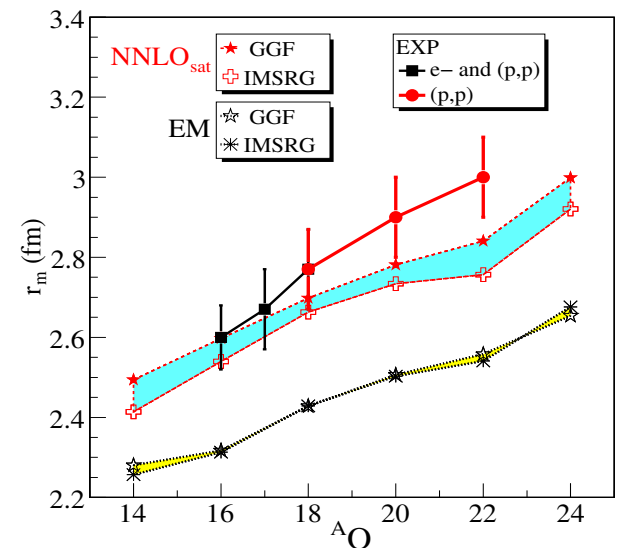
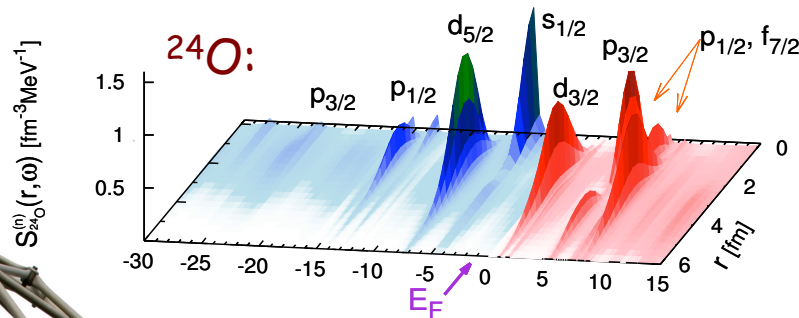
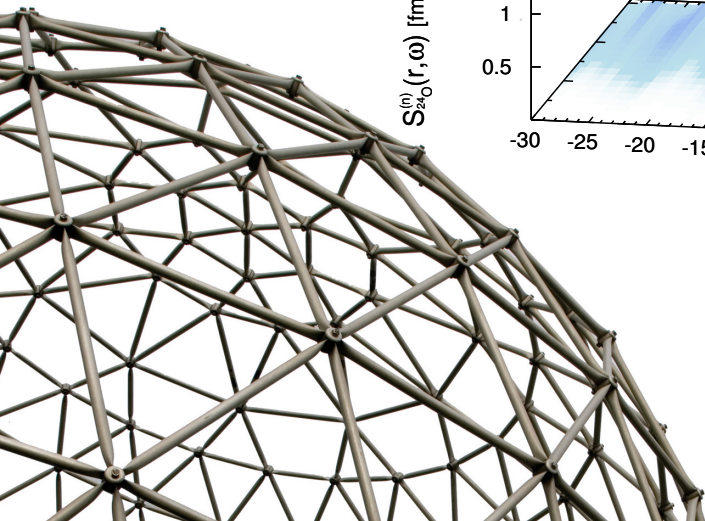


Neutron-rich nuclei from saturating chiral interactions

Carlo Barbieri — University of Surrey

Oct. 25th, 2016



Current Status of low-energy nuclear physics

Composite system of interacting fermions

Binding and limits of stability

Coexistence of individual and collective behaviors

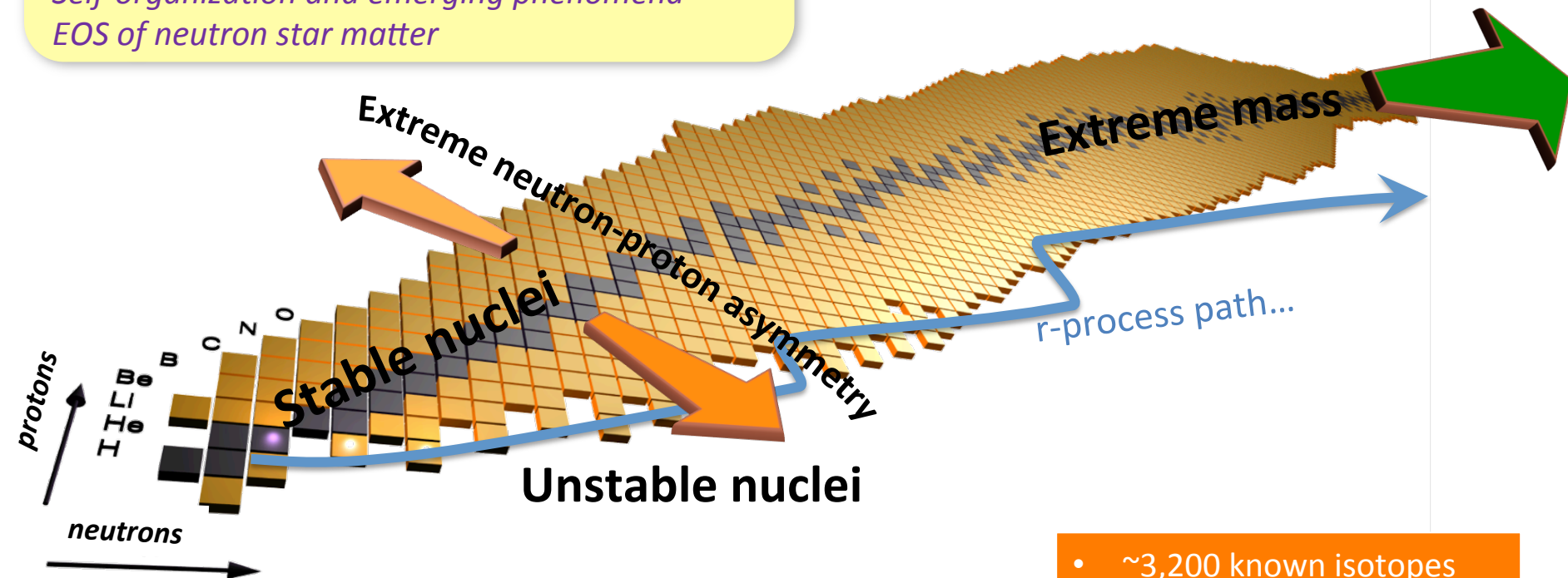
Self-organization and emerging phenomena

EOS of neutron star matter

Experimental

programs

RIKEN, FAIR, FRIB



- ~3,200 known isotopes
- ~7,000 predicted to exist
- Correlation characterised in full for ~283 stable

Nature **473**, 25 (2011); **486**, 509 (2012)

Current Status of low-energy nuclear physics

Composite system of interacting fermions

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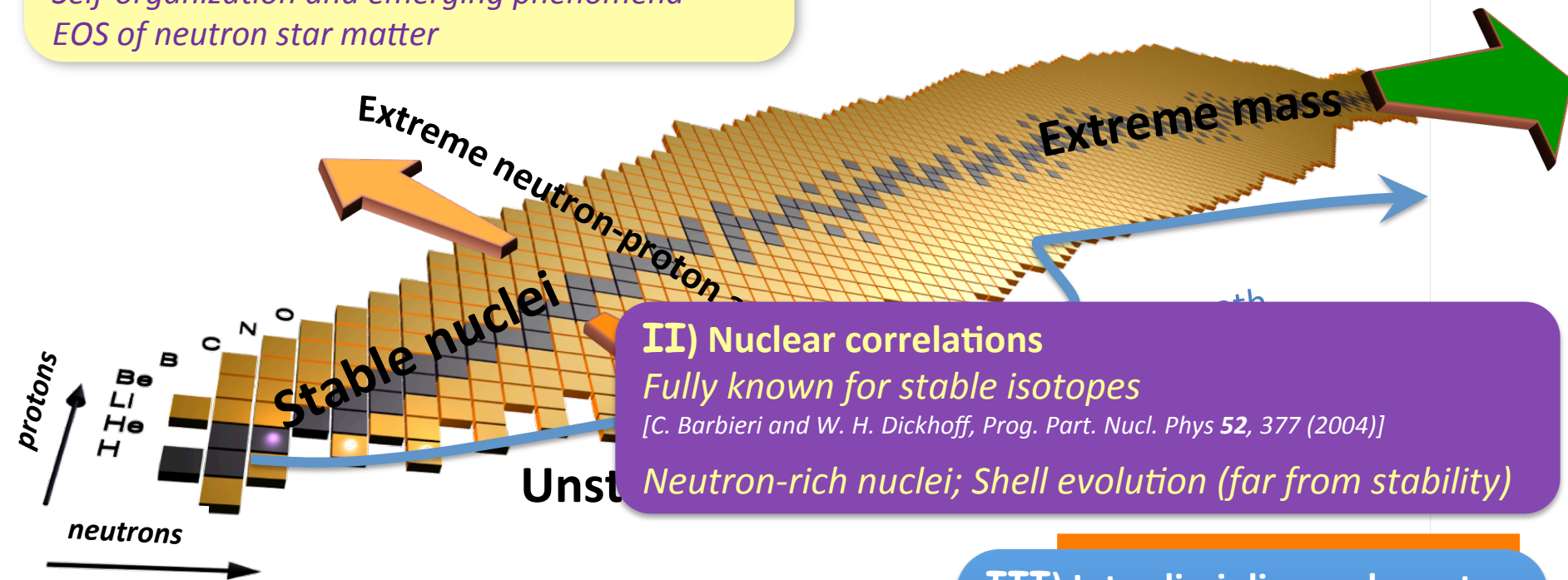
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Experimental

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II) Nuclear correlations

Fully known for stable isotopes

*[C. Barbieri and W. H. Dickhoff, Prog. Part. Nucl. Phys **52**, 377 (2004)]*

Neutron-rich nuclei; Shell evolution (far from stability)

I) Understanding the nuclear force

QCD-derived; 3-nucleon forces (3NFs)

First principle (ab-initio) predictions

III) Interdisciplinary character

Astrophysics

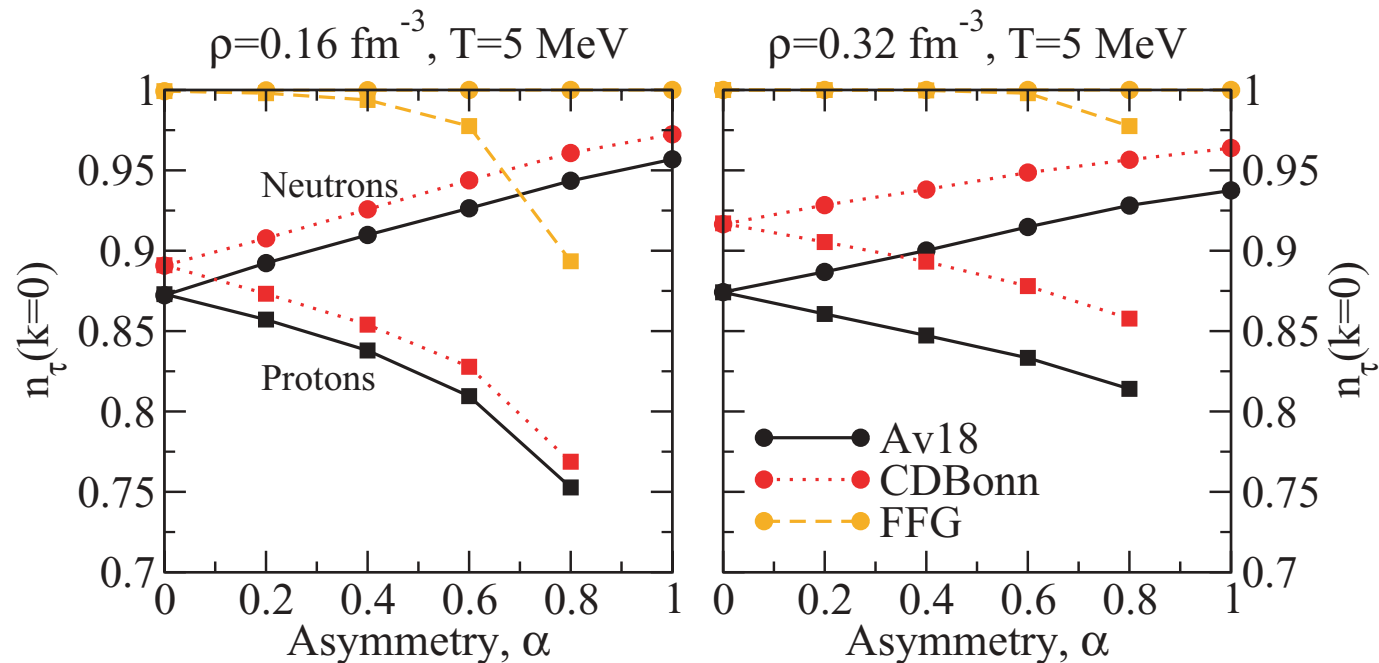
Tests of the standard model

Other fermionic systems:

ultracold gasses; molecules;

Correlations in asymmetric matter

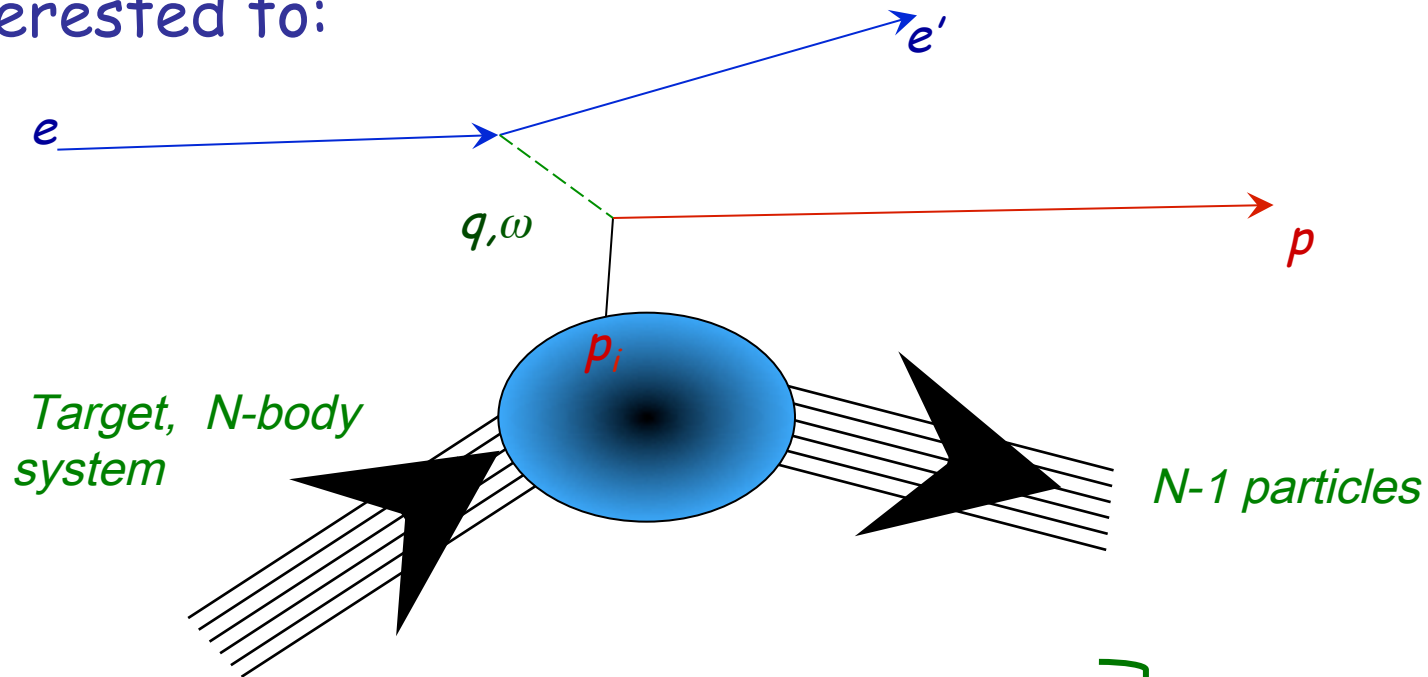
- Different correlations nuclear matter (stronger!) and in neutron matter
- 3 nucleon forces not negligible at high asymmetry



[Rios, Polls, Dickhoff, Phys Rev C 79, 064308 (2009)]

Spectroscopy via knock out reactions - *basic idea*

Use a probe (ANY probe) to eject the particle we are interested to:



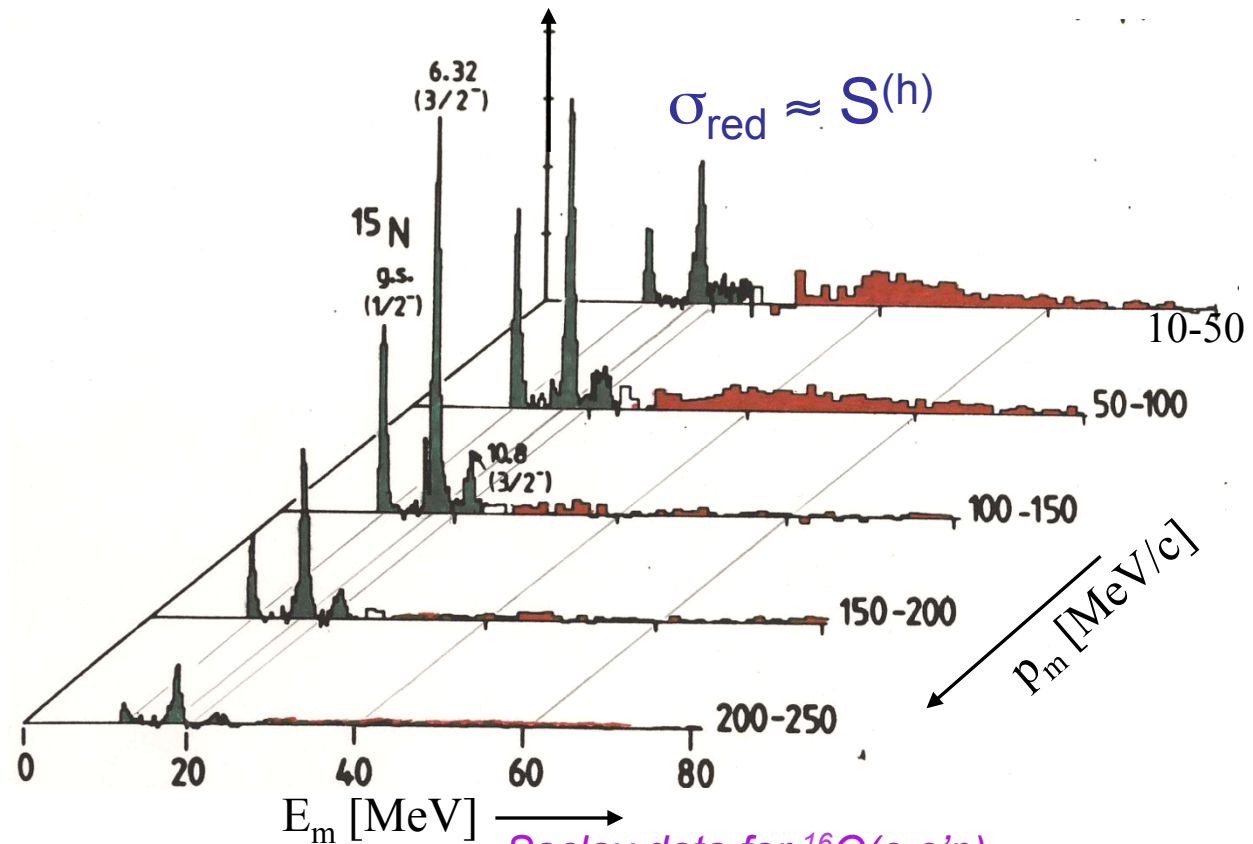
Basic idea:

- we know, e , e' and p
- "get" *energy and momentum of p_i* :
$$p_i = k_e' + k_p - k_e$$
$$E_i = E_e' + E_p - E_e$$

Better to choose
large transferred
momentum and weak
probes!!!

Concept of correlations

Spectral function: distribution of momentum (p_m) and energies (E_m)



Saclay data for $^{16}\text{O}(e,e'p)$

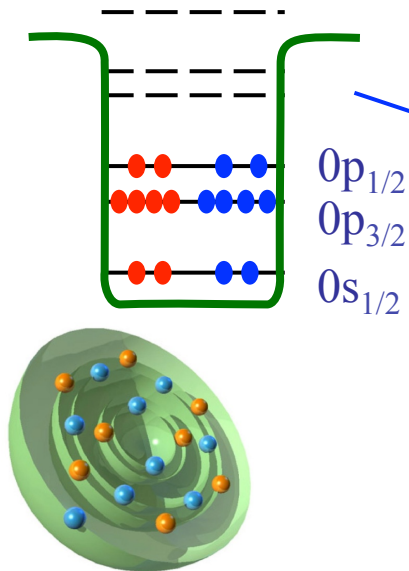
[Mougey et al., Nucl. Phys. A335, 35 (1980)]

Understood for a few stable closed shells:

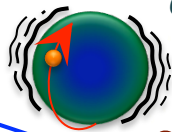
[CB and W. H. Dickhoff, Prog. Part. Nucl. Phys 52, 377 (2004)]

Concept of correlations

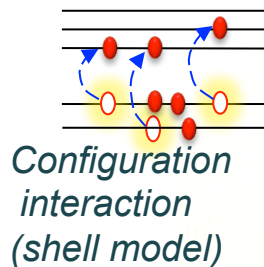
independent
particle picture



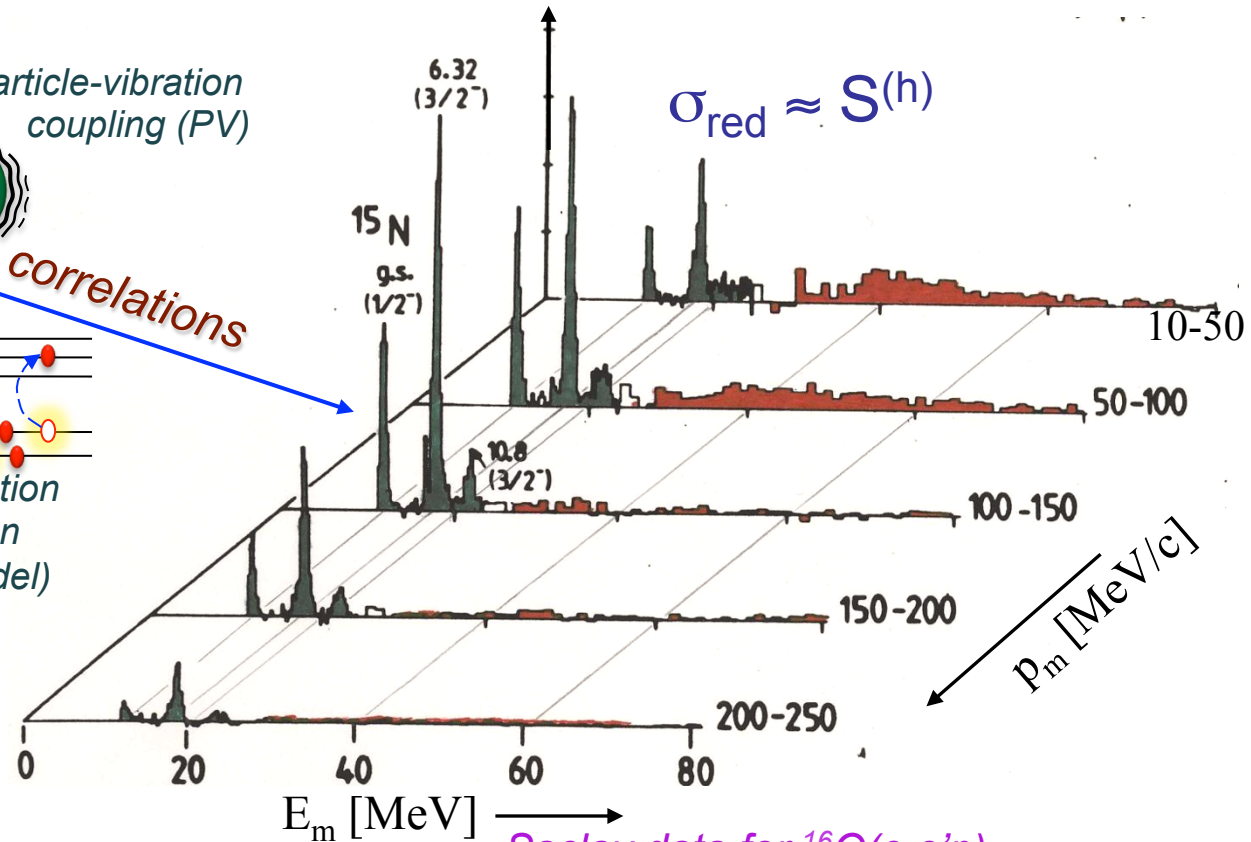
Particle-vibration
coupling (PV)



correlations



Spectral function: distribution of
momentum (p_m) and energies (E_m)



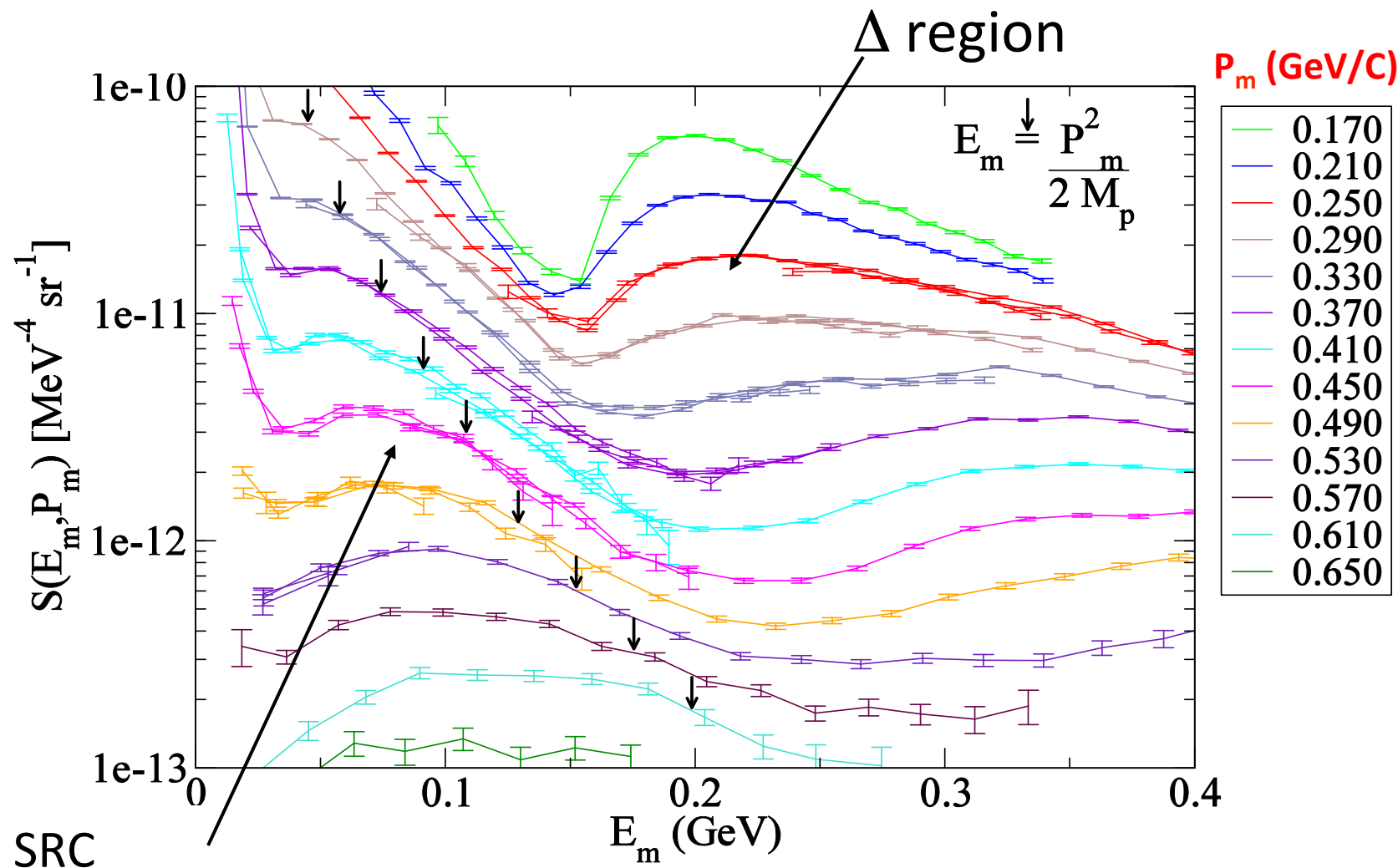
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[Mougey et al., Nucl. Phys. A335, 35 (1980)]

Understood for a few stable closed shells:

[CB and W. H. Dickhoff, Prog. Part. Nucl. Phys 52, 377 (2004)]

Spectral strength of ^{12}C from exp. E97-006



SRC
correlations

D.Rohe, et. al, Eur. Phys. J. A17, 349 (2003),
Phys Rev. Lett. 93 182501 (2004).

Concept of correlations

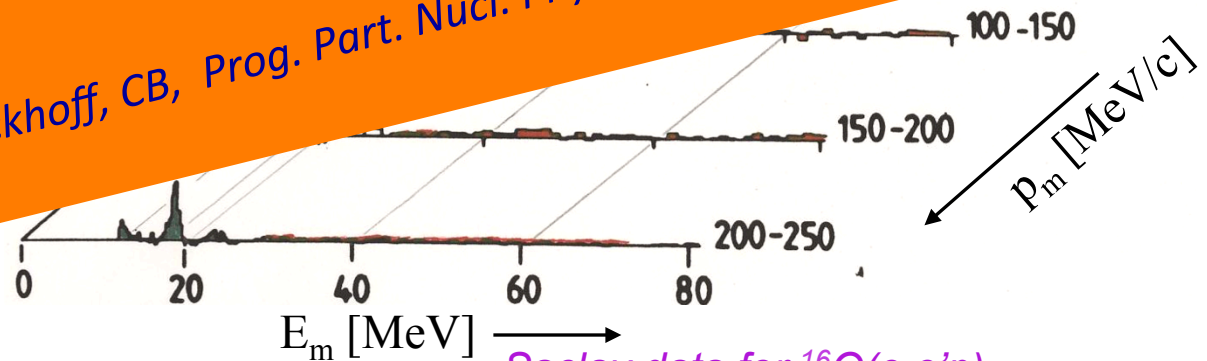
independent
particle picture

Spectral function: distribution of
momentum (p_m) and energy (E_m)

Particle-vibration
coupling

So far, fully characterised only for closed-shell and
stable isotopes... (!)

[W. Dickhoff, CB, Prog. Part. Nucl. Phys. **52**, 377 (2004)]



[Mougey et al., Nucl. Phys. A335, 35 (1980)]

Understood for a few stable closed shells:

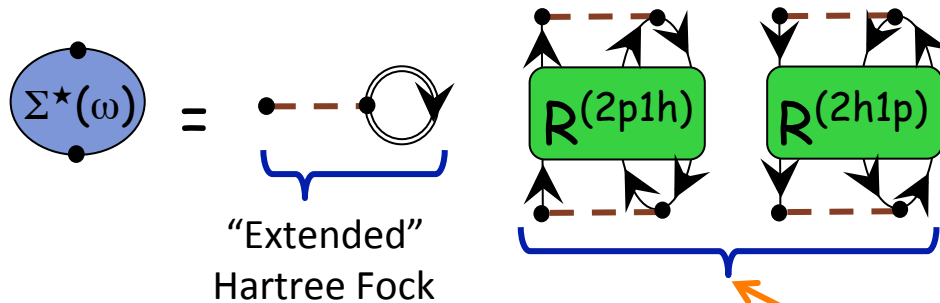
[CB and W. H. Dickhoff, Prog. Part. Nucl. Phys **52**, 377 (2004)]

Ab-Initio SCGF approaches

The FRPA Method in Two Words

Particle vibration coupling is the main cause driving the distribution of particle strength—on both sides of the Fermi surface...

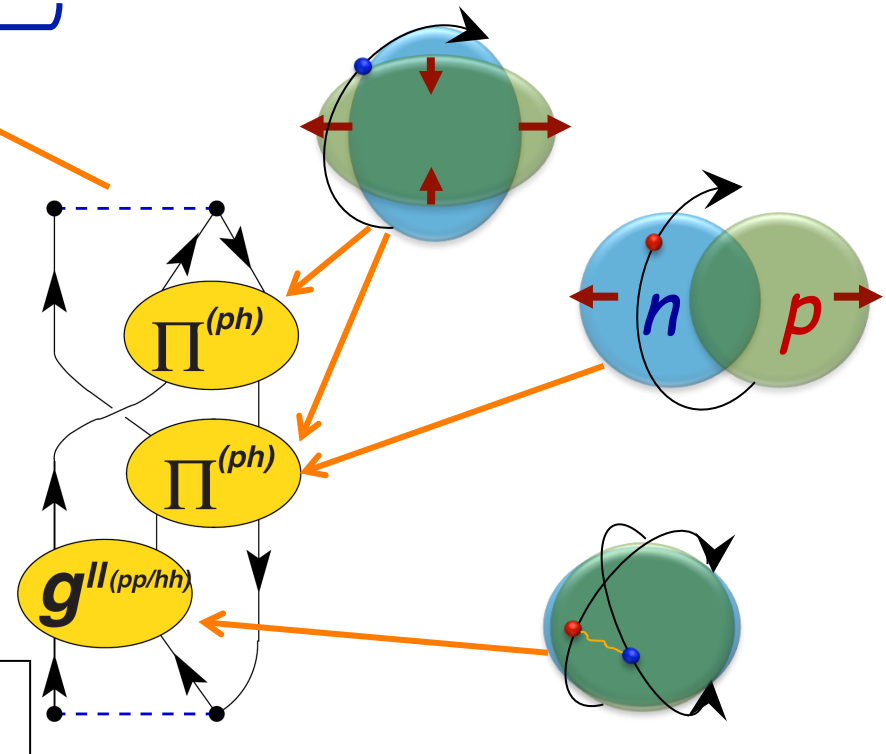
CB et al.,
Phys. Rev. C63, 034313 (2001)
Phys. Rev. A76, 052503 (2007)
Phys. Rev. C79, 064313 (2009)



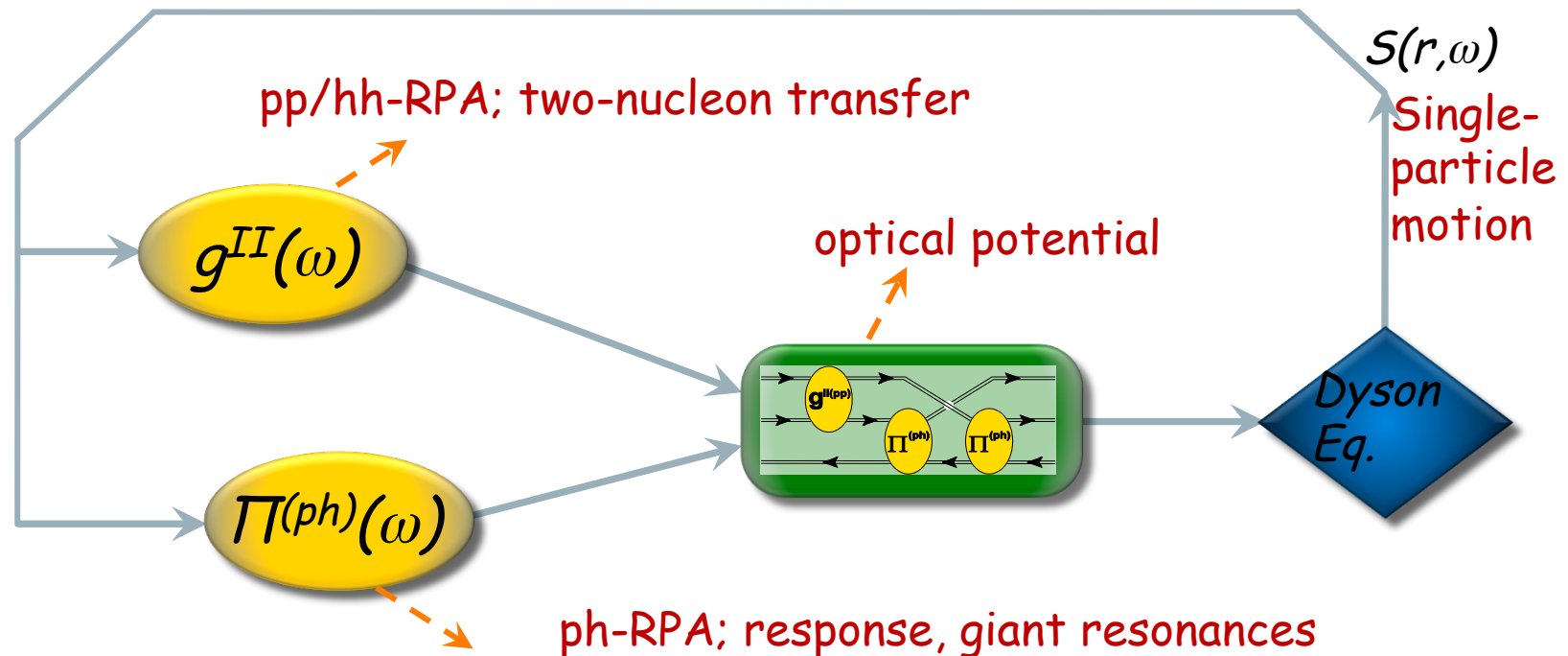
• A complete expansion requires all types of particle-vibration coupling

...these modes are all resummed exactly and to all orders in a *ab-initio* many-body expansion.

• The Self-energy $\Sigma^*(\omega)$ yields both single-particle states and scattering

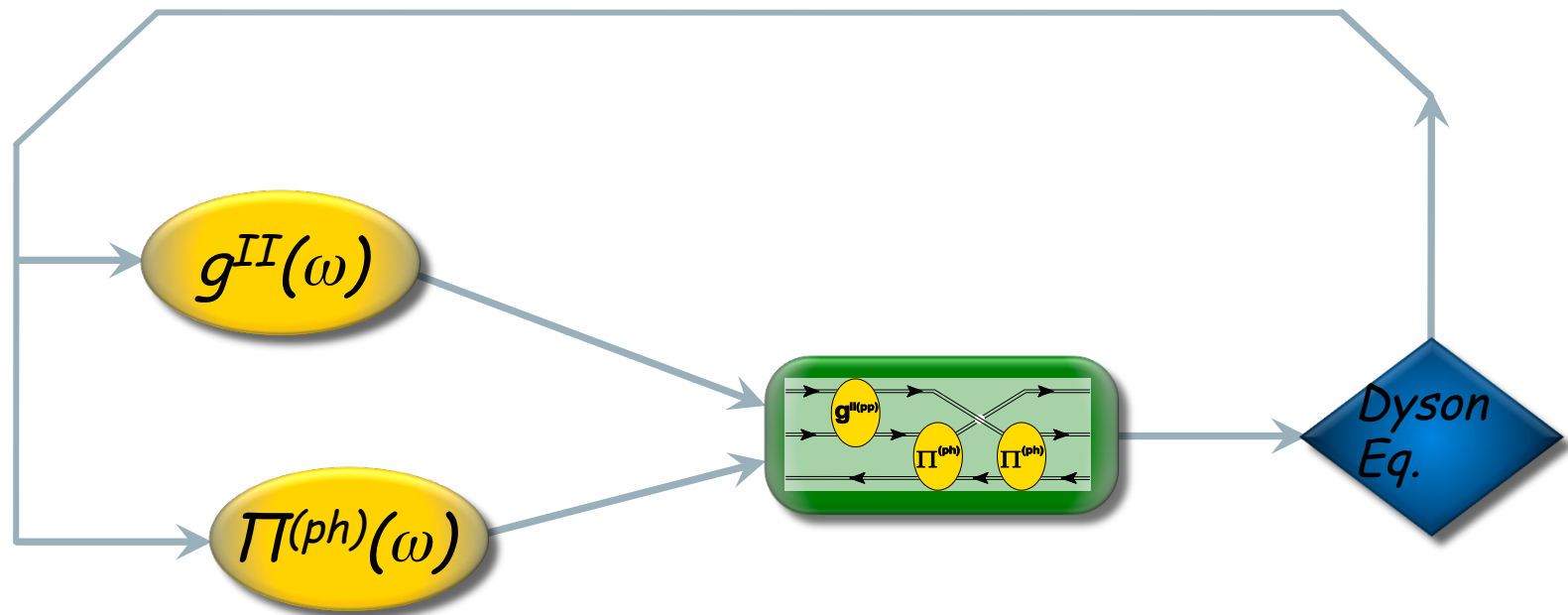


Self-Consistent Green's Function Approach



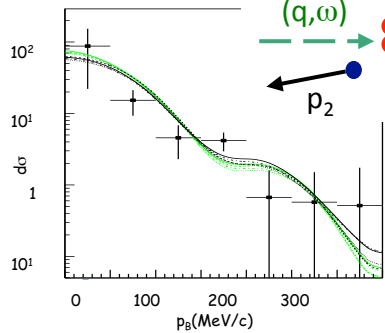
- Global picture of nuclear dynamics
- Reciprocal correlations among effective modes
- Guaranties *macroscopic conservation laws*

Self-Consistent Green's Function Approach

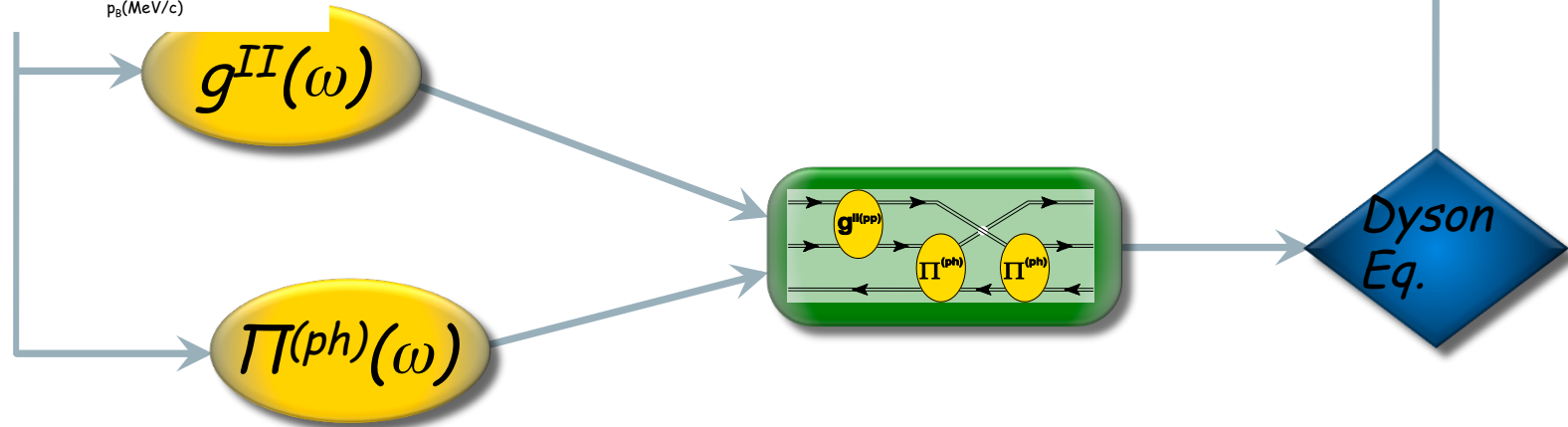


Self-Consistent Green's Function Approach

$^{16}\text{O}(e,e'pn)^{14}\text{N}$ @ MAINZ

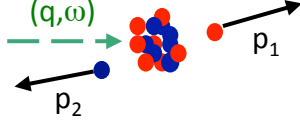
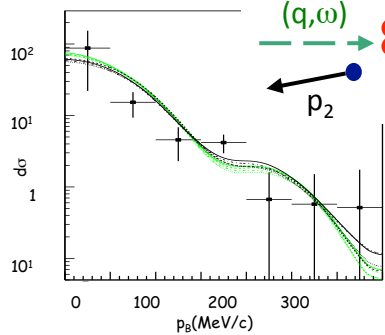


[C. B., C. Giusti, et al.
Phys Rev. C70, 014606 (2004)
D. Middleton, et al.
arXiv:0907.1758; EPJA in print]



Self-Consistent Green's Function Approach

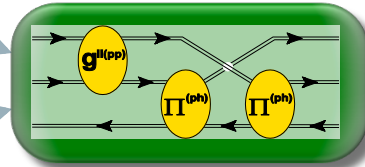
$^{16}\text{O}(e,e'pn)^{14}\text{N}$ @ MAINZ



[C. B., C. Giusti, et al.
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$g^{II}(\omega)$

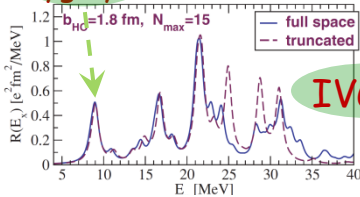
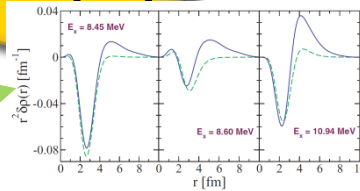
$\Pi^{(ph)}(\omega)$



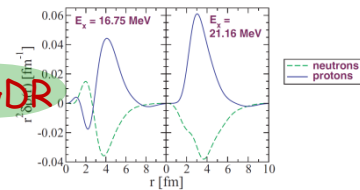
Dyson Eq.

Isovector response
for ^{32}Ar , ^{34}Ar

Proton
Pygmy



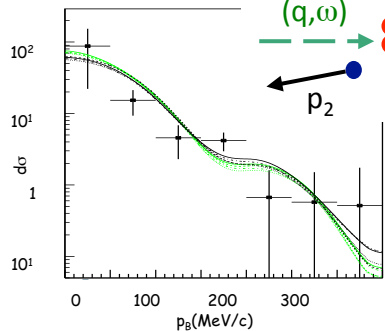
IVGDR



[C. B., K. Langanke, et al., Phys. Rev. C77, 024304 (2008)]

Self-Consistent Green's Function Approach

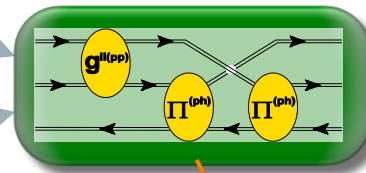
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$g^{II}(\omega)$

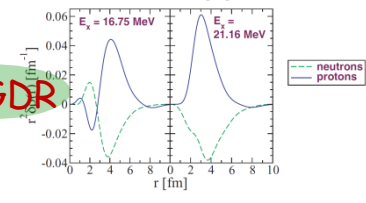
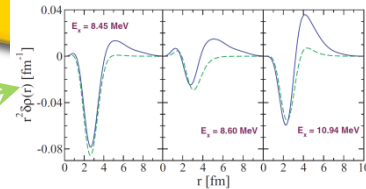
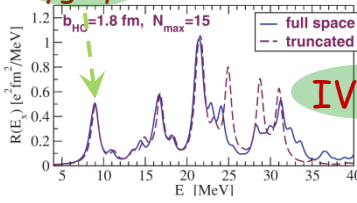
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Dyson Eq.

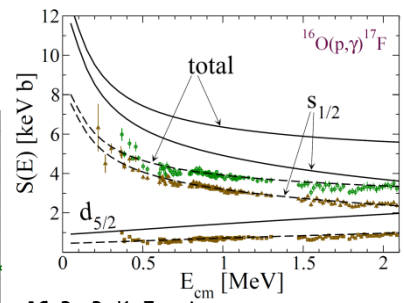
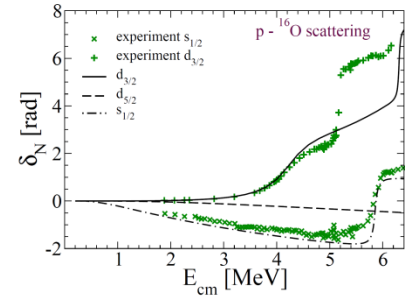
Isovector response
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IVGDR

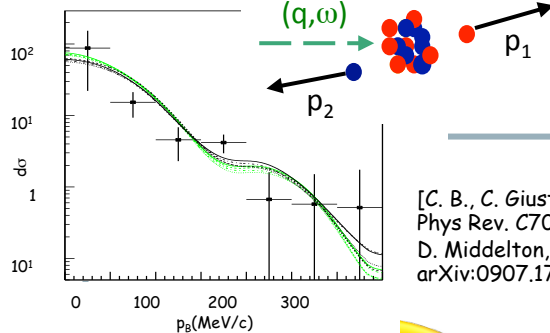
$^{16}\text{O}(p,\gamma)$



[C. B., B. K. Jennings
Nucl. Phys A758, 395c (2005)
Phys Rev. C72, 014613 (2005)]

Self-Consistent Green's Function Approach

$^{16}\text{O}(e,e'pn)^{14}\text{N}$ @ MAINZ



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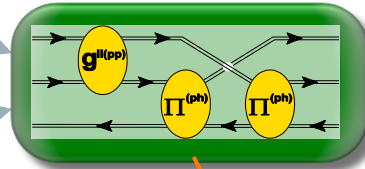
Ionization energies/
affinities, in atoms

[CB, D. Van Neck,
AIP Conf. Proc. 1120, 104 ('09) & in prep]

		Hartree-Fock	FRPac	Experiment [16, 17]
He:	1s	0.918 (+14)	0.9008 (-2.9)	0.9037
Be ²⁺ :	1s	5.6672 (+116)	5.6551 (-0.5)	5.6556
Be:	2s	0.3093 (-34)	0.3224 (-20.2)	0.3426
	1s	4.733 (+200)	4.5405 (+8)	4.533
Ne:	2p	0.852 (+57)	0.8037 (+11)	0.793
	1s	1.931 (+149)	1.7967 (+15)	1.782
Mg ²⁺ :	2p	3.0068 (+56.9)	2.9537 (+3.8)	2.9499
	1s	4.4827	4.3589	
Mg:	3s	0.253 (-28)	0.280 (-1)	0.281
	2p	2.282 (+162)	2.137 (+17)	2.12
Ar:	3p	0.591 (+12)	0.579 (±0)	0.579
	3s	1.277 (+202)	1.065 (-10)	1.075
	3s		1.544	
	2p	9.571 (+411)	9.219 (+59)	9.160

$g^{II}(\omega)$

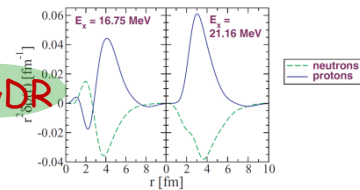
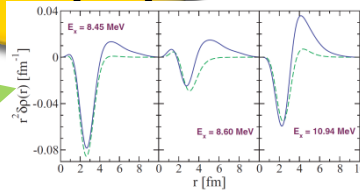
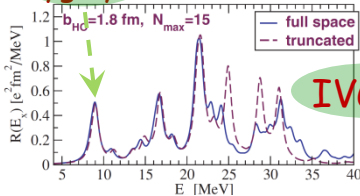
$\Pi^{(ph)}(\omega)$



Dyson
Eq.

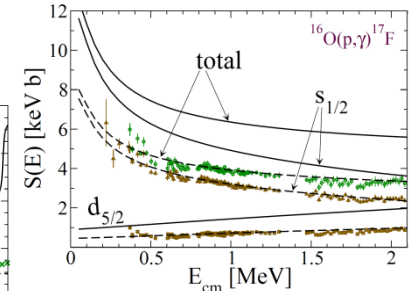
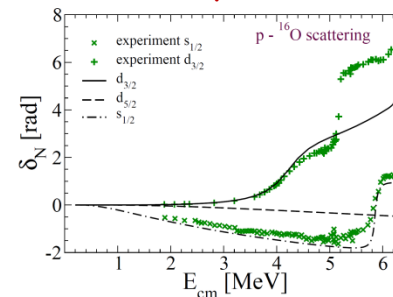
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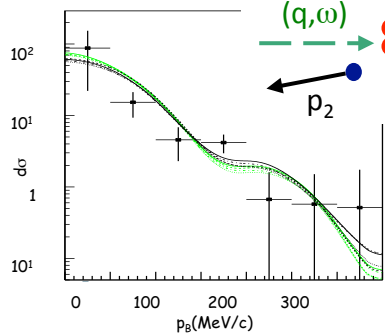


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SURREY

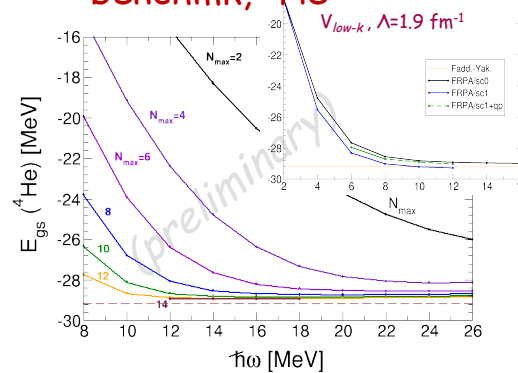
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D. Middleton, et al.
arXiv:0907.1758; EPJA in print]

Binding energy benchmk, ^4He [C. B., arXiv:0909.0336]



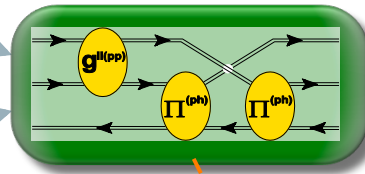
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$g^{II}(\omega)$

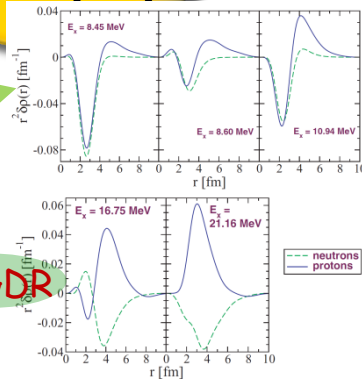
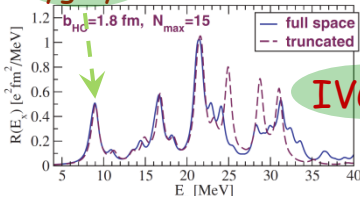
$\Pi^{(ph)}(\omega)$



Dyson
Eq.

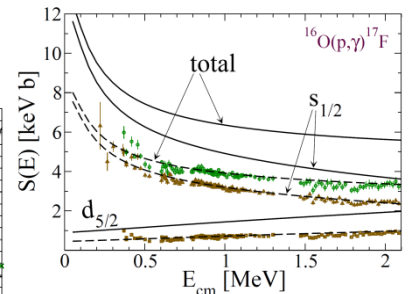
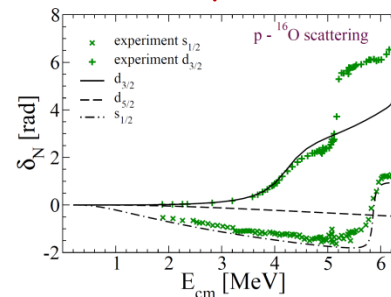
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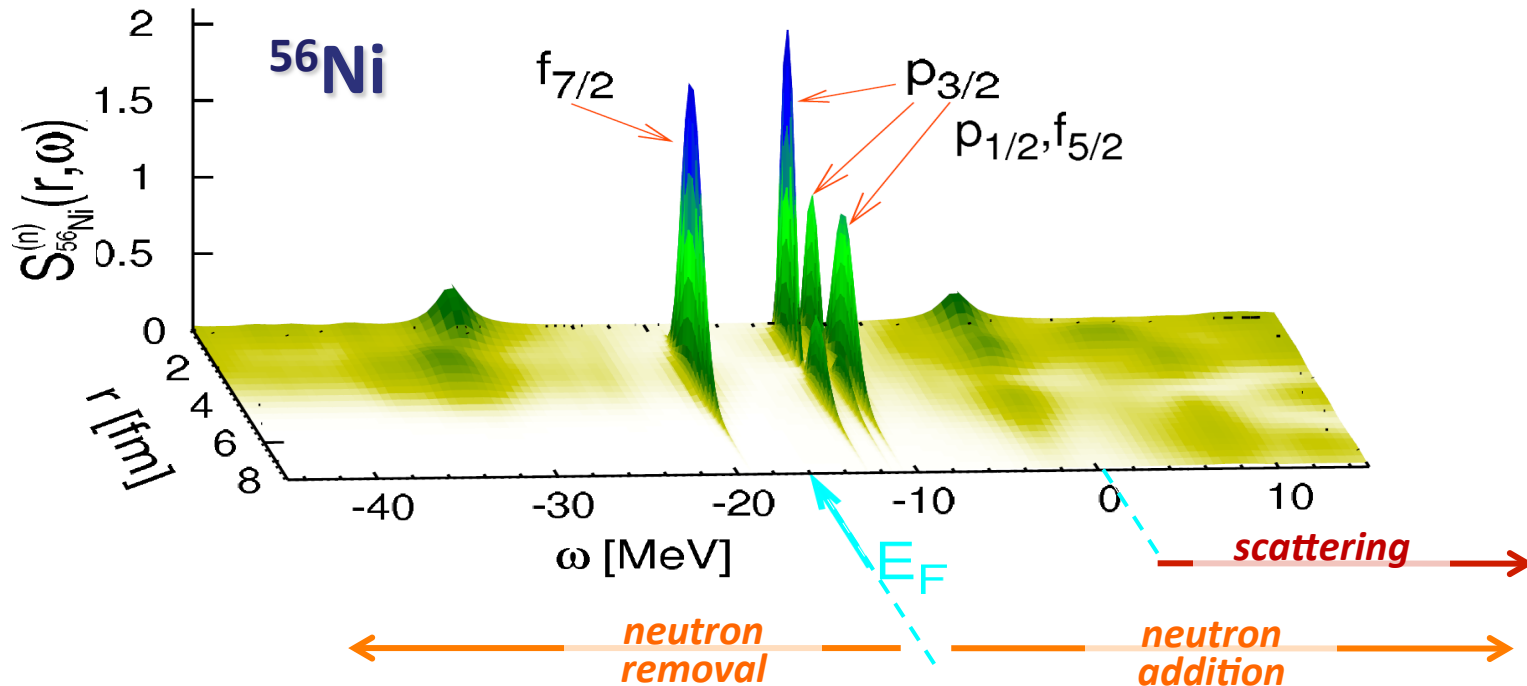


[C. B., B. K. Jennings
Nucl. Phys. A758, 395c (2005)
Phys. Rev. C72, 014613 (2005)]



One-nucleon spectral function

$$S^{p,h}(r, \omega) = \mp \frac{1}{\pi} \text{Im } g(r = r'; \omega)$$



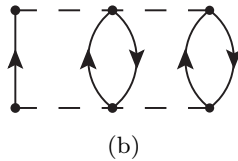
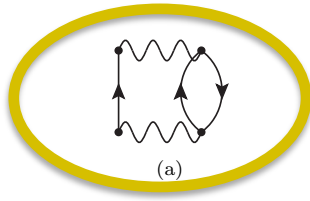
Distribution of particle and hole neutron states in ^{56}Ni

W. Dickhoff, CB, Prog. Part. Nucl. Phys. 53, 377 (2004)
CB, M.Hjorth-Jensen, Pys. Rev. C**79**, 064313 (2009)

Gorkov and 3-nucleon forces

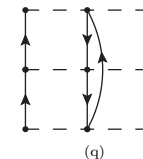
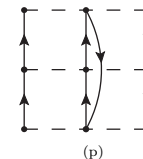
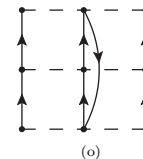
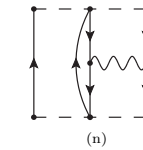
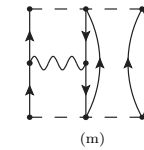
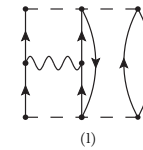
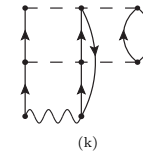
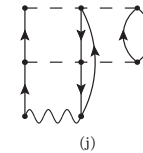
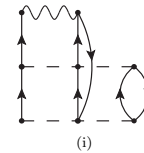
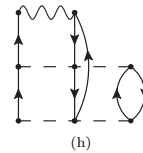
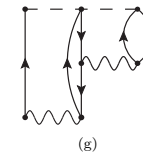
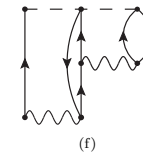
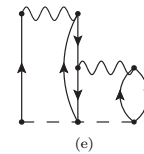
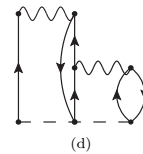
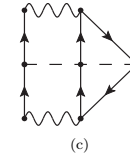
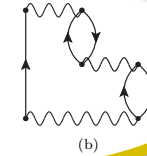
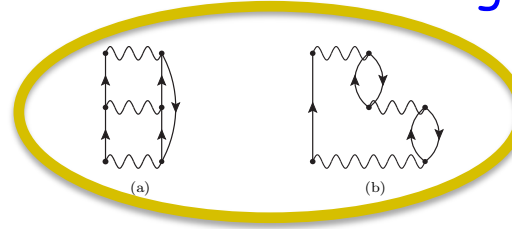
Inclusion of NNN forces

- Second order PT diagrams with 3BFs:



A. Carbone, CB, et al., *Phys. Rev. C* **88**, 054326 (2013)
and F. Raimondi, CB, in preparation (2016).

- Third order PT diagrams with 3BFs:



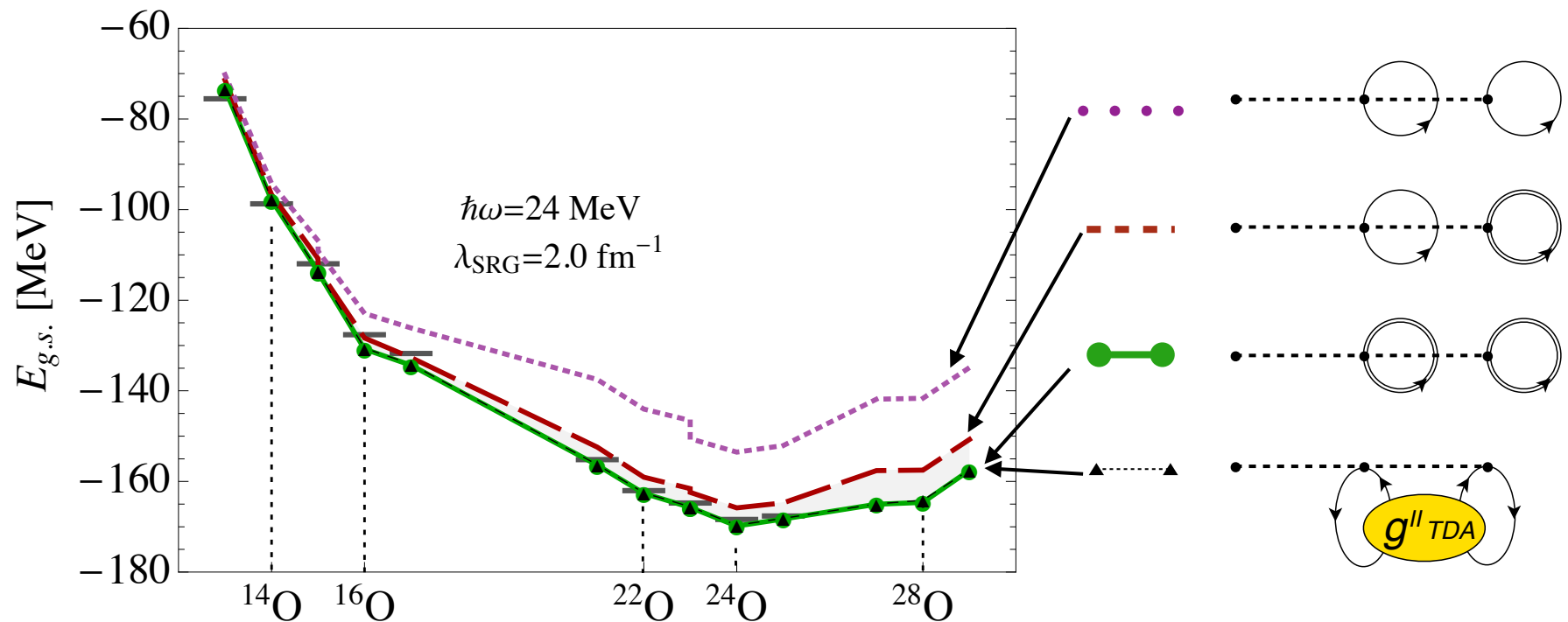
→ Use of irreducible 2-body interactions

→ Need to correct the Koltun sum rule (for energy)

FIG. 5. 1PI, skeleton and interaction irreducible self-energy diagrams appearing at 3^{rd} -order in perturbative expansion (7), making use of the effective hamiltonian of Eq. (9).

3N forces in FRPA/FTDA formalism

→ Ladder contributions to static self-energy are negligible (in oxygen)



Gorkov and symmetry breaking approaches

V. Somà, CB, T. Duguet, , Phys. Rev. C **89**, 024323 (2014)

V. Somà, CB, T. Duguet, Phys. Rev. C **87**, 011303R (2013)

V. Somà, T. Duguet, CB, Phys. Rev. C **84**, 064317 (2011)

➤ Ansatz $\dots \approx E_0^{N+2} - E_0^N \approx E_0^N - E_0^{N-2} \approx \dots \approx 2\mu$

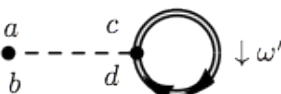
➤ Auxiliary many-body state $|\Psi_0\rangle \equiv \sum_N^{\text{even}} c_N |\psi_0^N\rangle$

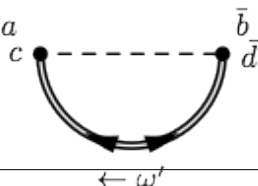
➤ Mixes various particle numbers

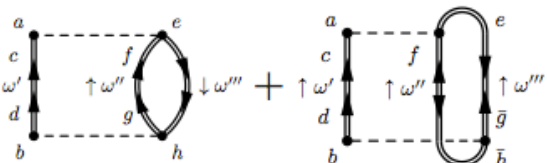
➤ Introduce a “grand-canonical” potential $\Omega = H - \mu N$

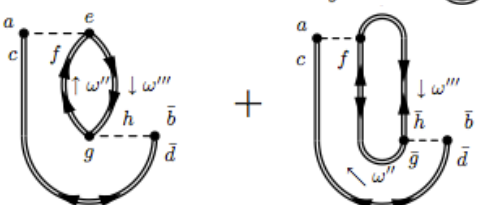
➔ $|\Psi_0\rangle$ minimizes $\Omega_0 = \langle \Psi_0 | \Omega | \Psi_0 \rangle$ under the constraint $N = \langle \Psi_0 | N | \Psi_0 \rangle$

➤ This approach leads to the following Feynman diagrams:

$$\Sigma_{ab}^{11(1)} =$$


$$\Sigma_{ab}^{12(1)} =$$


$$\Sigma_{ab}^{11(2)}(\omega) =$$


$$\Sigma_{ab}^{12(2)}(\omega) =$$


Approaches in GF theory

Truncation
scheme:

Dyson formulation
(closed shells)

Gorkov formulation
(semi-/doubly-magic)

1st order:

Hartree-Fock

HF-Bogolioubov

2nd order:

2nd order

2nd order (w/ pairing)

...

...

3rd and all-orders
sums,
P-V coupling:

ADC(3)
FRPA
etc...

G-ADC(3)
...work in progress

Approaches in GF theory

Truncation
scheme:

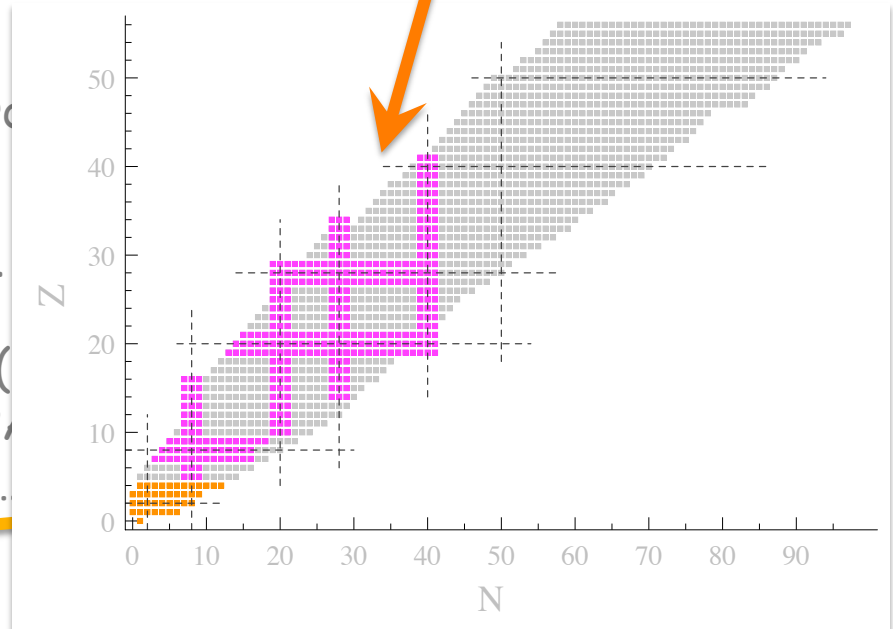
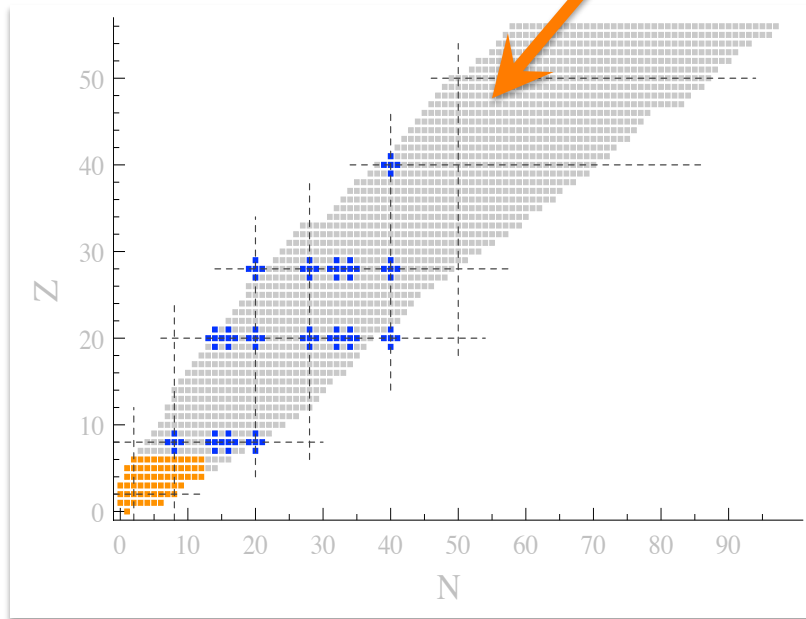
Dyson formulation
(closed shells)

Gorkov formulation
(semi-/doubly-magic)

1st order:

Hartree-Fock

HF-Bogoliubov



Ab-initio Nuclear Computation & BcDor code

BoccaDorata code:

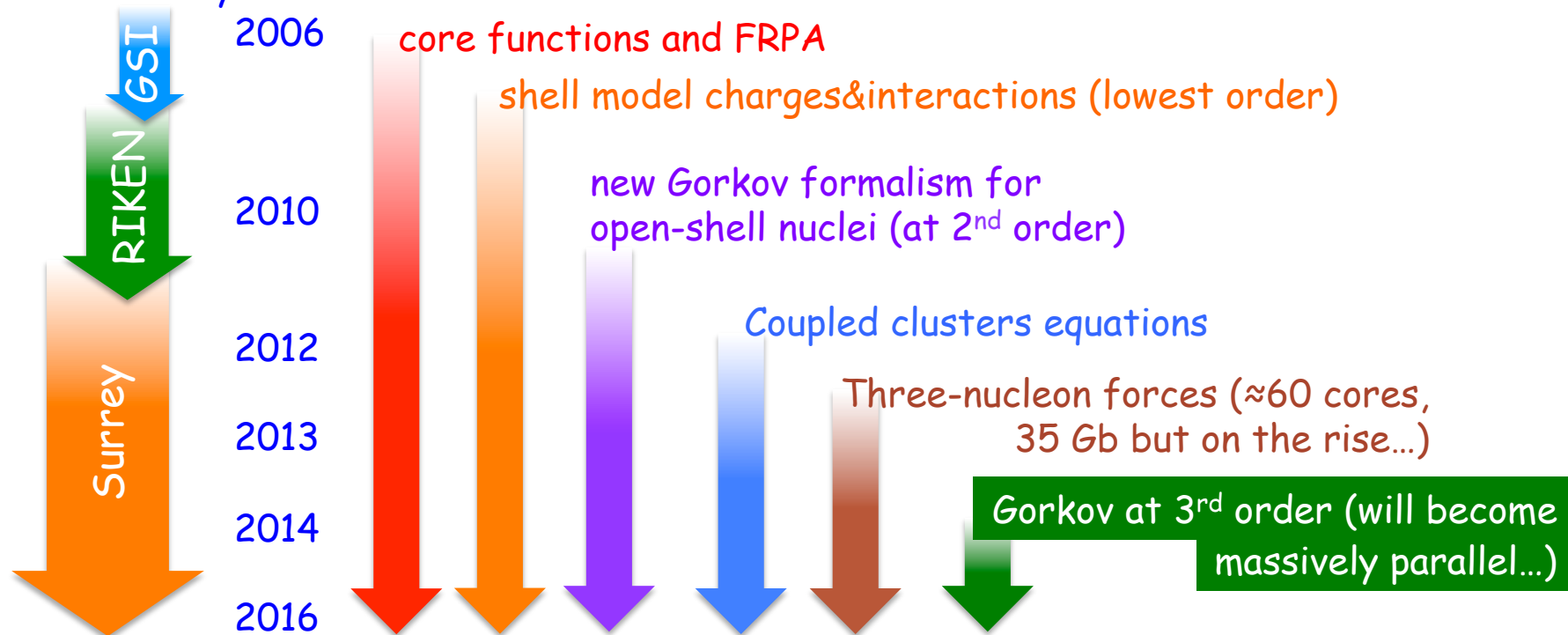
(C. Barbieri 2006-16

V. Somà 2010-15

A. Cipollone 2011-14)

- Provides a *C++ class library* for handling many-body propagators ($\approx 40,000$ lines, MPI&OpenMP based).
- Allows to solve for nuclear spectral functions, many-body propagators, RPA responses, coupled cluster equations and effective interaction/charges for the shell model.

Code history:

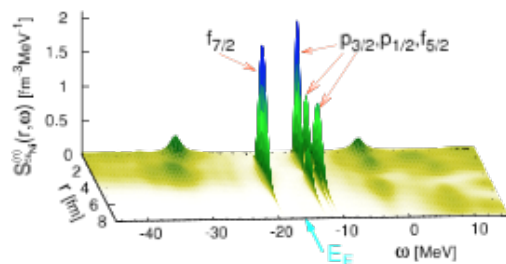
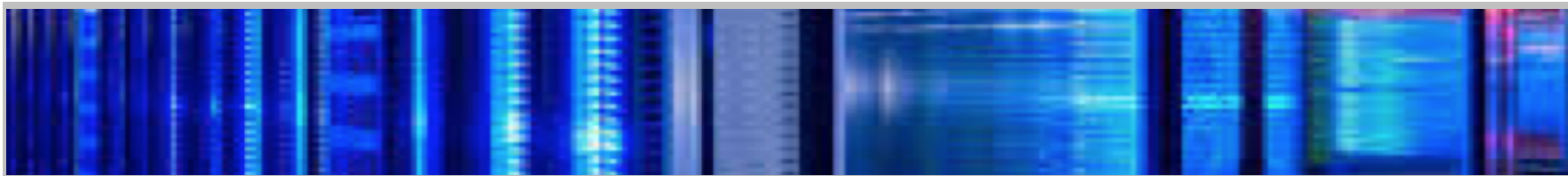


... applications ...

Ab-initio Nuclear Computation & BcDor code

<http://personal.ph.surrey.ac.uk/~cb0023/bcdor/>

Computational Many-Body Physics



Download

Documentation

Welcome

From here you can download a public version of my self-consistent Green's function (SCGF) code for nuclear physics. This is a code in J-coupled scheme that allows the calculation of the single particle propagators (a.k.a. one-body Green's functions) and other many-body properties of spherical nuclei.

This version allows to:

- Perform Hartree-Fock calculations.
- Calculate the correlation energy at second order in perturbation theory (MBPT2).
- Solve the Dyson equation for propagators (self consistently) up to second order in the self-energy.
- Solve coupled cluster CCD (doubles only!) equations.

When using this code you are kindly invited to follow the creative commons license agreement, as detailed at the weblinks below. In particular, we kindly ask you to refer to the publications that led the development of this software.

Relevant references (which can also help in using this code) are:

- Prog. Part. Nucl. Phys. 52, p. 377 (2004),
- Phys. Rev. A76, 052503 (2007),
- Phys. Rev. C79, 064313 (2009),
- Phys. Rev. C89, 024323 (2014)

Chiral interactions for mid-mass isotopes

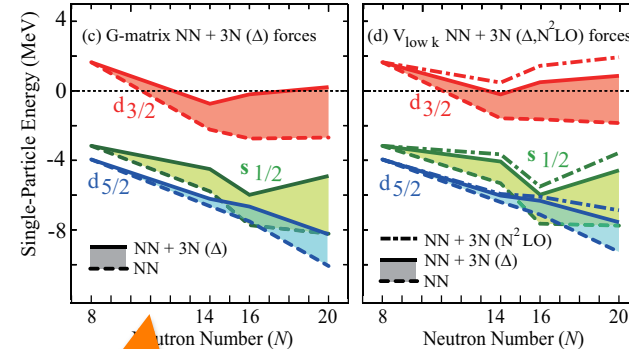
Modern realistic nuclear forces

Chiral EFT for nuclear forces:

	2N forces	3N forces	4N forces
LO $\mathcal{O}\left(\frac{Q^0}{\Lambda^0}\right)$			
NLO $\mathcal{O}\left(\frac{Q^2}{\Lambda^2}\right)$			
N ² LO $\mathcal{O}\left(\frac{Q^3}{\Lambda^3}\right)$			
N ³ LO $\mathcal{O}\left(\frac{Q^4}{\Lambda^4}\right)$			

(3NFs arise naturally at N2LO)

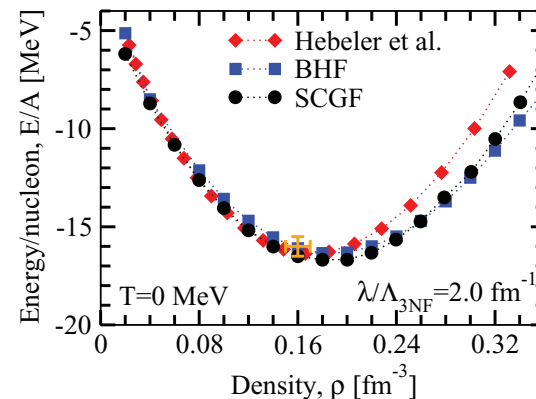
Single particle spectrum at E_{fermi} :



[T. Otsuka et al.,
Phys. Rev. Lett. **105**,
032501 (2010)]

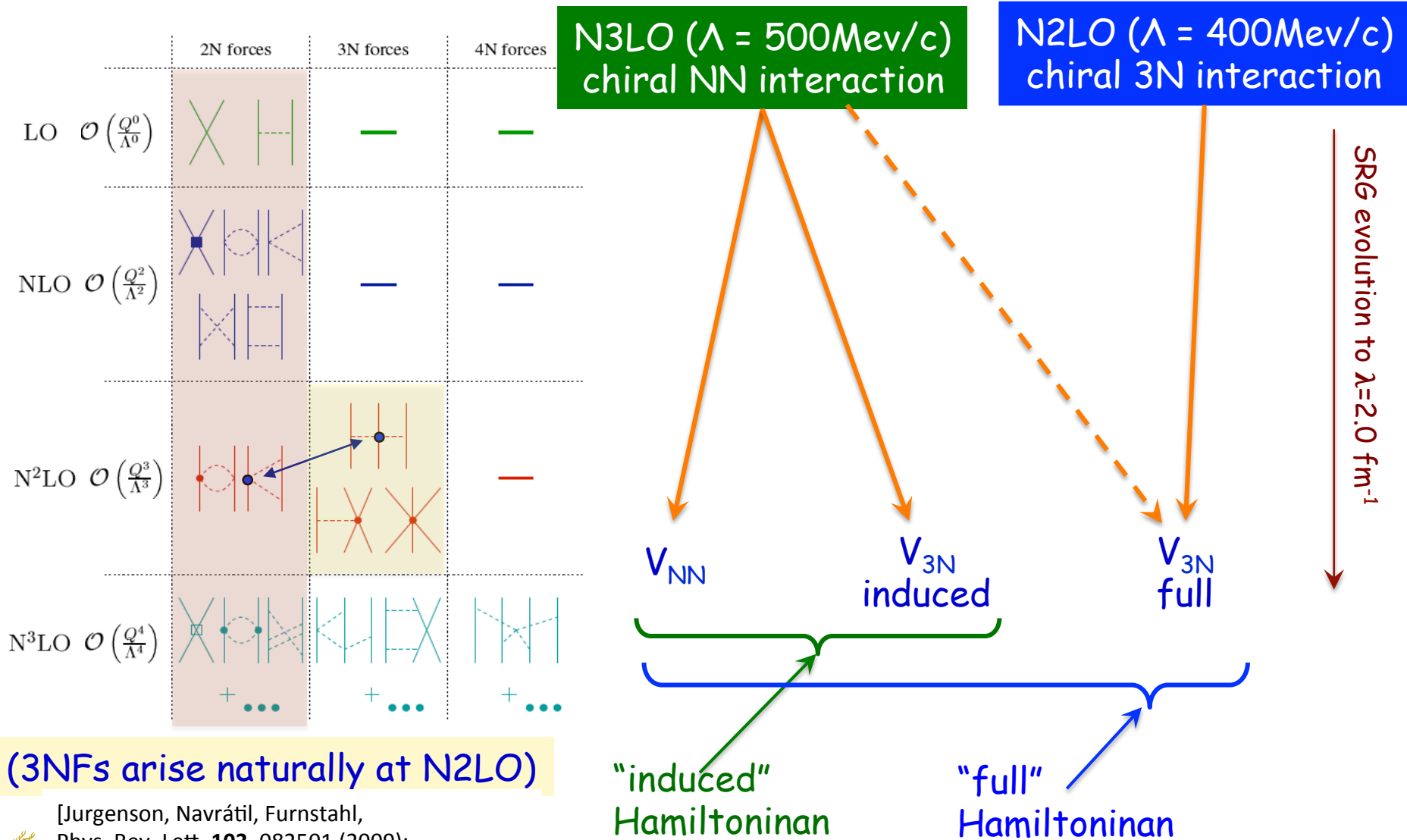
Need at LEAST 3NF!!!
("cannot" do RNB physics without...)

Saturation of nuclear matter:

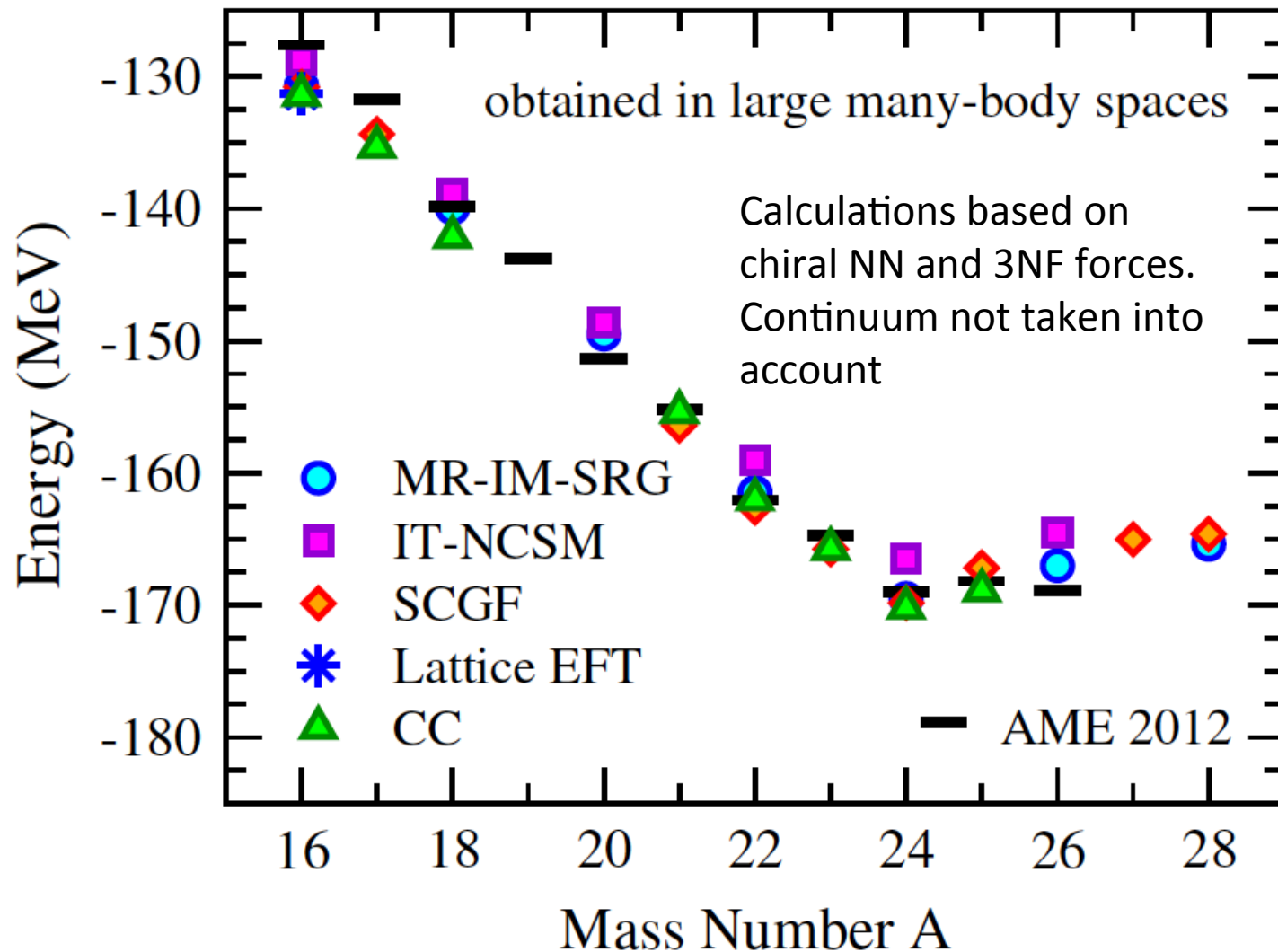


[A. Carbone et al.,
Phys. Rev. C **88**, 044302 (2013)]

Chiral Nuclear forces - SRG evolved

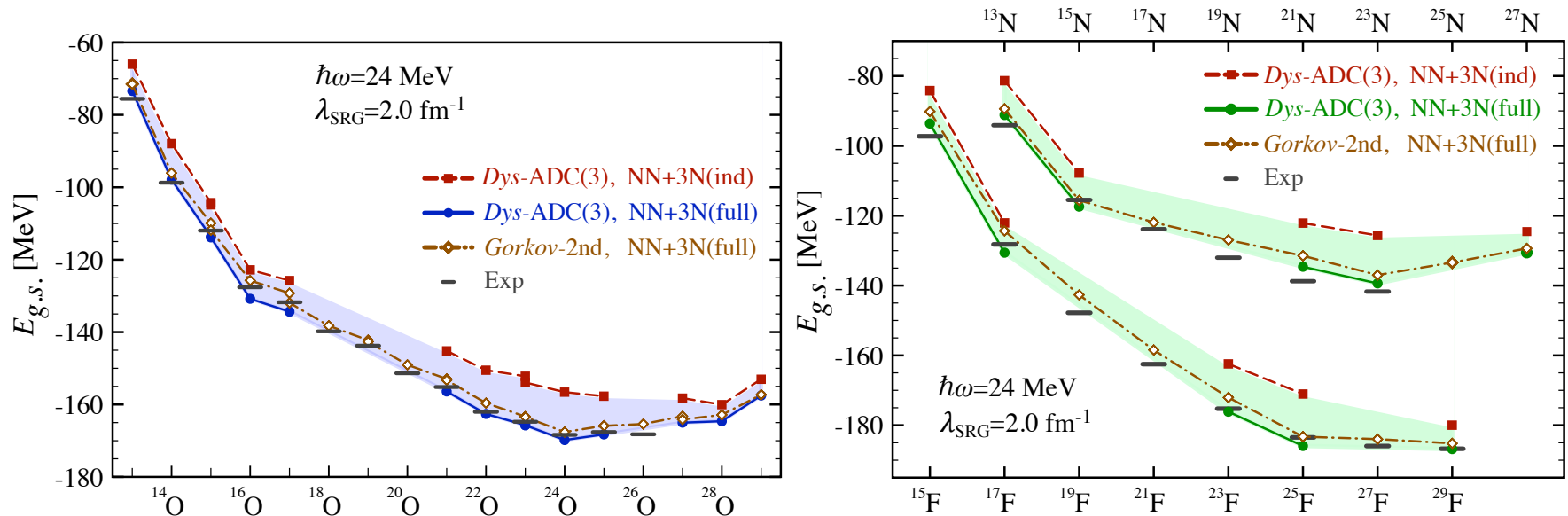


Benchmark of *ab-initio* methods in the oxygen isotopic chain



Results for the N-O-F chains

A. Cipollone, CB, P. Navrátil, Phys. Rev. Lett. **111**, 062501 (2013)
and Phys. Rev. C **92**, 014306 (2015)



→ 3NF crucial for reproducing binding energies and driplines around oxygen

→ cf. microscopic shell model [Otsuka et al, PRL**105**, 032501 (2010).]

Inversion of $d_{3/2}-s_{1/2}$ at $N=28$

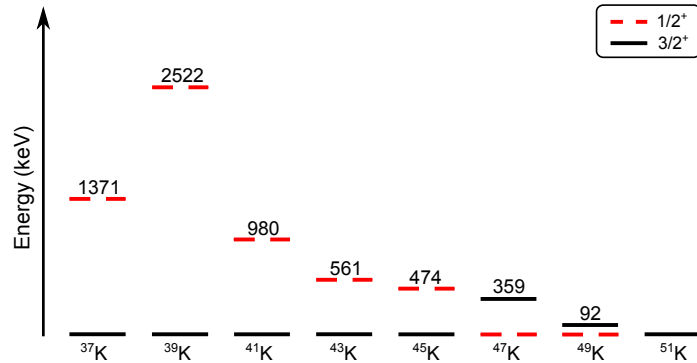


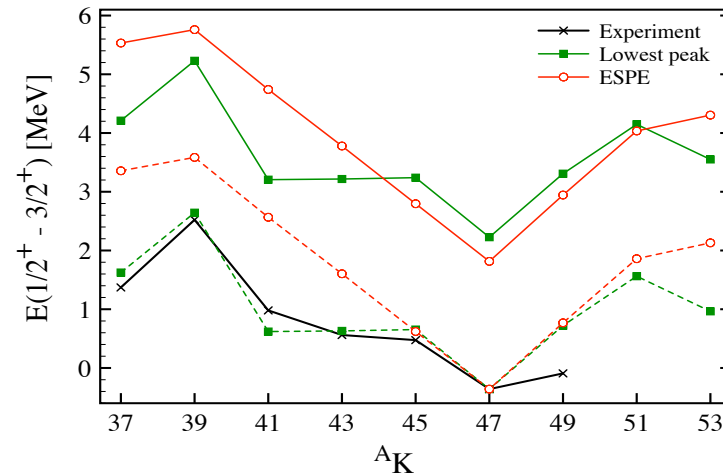
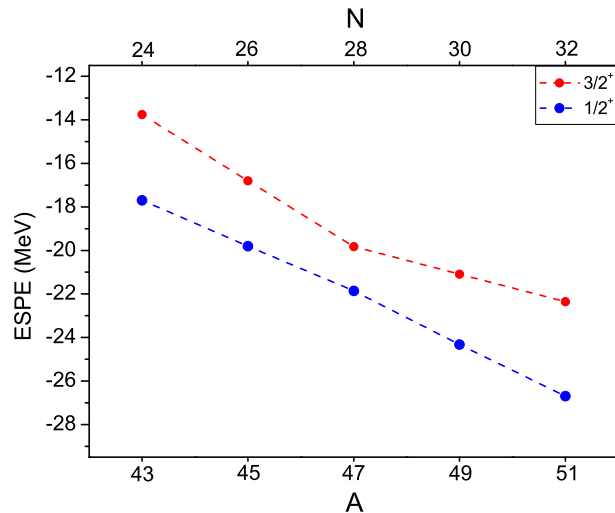
FIG. 1. (color online) Experimental energies for $1/2^+$ and $3/2^+$ states in odd-A K isotopes. Inversion of the nuclear spin is obtained in $^{47,49}\text{K}$ and reinversion back in ^{51}K . Results are

J. Papuga, et al., Phys. Rev. Lett. **110**, 172503 (2013);
J. Papuga, CB, et al., Phys. Rev. C **90**, 034321 (2014)

$A\text{K}$ isotopes

Laser spectroscopy @ ISOLDE

Change in separation described by chiral $\text{NN}_{[\text{EM}(500)]} + 3\text{NF}_{[\text{N2LO}(400)]}$:



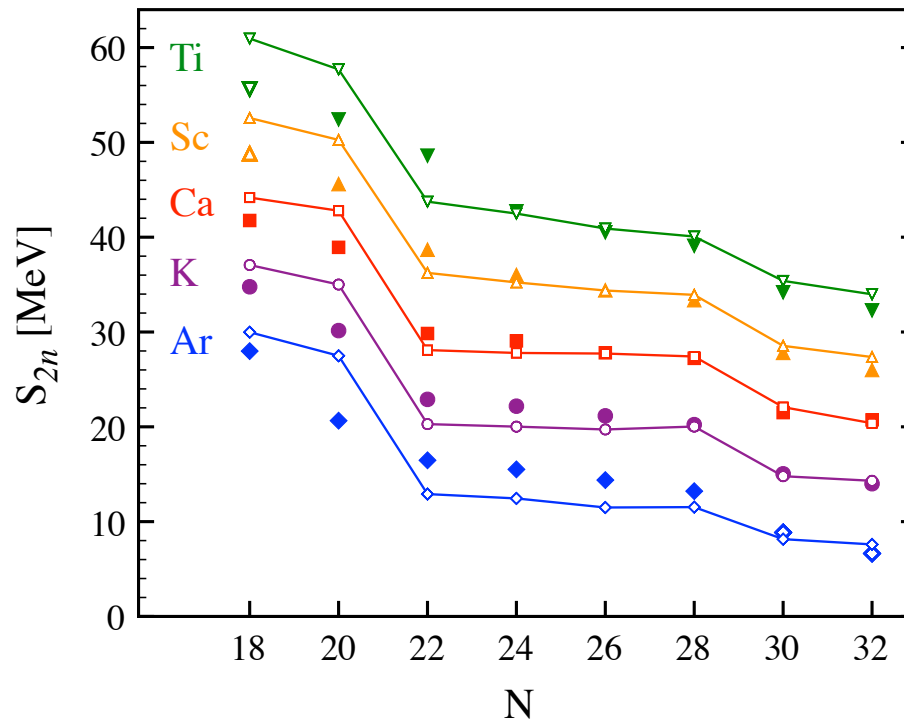
ESPE: "centroid" energies

(Gorkov calculations at 2nd order)

Neighbouring *Ar*, *K*, *Ca*, *Sc*, and *Ti* chains

V. Somà, CB *et al.* Phys. Rev. C **89**, 061301R (2014)

Two-neutron separation energies predicted by chiral $\text{NN}_{[\text{EM}(500)]} + 3\text{NF}_{[\text{N}^2\text{LO}(400)]}$:

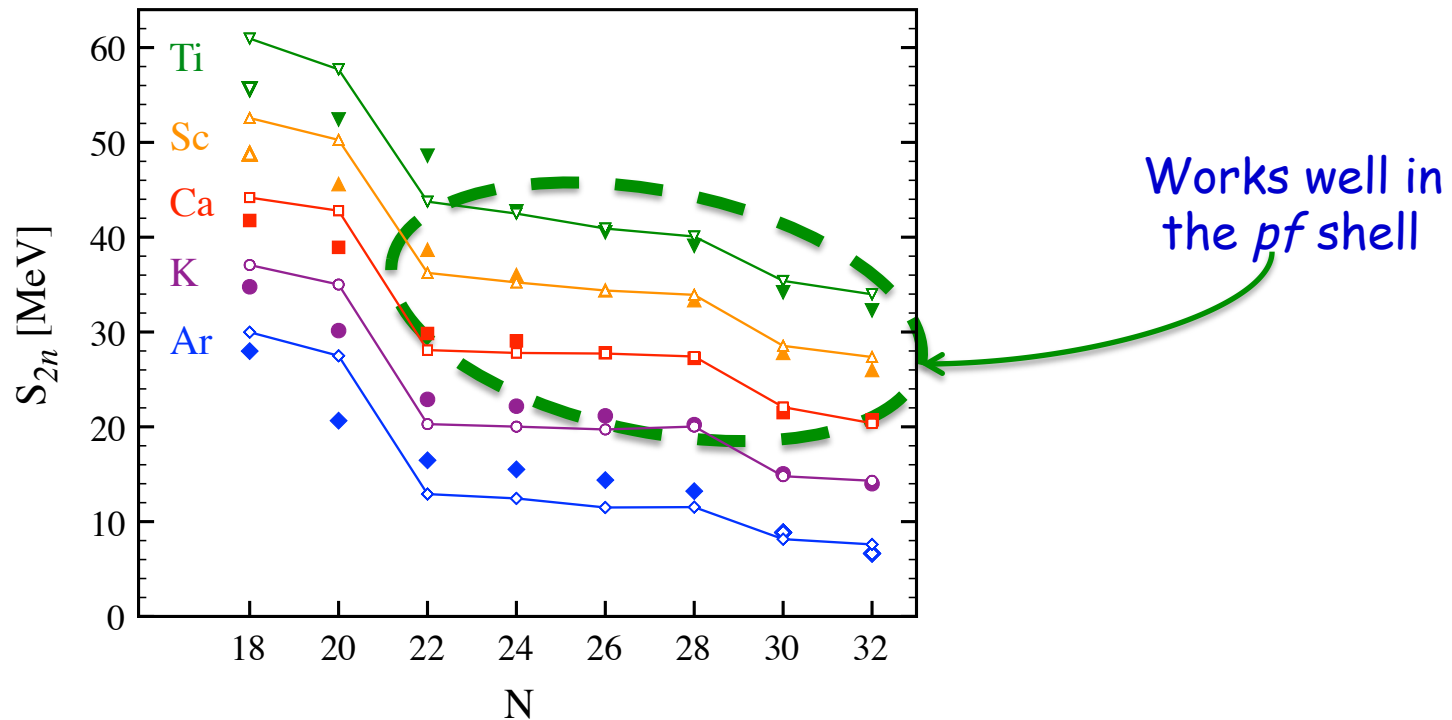


→ First *ab-initio* calculation over a contiguous portion of the nuclear chart—open shells are now possible through the Gorkov-GF formalism

Neighbouring *Ar*, *K*, *Ca*, *Sc*, and *Ti* chains

V. Somà, CB *et al.* Phys. Rev. C **89**, 061301R (2014)

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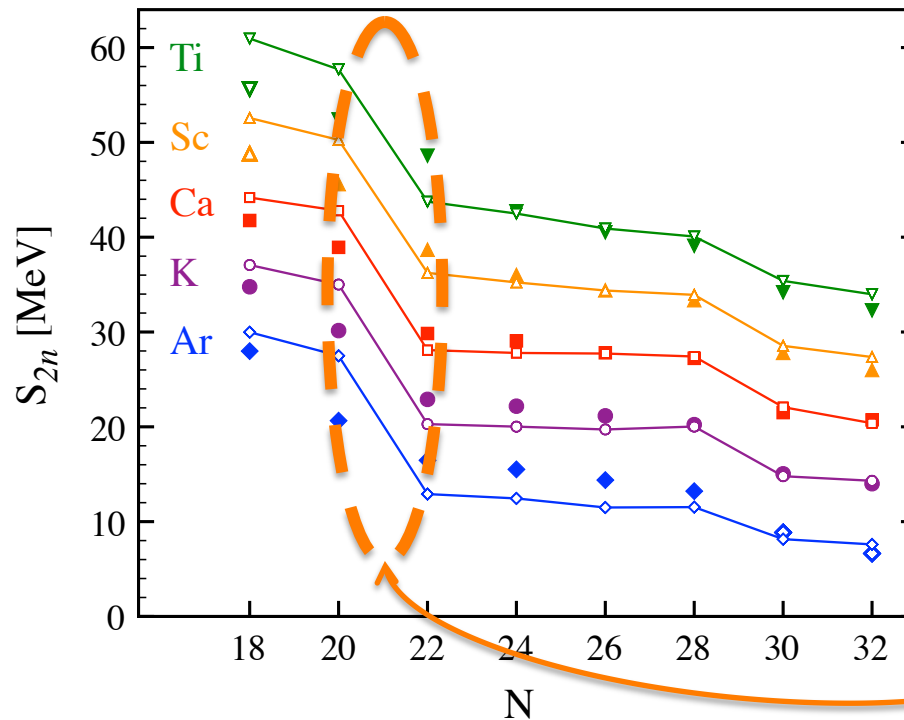


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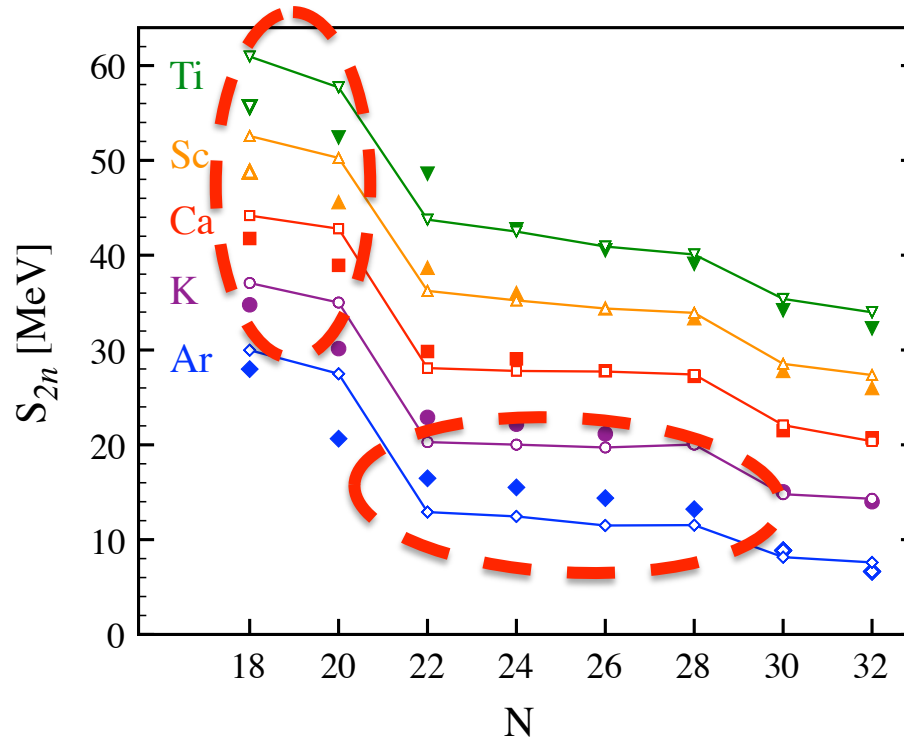
Over estimated
 $N=20$ and $Z=20$ gaps

→ First *ab-initio* calculation over a contiguous portion of the nuclear chart—open shells are now possible through the Gorkov-GF formalism

Neighbouring *Ar*, *K*, *Ca*, *Sc*, and *Ti* chains

V. Somà, CB *et al.* Phys. Rev. C **89**, 061301R (2014)

Two-neutron separation energies predicted by chiral $\text{NN}_{[\text{EM}(500)]} + 3\text{NF}_{[\text{N}^2\text{LO}(400)]}$:

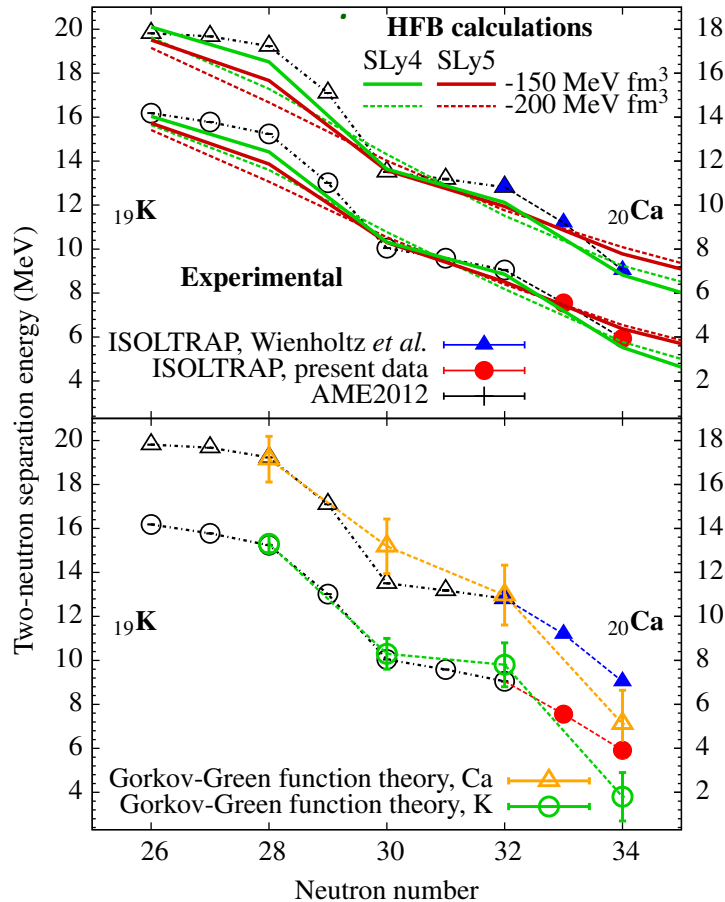


Lack of deformation due to quenched cross-shell quadrupole excitations

→ First *ab-initio* calculation over a contiguous portion of the nuclear chart—open shells are now possible through the Gorkov-GF formalism

Two-neutron separation energies for neutron rich K isotopes

M. Rosenbusch, CB, et al., PRL114, 202501 (2015)



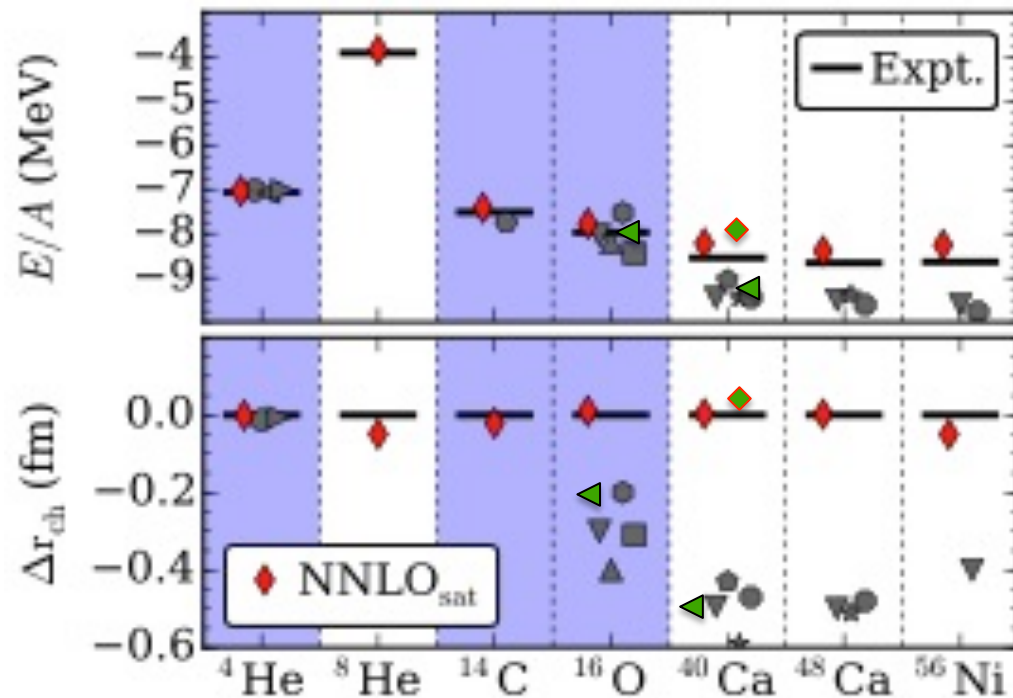
Measurements
@ ISOLTRAP

Theory tend to overestimate the gap at $N=34$, but overall good

→ Error bar in predictions are from extrapolating the many-body expansion to convergence of the model space.

NNLO-sat : a global fit up to $A \approx 24$

A. Ekström *et al.* Phys. Rev. C **91**, 051301(R) (2015)



- Constrain NN phase shifts

- Constrain radii and energies up to $A \leq 24$

→ Provides saturation up to large masses!



NNLOsat (V2 + W3) -- Grkv 2nd ord.

From SCGF:



V2-N3LO(500) + W3-NNLO(400MeV/c) w/ SRG at 2.0 fm^{-1}

A. Cipollone, CB, P. Navrátil, Phys. Rev. Lett. **111**, 062501 (2013)

V. Somà, CB *et al.* Phys. Rev. C **89**, 061301R (2014)



Radii and Binding Energies in Oxygen Isotopes: A Challenge for Nuclear Forces

V. Lapoux,^{1,*} V. Somà,¹ C. Barbieri,² H. Hergert,³ J. D. Holt,⁴ and S. R. Stroberg⁴

- New fits of chiral interactions (NNLO_{sat}) highly improve comparison to data

- Deficiencies remain for neutron rich isotopes

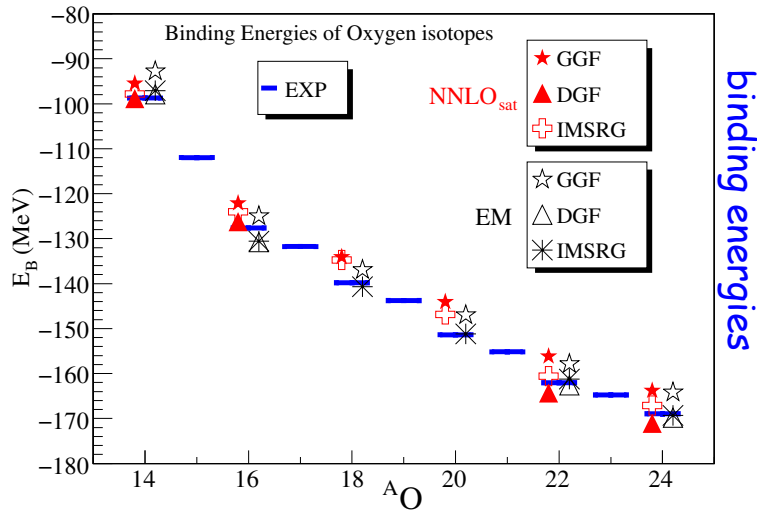
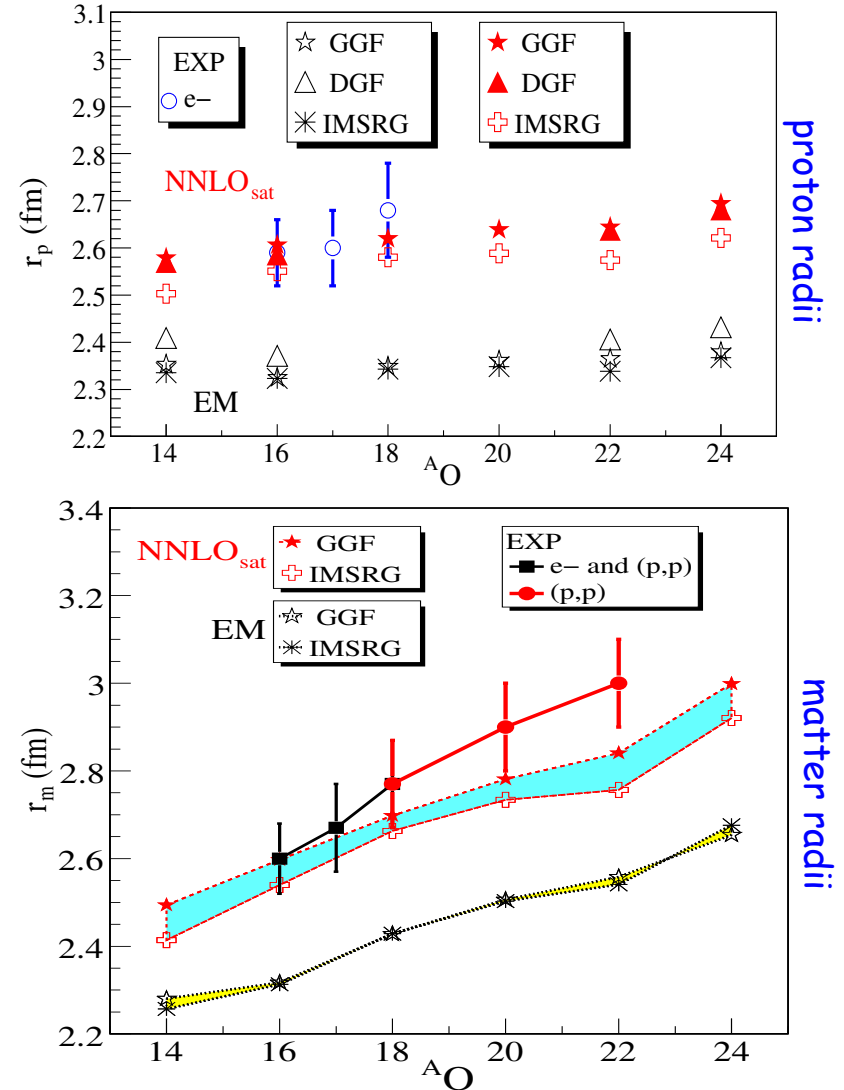


FIG. 1. Oxygen binding energies. Results from SCGF and IMSRG calculations performed with EM [20–22] and NNLO_{sat} [26] interactions are displayed along with available experimental data.

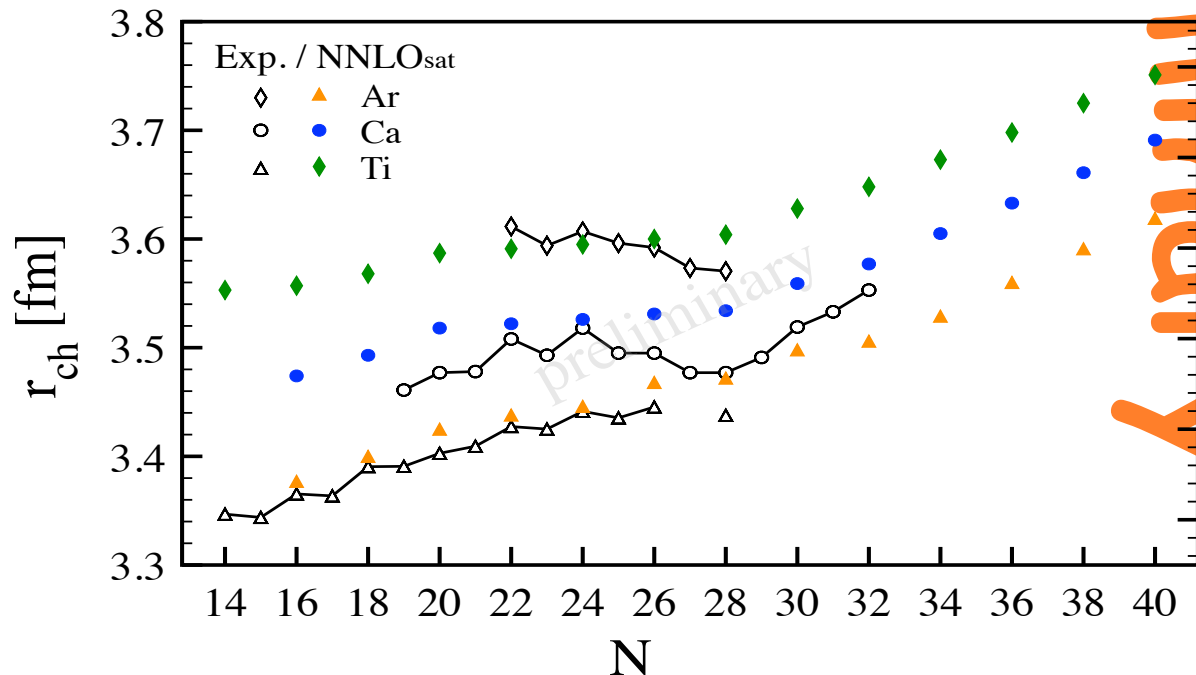
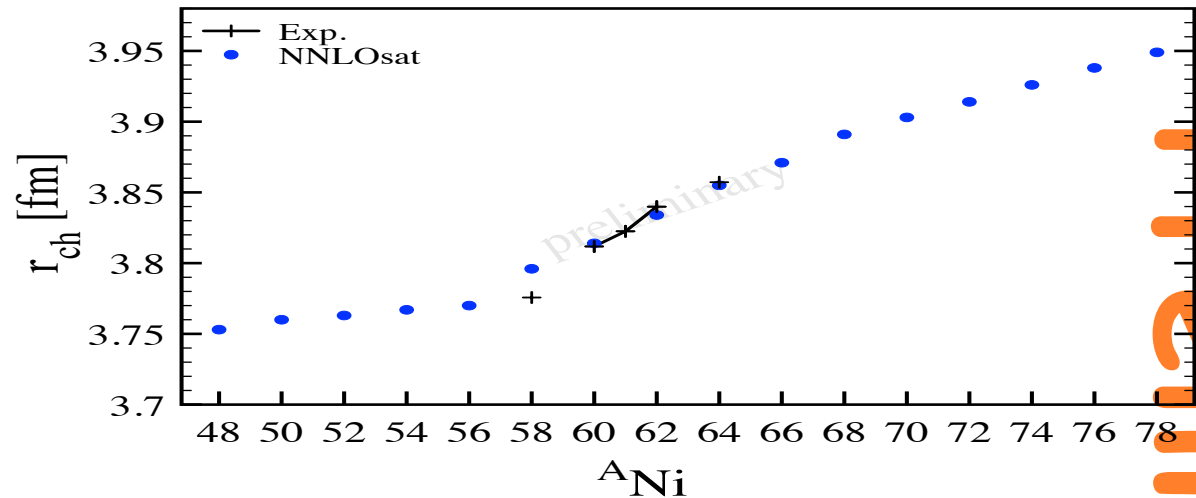


charge radii in the pf shell

Size of radii not perfect but remains overall correct throughout the *pf* shell with NNLO-sat.

This suggests that saturation is indeed under control.

→ Improvements of many-body truncations beyond 2nd order Gorkov will also be relevant.
(work in progress!)

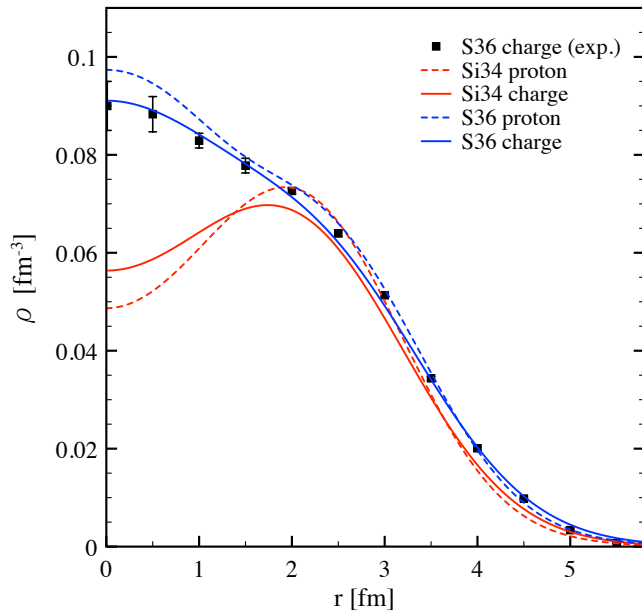


Preliminary

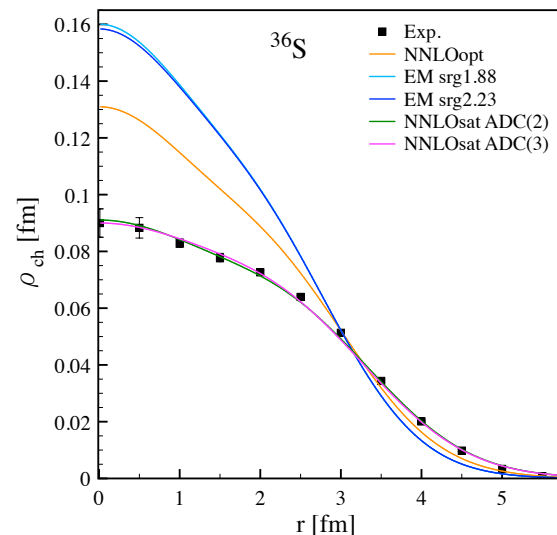
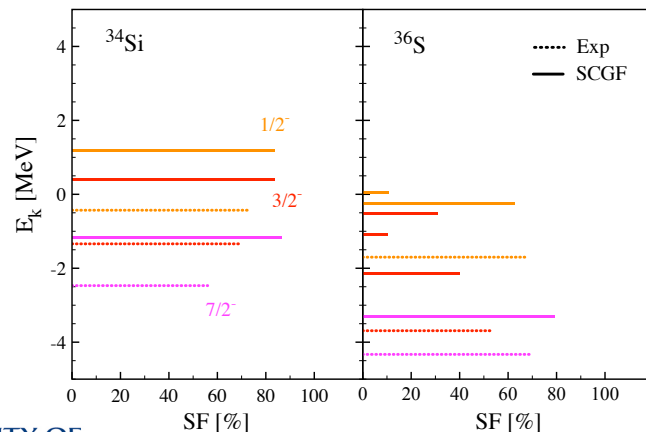
Bubble nuclei... ^{34}Si prediction

[Simon Lecluse, V. Somà, T. Duguet, CB, P. Navrátil]

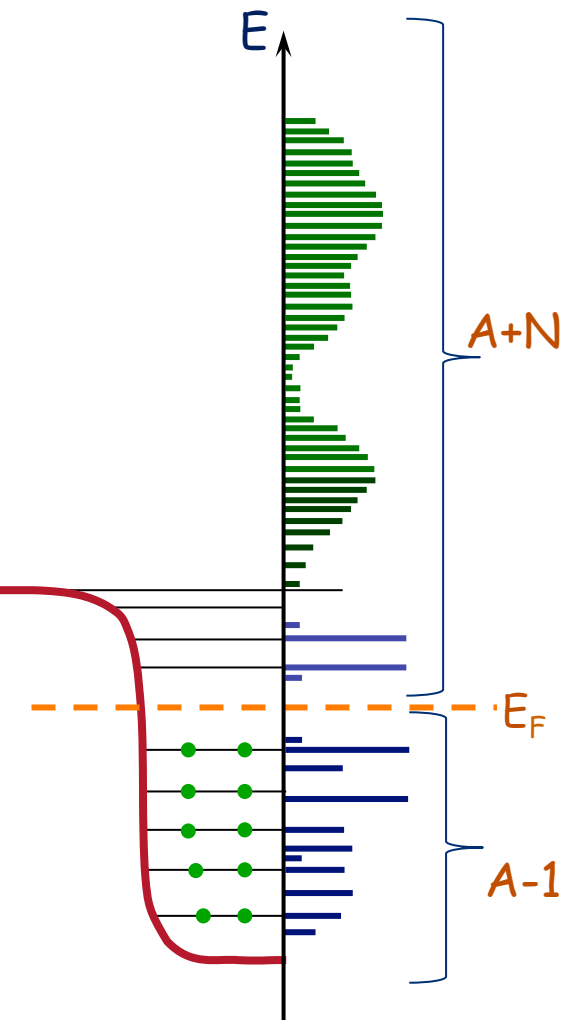
- ^{34}Si is unstable, charge distribution still unknown
- Suggested central depletion from mean-field simulations
- *Ab-initio* theory confirms predictions



Validated by charge distributions and neutron quasiparticle spectra:



Ab-initio optical potentials



Nuclear self-energy $\Sigma^*(\mathbf{r}, \mathbf{r}'; \varepsilon)$

- contains *both particle* and *hole* props.
- it is proven to be a **Feshbach opt. pot**
→ in general it is *non-local* !
- must* satisfy the **dispersion relation**:

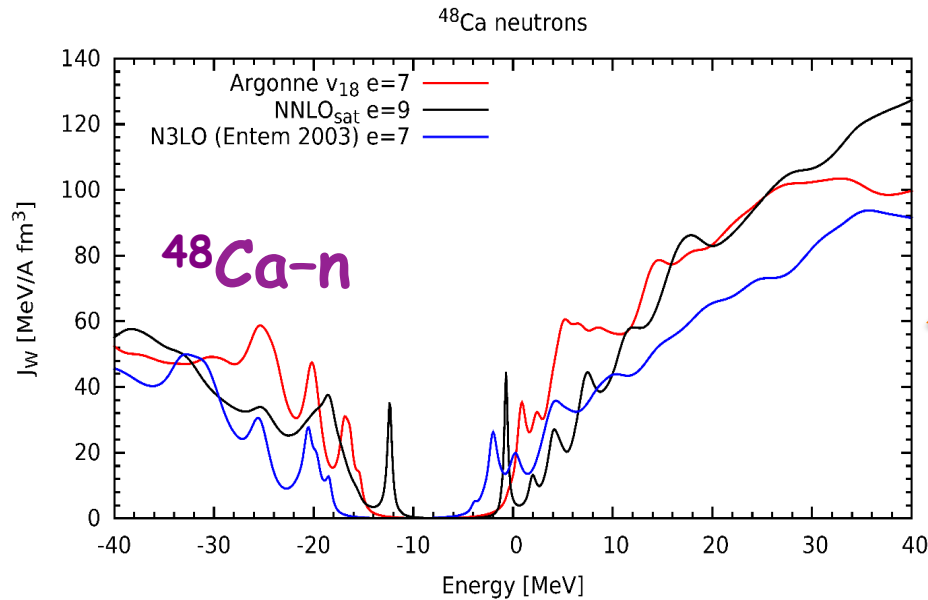
$$\Sigma^*(\mathbf{r}, \mathbf{r}'; \varepsilon) = \Sigma_{\alpha\beta}^{HF} - \frac{1}{\pi} \int_{\varepsilon_T^>}^{\infty} dE' \frac{\text{Im } \Sigma^*(\mathbf{r}, \mathbf{r}'; E')}{\varepsilon - E' + i\eta} + \frac{1}{\pi} \int_{-\infty}^{\varepsilon_T^<} dE' \frac{\text{Im } \Sigma^*(\mathbf{r}, \mathbf{r}'; E')}{\varepsilon - E' - i\eta}$$

$$\frac{1}{x \pm i\eta} = \mathcal{P} \frac{1}{x} \mp i\pi\delta(x)$$

$$\theta(\pm\tau) = \mp \lim_{\eta \rightarrow 0^+} \frac{1}{2\pi i} \int_{-\infty}^{+\infty} d\omega \frac{e^{-i\omega\tau}}{\omega \pm i\eta}$$

proper boundary conditions are driven by the causality principle

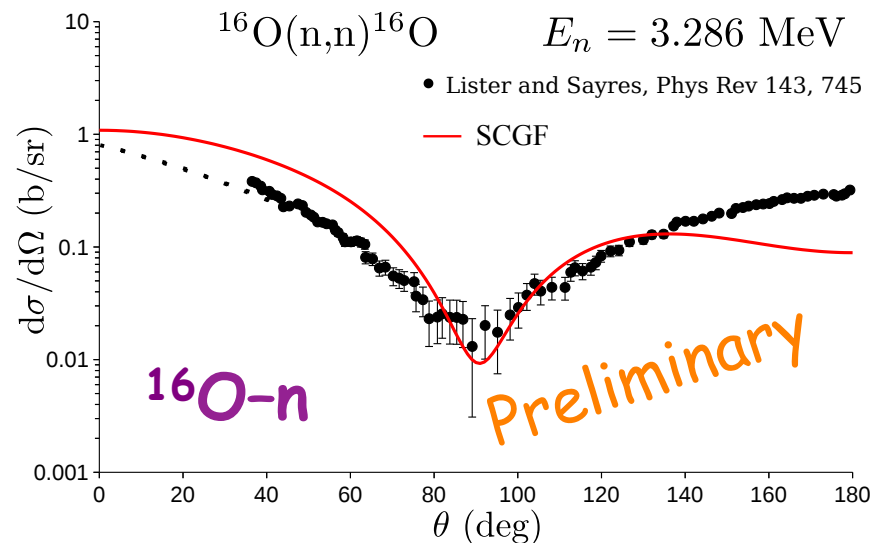
Ab-initio optical potentials



Overall absorption from OP
 (integral of imaginary part)

Low energy scattering

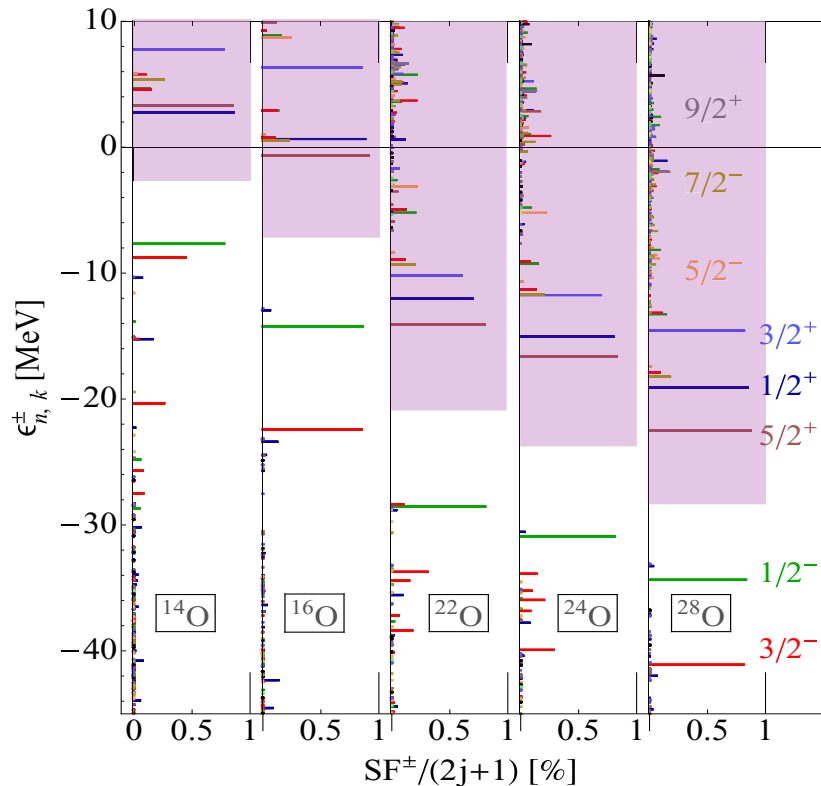
[Andrea Idini, CB, in prep]



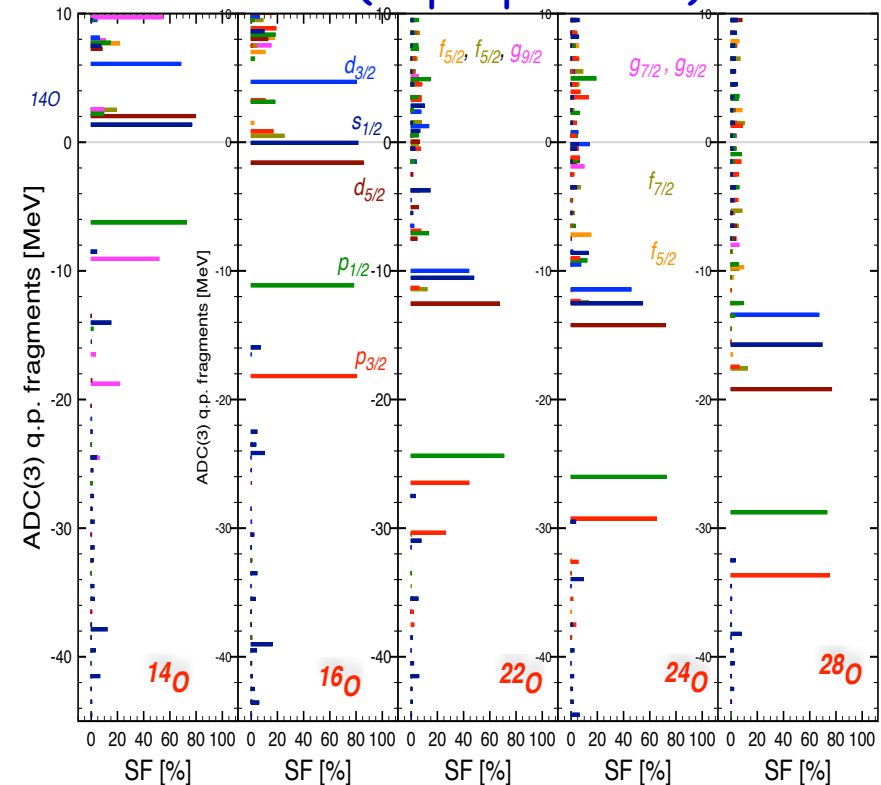
Proton spectral strength in Oxygen

A. Cipollone, CB, P. Navrátil, Phys. Rev. Lett. **111**, 062501 (2013)
and Phys. Rev. C **92**, 014306 (2015)
and *in preparation*

EM-N3LO(500)/3NF-NNLO(400)



NNLO-sat (in preparation)

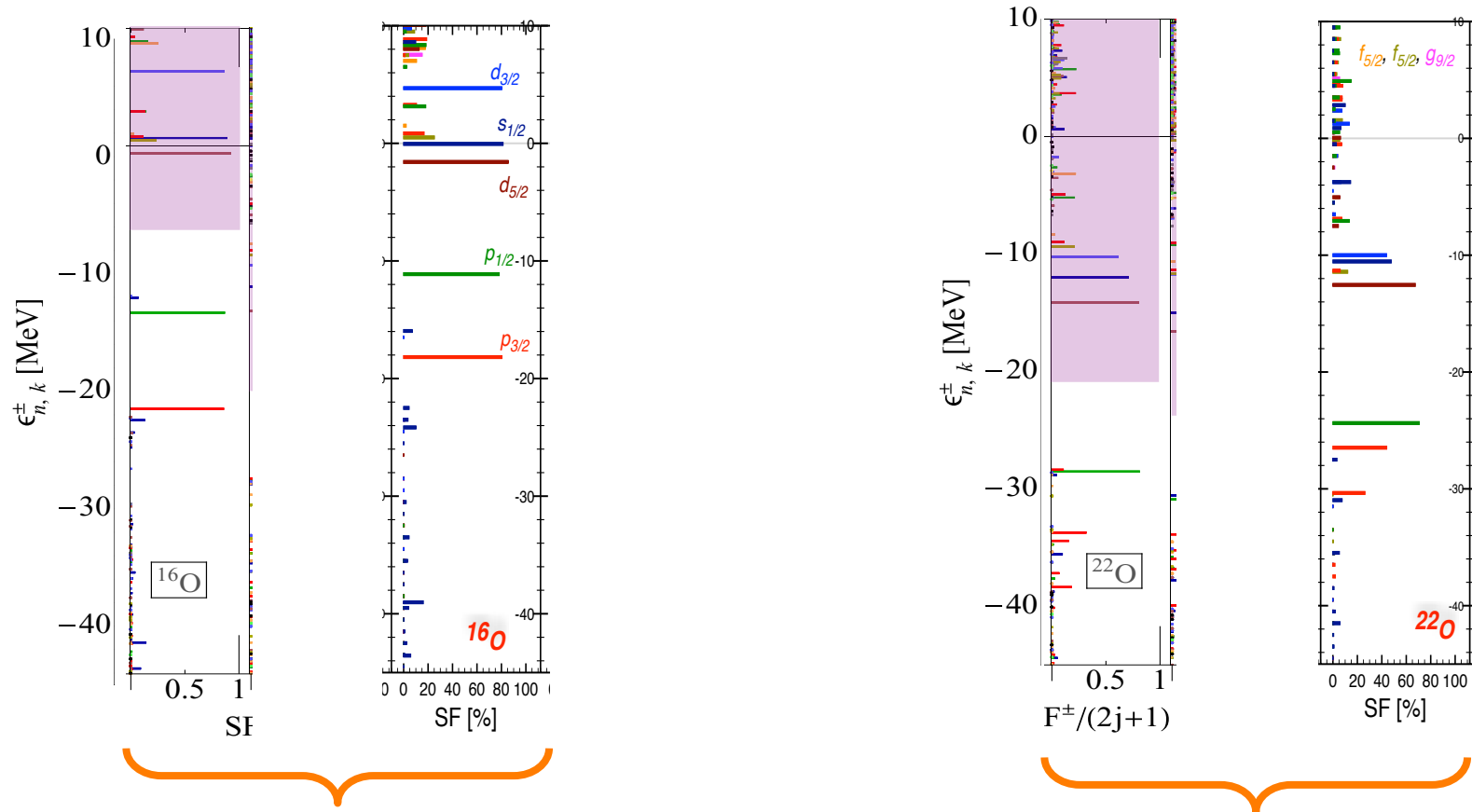


Spectroscopic factors

Proton spectral strength in Oxygen

A. Cipollone, CB, P. Navrátil, Phys. Rev. Lett. **111**, 062501 (2013)
and Phys. Rev. C **92**, 014306 (2015)
and *in preparation*

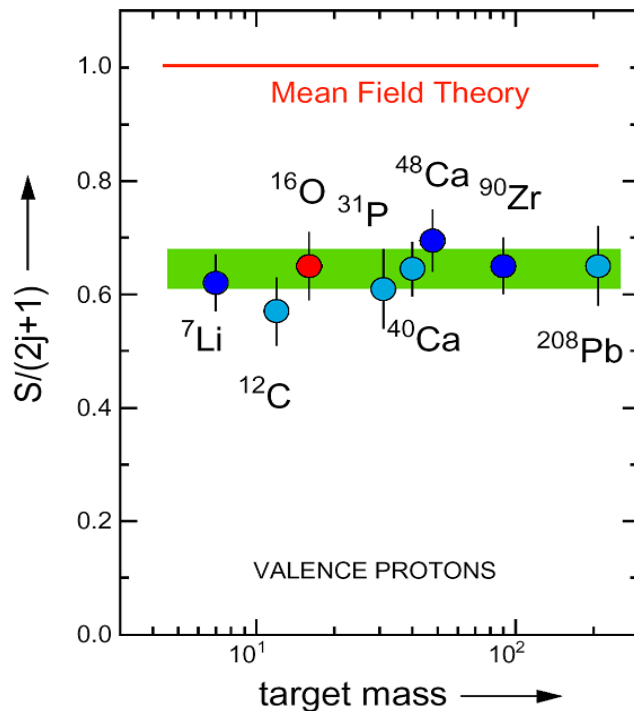
More in detail:



Quenching of SF in stable nuclei

Nucl. Phys. A553 (1993) 297c

NIKHEF:



A common *misconception* about SRC :

"The quenching is constant over all stable nuclei, so it must be a short-range effect"

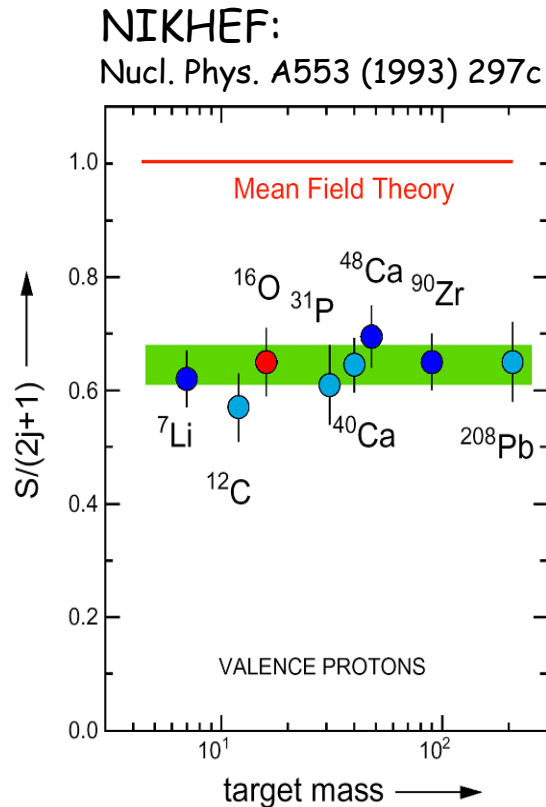


*Actually, **NO!***

All calculations show that SRC have just a small effect at the Fermi surface. And the correlation to the experimental p-h gap is much more important.

[W. Dickhoff, CB, Prog. Part. Nucl. Phys. **52**, 377 (2004)]

Quenching of SF in stable nuclei



- Short-range correlations oriented methods:

- VMC [Argonne, '94]
- GF(SRC) [St.Louis-Tübingen '95]
- FHNC/SOC [Pisa '00]

$S_{p1/2}$

$S_{p3/2}$

0.90

0.91

0.90

0.89

- Including particle-phonon couplings:

- GF(FRPA) [St.Louis '01]

[CB et al., Phys. Rev. C **65**, (02)]

0.77

0.72

- Experiment:

0.63

0.67 ± 0.07
(estimated uncertainty)

SRC are present and verified experimentally

BUT they are NOT the dominant mechanism for quenching SF!!!

Quenching of absolute spectroscopic factors

[CB, Phys. Rev. Lett. **103**, 202520 (2009)]

...with analogous conclusions for ^{48}Ca

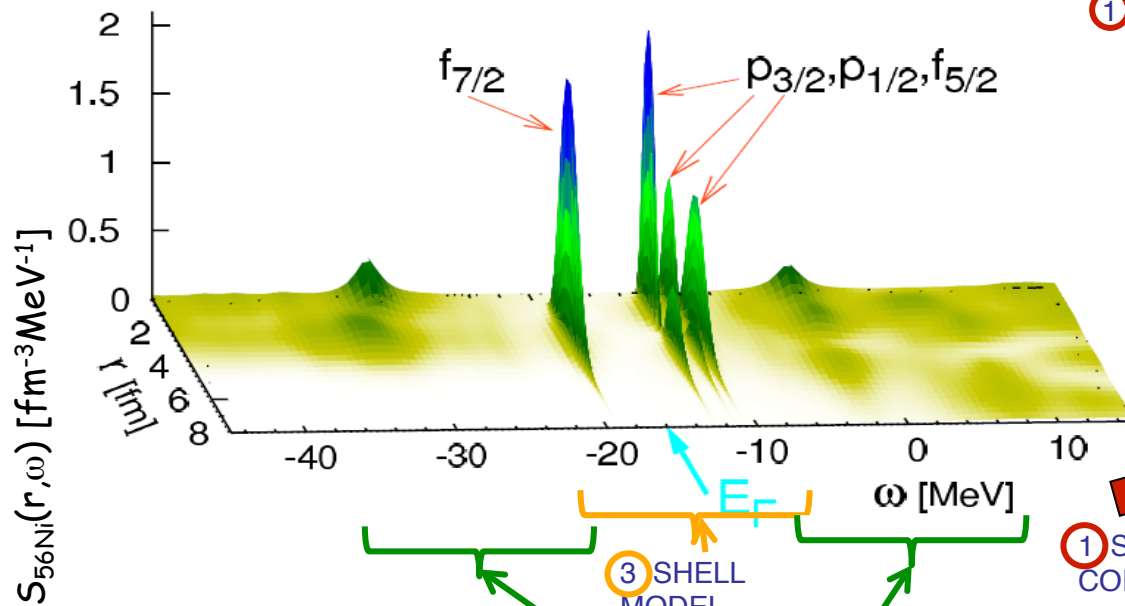
Overall quenching of *spectroscopic factors* is driven by:

SRC → ~10%
part-vibr. coupling → dominant
"shell-model" → in open shell

^{57}Ni

^{55}Ni

	10 osc. shells			Exp. [30]	1p0f space		
	FRPA (SRC)	full FRPA	FRPA + ΔZ_α		FRPA	SM	ΔZ_α
^{57}Ni :							
$\nu 1p_{1/2}$	0.96	0.63	0.61		0.79	0.77	-0.02
$\nu 0f_{5/2}$	0.95	0.59	0.55		0.79	0.75	-0.04
$\nu 1p_{3/2}$	0.95	0.65	0.62	0.58(11)	0.82	0.79	-0.03
^{55}Ni :							
$\nu 0f_{7/2}$	0.95	0.72	0.69		0.89	0.86	-0.03
	①	② + ③			③		



$$Z_\alpha = \int d^3r |\psi_\alpha^{overlap}(\mathbf{r})|^2 = \frac{1}{1 - \left. \frac{\partial \Sigma_{\hat{\alpha}\hat{\alpha}}(\omega)}{\partial \omega} \right|_{\omega=\epsilon_\alpha}}$$

① SHORT RANGE CORRELATIONS

② PARTICLE-VIBRATION COUPLING

③ SHELL MODEL

Dependence of Spect. Fact. from p-h gap

N3LO needs a monopole correction to fix the p-h gap:

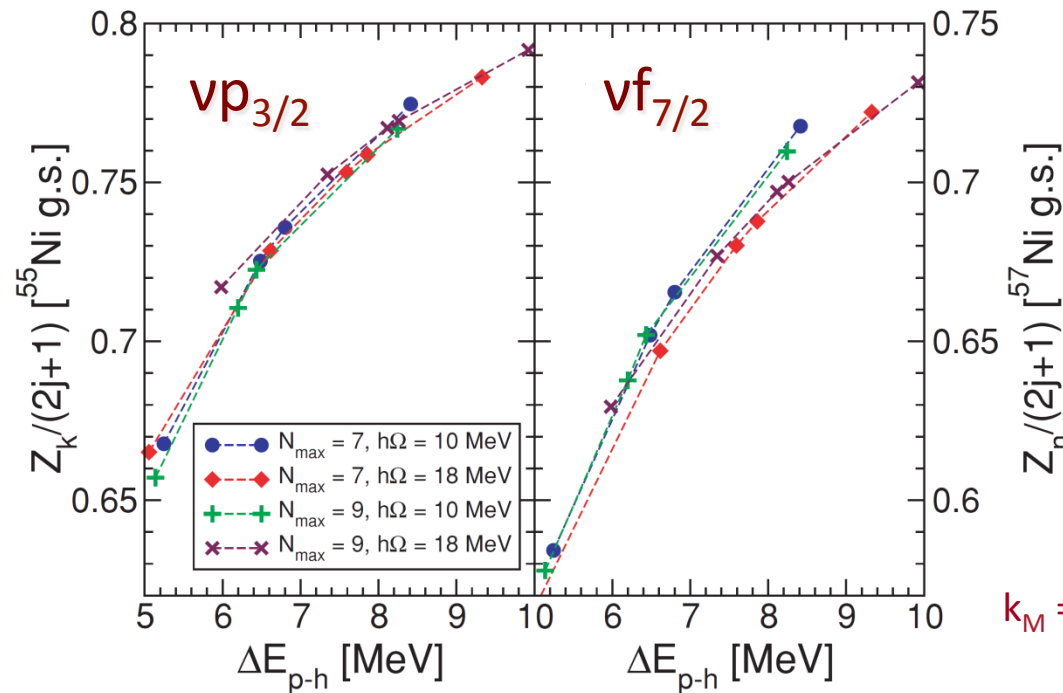
$$\left\{ \begin{array}{l} \Delta V_{fr}^T \rightarrow \Delta V_{fr}^T - (-1)^T \kappa_M, \\ \Delta V_{ff}^T \rightarrow \Delta V_{ff}^T - 1.5(1 - T) \kappa_M, \end{array} \right.$$

$$r \equiv p_{3/2}, p_{1/2}, f_{5/2}$$

$$f \equiv f_{7/2}$$

Experimental Eph

is found for $\kappa_M = 0.57$



$\kappa_M = 0.4-0.7$ MeV

small $\kappa_M \leftarrow$

\rightarrow large κ_M

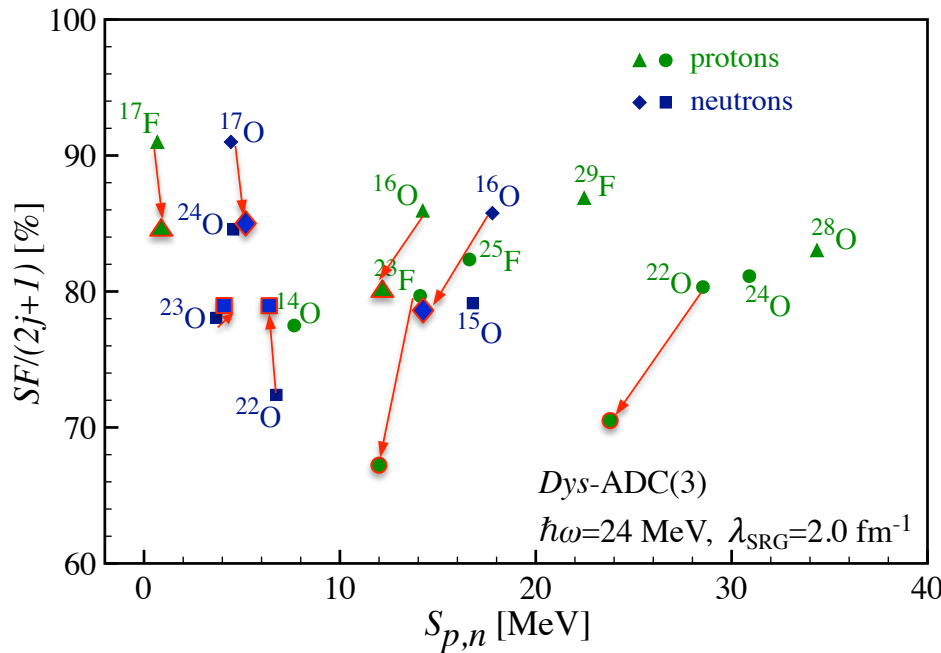
N3LO interaction + monopole corr.

[CB, M.Hjorth-Jensen, Pys.Rev.C79, 064313 (2009)]

Z/N asymmetry dependence of SFs - Theory

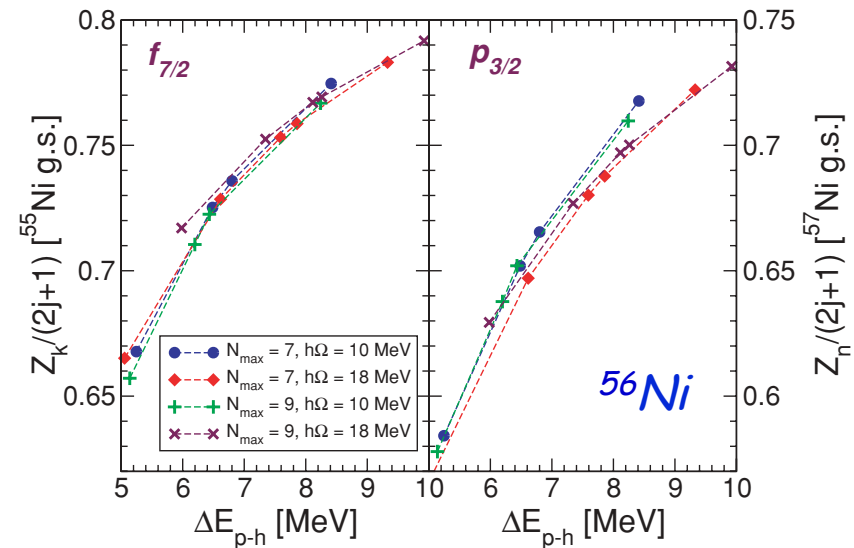
Ab-initio calculations explain (a very weak) the Z/N dependence but the effect is much lower than suggested by direct knockout

Rather the quenching is high correlated to the gap at the Femi surface.



A. Cipollone, CB, P Navrátil
Phys. Rev. C **92**, 014306 (2015)

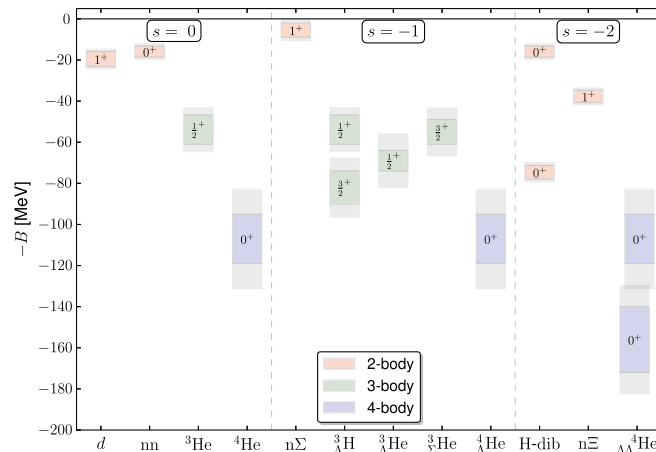
Spectroscopic factor are strongly correlated to p-h gaps:



CB, M. Hjorth-Jensen,
Phys. Rev. C **79**, 064313 (2009)

Study of nuclear interactions from Lattice QCD

Other paths in LQCD, see:



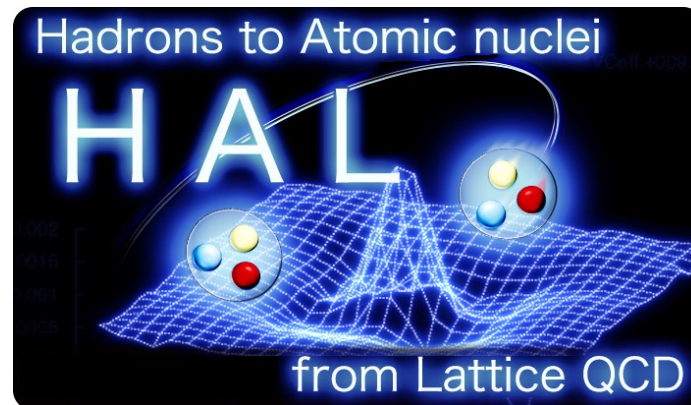
nuclei, the deuteron and the dineutron in $2+1$ flavor QCD with $m_\pi = 0.51$ GeV and $m_N = 1.32$ GeV. The bound states are distinguished from the attractive scattering states by investigating the spatial volume dependence of the energy shift ΔE_L . In the infinite spatial volume limit we obtain

$$-\Delta E_\infty = \begin{cases} 43(12)(8) & \text{MeV for } ^4\text{He}, \\ 20.3(4.0)(2.0) & \text{MeV for } ^3\text{He}, \\ 11.5(1.1)(0.6) & \text{MeV for } ^3\text{S}_1, \\ 7.4(1.3)(0.6) & \text{MeV for } ^1\text{S}_0. \end{cases} \quad (17)$$

PACS-CS PRD **86**, 074514 (2012)

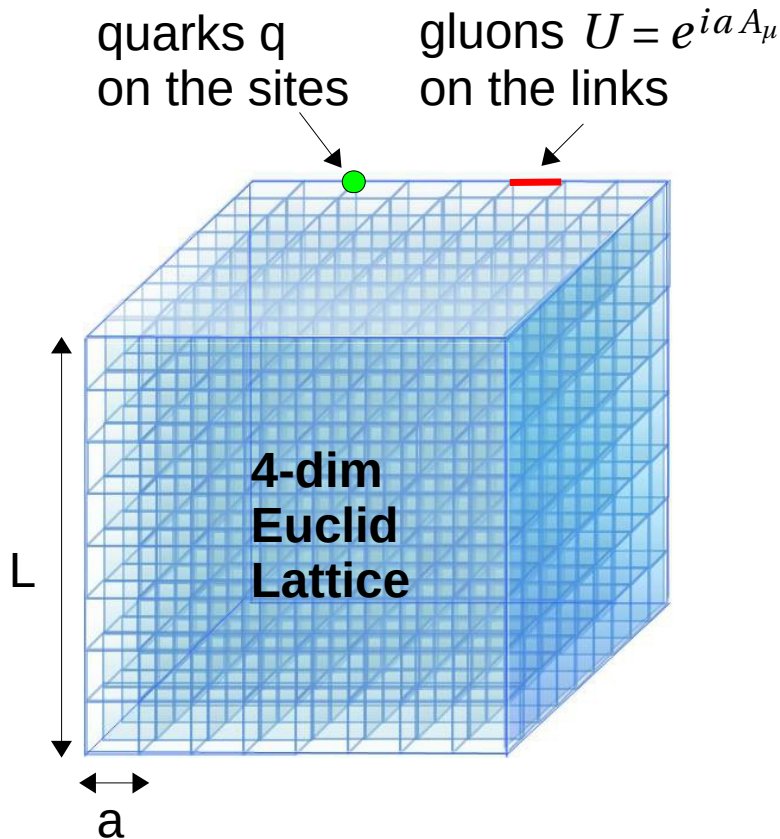
Study of nuclear interactions from Lattice QCD

In collaboration with:



Lattice QCD

$$L = -\frac{1}{4} G_{\mu\nu}^a G_a^{\mu\nu} + \bar{q} \gamma^\mu (i \partial_\mu - g t^a A_\mu^a) q - m \bar{q} q$$



Vacuum expectation value

$$\begin{aligned} \langle O(\bar{q}, q, U) \rangle &= \int dU d\bar{q} dq e^{-S(\bar{q}, q, U)} O(\bar{q}, q, U) \\ &= \int dU \det D(U) e^{-S_U(U)} O(\overset{\text{quark propagator}}{D^{-1}(U)}) \\ &= \lim_{N \rightarrow \infty} \frac{1}{N} \sum_{i=1}^N O(D^{-1}(U_i)) \end{aligned}$$

path integral

$\{U_i\}$: ensemble of gauge conf. U
generated w/ probability $\det D(U) e^{-S_U(U)}$

- ★ Well defined (regularized)
- ★ Manifest gauge invariance

- ★ Fully non-perturbative
- ★ Highly predictive

The HAL-QCD Method

Define a general potential $U(\mathbf{r}, \mathbf{r}')$ which is **non-local** but **energy independent** up to inelastic threshold, such that:

$$-\frac{\nabla^2}{2\mu} \varphi_{\vec{k}}(\vec{r}) + \int d\vec{r}' U(\vec{r}, \vec{r}') \varphi_{\vec{k}}(\vec{r}') = E_{\vec{k}} \varphi_{\vec{k}}(\vec{r})$$

for the **Nambu-Bethe-Salpeter (NBS)** wave function,

$$\varphi_{\vec{k}}(\vec{r}) = \sum \langle 0 | B_i(\vec{x} + \vec{r}, t) B_j(\vec{x}, t) | B = 2, \vec{k} \rangle$$

Operationally, measure the **4-pt function** on the QCD Lattice

$$\psi(\vec{r}, t) = \sum_{\vec{x}} \langle 0 | B_i(\vec{x} + \vec{r}, t) B_j(\vec{x}, t) J(t_0) | 0 \rangle = \sum_{\vec{k}} A_{\vec{k}} \varphi_{\vec{k}}(\vec{r}) e^{-W_{\vec{k}}(t-t_0)} + \dots$$

and extract $U(\mathbf{r}, \mathbf{r}')$ from:

$$\left\{ 2M_B - \frac{\nabla^2}{2\mu} \right\} \psi(\vec{r}, t) + \int d\vec{r}' U(\vec{r}, \vec{r}') \psi(\vec{r}', t) = -\frac{\partial}{\partial t} \psi(\vec{r}, t)$$

A **local potential** $V(\mathbf{r})$ is then obtained through a derivative expansion of $U(\mathbf{r}, \mathbf{r}')$, which **must give the same observables** of the LQCD simulation:

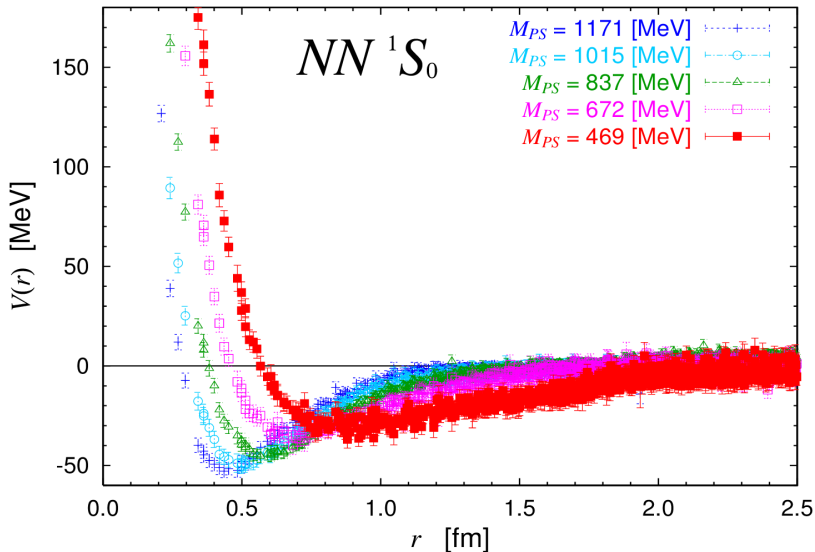
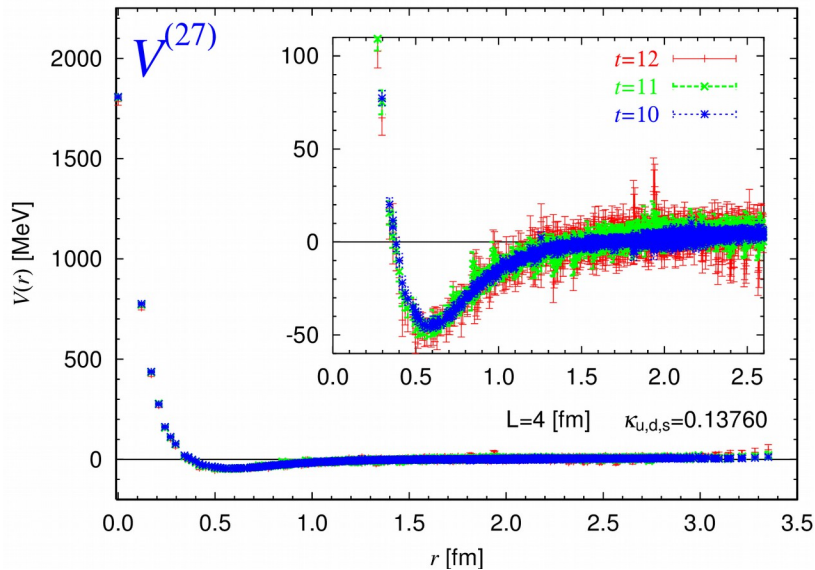
$$U(\vec{r}, \vec{r}') = \delta(\vec{r} - \vec{r}') V(\vec{r}, \nabla) = \delta(\vec{r} - \vec{r}') \{ V(\vec{r}) + \mathcal{O}(\nabla) + \mathcal{O}(\nabla^2) + \dots \}$$

$$\rightarrow V(\vec{r}) = \frac{1}{2\mu} \frac{\nabla^2 \psi(\vec{r}, t)}{\psi(\vec{r}, t)} - \frac{\frac{\partial}{\partial t} \psi(\vec{r}, t)}{\psi(\vec{r}, t)} - 2M_B$$

Tensor/Yukawa
force in S-D

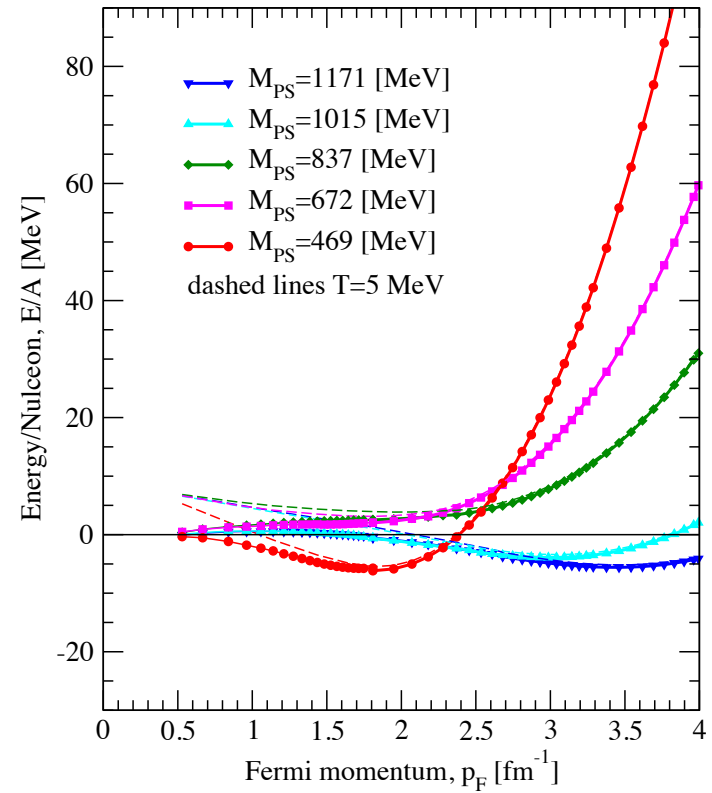
Spin-orbit
force, P waves

Two-Nucleon HAL potentials



Quark mass dependence of $V(r)$ for NN partial wave (1S_0 , 3S_1 , 3S_1 - 3D_1)

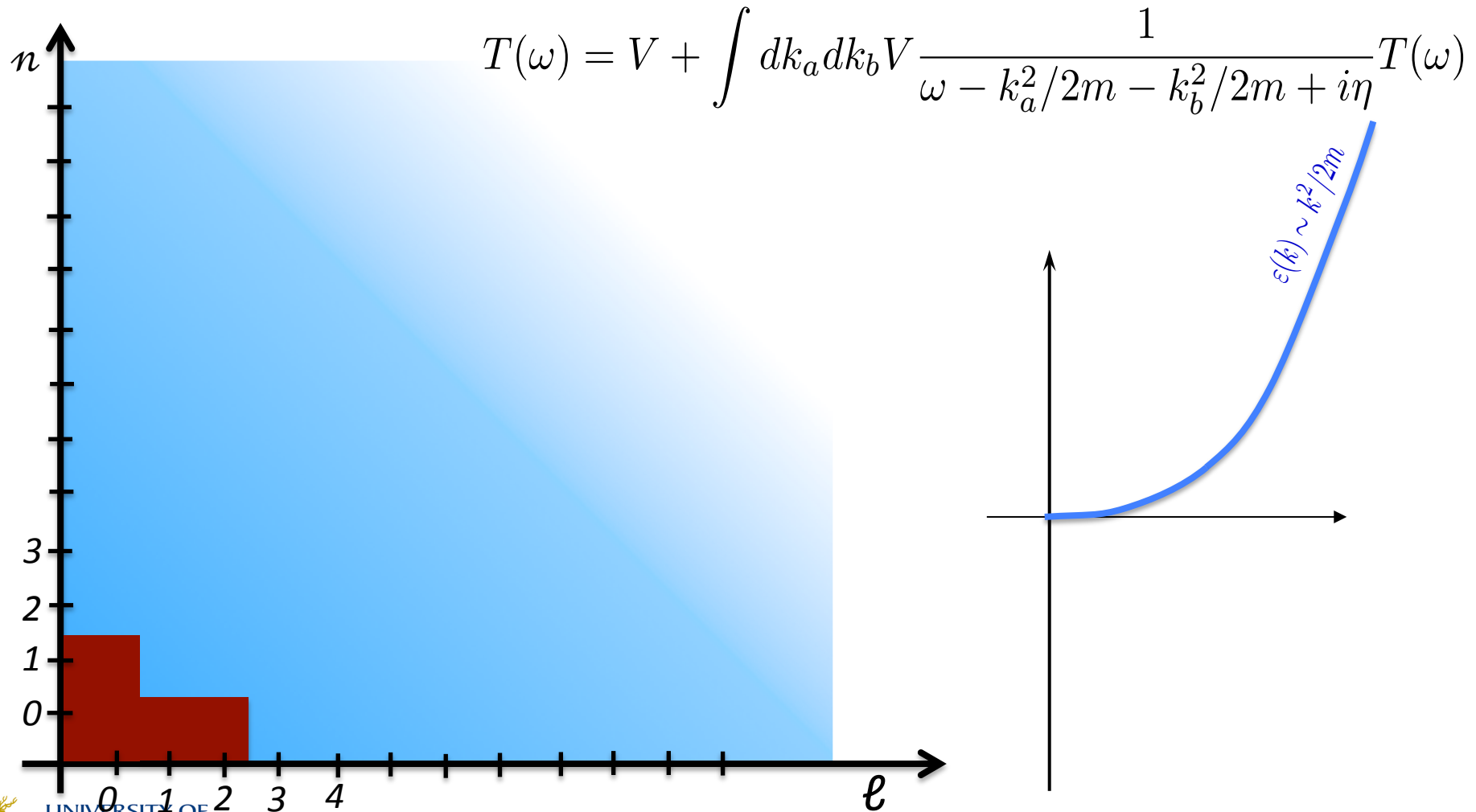
→ Potentials become stronger m_π as decreases.



T. Inoue *et al.*,
Phys. Rev. Lett. **111** 112503 (2013).

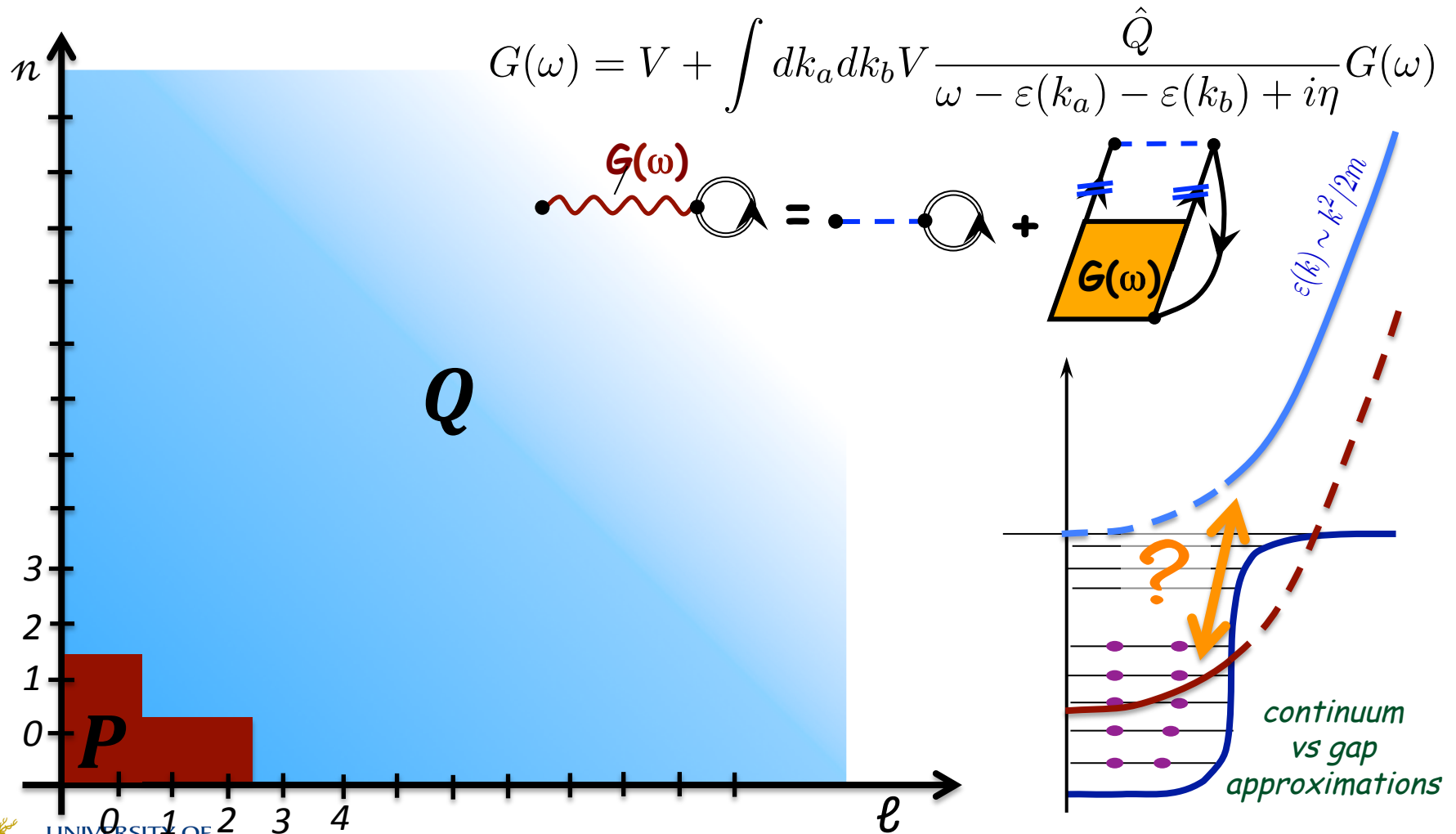
Analysis of Brueckner HF

Scattering of two nucleon in free space:



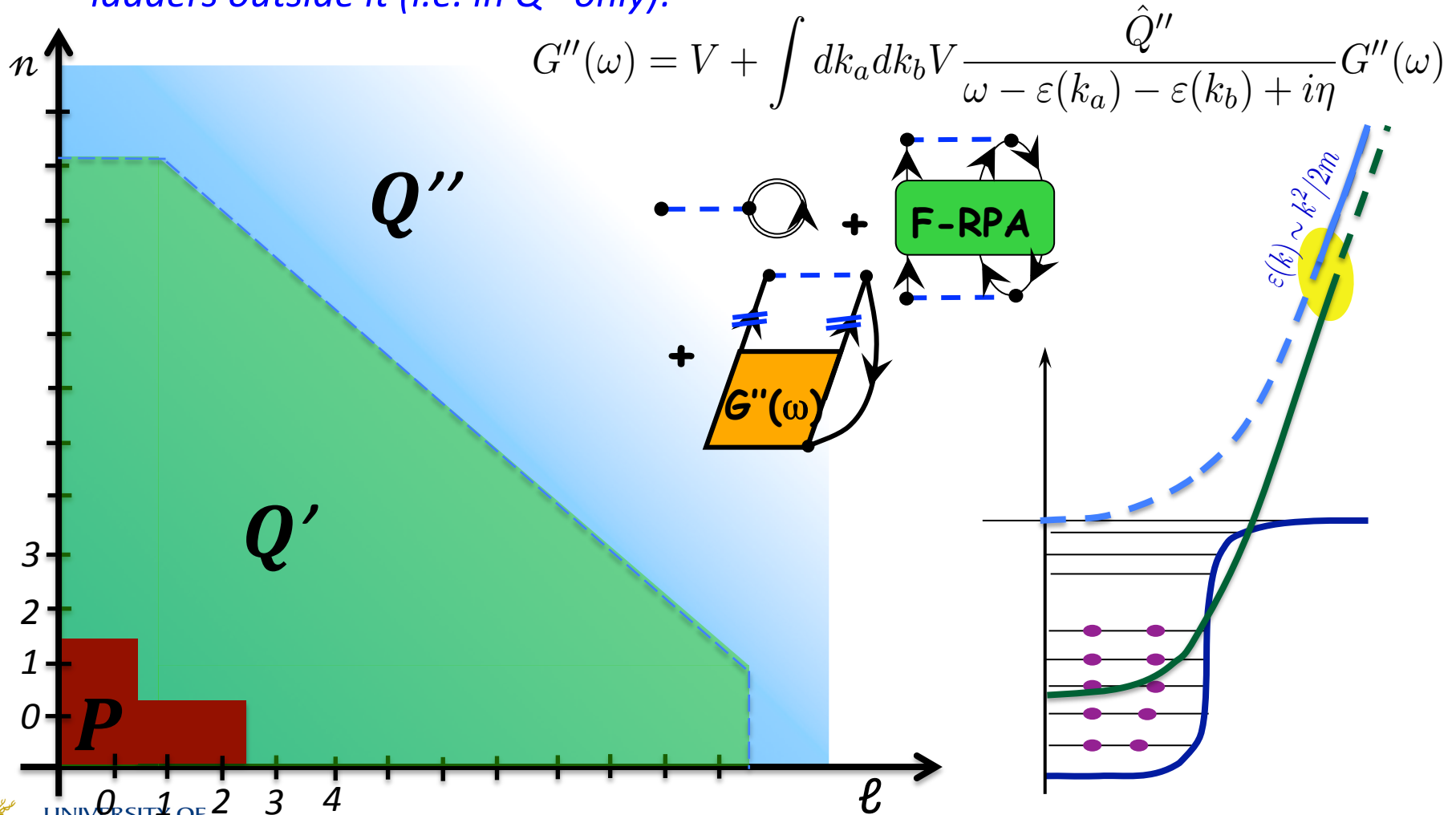
Analysis of Brueckner HF

Scattering of two nucleons outside the Fermi sea (\rightarrow BHF):



Mixed SCGF-Brueckner approach

Solve full many-body dynamics in model space ($P+Q'$) and the Goldstone's ladders outside it (i.e. in Q'' only):



Treating short-range corr. with a G-matrix

- The short-range core can be treated by summing ladders outside the model space:

$$\Sigma_{\alpha\beta}^{\text{MF}}(\omega) = i \sum_{\gamma\delta} \int \frac{d\omega'}{2\pi} G_{\alpha\gamma, \delta\beta}(\omega + \omega') g_{\delta\gamma}(\omega') = \text{diagram}$$

The diagram shows a red wavy line labeled $G(\omega)$ connecting two black dots. The right dot is part of a self-energy loop (a circle with an arrow).

$$\Sigma^*(\mathbf{r}, \mathbf{r}'; \omega) = \Sigma^{\text{MF}}(\mathbf{r}, \mathbf{r}'; \omega) + \tilde{\Sigma}(\mathbf{r}, \mathbf{r}'; \omega) .$$

$$Z_\alpha = \int d\mathbf{r} |\psi_\alpha^{A\pm 1}(\mathbf{r})|^2 = \frac{1}{1 - \frac{\partial \Sigma_{\hat{a}\hat{a}}^*(\omega)}{\partial \omega}} \bigg|_{\omega = \pm(E_\alpha^{A\pm 1} - E_0^A)}$$

Two contributions to the derivative:

- $\Sigma_{\alpha\beta}^{\text{MF}}(\omega)$ is due to scattering to (high-k) states in the Q space
- $\Sigma(\mathbf{r}, \mathbf{r}'; \omega)$ accounts for low-energy (long range) correlations

(Galitskii-Migdal-Boffi-) Koltun sumrule

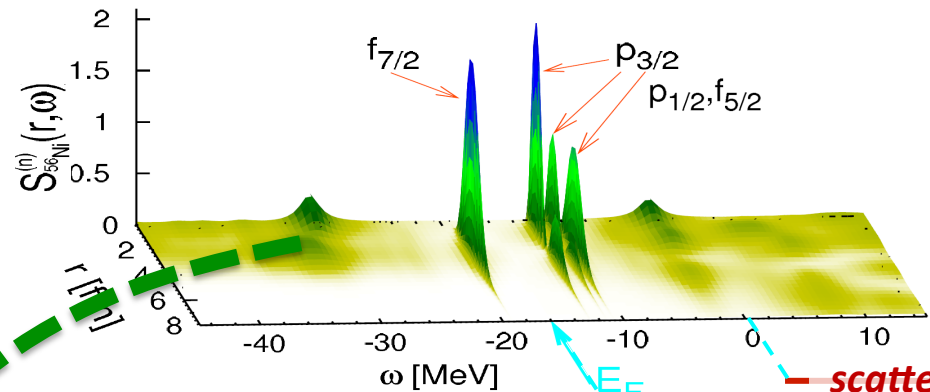
✱ Koltun sum rule (with NNN interactions):

$$\sum_{\alpha} \frac{1}{\pi} \int_{-\infty}^{\epsilon_F} d\omega \omega \operatorname{Im} G_{\alpha\alpha}(\omega) = \langle \Psi_0^N | \hat{T} | \Psi_0^N \rangle + 2 \langle \Psi_0^N | \hat{V} | \Psi_0^N \rangle + 3 \langle \Psi_0^N | \hat{W} | \Psi_0^N \rangle$$

two-body

three-body

$$E_0^N = \frac{1}{2\pi} \int_{-\infty}^{\epsilon_F} d\omega \sum_{\alpha\beta} (T_{\alpha\beta} + \omega \delta_{\alpha\beta}) \operatorname{Im} G_{\beta\alpha}(\omega) - \frac{1}{2} \langle \Psi_0^N | \hat{W} | \Psi_0^N \rangle$$

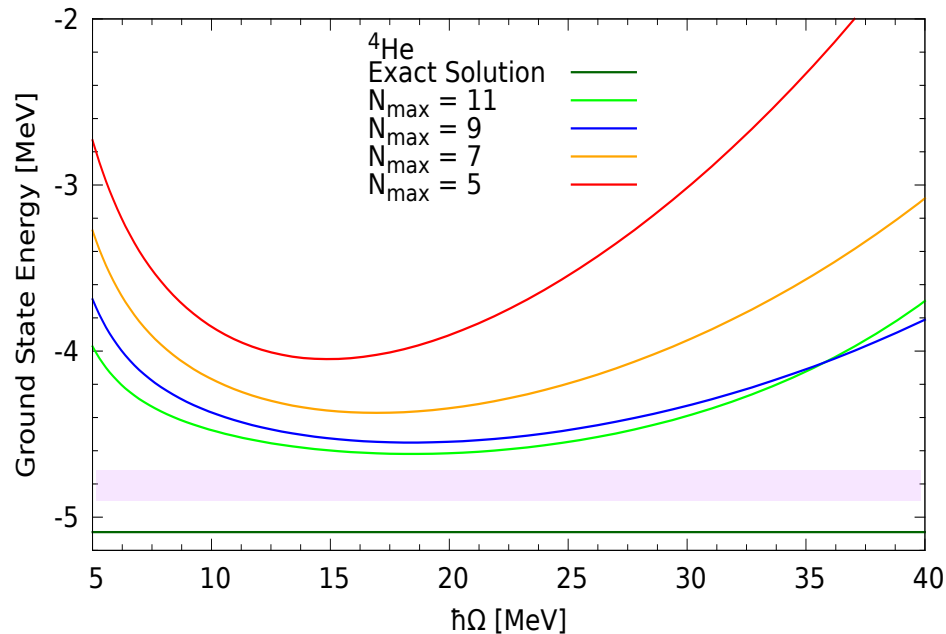


High-k and missing energy tail from SRC... (currently neglected in calculating Koltun SR)

← neutro
n
remova
l

→ scattering
neutro
n
additio
n

Benchmark on ^4He

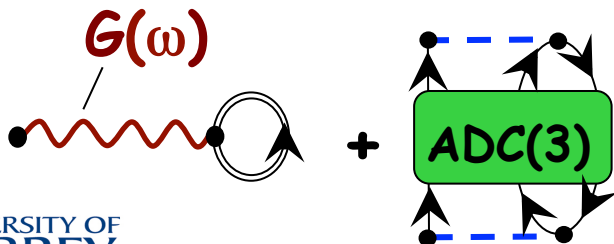


Can benchmark the Gmtx+ADC(3) method on light ^4He , where exact solutions are possible:

	$G(\omega) + \text{ADC}(3)$	Exact
HALQCD @ $m_\pi \approx 470\text{MeV}$	4.8(1) MeV	5.09 MeV ¹

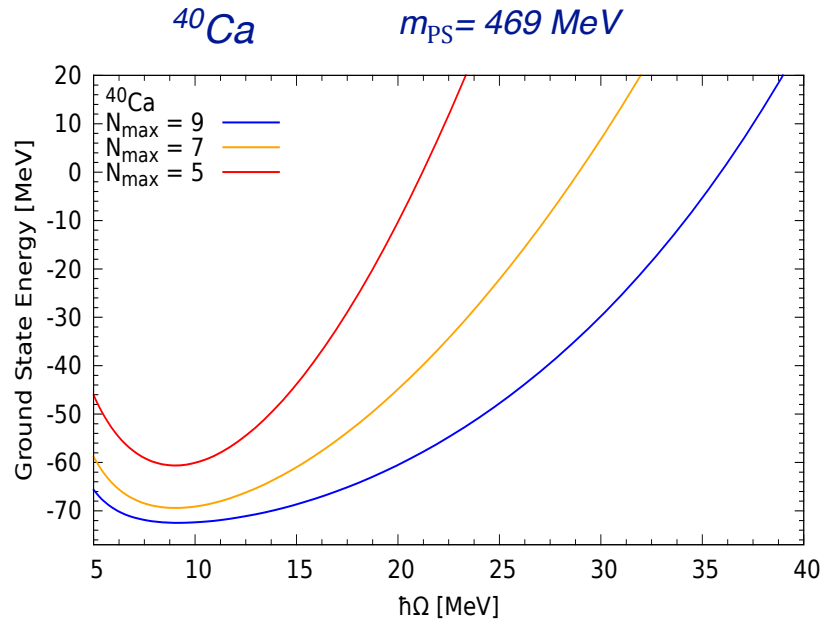
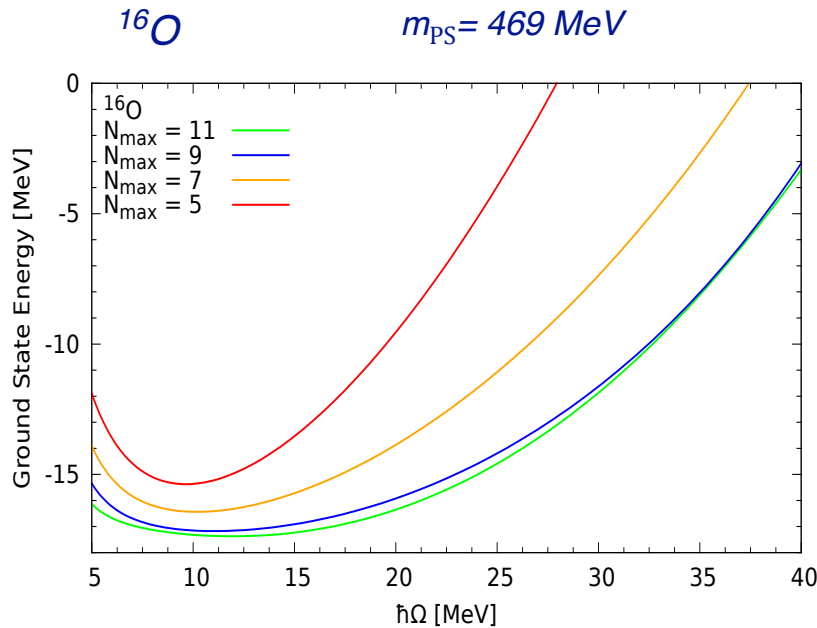
¹H. Nemura *et al.*, Int. J. Mod. Phys. E **23**, 1461006 (2014)

→ Can expect accuracy on binding energies at about 10%



$$G''(\omega) = V + \int dk_a dk_b V \frac{\hat{Q}''}{\omega - \varepsilon(k_a) - \varepsilon(k_b) + i\eta} G''(\omega)$$

Binding of ^{16}O and ^{40}Ca :



Binding energies are $\sim 17 \text{ MeV}$ ^{16}O and $70\text{--}75 \text{ MeV}$ for ^{40}Ca . Possibly being underestimated by 10%

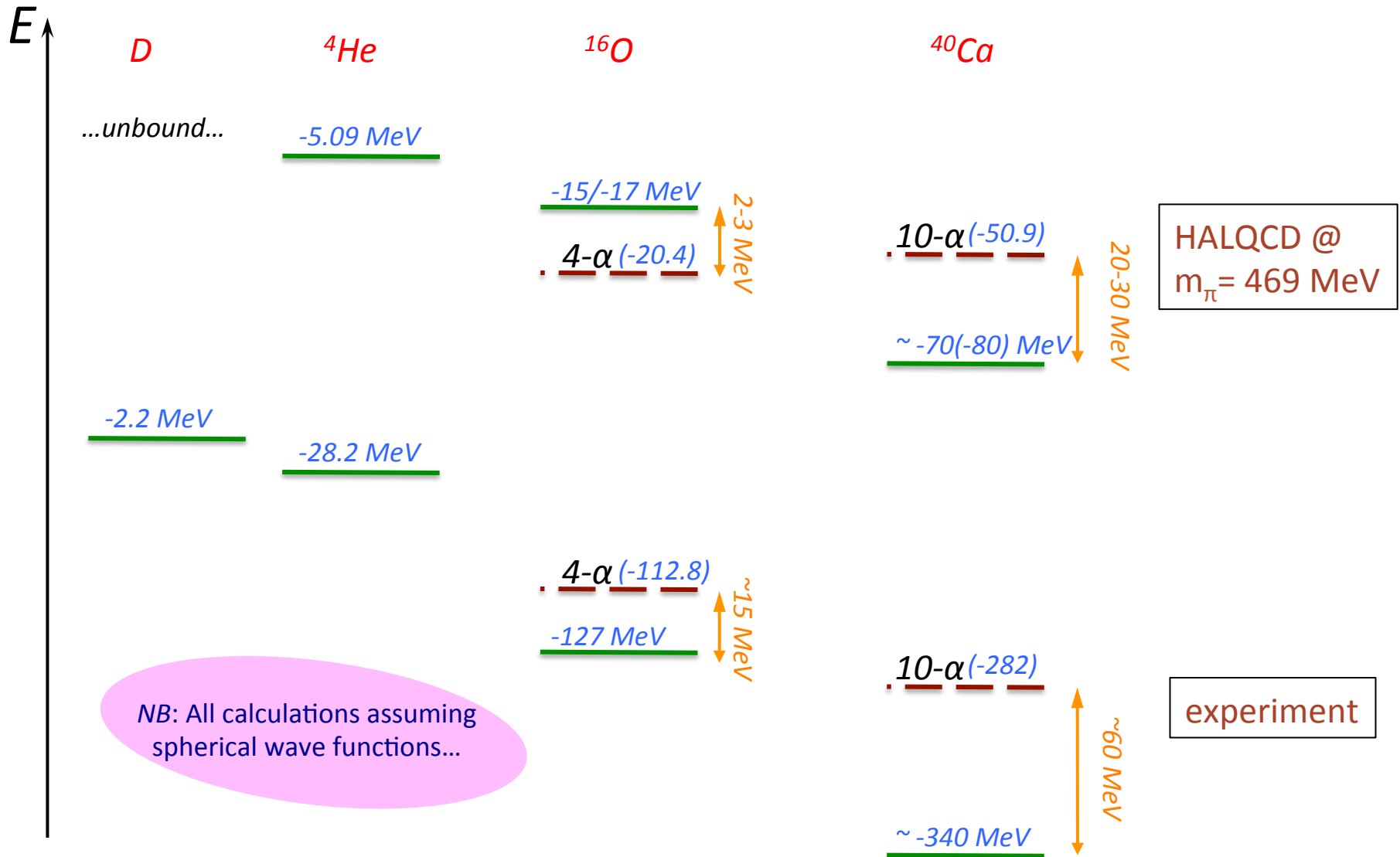
→ ^{16}O at $m_{\pi} \approx 470 \text{ MeV}$ is unstable toward $4\text{-}\alpha$ breakup!

	Oxygen-16	Calcium-40
$G(\omega) + \text{ADC}(3)$	$-17.4(3) \text{ MeV}$	$-75.4(7) \text{ MeV}$
Separate ^4He clusters	-20.36 MeV	-50.9 MeV

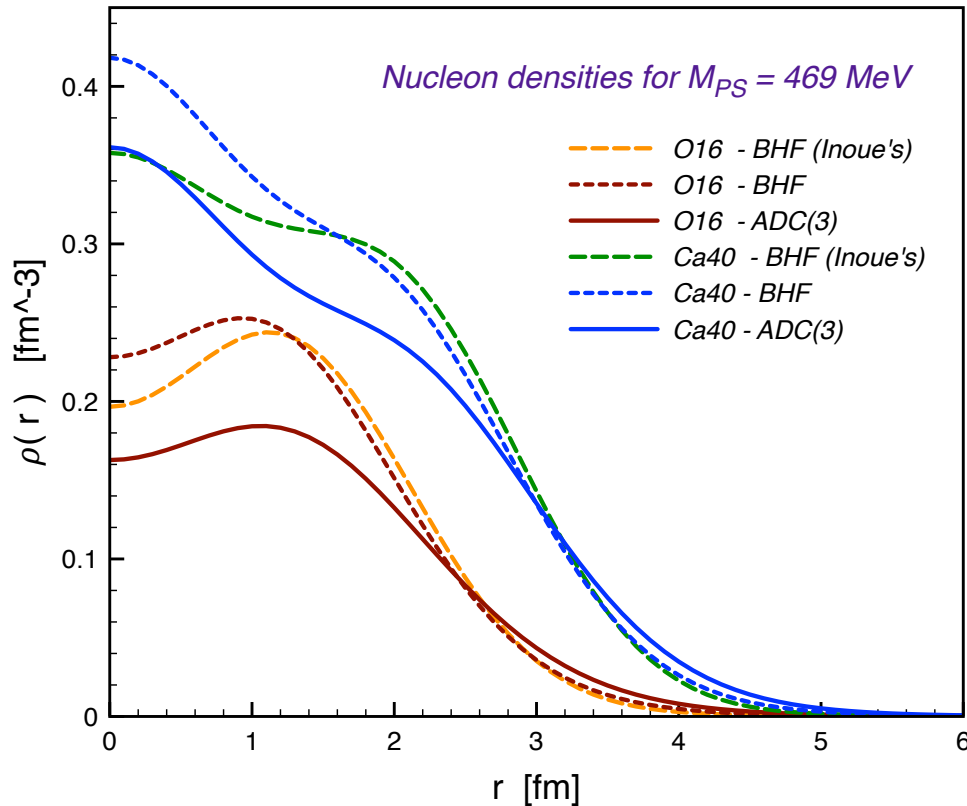
[C.S. McIlroy, CB, HAL coll., in prep]

TABLE II. Comparison of ^{16}O and ^{40}Ca ground state energies using $M_{\text{PS}} = 469 \text{ MeV}/c^2$.

Results for binding



Matter distribution of ^{16}O and ^{40}Ca :

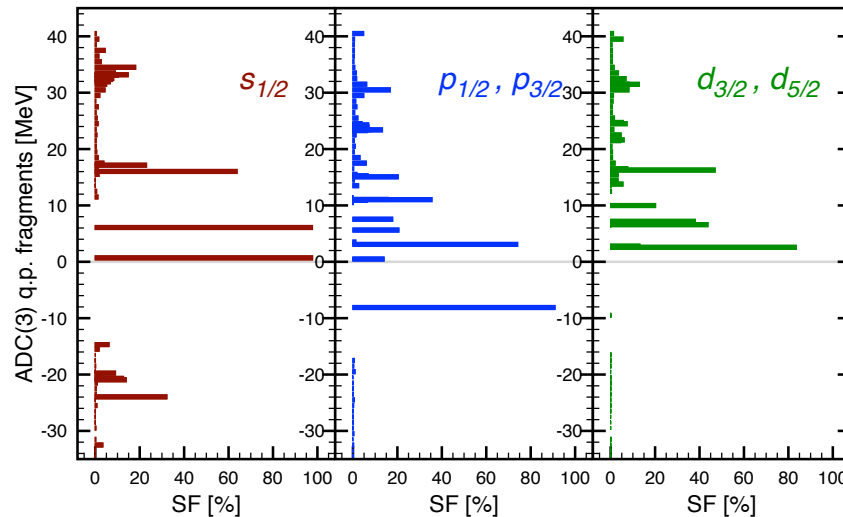
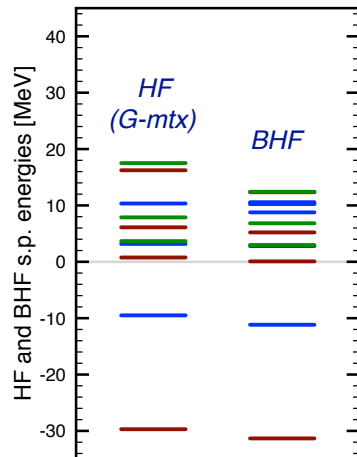


Calculated matter radii at $m_\pi \approx 470 \text{ MeV}$:

		^{16}O	^{40}Ca
$r_{pt\text{-}matter}$:	BHF [18]	2.35 fm	2.78 fm
	GHF	2.39 fm	2.78 fm
	$G(\omega) + \text{ADC}(3)$	2.64 fm	2.97 fm
r_{charge} :	$G(\omega) + \text{ADC}(3)$	2.79 fm	3.10 fm
	Experiment [44, 45]	2.73 fm	3.48 fm

[C.S.McIlroy, CB, HAL coll., in prep]

Spectral strength in ^{16}O and ^{40}Ca :



Particle-hole gaps

^{16}O

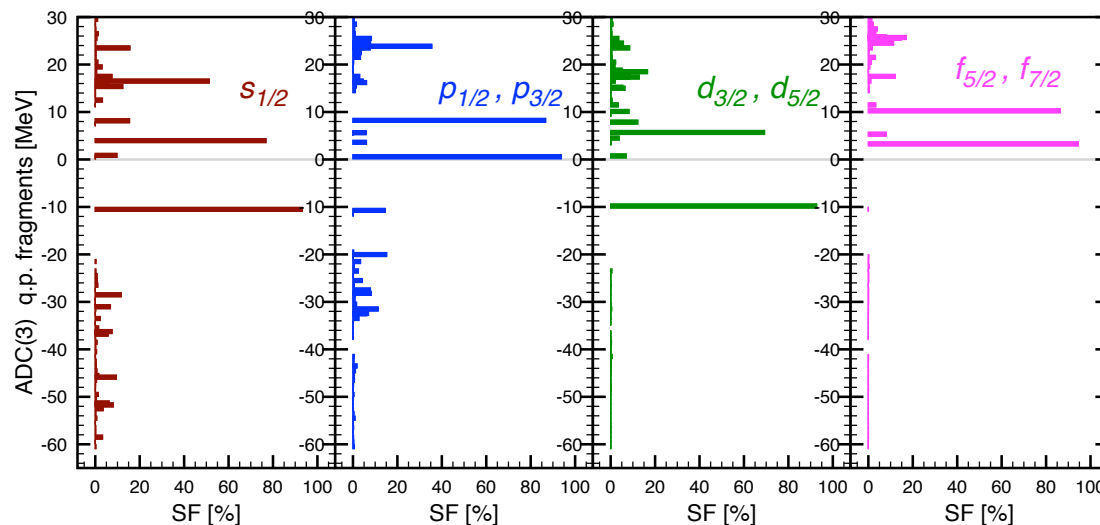
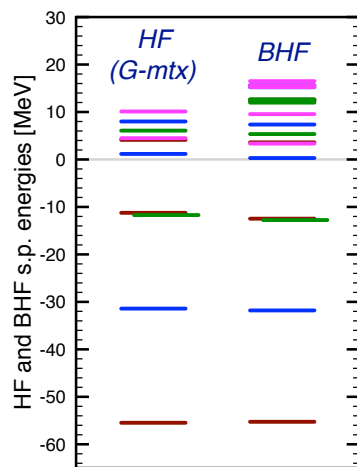
$m_\pi = 469$ MeV: ~ 8 MeV

Expt (phys m_π): 11.5 MeV

^{40}Ca

$m_\pi = 469$ MeV: ~ 10 MeV

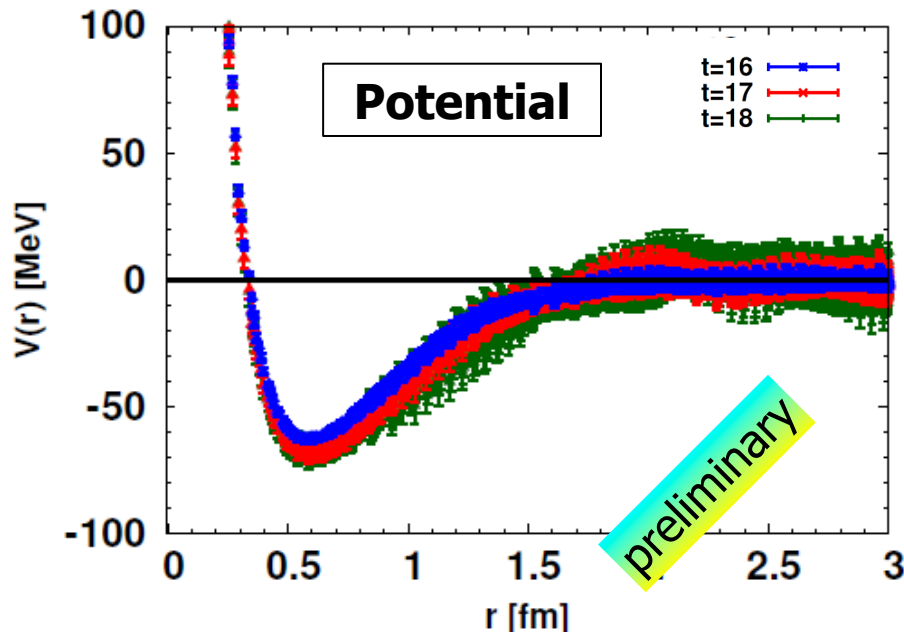
Expt (phys m_π): 7.5 MeV



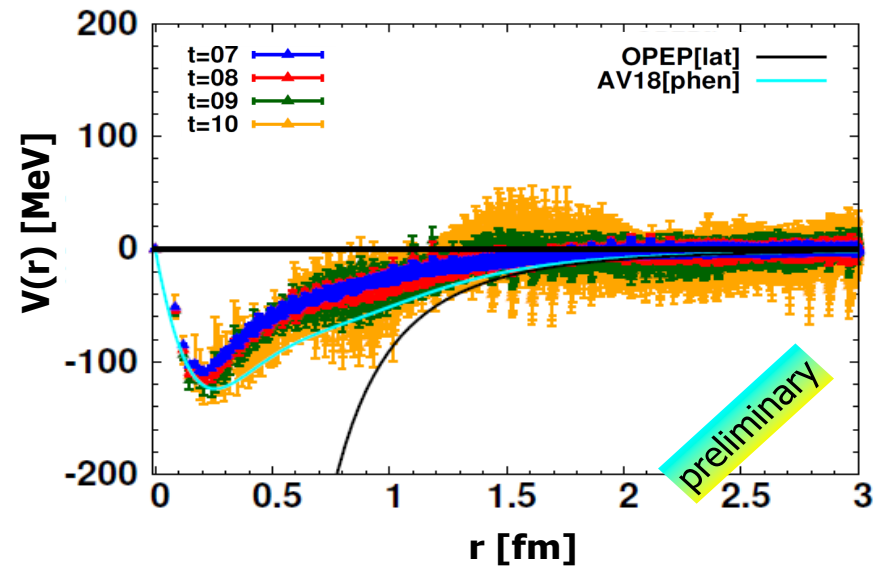
Future application for Y s in nuclei now possible

- Physical mass now under reach ($m_\pi \approx 145$ MeV) for hyperons
- Need to improve on statistic for the NN sector

$\Omega\Omega$ potential



$NN(^3S_1)$ tensor potential

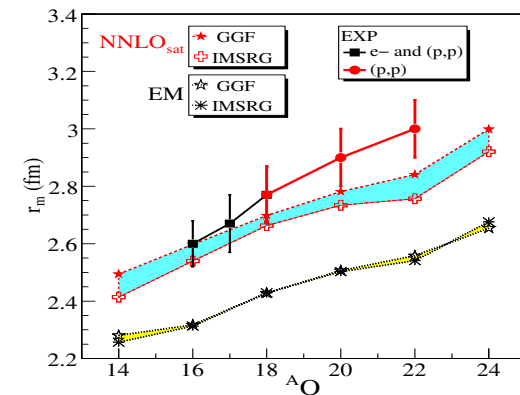
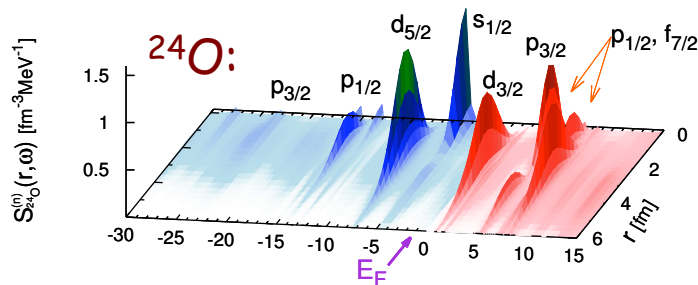


HALQCD coll. -- Talk of **S. Aoki** at Kavli institute, Oct. 2016

Summary

Mid-masses and chiral interactions:

- Leading order 3NF are crucial to predict many important features that are observed experimentally (drip lines, saturation, orbit evolution, etc...)
- Experimental binding is predicted accurately up to the lower sd shell ($A \approx 30$) but deteriorates for medium mass isotopes (Ca and above) with roughly 1 MeV/A over binding.
- New fits of chiral interaction are promising for low-energy observables
- Comparison of spectroscopic strength with experiment is much improved...
- Nuclear forces from Lattice-QCD approaching physical pion mass



Thank you for your attention!!!

Collaborators



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energie atomique • energies alternatives

V. Somà, T. Duguet



**W.H. Dickhoff,
S. Waldecker**



A. Carbone



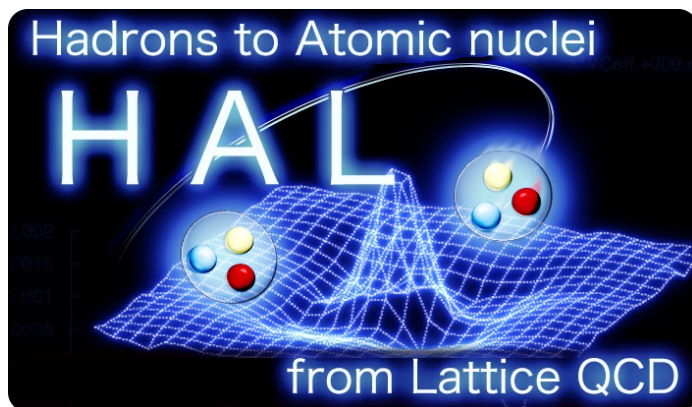
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Univ. Birjand
Univ. Tsukuba
Stony Brook Univ.
YITP Kyoto Univ.