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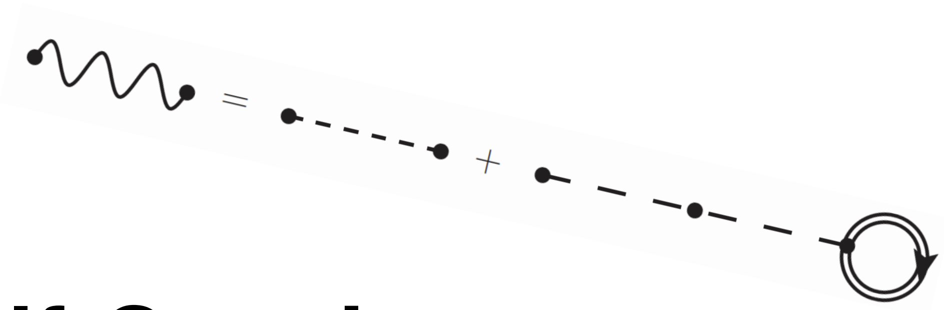
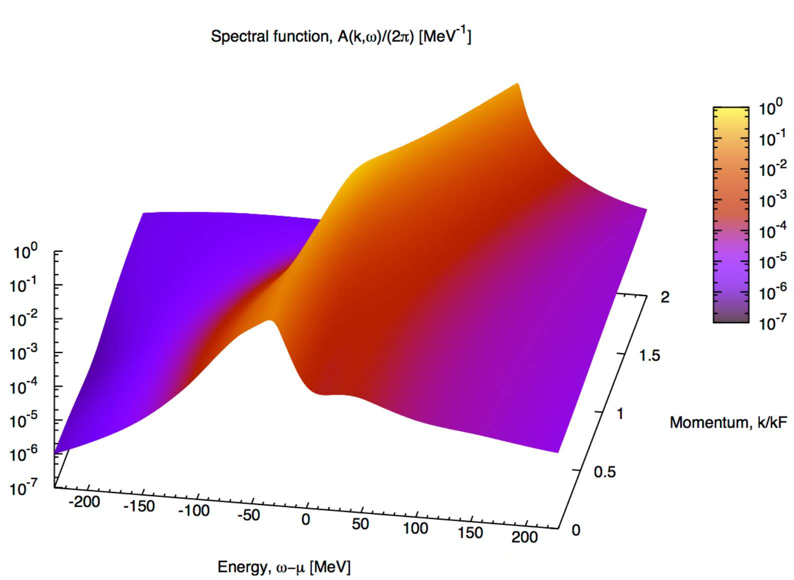
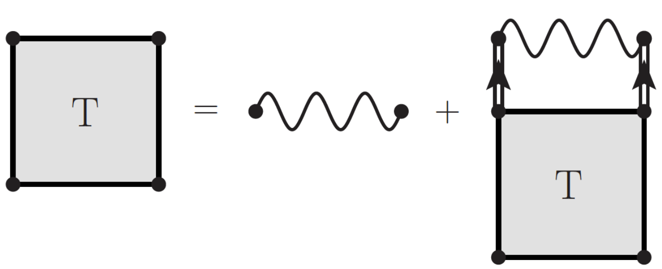


Nuclear Matter from a Self-Consistent Green's Function Approach

Arianna Carbone

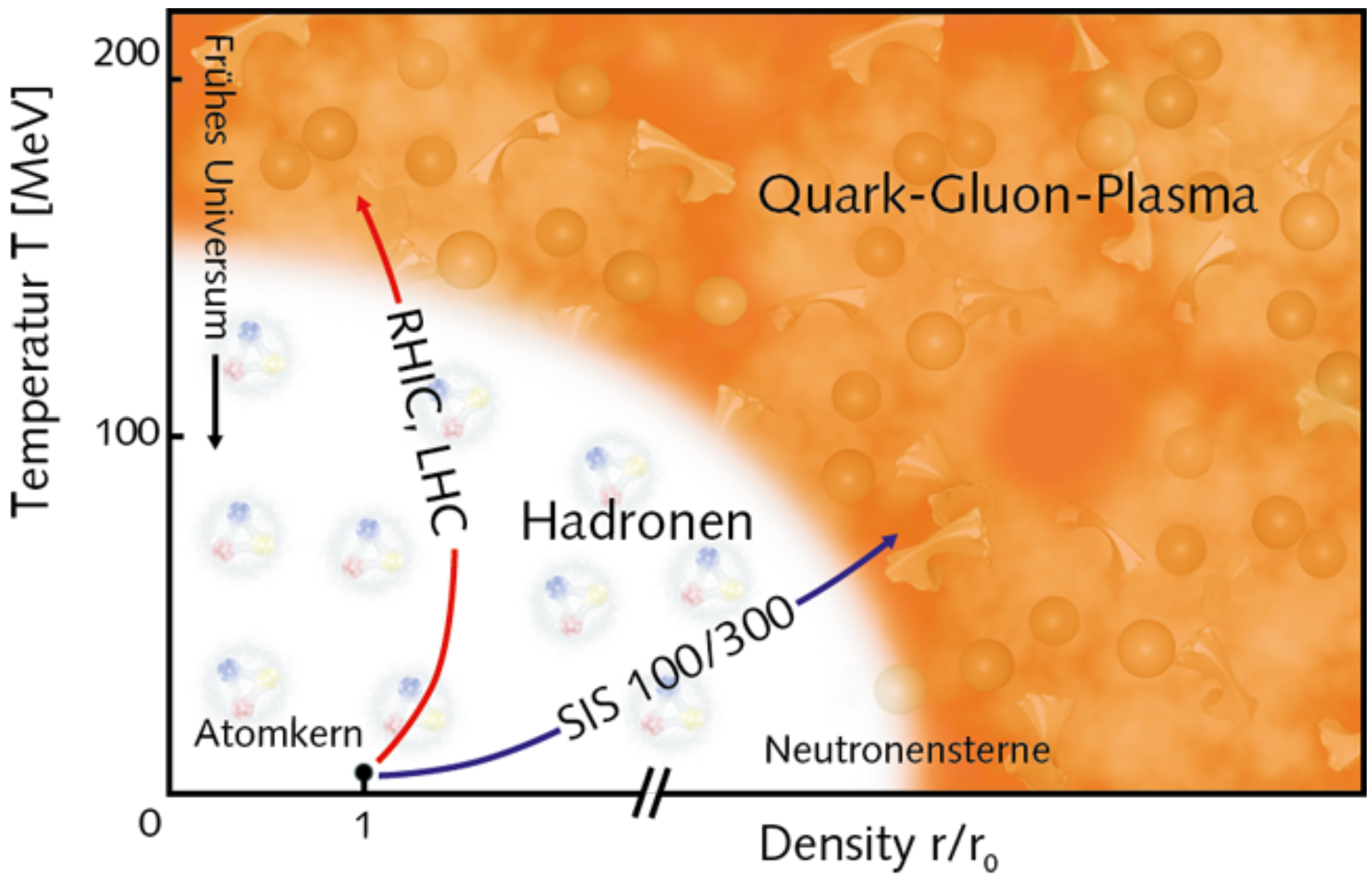
NPCSM - YITP, Kyoto - 25 October 2016





Nuclear matter covers wide ranges of density and temperature

The phase diagram of **hadronic matter**



https://www.gsi.de/en/start/fair/forschung_an_fair/kernmateriephysik.htm

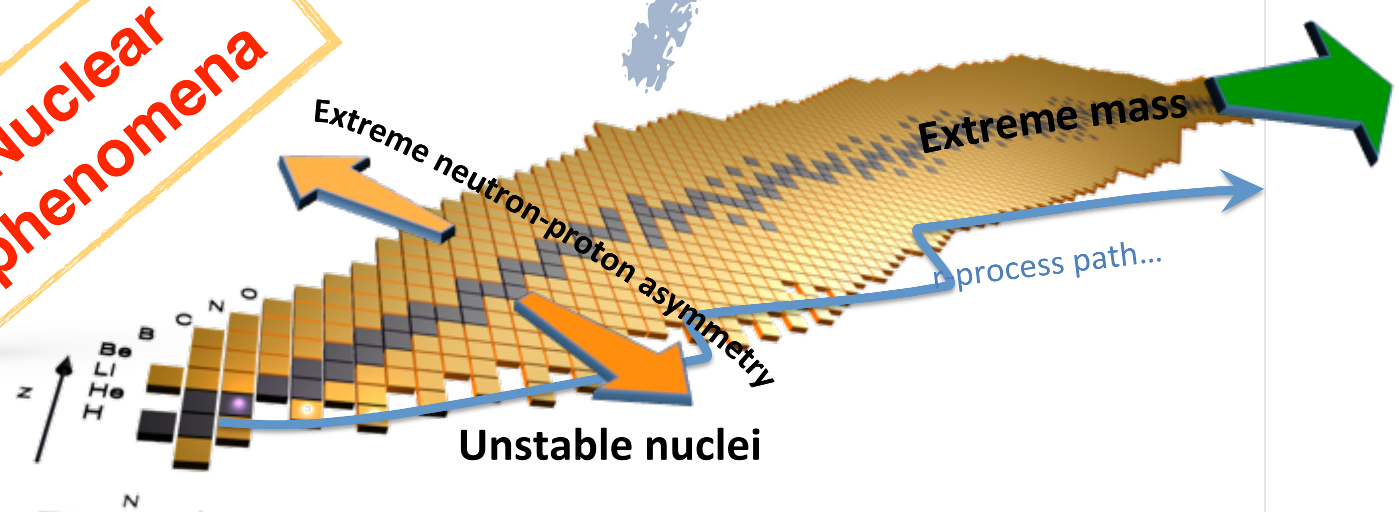
- Matter interacting via the **strong force** appears in diverse forms
- Experiments try to fill the phase diagram puzzle
- Radioactive beam facilities will probe the **neutron-rich region**
- What's the contribution from nuclear many-body theory?

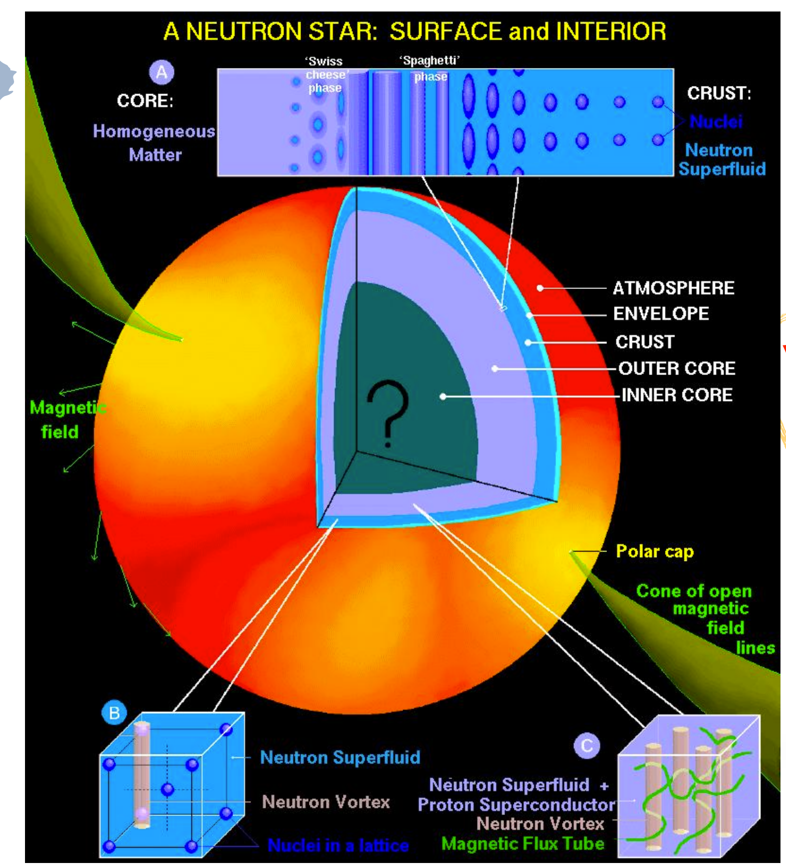
The nuclear many-body problem

- Build reliable methods with predictive power
- The study of exotic nuclei is probing the limits of the nuclear landscape
- Constraining the equation of state (EOS) of infinite matter has a high astrophysical impact

From nuclei to nuclear matter

Nuclear phenomena





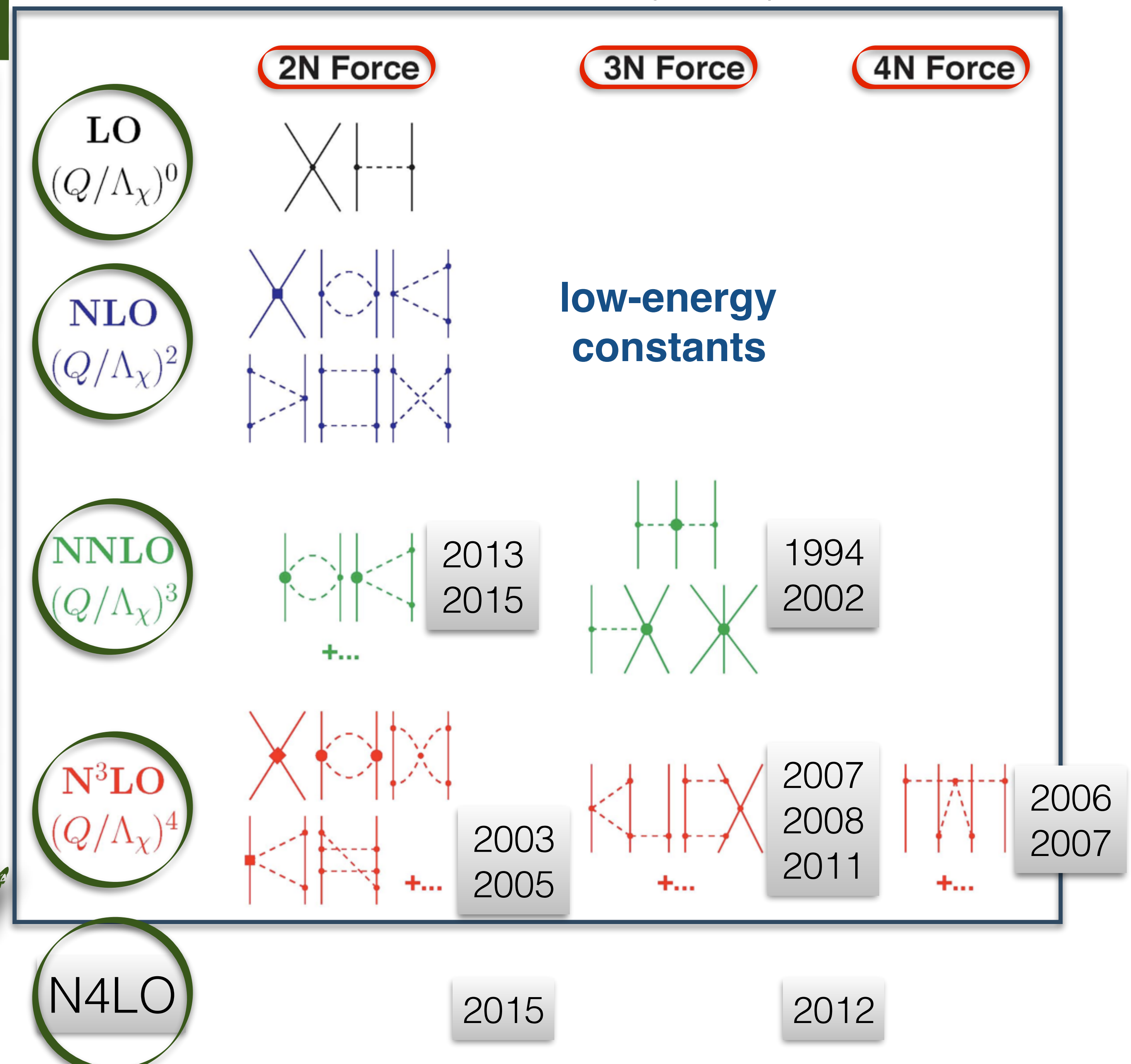
Astrophysics

Why nuclear matter from chiral EFT?

Epelbaum *et al.*, Rev. Mod. Phys. 81, 1773(2009)
Machleidt *et al.*, Phys. Rep. 503, 1 (2011)

Power counting

- Effective theory of QCD
- Nucleons & pions as d.o.f.
- Power counting expansion
- Hierarchy of many-body forces
- Theoretical uncertainties



20 years of ongoing improvement

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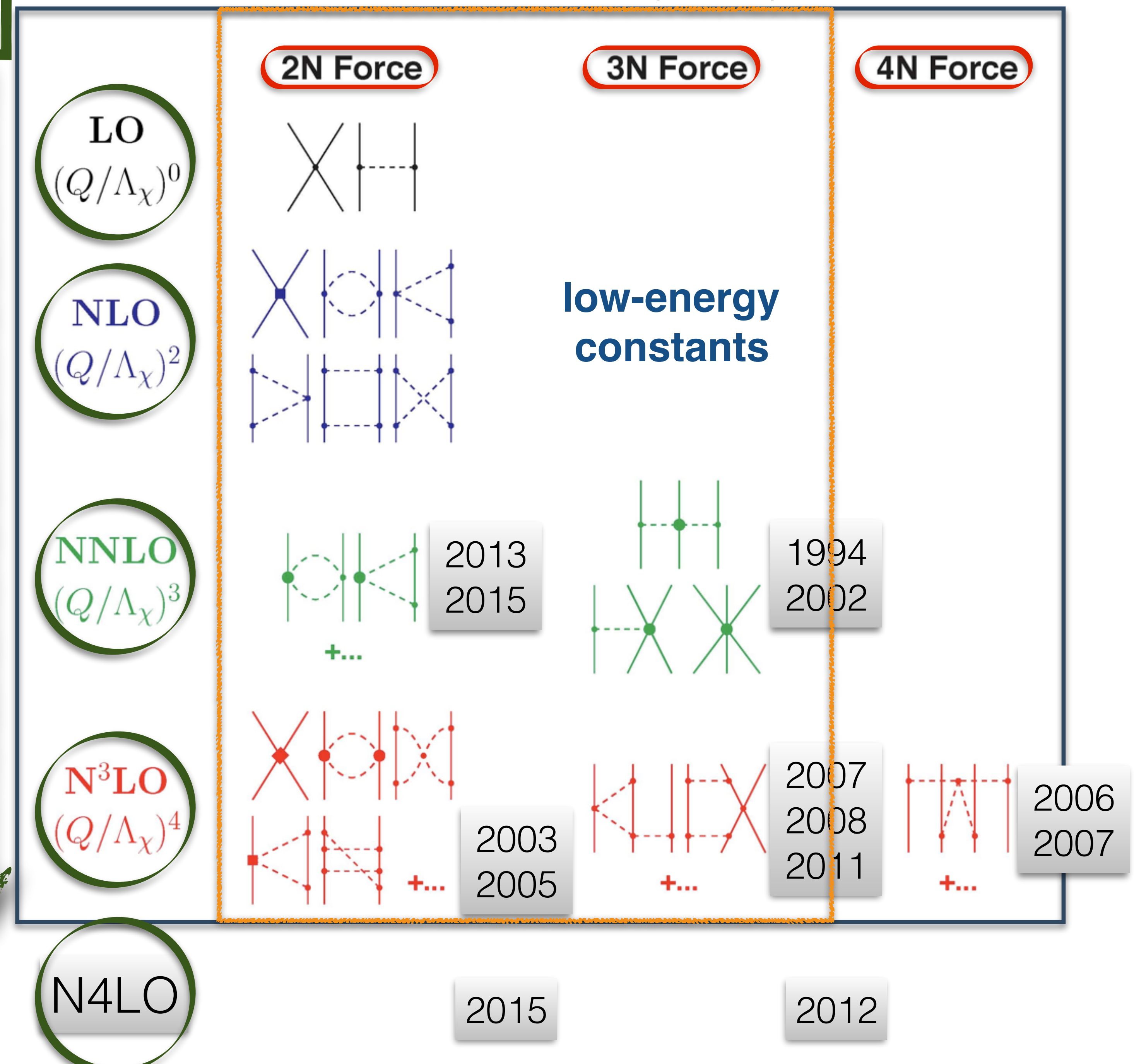


20 years of ongoing improvement

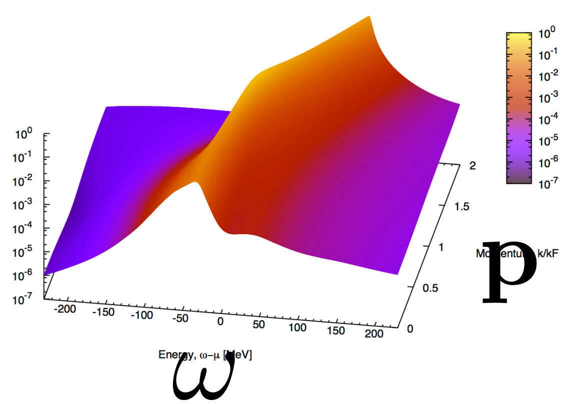
Self-consistent Green's functions

Dickhoff & Barbieri, PPNP **52**, 377 (2004)

- The Green's function as a tool to solve the nuclear many-body problem:

$$G_{\alpha\beta}(\omega) = \sum_m \frac{\langle \Psi_0^N | a_\alpha | \Psi_m^{N+1} \rangle \langle \Psi_m^{N+1} | a_\beta^\dagger | \Psi_0^N \rangle}{\omega - (E_m^{N+1} - E_0^N) + i\eta} + \sum_n \frac{\langle \Psi_0^N | a_\beta^\dagger | \Psi_n^{N-1} \rangle \langle \Psi_n^{N-1} | a_\alpha | \Psi_0^N \rangle}{\omega - (E_0 - E_n^{N-1}) + i\eta}$$

Spectral Function



- Self-consistent nonperturbative method:

Dyson equation

- Breakthrough: full formal extension to consistently include 3BFs

Carbone, Cipollone, Barbieri, Rios, Polls, PRC **88**, 054326 (2013)

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particle exits — *particle enters* — *hole exits* — *hole enters*

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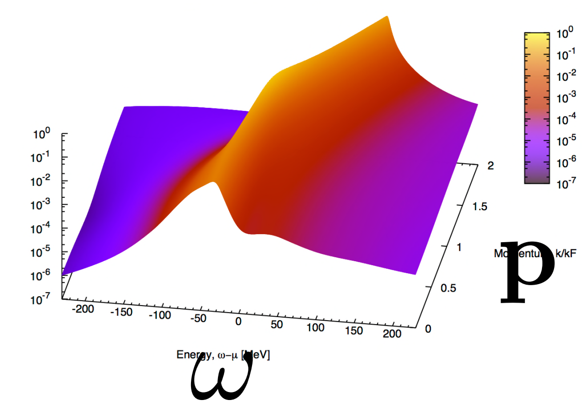
energy with an added particle — *energy with a removed particle*

Spectral Function



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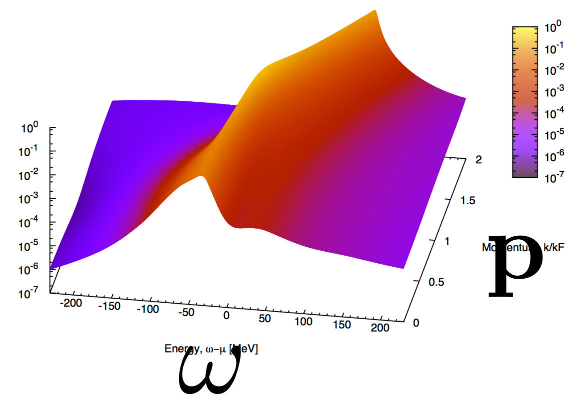

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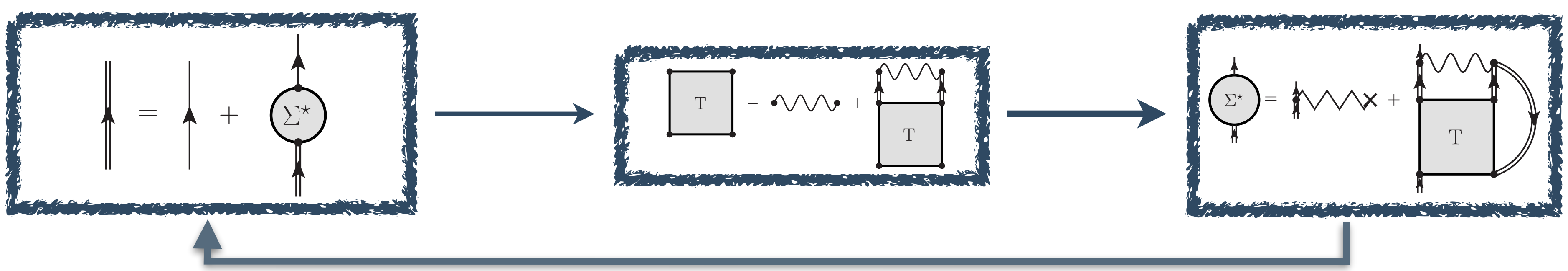


Dyson equation

- Breakthrough: full formal extension to consistently include 3BFs

Carbone, Cipollone, Barbieri, Rios, Polls, PRC **88**, 054326 (2013)

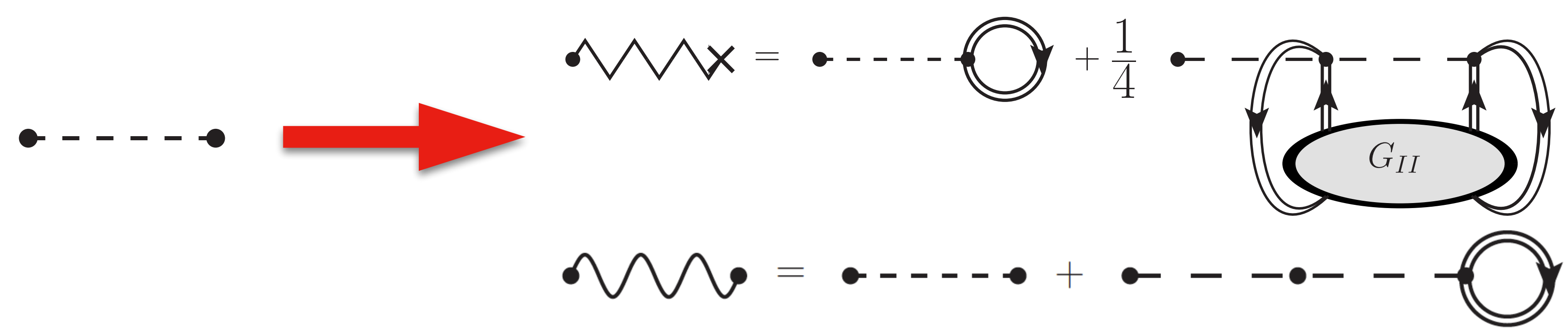
Extended SCGF approach

Carbone, Cipollone, Barbieri, Rios, Polls, PRC **88**, 054326 (2013)

2B → 2B + 3B

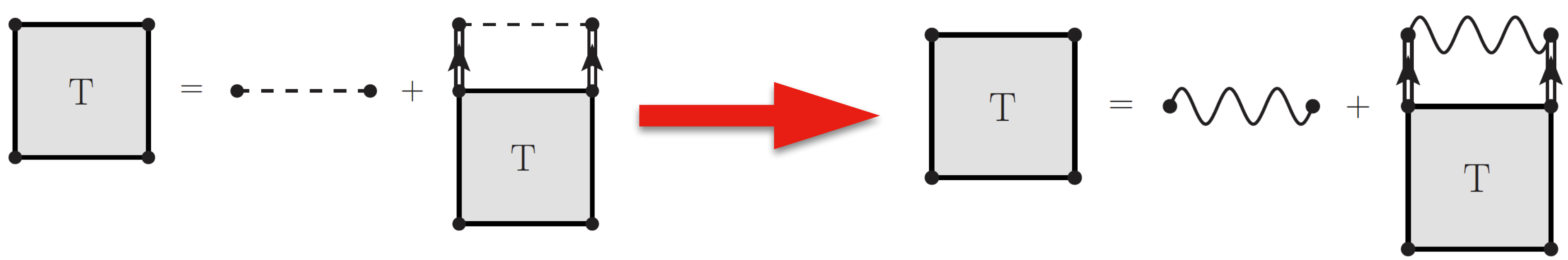
1. define **effective interactions** to include correctly 3B terms, **dressed normal ordering**:

Interaction



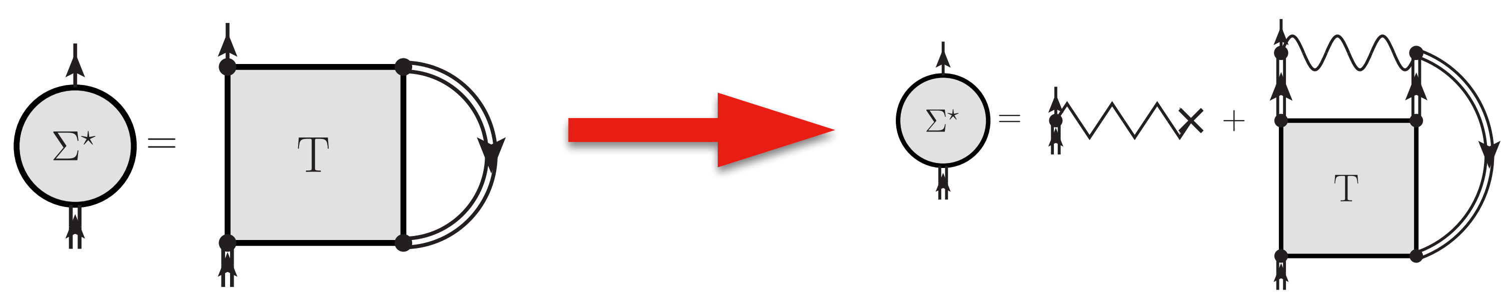
2. calculate T-matrix with effective 2B term, **modified ladder approximation**:

T-matrix



3. calculate self-energy distinguishing the effective terms, **correct diagrams counting**:

Self-energy

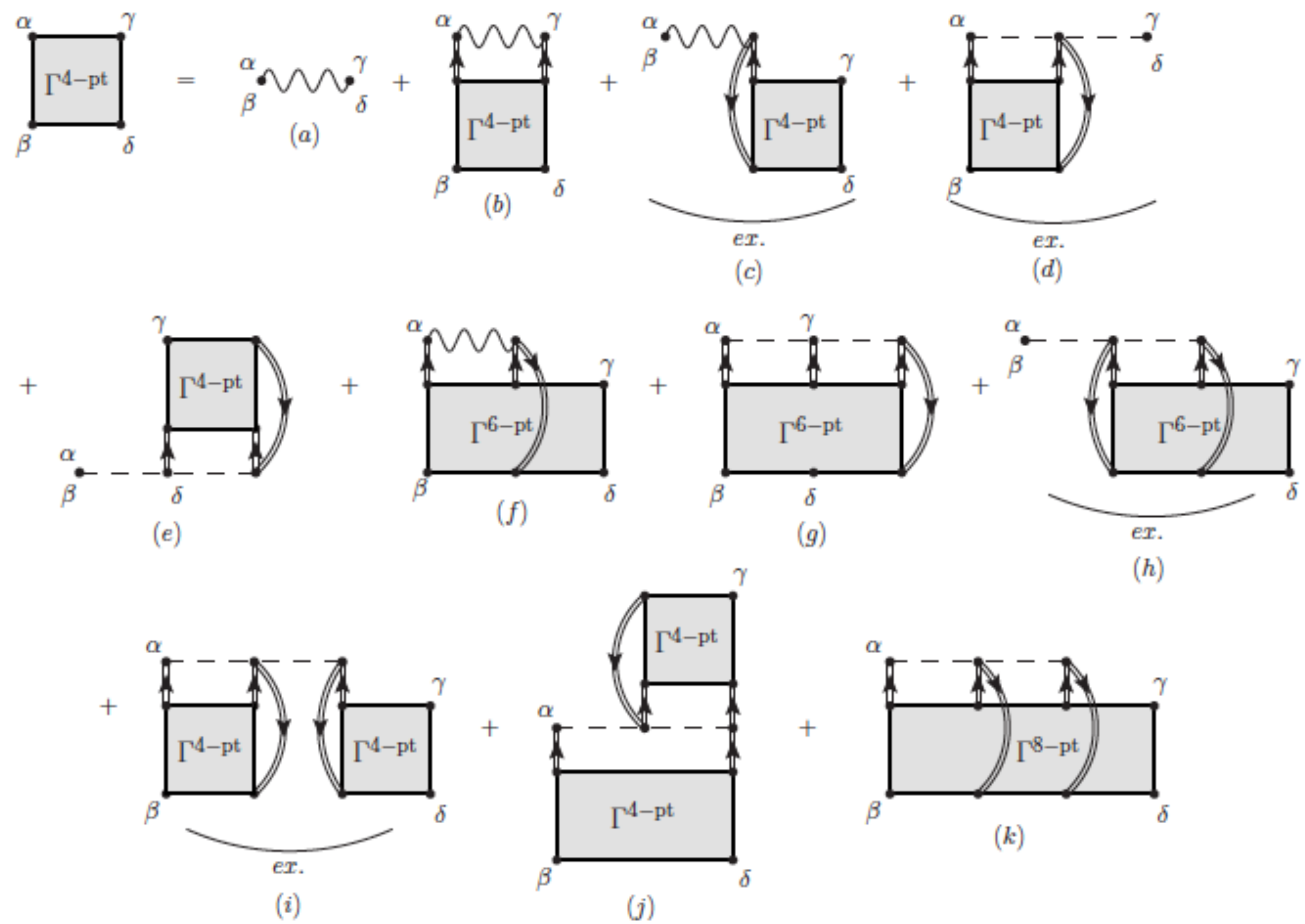


The 4-pt vertex function

Carbone, Cipollone, Barbieri, Rios, Polls, PRC **88**, 054326 (2013)

Obtain the interacting vertex function including 3body forces:

The 4-pt vertex function:



It's an equation including the 4-pt, 6-pt and 8-pt interacting vertex functions!

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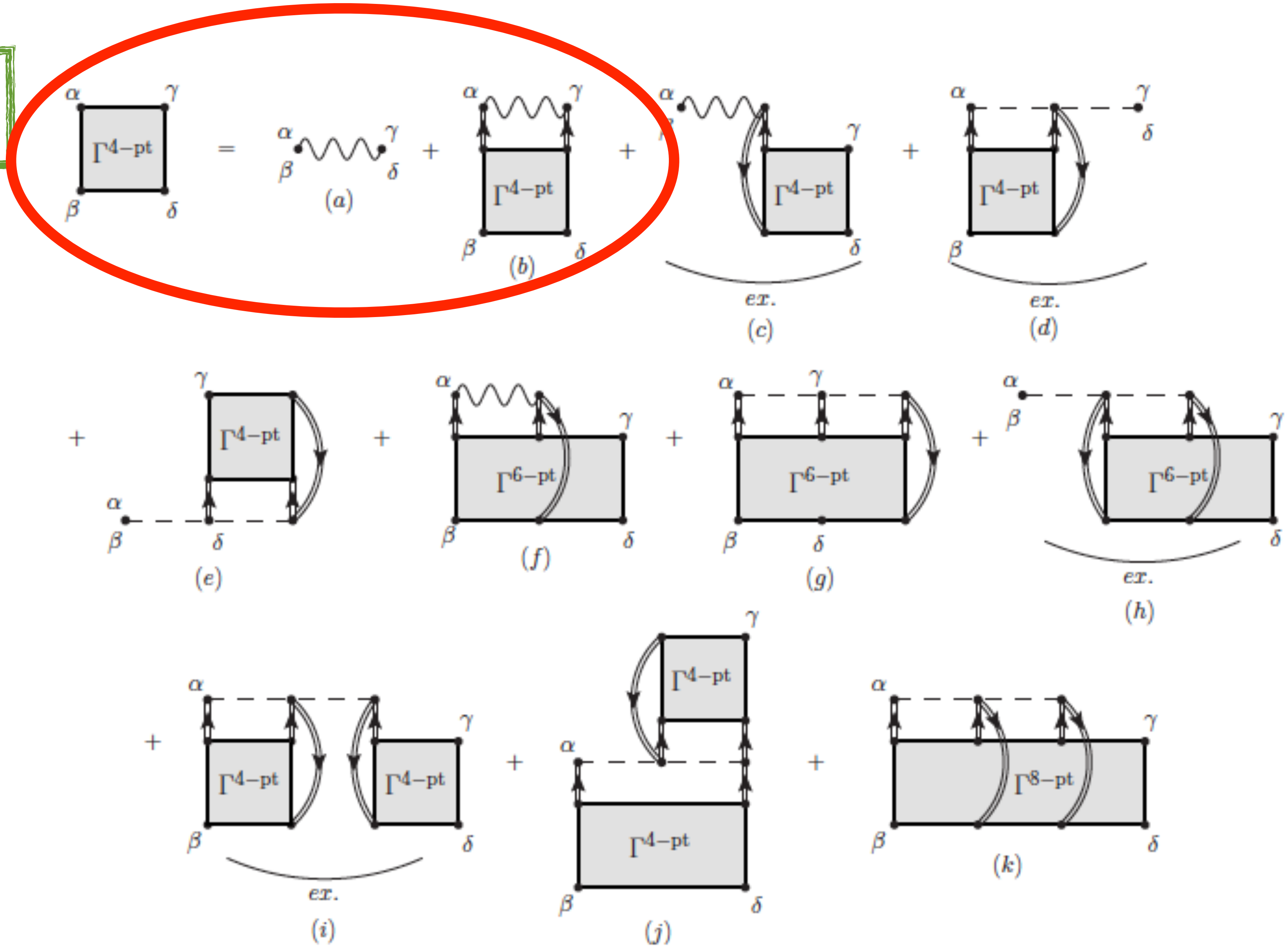


It's an equation including the 4-pt, 6-pt and 8-pt interacting vertex functions!

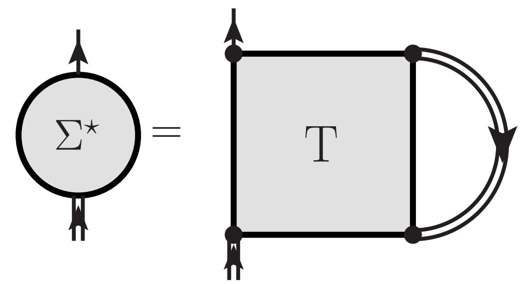
The single-particle self-energy

Carbone, Cipollone, Barbieri, Rios, Polls, PRC **88**, 054326 (2013)

Calculate self-energy paying attention to the effective terms

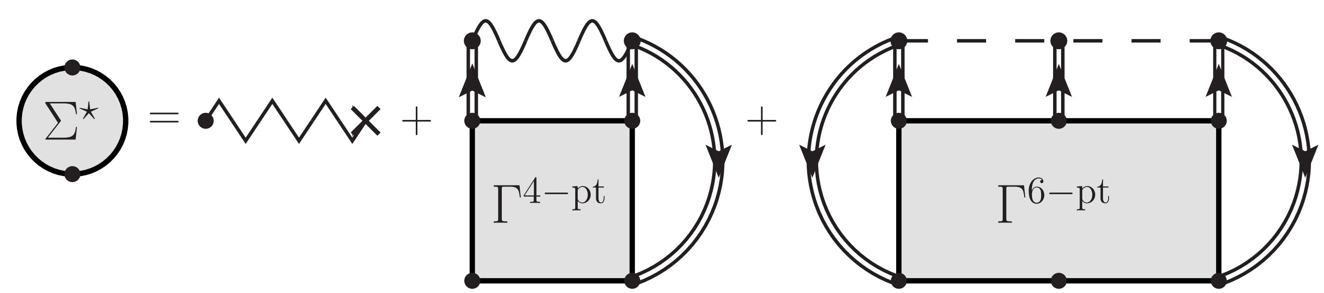
Self-energy:

2B



2B + 3B

Residual 3NF



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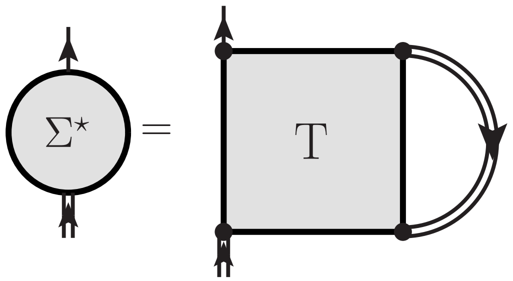
Carbone, Cipollone, Barbieri, Rios, Polls, PRC **88**, 054326 (2013)

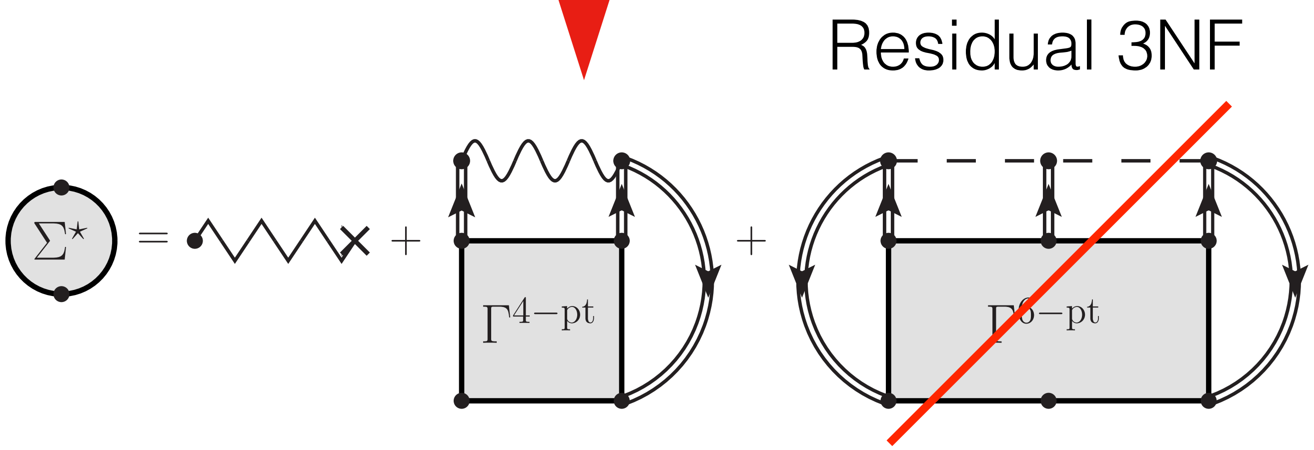
Calculate self-energy paying attention to the effective terms

Self-energy:

2B



2B + 3B



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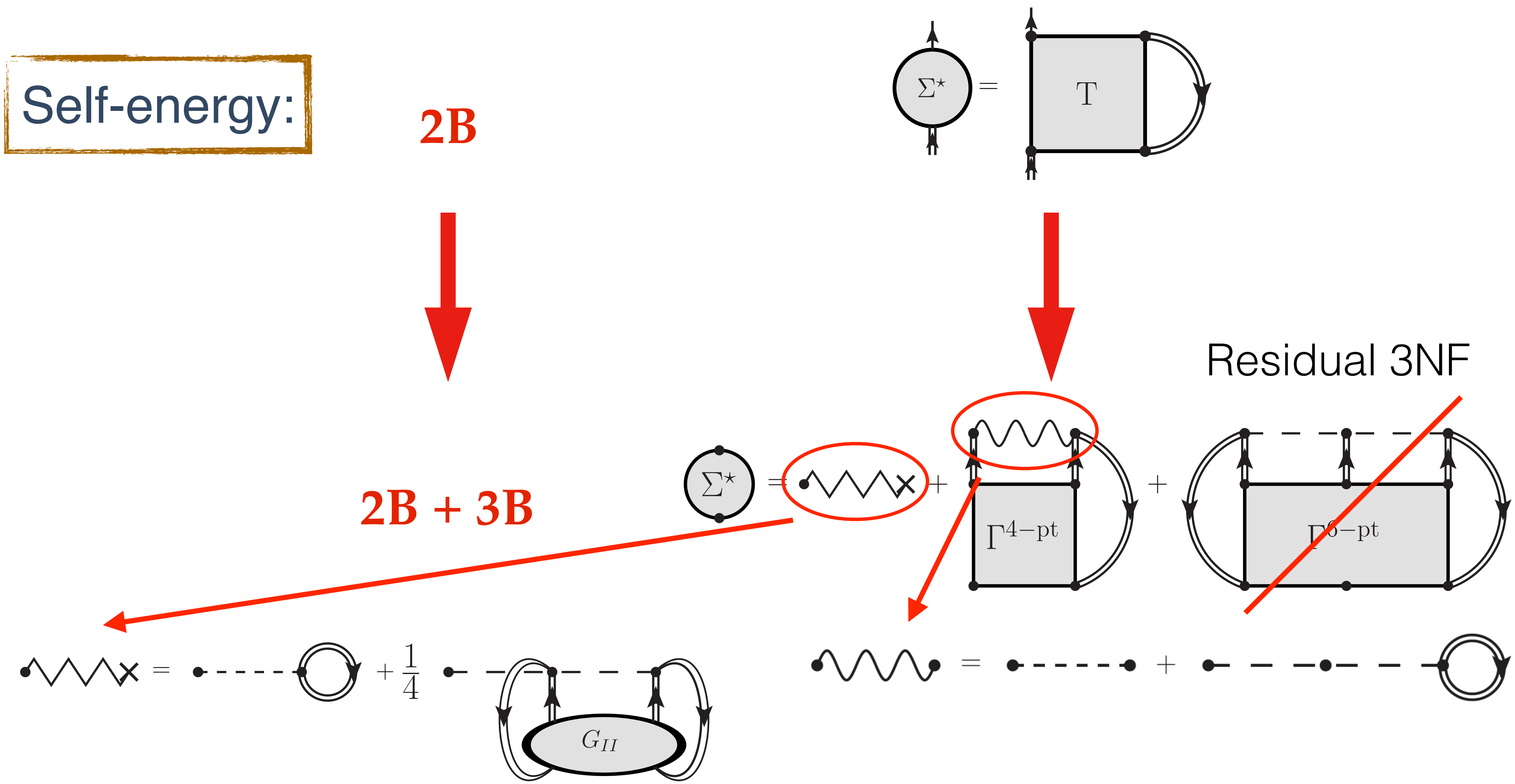

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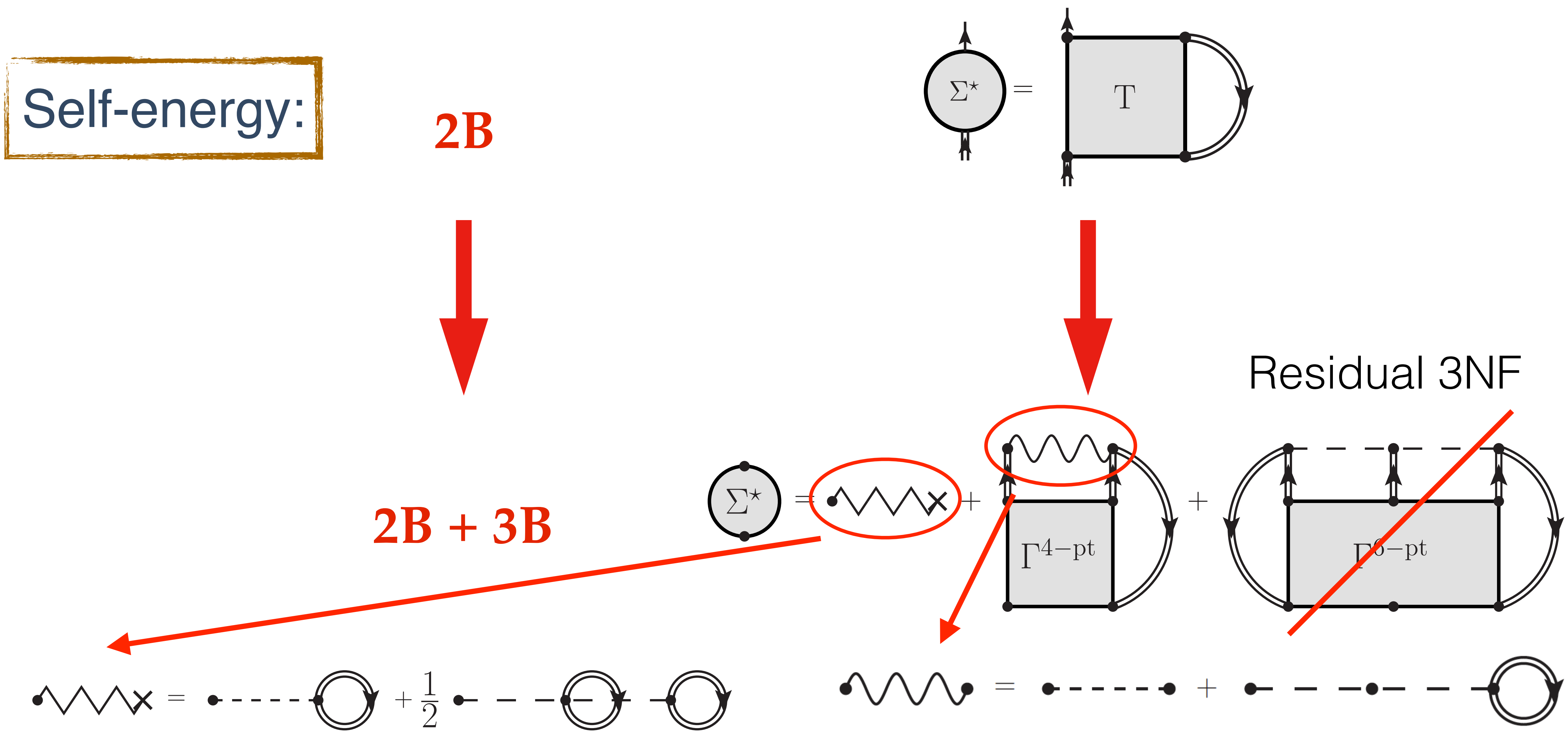
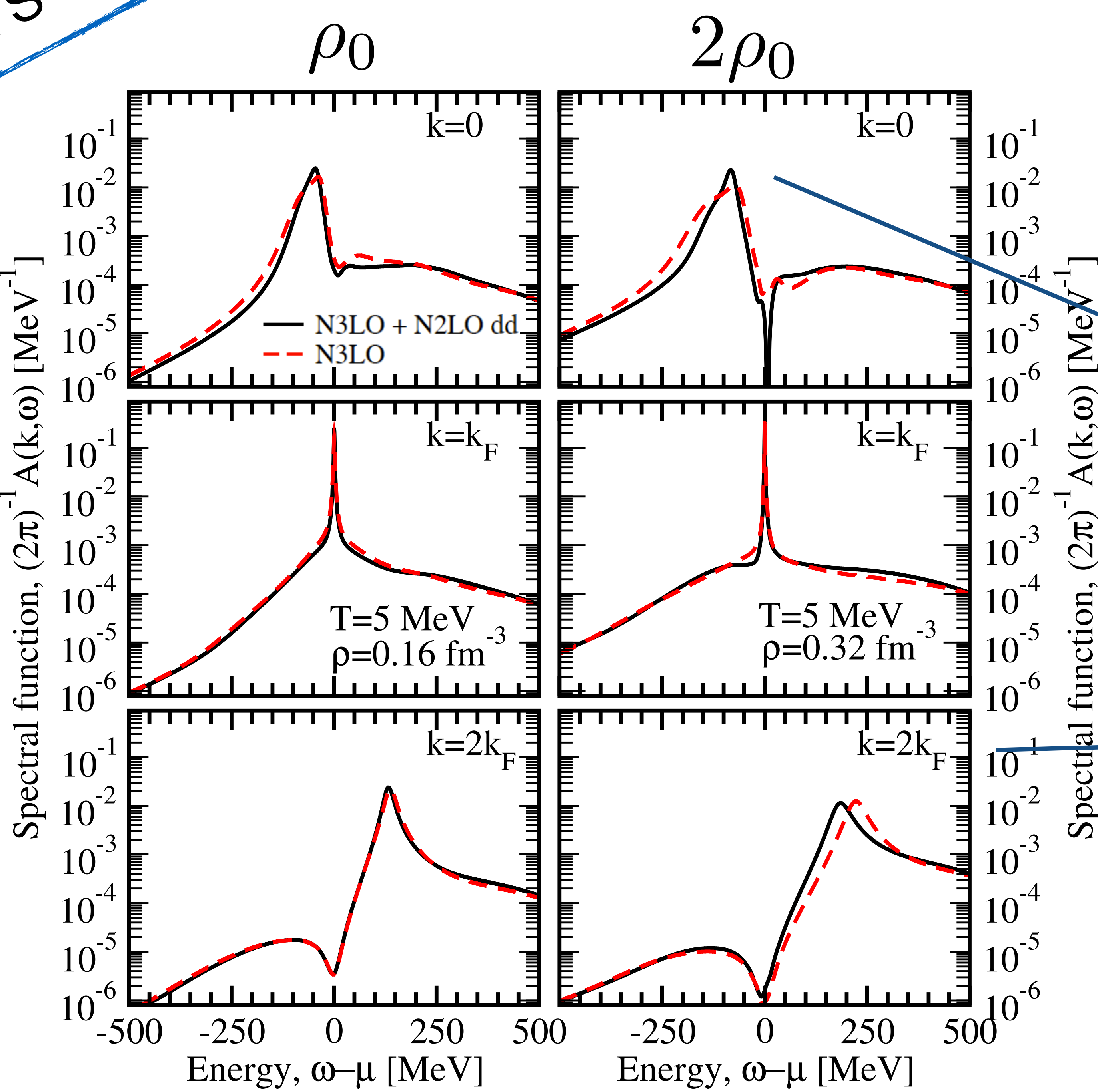


NN vs NN + 3N

The spectral function A(p,w)



Carbone, Rios, Polls, PRC 88, 044302 (2013)

Slight 3BF effect in general....

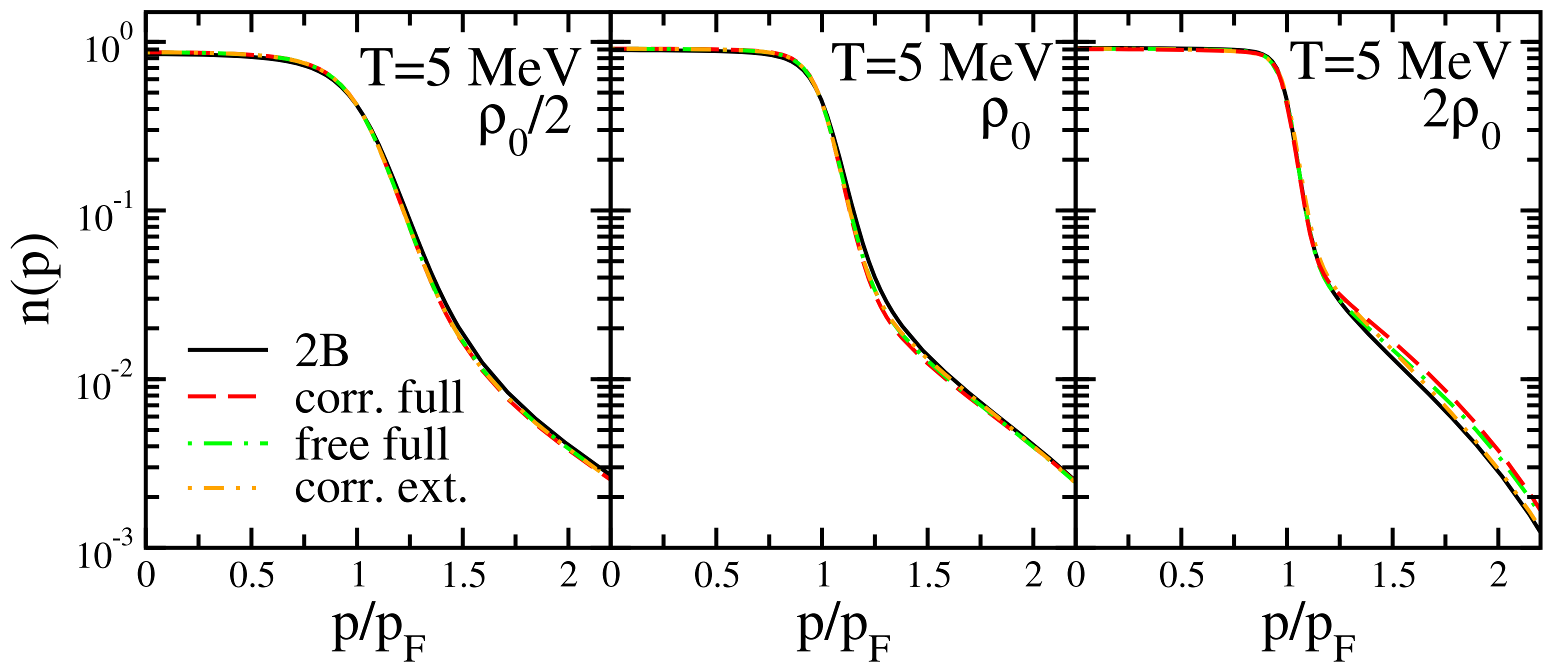
Narrower quasi-parcicle peak at low momenta

Lower energy for quasi-particle with 3NF because of the rescaling

Momentum distribution n(p)

Carbone, Rios, Polls, PRC 90, 054322 (2014)

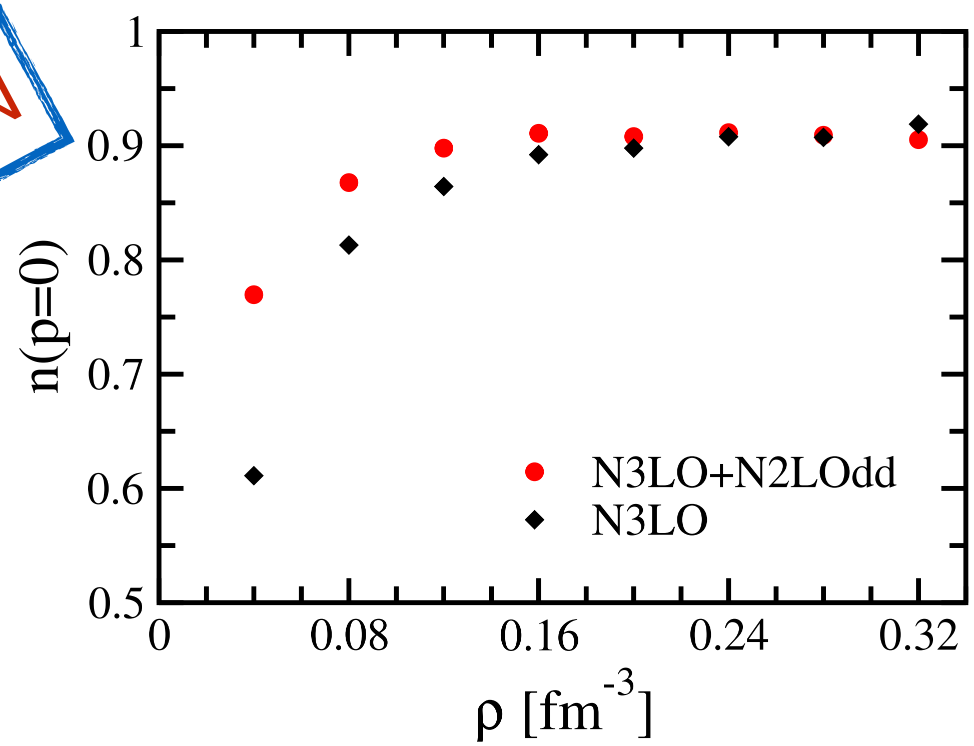
density →



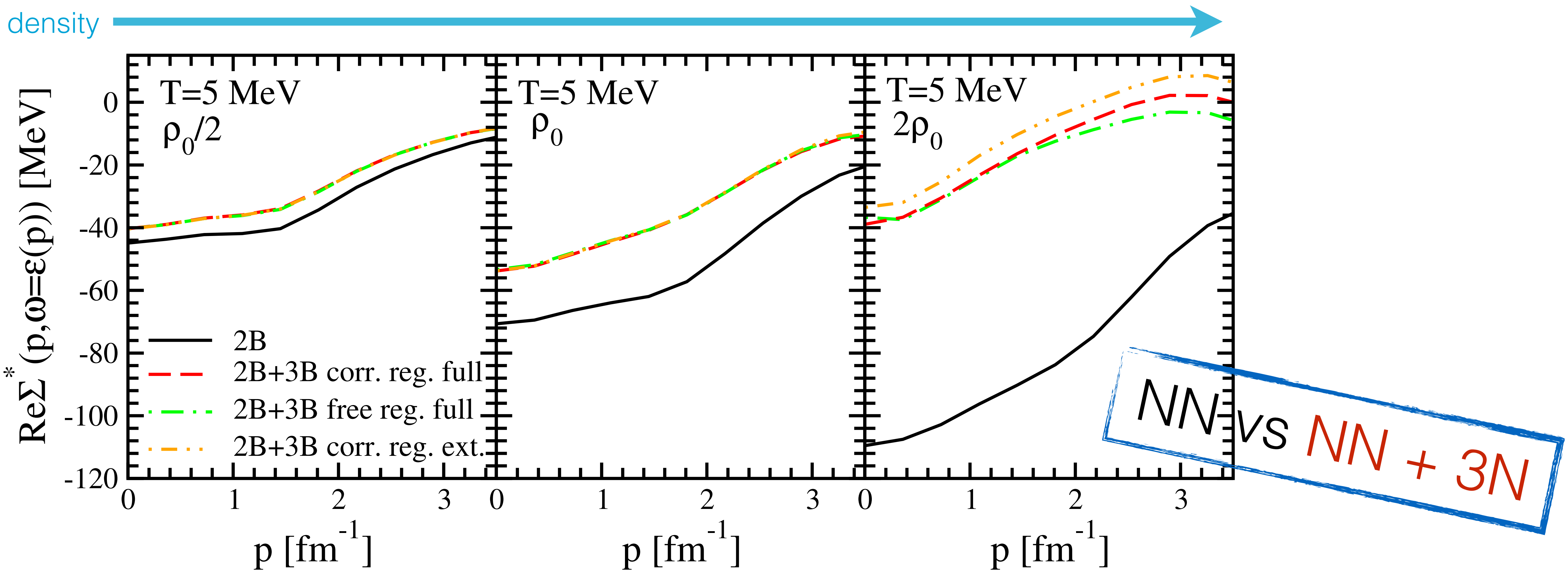
$$n(p) = \int \frac{d\omega}{2\pi} \mathcal{A}(p,\omega) f(\omega)$$

- small changes due to 3BF
- depletion density-dependent
- 3NFs affects depletion
- high-momentum components

NN vs NN + 3N



Single-particle potential



$$\varepsilon_{qp}(p) = \frac{p^2}{2m} + \mathrm{Re}\Sigma^\star(p, \varepsilon_{qp}(p))$$

Single-particle spectra

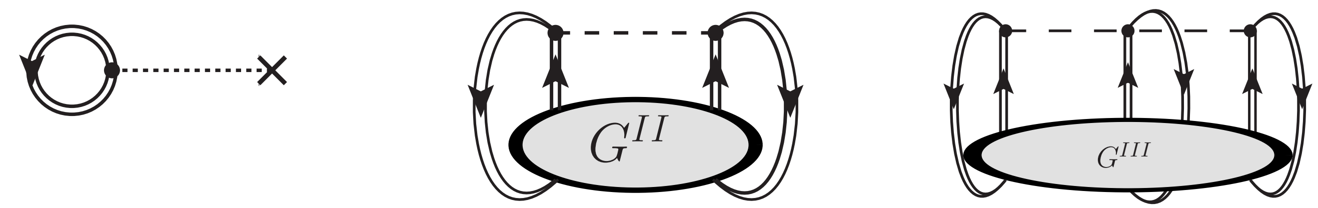
- strong effect of 3-body forces
- repulsion rises with density
- modifications due to averaging procedure visible at high density

Define a new sum rule

Carbone, Cipollone, Barbieri, Rios, Polls, PRC **88**, 054326 (2013)

- Total energy of the system with three-body forces:

$$E^N = \langle\Psi^N|\hat{H}|\Psi^N\rangle = \langle\Psi^N|\hat{T}|\Psi^N\rangle + \langle\Psi^N|\hat{V}|\Psi^N\rangle + \langle\Psi^N|\hat{W}|\Psi^N\rangle$$



- Galitskii-Migdal-Koltun sumrule modified:

$$\sum_\alpha \int_{-\infty}^{E^N - E^{N-1}} \mathrm{d}\omega\, \omega \frac{1}{\pi} \mathrm{Im}\, G_{\alpha\alpha}(\omega) = \langle\Psi^N|\hat{T}|\Psi^N\rangle + 2\langle\Psi^N|\hat{V}|\Psi^N\rangle + 3\langle\Psi^N|\hat{W}|\Psi^N\rangle$$

$$\frac{E}{A} = \frac{\nu}{\rho} \int \frac{\mathrm{d}^3 p}{(2\pi)^3} \int \frac{\mathrm{d}\omega}{2\pi} \frac{1}{2} \left\{ \frac{p^2}{2m} + \omega \right\} \mathcal{A}(p,\omega) f(\omega) - \frac{1}{2}\langle\Psi^N|\hat{W}|\Psi^N\rangle$$

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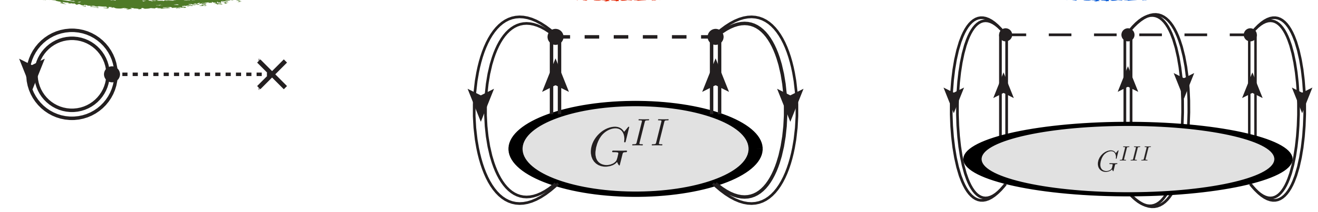

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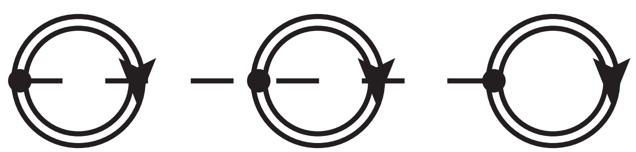
Modified Koltun sum rule

Carbone, Cipollone, Barbieri, Rios, Polls, PRC **88**, 054326 (2013)

- Written in other words, we are calculating:

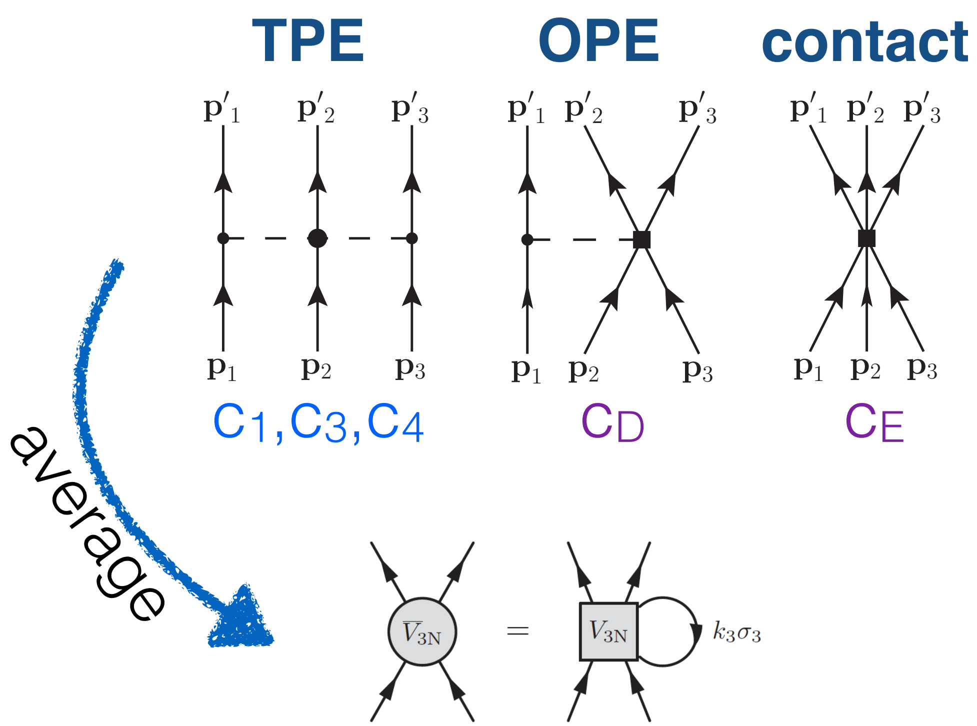
$$\frac{E}{A} = \frac{\nu}{\rho} \int \frac{d^3p}{(2\pi)^3} \int \frac{d\omega}{2\pi} \frac{1}{2} \left\{ \frac{p^2}{2m} + \omega \right\} \mathcal{A}(p,\omega) f(\omega) - \frac{1}{2} \langle \Psi^N | \hat{W} | \Psi^N \rangle$$



1st order fully dressed

$$\frac{E}{A} = \frac{\nu}{\rho} \int \frac{d^3p}{(2\pi)^3} \int \frac{d\omega}{2\pi} \frac{1}{2} \left\{ \frac{p^2}{2m} + \omega - \frac{1}{3} \Sigma^{3NF}_{HF}(p) \right\} \mathcal{A}(p,\omega) f(\omega)$$

The need for 3-body nuclear forces

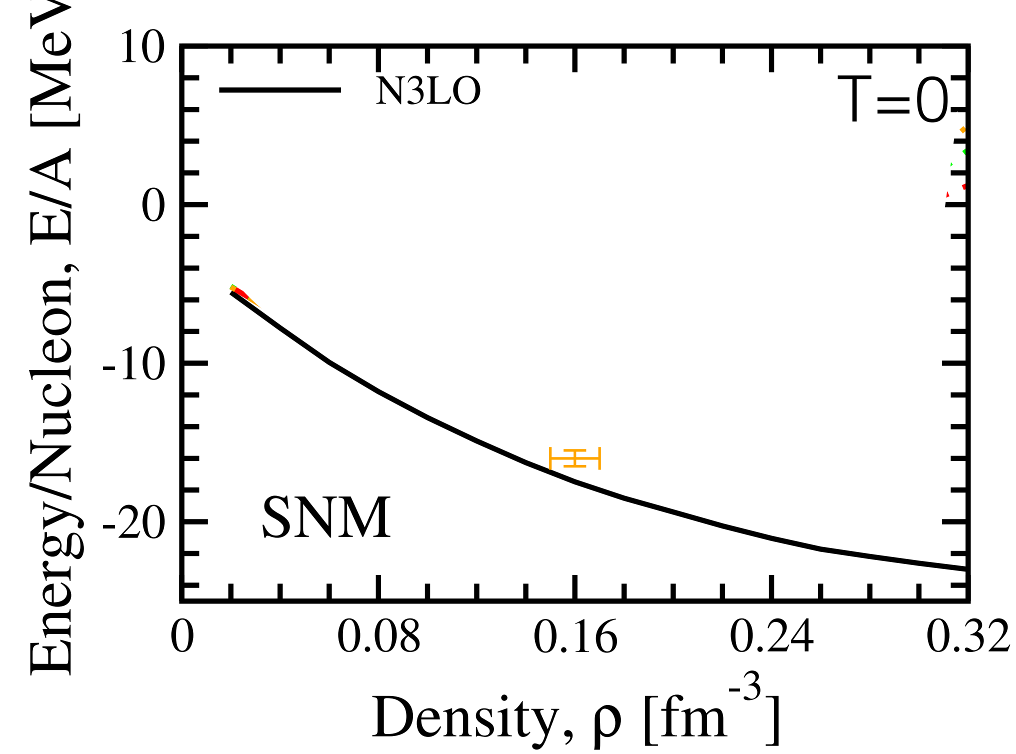


J.W. Holt *et al.*, PRC 81, 024002 (2010)
Hebeler *et al.*, PRC 82, 014314 (2010)
Carbone *et al.*, PRC 90, 054322 (2014)

The Koltun sumrule

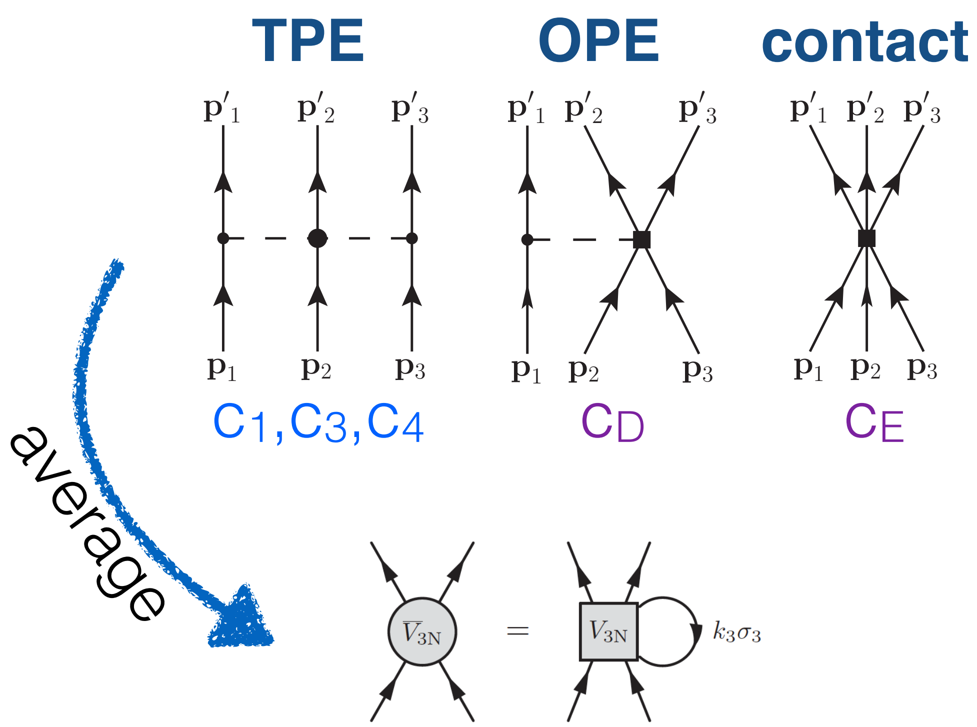
$$\frac{E}{A} = \frac{\nu}{\rho} \int \frac{d^3p}{(2\pi)^3} \int \frac{d\omega}{2\pi} \frac{1}{2} \left\{ \frac{p^2}{2m} + \omega \right\} \mathcal{A}(p,\omega) f(\omega) - \frac{1}{2} \langle \Psi^N | \hat{W} | \Psi^N \rangle$$

Self-consistent Green's functions



Carbone *et al.*, PRC 90, 054322 (2014)

The need for 3-body nuclear forces



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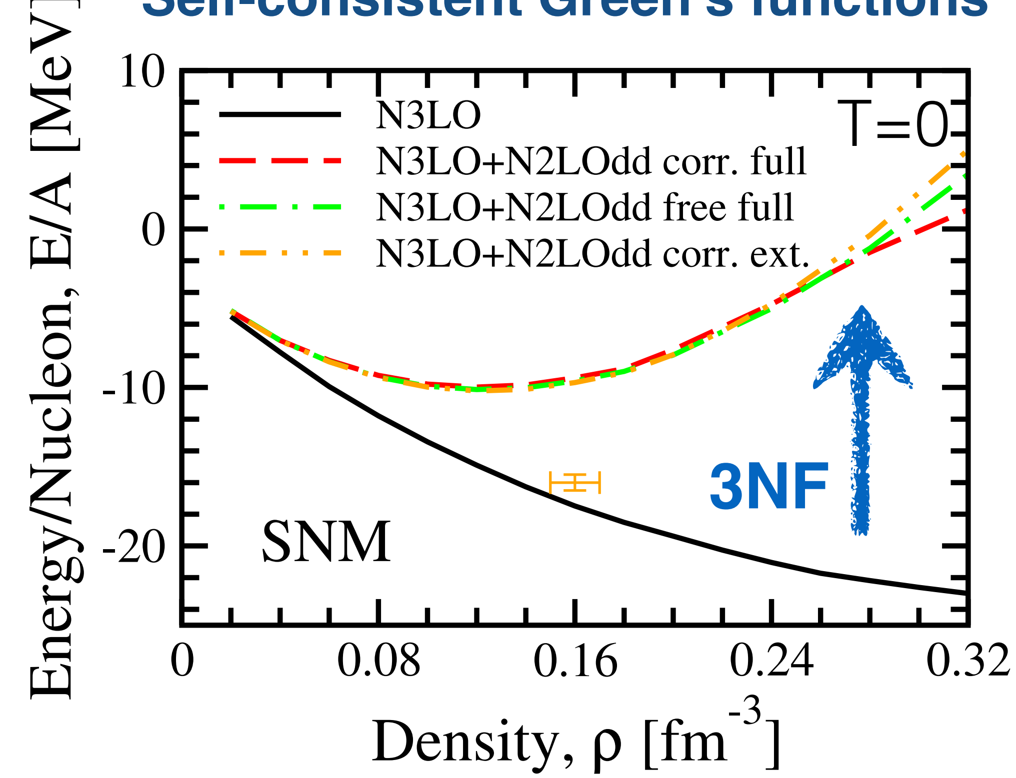
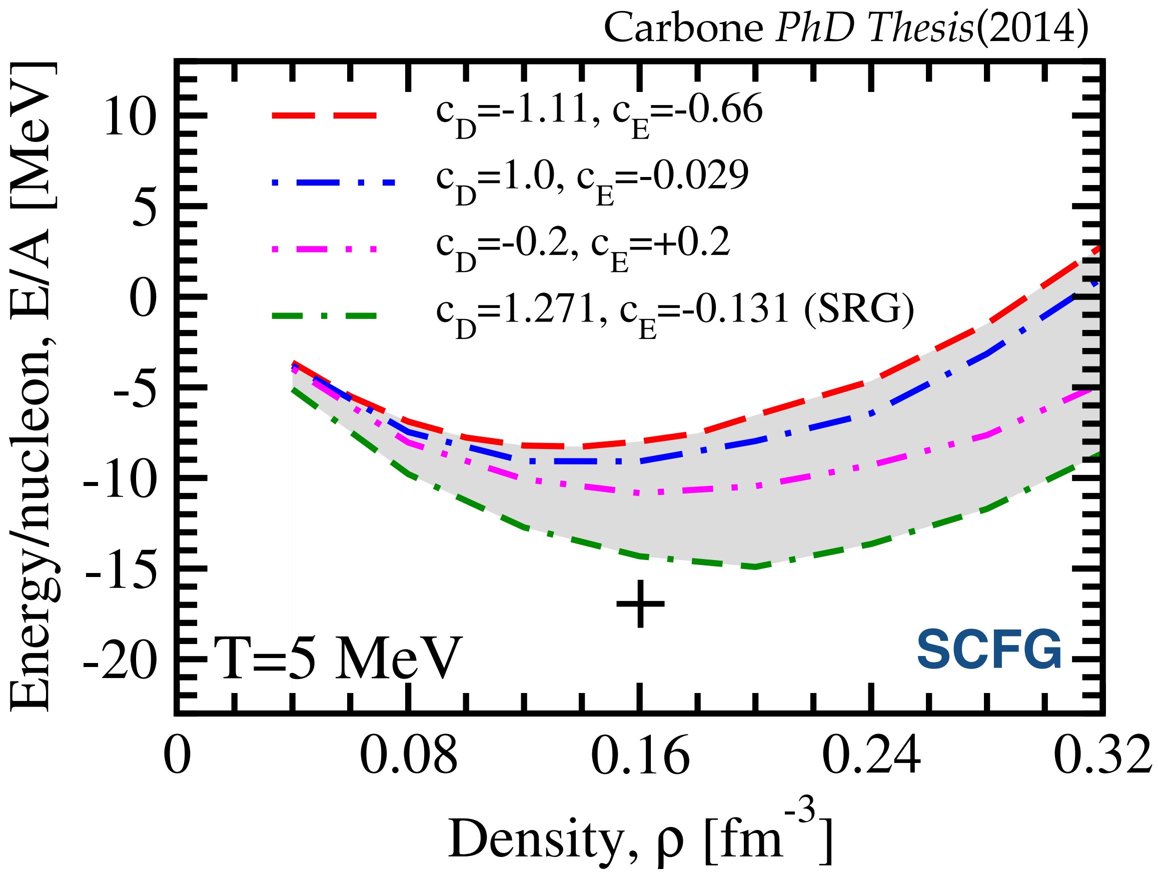
- Overall repulsion due to 3BF
- Improved prediction of saturation density
- Small averaging dependence
- However saturation energy underbound

Self-consistent Green's functions



Carbone *et al.*, PRC 90, 054322 (2014)

Uncertainties due to fitting procedures

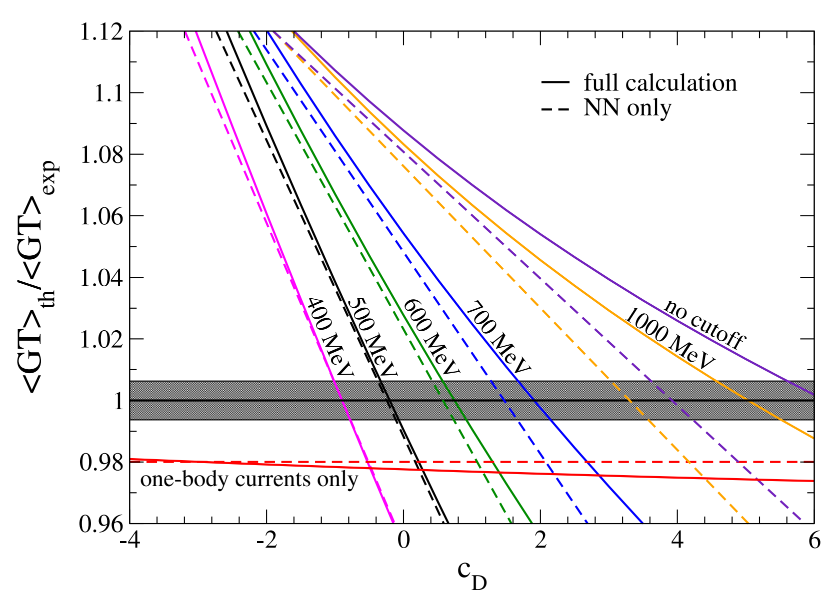


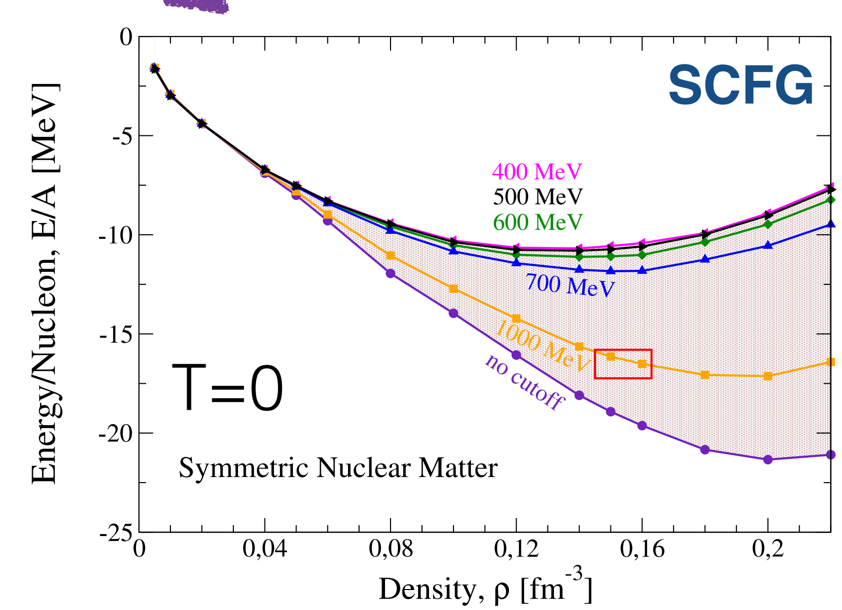
Some low-energy constants are fit to few-body properties

(binding energies, nuclear radii, etc.)

- Band gives theoretical uncertainties
- Uncertainty increases with density

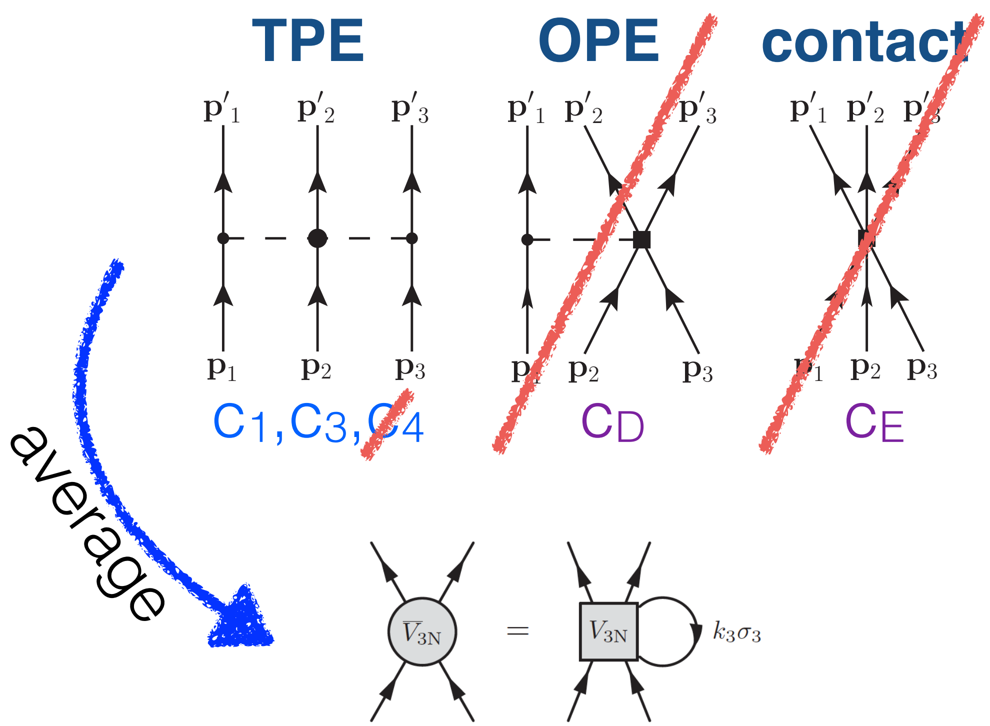
- Triton beta-decay is precisely known
- Visible dependence on the current cutoff



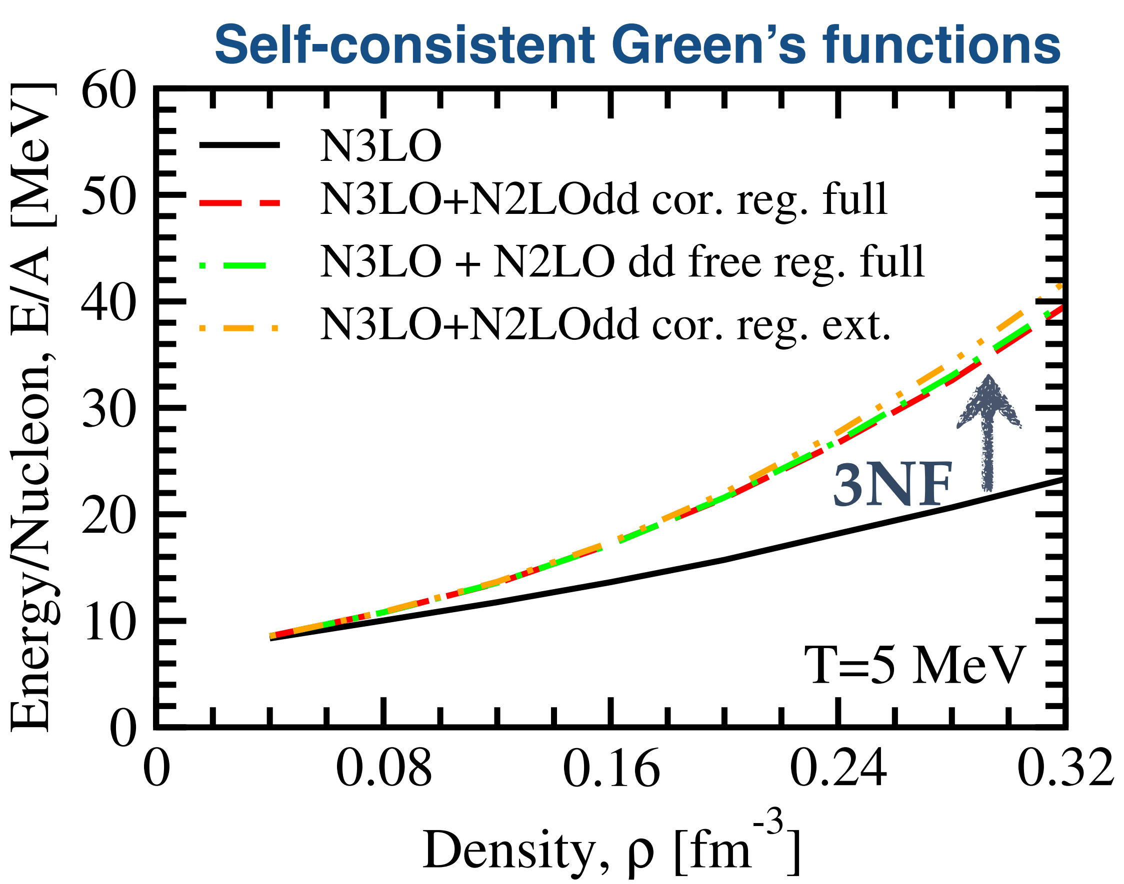


Klos, Hebeler, Menéndez, Carbone, Schwenk *(in preparation)*

How neutron matter energy stiffens



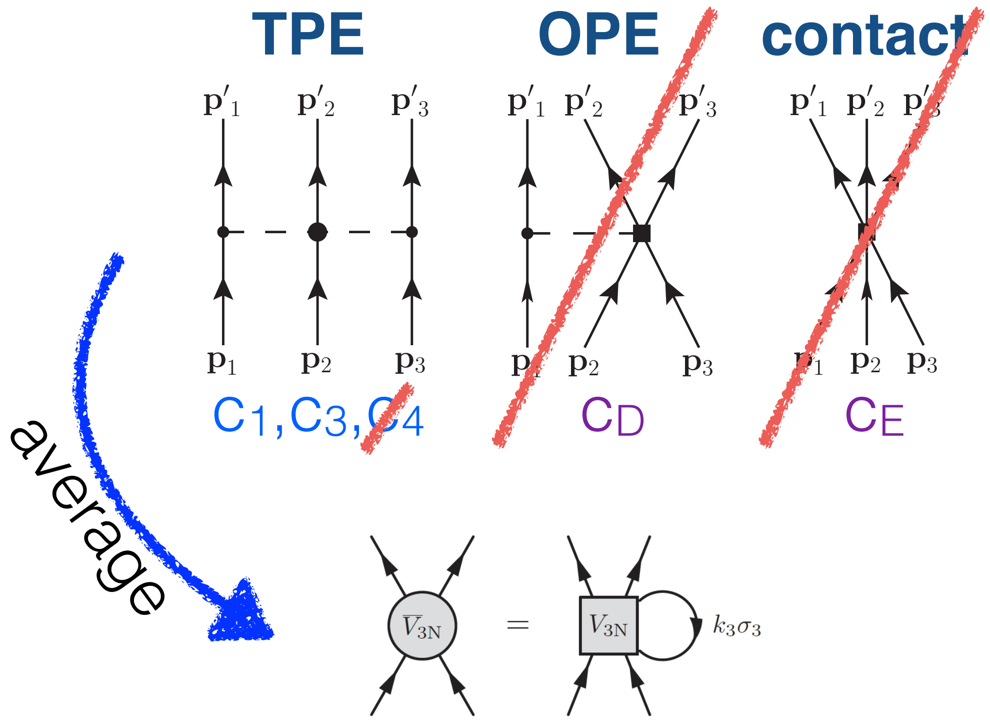
- 3NFs fully predicted
- no need to fit to few-body properties



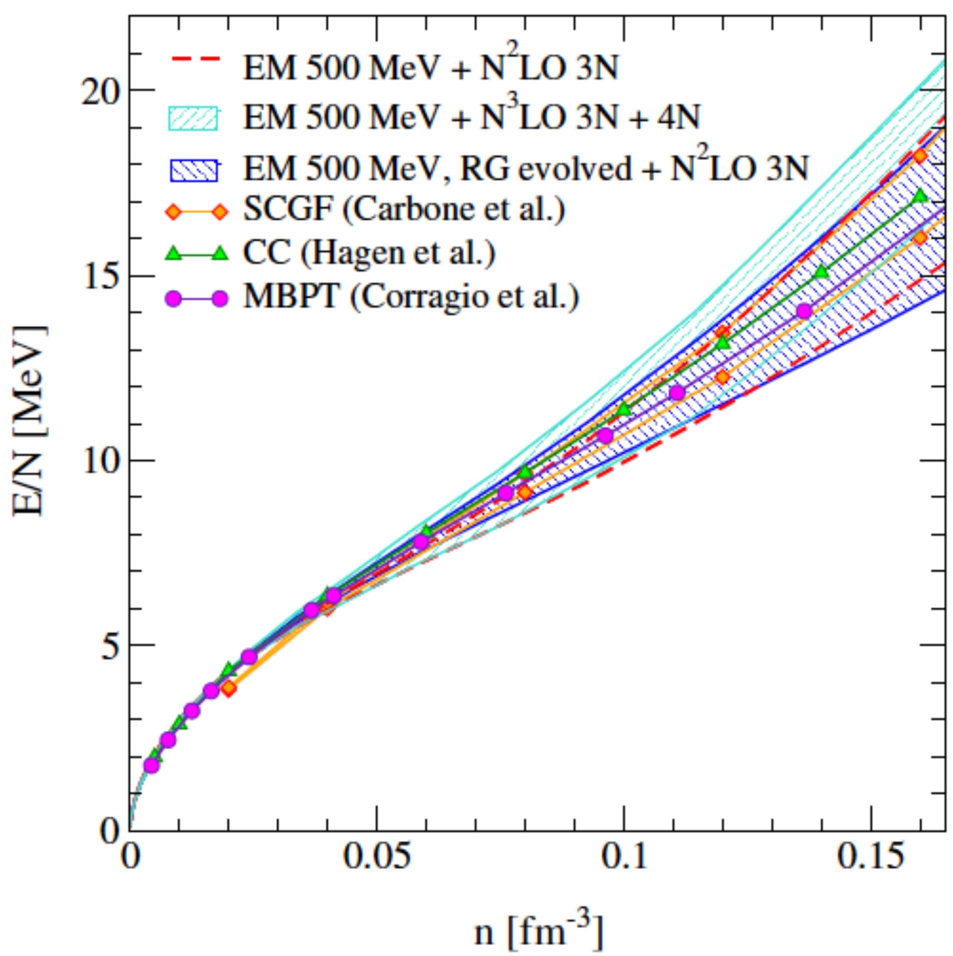
Carbone *et al.*, PRC 90, 054322 (2014)

- Global repulsive effect due to 3NFs
- Repulsion of 4 MeV at 0.16 fm^{-3} to 15 MeV at 0.32 fm^{-3}
- Small dependence on averaging procedures

Many-body methods comparison



Hebeler *et al.*, Ann. Rev. Nucl. Part. Sci. 65, 457 (2015)



- Low-density neutron matter perturbative
- Bands from c1 and c3 uncertainties
- First calculations including N3LO 3N at HF

Remarkable agreement between many-body methods and different Hamiltonians

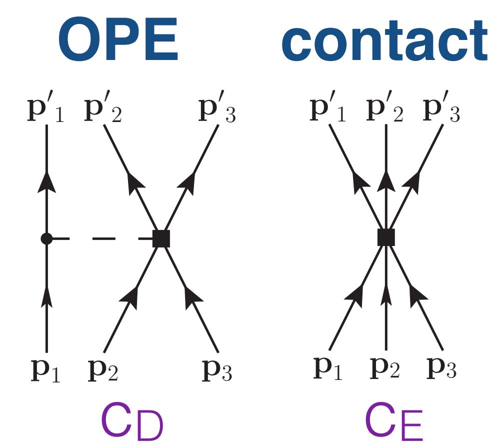
Further results:
AFDMC - Gezerlis et al., PRC 90, 054323 (2014)
Lattice EFT - Epelbaum et al., EPJA 40, 199 (2009)
In-medium Chiral PT - J.W. Holt et al., PPNP 73, 35 (2013),
Lacour et al., Ann. Phys. 326, 241 (2011)
MBPT Wellenhofer et al, PRC 92, 015801 (2015)

Chiral forces within the SCGF approach

- Choose five different chiral Hamiltonians
- Test how they behave in SNM
- Check microscopic and bulk properties
- Predict PNM and estimate the symmetry energy
- Study SNM finite-T properties and the liquid-gas phase transition

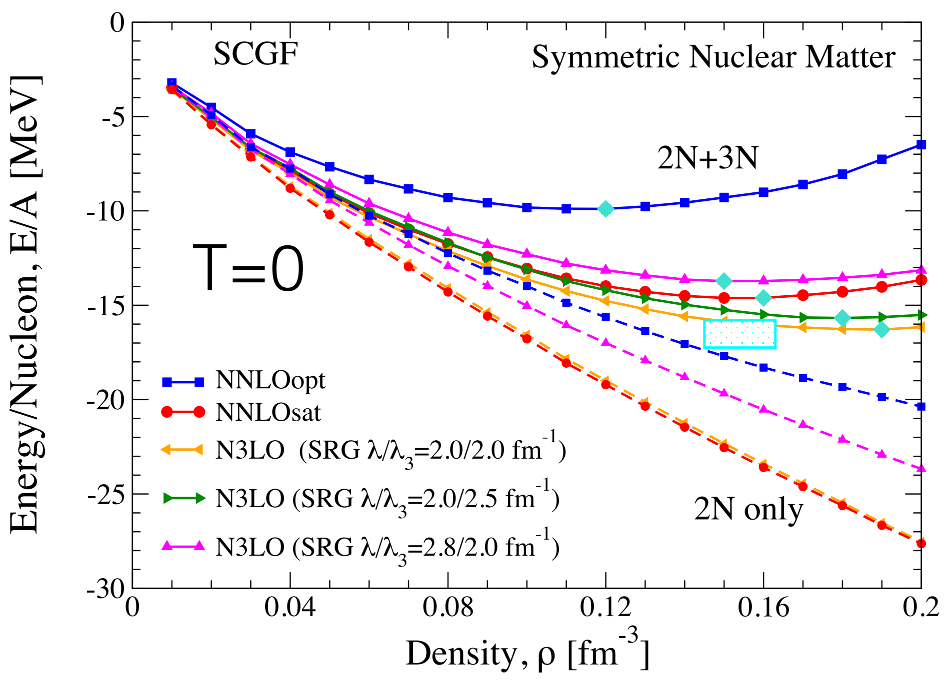
Saturation point according to different Hamiltonians

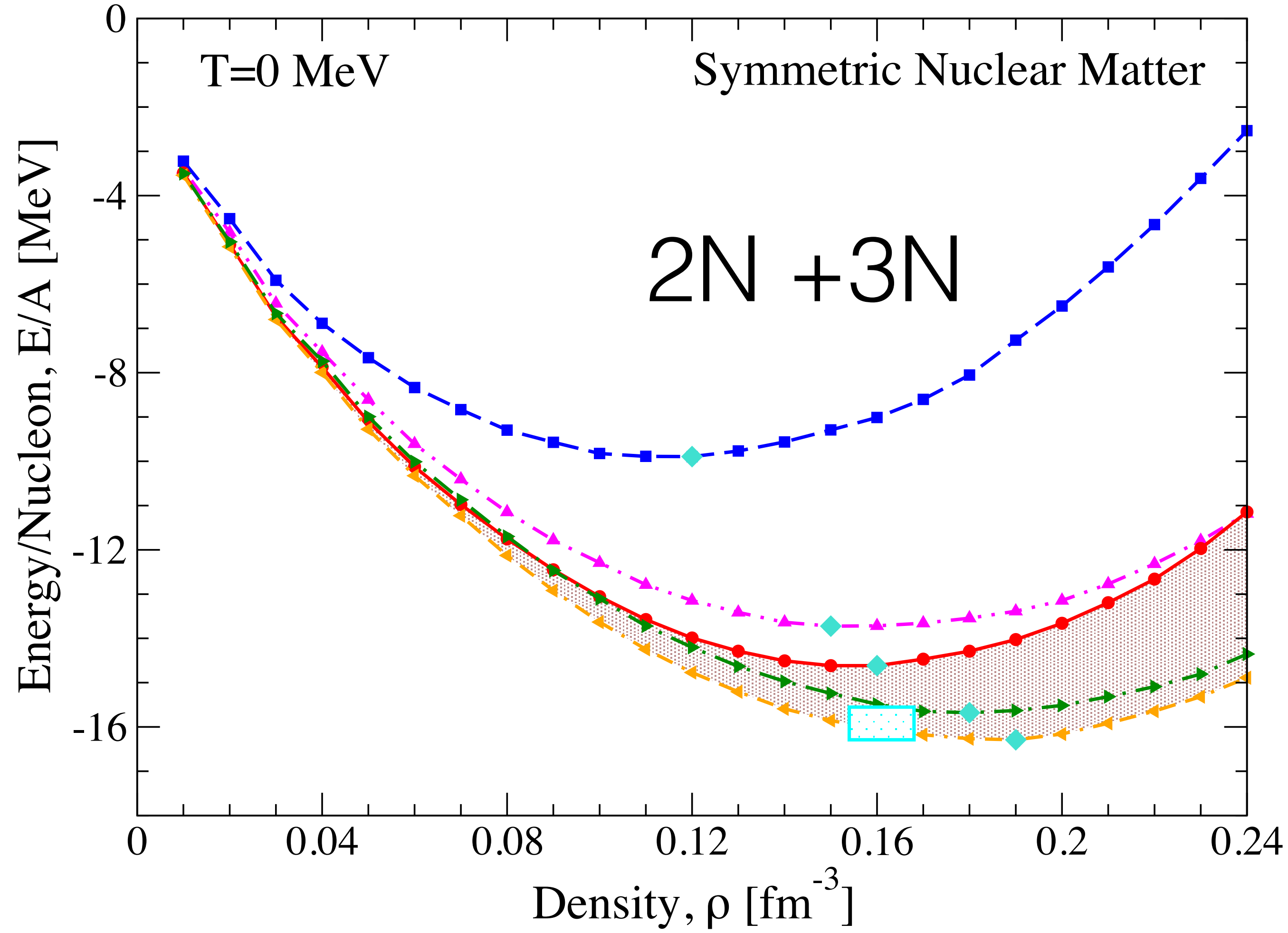
- N3LO EM500+SRG, 3NFs fit to ^{3}H BEs, ^{4}He r_m
- N2LOopt (POUNDERS), 3NFs fit to ^{3}H,^{3}He BEs
- N2LOsat (POUNDERS), NN+3N fit to ^{3}H,$^{3,4}He$,^{14}C,^{16}O BEs,r_{ch}, etc.



	N3LO 1	N3LO 2	N3LO 3	NNLOopt	NNLOsat
cD	1.271	-0.292	1.278	-2	0.81680589
cE	-0.131	-0.592	-0.078	-0.791	−0.03957471

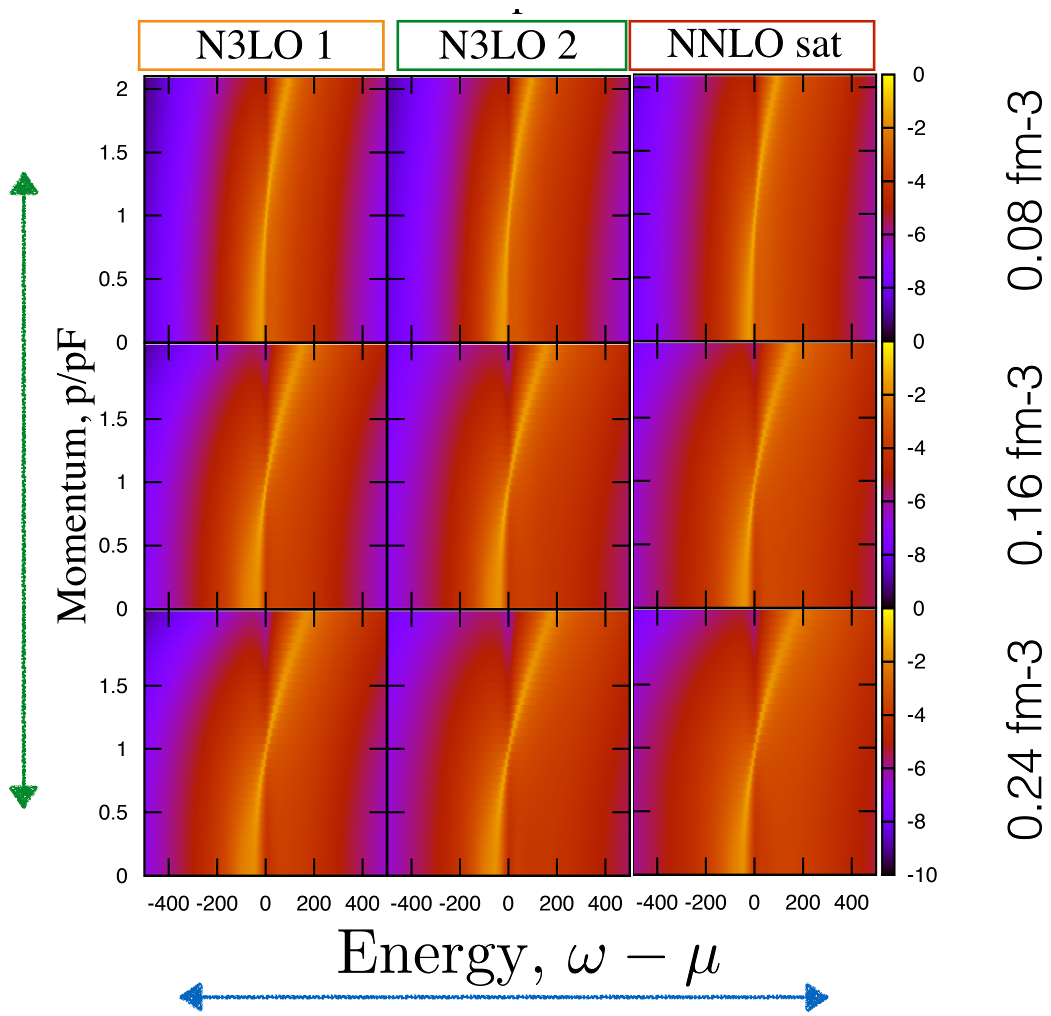
- 3NF make a difference from the 2NF only case





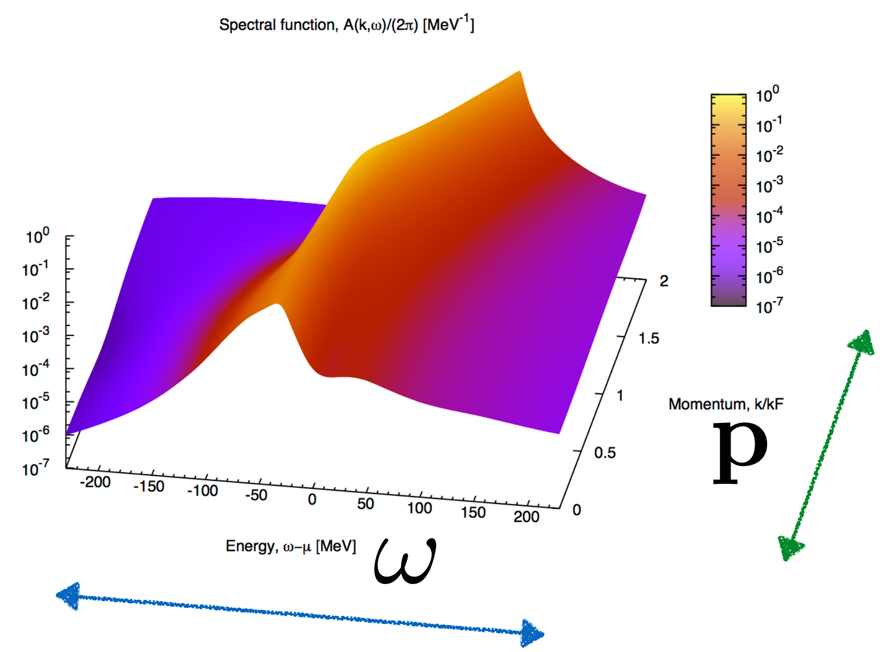
Carbone (*in preparation*)

Microscopic properties: the spectral function

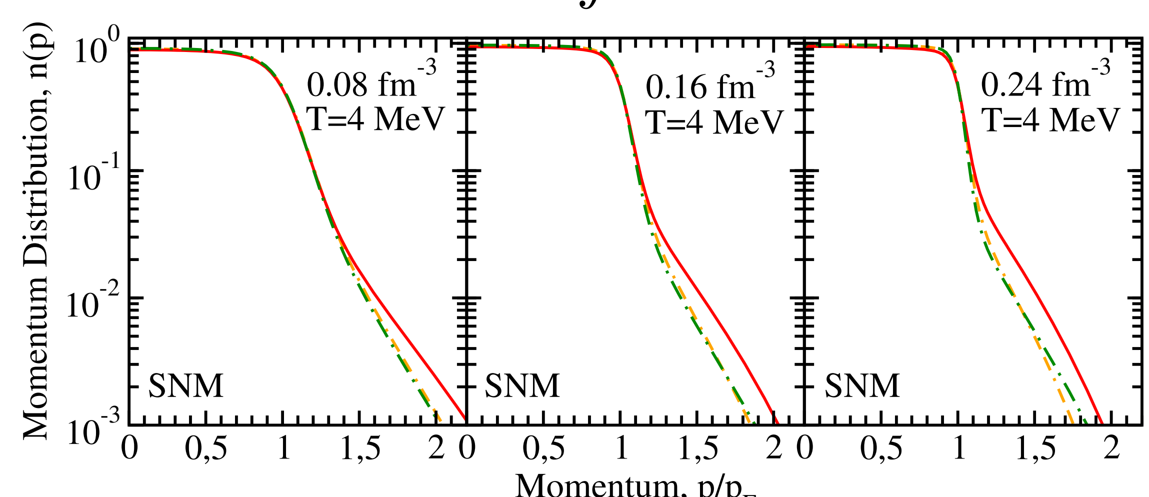


- energy tails affected by the cutoff on the NN force
- high-momentum region also affected by cutoff and density dependence
- effects clearly visible in momentum distribution

- full description beyond quasiparticle

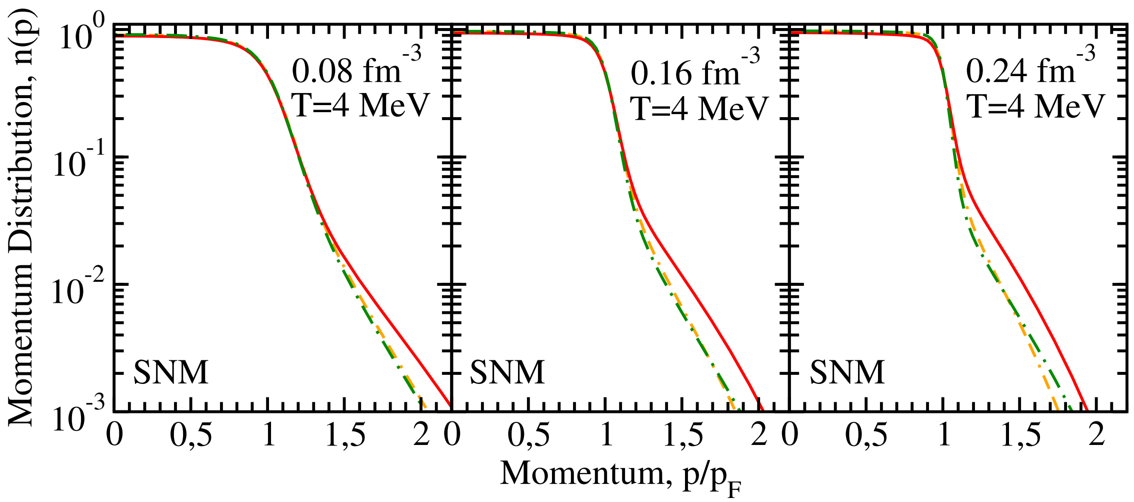


$$n(p) = \int \frac{\mathrm{d}\omega}{2\pi} \mathcal{A}(p,\omega) f(\omega)$$



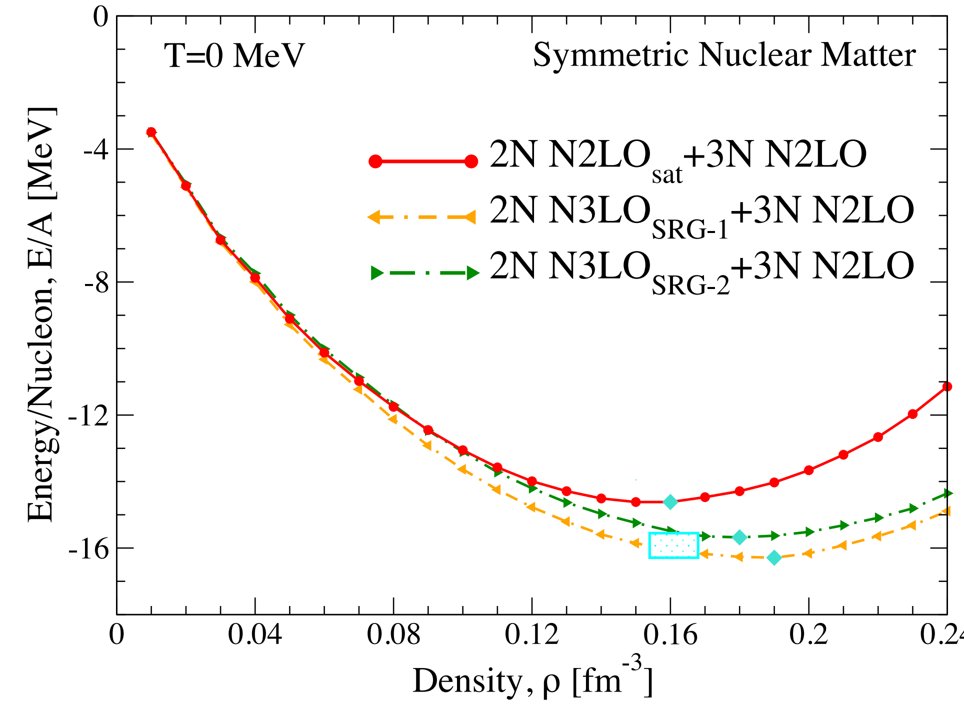
Carbone *(in preparation)*

From microscopic… to macroscopic



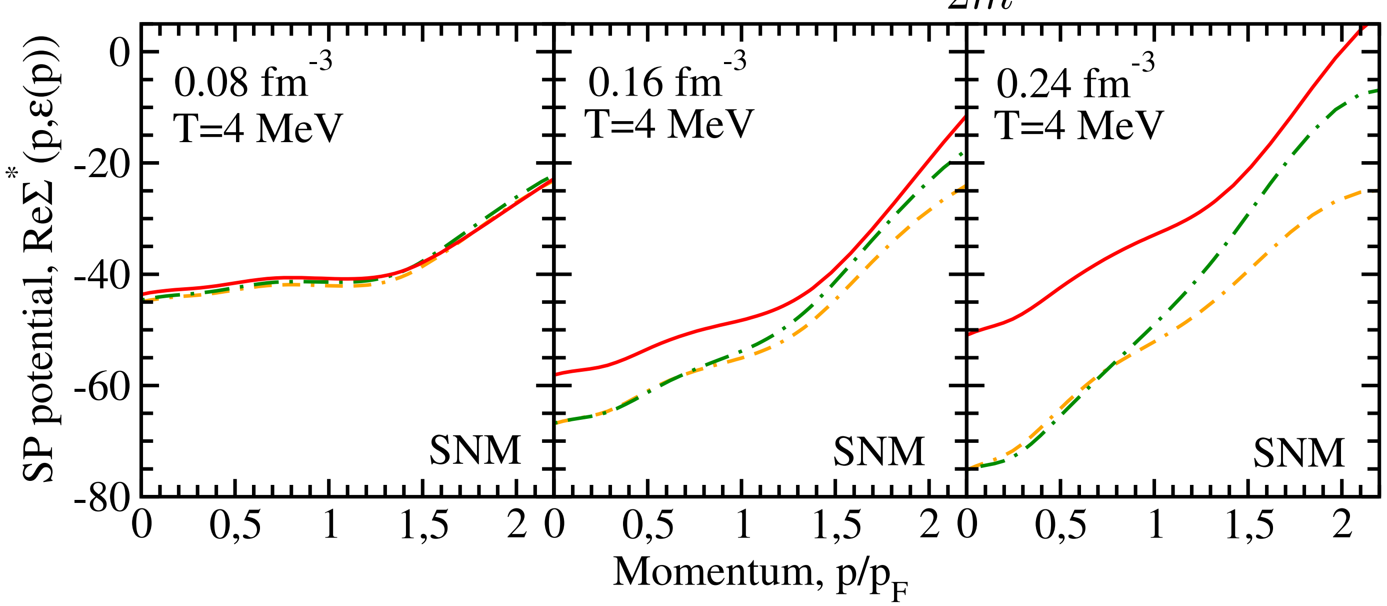
$$n(p) = \int \frac{d\omega}{2\pi} \mathcal{A}(p,\omega) f(\omega)$$

…start seeing the big picture



the big picture

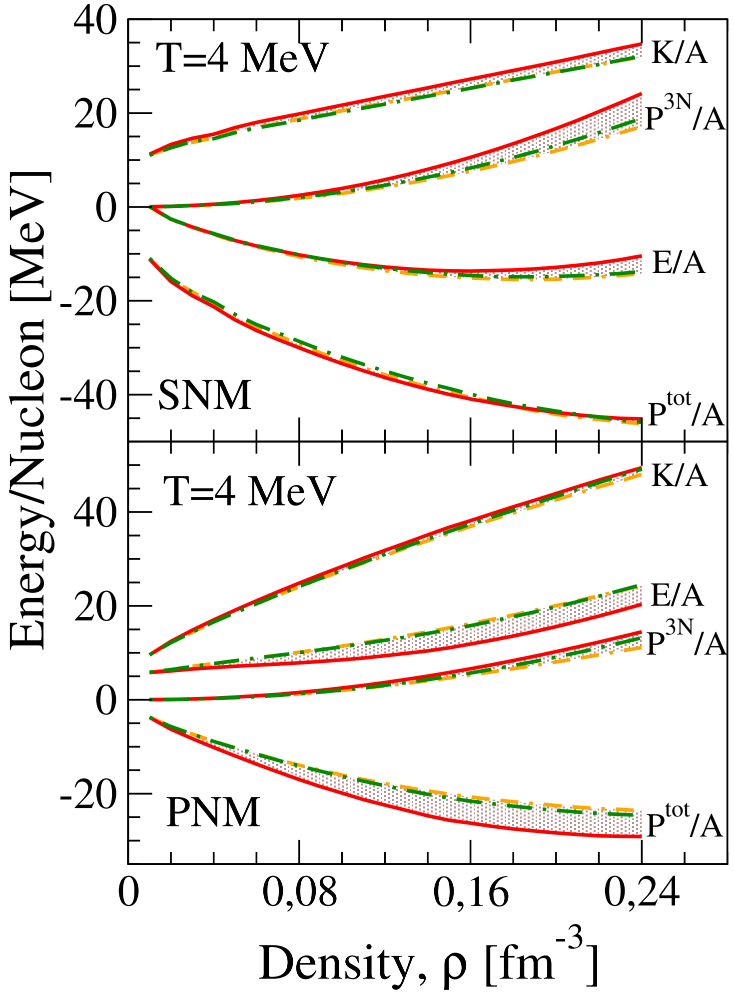
$$\varepsilon_{qp}(p) = \frac{p^2}{2m} + \mathrm{Re}\Sigma^\star(p, \varepsilon_{qp}(p))$$



Carbone *(in preparation)*

- N2LOsat high-momentum states
- 3NF effects as dens. increases
- N2LOsat more repulsive
- Visible effect of 3NF cutoff

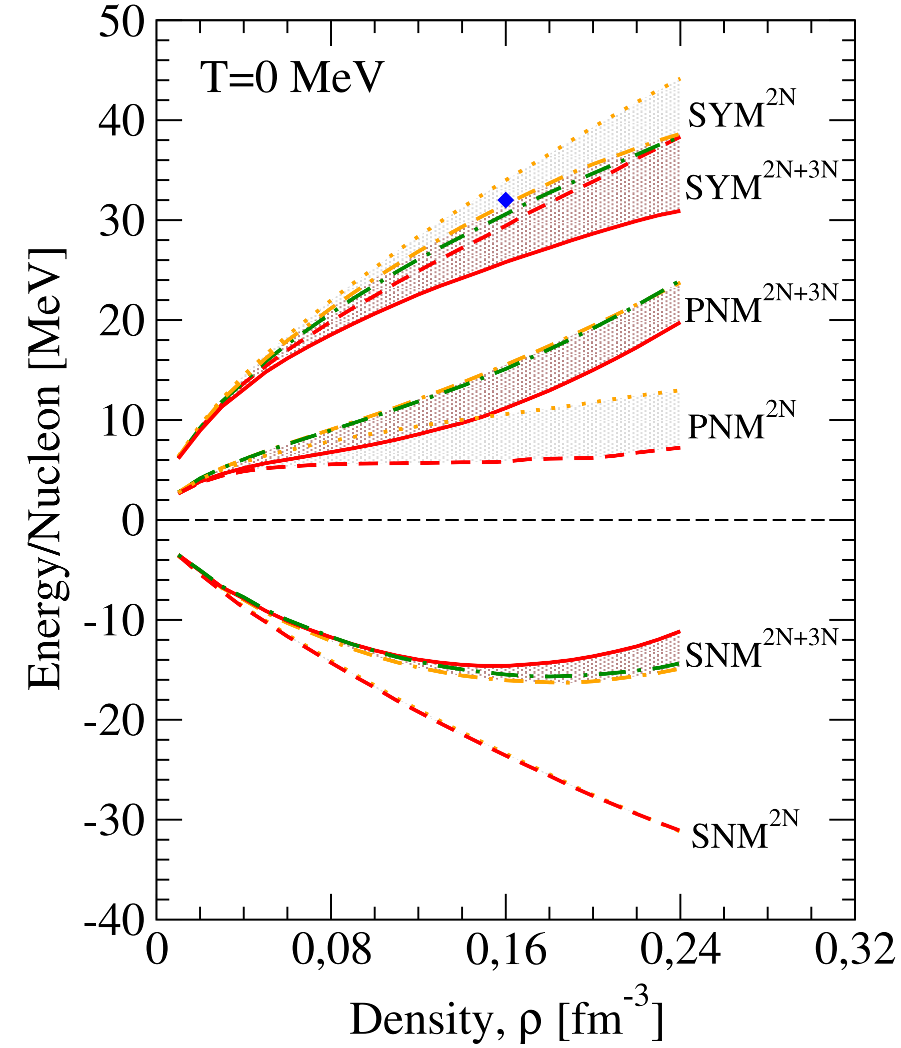
Pure Neutron Matter and Symmetry Energy



SNM Kin.
SNM 3N Pot.
SNM Tot.
SNM NN+3N Pot.
PNM Kin.
PNM Tot.
PNM 3N Pot.
PNM NN+3N Pot.

$$\frac{S}{A}(\rho) = \frac{E_{\rm PNM}}{A}(\rho) - \frac{E_{\rm SNM}}{A}(\rho)$$

	SRG1	SRG2	SAT
Sv (MeV)	31.57	30.59	25.81
L (MeV)	49.27	48.69	32.70

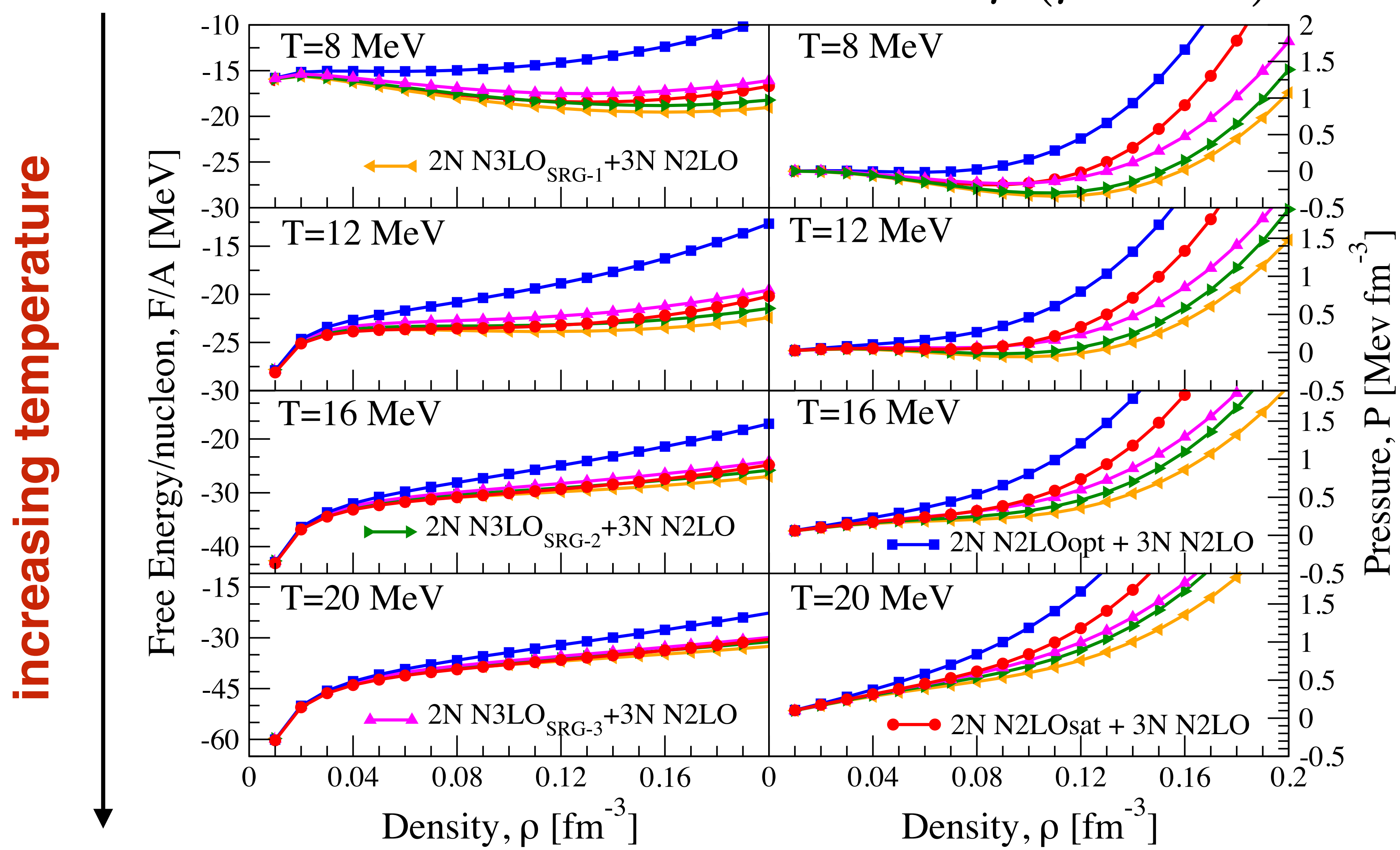


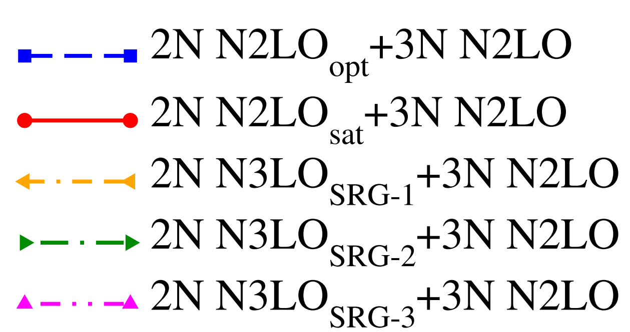
Carbone *(in preparation)*

- N2LOsat higher Kin. Energy
- N2LOsat higher 3N Pot. Energy
- NN+3N Pot. Energy differs
- PNM Tot. more attractive with N2LOsat

Free energy and pressure at varying temperature

$$F = E - TS \qquad P = \rho(\mu - F)$$



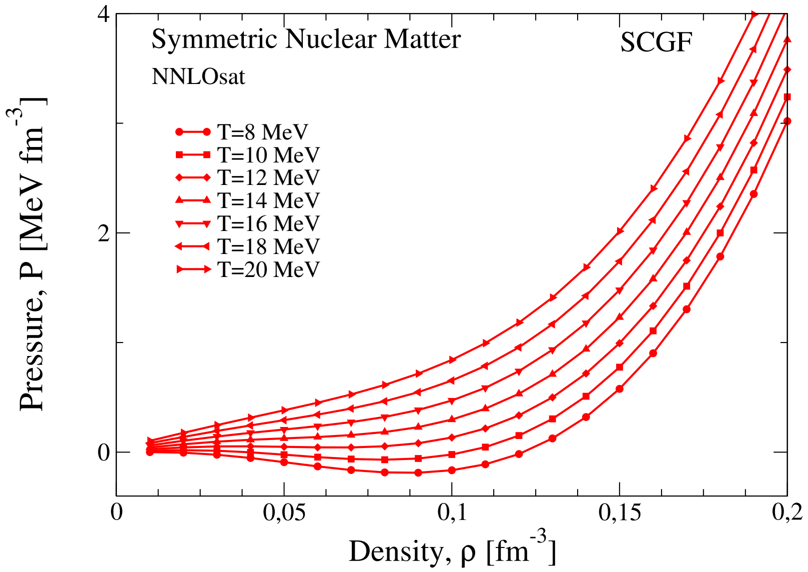


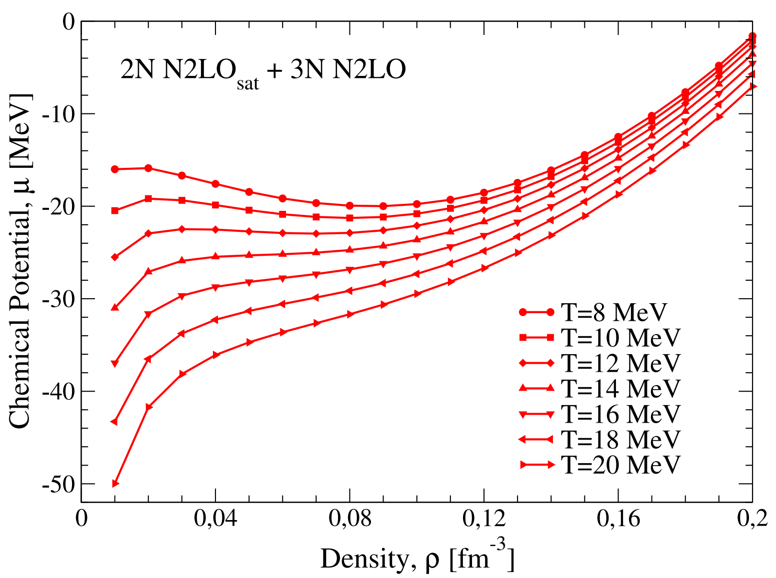
Carbone, Rios, Polls *(in preparation)*

- similar behaviour to zero T energy
- N2LOopt most repulsive
- less difference between other potentials
- liquid-gas phase transition

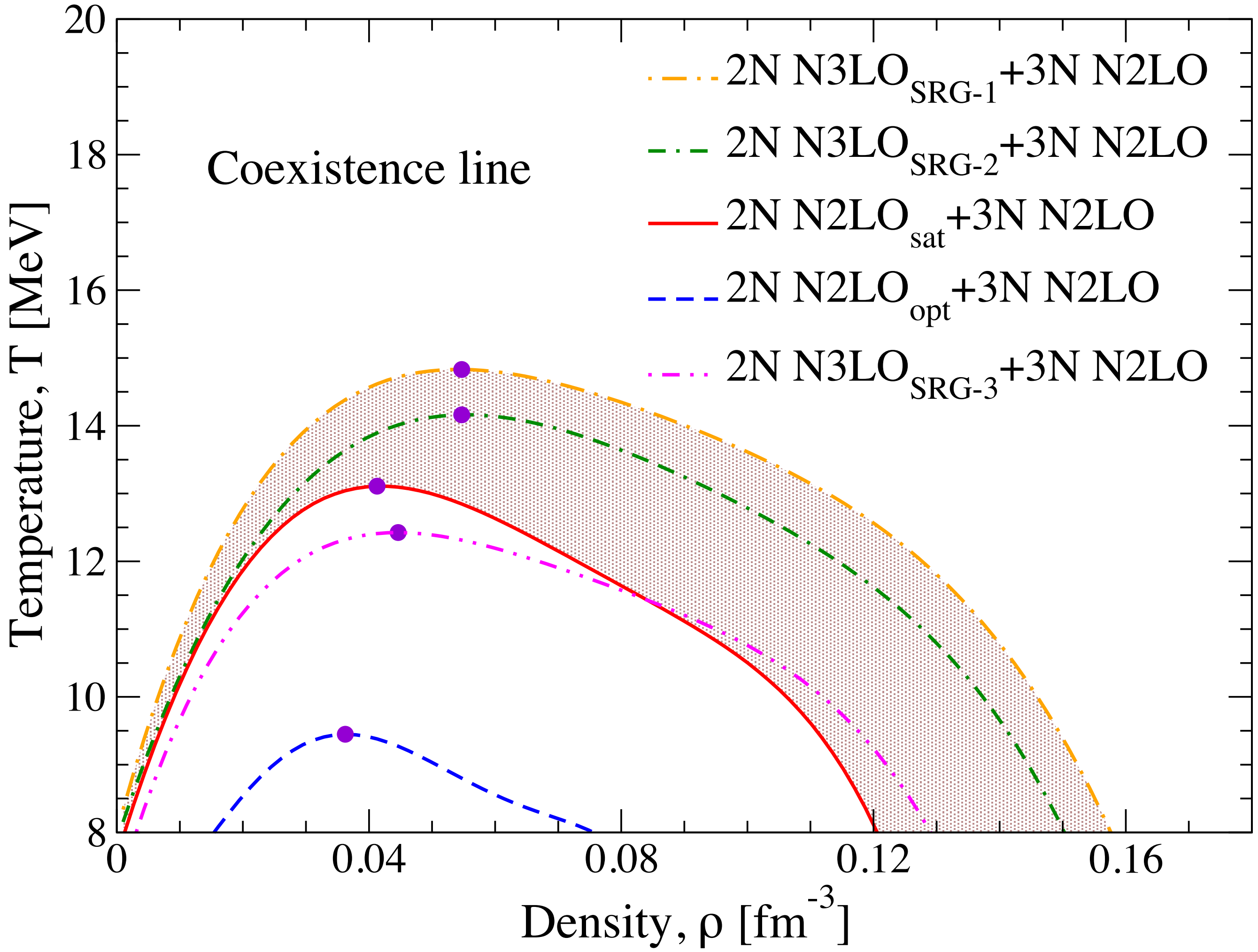
The liquid-gas phase transition and critical point

N2LOsat





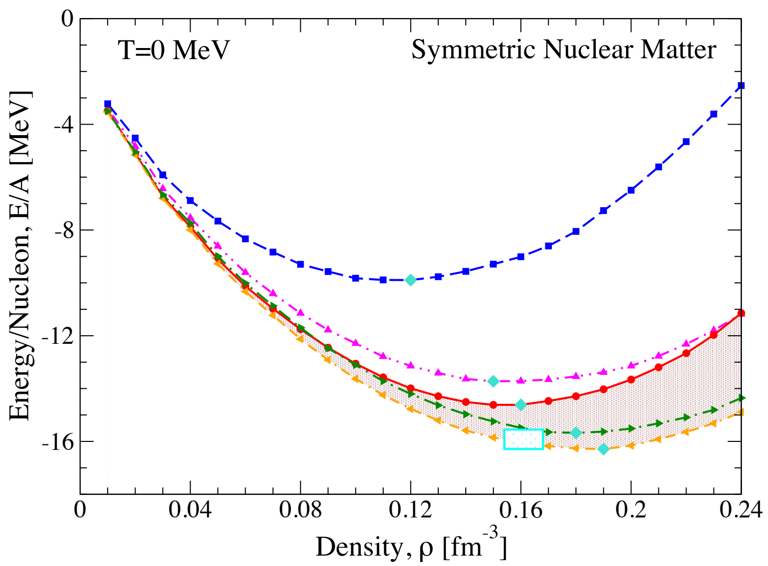
$$\mu(\rho_g) = \mu(\rho_l) \qquad P(\rho_g) = P(\rho_l)$$

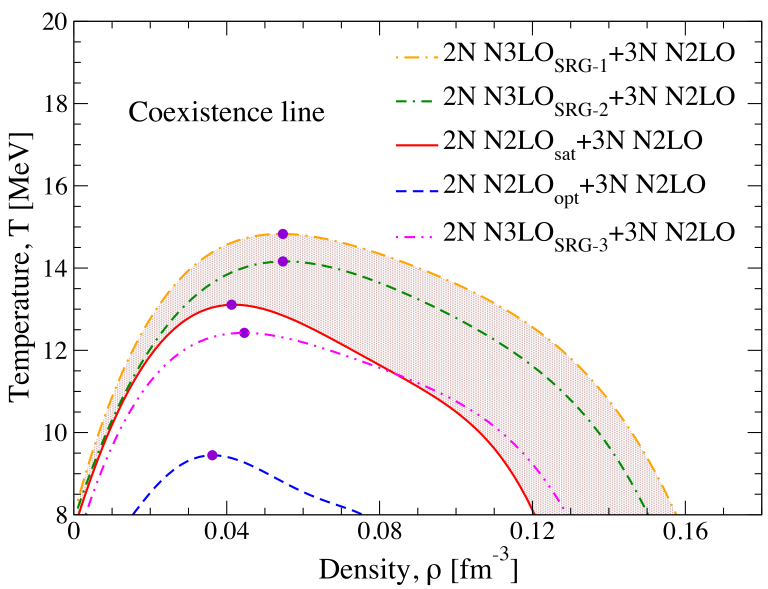


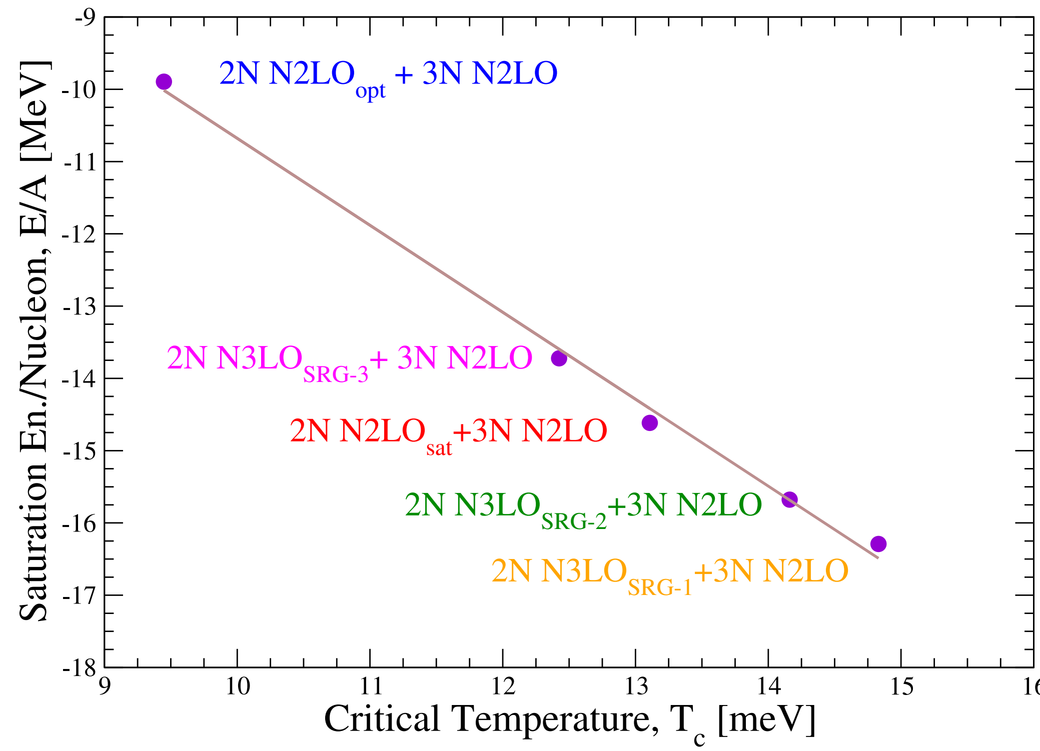
Carbone, Rios, Polls *(in preparation)*

- Coexistence line: equilibrium between a gas and a liquid phase
- Lower critical temperature respect to estimated experimental value ~ T=18 MeV

Saturation Energy vs Critical Temperature





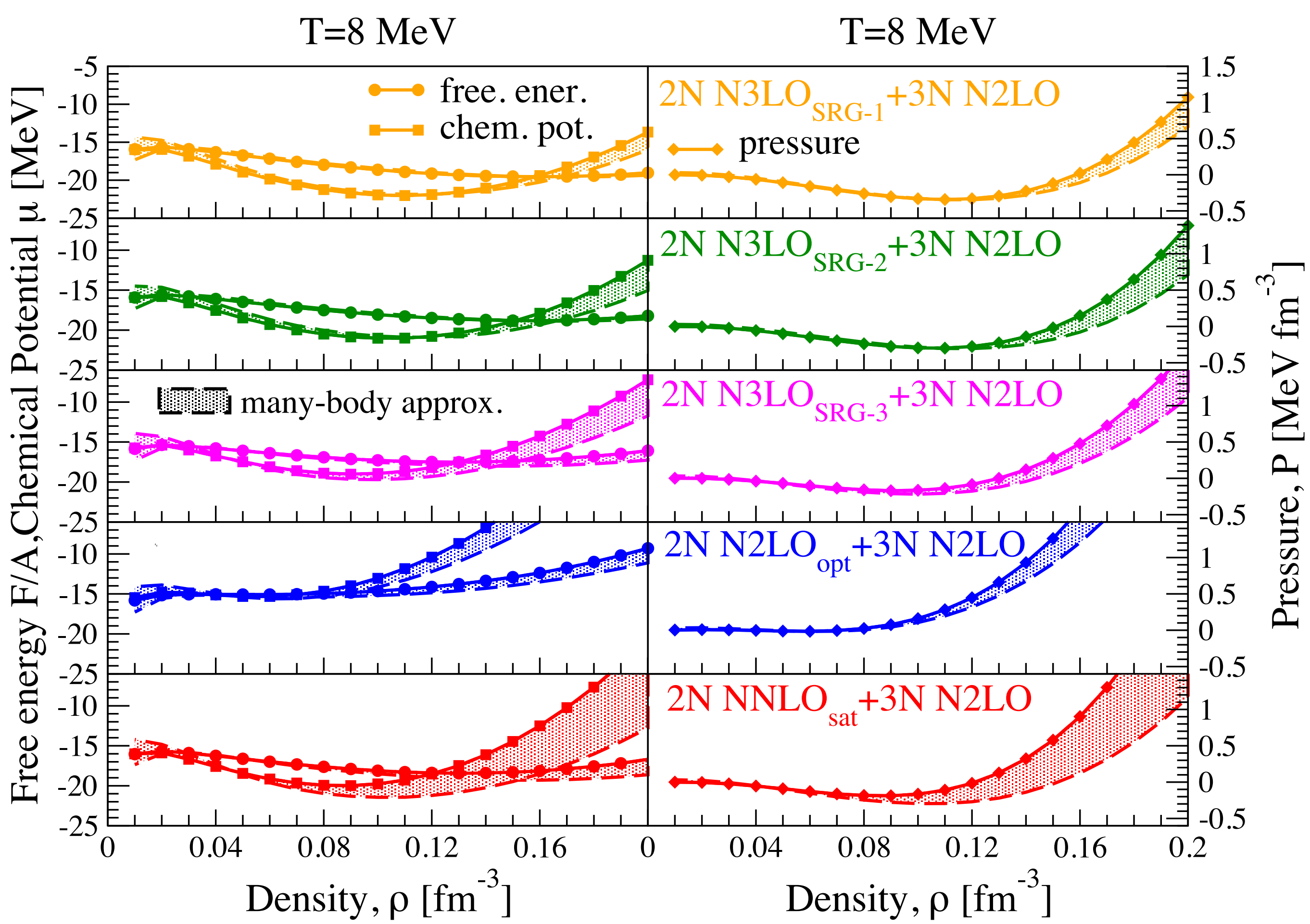


SCGF	ρ_c [fm^{-3}]	T$_c$ [MeV]	ρ_0 [fm^{-3}]	$\frac{E_0}{N}$ [MeV]	$\frac{m^*}{m}$
N3LO$_{\mathrm{SRG-1}}$	0.05	14.8	0.19	-16.3	0.85
N3LO$_{\mathrm{SRG-2}}$	0.05	14.2	0.18	-15.7	0.81
N3LO$_{\mathrm{SRG-3}}$	0.04	12.4	0.15	-13.7	0.90
NNLO$_{\mathrm{opt}}$	0.04	9.4	0.12	-9.9	0.90
NNLO$_{\mathrm{sat}}$	0.04	13.1	0.16	-14.6	0.90

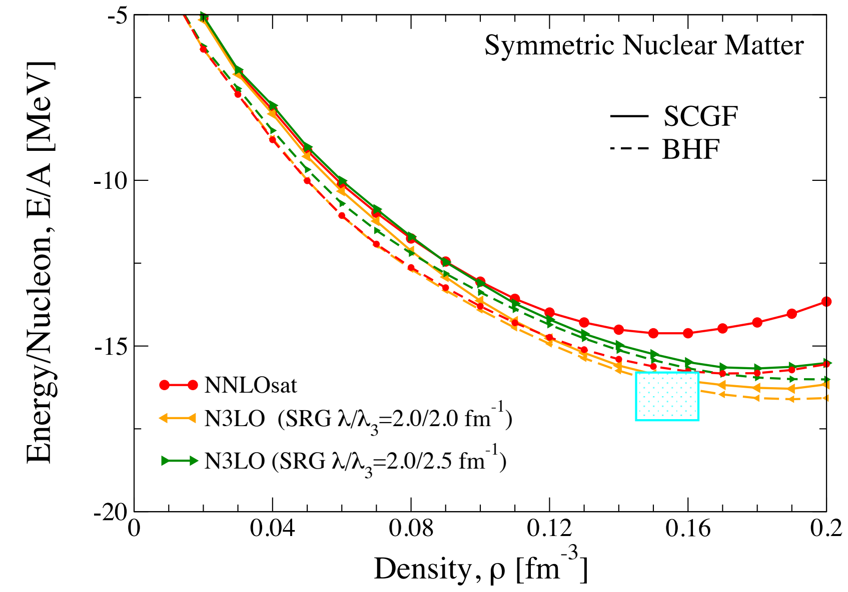
Carbone, Rios, Polls *(in preparation)*

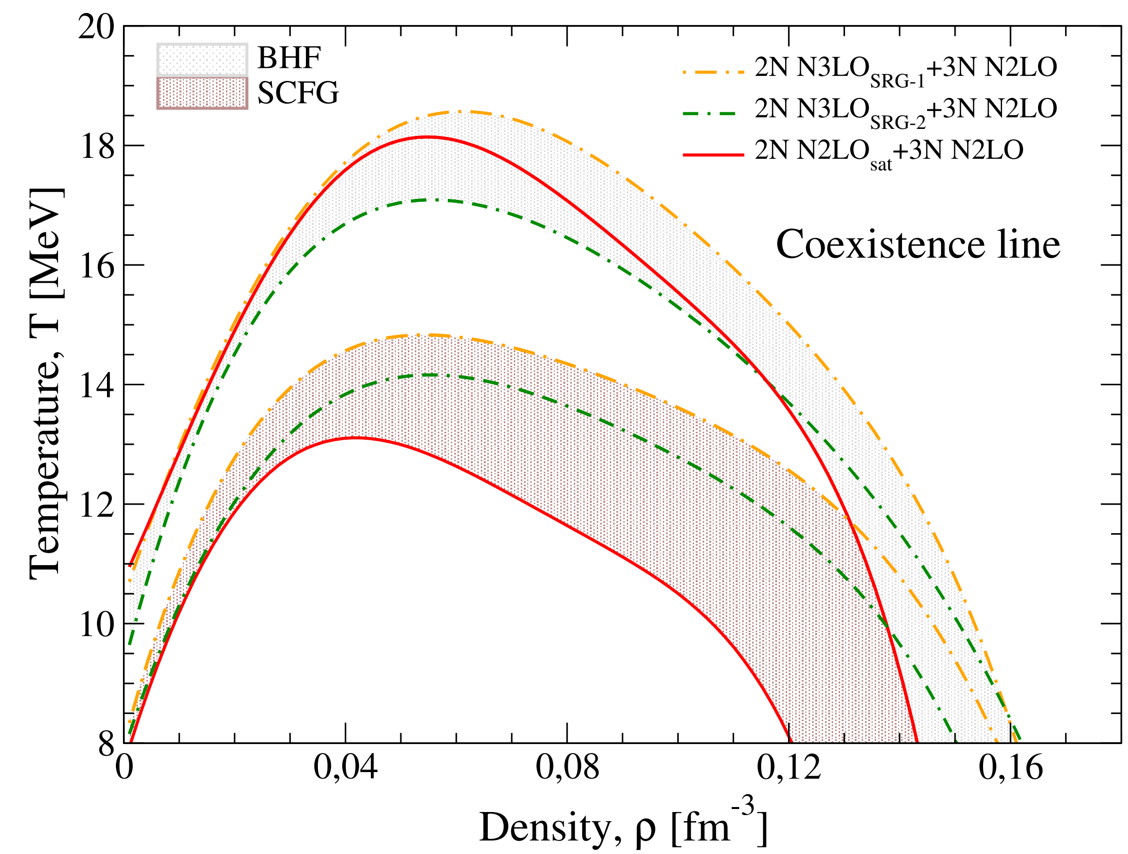
- Remarkable linear correlation between saturation energy and critical temperature

Many-Body approximation uncertainties



Carbone, Rios, Polls *(in preparation)*



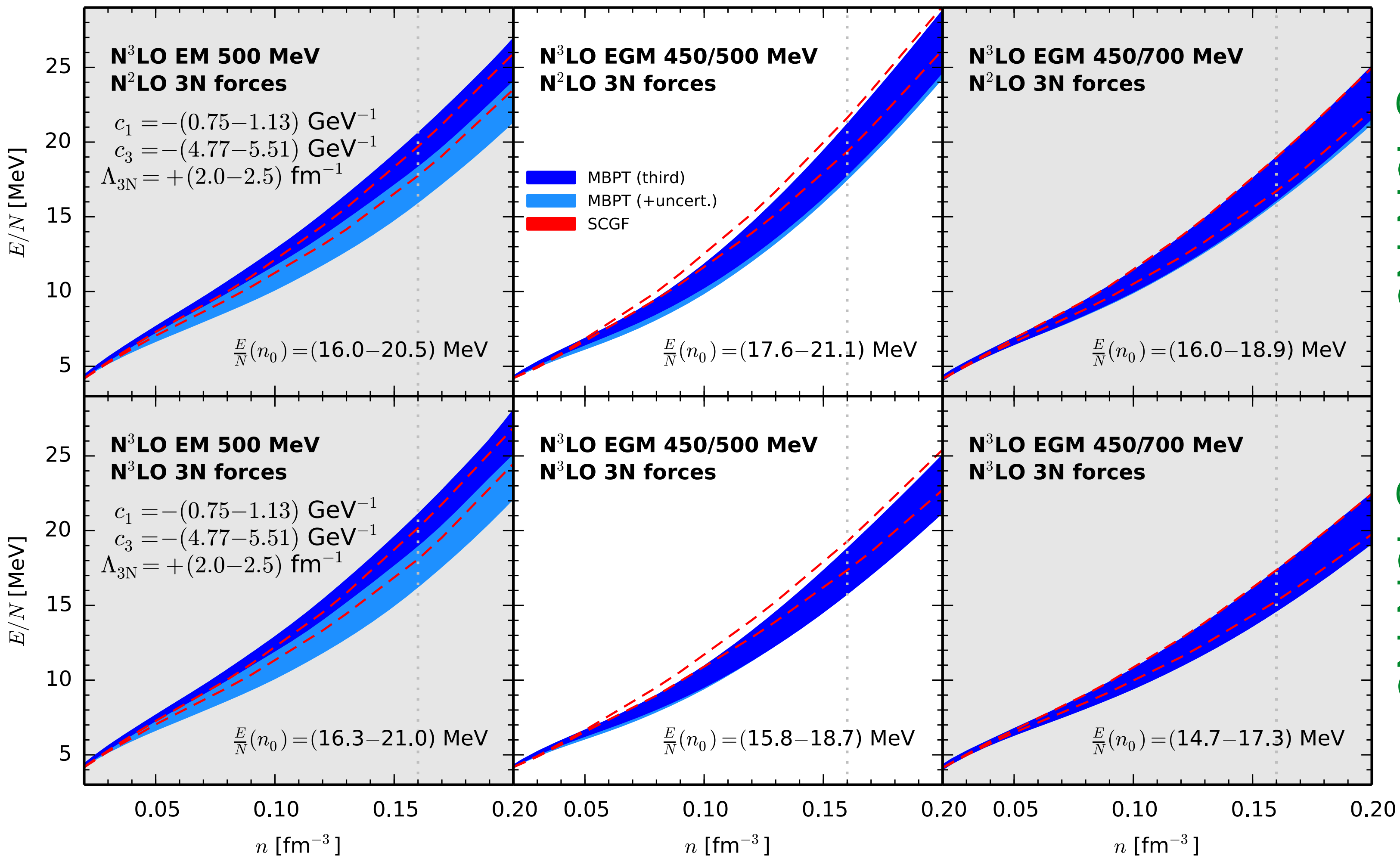


- SCGF vs BHF
- Chemical pot. provides higher uncertainty
- N2LOsat biggest spread
- Small difference at T=0 causes bigger spread for Tc

Pure neutron matter at 2N + 3N at N3LO

Improved 3NF matrix elements Hebeler et al. 2015
Partial-wave based average Drischler 2014-2015

EM N3LO500 | EGM 450/500 | EGM 450/700



3N N2LO

3N N3LO

Drischler, Carbone, Hebeler, Schwenk *(arXiv:1608.05615, PRC accepted)*

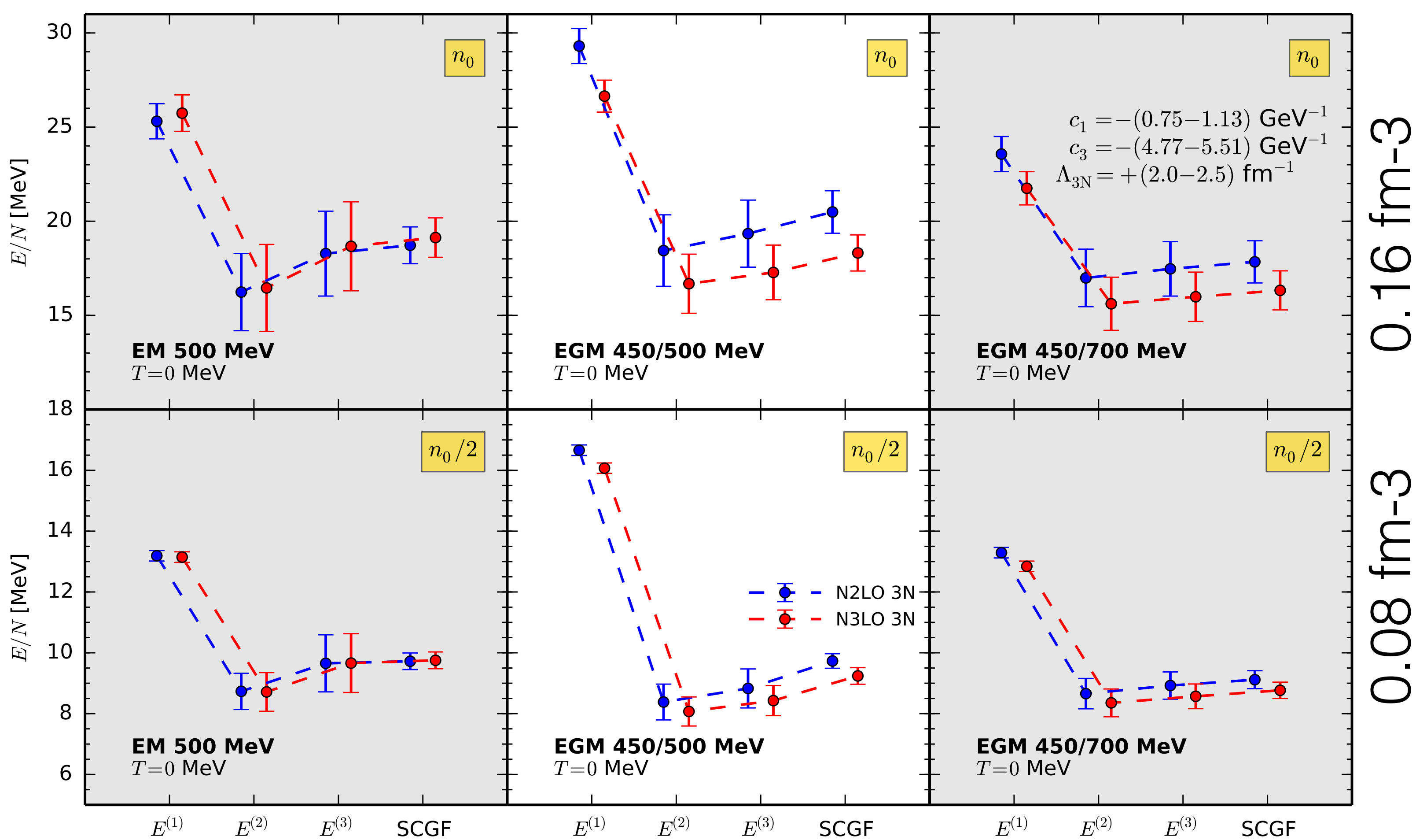
- **3rd order MBPT** vs **SCGF**
- bands: LECs and cutoff
- no major effect of 3NN3LO
- EM500 less perturbative
- 3rd order MBPT very well converged for EGMs

Check C. Drischler's poster next week!

Pure neutron matter at N3LO: many-body convergence

Improved 3NF matrix elements Hebeler et al. 2015
Partial-wave based average Drischler 2014-2015

EM N3LO500 EGM 450/500 EGM 450/700



- **3NN2LO** vs **3NN3LO**
- bands: LECs and cutoff
- shift due 3NN3LO
- EM500 less perturbative
- 3rd order MBPT very well converged for EGMs

Drischler, Carbone, Hebeler, Schwenk *(arXiv:1608.05615, PRC accepted)*

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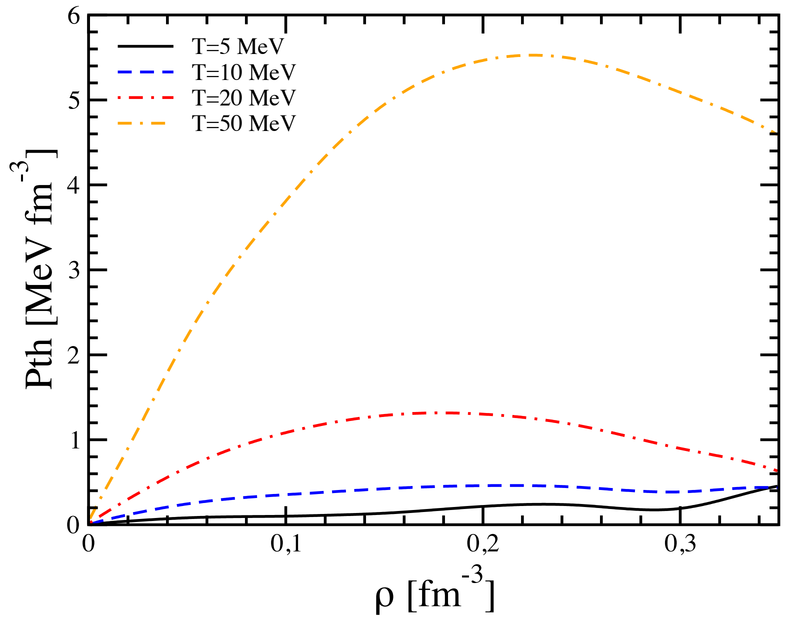
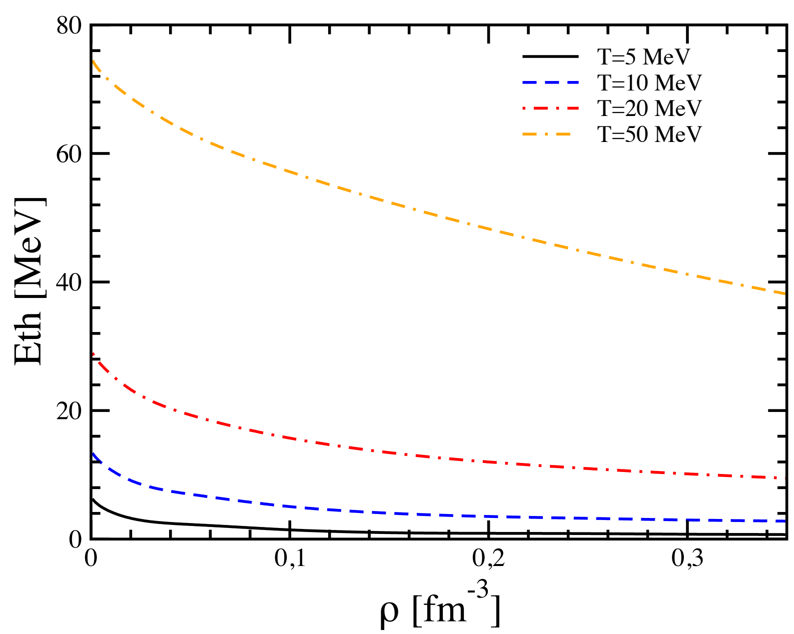
Thermal effects for supernovae simulations

$$P_{\text{th}} = P(T) - E_0$$

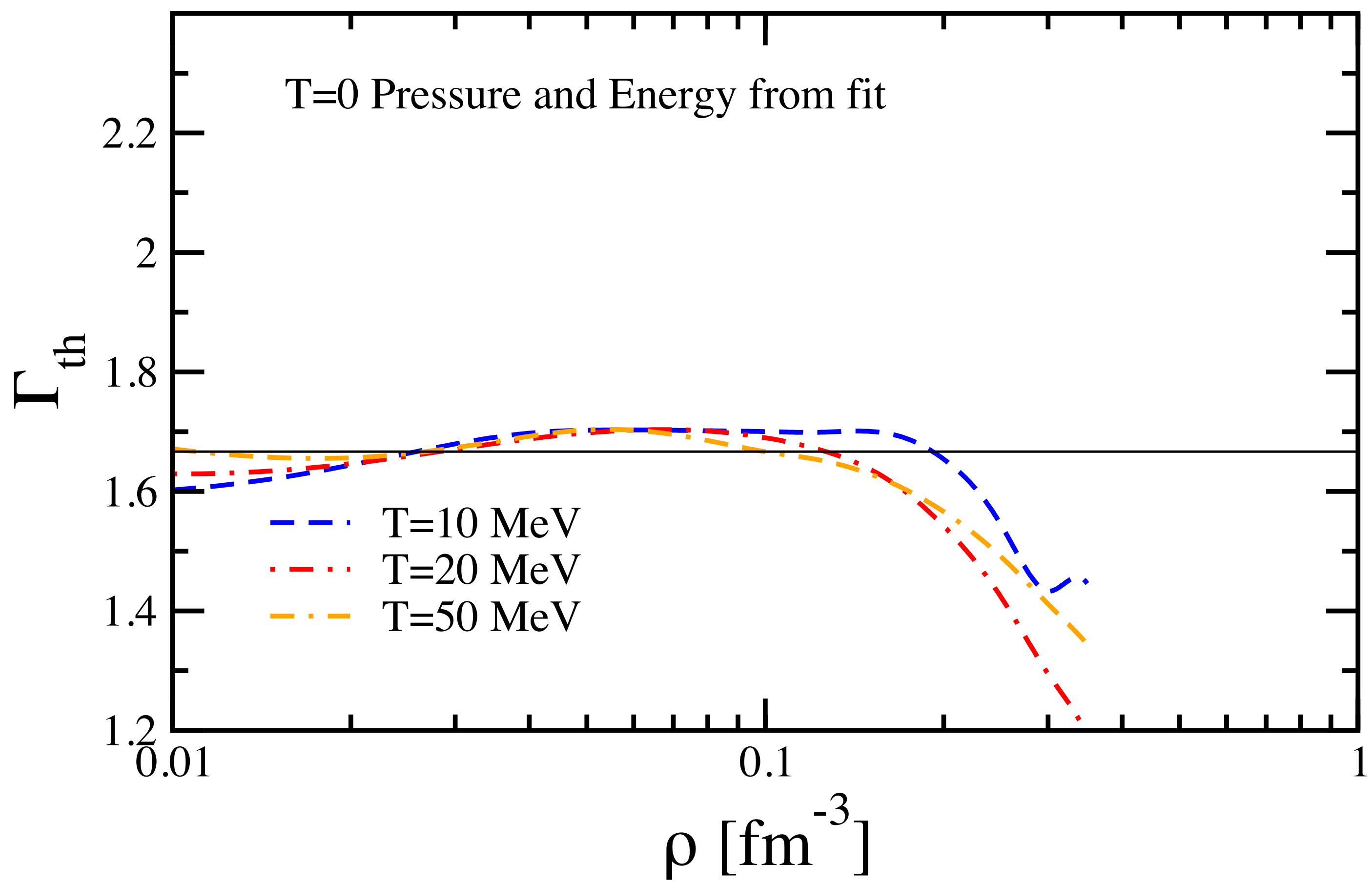


$$E_{\text{th}} = E(T) - E_0$$

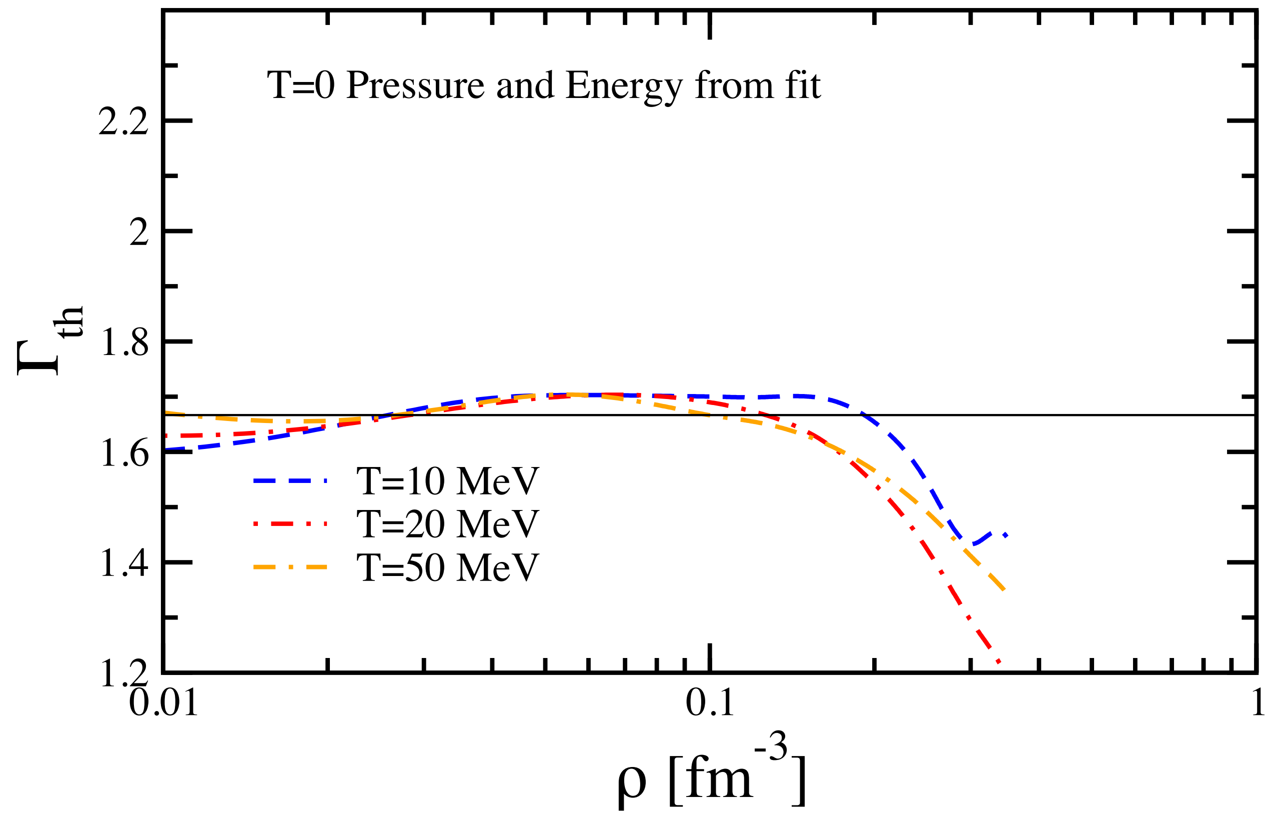


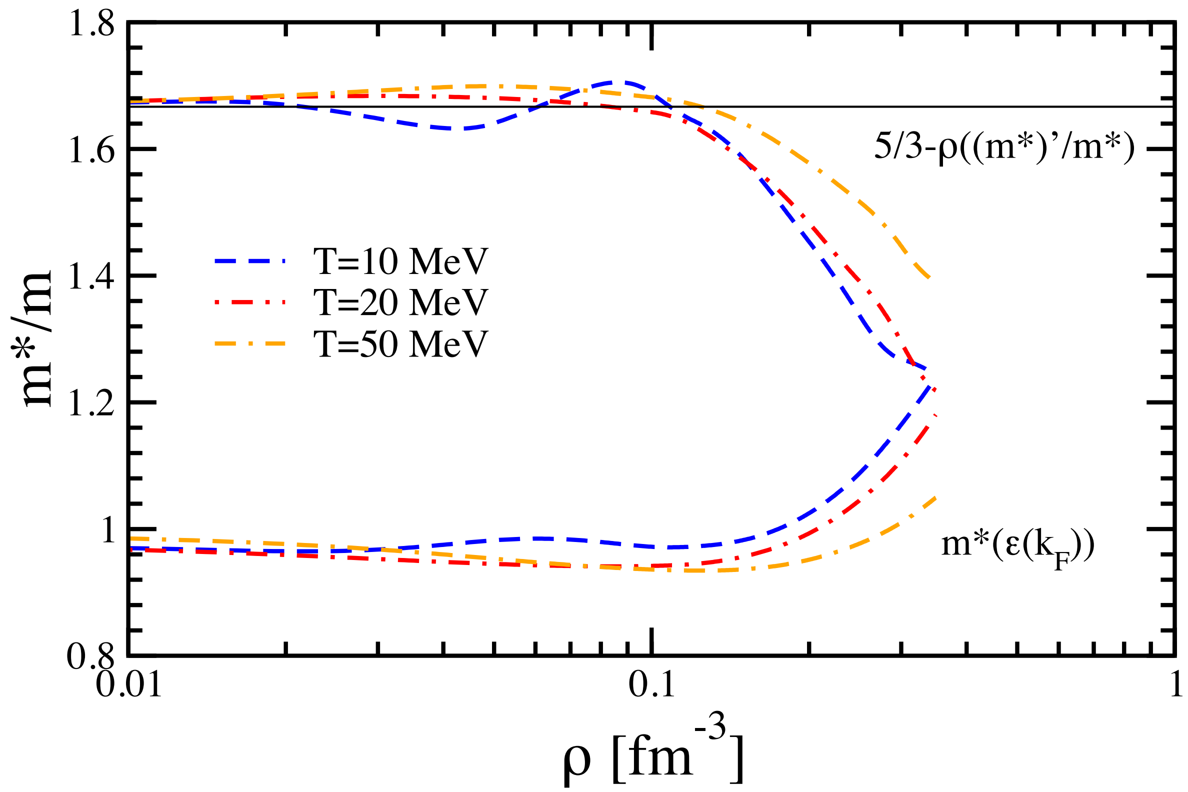
- Thermal index includes finite-T effects
- Pth decreases after certain density
- Eth decreases monotonically
- Gamma increases then decreases after sat. density

$$\Gamma_{\text{th}} = 1 + \frac{P_{\text{th}}}{E_{\text{th}}}$$

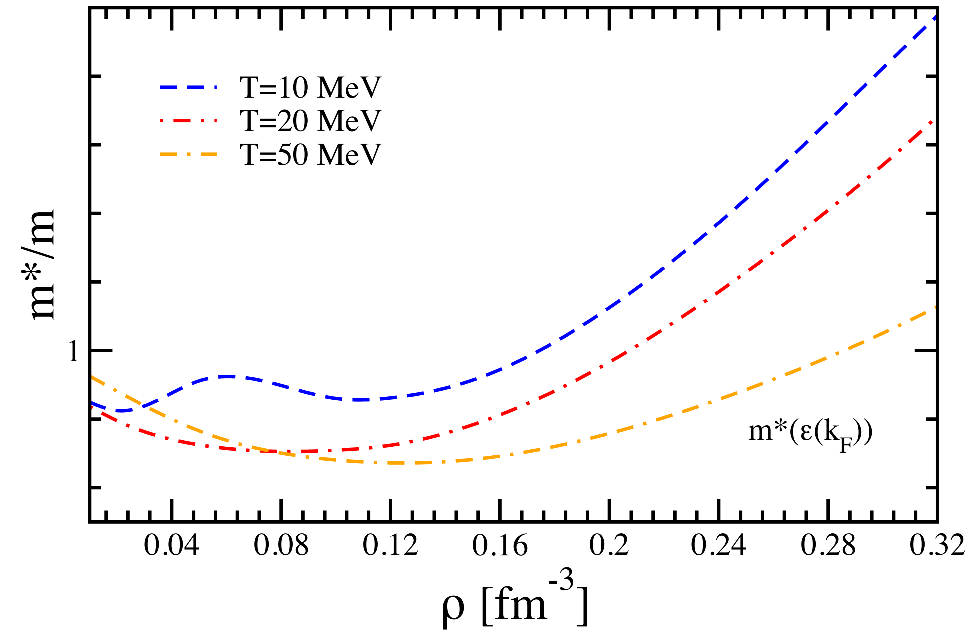


The effective mass

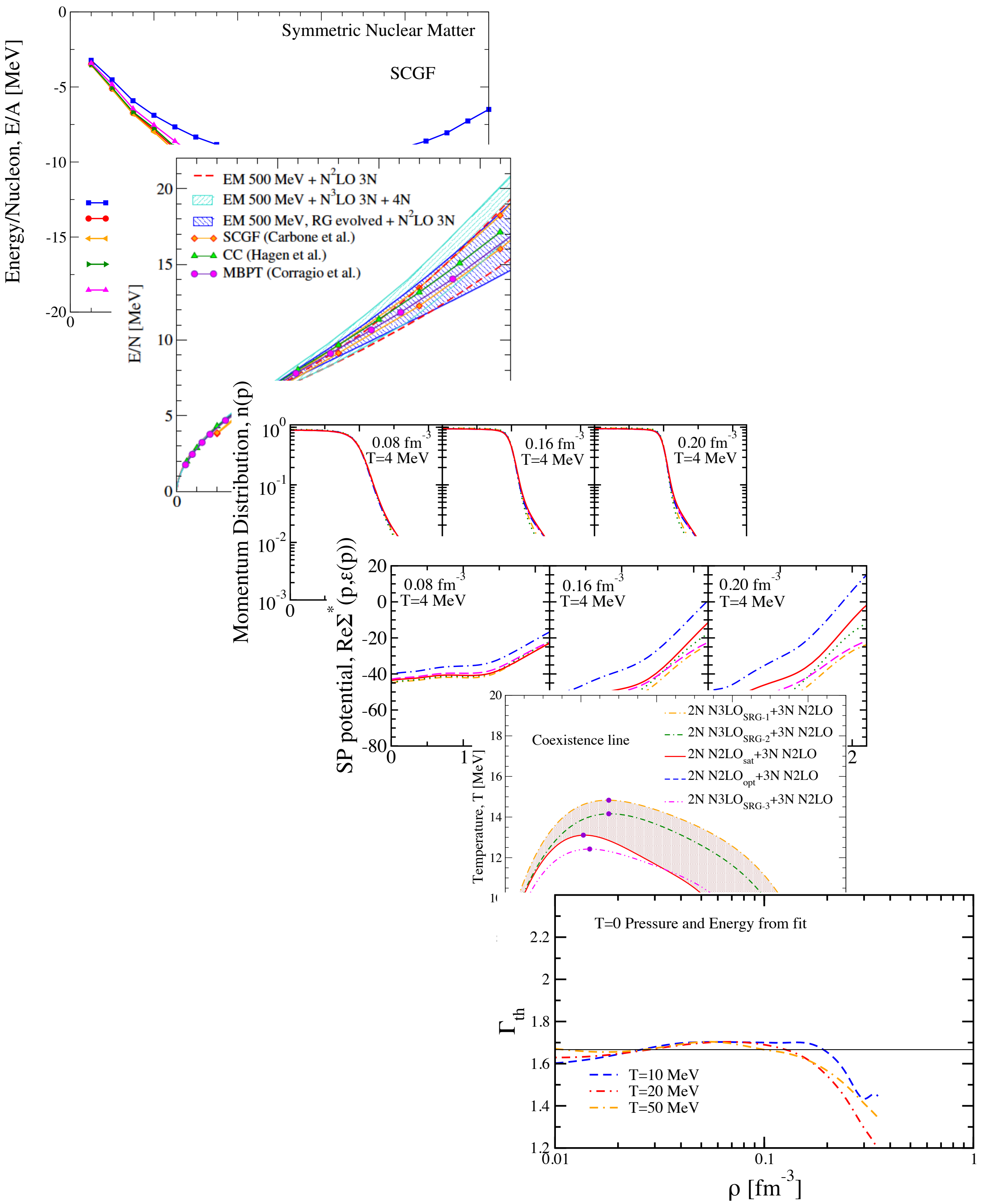




- Gamma index depends on effective mass derivative
- increase of effective mass, decrease of thermal index



Summary



- A consistent extension of the SCGF method to include 3NFs is accomplished
- Nuclear and neutron matter with theoretical uncertainties can be calculated reliably using *ab initio* methods based on chiral Hamiltonians
- A thorough microscopic description within the SCGF method is available
- Small overall effect of 3NFs on the momentum distribution, strong repulsive effect on bulk properties
- Critical temperature can be estimated
- Gamma thermal index obtainable microscopically

Outlook

- Chiral EFT Hamiltonian: power counting, theoretical uncertainties, limits of chiral EFT, etc.
- Many-body approximation methods: include irreducible 3B terms, improve the effective interactions, include particle-hole diagrams, asymmetric matter, etc.
- Reliable finite temperature results from *ab initio* theory: high astrophysical impact (EOSs at finite T, dynamics of neutron star merger simulations, core-collapse supernovae, etc.)

Collaborators:

Universitat de Barcelona — A. Polls

University of Surrey — C. Barbieri, A. Rios

Technische Universität Darmstadt — C. Drischler, P. Klos, K. Hebeler, A. Schwenk

Thank you for your attention!

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