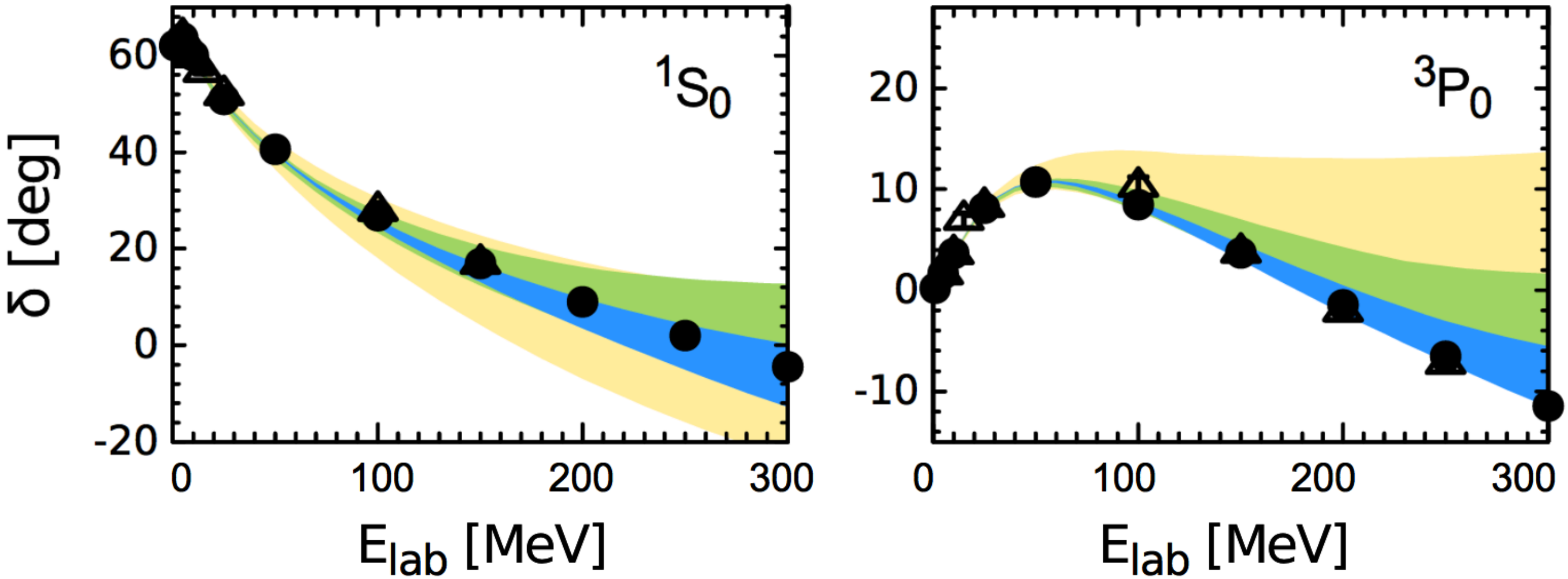
- Pions and nucleons
- Long-range physics explicit
- Short-range physics expanded in general operator basis
- High-momentum physics absorbed into short-range couplings, fit to experiment (phase shifts)



Second scale: cutoff Λ (resolution):

- Interactions Λ-dependent

Chiral effective field theory for nuclear forces

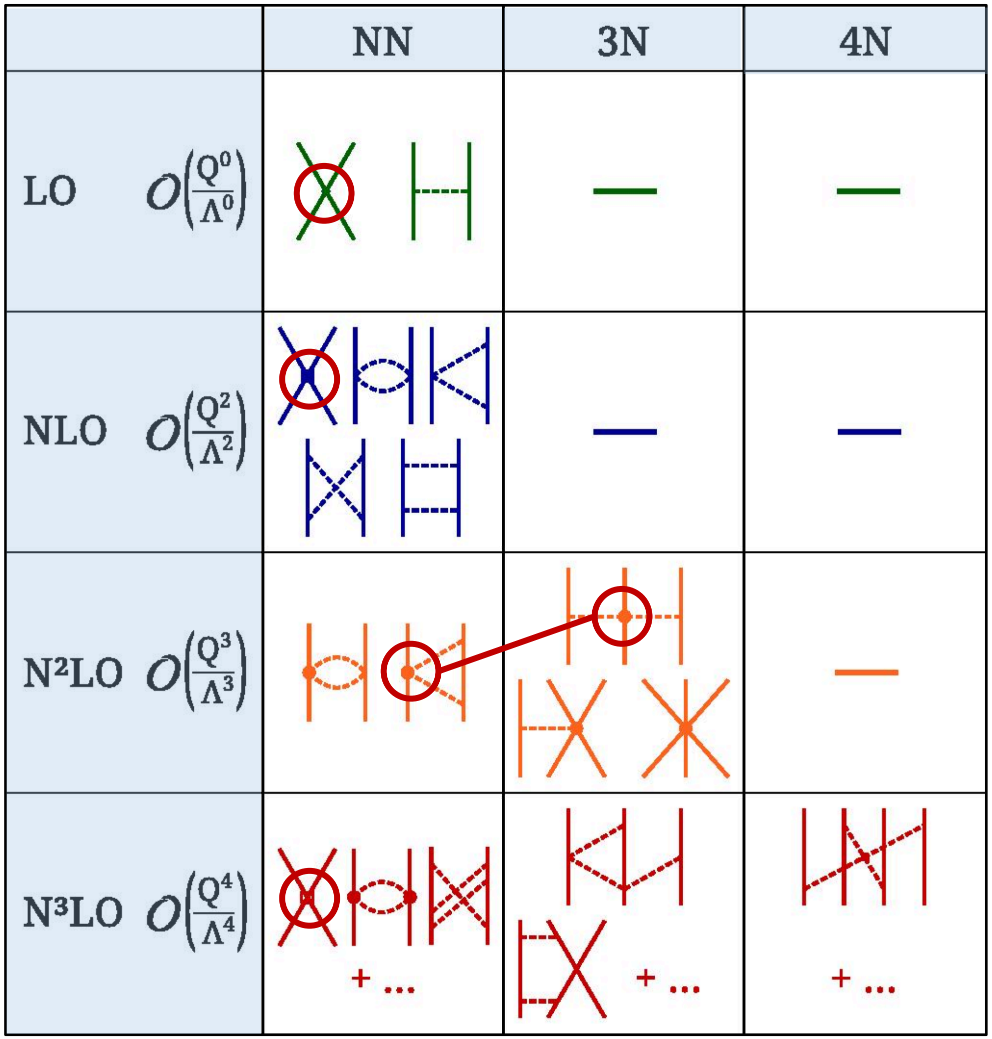


Epelbaum *et al.*, Eur. Phys. J (2015)

Systematic expansion of the nuclear forces:

- Can work to desired accuracy
- Can obtain systematic error estimates

Chiral effective field theory for nuclear forces



Many-body forces:

- Have been found to be crucial ingredient to describe nuclear physics

Natural hierarchy of nuclear forces:

- Two-body (NN) forces start at first order
- Three-body (3N) forces start at third order (2 LECs)

Fitting:

- NN forces in NN system (NN phase shifts, …)
- 3N forces in 3N/4N system (Binding energies, radii, …)

Chiral effective field theory for nuclear forces

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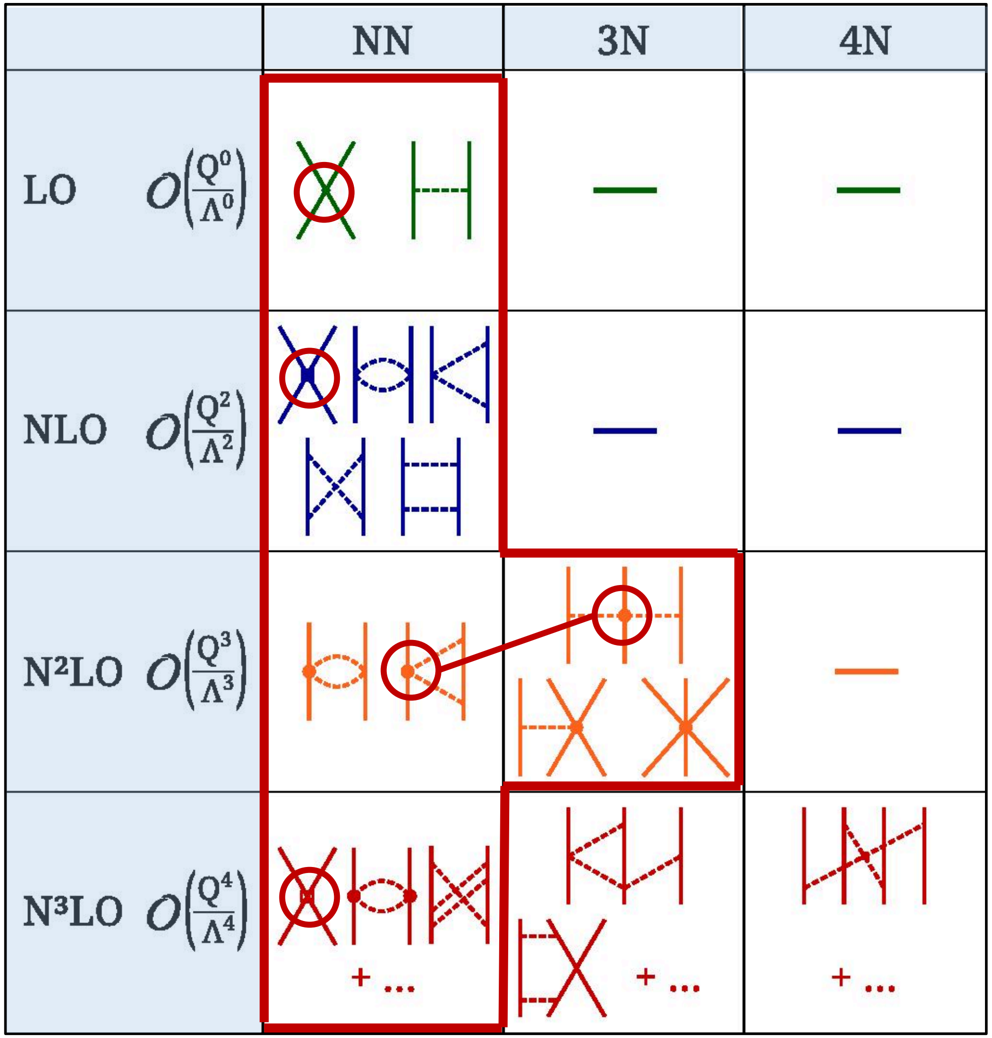
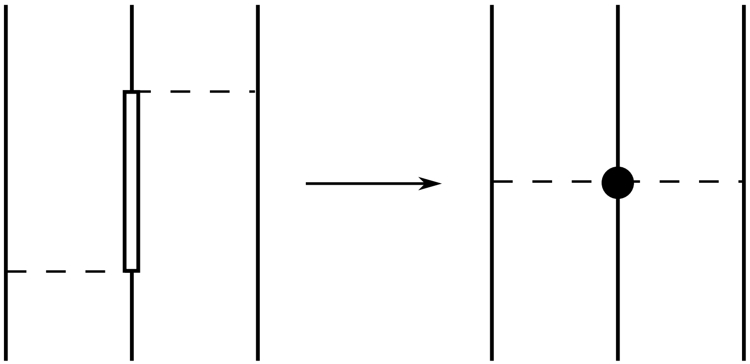


Consistent interactions:

- Same couplings for two-nucleon and many-body sector
- In contrast to phenomenological interactions



Chiral effective field theory for nuclear forces

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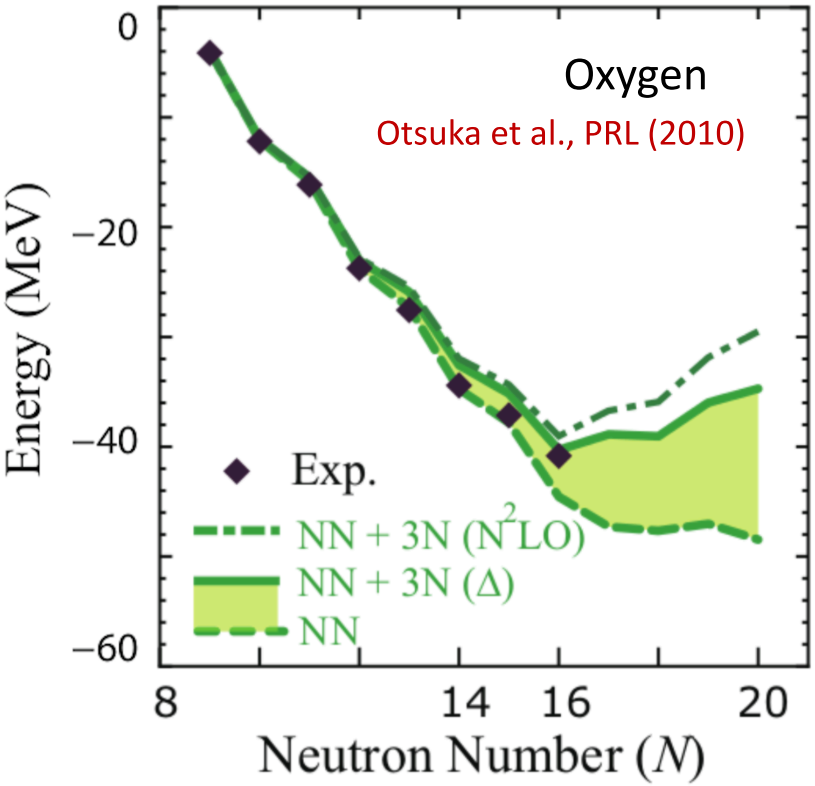
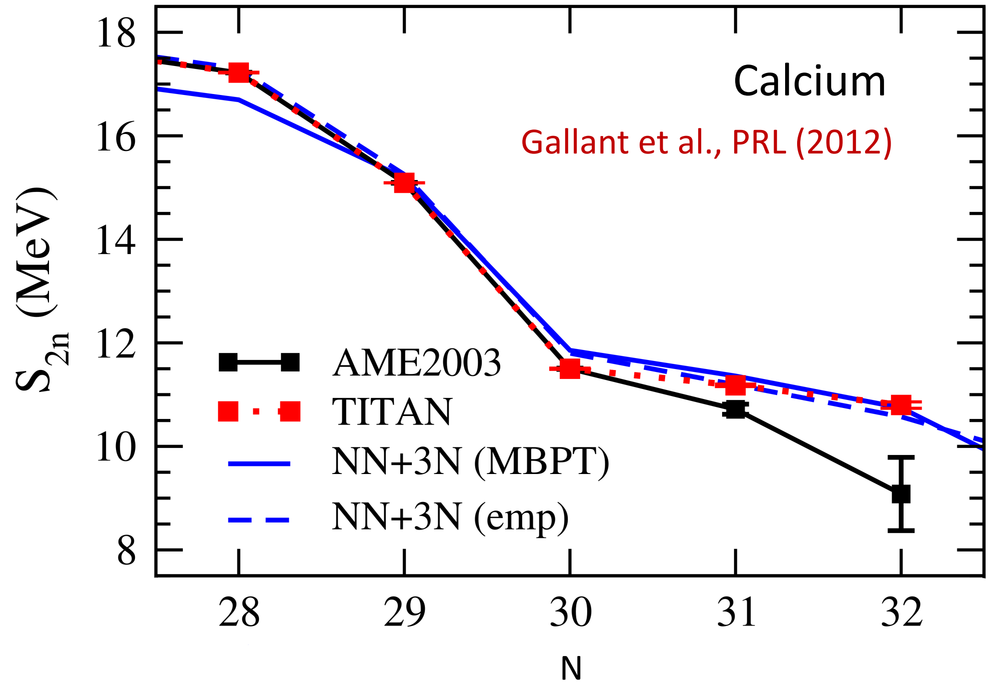
Many-body forces are crucial:





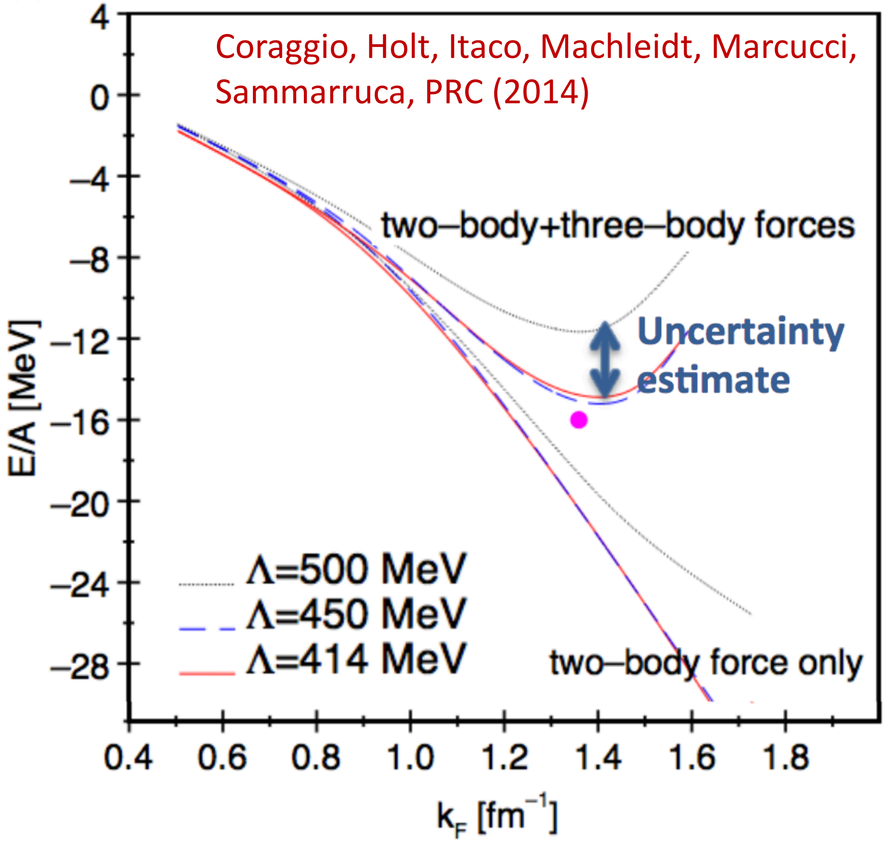
NN + 3N forces:

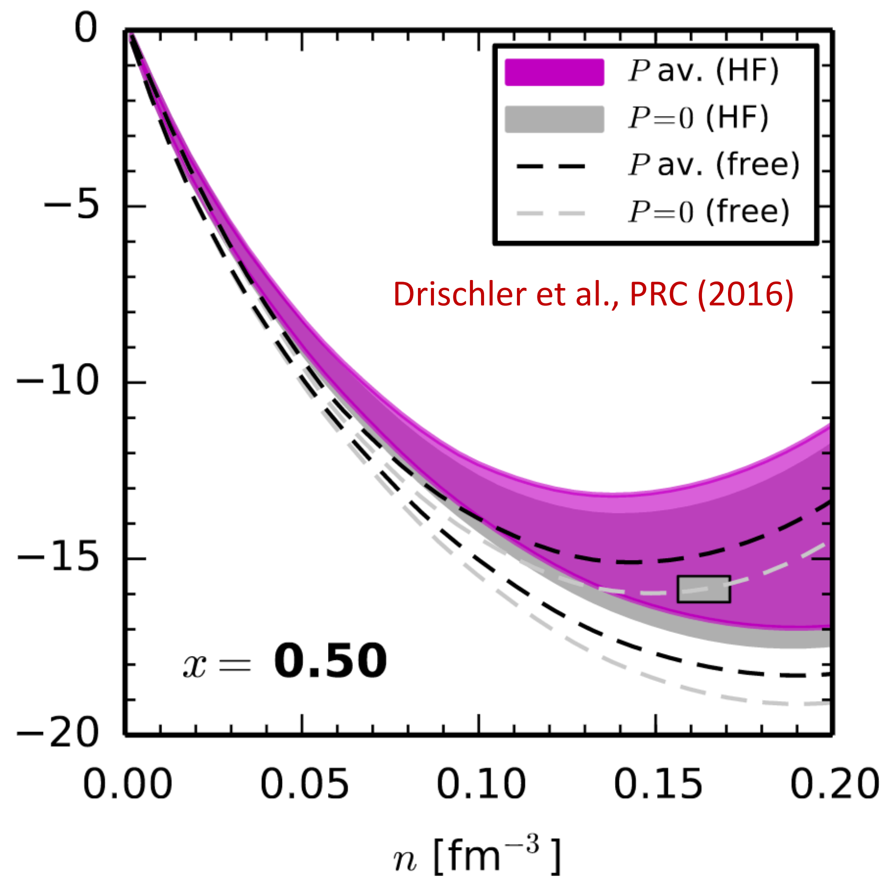
- Give correct physics of neutron-rich nuclei

See also Hebeler et al., ARNPS (2015)

Chiral effective field theory for nuclear forces

Many-body forces are crucial:

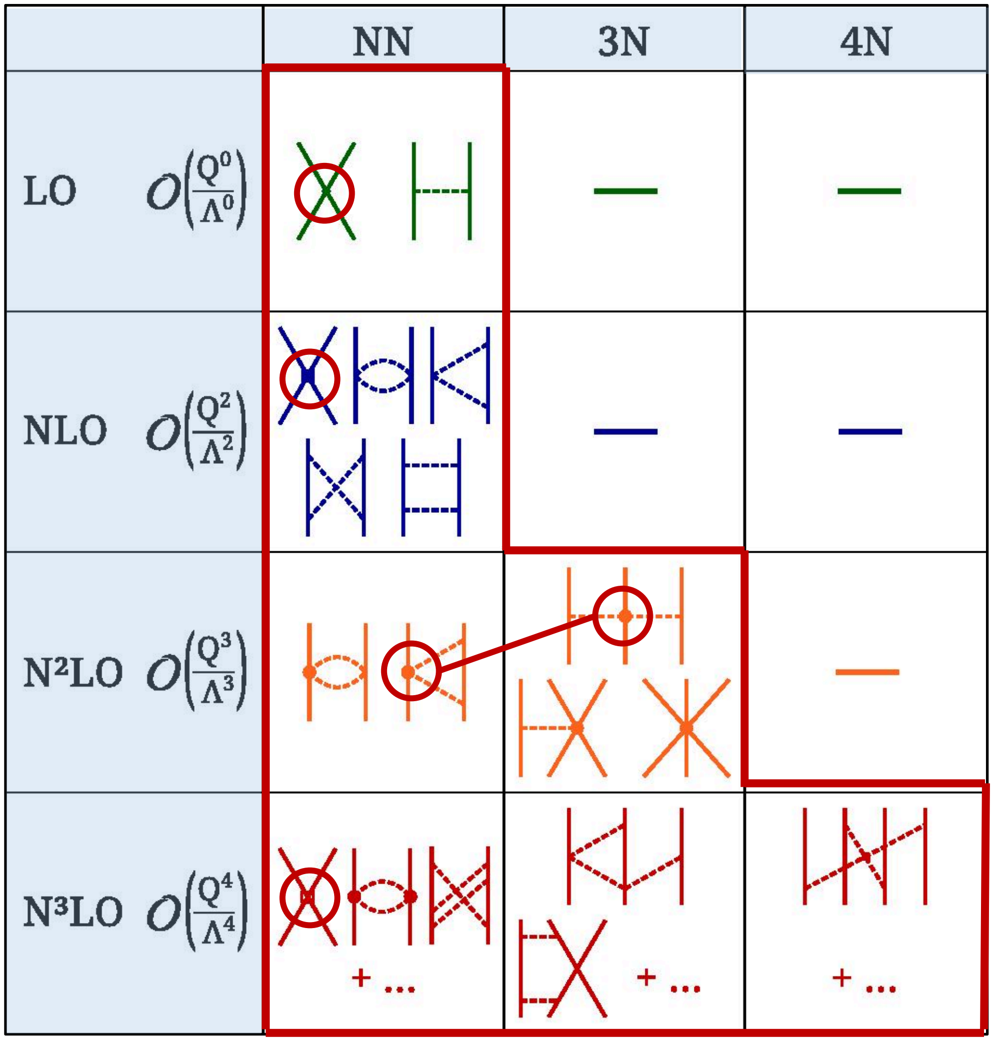




NN + 3N forces:

- Give correct saturation with theoretical uncertainties in nuclear matter
  Drischler et al., PRC (2016)

Chiral effective field theory for nuclear forces



Neutron matter:

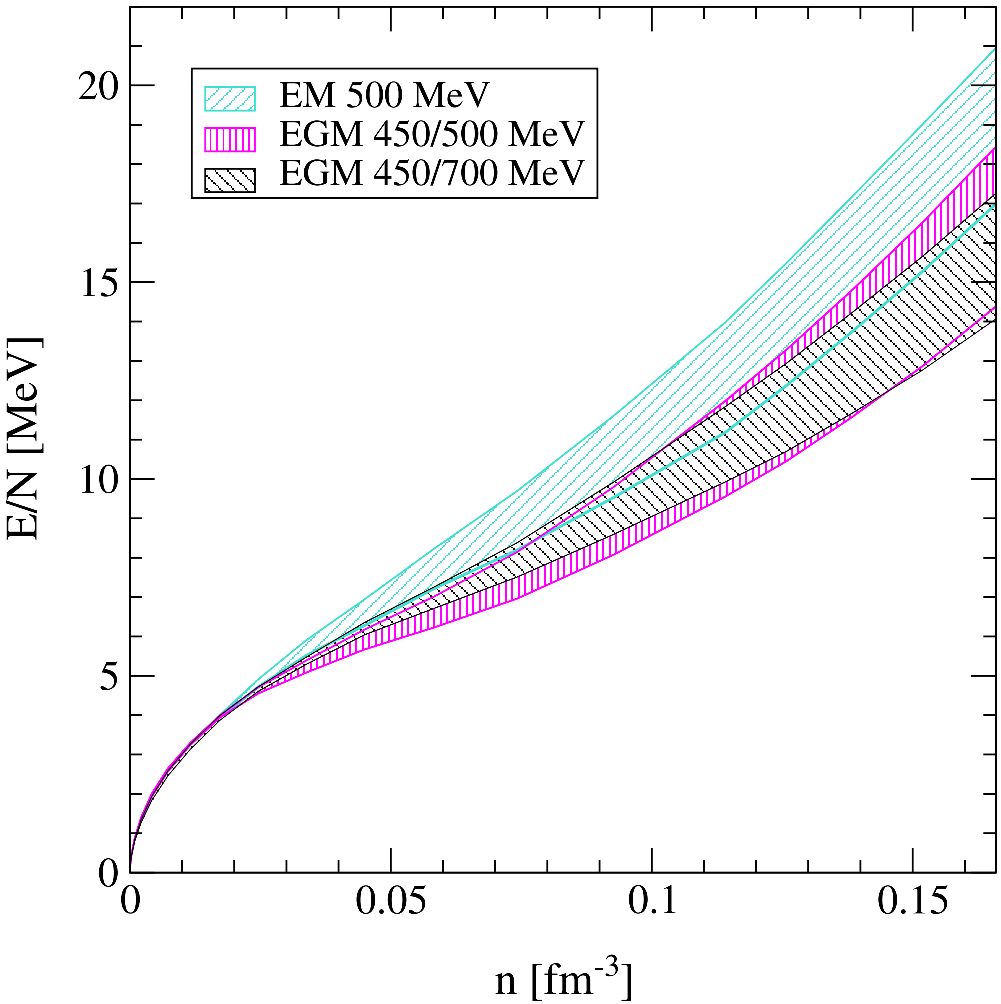
- Complete calculation at N^{3}LO using many-body perturbation theory (MBPT)

  IT, Krüger, Hebeler, Schwenk, PRL (2013)

Calculation is simpler in neutron matter:

- Only certain parts of the many-body forces contribute
- Chiral many-body forces **completely predicted** from NN sector

Neutron matter



IT, Krüger, Hebeler, Schwenk, PRL (2013)

Bands:

- Include several sources of uncertainty:
  - Chiral Hamiltonians (cutoff, 3N LECs)
  - Many-body method

NN interactions:

- E/N at saturation density:

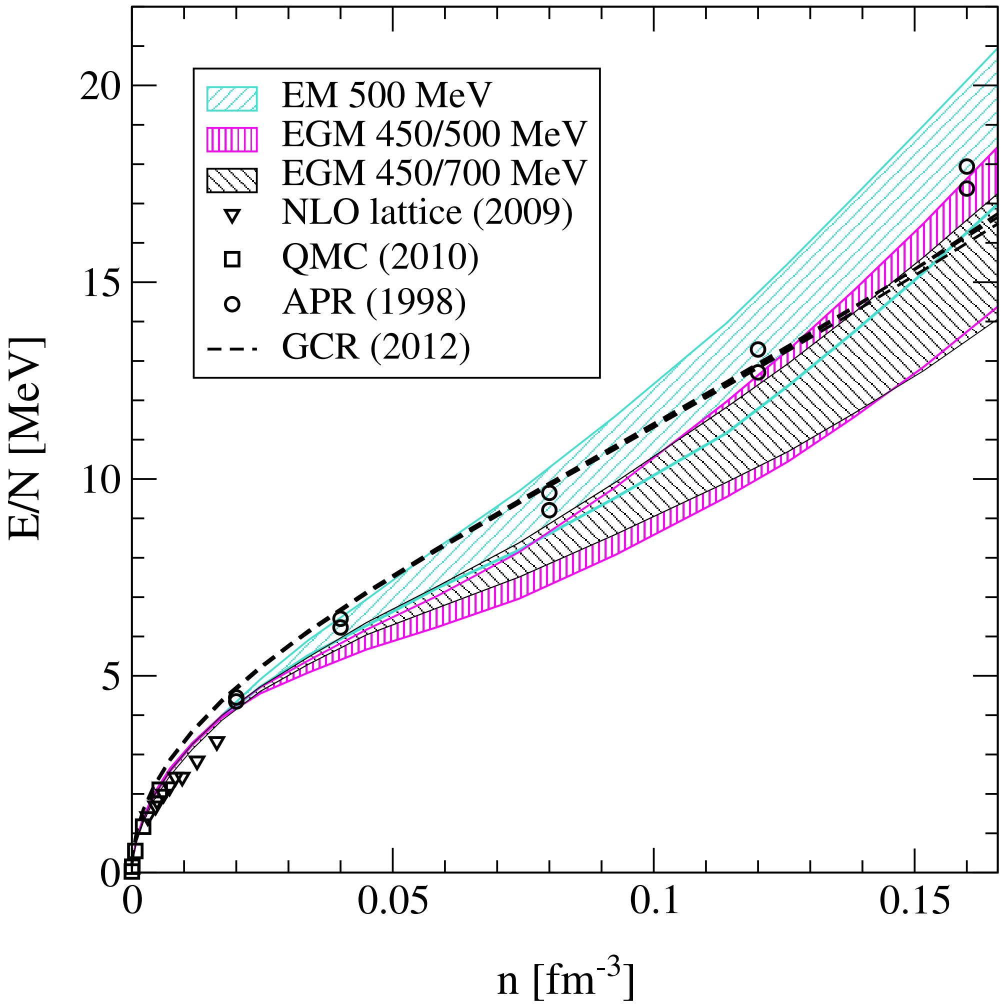
12-15 MeV

NN+3N interactions:

- Have large impact on energy and uncertainty:

14-21 MeV

Neutron matter



IT, Krüger, Hebeler, Schwenk, PRL (2013)

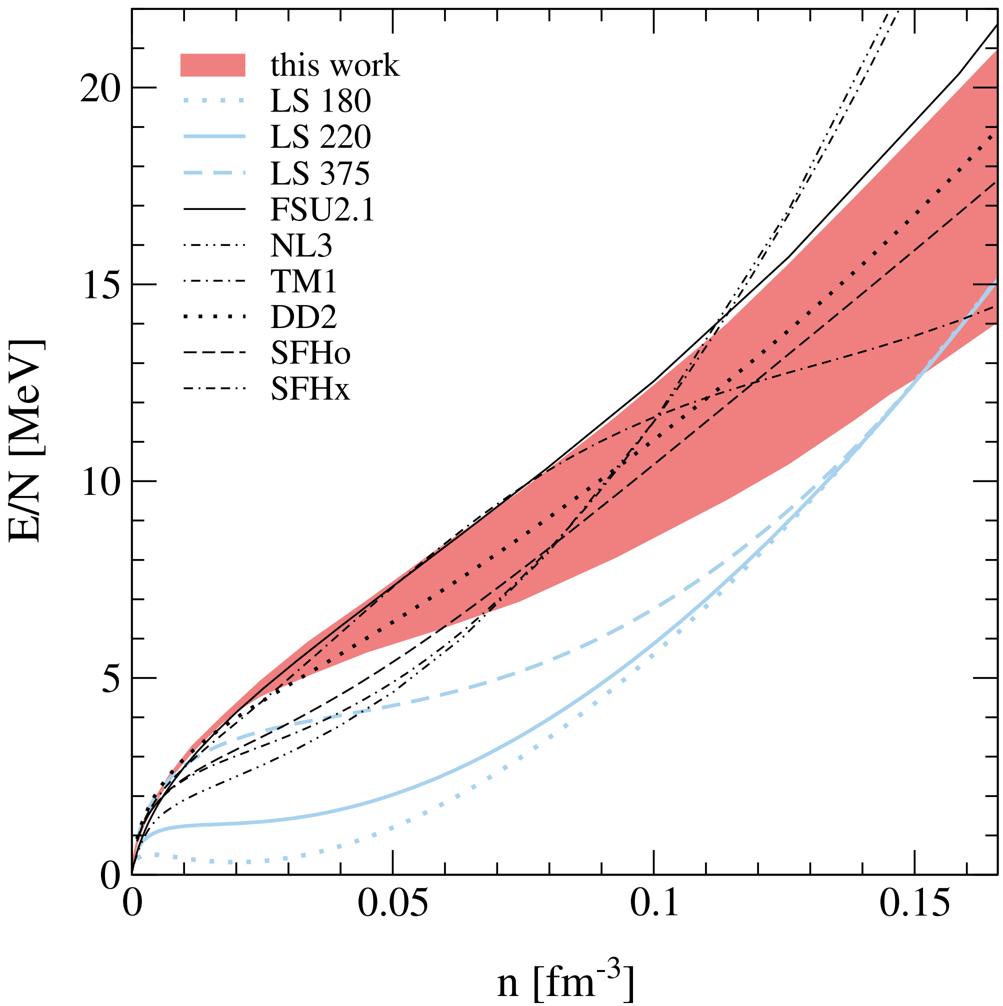
Good agreement with other calculations

- but in those no theoretical uncertainties

Akmal *et al.*, PRC (1998)
Gandolfi *et al.*, PRC (2012)

Chiral EFT puts constraints on neutron matter EOS

Neutron matter



Lines from Hempel, Lattimer, G. Shen

Good agreement with other calculations

- but in those
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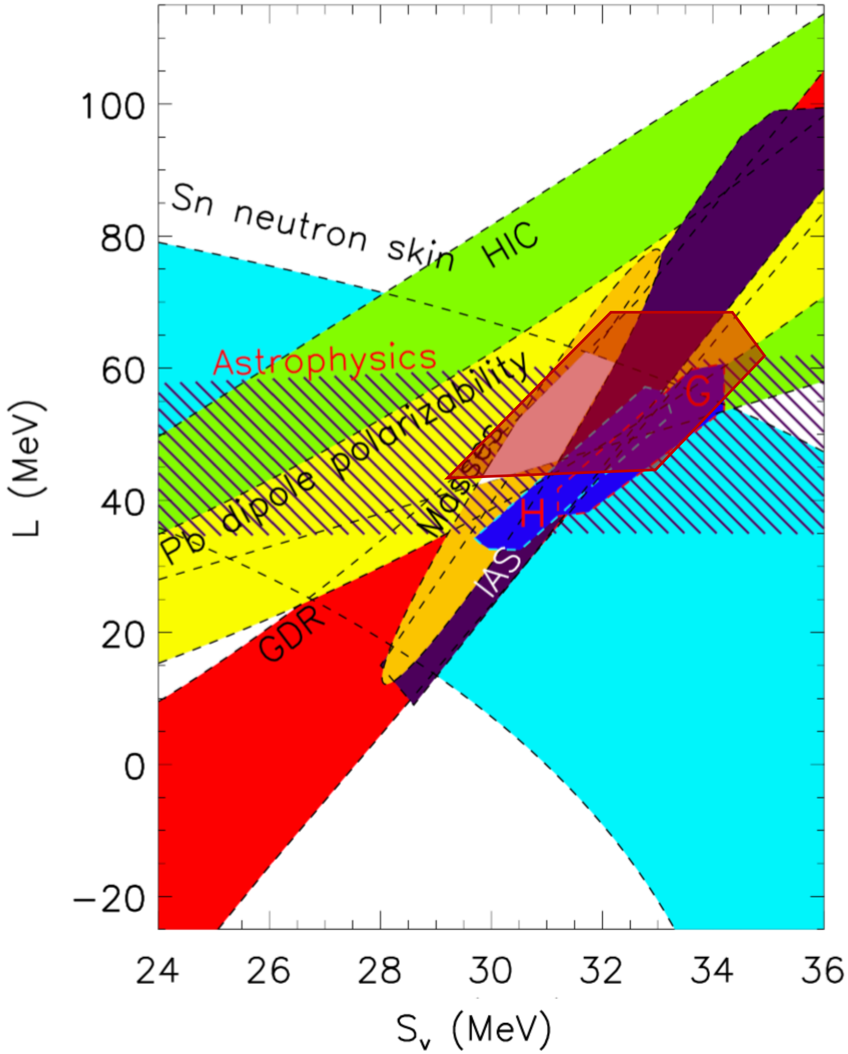
Akmal *et al.*, PRC (1998)
Gandolfi *et al.*, PRC (2012)

Chiral EFT puts constraints on neutron matter EOS

Symmetry energy and L parameter



Lattimer, Lim, ApJ (2013)

Put constraints on symmetry energy and its density dependence L:

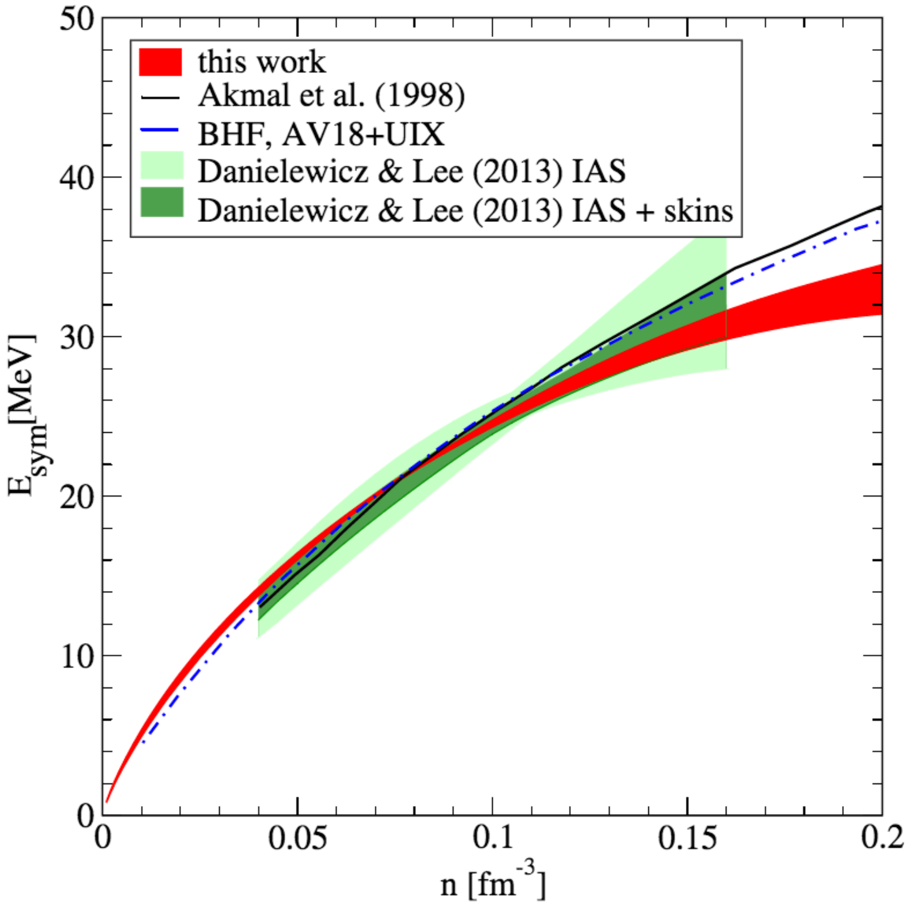
$$S_v(n) = \frac{1}{8}\frac{\partial^2}{\partial x^2}\frac{E}{A}(n,x)\bigg|_{x=1/2},$$

$$L(n_0) = 3n_0 \frac{\partial}{\partial n} S_V(n)\bigg|_{n_0},$$

- $S_V = 28.9 - 34.9$ MeV
- $L = 43.0\ - 66.6$ MeV

Good agreement with experimental constraints

Symmetry energy and L parameter



Drischler, Soma, Schwenk, PRC (2014)

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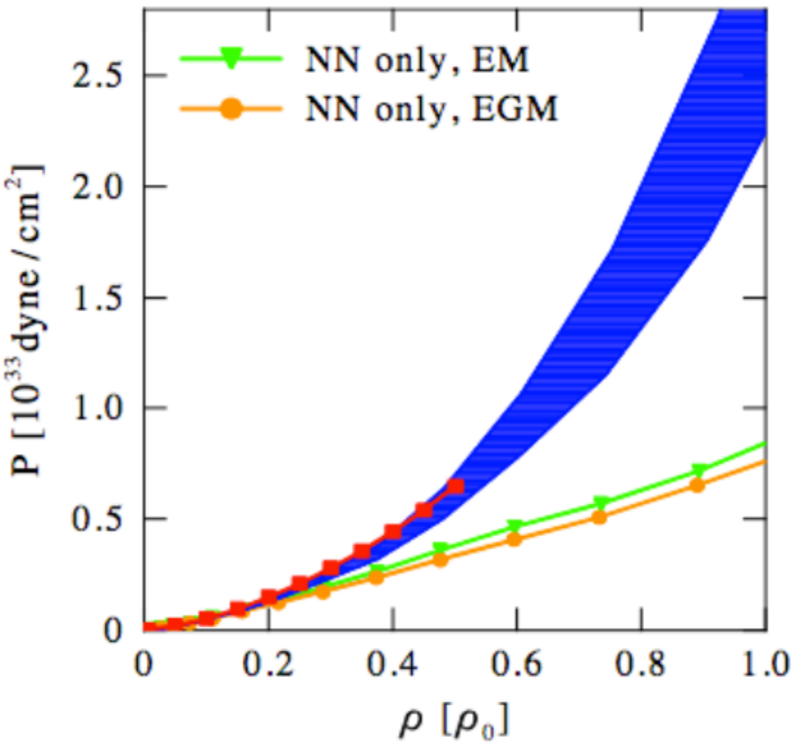
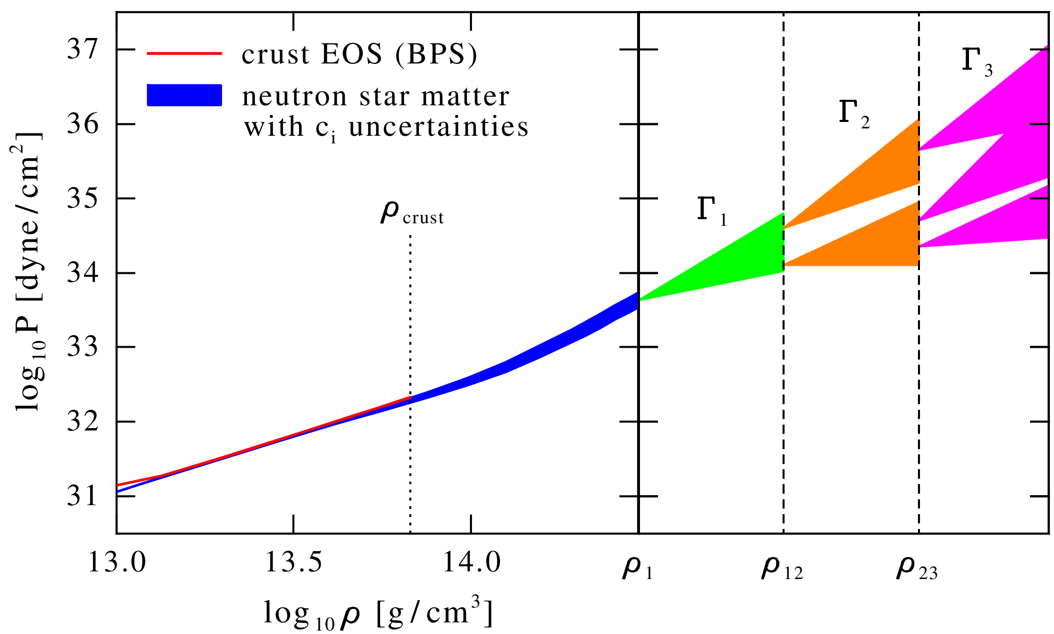
Good agreement with experimental constraints

Neutron Stars

Equation of state for neutron star matter: extend results to small $Y_{e,p}$

Hebeler, Lattimer, Pethick, Schwenk, PRL (2010) and APJ (2013)



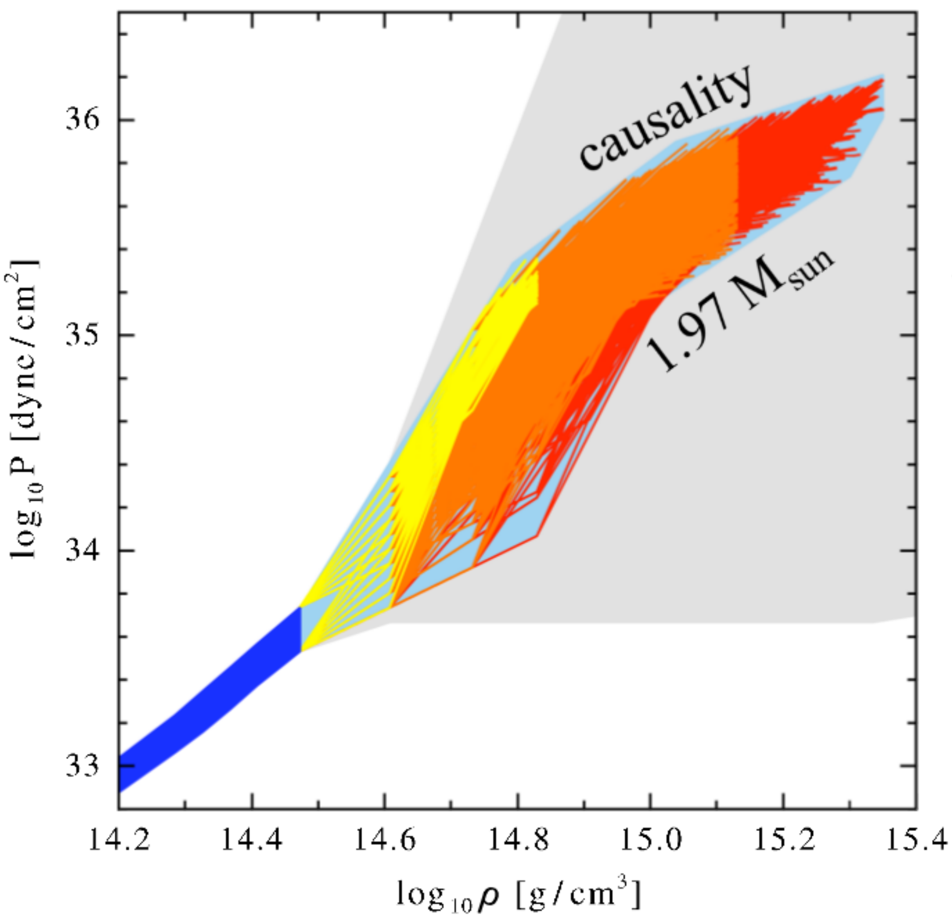


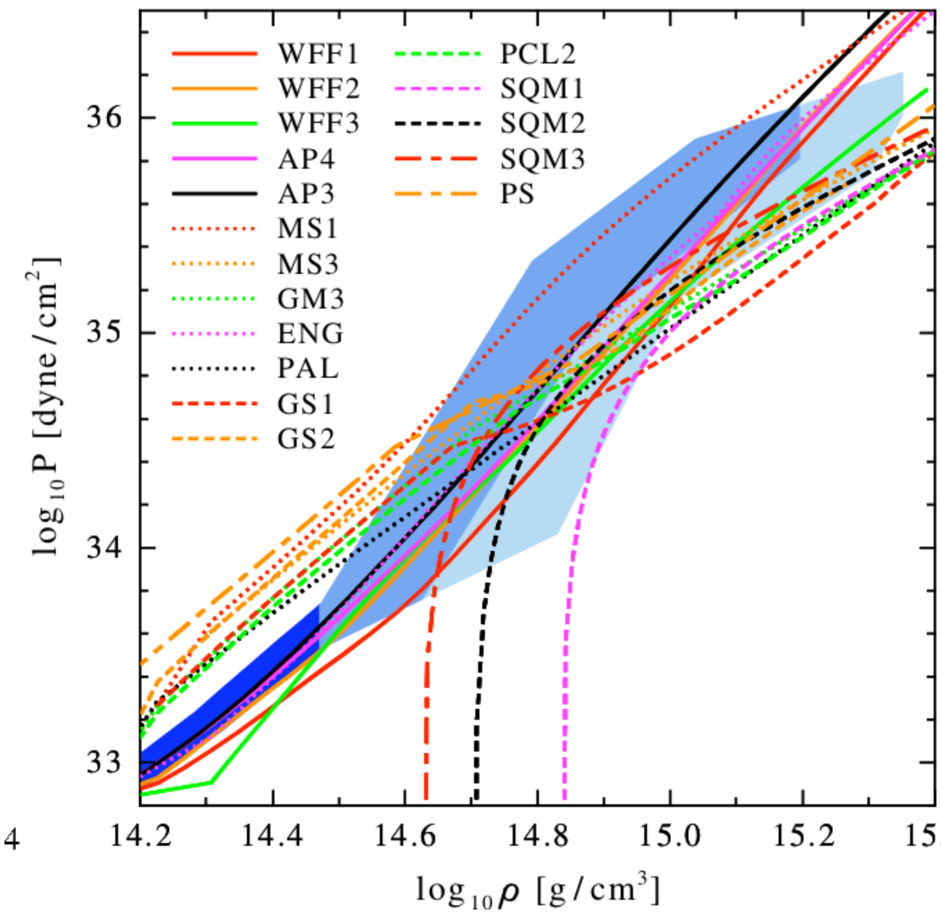
Agrees with standard crust EOS after inclusion of many-body forces

Extend to higher densities using polytropic expansion

Neutron Stars

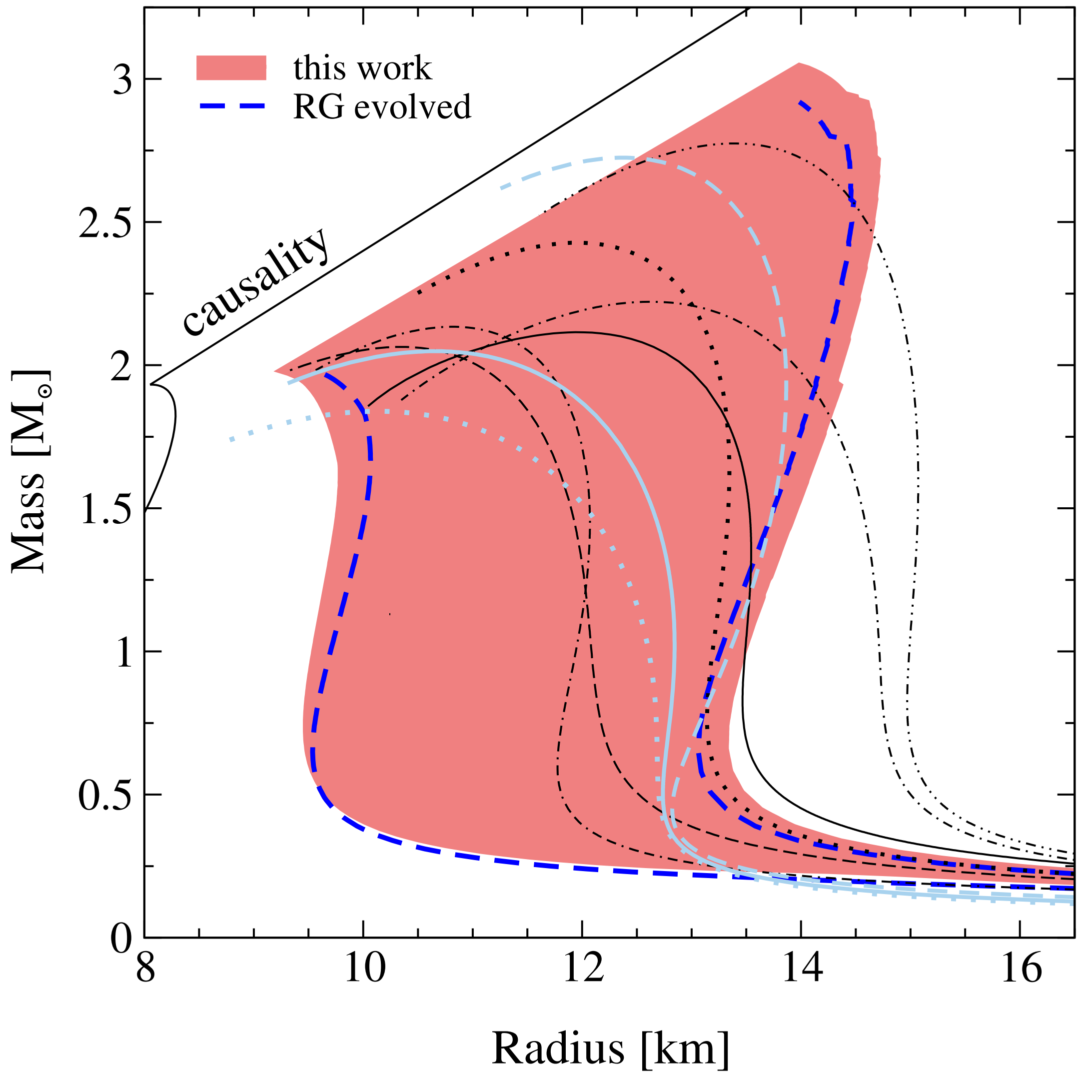
Constrain resulting EOS: causality and observed 1.97 $M_{\odot}$ neutron star





Hebeler, Lattimer, Pethick, Schwenk, PRL (2010) and APJ (2013)

Neutron Stars



Radius for 1.4 $M_\odot$ neutron star:

- $R = 9.7 - 13.9$ km

Maximum mass neutron star:

- $M_{max} \leq 3.05 M_\odot$ (14 km)

Uncertainties from many-body forces and polytropic expansion

- How to reduce uncertainties?

IT, Krüger, Hebeler, Schwenk, PRL (2013)

Neutron Stars

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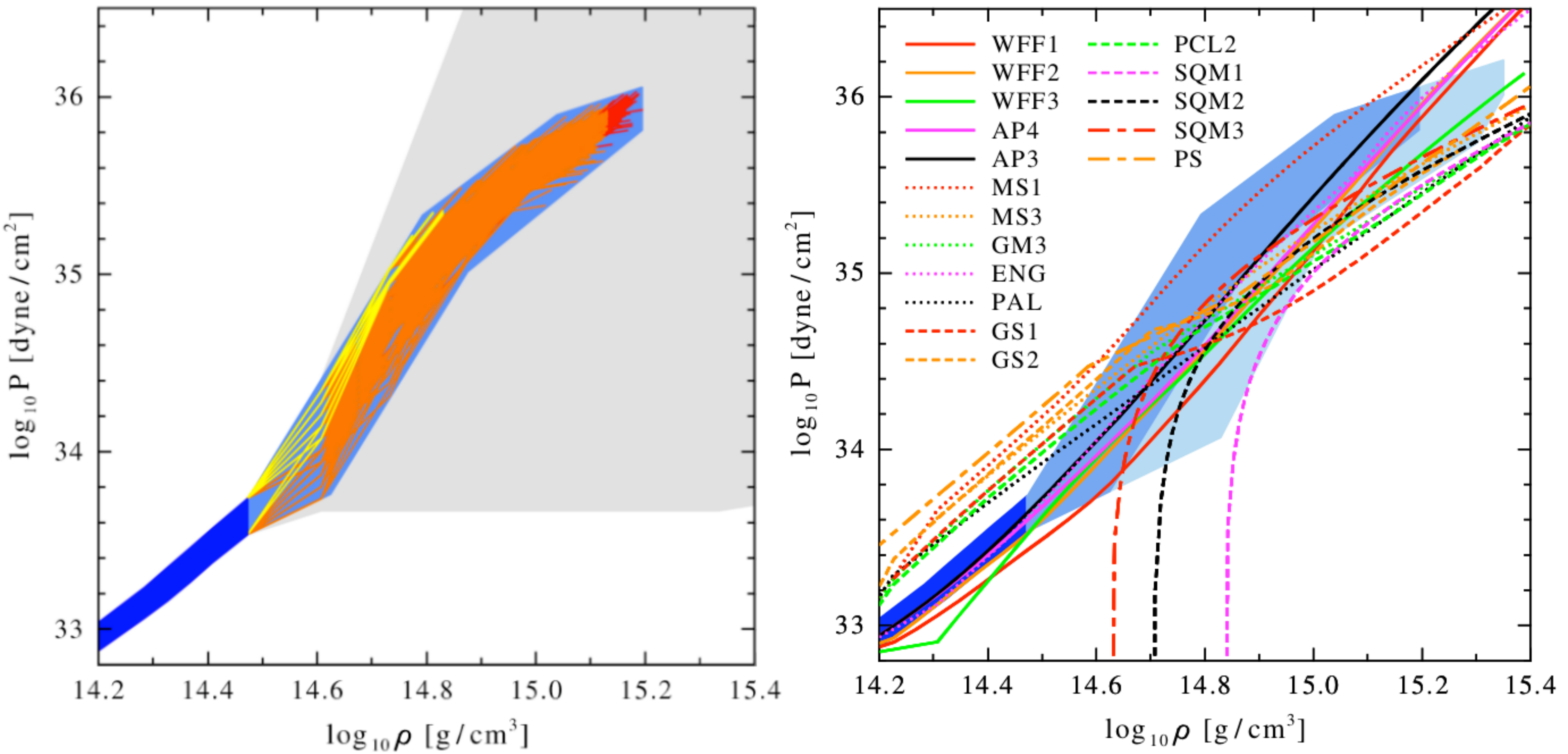
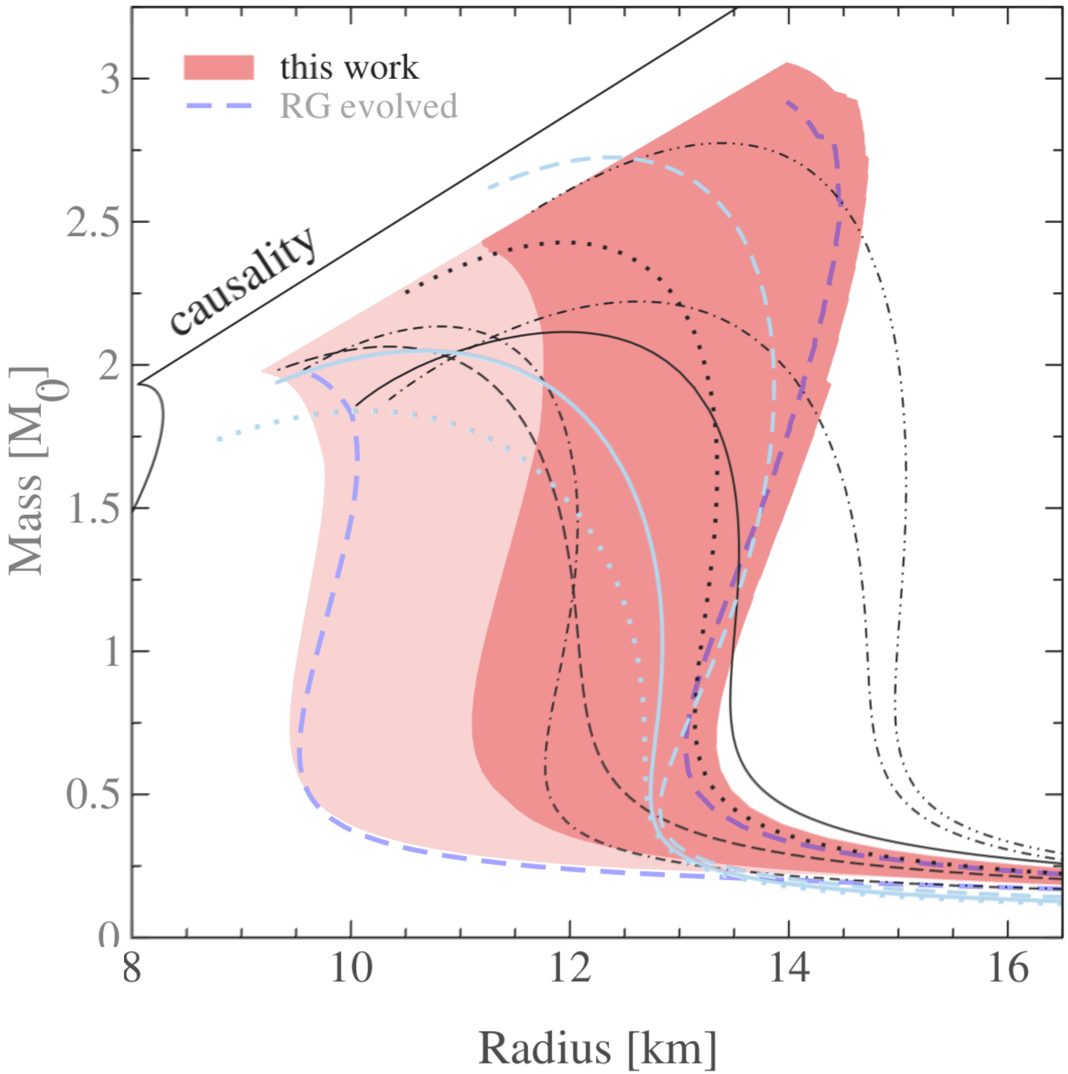
If a 2.4 $M_\odot$ neutron star was observed:



Hebeler et al., PRL (2010) and APJ (2013)

Neutron Stars



Radius for 1.4 $M_\odot$ neutron star:

- $R = 11.5 - 13.9$ km

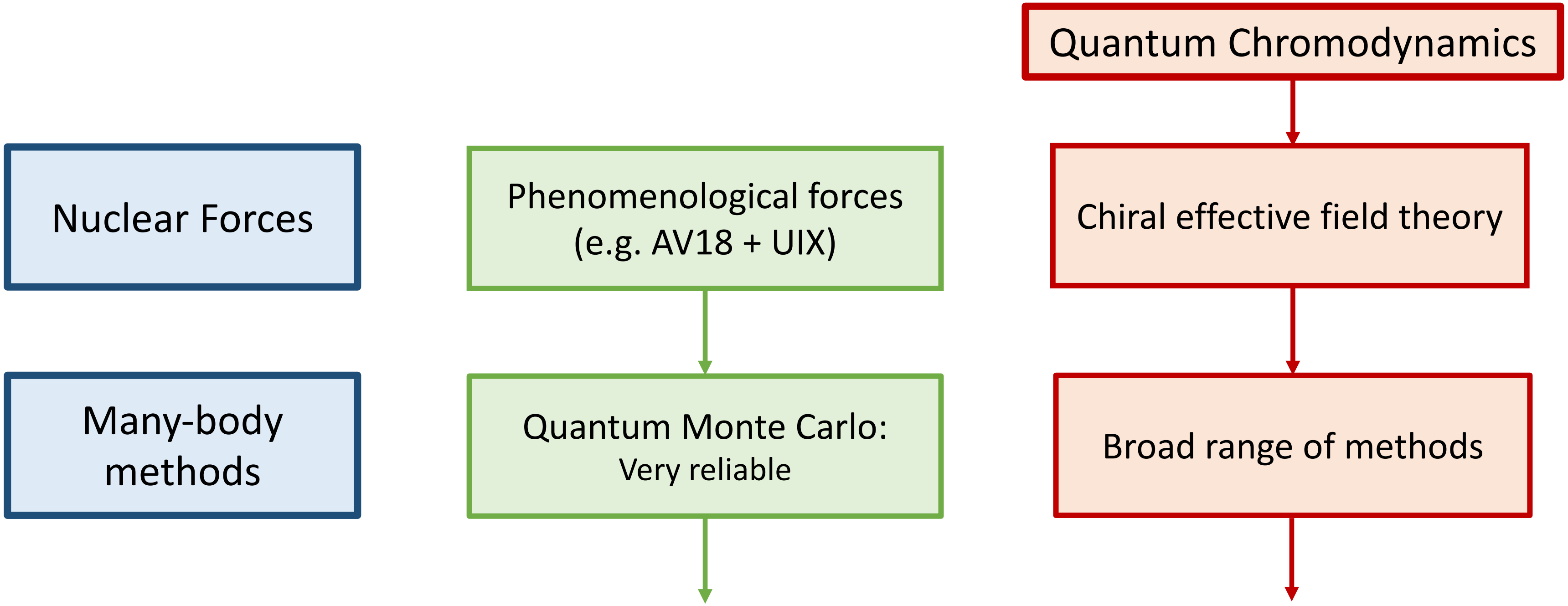
Maximum mass neutron star:

- $M_{max} \leq 3.05 M_\odot$ (14 km)

Uncertainties from many-body forces and polytropic expansion

IT, Krüger, Gezerlis, Hebeler, Schwenk (2013)

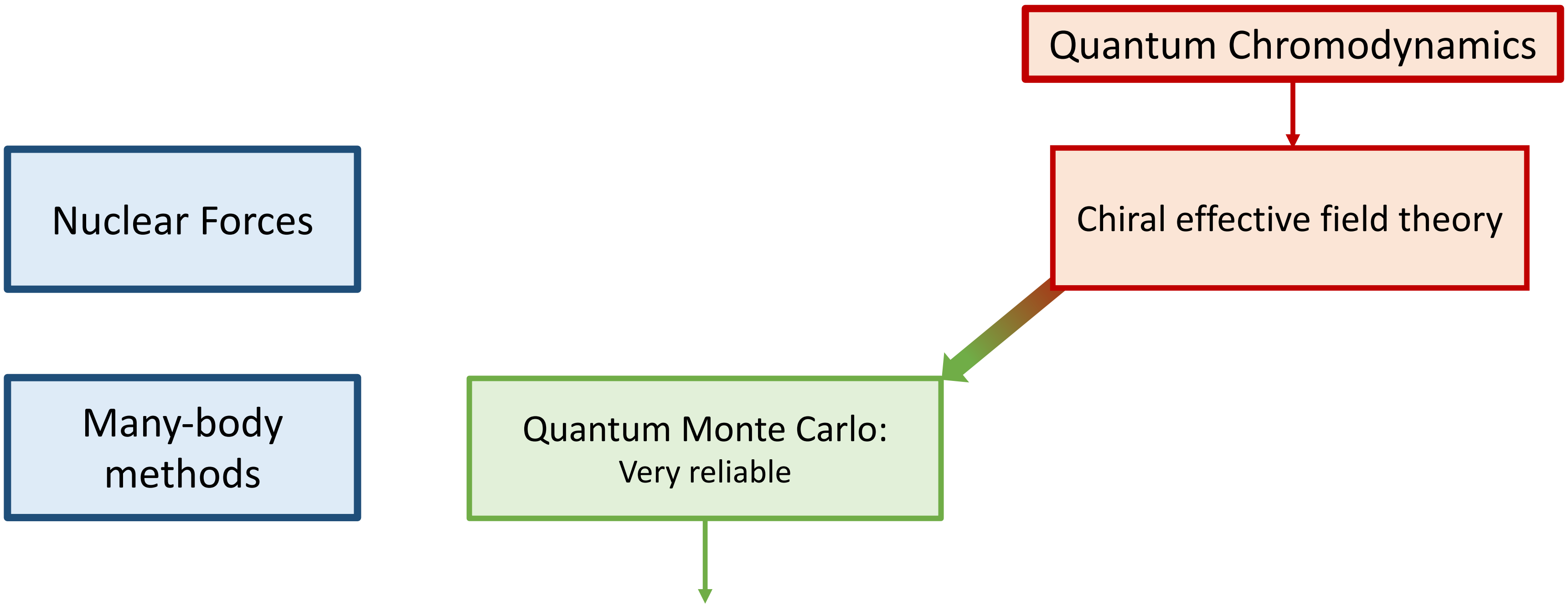
Improving neutron-matter band



Status:

- Sizeable uncertainty for chiral EFT calculations of neutron matter
- Phenomenological interactions provide a good description of light nuclei and nulear matter
- But it is not clear how to systematically improve their quality
- No systematic uncertainty estimates

Improving neutron-matter band



Status:

- Sizeable uncertainty for chiral EFT calculations of neutron matter

Goal: Combine QMC methods and chiral EFT

- Minimize uncertainty to enable precision studies of nuclear matter
- Check convergence of MBPT calculations and other approaches

Improving neutron-matter band

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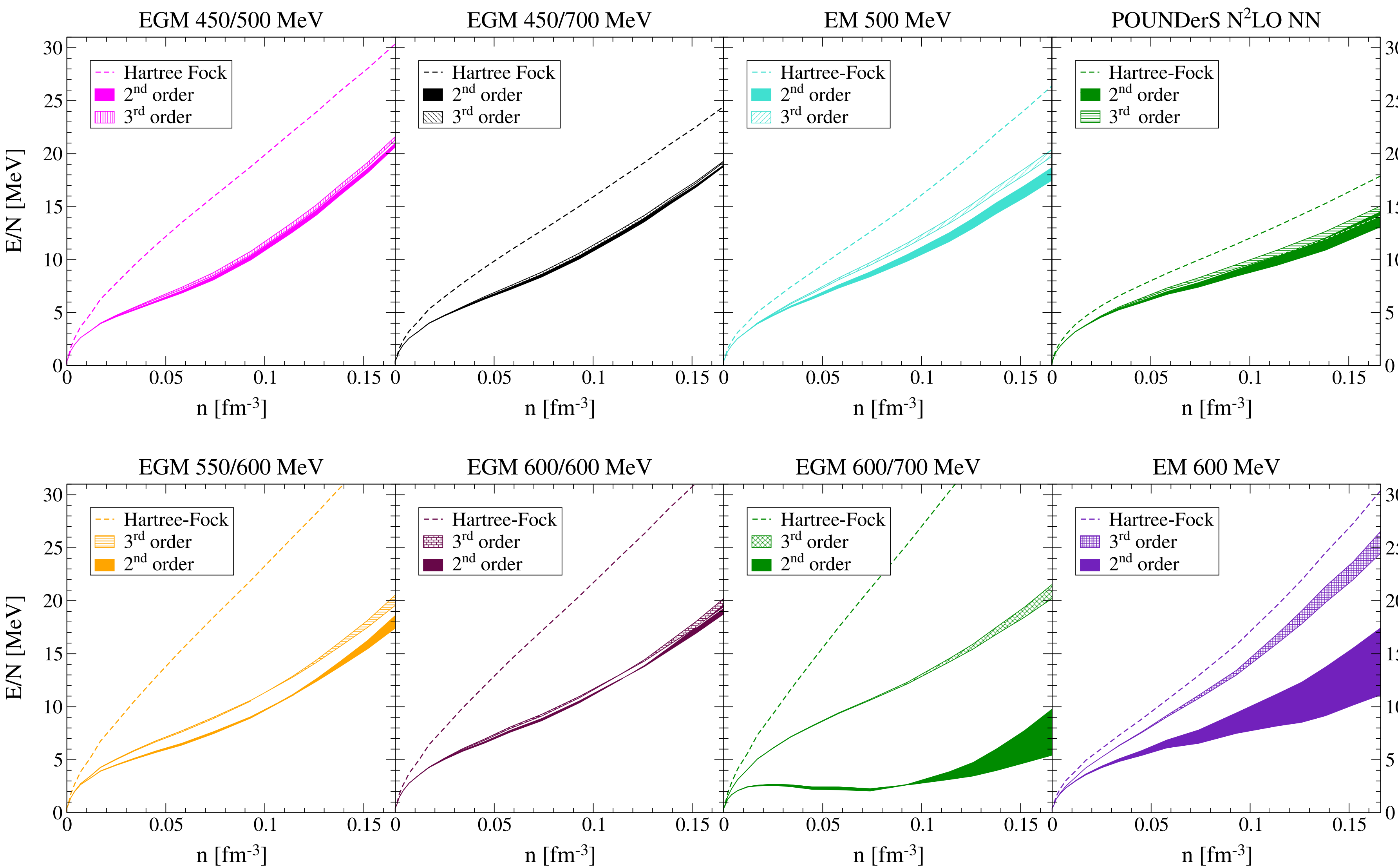


Quantum Monte Carlo method

Solve the many-body Schrödinger equation

$$H\left|\psi\right\rangle = -\frac{\partial}{\partial\tau}\left|\psi\right\rangle, \qquad \tau = it$$

$$\psi(R,\tau) = \int dR'^{3N} \langle R| e^{-(T+V)\tau} |R'\rangle \psi(R',0)$$

Basic steps:

- Choose trial wavefunction which overlaps with the ground state

$$|\psi(R,0)\rangle = |\psi_T(R,0)\rangle = \sum_i c_i |\phi_i\rangle \rightarrow \sum_i c_i e^{-(E_i-E_0)\tau} |\phi_i\rangle$$

- Evaluate propagator for small timestep $\Delta\tau$, feasible only for local potentials
- Make consecutive small time steps using Monte Carlo techniques to project out ground state

$$|\psi(R,\tau)\rangle \rightarrow |\phi_0\rangle \quad \text{for} \quad \tau \rightarrow \infty$$

More details:
Carlson, Gandolfi, Pederiva, Pieper, Schiavilla, Schmidt, Wiringa, RMP (2015)

Quantum Monte Carlo method

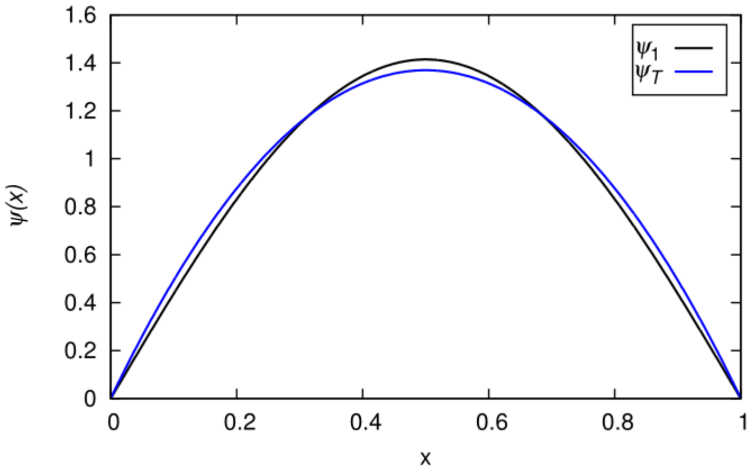
Particle in a 1D box, solution:

$$\psi_n(x) = \sqrt{2}\sin(n\pi x), \quad E_n = \frac{n^2\pi^2}{2}$$

Basic steps:

- Choose parabolic trial wavefunction which overlaps with the ground state

Animation by Joel Lynn, TU Darmstadt



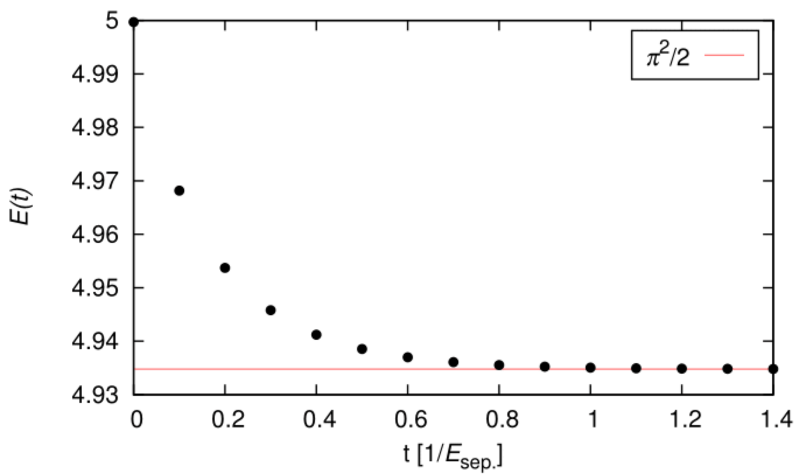
Quantum Monte Carlo method

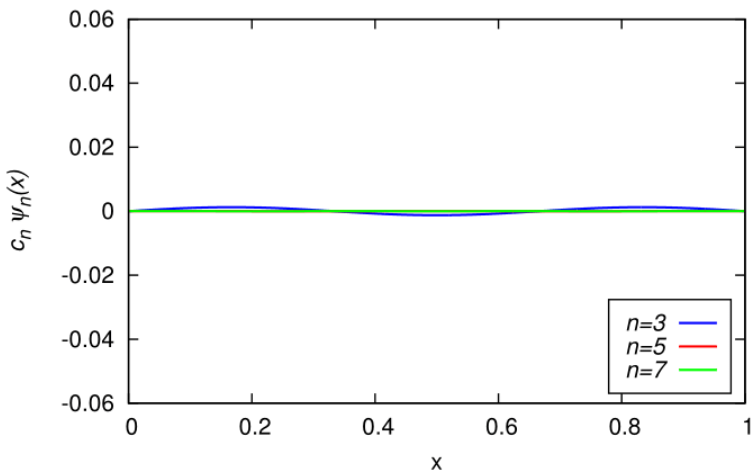
Particle in a 1D box, solution:

$$\psi_n(x) = \sqrt{2}\sin(n\pi x), \quad E_n = \frac{n^2\pi^2}{2}$$

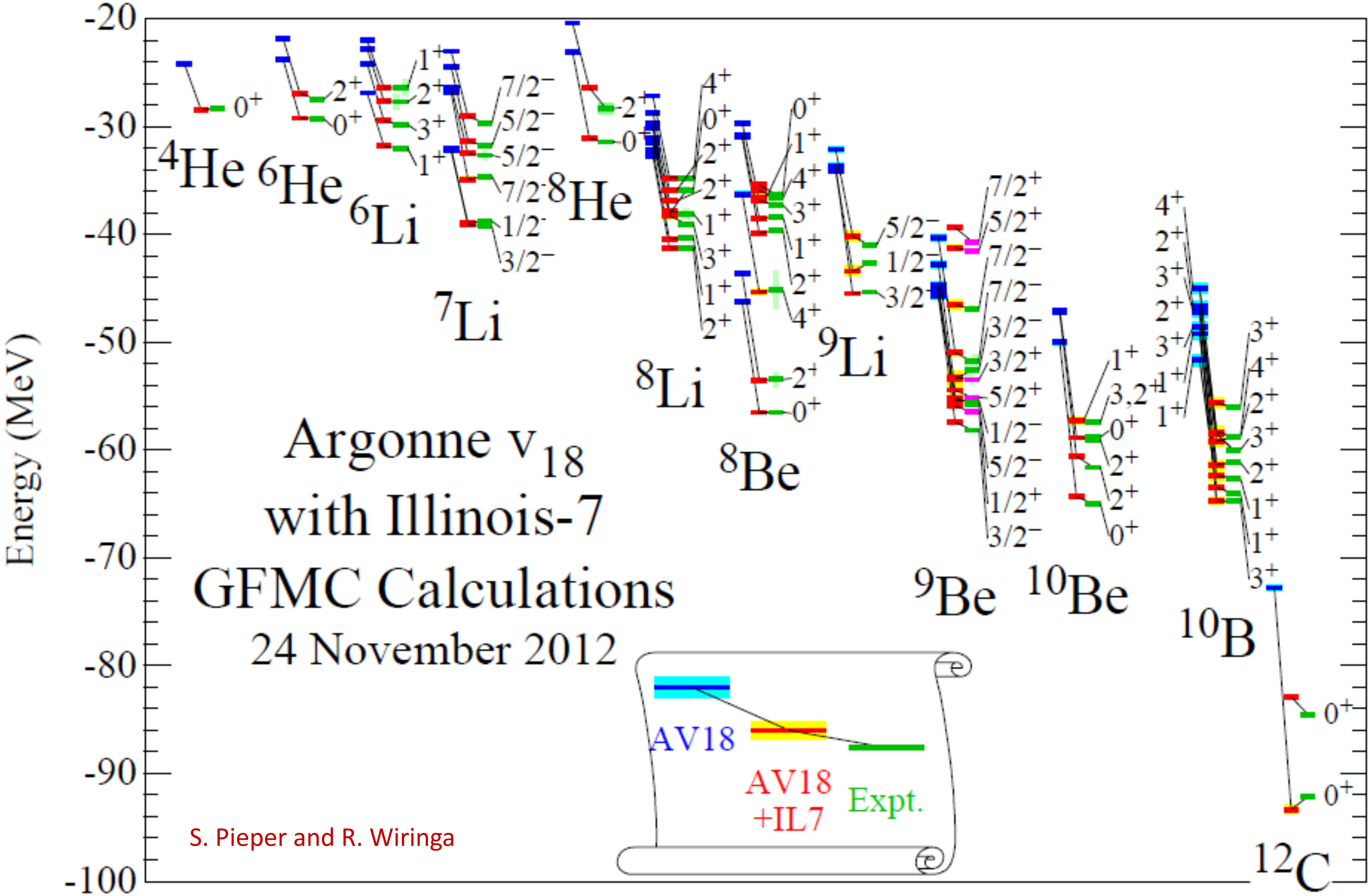
- Make consecutive small timesteps, $\tau = 1.4\left(\frac{1}{E_{\rm sep}}\right)$

  Animation by Joel Lynn, TU Darmstadt





Quantum Monte Carlo method



Local chiral interactions

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Quantum Chromodynamics

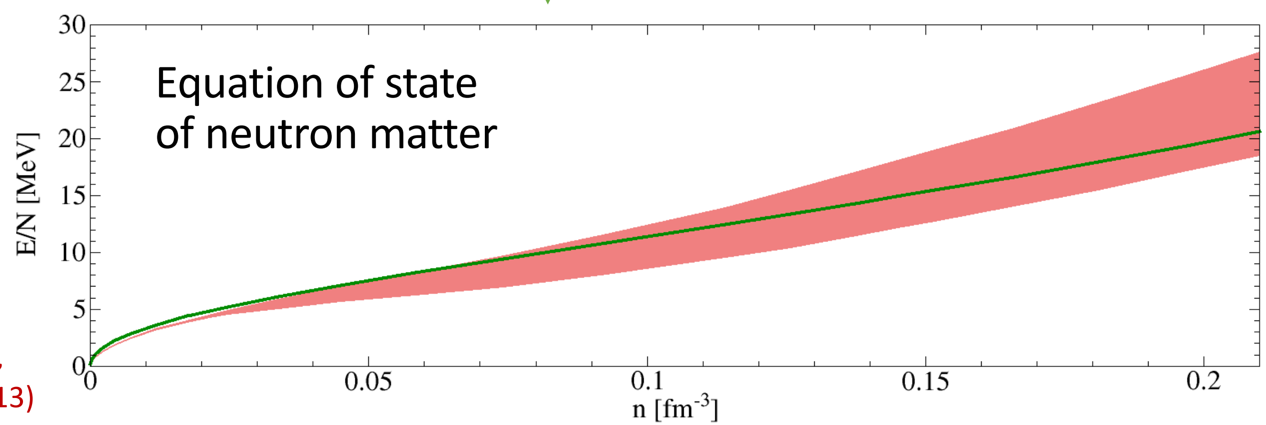
Nuclear Forces

Chiral effective field theory:
Systematic, generally nonlocal

Many-body methods

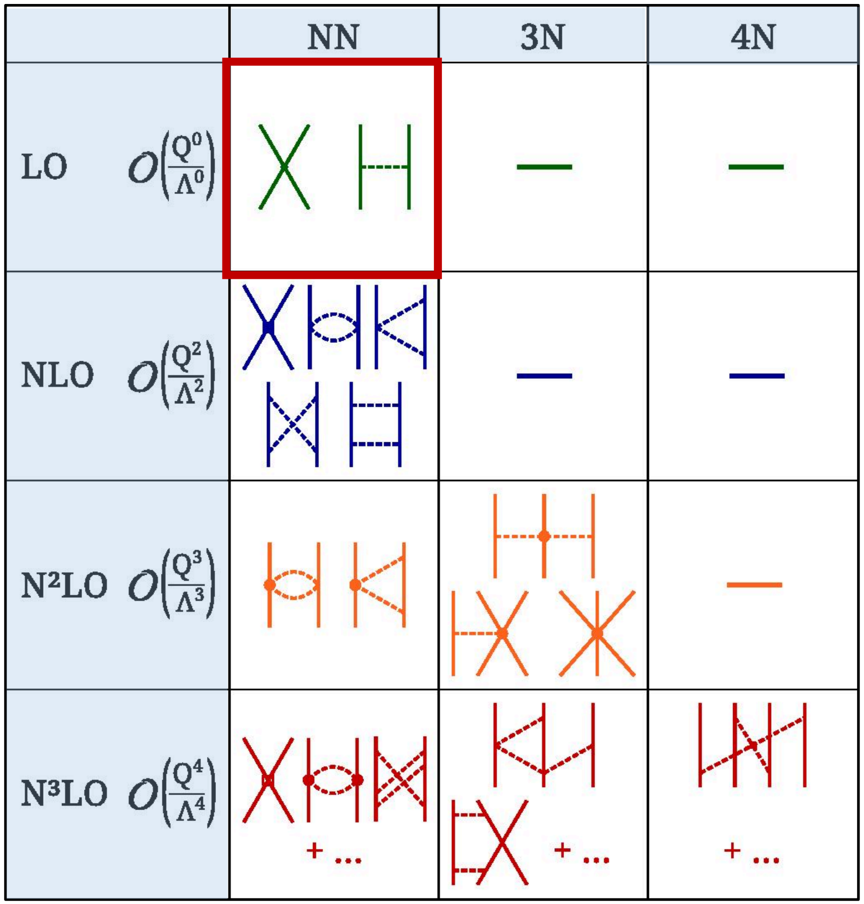
Quantum Monte Carlo:
Precise, needs local interactions

Problems:
1) Regulator
→ choose local regulator
2) Contact operators
→ use Fierz freedom



Krüger, IT, Hebeler, Schwenk, PRC (2013)

Local chiral interactions



Weinberg, van Kolck, Kaplan, Savage, Wise,
Epelbaum, Kaiser, Machleidt, Meißner, Hammer ...

- Leading order $V^{(0)} = V_{\text{cont}}^{(0)} + V^{\text{OPE}}$
- Pion exchange local → local regulator

$$\boxed{f_{\text{long}}(r) = 1 - \exp(-r^4/R_0^4)}$$

- Contact potential:

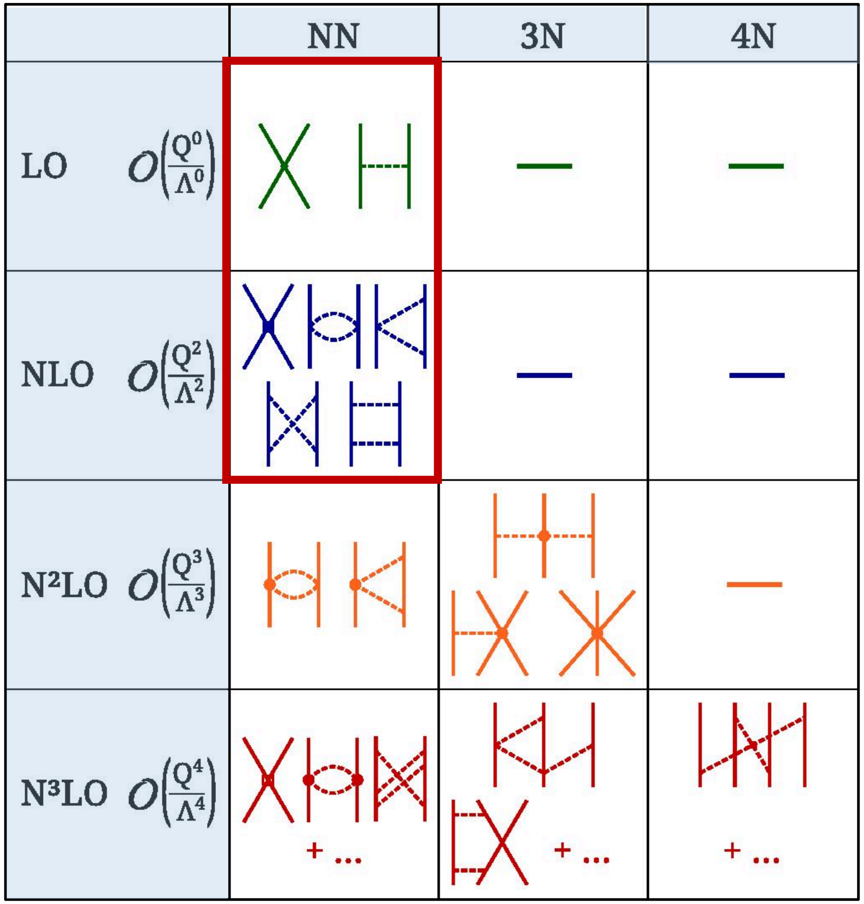
$$V_{\text{cont}}^{(0)} = \alpha_1 \mathbf{1} + \alpha_2 \sigma_1 \cdot \sigma_2 + \alpha_3 \tau_1 \cdot \tau_2 + \alpha_4 \sigma_1 \cdot \sigma_2 \tau_1 \cdot \tau_2$$

→ Only two independent (Pauli principle)

$$V_{\text{cont}}^{(0)} = C_S \mathbf{1} + C_T \sigma_1 \cdot \sigma_2$$

$$\boxed{f_{\text{short}}(r) = \alpha \exp(-r^4/R_0^4)}$$

Local chiral interactions

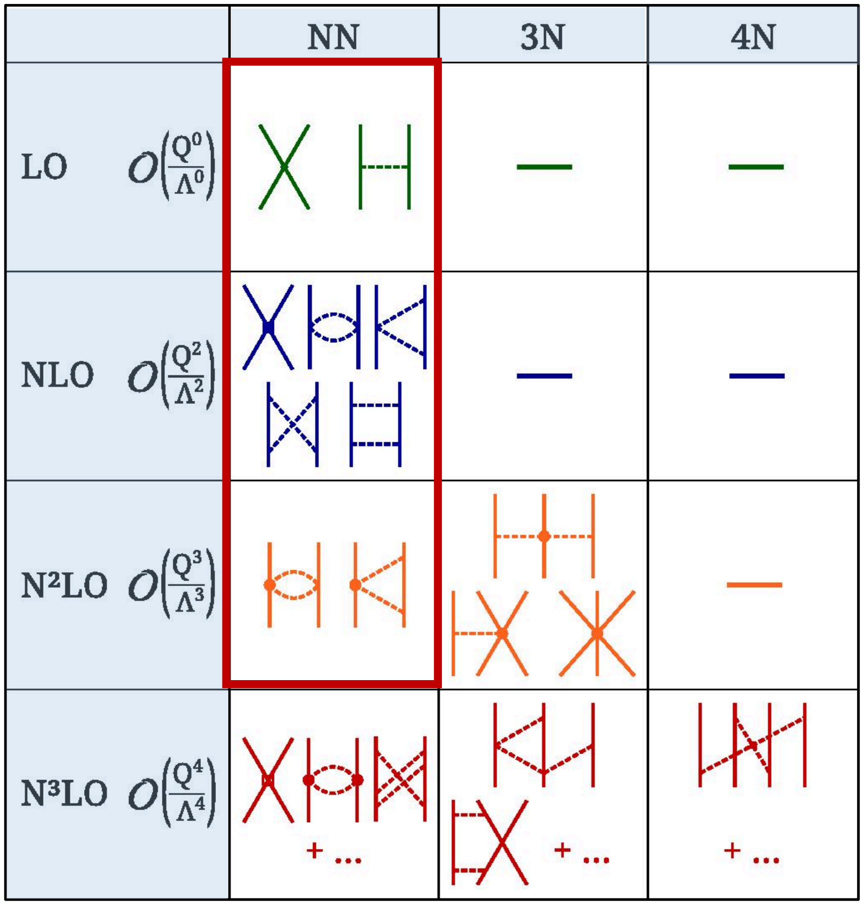


- Choose local set of short-range operators at NLO (7 out of 14)

$$
\begin{aligned}
V_{\text{cont}}^{(2)} = \; & \gamma_1 q^2 + \gamma_2 q^2 \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 + \gamma_3 q^2 \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 \\
& + \gamma_4 q^2 \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 \\
& + \gamma_5 k^2 + \gamma_6 k^2 \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 + \gamma_7 k^2 \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 \\
& + \gamma_8 k^2 \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 \\
& + \gamma_9 (\boldsymbol{\sigma}_1 + \boldsymbol{\sigma}_2)(\mathbf{q} \times \mathbf{k}) \\
& + \gamma_{10} (\boldsymbol{\sigma}_1 + \boldsymbol{\sigma}_2)(\mathbf{q} \times \mathbf{k}) \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 \\
& + \gamma_{11} (\boldsymbol{\sigma}_1 \cdot \mathbf{q})(\boldsymbol{\sigma}_2 \cdot \mathbf{q}) \\
& + \gamma_{12} (\boldsymbol{\sigma}_1 \cdot \mathbf{q})(\boldsymbol{\sigma}_2 \cdot \mathbf{q}) \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 \\
& + \gamma_{13} (\boldsymbol{\sigma}_1 \cdot \mathbf{k})(\boldsymbol{\sigma}_2 \cdot \mathbf{k}) \\
& + \gamma_{14} (\boldsymbol{\sigma}_1 \cdot \mathbf{k})(\boldsymbol{\sigma}_2 \cdot \mathbf{k}) \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 \,.
\end{aligned}
$$

Weinberg, van Kolck, Kaplan, Savage, Wise,
Epelbaum, Kaiser, Machleidt, Meißner, Hammer ...

Local chiral interactions



- Choose local set of short-range operators at NLO (7 out of 14)
- Pion exchanges up to N²LO are local
- This freedom can be used to remove all nonlocal operators up to N²LO

  Gezerlis, IT, Epelbaum, Gandolfi, Hebeler, Nogga, Schwenk, PRL (2013)
  Gezerlis, IT, Epelbaum, Freunek, Gandolfi, Hebeler, Nogga, Schwenk, PRC (2014)
- LECs fit to phase shifts

Weinberg, van Kolck, Kaplan, Savage, Wise,
Epelbaum, Kaiser, Machleidt, Meißner, Hammer ...

Phaseshifts for local potentials

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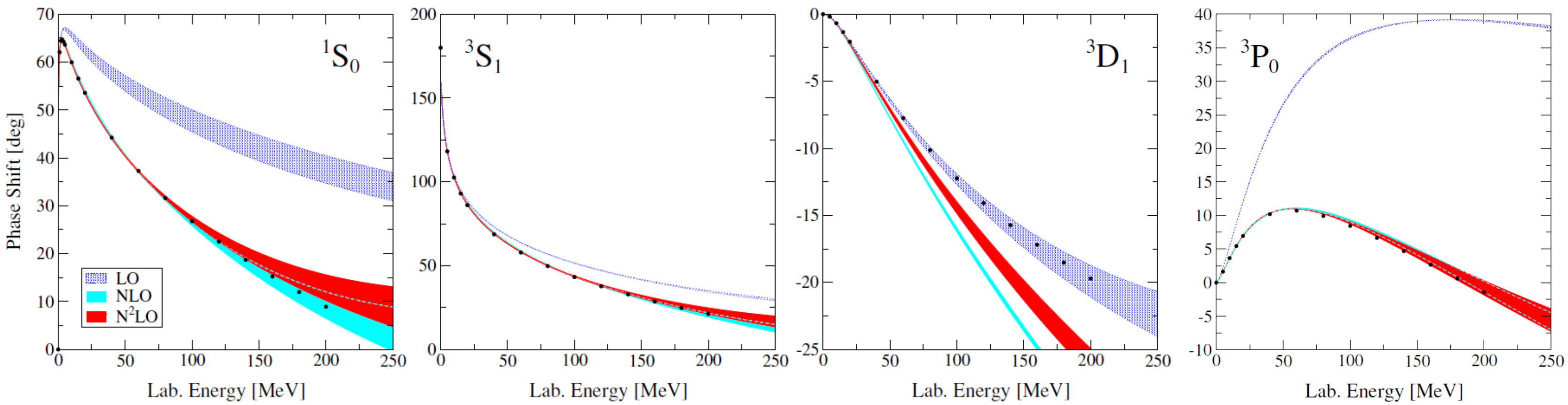
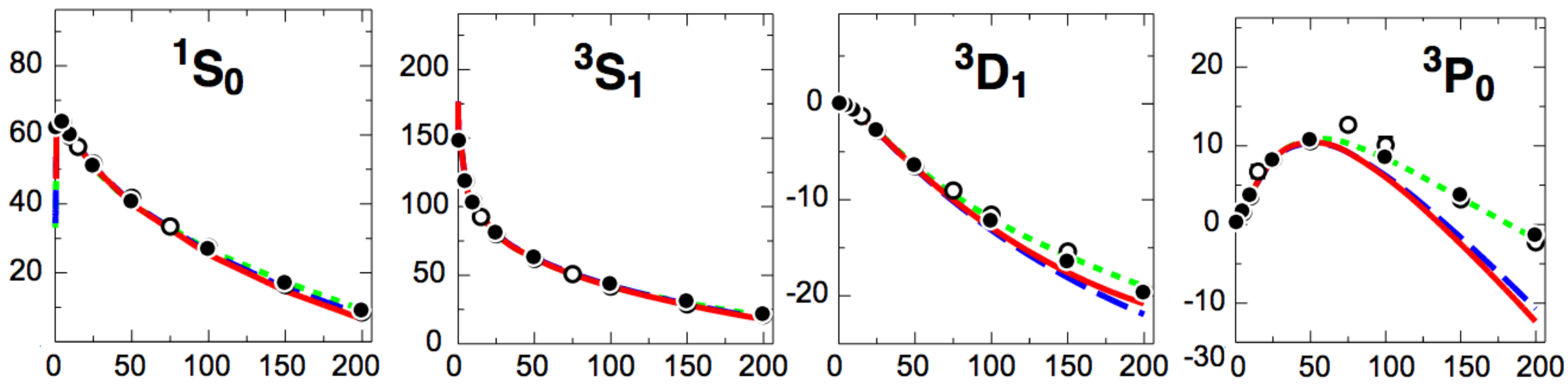


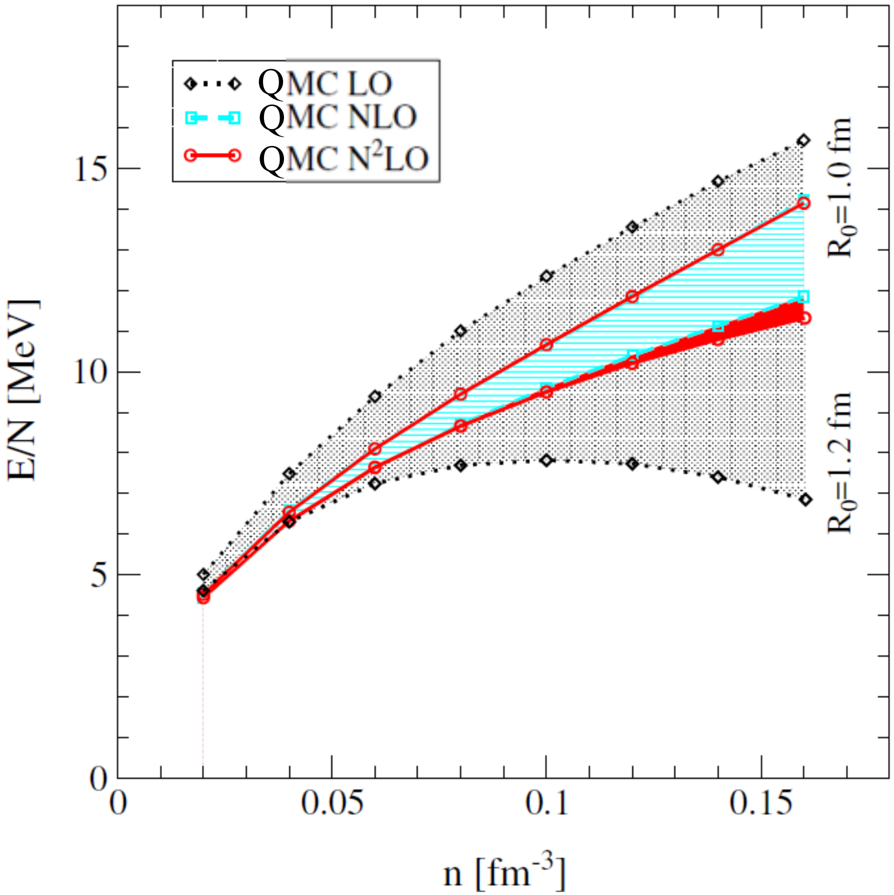
Gezerlis, IT, Epelbaum, Freunek, Gandolfi, Hebeler, Nogga, Schwenk, PRC (2014)

Compare to NNLO$_{opt}$:



Ekström et al., PRL (2013)

QMC results for NN forces

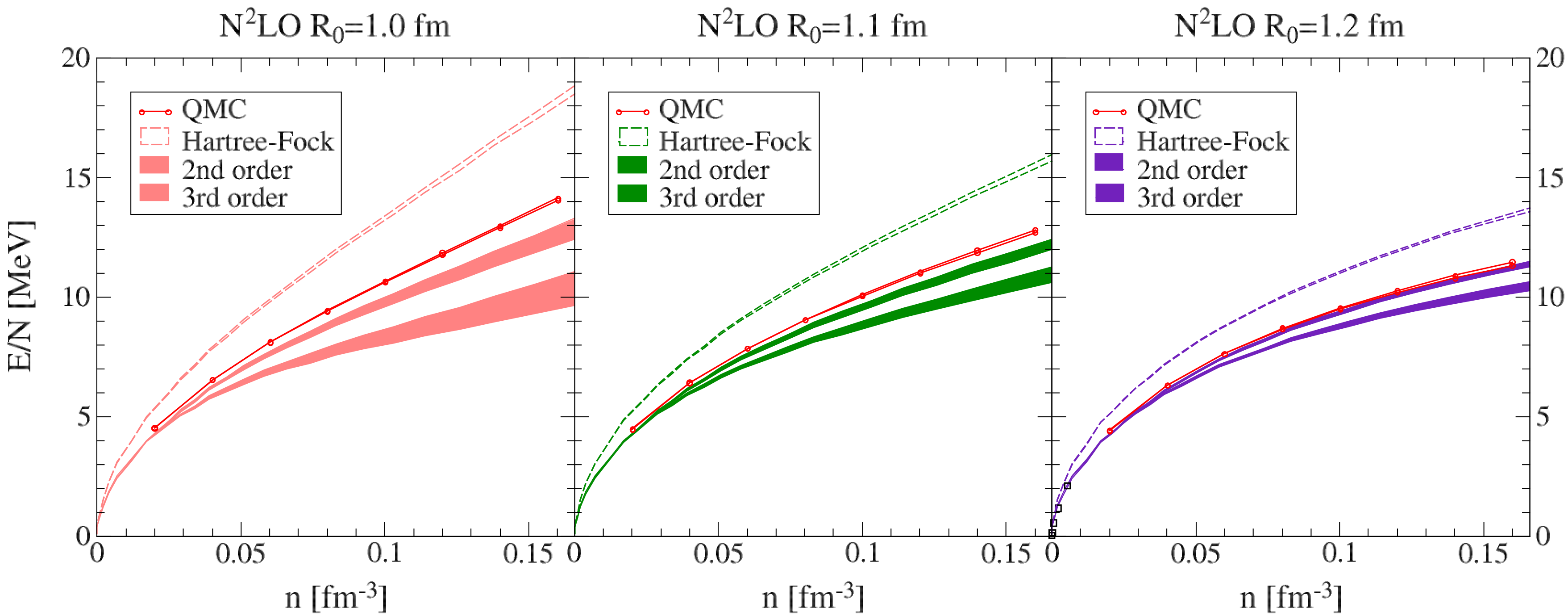


Gezerlis, IT, Epelbaum, Freunek, Gandolfi, Hebeler, Nogga, Schwenk, PRL (2013) and PRC (2014)

NN-only calculation:

- QMC:
  Statistical uncertainty of points negligible
- Bands include NN cutoff variation
  $R_0 = 1.0 - 1.2$ fm
- Order-by-order convergence up to saturation density

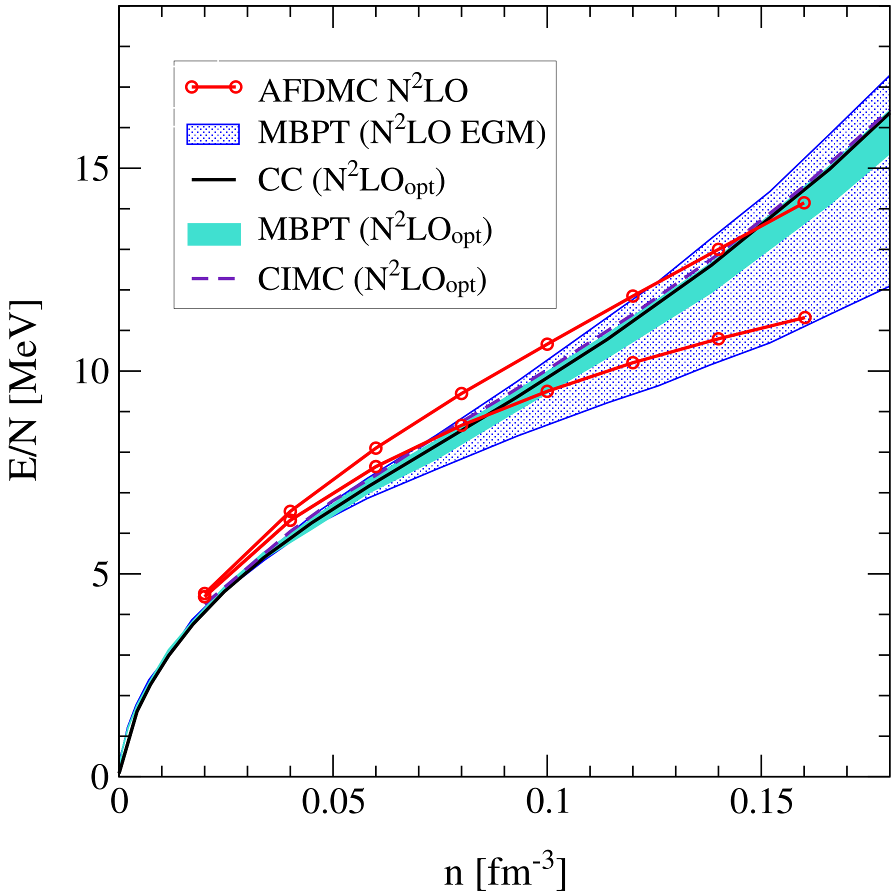
Benchmark of MBPT



Gezerlis, IT, Epelbaum, Freunek, Gandolfi, Hebeler, Nogga, Schwenk, PRC (2014)

Many-body perturbation theory:
- Excellent agreement with QMC for soft potentials ($R_0 = 1.2$ fm)
- Validates perturbative calculations for those interactions

QMC results for NN forces



Gezerlis, IT, Epelbaum, Freunek, Gandolfi, Hebeler, Nogga, Schwenk, PRC (2014)

NN-only calculation

- Good agreement with other approaches:

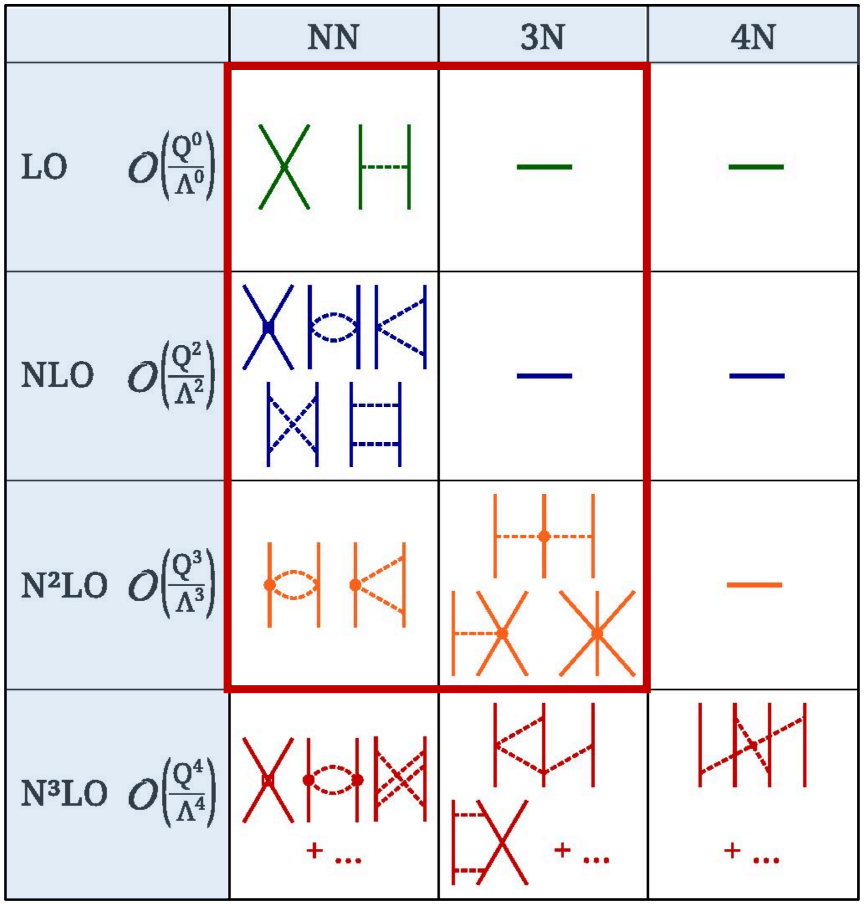
MBPT with N^{2}LO EGM
IT, Krüger, Hebeler, Schwenk, PRL (2013)

CC with N^{2}LO$_{opt}$
Hagen, Papenbrock, Ekström, Wendt, Baardsen, Gandolfi, Hjorth-Jensen, Horowitz, PRC (2013)

MBPT with N^{2}LO$_{opt}$
IT, Krüger, Gezerlis, Hebeler, Schwenk, NTSE (2013)

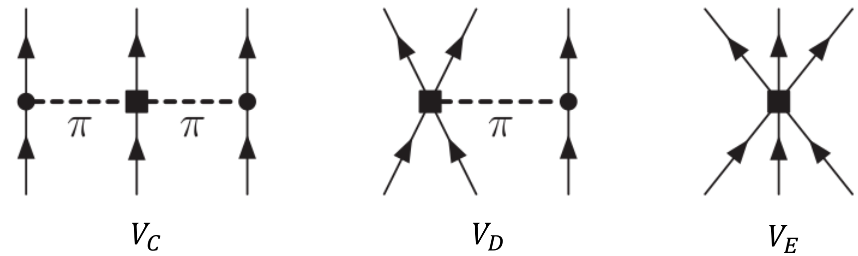
CIMC with N^{2}LO$_{opt}$
Roggero, Mukherjee, Pederiva, PRL (2014)

QMC with chiral 3N forces



Weinberg, van Kolck, Kaplan, Savage, Wise,
Epelbaum, Kaiser, Machleidt, Meißner, Hammer ...

Next: inclusion of leading 3N forces

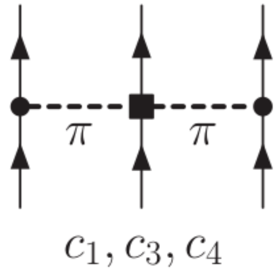


Three topologies:

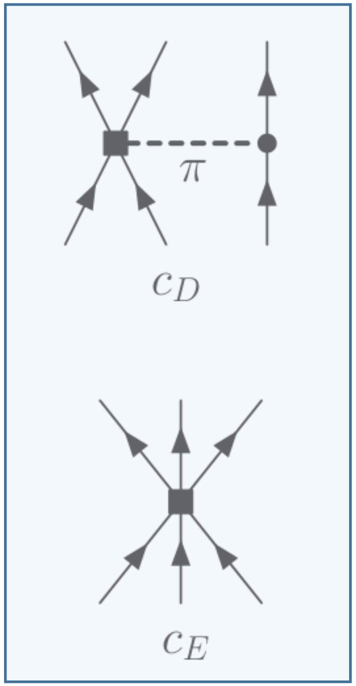
- Two-pion exchange V_C
- One-pion-exchange contact V_D
- Three-nucleon contact V_E

Only two new couplings: c_D and c_E

QMC with chiral 3N forces



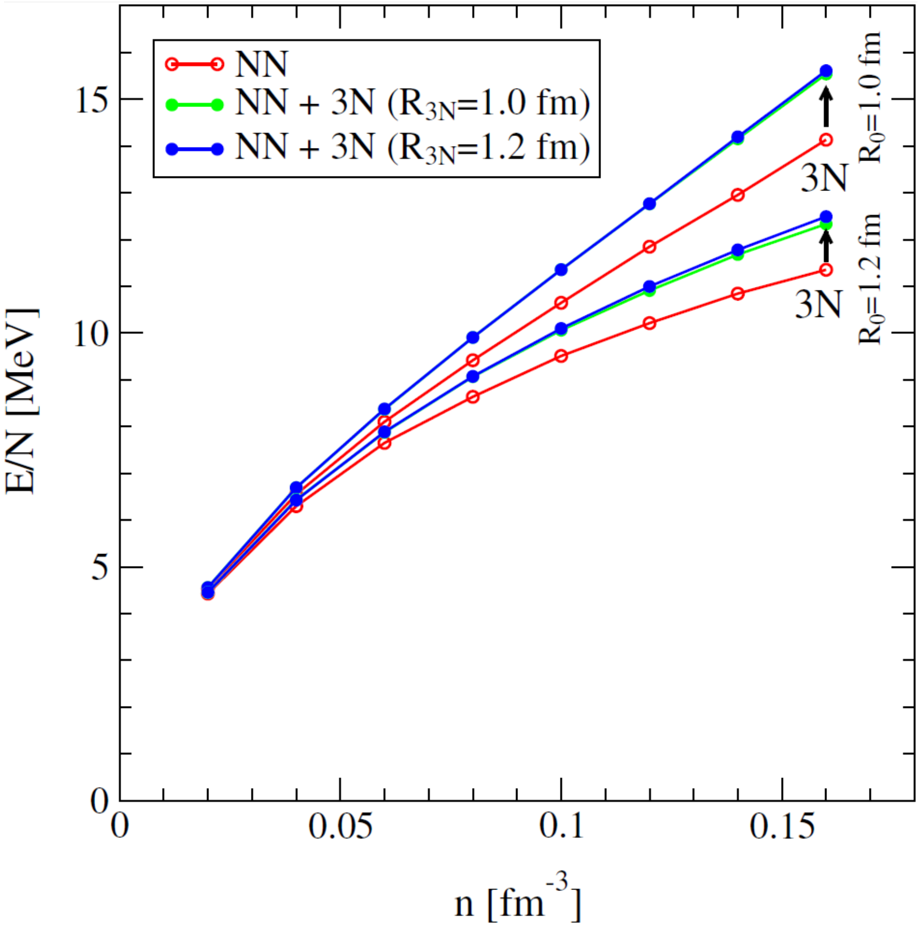
c_1 term: Tucson-Melbourn S-wave interaction
$c_{3,4}$ term: Fujita-Miyazawa interaction



Two-pion-exchange most important in PNM,
usually V_D and V_E vanish in neutron matter:
c_D due to spin-isospin structure,
c_E due to Pauli principle
see also Hebeler, Schwenk, PRC (2010)

Only true for regulator symmetric in particle labels like commonly used nonlocal regulators, not for local regulators

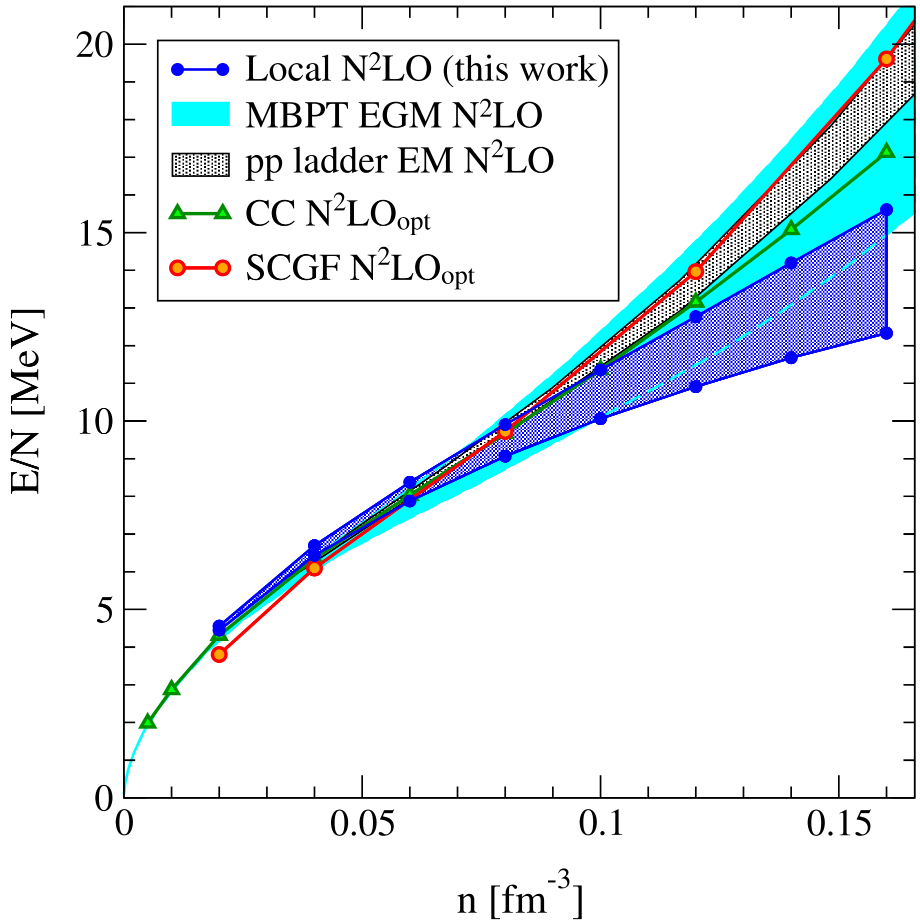
QMC results with 3N TPE



IT, Gandolfi, Gezerlis, Schwenk, PRC (2016)

- Only three-nucleon two-pion exchange $\sim c_1$ and c_3
- Auxiliary-field diffusion Monte Carlo:
  - NN + 3N TPE forces
  - $R_0 = 1.0 - 1.2$ fm
  - $R_{3N} = 1.0 - 1.2$ fm
- 3N cutoff dependence small
- TPE 3N contributions ≈ 1 – 2 MeV at n_0

QMC results with 3N TPE



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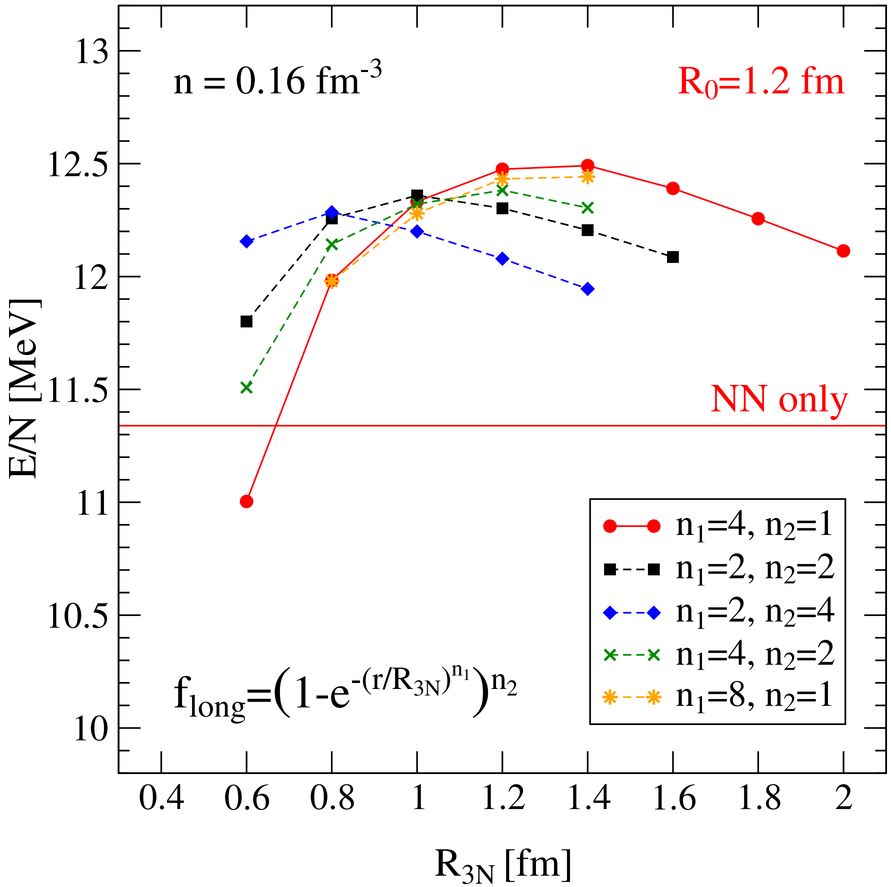
- smaller than for nonlocal regulators

QMC results with 3N TPE



IT, Gandolfi, Gezerlis, Schwenk, PRC (2016)

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  - NN + 3N TPE forces
  - $R_0 = 1.0 - 1.2$ fm
  - $R_{3N} = 1.0 - 1.2$ fm
- 3N cutoff dependence small
- TPE 3N contributions ≈ 1 – 2 MeV at n_0
- smaller than for nonlocal regulators
- Independent of exact regulator form

Local NN forces in HF

- Example: two-body regulators at Hartree-Fock:

$$f_{\text{reg}}^{MSL} = \exp\left(-\left(\frac{q}{\Lambda}\right)^{2n}\right), \qquad f_{\text{reg}}^{MSNL} = \exp\left(-2\left(\frac{p}{\Lambda}\right)^{2n}\right)$$

- After antisymmetrization we have a direct and an exchange term.

Direct term:

$$q = p - p' = 0 \;\rightarrow\; f_{\text{reg}}^{MSL} = 1, \qquad f_{\text{reg}}^{MSNL} = \exp\left(-2\left(\frac{p}{\Lambda}\right)^{2n}\right)$$

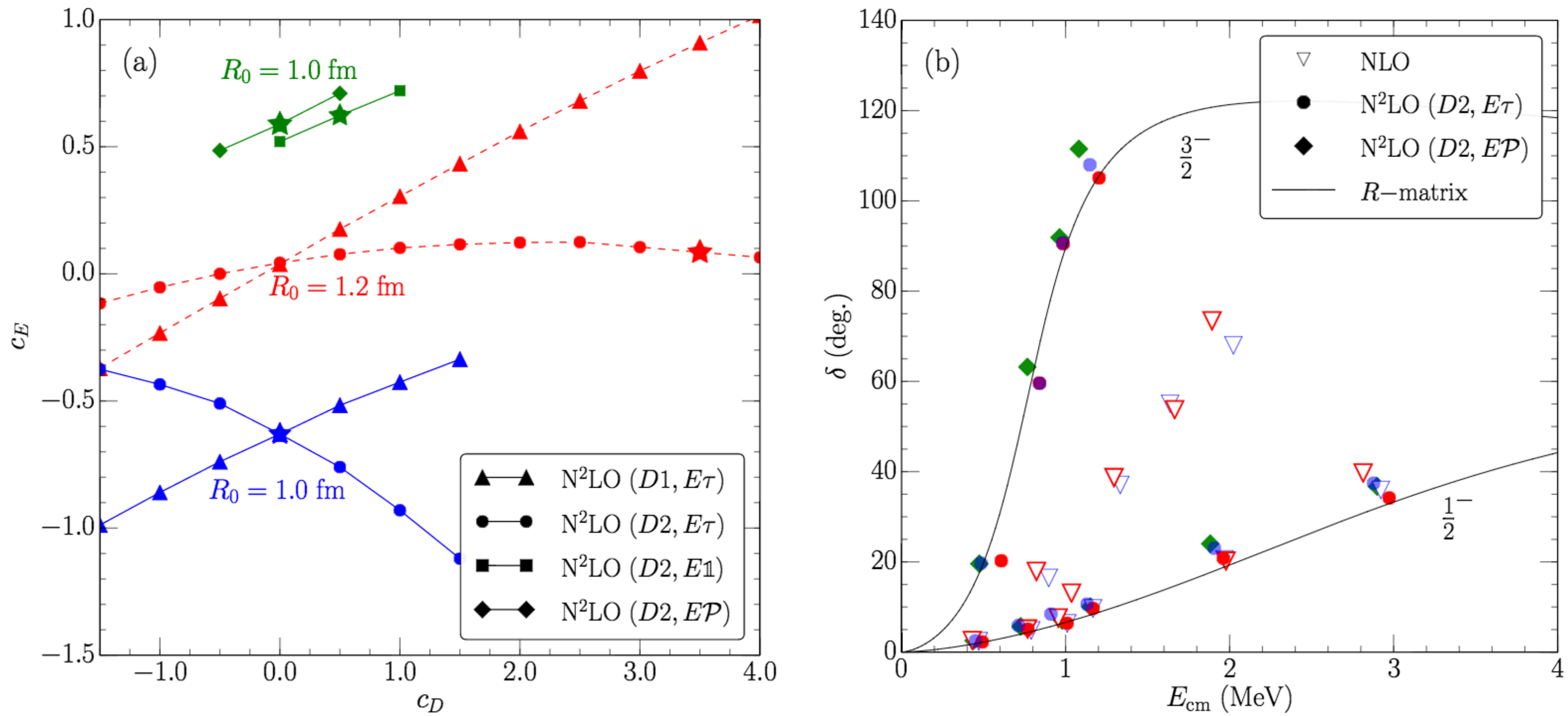
Exchange term:

$$q = p - p' = 2p \;\rightarrow\; f_{\text{reg}}^{MSL} = \exp\left(-\left(\frac{2p}{\Lambda}\right)^{2n}\right), f_{\text{reg}}^{MSNL} = \exp\left(-2\left(\frac{p}{\Lambda}\right)^{2n}\right)$$

- Spin-dependent interactions at Hartree-Fock: only exchange term survives
- Effective cutoff smaller for local regulators!

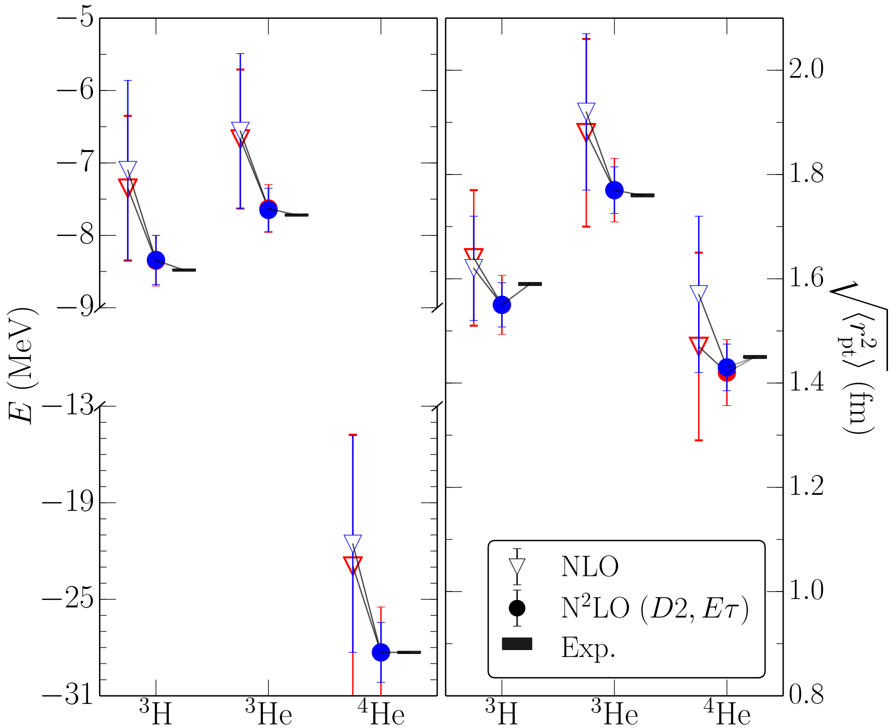
Fits of 3N LECs

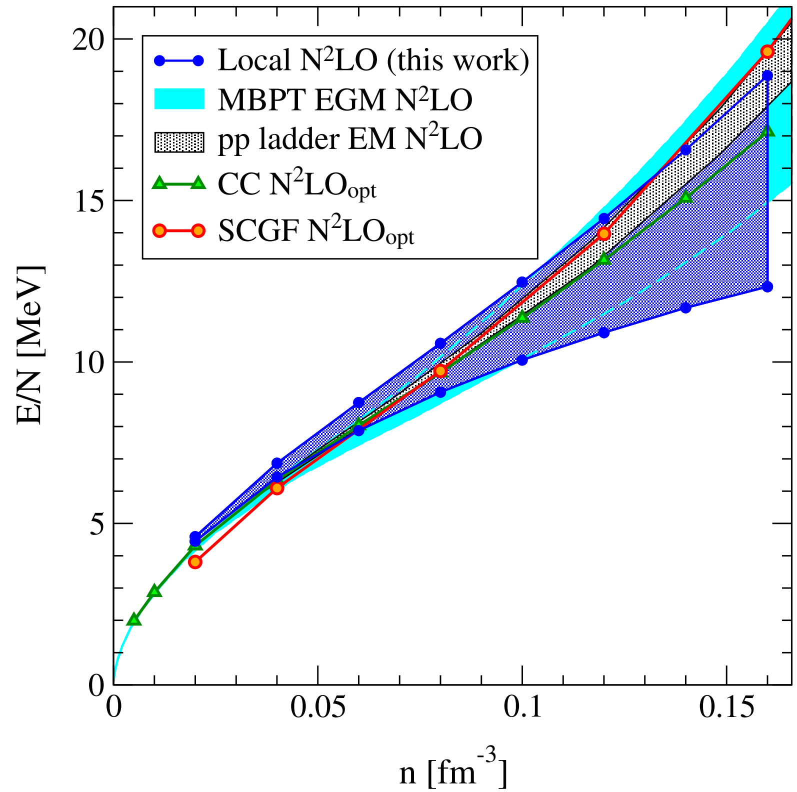
- Fit c_E and c_D to ^{4}He binding energy and n-α scattering



Lynn, IT, Carlson, Gandolfi, Gezerlis, Schmidt, Schwenk, PRL (2016)

Results





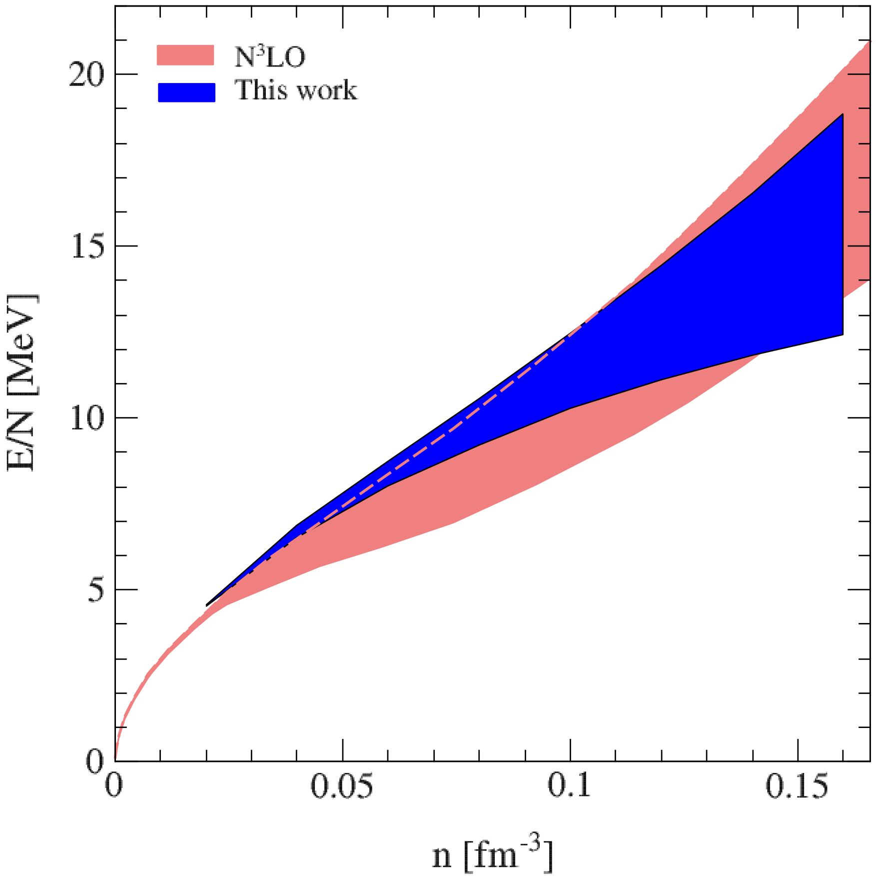
Lynn, IT, Carlson, Gandolfi, Gezerlis, Schmidt, Schwenk, PRL (2016)

- Chiral interactions at N^{2}LO simultaneously reproduce the properties of A=3, 4, 5 systems and of neutron matter
- Commonly used phenomenological 3N interactions fail for neutron matter

  Sarsa, Fantoni, Schmidt, Pederiva, PRC (2003)

Results

Comparing to N^3LO calculation:



IT, Krüger, Hebeler, Schwenk, PRL (2013)
Lynn, IT, Carlson, Gandolfi, Gezerlis, Schmidt, Schwenk, PRL (2016)

Chiral EFT forces with the Quantum Monte Carlo method:

- Energies agree well with MBPT result within uncertainty bands
- Many-body uncertainty negligible
- uncertainties comparable but QMC band only at N^2LO and includes also hard interactions

- Improve local chiral interactions:
  - Develop N^3LO potentials

Next step: N^{3}LO

Improve local chiral interactions:

- Develop maximally local N^{3}LO potentials
- Inclusion of Delta degree of freedom

- Problem: only 8 out of 30 possible operators local

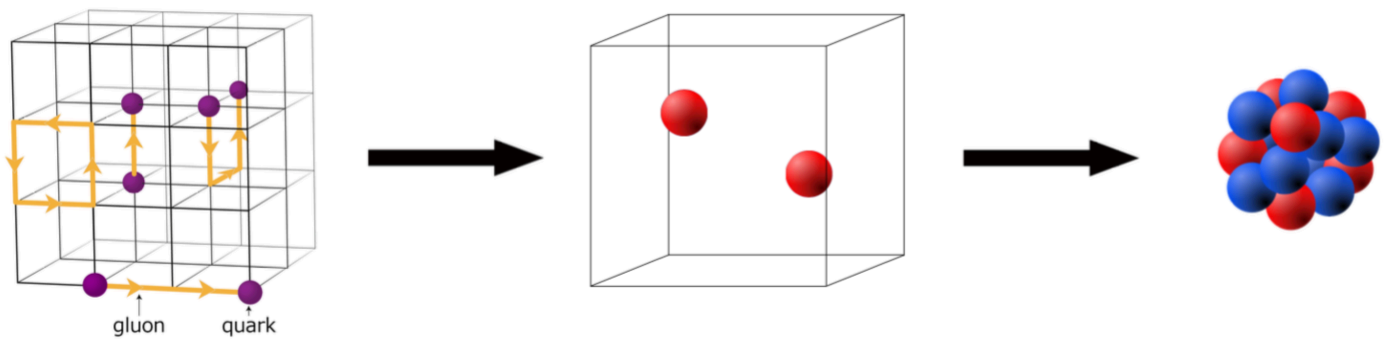
$$
\begin{aligned}
V_{\text{cont}}^{(4)} = \quad & D_1\, \boldsymbol{q}^4 + D_2\, \boldsymbol{q}^4 \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 + D_3\, \boldsymbol{q}^4 \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 + D_4\, \boldsymbol{q}^4 \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 \\
& + D_5\, \boldsymbol{k}^4 + D_6\, \boldsymbol{k}^4 \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 + D_7\, \boldsymbol{k}^4 \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 + D_8\, \boldsymbol{k}^4 \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 \\
& + D_9\, \boldsymbol{q}^2 \boldsymbol{k}^2 + D_{10}\, \boldsymbol{q}^2 \boldsymbol{k}^2 \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 + D_{11}\, \boldsymbol{q}^2 \boldsymbol{k}^2 \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 + D_{12}\, \boldsymbol{q}^2 \boldsymbol{k}^2 \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 \\
& + D_{13}\, (\boldsymbol{q} \times \boldsymbol{k})^2 + D_{14}\, (\boldsymbol{q} \times \boldsymbol{k})^2 \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 + D_{15}\, (\boldsymbol{q} \times \boldsymbol{k})^2 \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 + D_{16}\, (\boldsymbol{q} \times \boldsymbol{k})^2 \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 \\
& + \frac{i}{2} D_{17}\, \boldsymbol{q}^2 (\boldsymbol{\sigma}_1 + \boldsymbol{\sigma}_2) \cdot (\boldsymbol{q} \times \boldsymbol{k}) + \frac{i}{2} D_{18}\, \boldsymbol{q}^2 (\boldsymbol{\sigma}_1 + \boldsymbol{\sigma}_2) \cdot (\boldsymbol{q} \times \boldsymbol{k}) \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 \\
& + \frac{i}{2} D_{19}\, \boldsymbol{k}^2 (\boldsymbol{\sigma}_1 + \boldsymbol{\sigma}_2) \cdot (\boldsymbol{q} \times \boldsymbol{k}) + \frac{i}{2} D_{20}\, \boldsymbol{k}^2 (\boldsymbol{\sigma}_1 + \boldsymbol{\sigma}_2) \cdot (\boldsymbol{q} \times \boldsymbol{k}) \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 \\
& + D_{21}\, \boldsymbol{q}^2\, \boldsymbol{\sigma}_1 \cdot \boldsymbol{q}\, \boldsymbol{\sigma}_2 \cdot \boldsymbol{q} + D_{22}\, \boldsymbol{q}^2\, \boldsymbol{\sigma}_1 \cdot \boldsymbol{q}\, \boldsymbol{\sigma}_2 \cdot \boldsymbol{q}\, \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 \\
& + D_{23}\, \boldsymbol{k}^2\, \boldsymbol{\sigma}_1 \cdot \boldsymbol{q}\, \boldsymbol{\sigma}_2 \cdot \boldsymbol{q} + D_{24}\, \boldsymbol{k}^2\, \boldsymbol{\sigma}_1 \cdot \boldsymbol{q}\, \boldsymbol{\sigma}_2 \cdot \boldsymbol{q}\, \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 \\
& + D_{25}\, \boldsymbol{q}^2\, \boldsymbol{\sigma}_1 \cdot \boldsymbol{k}\, \boldsymbol{\sigma}_2 \cdot \boldsymbol{k} + D_{26}\, \boldsymbol{q}^2\, \boldsymbol{\sigma}_1 \cdot \boldsymbol{k}\, \boldsymbol{\sigma}_2 \cdot \boldsymbol{k}\, \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 \\
& + D_{27}\, \boldsymbol{k}^2\, \boldsymbol{\sigma}_1 \cdot \boldsymbol{k}\, \boldsymbol{\sigma}_2 \cdot \boldsymbol{k} + D_{28}\, \boldsymbol{k}^2\, \boldsymbol{\sigma}_1 \cdot \boldsymbol{k}\, \boldsymbol{\sigma}_2 \cdot \boldsymbol{k}\, \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 \\
& + D_{29}\, \left((\boldsymbol{\sigma}_1 + \boldsymbol{\sigma}_2) \cdot (\boldsymbol{q} \times \boldsymbol{k})\right)^2 + D_{30}\, \left((\boldsymbol{\sigma}_1 + \boldsymbol{\sigma}_2) \cdot (\boldsymbol{q} \times \boldsymbol{k})\right)^2 \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 \qquad (34)
\end{aligned}
$$

- But: work in progress!

Finite Volume Calculations

Motivation:

- Lattice QCD is the only ab initio method available to solve QCD directly at low energies but computational costs too high to compute more than a few particles
- Connect ab-initio nuclear physics to the underlying theory of QCD by studying, e.g., few-neutron systems in a box



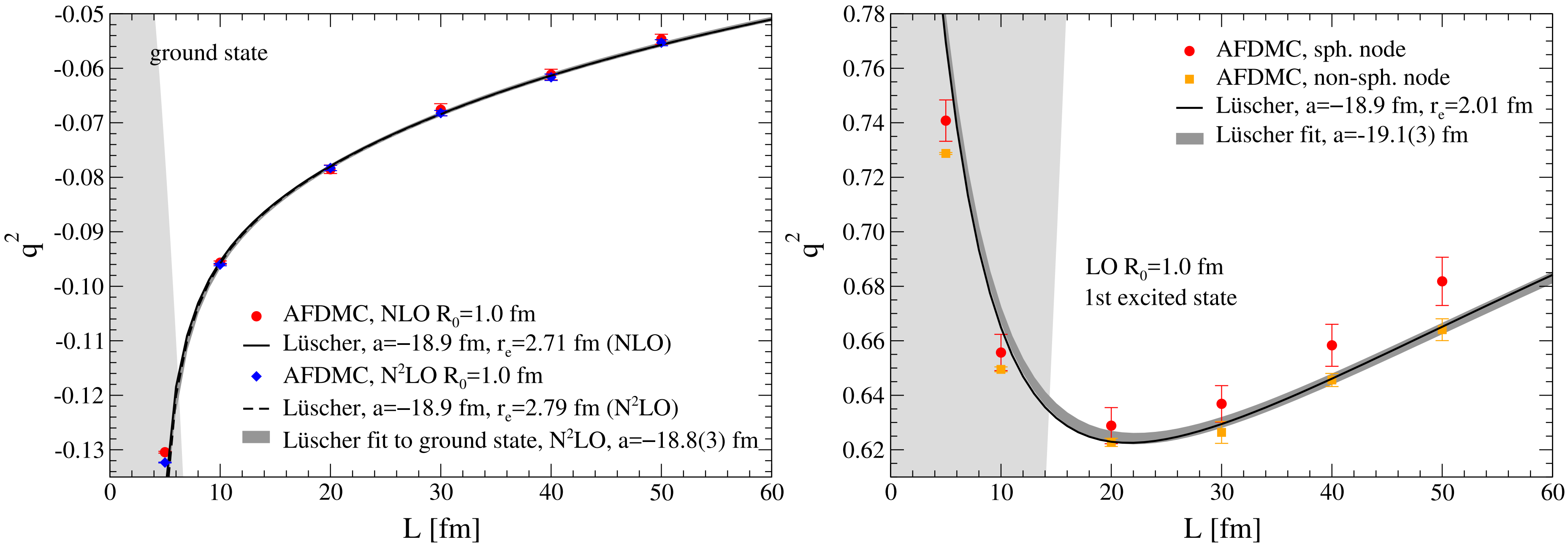
Credit: P. Klos

- Long-term goal: Matching of chiral EFT couplings to lattice QCD results
- Enable chiral EFT predictions from first principles

Finite Volume Calculations

Use Luescher formula to extract infinite-volume scattering data from finite volume calculations:

$$p\cot\delta_0(p) = \frac{1}{\pi L} S\left(\left(\frac{Lp}{2\pi}\right)^2\right) \qquad S(\eta) = \lim_{\Lambda\to\infty}\left(\sum_{|\mathbf{j}|<\Lambda} \frac{1}{\mathbf{j}^2-\eta} - 4\pi\Lambda\right)$$



Klos, Lynn, IT, Gandolfi, Gezerlis, Hammer, Hoferichter, Schwenk, arXiv:1604:01387, accepted for PRC

- Easy to extend to larger systems or, e.g., systems with hyperons

Summary

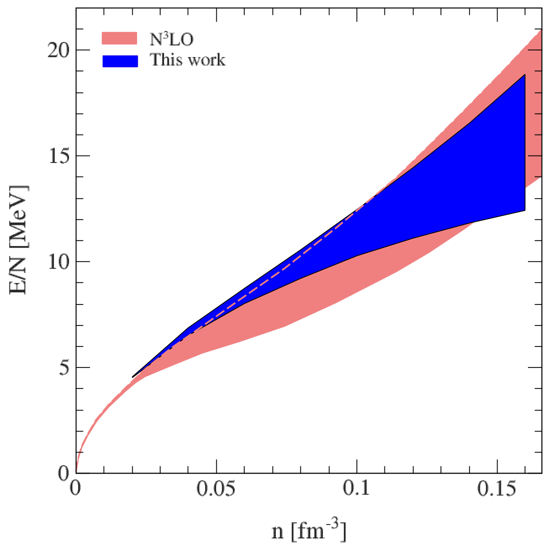
Chiral effective field theory:

- Provides constraints on symmetry energy, neutron star EOS
- Improvement of neutron-matter EOS work in progress
- Using QMC methods with higher order interactions expected to reduce theoretical uncertainties by a factor of two

  Gezerlis, IT, Epelbaum, Gandolfi, Hebeler, Nogga, Schwenk, PRL (2013) & PRC (2014)

  IT, Gandolfi, Gezerlis, Schwenk, PRC (2016)

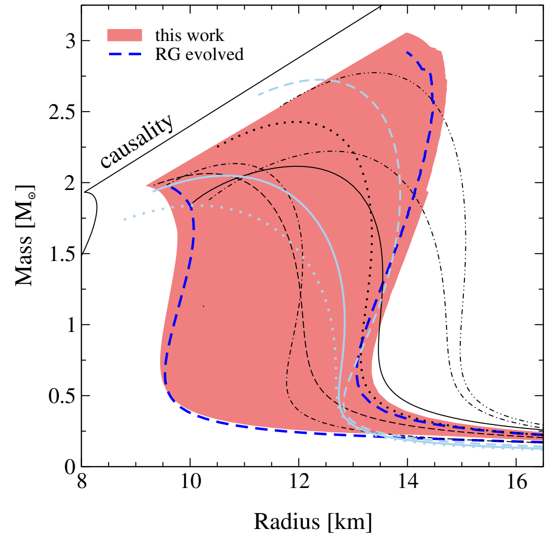
  Lynn, IT, Carlson, Gandolfi, Gezerlis, Schmidt, Schwenk, PRL (2016)



Constraints on symmetry energy and neutron stars:

- $S_V = 28.9 - 34.9$ MeV
- $L = 43.0 - 66.6$ MeV
- Radius for 1.4 $M_\odot$ neutron star: 9.7 – 13.9 km

  IT, Krüger, Hebeler, Schwenk, PRL & PRC (2013)



Thanks

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- Universität Bochum: E. Epelbaum
- Ohio State University: A. Dyhdalo, D. Furnstahl
- Los Alamos National Laboratory: J. Carlson, S. Gandolfi
- University of Guelph: A. Gezerlis
- Forschungszentrum Jülich: A. Nogga
- Institute for Nuclear Theory: M. Hoferichter

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ありがとうございました!