Unified Description of Equilibrium and Non-Equilibrium Properties of Hot Nuclear Matter

Omar Benhar

INFN and Department of Physics, "Sapienza" University
I-00185 Roma, Italy

NPCSM Workshop

Yukawa Institute for Theoretical Physics, Kyoto
October 24, 2016

Outline

- Preamble. Why do we need a unified description of equilibrium and non-equilibrium properties? The CFS instability of rotating stars as an example.
- Many-body theory of nuclear matter:
  - Correlated Basis Functions (CBF) formalism;
  - Bridging the gap between the *ab initio* and effective interaction approaches: the CBF effective interaction;
- Nucler Matter properties:
  - Equation of state of cold and hot matter;
  - Transport coefficients;
  - Superfuid gap;
  - Neutrino mean free path.
- Summary & Outlook

PREAMBLE: THE CFS INSTABILITY

- In the 1960s and 1970s, Chandrasekhar Friedman & Schutz demostrated that, if their core was made of a perfect fuid, rotating compact stars would become unstable against gravitational wave emission.
- The determination of the stability region of rapidly rotating stars requires the understanding of a variety of equilibrium and non-equilibrium properties of neutron star matter, including:
  - the Equation of State (EoS), dictating the composition of matter and the equilibrium properties of the star: mass and radius;
  - the shear and bulk viscosity coefficients, depending on the nucleon-nucleon scattering rate and from the deviations from chemical equilibrium, respectively;
  - the superfluid and superconducting gaps, the appearance of which strongly affects the role played by shear viscosity.
- The development of a *consistent* treatment of all the above properties is an outstanding challenge for nuclear many-body theory

THE PARADIGM OF NUCLEAR MANY-BODY THEORY

- ⋆ Nuclear matter is described as a collection of pointlike protons and neutrons interacting through the hamiltonian

$$H = \sum_i \frac{p_i^2}{2m} + \sum_{j>i} \mathrm{v}_{ij} + \sum_{k>j>i} \mathrm{V}_{\mathrm{ijk}}$$

- ⋆ Note: the independent particle approximation underlying the nuclear shell model, amounts to replacing

$$\sum_{j>i} \mathrm{v}_{ij} + \sum_{k>j>i} \mathrm{V}_{\mathrm{ijk}} \to \sum_i U_i \ ,$$

- ⋆ while being able to explain a number of nuclear properties, independent particle models fail to take into account nucleon-nucleon correlations, which are long known to play a significant role[1].

[1] A quote from Blatt & Weiskopf (AD 1952): "The limitation of any independent particle model lies in its inability to encompass the correlation between the positions and spins of the various particles in the system"

The *ab initio* Approach

- ⋆ The potentials are determined by a fit to the properties of the *exactly solvable* two- and three-nucleon systems
  - v_{ij} strongly constrained by deuteron properties and nucleon-nucleon (NN) scattering data, and reduces to Yukawa's one-pion exchange potential at large distances. The ANL v_{18} model, as an example

$$\mathrm{v}_{ij} = \sum_{p=1,18} \mathrm{v}_p(r_{ij}) O^p_{ij}$$

$$O^p_{ij} = [\mathbb{1}, (\boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j), S_{ij}, \mathbf{L}\cdot\mathbf{S}, \mathbf{L}^2, \mathbf{L}^2(\boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j), (\mathbf{L}\cdot\mathbf{S})^2] \otimes [\mathbb{1}, (\boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j)] \,,$$
$$[1, (\boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j), S_{ij}] \otimes T_{ij} \;\;,\;\; (\tau_{zi} + \tau_{zj})$$

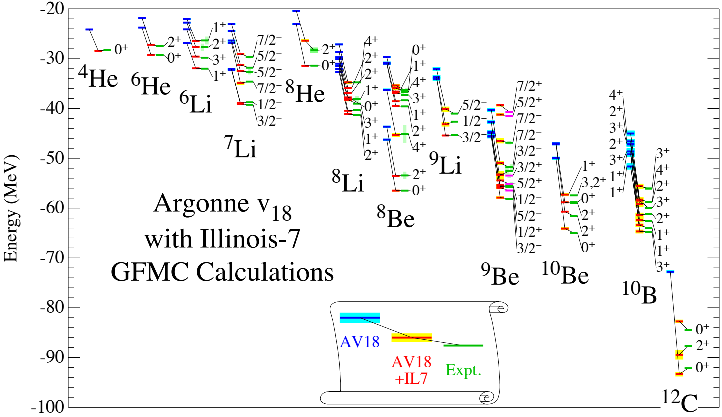
  - The three-nucleon potential is determined fitting the properties of the three-nucleon system

$$V_{ijk} = V^{2\pi}_{ijk} + V^{\mathrm{R}}_{ijk}$$

- ⋆ Recently, consistent models of v_{ij} and V_{ijk} have been also derived within a formalism inspired to chiral perturbation theory

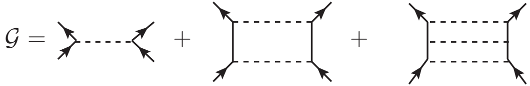
Results for light nuclei

- ⋆ Energy of the ground and low-lying states of light nuclei obtained from Green's Function Monte Carlo (GFMC) calculations performed by the ANL-LANL group. [J. Carlson, et al RMP 87, 1067 (20150]



Many-Body Theory of Nuclear Matter

- ⋆ Using the *ab initio* approach for the description of haevy nuclei and uniform matter involves severe difficulties.
- ⋆ Owing to the presence of a strong repulsive core, the matrix elements of the nuclear Hamiltonian between eigenstates of the Hamiltonian describing the non-interacting system are large. Perturbation theory *in this basis* is not applicable.
- ⋆ Alternative options
    - Replace the bare NN potential with a well behaved *effective interaction* that can be used in perturbation theory using the Fermi gas basis, e.g. the G-matrix:



    - Modify the basis states in such a way as to mitigate the effects of the repulsive core.

Correlated Basis Function (CBF) Formalism

- The eigenstates of the nuclear hamiltonian are approximated by the set of correlated states, obtained from the eigenstates of the Fermi Gas (FG) model

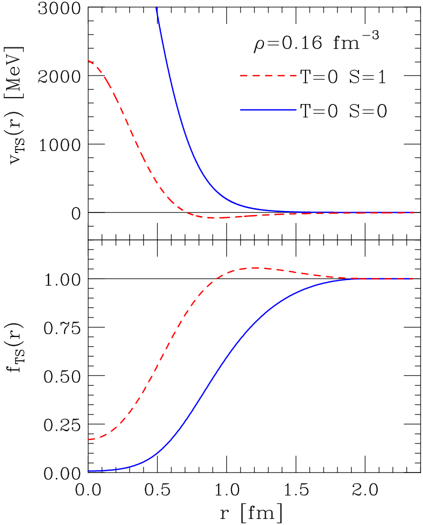
$$|n\rangle = \frac{F|n_{FG}\rangle}{\langle n_{FG}|F^\dagger F|n_{FG}\rangle^{1/2}} = \frac{1}{\sqrt{\mathcal{N}_n}} F\,|n_{FG}\rangle \quad , \quad F = \mathcal{S}\prod_{j>i} f_{ij}$$

- the structure of the two-nucleon correlation operator reflects the complexity of interaction

$$f_{ij} = \sum_p f_p(r_{ij}) O^p_{ij}$$

- the operators O^n_{ij} are the same as those entering the definition of the NN potential v_{ij}

NN Potential and Correlation Functions



Cluster Expansion and FHNC Equations

- The ground state expectation value of the hamiltonian is written as a sum of contributions associated with subsystems (clusters) consisiting of an increasing number of particles

$$\langle H \rangle = \frac{\langle 0|H|0\rangle}{\langle 0|0\rangle} = E_{FG} + \sum_{n\geq 2} (\Delta E)_n$$

- The relevant terms of the cluster expansion can be summed up at all orders solving a set of integral equations known as Fermi Hyper-Netted Chain (FHNC) equations
- the shapes of the $f_p(r_{ij})$ are determined form the minimization of the ground-state expectation value of the hamiltonian

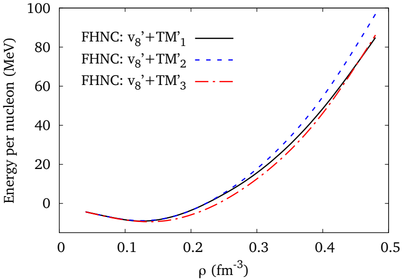
$$E_0 \geq \min_{\{F\}} \frac{\langle 0_{FG}|F^\dagger H F|0_{FG}\rangle}{\langle 0_{FG}|0_{FG}\rangle}$$

- Recently, accurate calculation of $\langle H \rangle$ of pure neutron matter have been also carried out using Quantum Monte Carlo techniques

Density-dependence of Nuclear Matter Energy

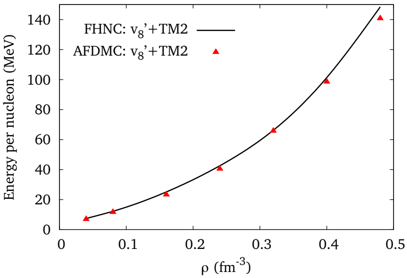
- ⋆ Binding energy per particle of isospin-symmetric nuclear matter (SNM) and pure neutron matter (PNM). FHNC and QMC (AFDMC) calculations. [A. Lovato et al, PRC 85, 024003 (1012)]

- SNM



	TM'_1	TM'_2	TM'_3
ρ_0 (fm^{-3})	0.12	0.13	0.14
E_0 (MeV)	-9.0	-8.8	-9.4
K (MeV)	266	243	249

- ⋆ PNM



Alternative Approach: the CBF Effective Interaction

- Within CBF, the effective interaction is defined through

$$\langle H \rangle = \frac{\langle 0|F^\dagger (T+V) F|0\rangle}{\langle 0|F^\dagger F|0\rangle} = \langle 0_{FG}|T + V_{\rm eff}|0_{FG}\rangle$$

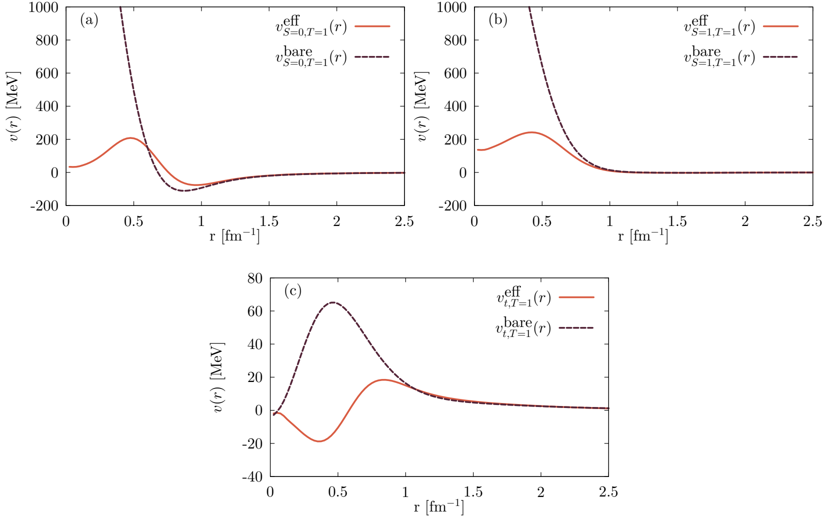
- At two-body cluster level

$$V_{\rm eff} = \sum_{j>i} {\rm v}_{\rm eff}(ij)$$

$${\rm v}_{\rm eff}(ij) = \frac{1}{m}\left(\boldsymbol{\nabla} f_{ij}\right)^2 + f_{ij}{\rm v}_{ij} f_{ij}$$

- Three-nucleon interactions can be taken into account extending the definition to include three-body cluster contributions
- The correlation functions are determined in such a way as to reproduce the value of $\langle H \rangle$ obtained from the FHNC approach

CBF Effective Interaction at SNM Equilibrium Density



Nuclear Matter Energy

- ⋆ The CBF effective interaction can be employed to carry out self-consistent Hartee-Fock calculations of the energy of cold nuclear matter at fixed baryon density n_B, for any values of the proton fraction $Y_p = n_p/n_B$
- ▸ The same firmalism can be extended to describe the regime in which thermal effect only alter the Fermi distributions, corresponding to $T = \beta^{-1} \ll m_\pi$, m_π being the pion mass ($\alpha = p, n$):

$$\frac{E}{N_B} = \sum_{\mathbf{k}\alpha} \frac{\mathbf{k}^2}{2m} n[e_\alpha(\mathbf{k})] + \frac{1}{2} \sum_{\mathbf{k}\alpha,\ \mathbf{k}'\alpha'} \langle \mathbf{k}\alpha\ \mathbf{k}'\alpha' | V_{\rm eff} | \mathbf{k}\alpha\ \mathbf{k}'\alpha' \rangle_A \, n[e_\alpha(\mathbf{k})] n[e_{\alpha'}(\mathbf{k}')]$$

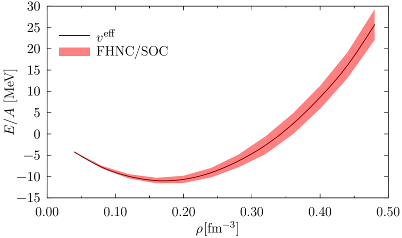
$$e_\alpha(\mathbf{k}) = \frac{\mathbf{k}^2}{2m} + \sum_{\mathbf{k}'\alpha'} \langle \mathbf{k}\alpha\ \mathbf{k}'\alpha' | V_{\rm eff} | \mathbf{k}\alpha\ \mathbf{k}'\alpha' \rangle_A \, n[e_{\alpha'}(\mathbf{k}')]$$

$$n[e_\alpha(\mathbf{k})] = \{1 + \beta[e_\alpha(\mathbf{k}) - \mu_\alpha]\}^{-1} \xrightarrow[\beta\to\infty]{} \theta[\mu_\alpha - e_\alpha(\mathbf{k})]$$

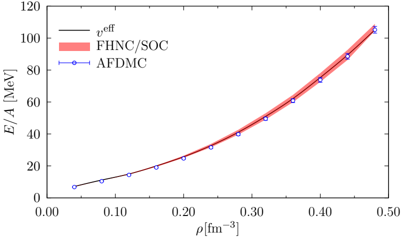
COLD NUCLEAR MATTER WITH $x_p = 0$ (PNM) & 1/2 (SNM)

- ⋆ Density-dependence of the energy per baryon of cold nuclear matter. CBF effective interaction derived from the AV6′ + UIX nuclear Hamiltonian

- ▶ SNM



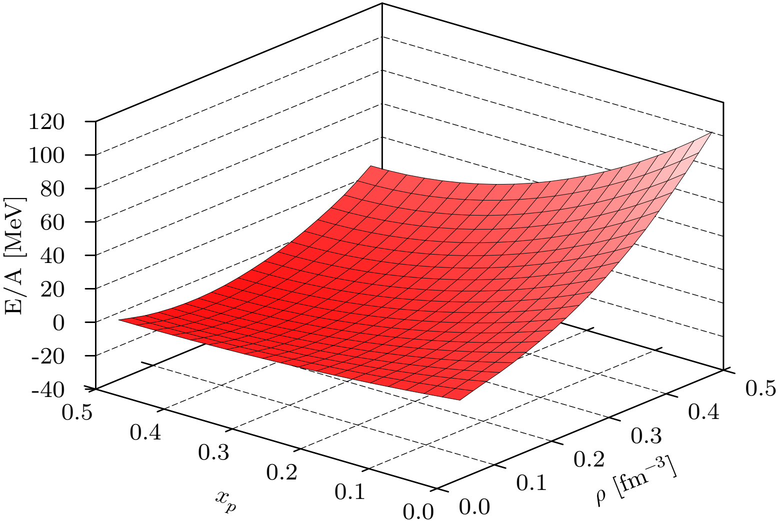
- ▶ PNM



- ⋆ Symmetry energy at equilibrium density of SNM:

$$S(n_0) = 30.9 \text{ MeV}$$

COLD NUCLEAR MATTER WITH ARBITRARY x_p



β-STABLE MATTER

- The composition of matter is determined by the conditions of
  - conservation of baryon number

$$n_n + n_p = n_B \quad \leftrightarrow x_n = 1 - x_p$$

  - charge neutrality

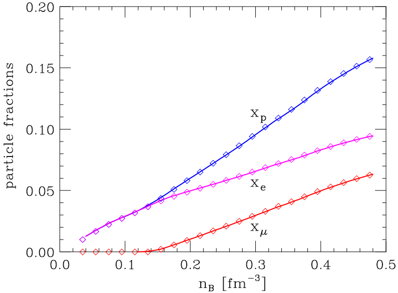
$$\sum_\ell n_\ell = n_e + n_\mu = n_p$$

  - β-equilibrium

$$\mu_n(n_B, x_n) = \mu_p(n_B, x_p) + \mu_e(n_B, x_e) \quad , \quad \mu_\alpha = \left(\frac{\partial E}{\partial N_\alpha}\right)_{N_{\beta \neq \alpha}}$$

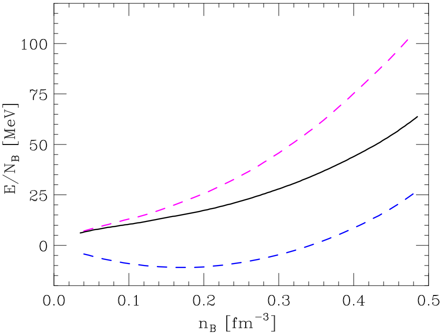
- For any given n_B and T the above equations yield the particle fractions

- Proton, electron and muon fractions as a function of baryon density



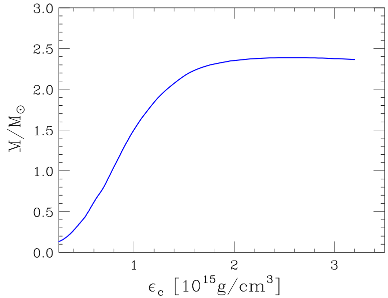
- The parabolic approximation for the interaction energy (diamonds) appears to be very accurate

- Density-dependence of the energy of cold β-stable matter

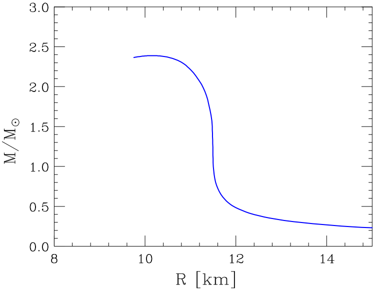


Neutron Star Properties

- Star mass as a function of central energy density



- Mass-radius relation



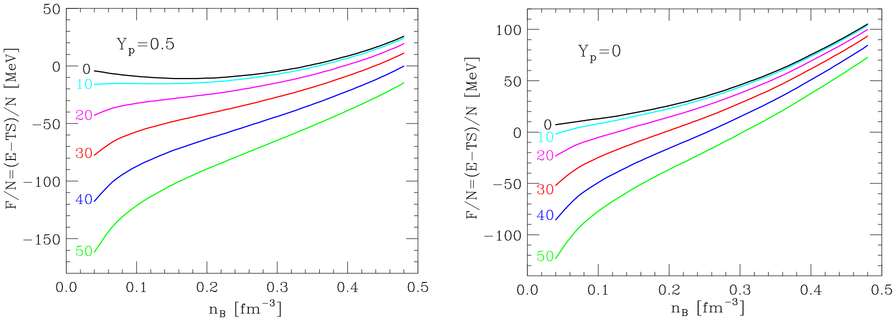
SNM and PNM at Finite Temperature

- Gibbs free energy

$$\frac{F}{N_B} = \frac{1}{N_B}(E - TS)$$

- Entropy

$$S = -\sum_{\mathbf{k}\alpha} \{n[e_\alpha(\mathbf{k})] \ln n[e_\alpha(\mathbf{k})] + [1 - n[e_\alpha(\mathbf{k})]] \ln [1 - n[e_\alpha(\mathbf{k})]]\}$$

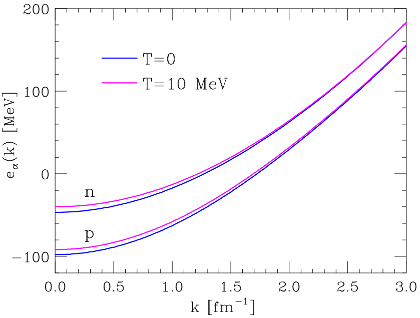


Spectra and Effective Masses in β-stable Matter

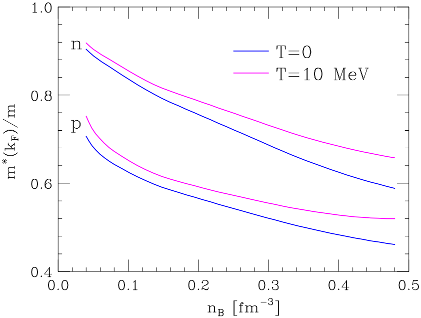
⋆ The effective masses are obtained from

$$\frac{1}{{m_\alpha}^\star} = \frac{1}{k}\,\frac{de_\alpha(k)}{dk}$$

▶ p and n spectra at $n_B = n_0$



▶ ${m_\alpha}^\star(k_{F_\alpha})/m$



Transport Properties

- Landau-Abrikosov-Khalaktnikov formalism: Boltzman equation

$$\frac{\partial n}{\partial t} + \frac{\partial n}{\partial \mathbf{r}}\frac{\partial \epsilon}{\partial \mathbf{p}} - \frac{\partial n}{\partial \mathbf{p}}\frac{\partial \epsilon}{\partial \mathbf{r}} = I(n)$$

$$n = n_0 + \delta n \quad , \quad n_0 = \{1 + \exp[\beta(\epsilon - \mu)]\}^{-1}$$

- The collision integral $I(n)$ depends on the probability of the *in medium* NN scattering process

$$W = \frac{16\pi^2}{m^{\star 2}} \left(\frac{d\sigma}{d\Omega}\right)$$

- The description of transport properties require dynamical models providing an accurate description of NN scattering in the nuclear medium, constrained by the available data in the zero-density limit

Shear Viscosity of PNM

- Abrikosov-Khalatnikov (AK) estimate of the shear viscosity in the low-temperature limit

$$\eta_{AK} = \frac{1}{5}\rho m^{\star} \mathrm{v}_F^2 \tau \, \frac{2}{\pi^2(1-\lambda_\eta)}$$

- Quasiparticle lifetime

$$\tau T^2 = \frac{8\pi^4}{m^{*3}} \, \frac{1}{\langle W \rangle} \, ,$$

- Angle-averaged collision probability

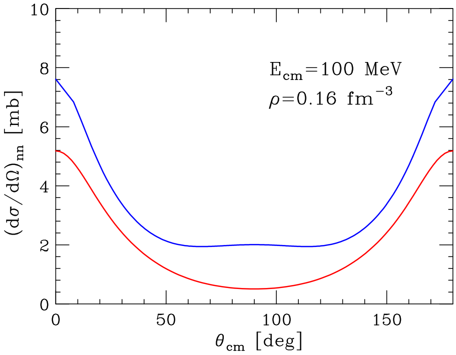
$$\langle W \rangle = \int \frac{d\Omega}{2\pi} \, \frac{W(\theta,\phi)}{\cos(\theta/2)} \quad , \quad \lambda_\eta = \frac{\langle W[1 - 3\sin^4(\theta/2)\sin^2\phi] \rangle}{\langle W \rangle}$$

In Medium Neutron-Neutron Cross Section

- From Fermi's golden rule

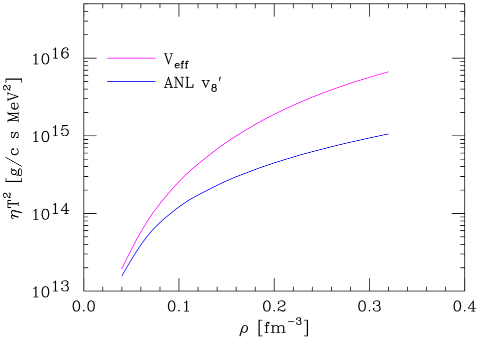
$$W(\mathbf{p}, \mathbf{p}') = 2\pi \left|\hat{\mathrm{v}}_{\mathrm{eff}}(\mathbf{p} - \mathbf{p}')\right|^2 \rho(\mathbf{p}')$$

$$\frac{d\sigma}{d\Omega_{\mathbf{p}'}} = \frac{m^{\star 2}}{16\pi^2} \left|\hat{\mathrm{v}}_{\mathrm{eff}}(\mathbf{p} - \mathbf{p}')\right|^2$$



SHEAR VISCOSITY OF PNM

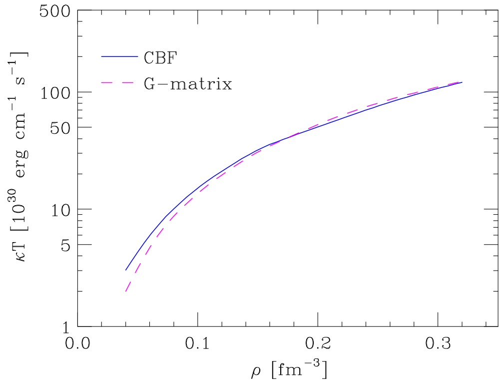
⋆ Density dependence of ηT^2 of PNM



⋆ Medium modifications of the scattering cross section increase ηT^2 by a factor $\sim 3-7$ @ $\rho/\rho_0 \sim 1-2$

Thermal Conductivity of PNM

- ⋆ Results from PRC 81, 024305 (2009). Three-nucleon interactions not taken into account.



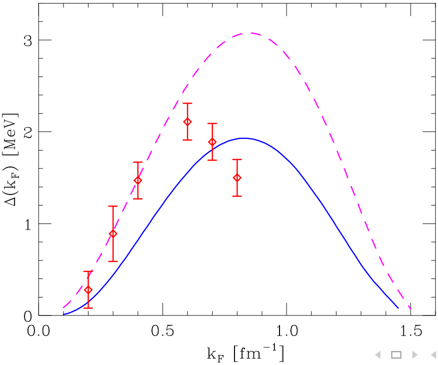
- ⋆ The transport coefficients computed using the CBF effective interaction is remarkably close to the result obtained within the G-matrix approach using the same bare NN potential.

Neutron Pairing in the $^1S_0$ Channel

- Gap equation

$$\Delta(k) = -\frac{1}{\pi} \int k'^2 dk' \frac{\mathrm{v}(k,k')\Delta(k')}{\left[(e(k')-\mu)^2 + \Delta^2(k')\right]^{1/2}}$$

$$\mathrm{v}(k,k') = \int r^2 dr\, j_0(kr) \mathrm{v}_{\mathrm{eff}}(r) j_0(k'r)$$



Nuclear Weak Responses at Low Energy

- density response

$$\mathcal{S}^{\rho} = \frac{1}{N} \sum_{n} |\langle 0|J_0|n\rangle\langle n|J_0|0\rangle \delta^{(4)}(P_0 + q - P_n)$$

- spin-density response ($\alpha,\ \beta = 1, \ldots 3$)

$$\mathcal{S}^{\rho} = \sum_{\alpha} \mathcal{S}^{\rho}_{\alpha\alpha}$$

$$\mathcal{S}^{\rho}_{\alpha\beta} = \frac{1}{N} \sum_{n} |\langle 0|J_{\alpha}|n\rangle\langle n|J_{\beta}|0\rangle \delta^{(4)}(P_0 + q - P_n)$$

- Neutral weak current

$$J_0 = \sum_i j_i^0 = \sum_i \mathrm{e}^{i\mathbf{q}\cdot\mathbf{x}_i} \quad , \quad J_{\alpha} = \sum_i j_i^{\mu} = \sum_i \mathrm{e}^{i\mathbf{q}\cdot\mathbf{x}_i} \sigma_{\alpha}$$

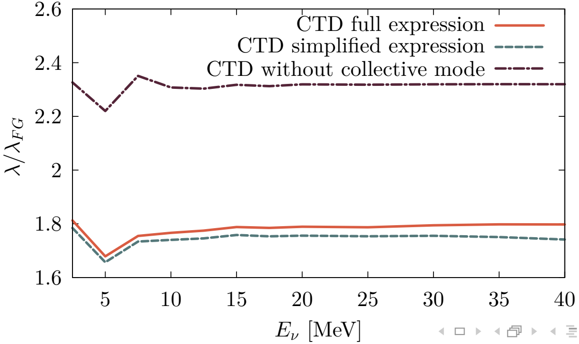
- The can be calculated using the CBF effective interaction and consistently defined effective current operators

Neutrino mean free path in neutron matter

- The mean free path of non degenerate neutrinos at zero temperature is obtained from

$$\frac{1}{\lambda} = \frac{G_F^2}{4}\rho \int \frac{d^3q}{(2\pi)^3} \left[(1+\cos\theta)S(\mathbf{q},\omega) + \mathbf{C_A^2}(\mathbf{3}-\cos\theta)\mathcal{S}(\mathbf{q},\omega)\right]$$

where S and $\mathcal{S}$ are the density (Fermi) and spin (Gamow Teller) response, respectively [A. Lovato et al, NPA 89, 025804 (2013); PRC 89, 025804 (2013)]



SUMMARY

- ⋆ The analysis of neutron star properties requires a variety of theoretical inputs, that must be *consistently* derived from the *same dynamical model* using a *unified formalism*

- ⋆ The CBF effective interaction *derived from a realistic nuclear Hamiltonians using the correlated bais function formalism and the cluster expansion technique* is a powerful tool to carry out perturbative calculations of a number of different quantities, ranging from the EoS to single particle properties, *in medium* scattering probabilities, superfluid and superconducting gaps and neutrino emission and absorption rates

- ⋆ Owing to the weak temperature depencence of the correlation functions, the extension of the CBF effective interaction approach to temperatures $T \ll m_\pi$, does not involve any conceptual difficulties.

- ▶ The properties of hot matter obtained using the CBF effective interaction will provide a valuable input for the next generation of studies of protoneutron stars.

Backup slides

Equation of State and Neutron Star Structure

- From the energy per baryon as a function of density, one can readily obtain the pressure and mass-energy density

$$P = n_B^2 \frac{\partial(E/N_B)}{\partial n_B} \quad , \quad \epsilon = \frac{1}{V}(E + N_B m) = n_B \left(\frac{E}{N_B} + m \right)$$

and the Equation of State (EoS) $P = P(\epsilon)$, providing the input for the solution ofthe Tolman-Oppenheimer-Volkof (TOV) equations, whose solutions correspond to equilibrium configurations of non-rotating stars

$$\frac{dP(r)}{dr} = -G \frac{[\epsilon(r) + P(r)/c^2][M(r) + 4\pi r^3 P(r)/c^2]}{r^2[1 - 2GM(r)/rc^2]}$$

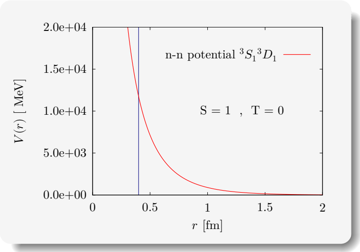
$$M(r) = 4\pi \int_0^r r' dr' \epsilon(r')$$

The hard-sphere model

The Fermi hard-sphere model: point-like spin one-half particles

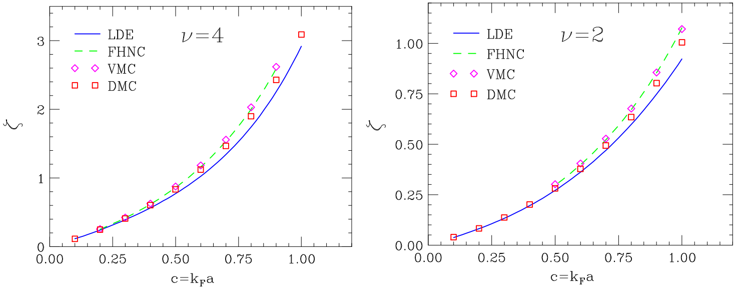
$$v(r) = \begin{cases} \infty & r < a \\ 0 & r > a \end{cases}$$

- ⋆ Valuable model to study properties of nuclear matter.
- ⋆ Purely repulsive potential to prevent the possibility of Cooper pairs formation.
- ⋆ A simple many-body system to investigate the validity and robustness of the assumptions of CBF effective interaction approach.



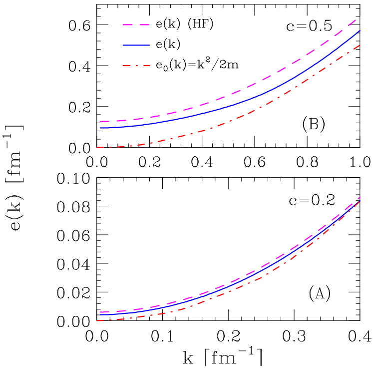
THE GROUND-STATE ENERGY

$$E_0 = \frac{3k_F^2}{10m}(1+\zeta)$$

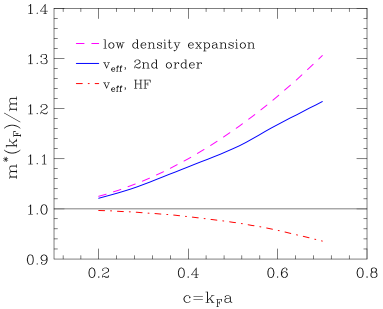


- The accuracy of the variational results depends on the quality of the trial wave function.
- Long-range statistical correlations effects in $f(r)$ much larger for $\nu = 2$ than for $\nu = 4$.
- DMC overcomes the limitations of the variational approach by using a projection technique on the trial wave function.

QUASIPARTICLE SPECTRUM



$$m^\star = \left[\frac{1}{k}\frac{de(k)}{dk}\right]^{-1}$$



$$\frac{de(k)}{dk} = \left[\frac{k}{m} + \frac{\partial}{\partial k}\mathrm{Re}\Sigma\left(k, E\right)\right]\left[1 - \frac{\partial}{\partial E}\mathrm{Re}\Sigma\left(k, E\right)\right]^{-1}_{E=e(k)}$$