Magnetic field effects on neutron stars and white dwarfs

arXiv:1609.05994
Mon.Not.Roy.Astron.Soc. 456 (2016) no.3, 2937-2945
Phys.Rev. D94 (2016) no.4, 044018
Mon.Not.Roy.Astron.Soc. 463 (2016) 571-579
Phys.Rev. D92 (2015) no.8, 083006

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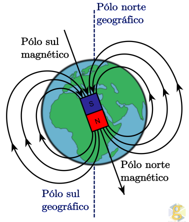
Nuclear Physics, Compact Stars, and Compact Star Mergers 2016
Kyoto, Japan

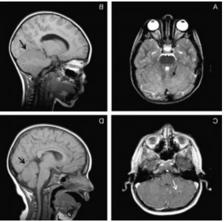
Plan of the talk

- Motivation
- Magnetized Neutron Stars: fully-general relativistic approach
  Langage Objet pour la RElativité NaumériquE (LORENE)
- Results
- Summary

Motivation: magnetic fields



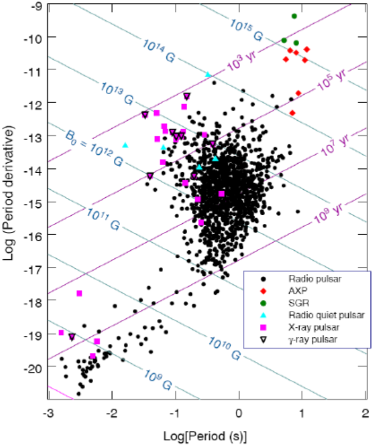
Earth: B$\sim$ 0.5 G



MR: B$\sim 10^3$ G

Atlas: B$\sim 10^{20}$ G

JP Ridley



Typical NS: $B_s \sim 10^{12}$ **G**

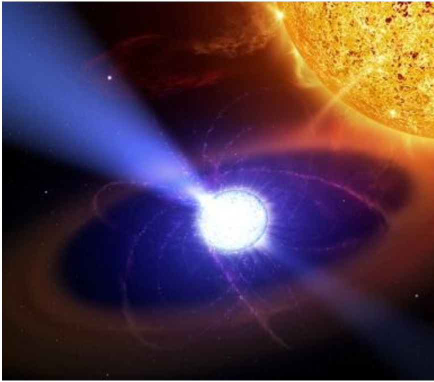
Magnetars: $B_s > 10^{14}$ **G**

♣ **Virial Theorem:** $B_c \sim 10^{18}$ **G**

Motivation: magnetic fields

I. Some white dwarfs are also associated with strong magnetic fields
II. From observations, the surface magnetic field: $B_s \sim 10^{6-9}\,\mathrm{G}$

♣ **Virial theorem:** $\mathbf{B_c} \sim \mathbf{10^{13}\,G}$



Origin? Duncan, Thompson, Kouveliotou

The Ultimate Convection Oven

THIS PICTURE LEAVES a basic question unanswered: Where did the magnetic field come from in the first place? The traditional assumption was: it is as it is, because it was as it was. That

I. fossil field ($B \sim 1/R^2$)

II. dynamo process

How to model highly magnetized stars

Einstein Equation

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = 8\pi G T_{\mu\nu}$$

Geometry

1. Spherical: TOV
2. Perturbation
3. Fully-GR

Energy Content

1. Matter: particles
2. Fields: magnetic field

Fully-General Relativistic Approach

• Stationary neutron stars with no magnetic-field-dependent EoS were studied by Bonazzola (1993), Bocquet (1995).

• magnetic fields effects in the EoS was presented in Chatterjee (2014), for a **quark EoS** and, later on, we took into considaration a much more complex system with **nucleons, hyperons, mixed phase with quarks, AMM** of all hadrons (even the uncharged ones) in Franzon (2015).

$$\Downarrow$$

B field in the EoS: effects mentioned above are negligible for calculating the final structure of highly magnetized neutron stars.

Mathematical setup

- The energy-momentum tensor:

$$T^{\mu\nu} = (e+p)u^\mu u^\nu + pg^{\mu\nu} + \frac{1}{\mu_0}\left(-b^\mu b^\nu + (b\cdot b)u^\mu u^\nu + \frac{1}{2}g^{\mu\nu}(b\cdot b)\right)$$

  where m and B are the lengths of the magnetization and magnetic field 4-vectors.

- In the rest frame of the fluid:

$$T^{\mu\nu} = \mathit{fluid} + \mathit{field}$$

$$T^{\mu\nu} = \begin{pmatrix} e+\frac{B^2}{2\mu_0} & 0 & 0 & 0 \\ 0 & p+\frac{B^2}{2\mu_0} & 0 & 0 \\ 0 & 0 & p+\frac{B^2}{2\mu_0} & 0 \\ 0 & 0 & 0 & p-\frac{B^2}{2\mu_0} \end{pmatrix}$$

Mathematical setup

- Stationary and axisymmetric space-time, the metric is written as:

$$ds^2 = -N^2 dt^2 + \Psi^2 r^2 \sin^2\theta (d\phi - N^\phi dt)^2 + \lambda^2 (dr^2 + r^2 d\theta^2)$$

  where N^ϕ, N, Ψ and λ are functions of (r, θ).
- A poloidal magnetic field satisfies the circularity condition:

$$A_\mu = (A_t, 0, 0, A_\phi)$$

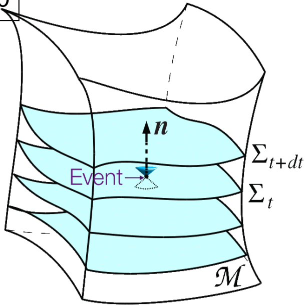
- The magnetic field components as measured by the observer $(\mathcal{O}_0)$ with n^μ velocity can be written as:

$$B_\alpha = -\frac{1}{2}\epsilon_{\alpha\beta\gamma\sigma} F^{\gamma\sigma} n^\beta = \left(0, \frac{1}{\Psi r^2 \sin\theta}\frac{\partial A_\phi}{\partial \theta}, -\frac{1}{\Psi \sin\theta}\frac{\partial A_\phi}{\partial r}, 0\right)$$

$$A_t, A_\phi \rightarrow Maxwell\ Equations.\ Static\,case:\ no\ electric\ field$$

3+1 foliation of space time

E. Gourgoulhon 2010



→ One decomposes any 4D tensor into a purely spatial part:

1. onto the **hypersurface** Σ_t with **3D** spatial metric $\gamma_{\mu\nu} := g_{\mu\nu} + n_\mu n_\nu$

and

2. a purely **timelike** part, orthogonal to Σ_t, $\gamma_{\mu\nu} n^\mu = 0$, and aligned with n^μ. A observer with n^μ is called Eulerian observer.

3+1 decomposition of $T_{\mu\nu}$

- Total energy density, $E = n^\mu n^\nu T_{\mu\nu}$: Bocquet (1995)

  $E = \Gamma^2(e+p) - p + \frac{1}{2\mu_0}(B^i B_i)$

- and the momentum density flux, $J_\alpha = -\gamma^\mu_\alpha n^\nu T_{\mu\nu}$, can be written as:

  $J_\phi = \Gamma^2(e+p)U$

- 3-tensor stress, $S_{\alpha\beta} = \gamma^\mu_\alpha \gamma^\nu_\beta T_{\mu\nu}$, components are given by:

  $S^r{}_r = p + \frac{1}{2\mu_0}(B^\theta B_\theta - B^r B_r)$

  $S^\theta{}_\theta = p + \frac{1}{2\mu_0}(B^r B_r - B^\theta B_\theta)$

  $S^\phi{}_\phi = p + \Gamma^2(e+p)U^2$

  with $\Gamma = (1-U^2)^{-\frac{1}{2}}$ the Lorenz factor and U the fluid velocity defined as:

$$U = \frac{\Psi r \sin\theta}{N}(\Omega - N^\phi)$$

Field equations: our 4 unknowns N, N^ϕ, Ψ, λ

- **Einstein equations:** $R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = 8\pi G T_{\mu\nu}$

  Bocquet (1995)

$$\Delta_3 \nu = \sigma_1$$
$$\tilde{\Delta}(N^\phi r \sin\theta) = \sigma_2$$
$$\Delta_2[(N\Psi - 1) r \sin\theta] = \sigma_3$$
$$\Delta_2(\nu + \alpha) = \sigma_4$$

  Each σ_i contains terms involving **matter** and non-linear **metric terms**.

- **Definitions:**

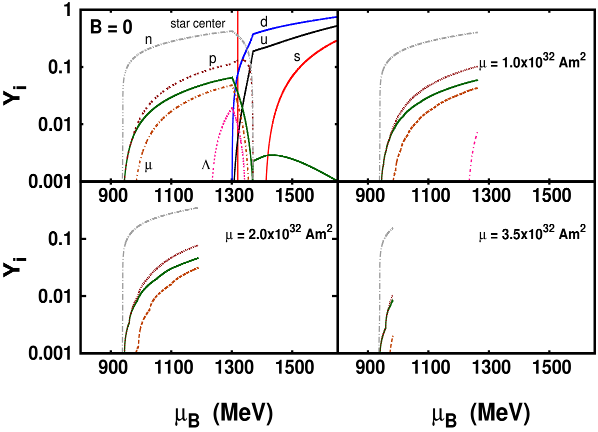
$$\nu = \ln N,\ \alpha = \ln \lambda,$$
$$\Delta_2 = \left(\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial^2\theta} \right)$$
$$\Delta_3 = \left(\frac{\partial^2}{\partial r^2} + \frac{2}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial^2\theta} + \frac{1}{r^2 \tan\theta}\frac{\partial}{\partial\theta} \right)$$
$$\tilde{\Delta}_3 = \Delta_3 - \frac{1}{r^2 \sin^2\theta}$$

Structure of the star

- **Mass**
  $M = \int \lambda^2 \Psi r^2 \times \left[N(E+S) + 2N^{\phi}\Psi(E+p)Ur\sin\theta \right] \sin\theta dr d\theta d\phi$

- **Circumferential Radius**
  $R_{circ} = \Psi(r_{eq}, \frac{\pi}{2}) r_{eq}$

Population change for a hybrid and cold NS star with $\mathrm{M_B} = 2.20\,\mathrm{M_\odot}$



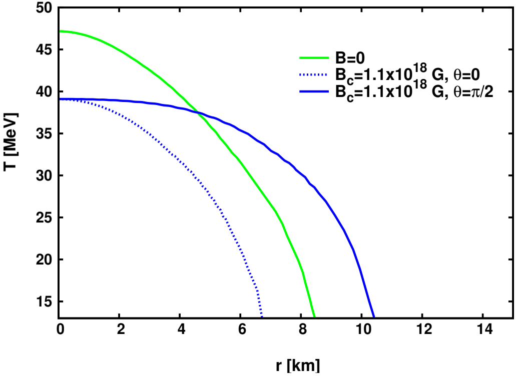
Hybrid stars containing nucleons, hyperons and quarks. See, e.g. Hempel M. at all (2013); Dexheimer V., Schramm S. (2008, 2010)

[B. Franzon at all, MNRAS (2015)]

→ As one increases the magnetic field, **the particle population changes** inside the star.

→ stars that possess strong magnetic fields might go through a **phase transition** later along their evolution.

Temperature distribution: hadronic PNS star with $\mathrm{M_B} = 2.35\,\mathrm{M_\odot}$ and $s_B = 2$, $Y_L = 0.4$



[B. Franzon, V. Dexheimer, S.Schramm PRD94 (2016) no.4, 044018]

→ magnetic field influences **temperature** distribution in star

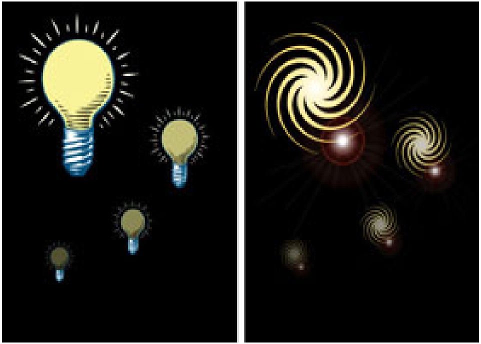
→ The same behaviour for **neutrino distribution** $n_{\nu_{e^-}} \times r$, but detailed **temporal evolution** necessary.

Properties of White Dwarfs

→ **Size** similar to Earth
→ **Densities** $10^{5-9}\,\mathrm{g/cm^3}$
→ Typical **composition**: C and/or O.
→ Gravity is balanced by **electron degenary pressure**
→ Masses are up to **1.4** $M_\odot$.

Progenitors of **Type Ia supernovae**: Chandrasekhar White Dwarfs

Standard Candles



EXPANSION OF THE UNIVERSE 2011

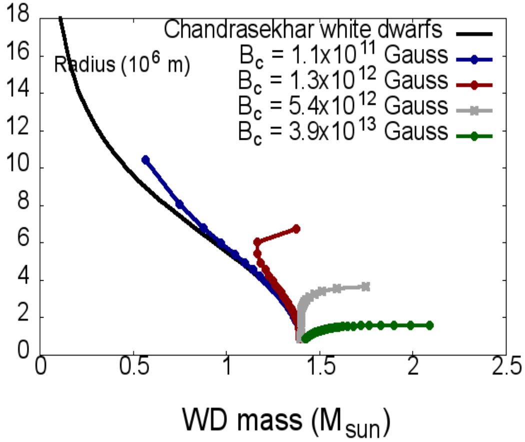
Saul Perlmutter

Brian P. Schmidt

Adam G. Riess

But, motivated by **observation** of supernova that appears to be **more luminous** than expected (e.g. SN 2003fg, SN 2006gz, SN 2007if, SN 2009dc), it has been argued that the **progenitor** of such super-novae should be a white dwarf with mass above the well-known Chandrasekhar limit: **2.0-2.8** $M_{\odot}$.

Mass-radius diagram for magnetized white dwarfs

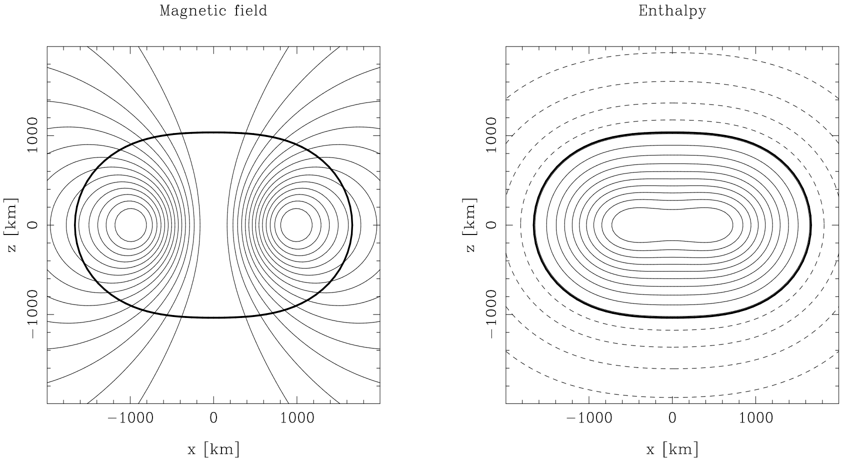


[B. Franzon and S.Schramm, Phys.Rev. D92 (2015) 083006]

→ **Magnetic field** effects can considerably increase the star masses and, therefore, might be the **source of superluminous SNIa**.

→ Recently, we included **beta decay** and **pyconuclear reactions** in the calculation: still mass well above 1.4 $M_\odot$, see [arXiv:1609.05994].

Deformation due to magnetic fields



→ **Microphysics** plays an important role. The critical density for pyconuclear fusion reactions limits the central white dwarf density and, as a consequence, its equatorial **radius cannot be smaller** than $R \sim 1600$ **km** for a mass of $\sim 2.0\,M_\odot$ [arXiv:1609.05994].

Summary

- Self-consistent stellar model including a poloidal magnetic field
- We have shown that high magnetic fields prevent the appearance of a quark and a mixed phase.
- Magnetic fiels can also change the temperature in the core of PNS, as well the neutrino distributions.
- Magnetized WD can be super-Chandrasekhar white dwarfs, whose masses are higher than 1.4 $M_{\odot}$
- Observables: distinct change in the cooling.

The End