Neutron star asteroseismology

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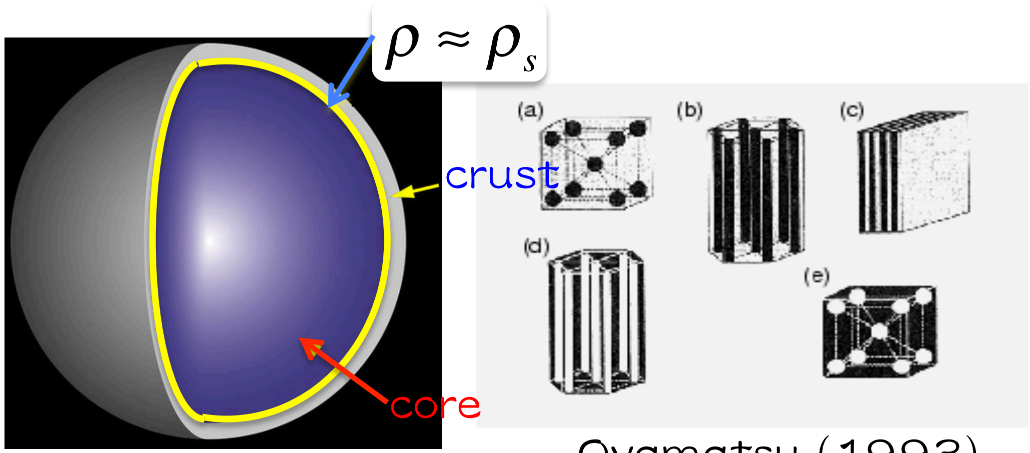
- Crustal oscillations and nuclear saturation parameters
- Gravitational wave asteroseismology in protoneutron stars

Crustal oscillations and nuclear saturation parameters

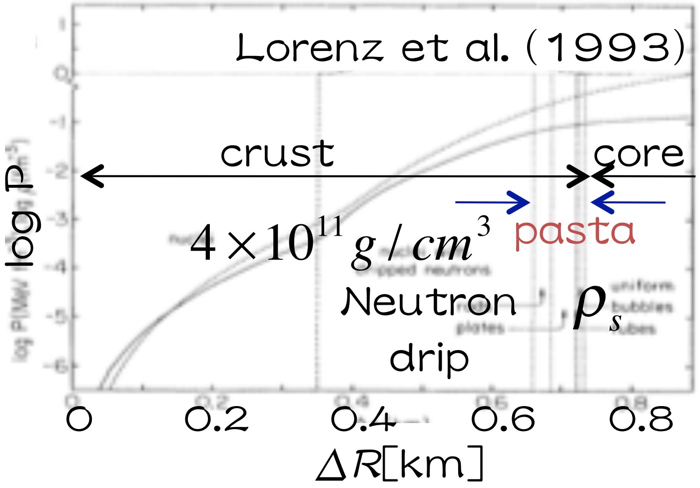
neutron stars

- Structure of NS
  - solid layer (crust)
  - nonuniform structure (pasta)
  - fluid core (uniform matter)
- Crust thickness ≲ 1km
  - strongly associated with nuclear saturation properties
- Constraint on EOS via observations of neutron stars
  - stellar mass and radius
  - stellar oscillations (& emitted GWs)

“(GW) asteroseismology”



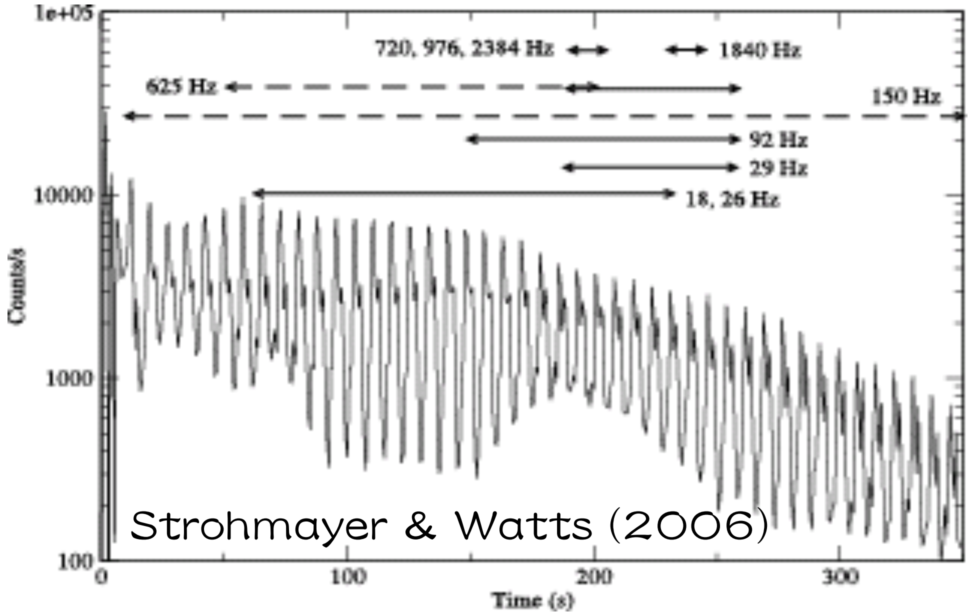
Oyamatsu (1993)



QPOs in SGRs

- Quasi-periodic oscillations (QPOs) in afterglow of giant flares from soft-gamma repeaters (SGRs)
  - SGR 0526-66 (5th/3/1979) : 43 Hz
  - SGR 1900+14 (27th/8/1998) : 28, 54, 84, 155 Hz
  - SGR 1806-20 (27th/12/2004) : 18, 26, 30, 92.5, 150, 626.5, 1837 Hz

  (Barat+ 1983, Israel+ 05, Strohmayer & Watts 05, Watts & Strohmayer 06)
  - additional QPO in SGR 1806-20 is found : 57Hz (Huppenkothen + 2014)



Strohmayer & Watts (2006)

- Crustal torsional oscillation ?
- Magnetic oscillations ?

- crustal torsional oscillations are confined only in crust

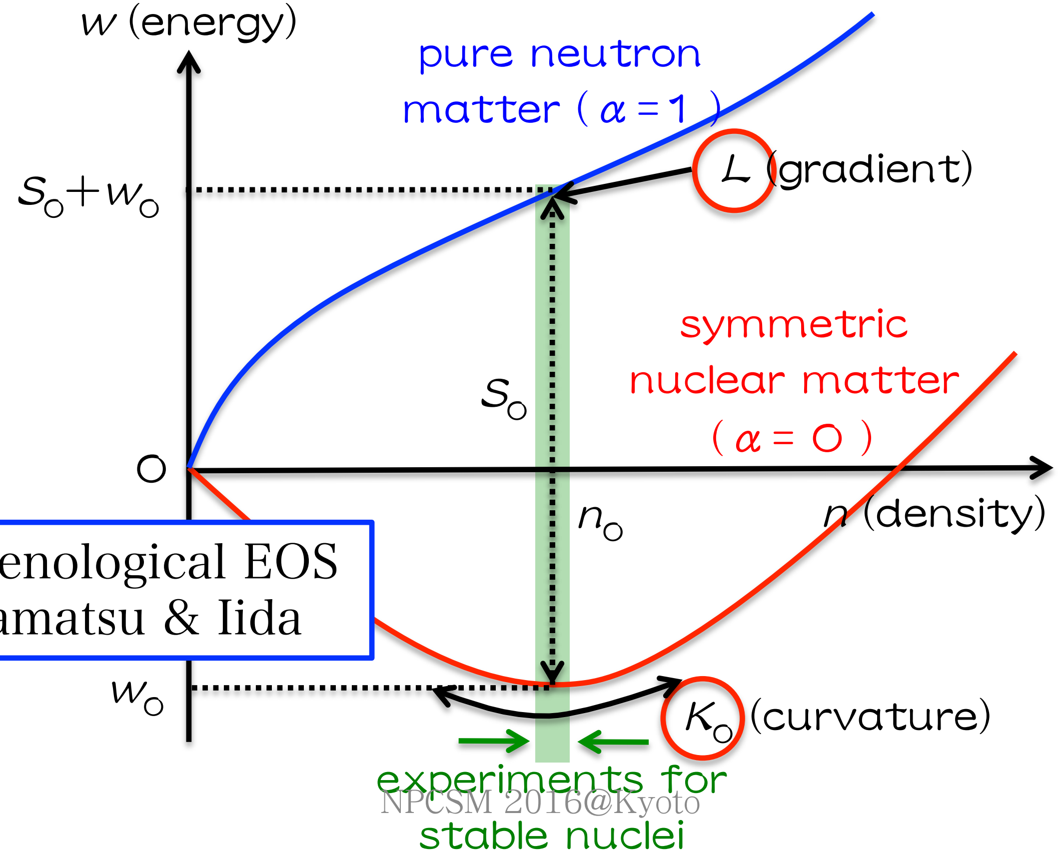
EOS near the saturation point

- Bulk energy per nucleon near the saturation point of symmetric nuclear matter at zero temperature;

incompressibility

symmetry parameter

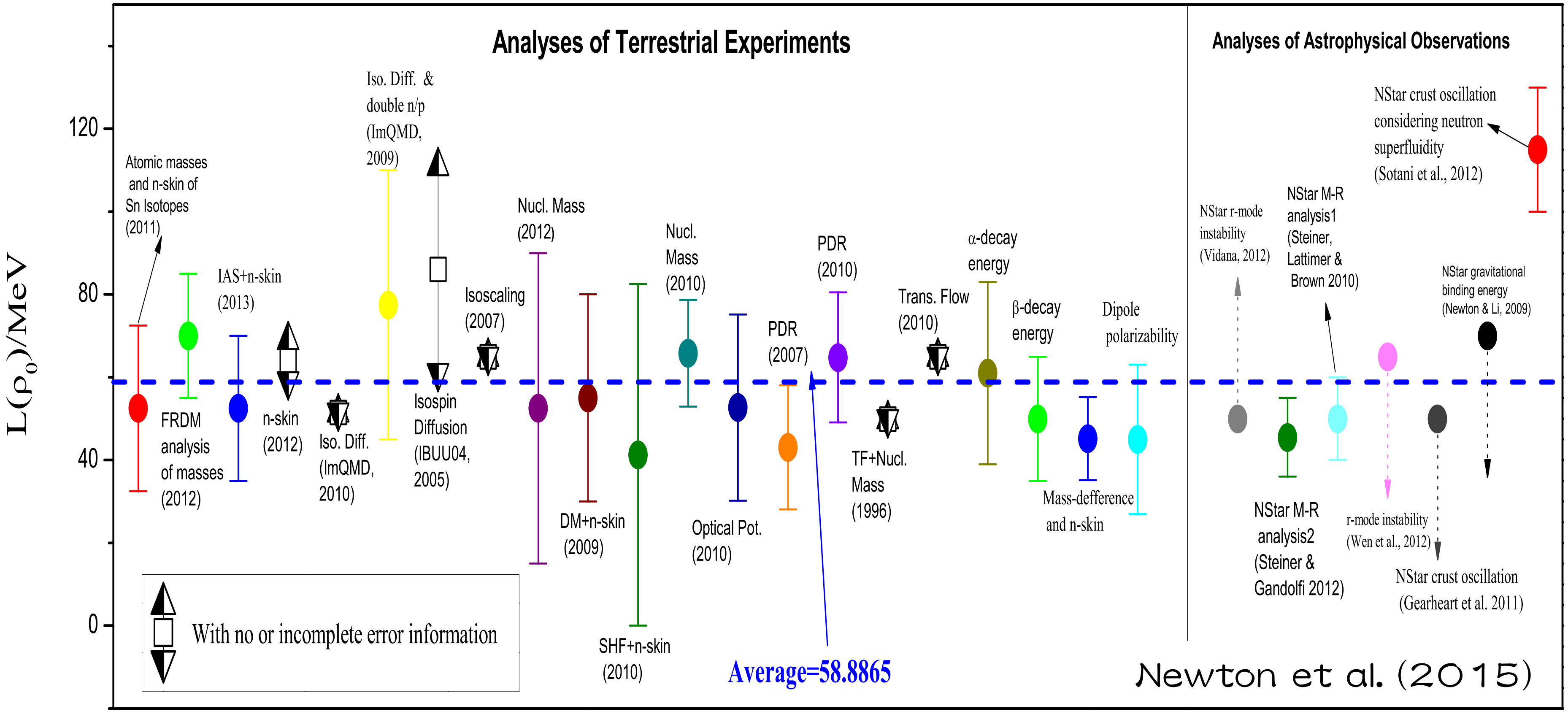
$$w = w_0 + \frac{K_0}{18n_0^2}(n-n_0)^2 + \left[S_0 + \frac{L}{3n_0}(n-n_0)\right]\alpha^2$$



phenomenological EOS
by Oyamatsu & Iida

$$P(n) = -n^2 \frac{dw}{dn}$$

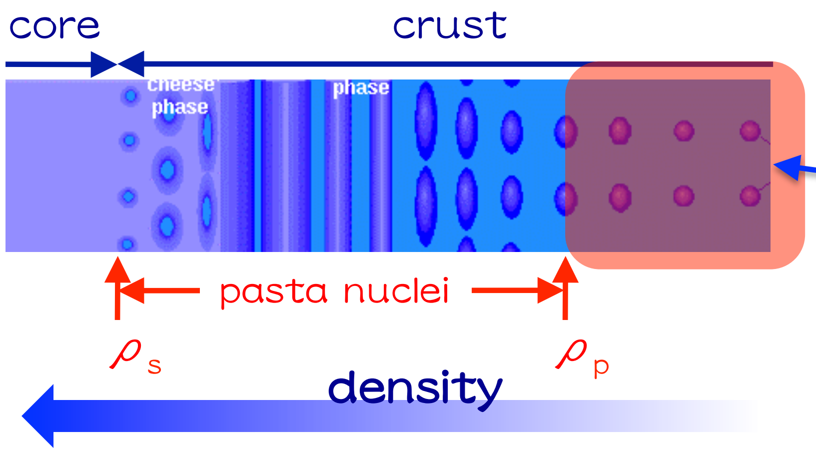
constraints on L



- most of constraints on L predict around **40 ≦ L ≦ 80 MeV**

in (our) previous works

- EOS for core region is still uncertain.
- To prepare the crust region, we integrate from $r=R$.
  - M, R : parameters for stellar properties
  - L, K_0 : parameters for curst EOS (Oyamatsu & Iida (2003), (2007))
    - → For $L \gtrsim$ 100MeV, pasta structure almost disappears
- In crust region, torsional oscillations are calculated.
  - considering the shear only in spherical nuclei.
  - frequency of fundamental oscillation $\propto v_s$ ($v_s^2 \sim \mu/H$)
  - calculated frequencies could be lower limit



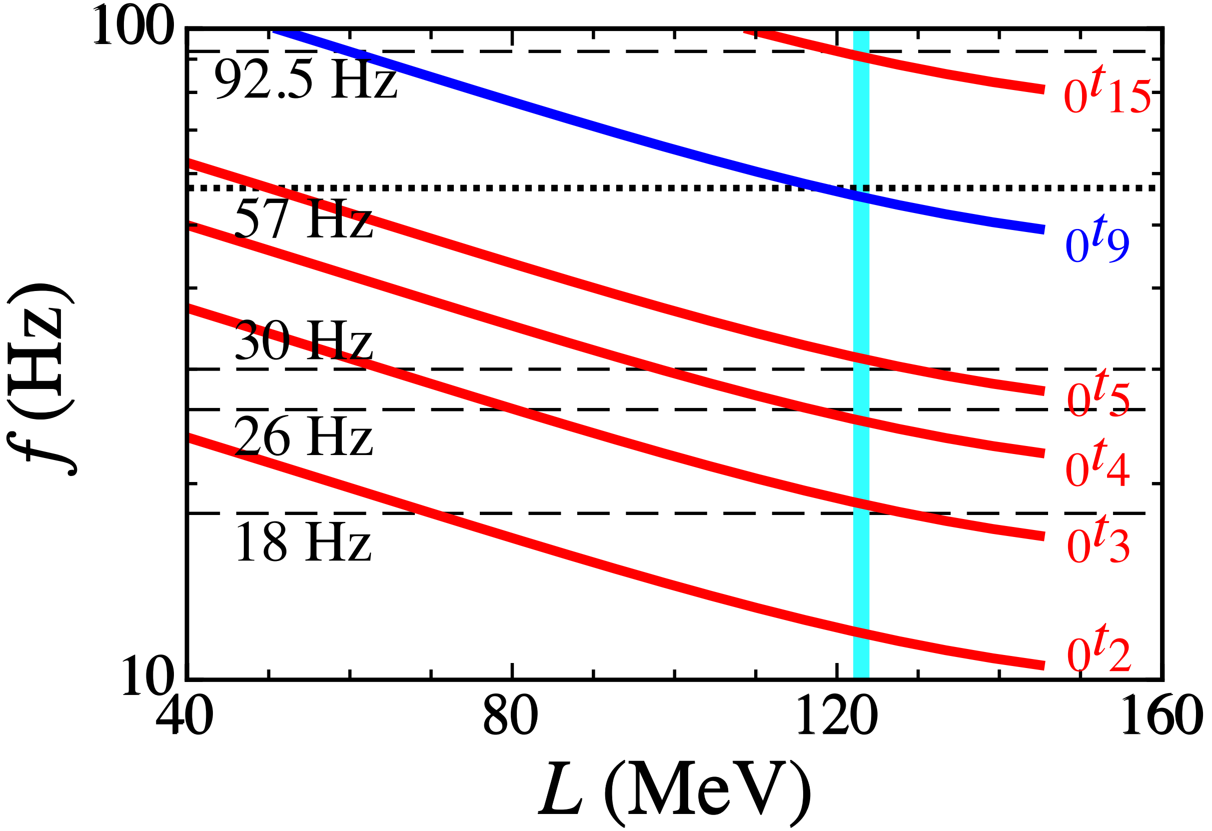
for bcc lattice

$$\mu = 0.1194\frac{n_i(Ze)^2}{a}$$

n_i : ion number density
Z : charge of nuclei
a : Wigner-Seitz radius

Identifications of SGR 1806-20

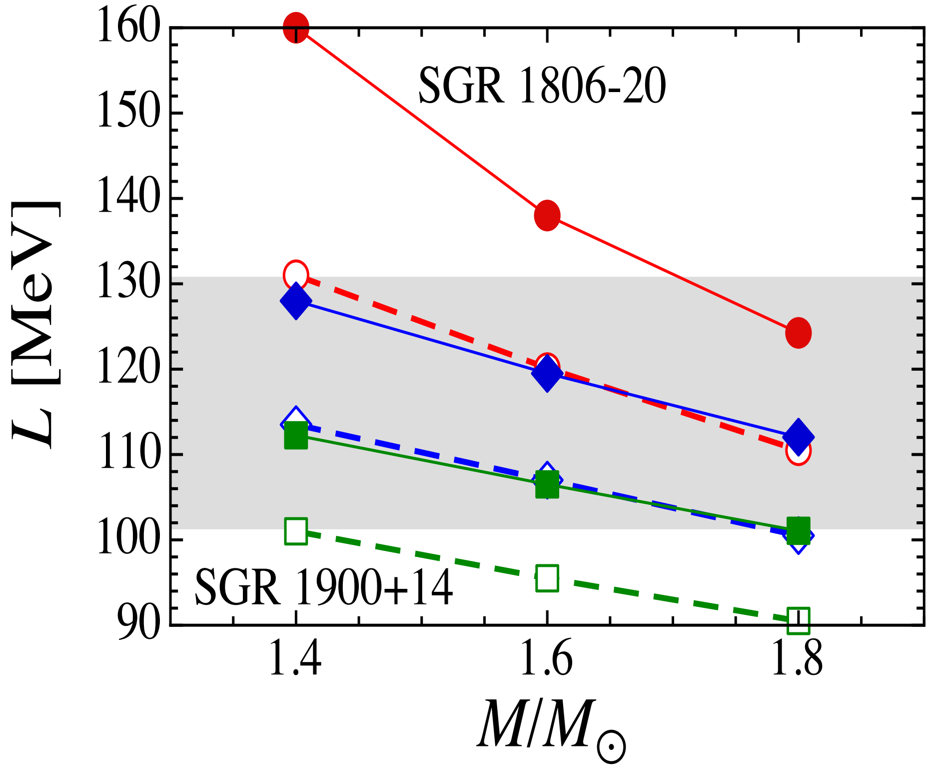
- discovery of new QPO from SGR1806-20, which is 57Hz
- for R = 12 km and $M = 1.4M_\odot$



$$\boxed{{}_0t_2 = c_0 - c_1 \frac{L}{100\ \text{MeV}} + c_2 \left(\frac{L}{100\ \text{MeV}}\right)^2}$$

constraint on *L* via QPO frequencies

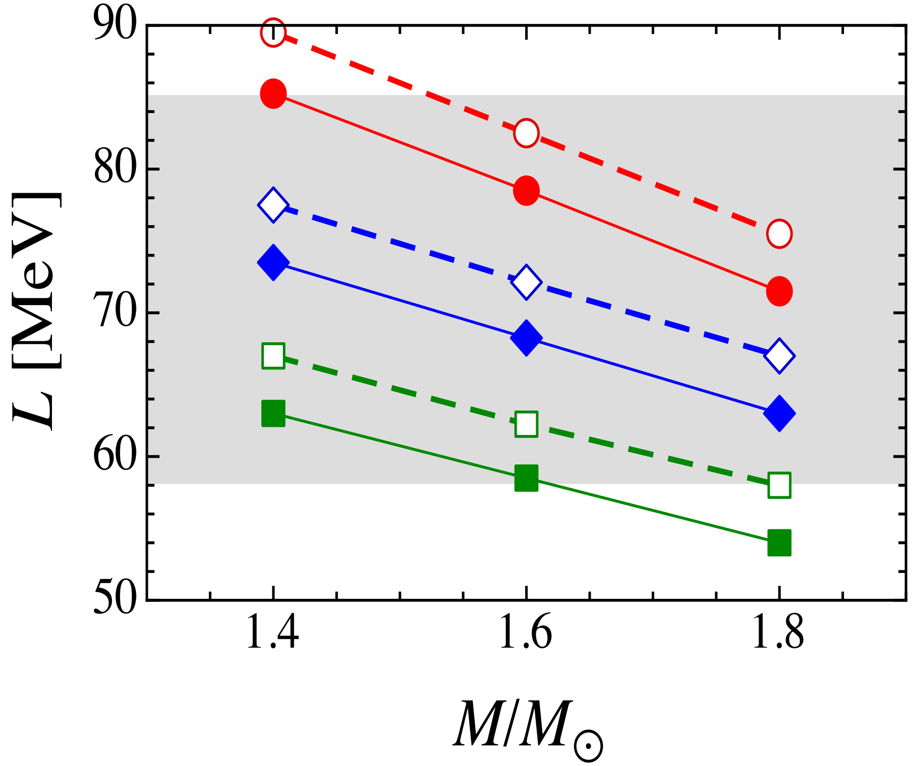
1) all QPOs come from crustal torsional oscillations



➡ $101.1 \leq L \leq 131.0$ MeV

cf) L = 40 ~ 80 MeV ??

2) QPOs except for 26Hz come from crustal torsional oscillations

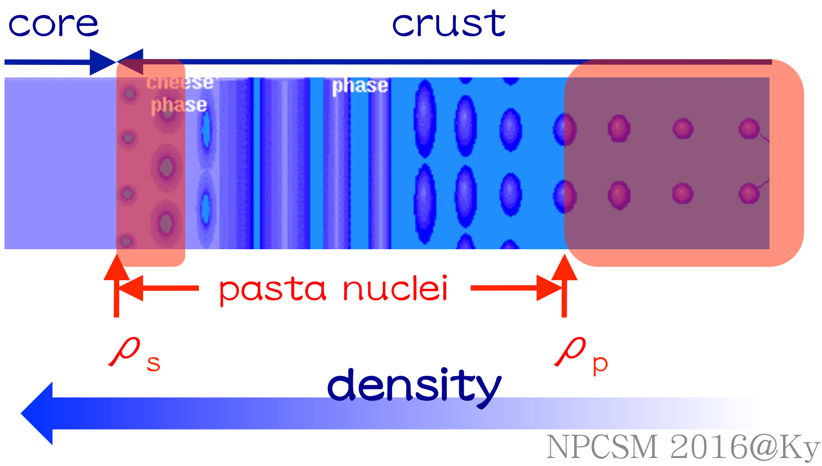


➡ $58.0 \leq L \leq 85.3$ MeV

need to prepare another oscillation mechanism to explain 26 Hz QPO !

as a possibility of 26Hz…

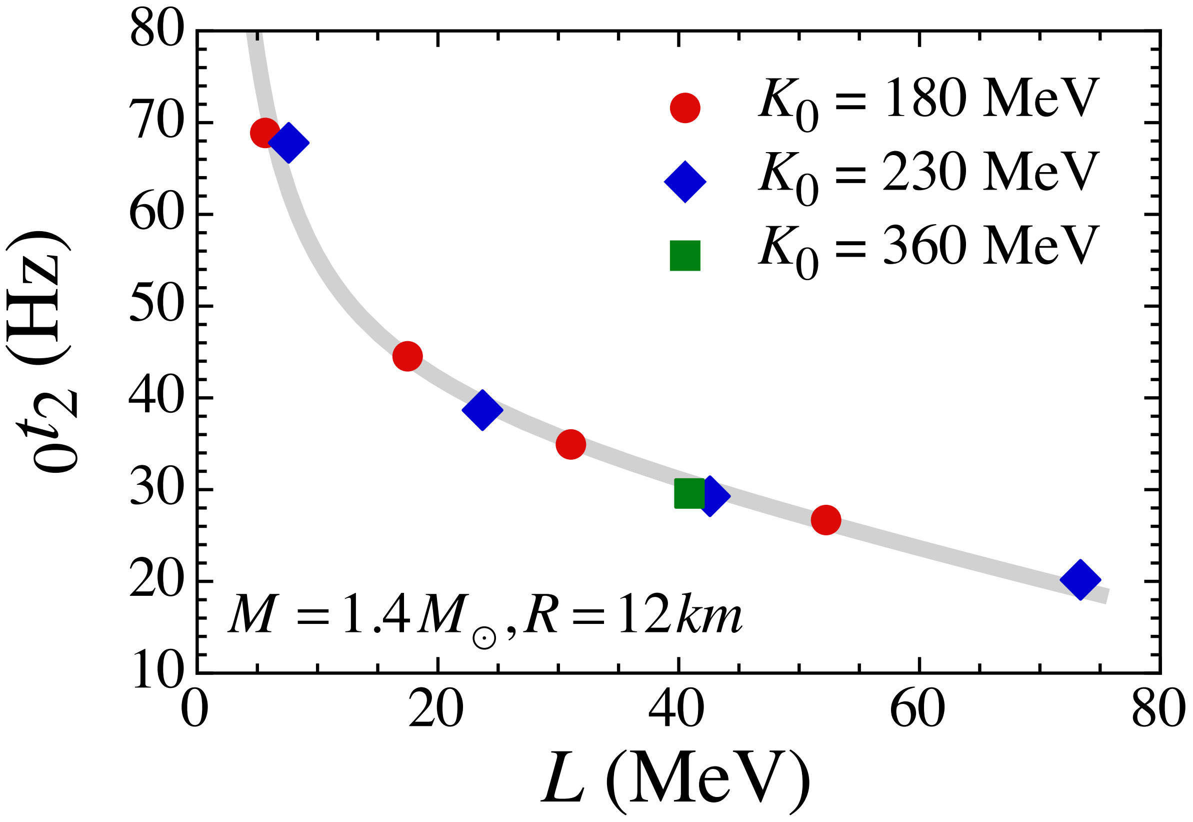
- we consider the oscillations in the pasta structure
- shear modulus in pasta phase
  - slab phase: shear is the 3rd order of displacement (Landau)
    → in the linear perturbation, oscillations in slab are negligible
  - two *independent* oscillations can be excited in different regions:
    - ① oscillations in spherical and cylindrical nuclei
    - ② oscillations in bubble and cylindrical-hole nuclei
  - as a first step, we consider only oscillations in bubble phase
    - for $L \gtrsim 75$MeV, bubble structure disappears



$$\mu = 0.1194\frac{n_i(Ze)^2}{a}$$

bubble oscillations 1

- in the case that all matter elements contribute to oscillations

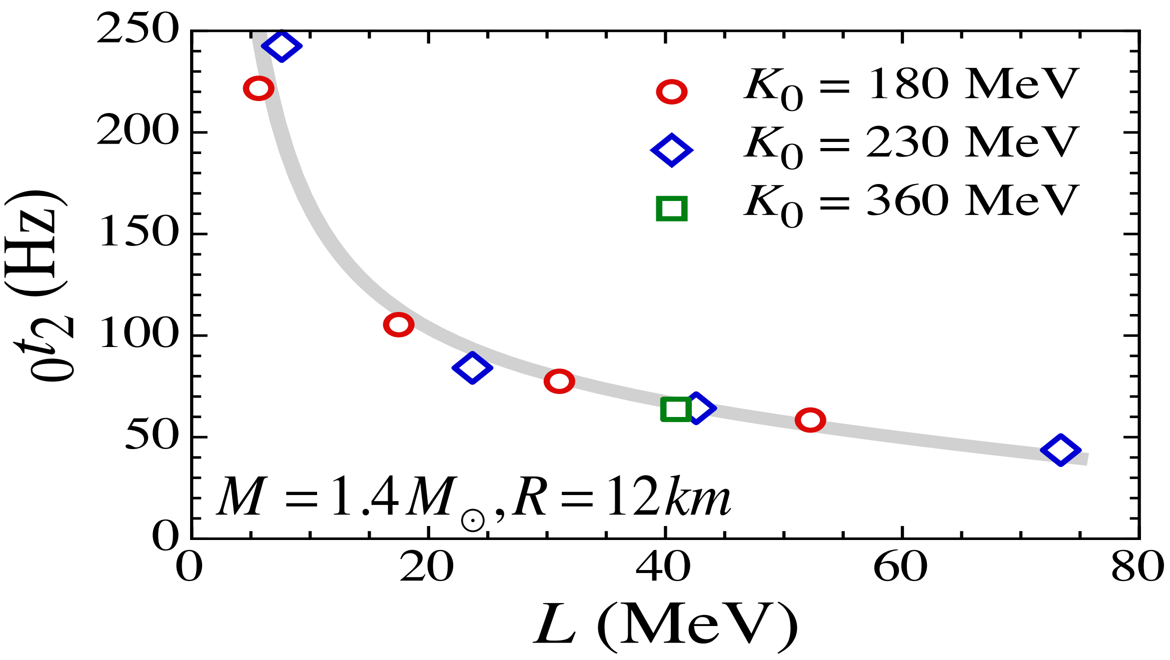


- the frequency is almost independent of the value of K_0

$_0t_2 = 205.5 / L + 37.73 - 0.2922 L$

bubble oscillations 2

- in the case that only matter elements inside the bubble contribute to oscillations

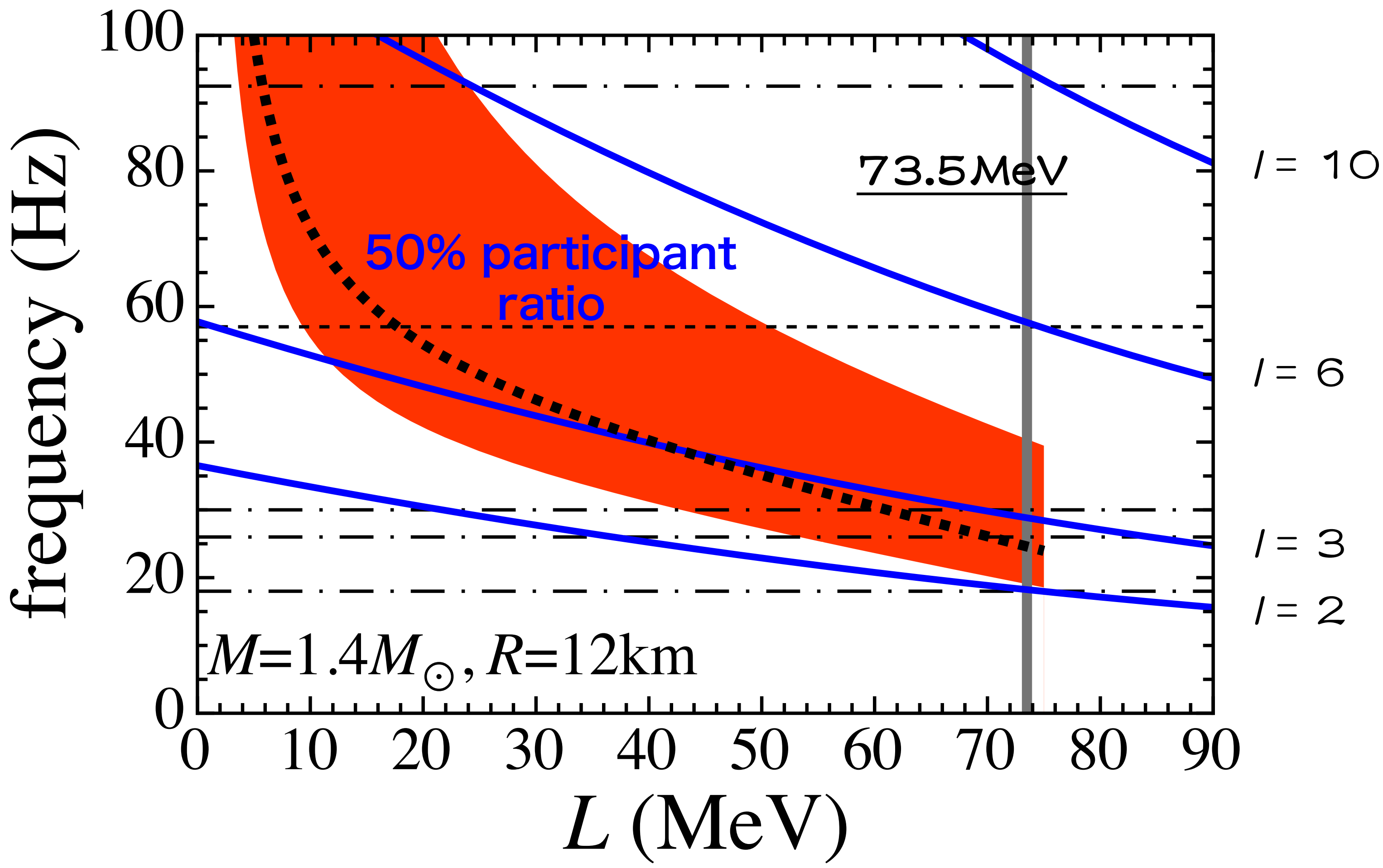


- again, the frequencies are almost independent of K_0

$${}_0t_2 = 1100 / L + 57.39 - 0.4345\, L$$

- frequencies strongly depend on the entrainment rate

comparison with QPOs



- Oscillation in bubble might be possible to correspond to 26Hz QPO, depending on the entrainment rate.

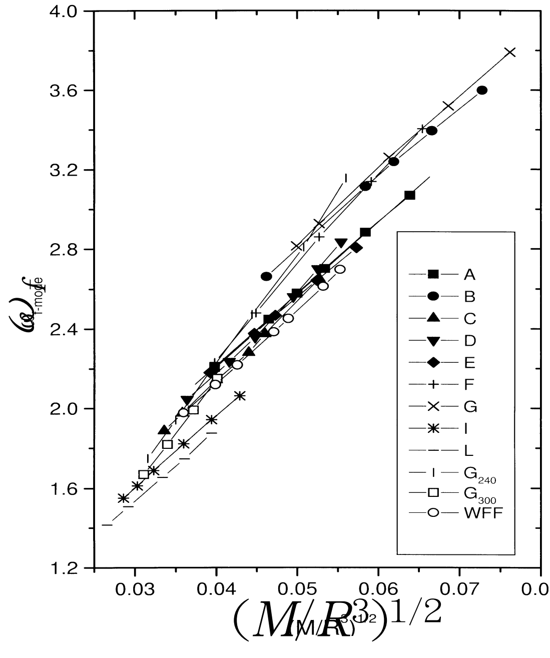
short summary

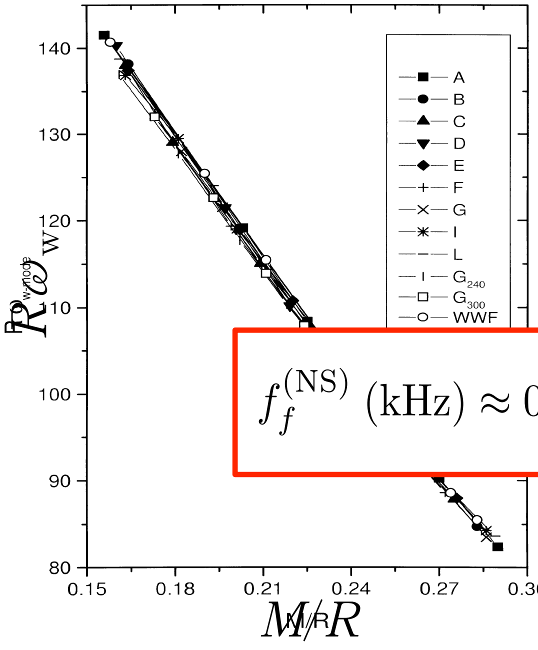
- We make a constraint on L by identifying the QPO frequencies with crustal torsional oscillations in neutron stars
  - $100 \lesssim L \lesssim 130$ MeV, if all QPOs come from torsional oscillations
  - $58 \lesssim L \lesssim 85$ MeV, if QPOs except for 26 Hz QPO coms from torsional oscillation
- as another possibility to produce the 26 Hz, we consider the torsional oscillations in bubble structure ($L \lesssim 75$ MeV)
  - frequencies strongly depend on the entrainment rate
  - even so, the frequency might be possible to correspond to 26 Hz with the suitable value of L to explain the other QPOs by torsional oscillations in the region composed of spherical nuclei
  - This may be an observational evidence for showing the existence of bubble phase in neutron star crust !

Gravitational wave asteroseismology in protoneutron stars

Gravitational wave asteroseismology

- oscillation spectra are important information for extracting the interior information of star
  - seismology on the Earth
  - helioseismology on the Sun
  - GW asteroseismology on compact stars
- several attempts have been done for cold neutron stars
  - one can get the stellar mass, radius, EOS, and magnetic properties…





via the observations of *f* and *w* mode oscillations, one could determine the *M* and *R within ~10% accuracy.*

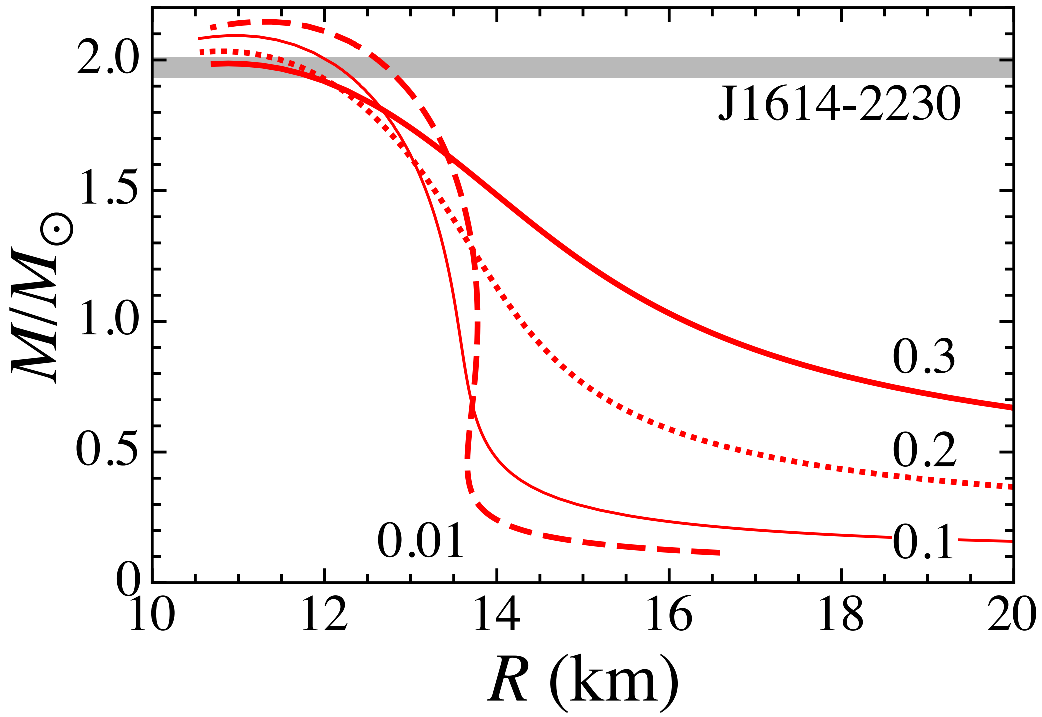
Andersson & Kokkotas (1998)

$$f_f^{(\mathrm{NS})}\ (\mathrm{kHz}) \approx 0.78 + 1.635 \left(\frac{M}{1.4M_\odot}\right)^{1/2} \left(\frac{R}{10\ \mathrm{km}}\right)^{-3/2}$$

For PNS, Muellar et al. (2013); Cerda-Duran et al. (2013); Fuller et al. (2015); Kuroda et al. (2016)

Protoneutron stars (PNSs)

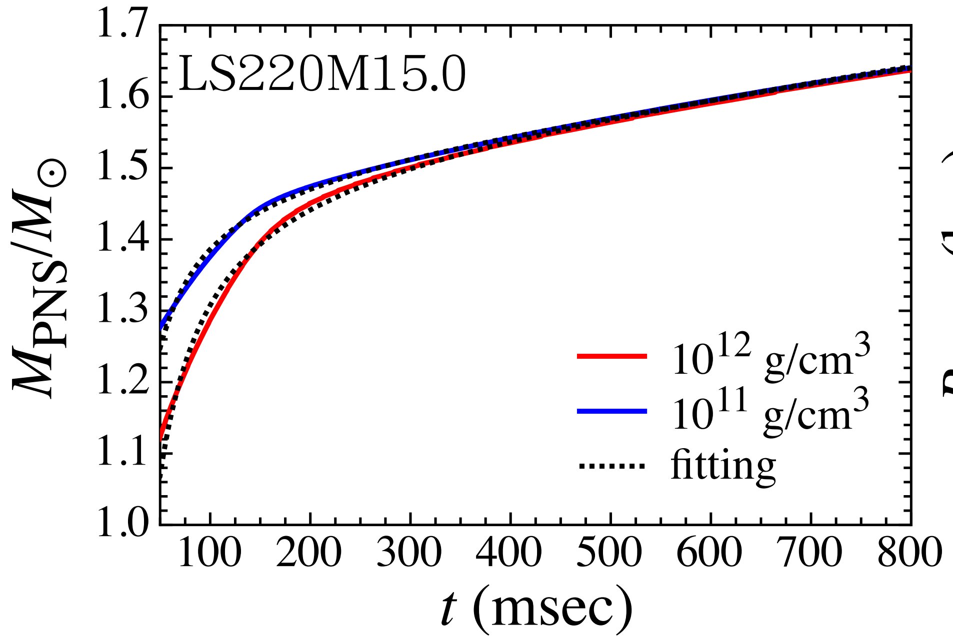
- Unlike cold neutron stars, to construct the PNS models, one has to prepare the profiles of Y_e and s.
  - for example, with LS220 and s =1.5 (k_B/baryon), but Y_e =0.01, 0.1, 0.2, and 0.3

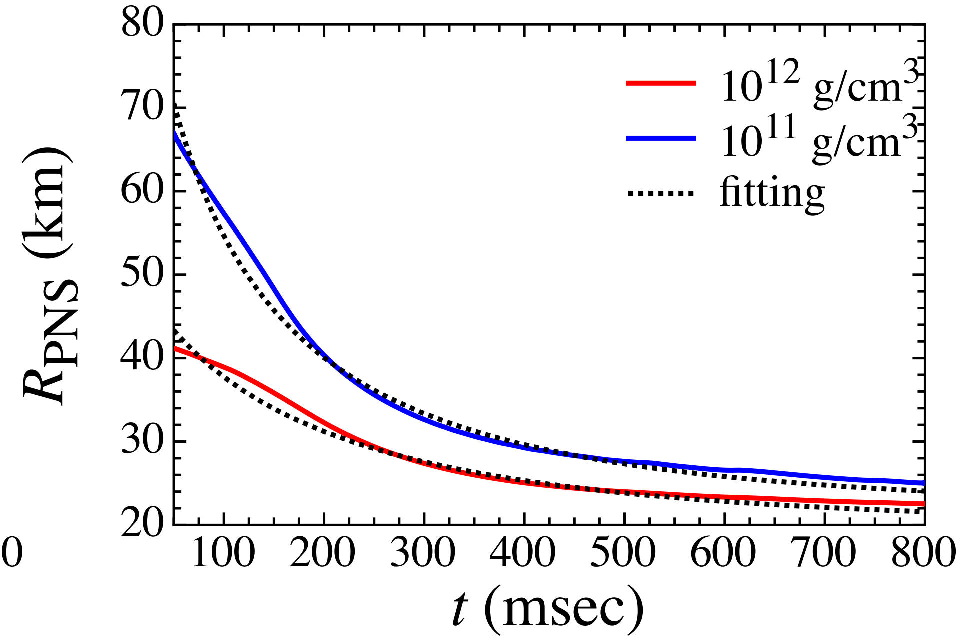


strategy

- calculate the 1D simulation of core-collapse supernova (by Takiwaki)
  - time evolutions of radius and mass of PNS are determined
  - (radius and mass of PNS are fitted by simple formula)
- PNS models are constructed in such a way that the radius and mass of PNS are equivalent to the expectation from the fitting
  - with the assumption that the PNS is quasi-static at each time step
  - with the profiles of Y_e and s
- calculate the eigenfrequencies as the eigenvalue problems on PNS models
  - dependence of the frequencies on the profiles of Y_e and s
  - dependence on the average density of PNS
  - dependence on the progenitor models
    - LS220 ($M_{pro}/M_\odot$ = 11.2, 15, 27, 40), Shen ($M_{pro}/M_\odot$ = 15)
  - we focus on the period of 1 sec. after core bounce.

evolutions of mass and radius





- fitted with

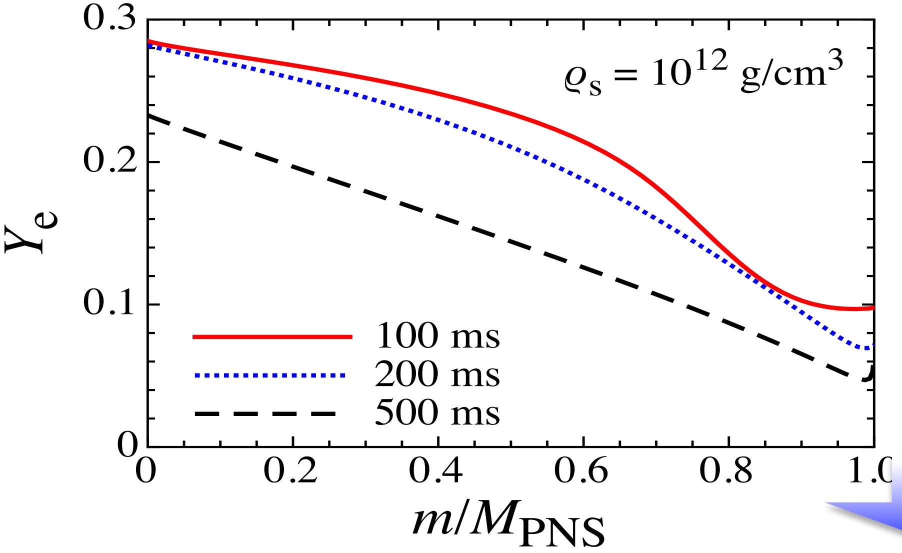
$$R_{\rm PNS}(t) = \frac{R_{\rm i}}{1+\left[1-\exp(-\frac{t}{\tau})\right]\left[\frac{R_{\rm i}}{R_{\rm f}}-1\right]}$$

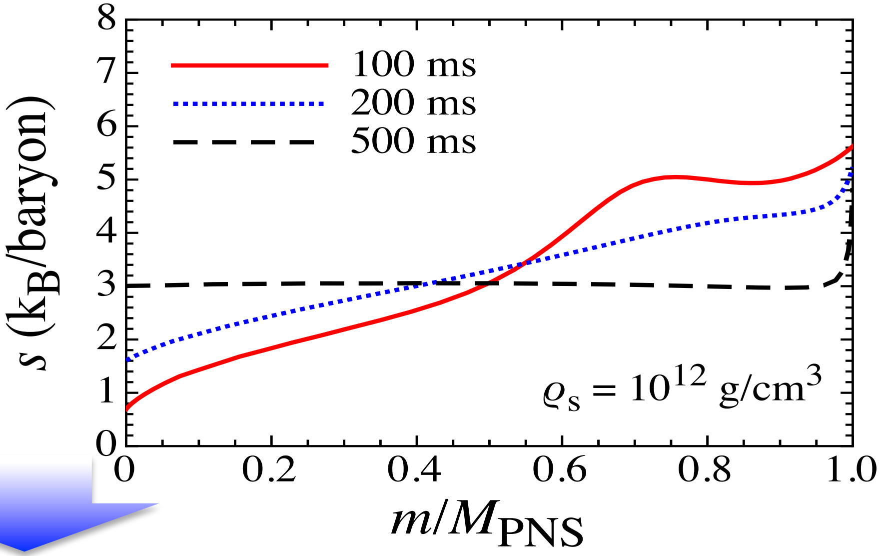
$$\frac{M_{\rm PNS}(t)}{M_\odot} = \frac{c_0}{t} + c_1 t + c_2$$

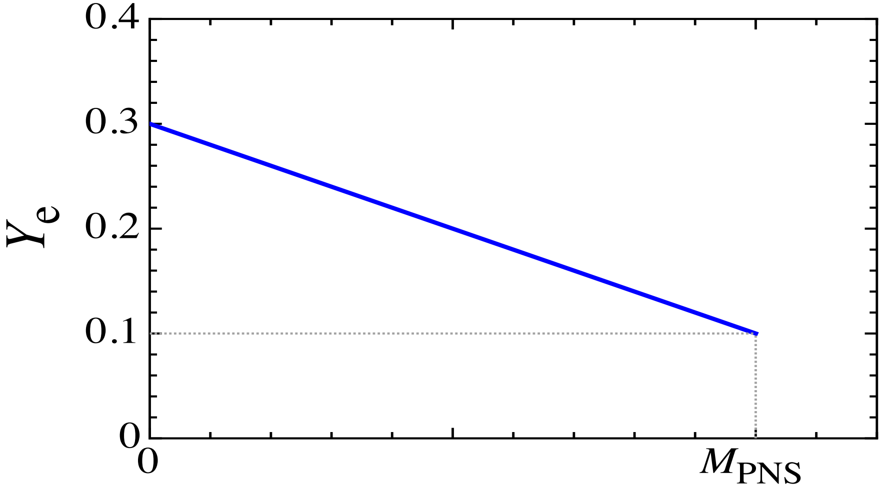
Scheck et al. (2006)

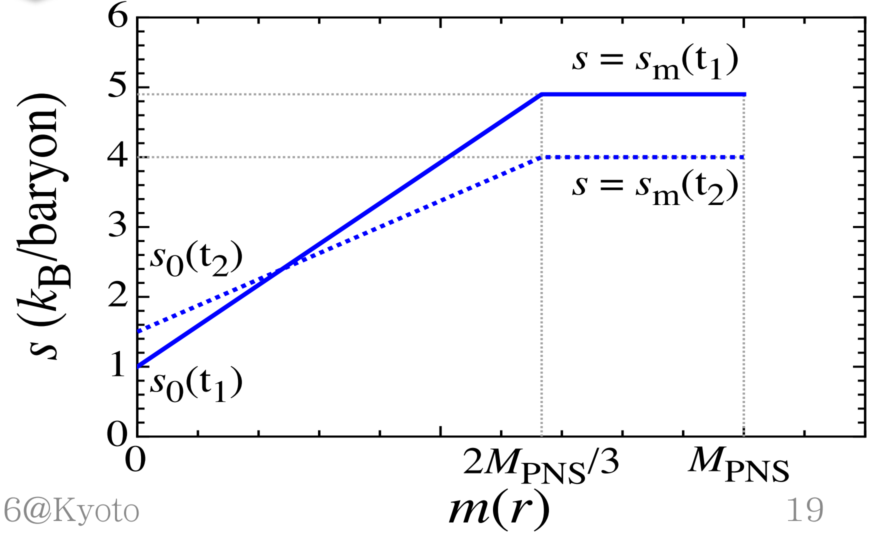
Y_e and *s* profiles

- the snap shot at t=100, 200, and 500ms after bounce from Takiwaki (TT).



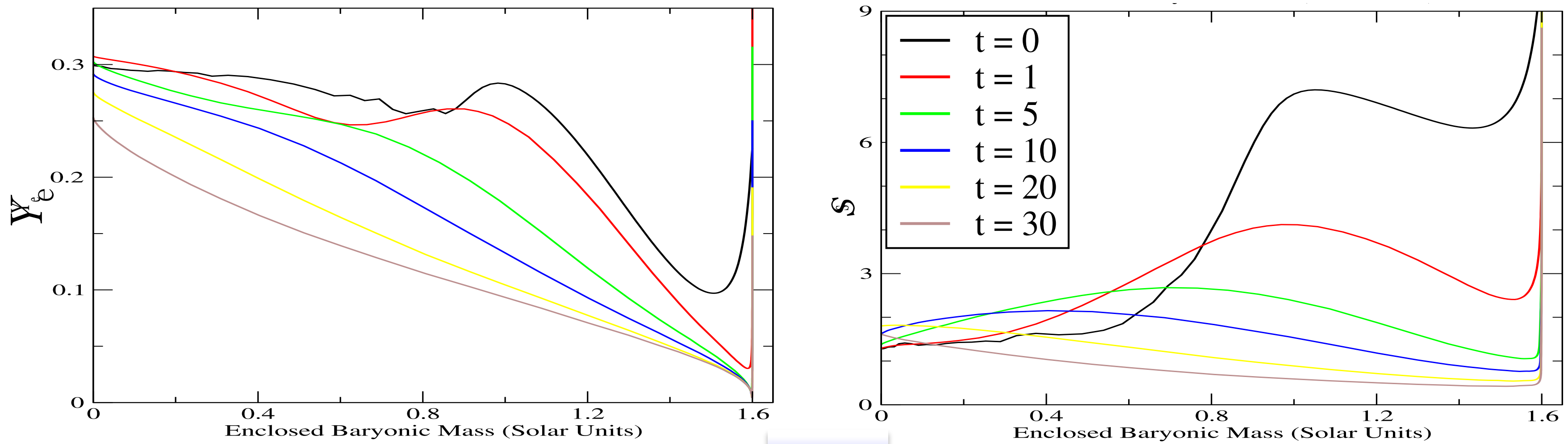


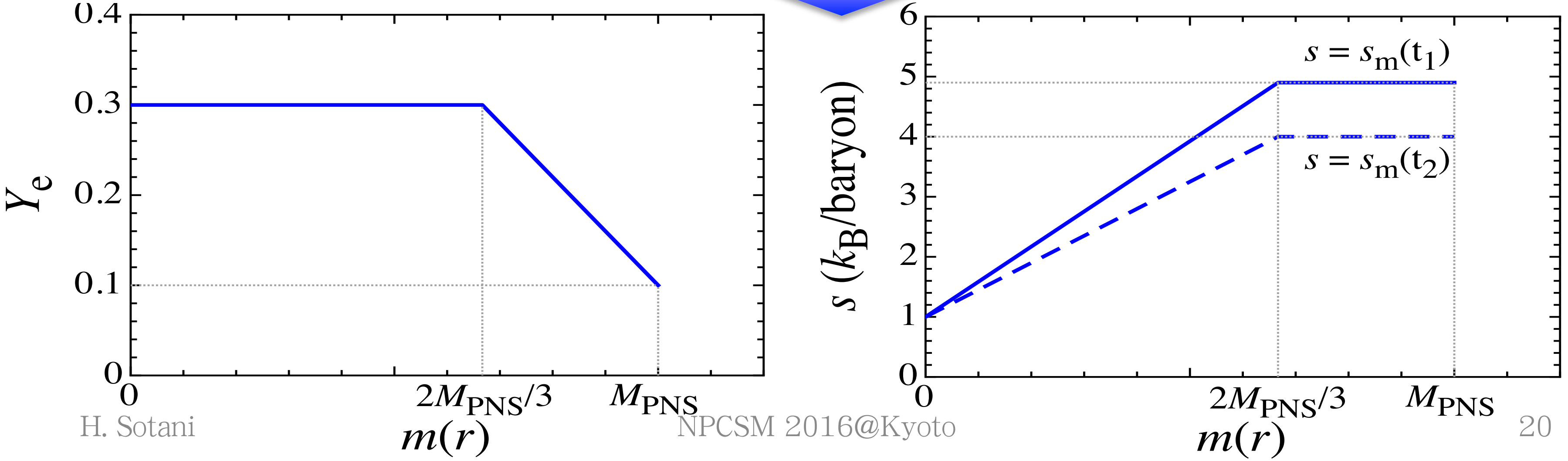




comparison with other results

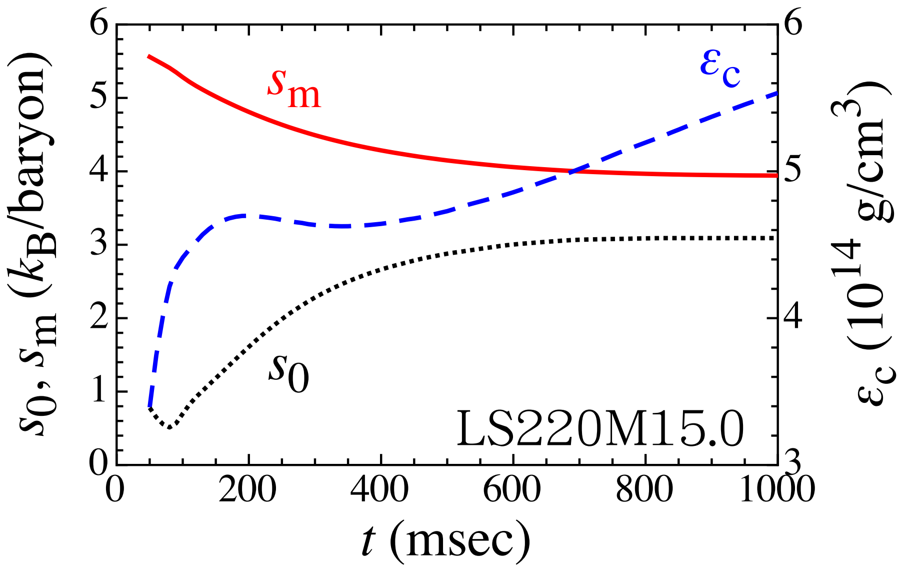
- results by Roberts (2012), where he has done the 1D simulations for long-term (RT).

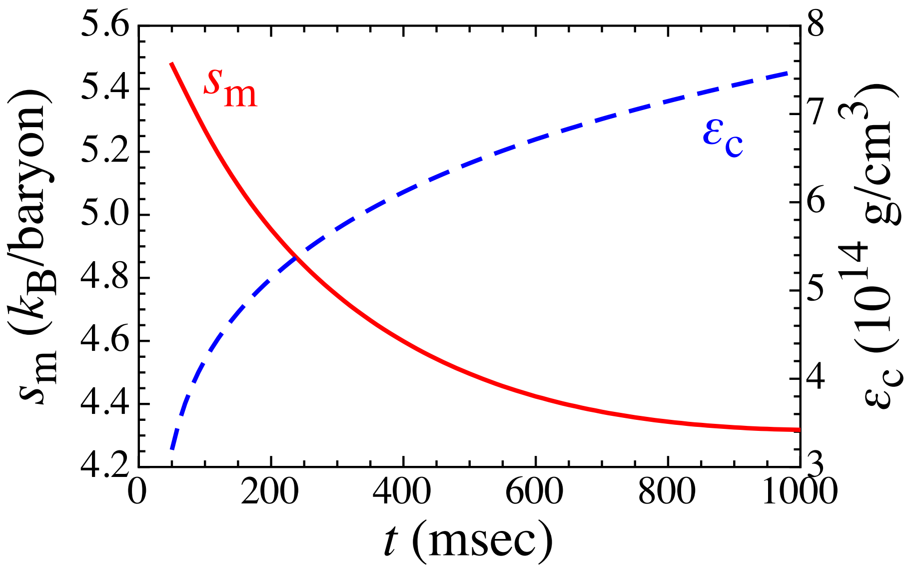




PNS models

- adopting two different profiles of Y_e and s inside the PNS, we construct the PNS models.
  - unknown parameter: ε_c & s_m
  - to reproduce the PNS models with given (M, R), ε_c and s_m are fixed.

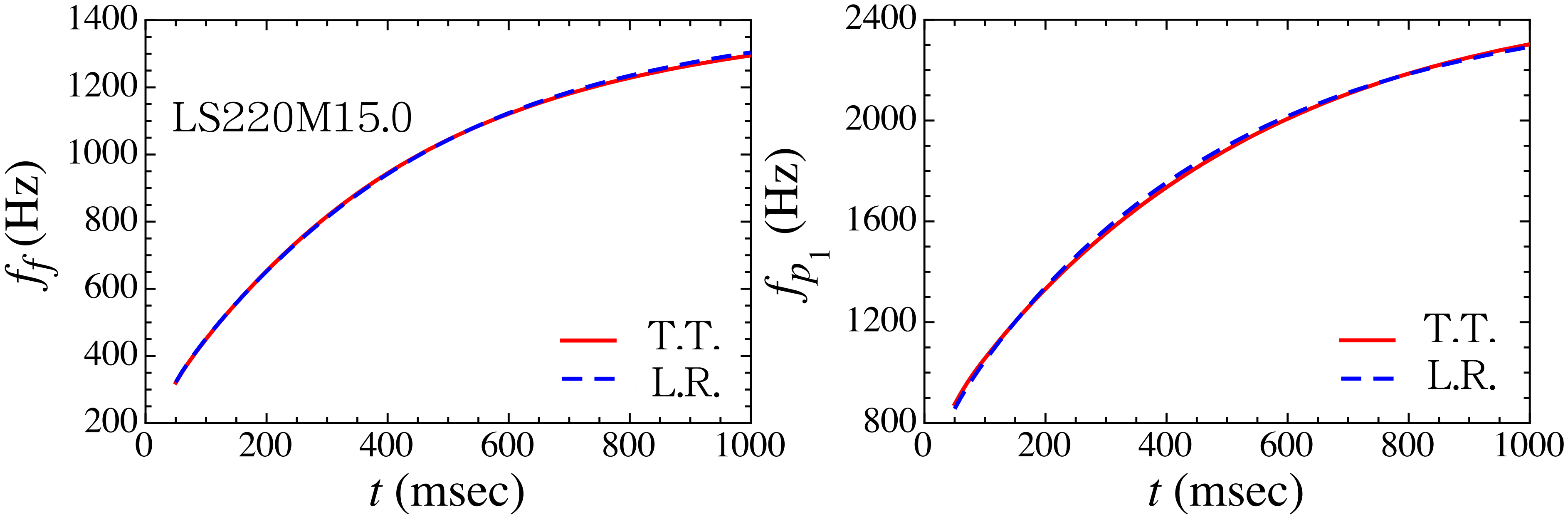




- evolutions of ε_c and s_m depend strongly on the profiles of Y_e and s.

oscillations in PNS

- with relativistic Cowling approximation
- omitting the entropy variation



- frequencies depend on mass and radius of PNS, but weakly depend on (Y_e, s) profiles.
- in the early stage, the typical frequencies of f-mode is ~ a few hundred hertz, which is good for gravitational wave detectors.

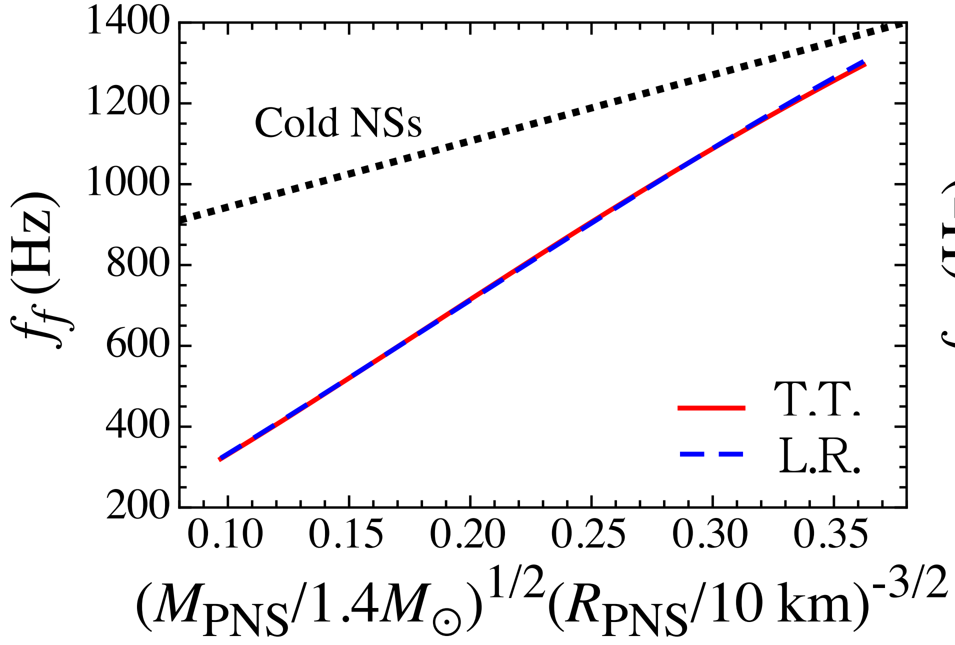
characterized by average density

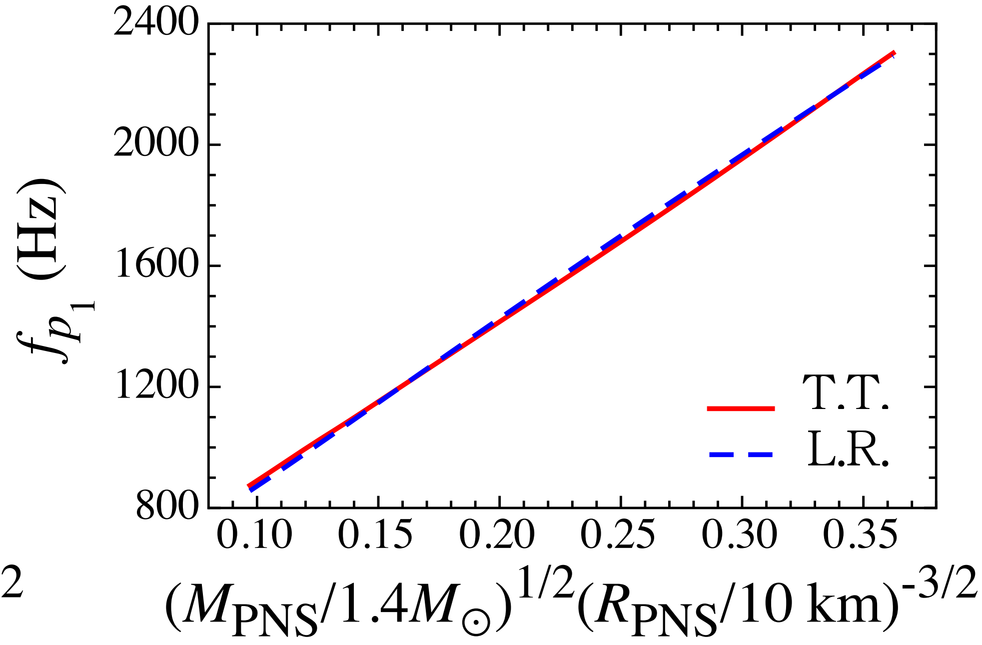
- frequencies of f-mode for cold neutron stars:

$$\boxed{f_f^{(\mathrm{NS})}\ (\mathrm{kHz}) \approx 0.78 + 1.635 \left(\frac{M}{1.4M_\odot}\right)^{1/2} \left(\frac{R}{10\ \mathrm{km}}\right)^{-3/2}}$$

Andersson & Kokkotas (1998)

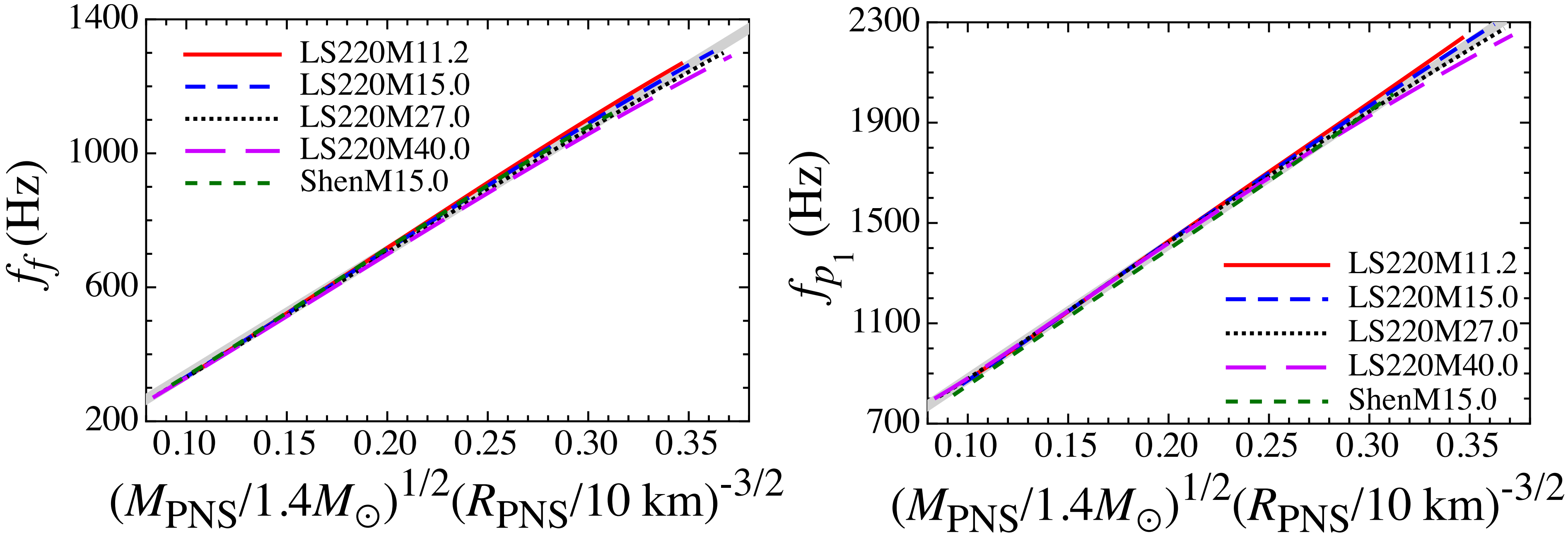
- Similarly, frequencies for PNS can be characterized by average density, but obviously different from those for neutron stars.





dependence on progenitor models

- results for LS220 with $M_{\rm pro}/M_\odot$=11.2, 15, 27, and 40, for Shen with $M_{\rm pro}/M_\odot$=15

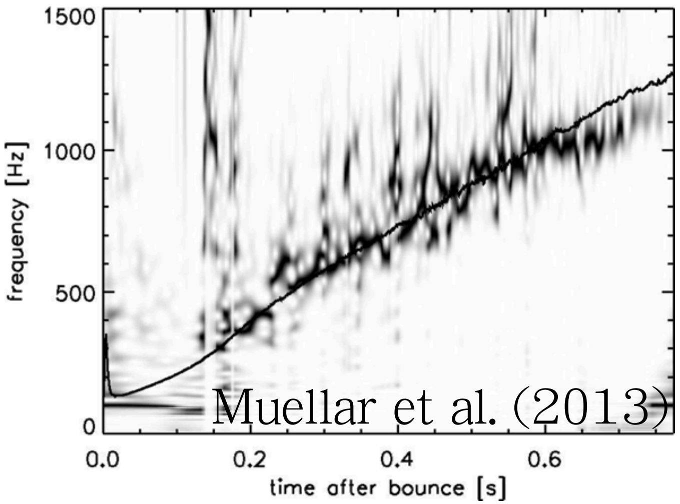


- progenitor model dependence is quite weak.

$$\boxed{f_i^{(\rm PNS)}({\rm Hz}) \approx c_i^0 + c_i^1 \left(\frac{M_{\rm PNS}}{1.4M_\odot}\right)^{1/2} \left(\frac{R_{\rm PNS}}{10\ {\rm km}}\right)^{-3/2}}$$

comparison with g-modes

- as characteristic GWs from core-collapse supernova, the excitation of g-modes around PNS has been reported (Muellar et al. (2013); Cerda-Duran et al. (2013))
  - due to the convection and the standing accretion-shock instability.

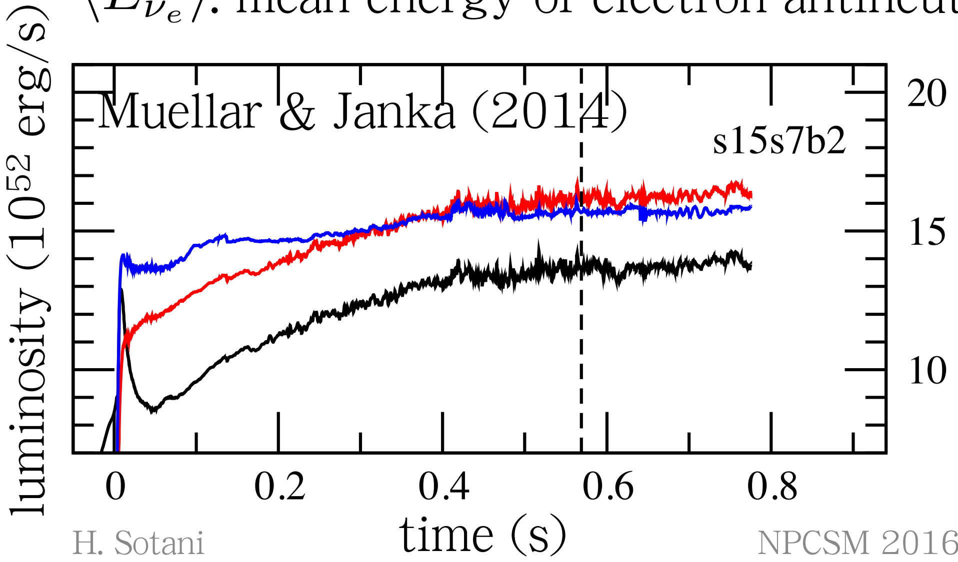


$$\boxed{f_g \approx \frac{1}{2\pi}\frac{GM_{\rm PNS}}{R_{\rm PNS}^2}\left(\frac{1.1m_n}{\langle E_{\bar{\nu}_e}\rangle}\right)^{1/2}\left(1-\frac{GM_{\rm PNS}}{c^2R_{\rm PNS}}\right)^2}$$

Muellar et al. (2013)

m_n: neutron mass

$\langle E_{\bar{\nu}_e}\rangle$: mean energy of electron antineutrinos



black: electron neutrinos

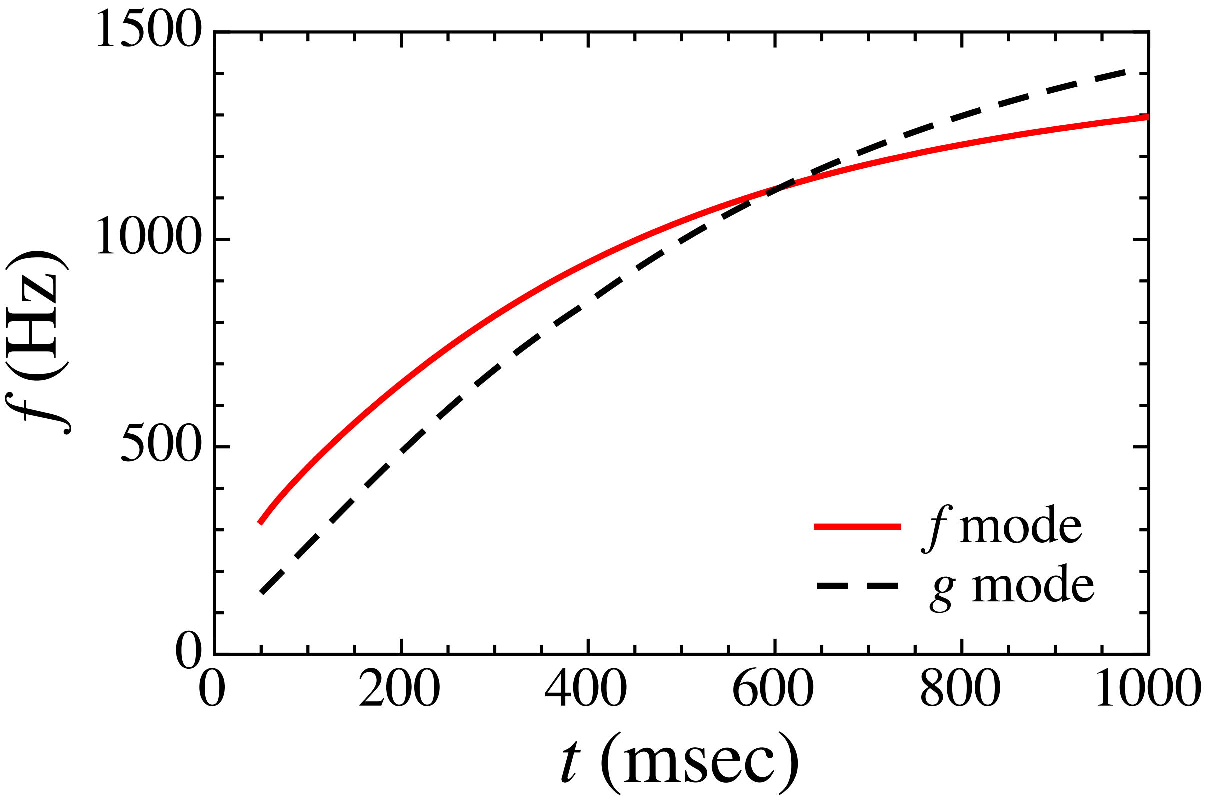
red: electron antineutrinos

blue: μ / τ neutrinos

$$\boxed{\langle E_{\bar{\nu}_e}\rangle = \begin{cases} 3t/400+13 & (0 \le t \le 400\ \text{msec}) \\ 16 & (400\ \text{msec} \le t) \end{cases}}$$

comparison with g-modes

- careful observing the gravitational wave spectra after core-collapse supernova, one might see the different sequences in spectra
  - which tells us the radius and mass of PNS



short summary

- We examine the frequencies of gravitational waves radiating from PNS after bounce.
- The PNS models are constructed in such a way that the mass and radius obtained from 1D simulation are reconstructed.
  - two different profiles of Y_e and s are considered
- frequencies of gravitational waves are almost independent from the profiles of Y_e and s, but sensitive to the mass and radius.
  - characterized by average density
  - different from that expected for cold neutron stars
- progenitor dependence is quite weak.
  - find an universal relation for frequencies of f- and p-modes as a function of average density
  - different dependence for g-mode around PNS
- one might be possible to determine the mass and radius of PNS via careful observations of time evolution of gravitational wave spectra.

conclusion

- NSs can become a Rosetta stone to understand the physics under the extreme states.
- Asteroseismology is one of the powerful tools for extracting an interior information.
  - crustal torsional oscillations
  - GWs from PNS

Thank you
for your attention!