INSTITUTE for NUCLEAR THEORY
PHYSICS/ASTRONOMY BLDG. RM C411 • BOX 351550
UNIVERSITY OF WASHINGTON • SEATTLE, WA 98195-1550 • USA
www.int.washington.edu • 206-685-3360

Stephan Stetina

Institute for Nuclear Theory
Seattle, WA 98105

Der Wissenschaftsfonds.

Instabilities of Relativistic Superfluids

NPCSM 2016, Yukawa Institute, Kyoto, Japan

M.G. Alford, A. Schmitt, S.K. Mallavarapu, A. Haber

[A. Haber, A. Schmitt, S. Stetina, PRD93, 025011 (2016)]
[S. Stetina, arXiv: 1502.00122 hep-ph]
[M.G. Alford, S. K. Mallavarapu, A. Schmitt, S. Stetina, PRD89, 085005 (2014)]
[M.G. Alford, S. K. Mallavarapu, A. Schmitt, S. Stetina, PRD87, 065001 (2013)]

Superfluidity in dense matter

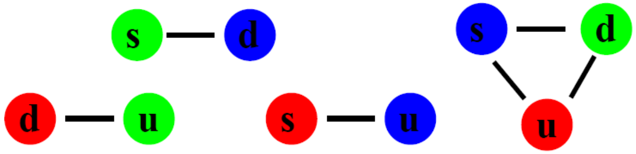
Microscopic vs macroscopic description of compact stars

derive hydrodynamics

- groundstate of dense matter
- quantum field theory
- Bose-Einstein condensate

- Pulsar glitches
- R-mode instability
- Asteroseismology
- (…)

learn about fundamental physics



Superfluidity in dense matter

Microscopic mechanism: Spontaneous Symmetry Breaking (SSB)

- Quark matter at asymptotically high densities:

  → colour superconductors break Baryon conservation $U(1)_B$

  [M. Alford, K. Rajagopal, F. Wilczek, NPB 537, 443 (1999)]

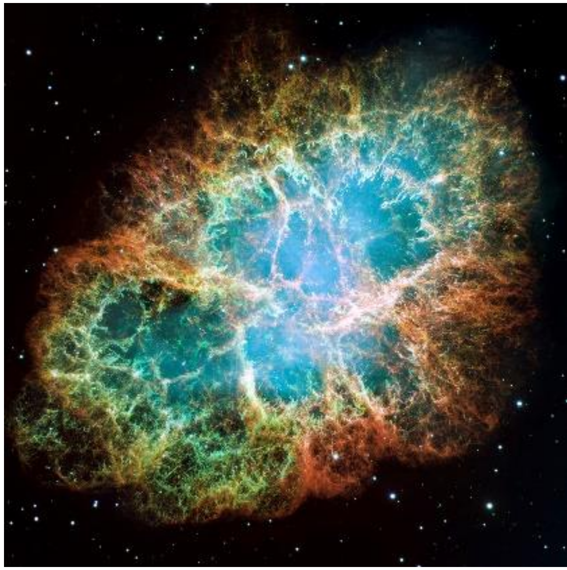
- Quark matter at intermediate densities:

  → meson condensate breaks conservation of strangeness $U(1)_S$

  [T. Schäfer, P. Bedaque, NPA, 697 (2002)]

- nuclear matter:

  → SSB of $U(1)_B$ (exact symmetry at any density)



Goal: translation between field theory and hydrodynamics

SSB in U(1) invariant model at finite T → superfluid coupled to normal fluid

SSB in U(1) x U(1) invariant model at T=0 → 2 coupled superfluids

Superfluidity from Quantum Field Theory

start from simple microscopic complex scalar field theory:

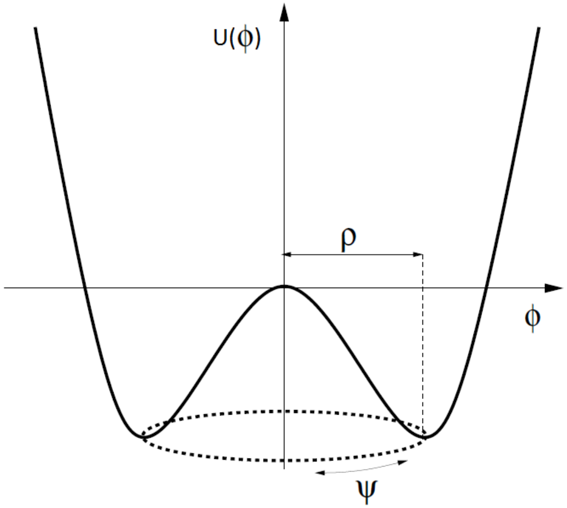
$$\mathcal{L} = \partial_\mu \varphi \partial^\mu \varphi^* - m^2 |\varphi|^2 - \lambda |\varphi|^4$$

- separate condensate\fluctuations:

$\varphi \to \varphi + \phi \qquad \phi = \rho\, e^{i\psi}$

→ superfluid related to condensate
[L. Tisza, Nature 141, 913 (1938)]

→ normal-fluid related to quasiparticles
[L. Landau, Phys. Rev. 60, 356 (1941)]



- static ansatz for condensate: (infinite uniform superflow) $\rho\,,\partial_\mu\psi = \text{const.}$

- Fluctuations $\delta\rho(\boldsymbol{x},t)$ and $\delta\psi(\boldsymbol{x},t)$ around the static solution determined by classical EOM, can be thermally populated

$$\rho = \rho\left(\partial_\mu\psi^2 - m^2 - \lambda\rho^2\right) \qquad \partial_\mu(\rho\partial^\mu\psi) = 0$$

→ Goldstone mode + massive mode

Hydrodynamics from Field Theory

Relativistic two fluid formalism at finite T (non dissipative)

[B. Carter, M. Khalatnikov, PRD 45, 4536 (1992)]

$j^\mu = n_s v_s^\mu + n_n v_n^\mu$ with: $v_s^\mu = \frac{\partial^\mu \psi}{\sigma}$ (superflow) $v_n^\mu = \frac{s^\mu}{s}$ (entropy flow)

$P = P_s + P_n$

connection to field theory at T=0:

$v_s^\mu = \partial^\mu \psi / \sigma$ $\sigma^2 = \partial_\mu \psi \partial^\mu \psi = \mu(1 - \boldsymbol{v}_s^2)$ $\mu_s = \partial_0 \psi$ $\boldsymbol{v}_s = -\nabla \psi / \mu_s$

derivation of hydrodynamic quantities at finite T: 2PI (CJT) formalism

effective Action: $\Gamma = \Gamma[\rho, S]$, $0 = \delta\Gamma/\delta\rho$, $0 = \delta\Gamma/\delta S$

→ **present results in normal fluid restframe**

[M.G. Alford, S. K. Mallavarapu, A. Schmitt, S. Stetina, PRD89, 085005 (2014)]
[M.G. Alford, S. K. Mallavarapu, A. Schmitt, S. Stetina, PRD87, 065001 (2013)]

Classification of excitations

elementary excitations

- **poles of the quasiparticle propagator**

→ energetic instabilities (negative quasiparticle energies)

collective modes (sound modes)

- **fluctuations in the *density* of elementary excitations**

→ equivalent to elementary excitations at T=0

→ introduce fluctuations for all hydrodynamic and thermodynamic quantities

$$x \to x_0 + \delta x(\boldsymbol{x}, t) \qquad x = \{\mathrm{P}_s\,, \mathrm{P}_n\,, n_s, n_n, \mu_s, \mathrm{T},\ \vec{v}_s\,\}$$

→ solutions to a given set of (linearized) hydro equations

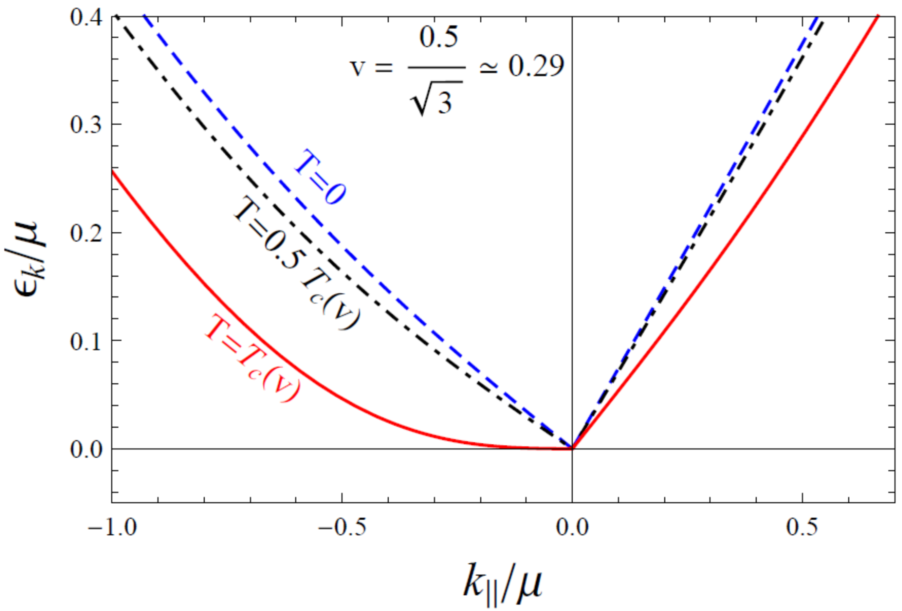
$$\partial_\mu j^\mu = 0\,, \quad \partial_\mu s^\mu = 0 \quad \text{and} \quad \partial_\mu T^{\mu\nu} = 0$$

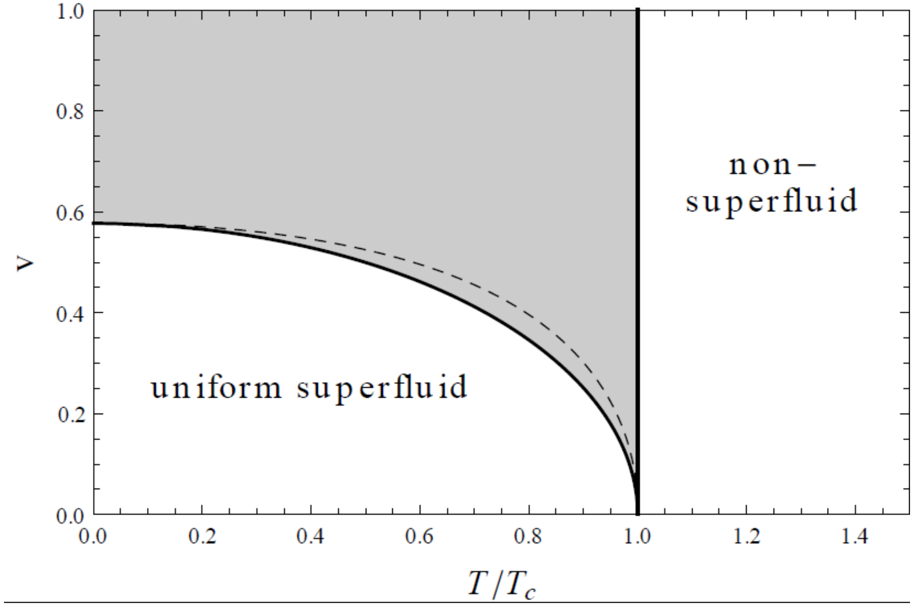
→ dynamic instabilities (complex sound modes)

Elementary excitations

→ critical temperature: condensate has “melted” completely

→ critical velocity: negative Goldstone dispersion relation (angular dependency)





Generalization of Landau critical velocity

- normal and super frame connected by Lorentz boost

- back reaction of condensate on Goldstone dispersion

sound excitations

- **Scale invariant limit**

→ pressure can be written as $\Psi = T^4\, h(T/\mu)$
[C. Herzog, P. Kovtun, and D. Son, Phys.Rev.D79, 066002 (2009)]

→ second sound still complicated! Compare e.g. to ^{4}He:

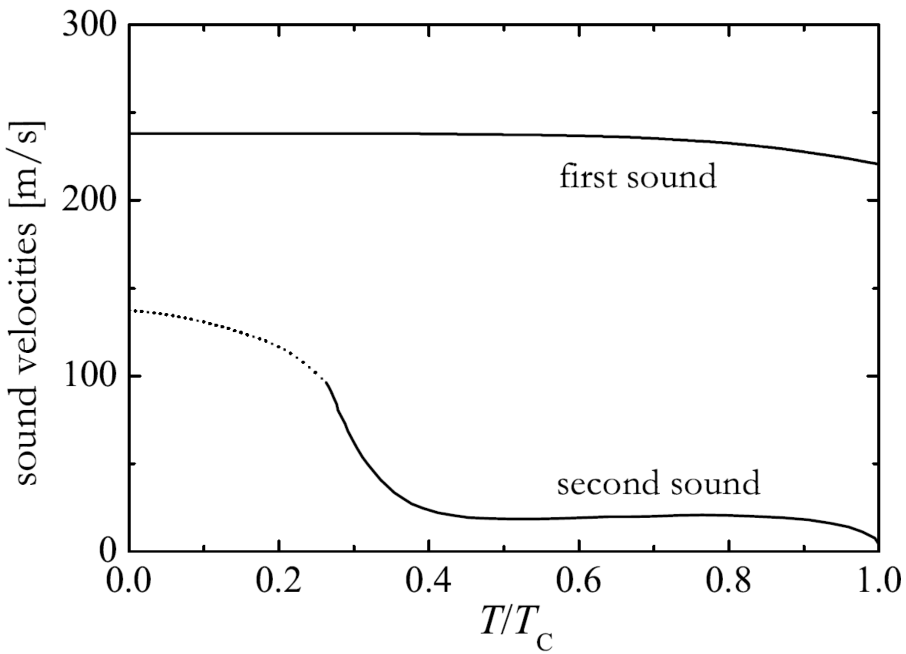
$$u_1^2 = \frac{1}{3}$$

$$u_2^2 = \frac{n_s s^2}{\mu n_n + Ts}\left(n\,\frac{\partial s}{\partial T} - s\,\frac{\partial n}{\partial \mu}\right)^{-1}$$

→ ratios of amplitudes

$$\left.\frac{\delta T}{\delta \mu}\right|_{u_1} = \frac{T}{\mu} \quad \text{(in phase)}$$

$$\left.\frac{\delta T}{\delta \mu}\right|_{u_2} = -\frac{n}{s} \quad \text{(out of phase)}$$

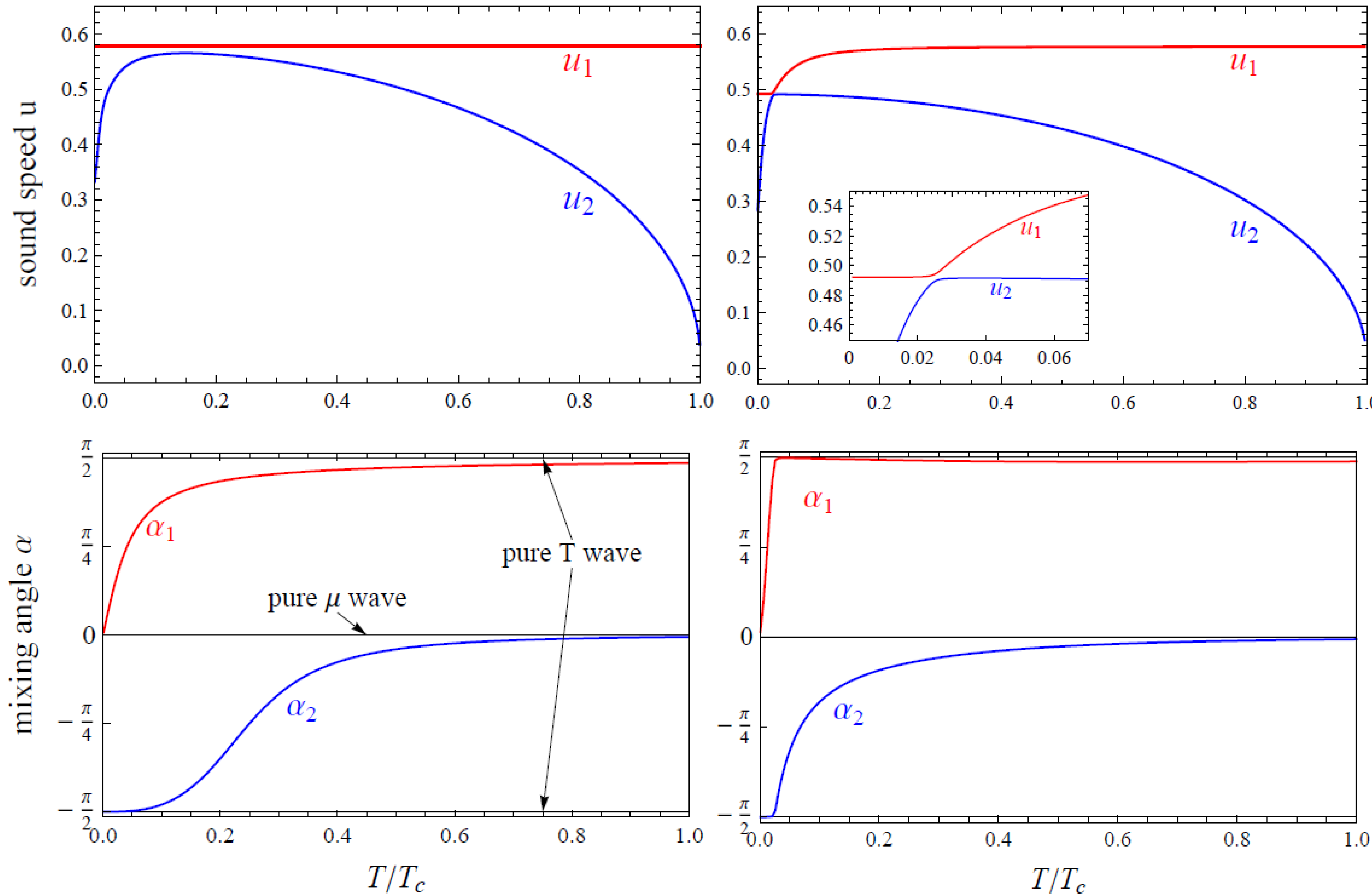


[E. Taylor, H. Hu, X. Liu, L. Pitaevskii, A. Griffin, S. Stringari, Phys. Rev. A 80, 053601 (2009)]

Role reversal, no superflow

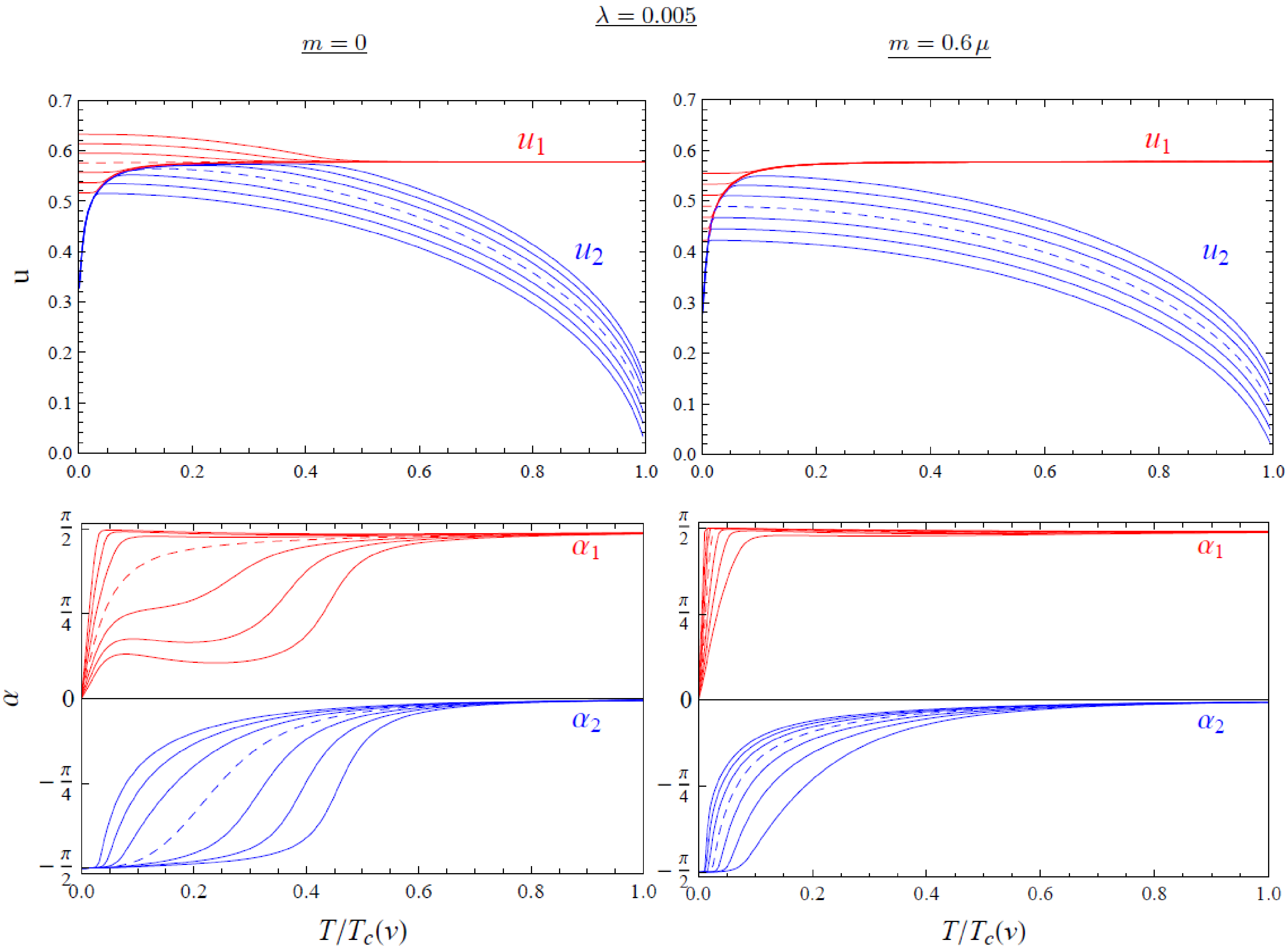
m={0 , 0.6 μ}

$$\alpha := \arctan\frac{\delta T}{\delta\mu}$$



Role reversal including superflow

$$\alpha := \arctan\frac{\delta T}{\delta \mu}$$



System of two coupled superfluids

$U(1) \times U(1)$ invariant microscopic model:

→ two coupled complex scalar fields

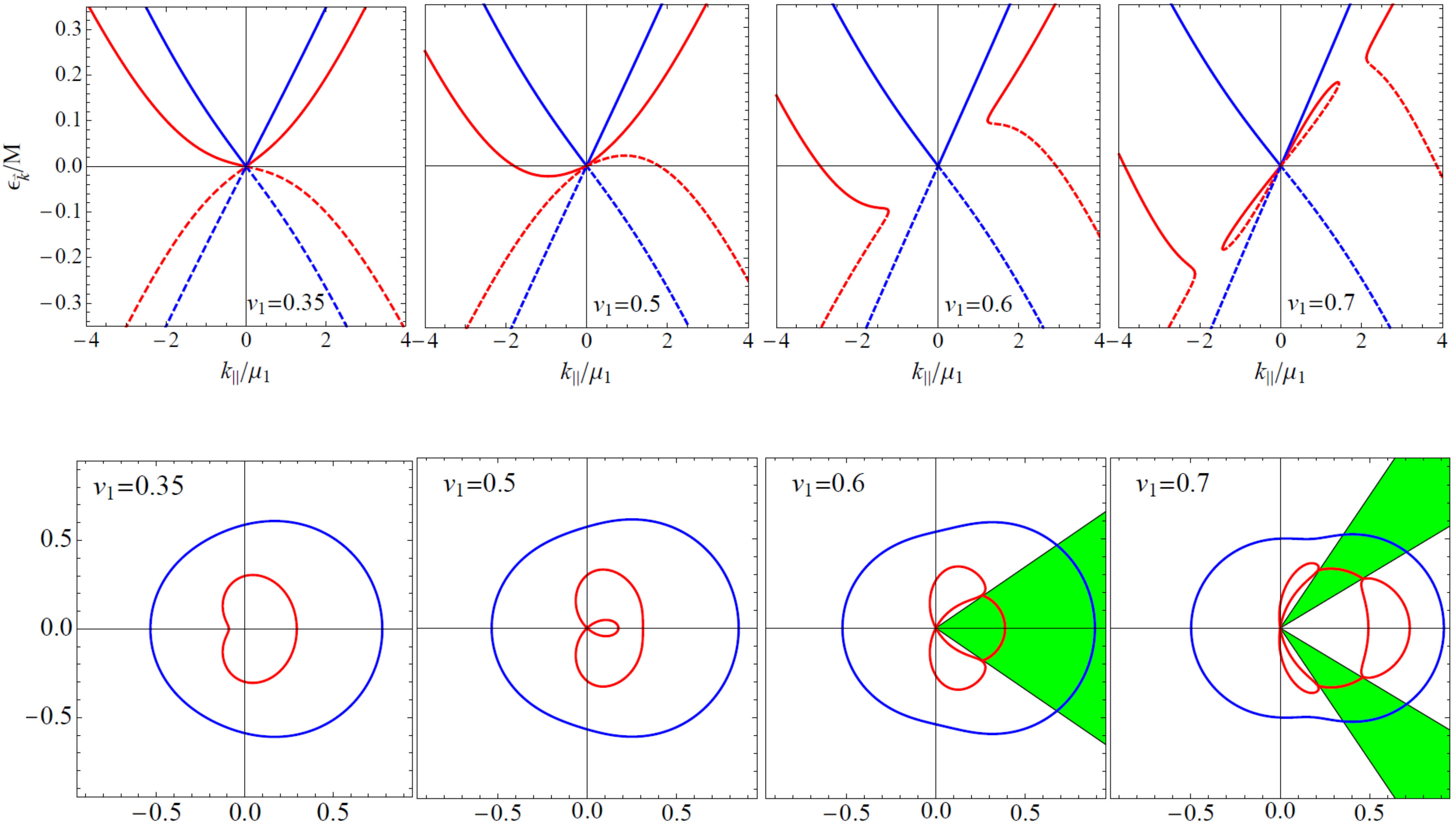
- quantum fields $\varphi_{1,2} \to \varphi_{1,2} + \phi_{1,2}$ $\quad\quad \phi_{1,2} = \rho_{1,2}\, e^{i\psi_{1,2}}$
- couplings: h $|\varphi_1|^2|\varphi_2|^2$, $\quad\quad$ g $\varphi_1\varphi_2^*\partial_\mu\varphi_1^*\partial^\mu\varphi_2 + c.c.$ (gradient coupling)

Relativistic two fluid formalism at T=0 (non dissipative)

- two conserved *charge* currents: $\partial_\mu\, j_1^\mu = 0\,,\ \ \partial_\mu\, j_2^\mu = 0$
- momenta: $\partial_\mu\psi_1\,,\ \ \partial_\mu\psi_2$

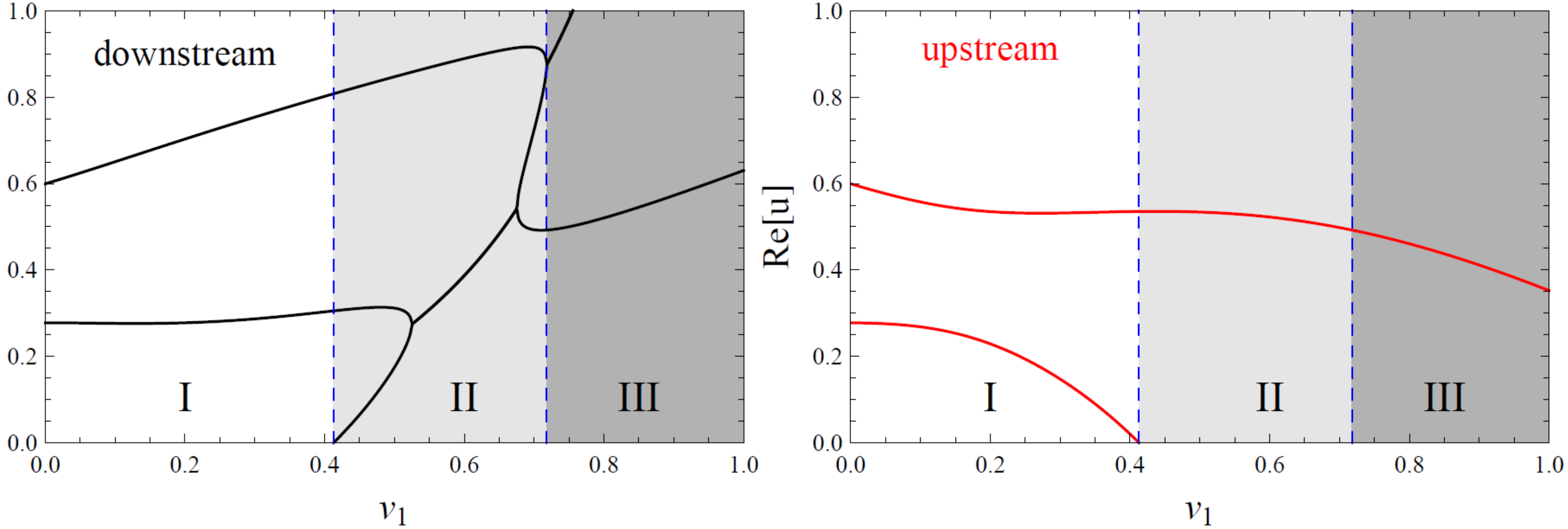
→ $\mu_1 = \partial_0\psi_1, \quad \mu_2 = \partial_0\psi_2, \quad \boldsymbol{v}_{s,1} = -\boldsymbol{\nabla}\,\psi_1/\mu_1\,, \quad \boldsymbol{v}_{s,2} = -\boldsymbol{\nabla}\,\psi_2/\mu_2$ etc.

Excitations in two coupled superfluids



Regions of stability of homogeneous SF

- **Energetic** instability (I)
- **Dynamical** instability (II)
- Single superfluid preferred (III)



[A. Haber, A. Schmitt, S. Stetina; Phys. Rev. D 93, 025011 (2016)]

Outlook

- excitations of *coupled superfluids at finite temperature* (3 component fluid)
  → study instabilities

- *impact of pairing*, start from Dirac Lagrangian

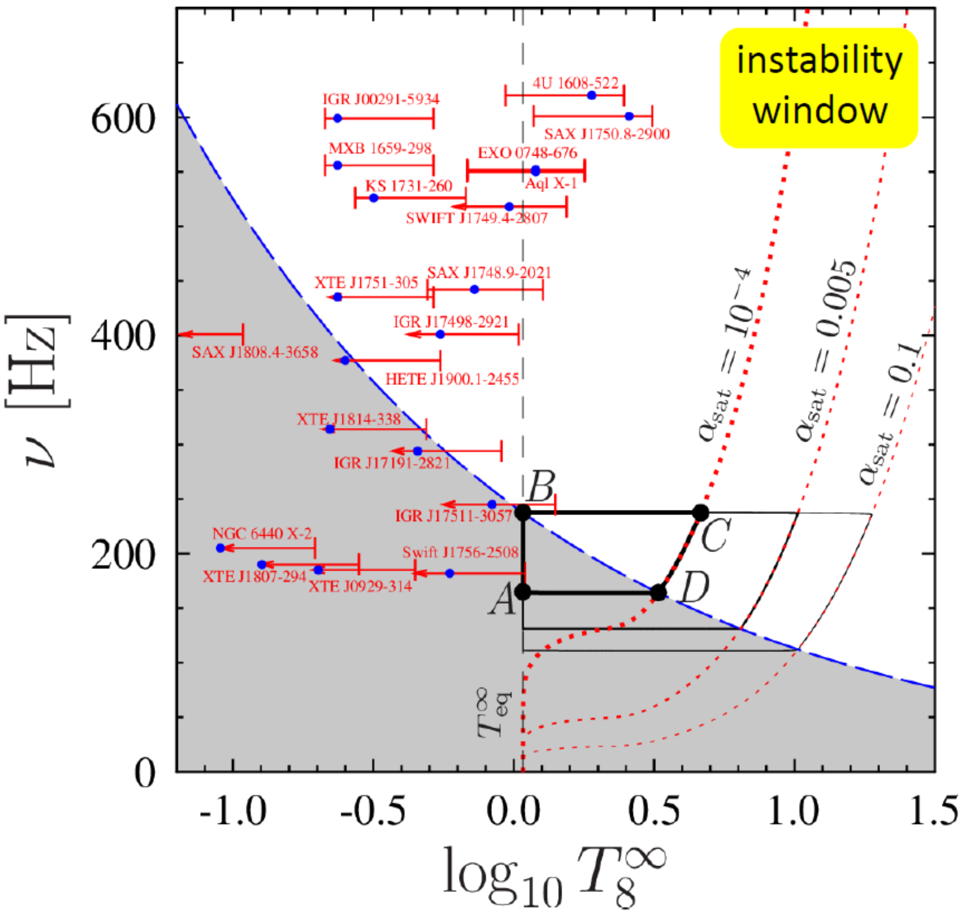
- consider *inhomogeneous condensates and vortices*
  → what happens to the energetic instability?

- add *dissipative terms*

- consider *explicit symmetry breaking*: what happens to superfluidity?

ありがとうございました (Thank you!)

Role reversal - comparison to r-modes



Conventional picture:

Amplitude of r-modes:

$$\partial_t \alpha = -\alpha\left(\tau_{grav}^{-1} + \tau_{diss}^{-1}\right)$$

τ_{grav} time scale of gravitational radiation

τ_{visc} time scale of viscous diss. (damping)

A→ B: - star spins up (accretion)

- T increase is balanced by ν cooling

B→ C: - unstable r-modes are excited

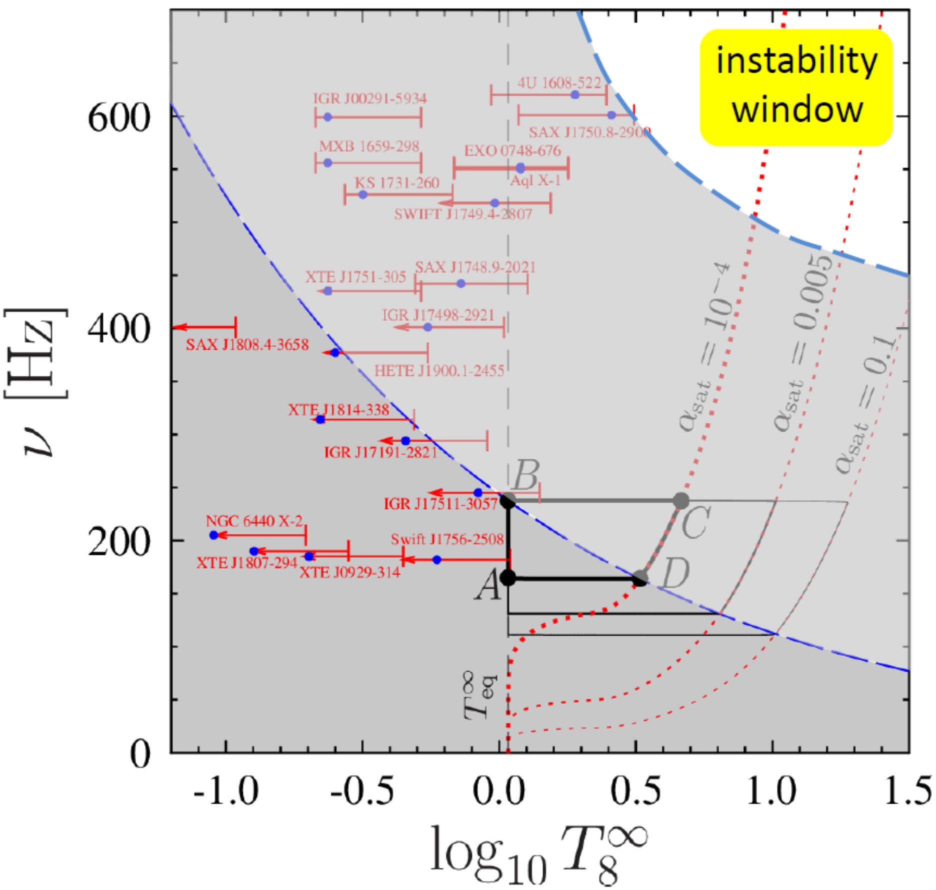
- r modes radiate gravitational waves (spin up stops)

- star heats up (viscous dissipation of r-modes)

[**images**: M. Gusakov, talk at “the structure and signals of neutron stars“ , 24. – 28.3. 2014, Florence, Italy]

Role reversal - comparison to r-modes

→ **why are fast spinning stars observed in nature?**



possible resolutions:

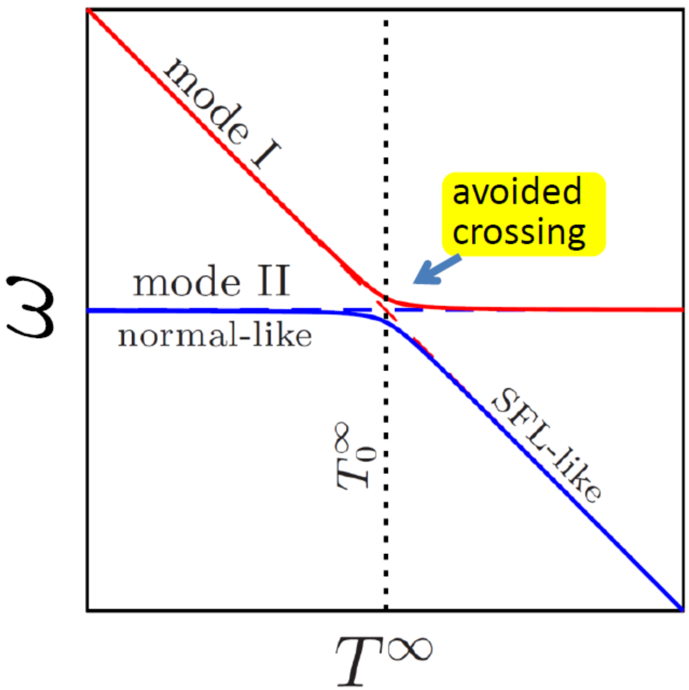
- Increase viscosity by a factor of 1000
  - all stars are in stable region (unrealistic for p, n, e^-, μ^-)
- Consider more exotic matter with high bulk viscosity (hyperons, quark matter)

→ **impact of superfluidity on r-modes?**

[M. Gusakov, A. Chugunov, E. Kantor Phys.Rev.Lett. 112 (2014) no.15, 151101]

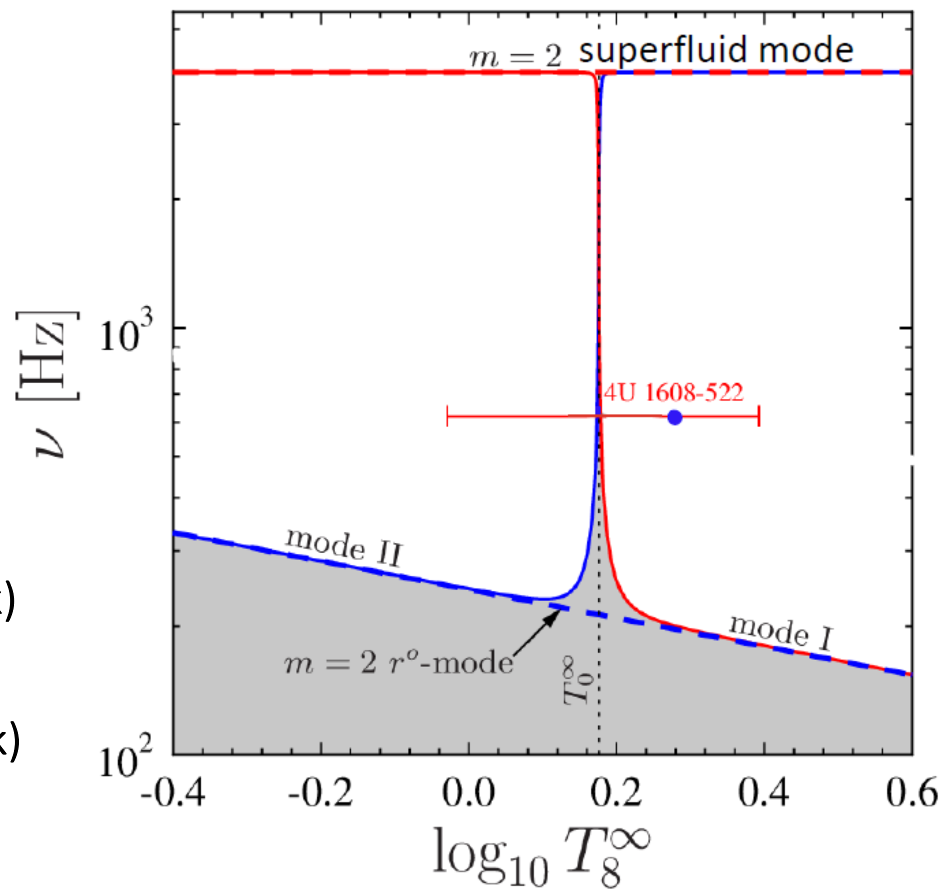
[**images**: M. Gusakov, talk at "the structure and signals of neutron stars" , 24. – 28.3. 2014, Florence, Italy]

Role reversal - comparison to r-modes



Excitation of normal fluid and superfluid modes

- **avoided crossing** if modes are coupled
- superfluid modes: **faster damping** $\tau_{diss}^{SFL} \ll \tau_{diss}^{normal}$



- **Close to avoided crossing:**

normal mode → SFL mode
(enhanced dissipation, left edge of stability peak)

SFL mode → normal mode
(reduced dissipation, right edge of stability peak)