FLUCTUATION EFFECTS IN THE INHOMOGENEOUS CHIRAL TRANSITION

T.T., T-G. Lee, R. Yoshiike (Kyoto U.)

Refs: S. Karasawa, T.-G. Lee and T.T., PTEP (2016).
T.T., T.-G. Lee and R. Yoshiike, in preparation

I. Introduction
II. Inhomogeneous chiral phase (iCP) within mean-field approximation
III. iCP in the magnetic field
IV. Fluctuation effects on the phase transition
V. Anomalous thermodynamic quantities
VI. Summary and concluding remarks

I. Introduction

Chiral transition in the QCD phase diagram

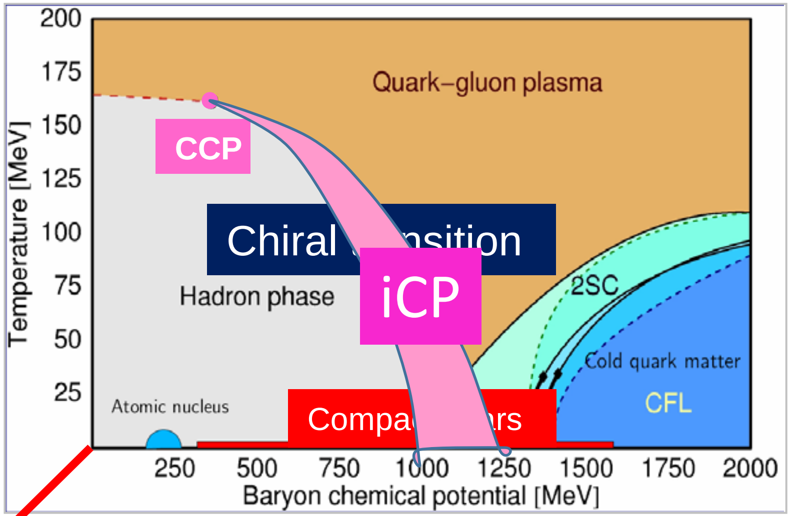
Generalized order parameter

$$M \equiv \langle \bar{q}q \rangle + i \langle \bar{q} i \gamma_5 \tau_3 q \rangle = \Delta(\mathbf{r}) \exp(i\theta(\mathbf{r}))$$

Inhomogeneous chiral phase (iCP)

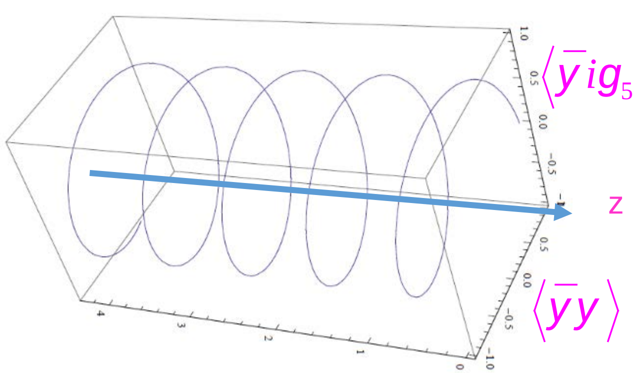
ex) Dual Chiral Density Wave (DCDW)

(T. T. and E. Nakano, hep-ph/0408294.
E. Nakano and T. T., PR**D 71** (2005) 114006.)



(B. Ruester)

H



$$\langle \bar{q}q \rangle = \Delta \cos(qz)$$

$$\langle \bar{q} i \gamma_5 \tau_3 q \rangle = \Delta \sin(qz)$$

II. Inhomogeneous chiral phase (iCP) within the mean-field approximation

Mean field approximation with NJL model (two flavor)

$$\mathcal{L}_{NJL} = \bar{\psi}(i\partial\!\!\!/ - m_c)\psi + G\left[(\bar{\psi}\psi)^2 + (\bar{\psi}i\gamma_5\boldsymbol{\tau}\psi)^2\right], \quad (m_c = 0)$$

$$\rightarrow \mathcal{L}_{MF} = \bar{\psi}\left(i\partial\!\!\!/ - \frac{1+\gamma_5\tau_3}{2}M(z) - \frac{1-\gamma_5\tau_3}{2}M^*(z)\right)\psi - \frac{M(z)^2}{4G},$$

Thermodynamic potential

$$\Omega(M(z);T,\mu) = -\frac{T}{V}\ln\int\mathcal{D}\bar{\psi}\mathcal{D}\psi\exp\left(\int d^4x\left(\mathcal{L}_{MF} + \mu\psi^\dagger\psi\right)\right)$$

$M(z)$: self-consistent mean-field solution

For 1D modulation for *M(z)*, general solutions have been known.

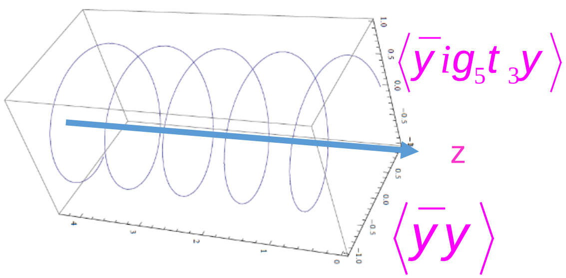
G.Basar and G.V.Dunne, PRL 100(2008) 2004004; PRD 78(2008) 065022.

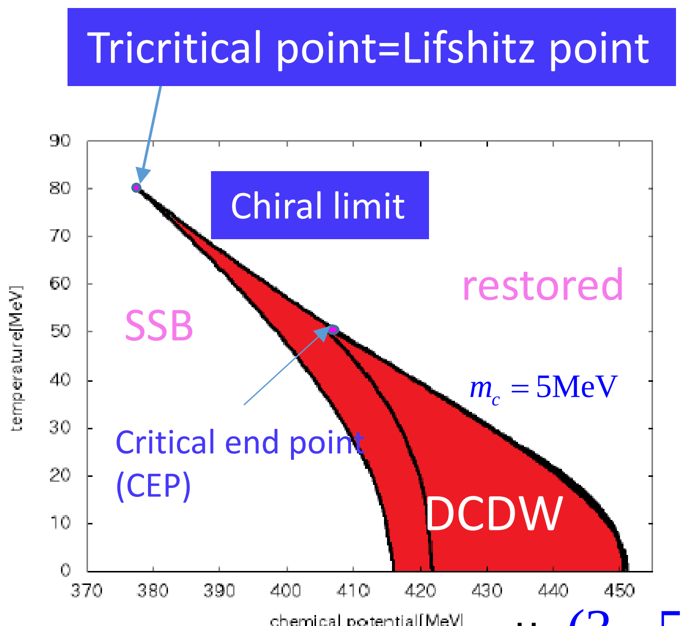
Twisted kink crystal condensate

Two popular configurations

$$\Delta(z) = \lambda e^{2iqz}$$

DCDW

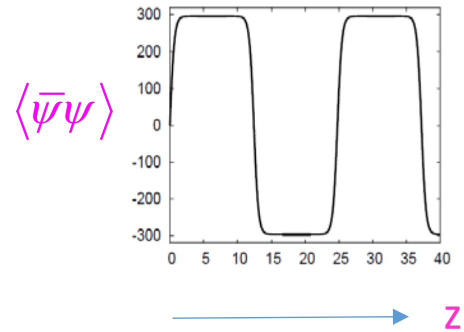


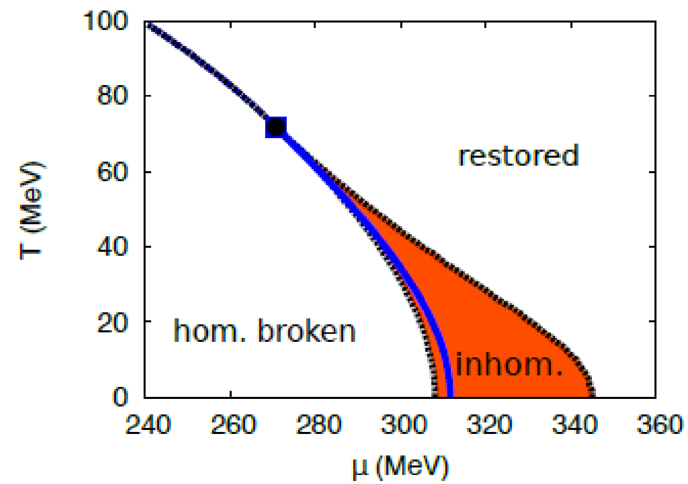


$\mu \sim (3-5)\rho_{\mathrm{nucl}}$

S. Karasawa and T.T., PRD92 (2015) 116004.

$$\Delta(z) = \lambda\left(\frac{2\sqrt{\nu}}{1+\sqrt{\nu}}\right)\mathrm{sn}\left(\frac{2\lambda z}{1+\sqrt{\nu}};\nu\right)$$



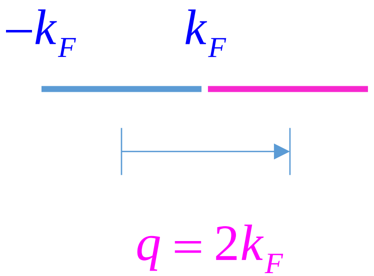


Real kink crystal (RKC)

(D.Nickel, PRL 103(2009) 072301; PR**D** 80(2009) 074025.)

(i) Mechanism should be owing to the nesting of the Fermi surface.

Cf. Charge density wave , spin density wave



A.W. Overhauser, PRL 4(1960) 462.
R.E. Peierls, *Quntum Theory of Solids* (1955)

FFLO state in superconductivity

P.Fulde and R.A. Ferrell, Phys. Rev. 135, A550 (1964)

A.I. Larkin and Yu N. Ovchinnikov, Sov. Phys. JETP 20, 762 (1965)

Cooper pairs

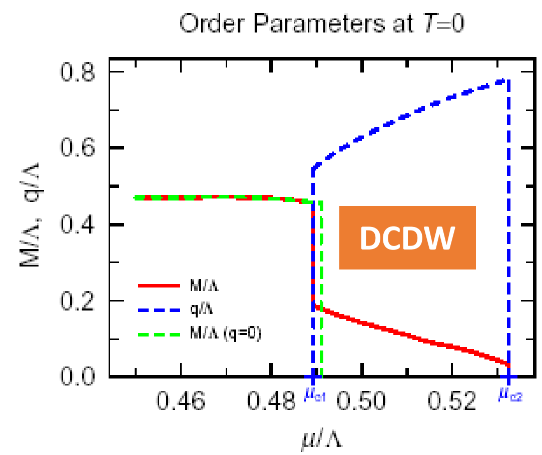
$$\langle \psi\psi \rangle = \Delta e^{i\mathbf{q}\cdot\mathbf{r}} \qquad \text{(FF)}$$

$$\langle \psi\psi \rangle = \Delta \cos(\mathbf{q}\cdot\mathbf{r}) \qquad \text{(LO)}$$

(ii) Model independent in 1+1 D due to the complete nesting.

Interaction strength comes in 1+2D or 1+3D case.

1+3D



ref. T.T. and E. Nakano, hep-ph/0408294
PRD71(2005)114006.

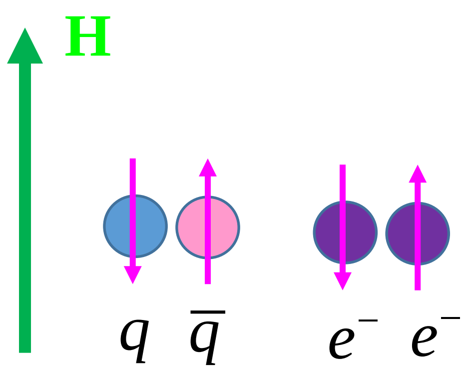
III. iCP in the magnetic field

Magnetic field $(H \neq 0)$

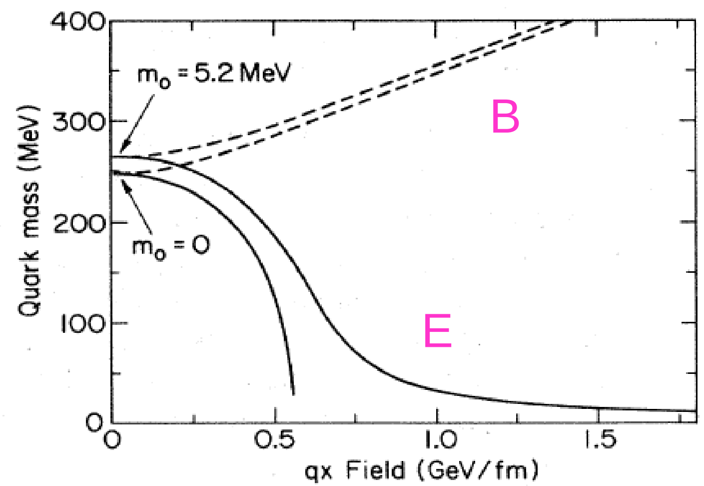
- High-energy H.I. Collisions $O(m_\pi^2 \sim 10^{17} G)$
- Compact stars $10^{12-15} G$
- Early Universe

Chiral transition

- Enhancement of SSB
or Magnetic catalysis



Cooper pair



S.P. Klevansky , Rev.Mod.Phys. 64 (1992) 649.

S. P. Klevansky and R.H. Lemmer, PRD 39 (1989) 3478.
H. Suganuma and T.T., Ann.Phys. 208 (1991) 371.
V.P. Gusynin, V.A. Milansky, I.A. Shovkovy, NPB 462 (1996) 249

- Dimensional reduction due to the Landau levels

Note: Chiral spiral (DCDW) is the most favorable phase in 1+1 dimensions.

(G.Baser et al., 2009)

Topological aspect of the iCP (phase degree of freedom)

ex) In the case of DCDW

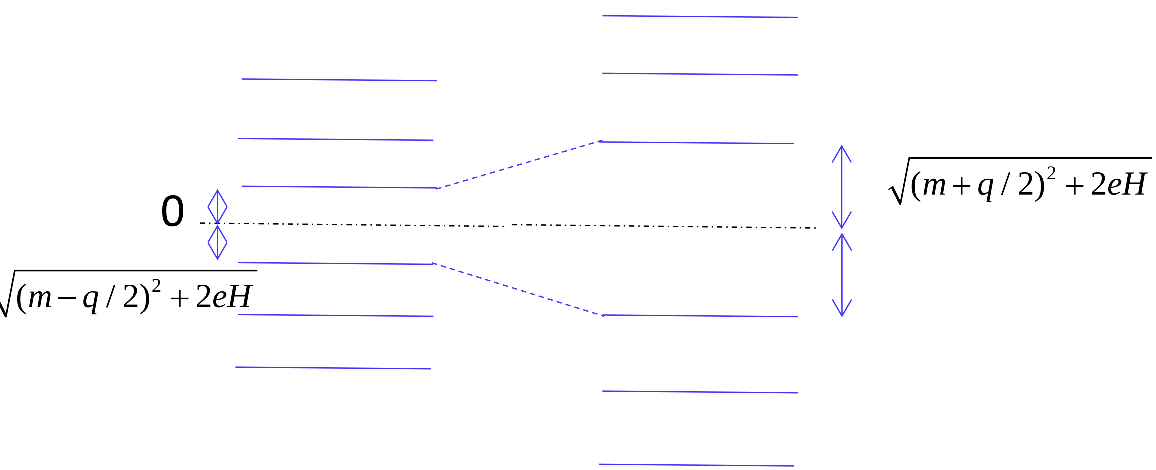
$$M(z) = -2G\Delta \exp(i\underline{qz})$$

$$\theta$$

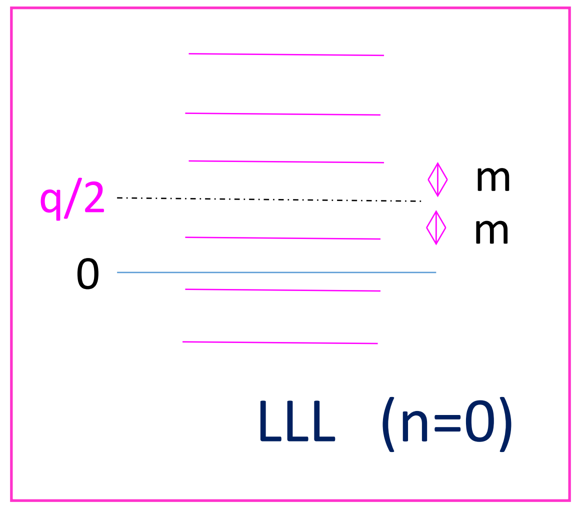
Energy spectrum:

$$E_{n,\zeta,\varepsilon}(p) = \varepsilon\sqrt{\left(\zeta\sqrt{m^2+p^2}+q/2\right)^2+2eHn}, \quad n=1,2,3,\ldots$$

$$E_{n=0,\varepsilon}(p) = \varepsilon\sqrt{m^2+p^2}+q/2, \quad \text{(Lowest Landau level(LLL))}$$



Spectral asymmetry



Cf: Landau levels:

$$E_{pe} = e\sqrt{m^2+|eH|(2n+1+s)+p^2}, \quad n=0,1,2,\ldots,$$

DCDW

$$E_{p,\zeta,\varepsilon} = \varepsilon\sqrt{p_\perp^2+\left(\zeta\sqrt{m^2+p_z^2}+q/2\right)^2}$$

Spectrum is symmetric

Both H and q are needed !

Spectral asymmetry leads to

T.T., K.Nishiyama and S. Karasawa, PLB 743(2015) 66.

Anomalous quark number $$N=-\frac{1}{2}\eta_H=\sum_{E_\lambda>0}1-\sum_{E_\lambda<0}1=\frac{eH}{2\pi}\frac{q}{2\pi}$$

Close relation to chiral anomaly in QCD

$$\Omega_{anom}=-\frac{e\mu}{4\pi^2}\int d^D x\mathbf{H}\nabla\theta$$

(D.T. Son and M.A. Stephanov, PRD 77(2008) 014021.)

Three consequences for the QCD phase diagram:

(i)Extension of the DCDW phase or novel Lifshitz point at $\mu=0$

(ii)Spontaneous magnetization

(iii)Stability of the 1D structure $\omega^2=\overline{a}k_z^2+\overline{b}(H)k_\perp^2$

(i)Extension of the DCDW phase and appearance of the new type of the condensate

Hybrid condensate

$$\Delta(z) = M(z)e^{iqz} = \underbrace{\frac{2m\sqrt{\nu}}{1+\sqrt{\nu}}\,\mathrm{sn}\left(\frac{2mz}{1+\sqrt{\nu}};\nu\right)}_{\text{RKC}}\underbrace{e^{iqz}}_{\text{DCDW}}$$

Note that this is one of the self-consistent solutions.

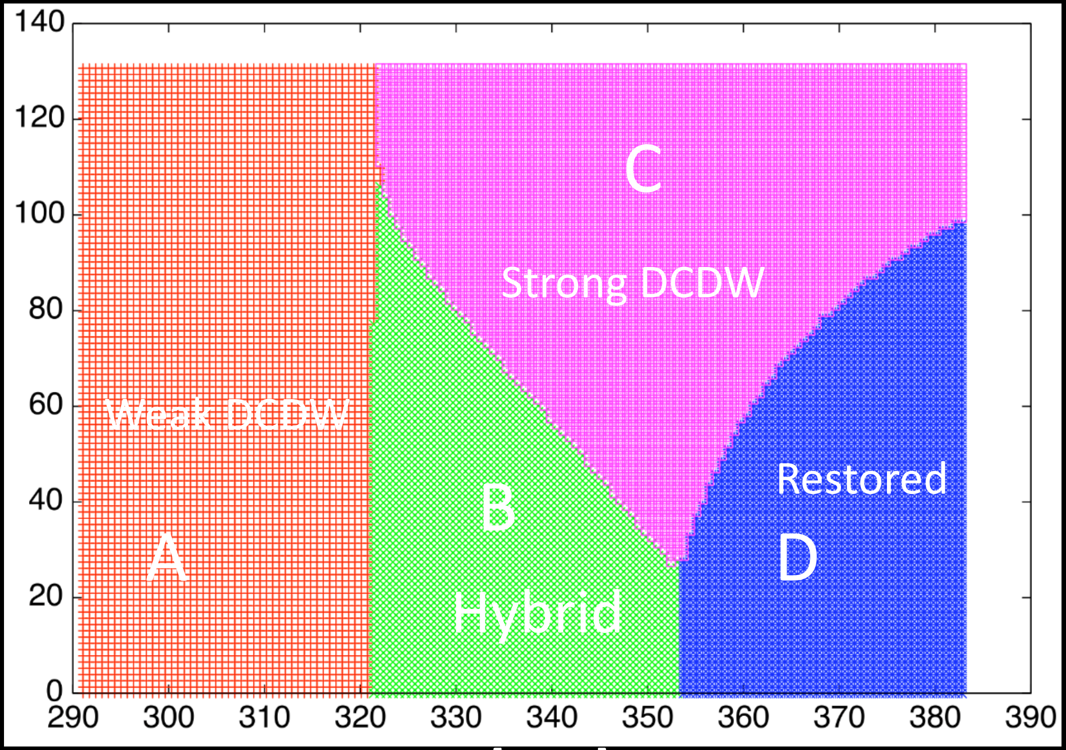
$$\Delta_{\mathrm{DCDW}}(z) \xleftarrow{\;\nu \to 1\;} \Delta(z) \xrightarrow{\;q \to 0\;} \Delta_{\mathrm{RKC}}(z)$$

This configuration is characterized by q,ν,m

Phase diagram

K.Nishiyama, S. Karasawa and T.T. ,PRD 92,036008 (2015)

- Phase Diagram at T=0



$\sqrt{eB}$[MeV]

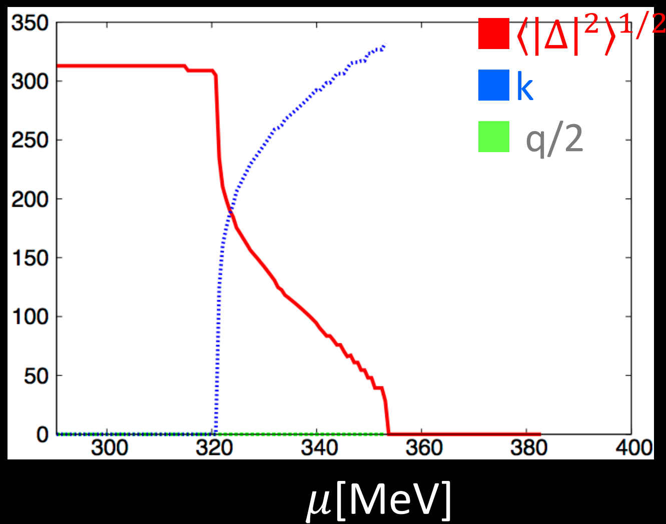
μ[MeV]

A: Weak DCDW phase
B: Hybrid
C: Strong DCDW phase
D Restored

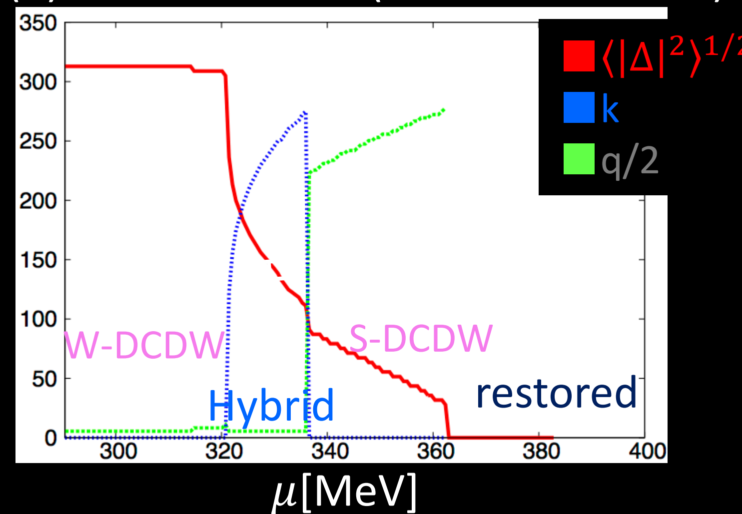
B=0, the order parameter is real. Homogeneous phase and RKC phase appear.
Weak B, the order parameter is complex but q is small
Strong B, DCDW is favored everywhere.

Order Parameter

(a) $B = 0$

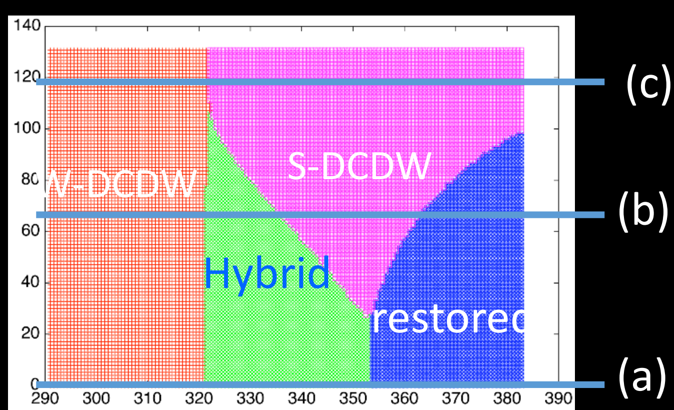


(b) $\sqrt{eB} = 70$MeV ($\sim 5 \times 10^{16}$ Gauss)

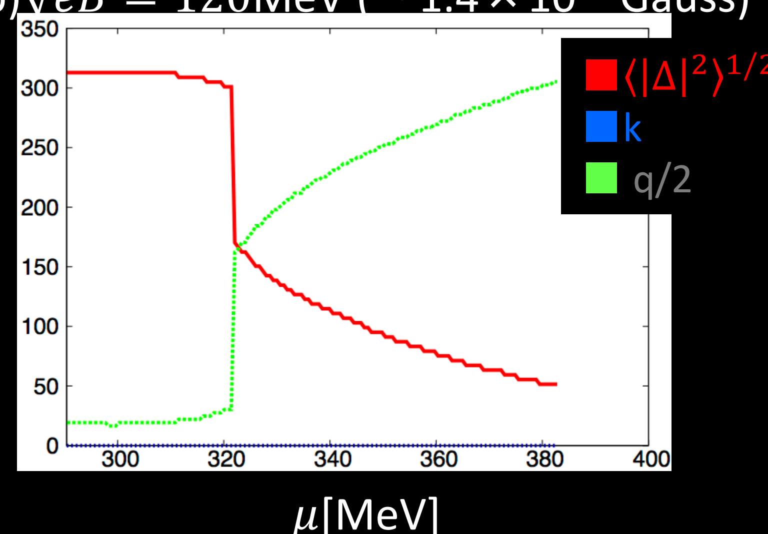


k is wavenumber of amplitude modulation

$$k = 2(1 + \sqrt{\nu})\mathrm{K}(\nu)/m$$



(b) $\sqrt{eB} = 120$MeV ($\sim 1.4 \times 10^{17}$ Gauss)



(ii)Spontaneous magnetization

Response to the weak B in the DCDW phase

Lagrangian in the external B

$$\mathcal{L} = \bar{\psi}\left(i\gamma^\mu D_\mu - m e^{i\gamma_5\tau^3 qz}\right)\psi - \frac{m^2}{4G} \qquad \mathbf{B}//\mathbf{q}//\hat{\mathbf{z}}$$

Thermodynamic potential

$\Omega(\mu, T, B; m, q)$ ※sufficiently weak B

Order parameters are considered as the free parameters.

Determination of the order parameters minimizing Ω : $m(\mu, T, B)\ \ q(\mu, T, B)$

Response to B of the minimized Ω

$$\Omega_{\min}(\mu, T, B) = \Omega^{(0)}_{\min}(\mu, T) + eB\,\Omega^{(1)}_{\min}(\mu, T) + (eB)^2\,\Omega^{(2)}_{\min}(\mu, T) + \cdots$$

Spontaneous magnetization

Magnetic susceptibility

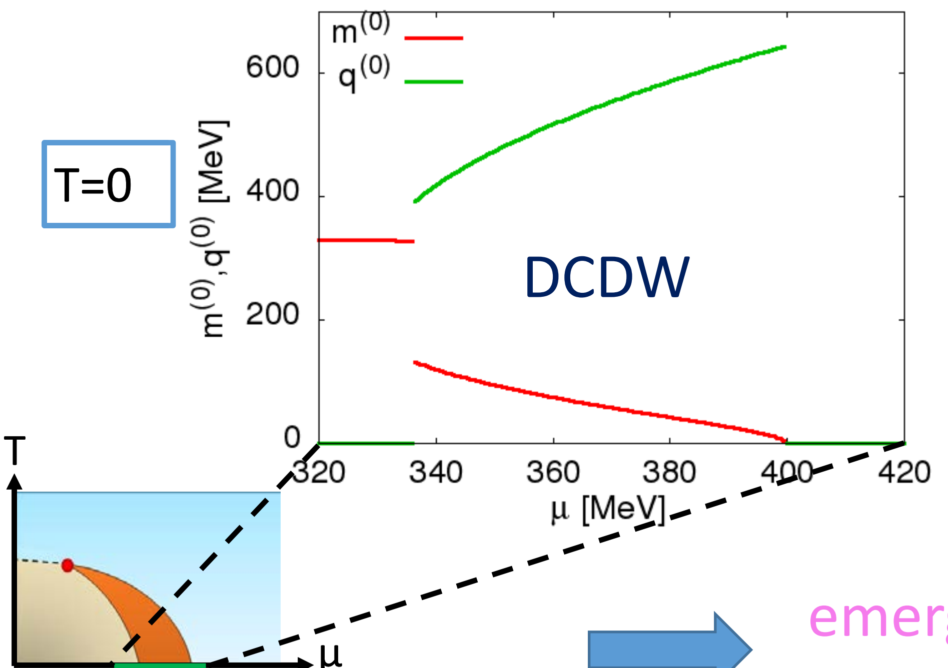
Spontaneous magnetization in the DCDW phase

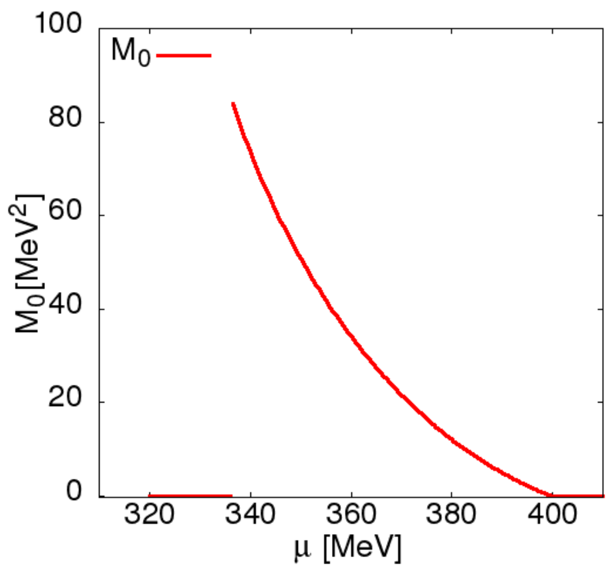
$$M_0 = -\left.\frac{\partial \Omega_{\min}}{\partial B}\right|_{B=0}$$

R. Yoshiike, K. Nishiyama and T. T., Phys. Lett. B751, 123 (2015)

which is given by spectral asymmetry in LLL.

T=0





$(3-5)\rho_{\text{nucl}}$

emergence of spontaneous magnetization

$$B_{\text{surf}} = \frac{8\pi}{3} M_0 \sim O(10^{16}) G$$

IV. Fluctuation effects on the phase transition to iCP

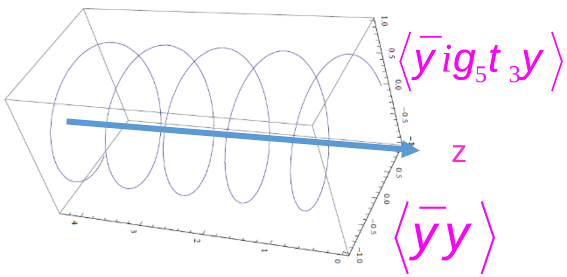
There are few works about fluctuation effects:

Stability of the one-dimensional structure
by the Nambu-Goldstone excitations in iCP

T.-G. Lee et al, PRD92,034024(2015)
Y. Hidaka et al., PRD92,(2015)

Ex) DCDW

Two kinds of spontaneous symmetry breaking (SSB)



- Translational and rotational symm.
- Chiral symmetry

- Anisotropic dispersion

$$\omega^2 = ak_z^2 + bk_\perp^4$$

Landau-Peierls instability

Quasi-Long-Range-Oder

- Correlation function

$$\langle \phi(z)\phi^*(0) \rangle \to \frac{1}{2}\Delta^2 \cos(qz)\left(\frac{z}{z_0}\right)^{-T/T_0}, z_0 = 2q/\Lambda^2$$

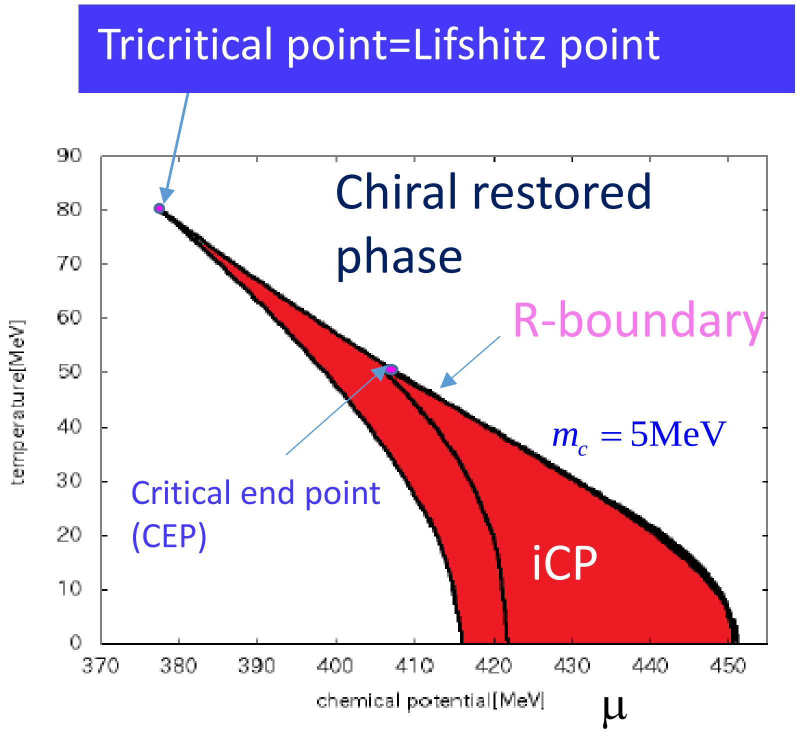
$$\langle \phi(\mathbf{r}_\perp)\phi^*(0) \rangle \to \frac{1}{2}\Delta^2 \left(\frac{r_\perp}{r_0}\right)^{-2T/T_0}, r_0 = \Lambda^{-1}$$

$(\phi : \bar{q}q \text{ or } \bar{q}i\gamma_5\tau_3 q)$

Note that system is stable in the presence of the magnetic field

$$\omega^2 = \bar{a}k_z^2 + \bar{b}(H)k_\perp^2$$

Here we discuss the fluctuation effects near the phase boundary.



S. Karasawa and T.T., PRD92 (2015) 116004.

Consider the right (R-) boundary, which is described by *the second order phase transition* within MFA.

➡ Brazovskii and Dyugaev effect

Partition function Z within the NJL model:

$$Z = \int Dq D\overline{q} e^{-S},$$

$$S = -\int_0^\beta d\tau \int d^3x \left[\overline{q} \left(-\gamma^0 \frac{\partial}{\partial \tau} + i\boldsymbol{\gamma} \cdot \nabla + \mu\gamma^0 \right) q + G \left[(\overline{q}q)^2 + (\overline{q} i \gamma_5 \tau q)^2 \right] \right]$$

Introduce the auxiliary (collective) fields: $\phi_a = (-2G\overline{q}q, -2G\overline{q}i\gamma_5\tau q)$

After integrating out the quark degrees of freedom we have an effective action *S*eff in terms of these collective fields:

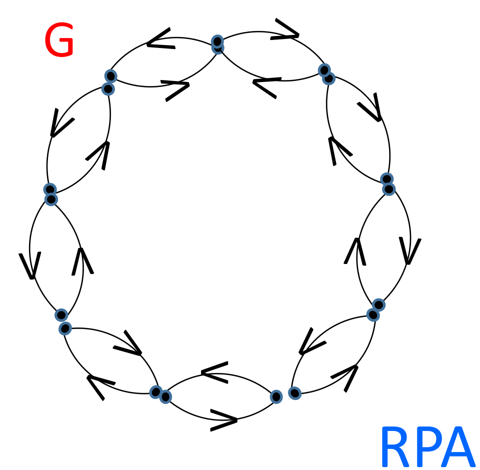
$$S_{\text{eff}} \approx \int d^4x \left[\frac{1}{2!}\Gamma^{(2)}\phi_a^2 + \frac{1}{4!}\Gamma^{(4)}(\phi_a^2)^2 + \ldots \right],$$

respecting $SU(2)_L \otimes SU(2)_R \simeq O(4)$ symmetry.

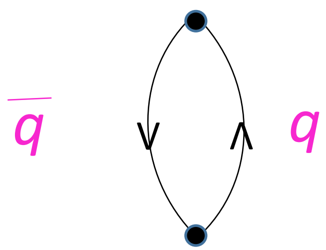
$$\phi_a = \langle \phi_{ps} \rangle + \xi_a \equiv \Phi(\Delta, q) + \xi_a$$

$$\Omega - \Omega_f = T \sum_q \sum_{\omega_n} \log\left(1 - 2G\overline{\Pi}^0_{ps}(q, \omega_n)\right)$$

$$+ \int d^3x \left[\frac{1}{2!}\overline{\Gamma}^{(2)} |\Phi(\mathbf{x})|^2 + \frac{1}{4!}\overline{\Gamma}^{(4)} |\Phi(\mathbf{x})|^4 + . \frac{1}{6!}\overline{\Gamma}^{(6)} |\Phi(\mathbf{x})|^6 .. \right]$$



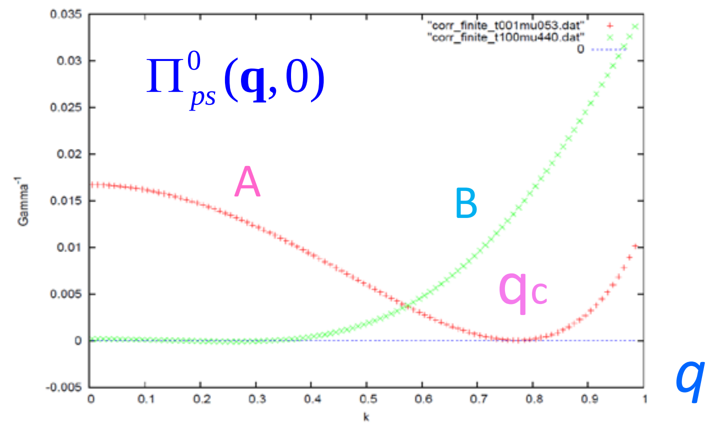
Chiral pair fluctuations in the chiral-restored phase

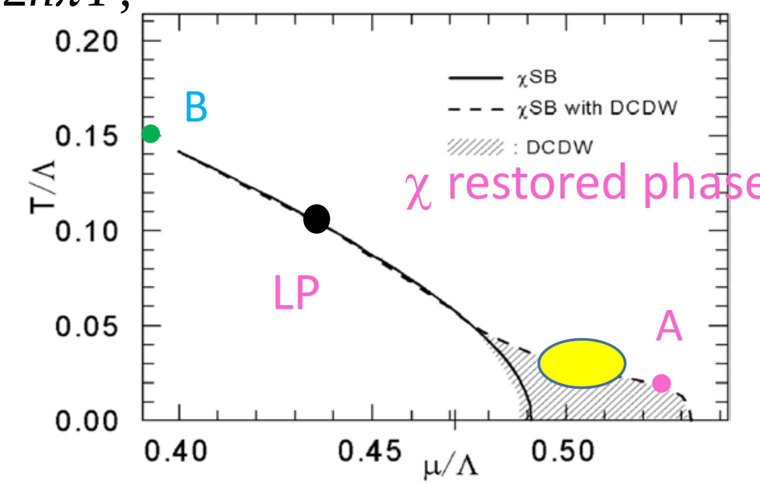


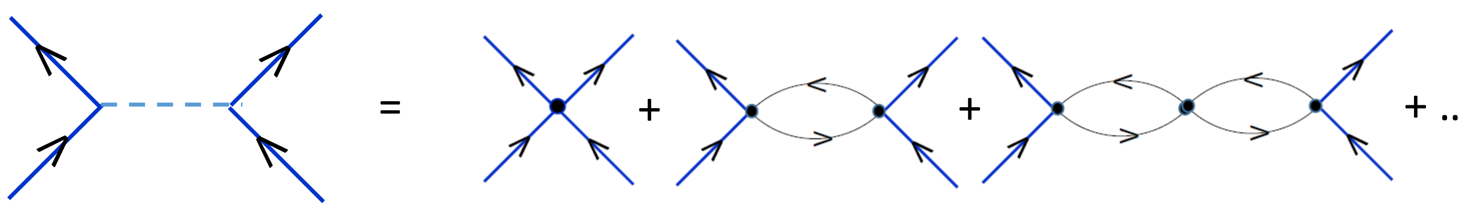
e.g. Pseudoscalar channel (Lindhard function)

$$\Pi^0_{ps}(\mathbf{q}, i\nu_n) = -T\sum_m \sum_{\mathbf{p}} \mathrm{tr}\left[i\gamma_5\tau_3 S_\beta(p+q) i\gamma_5\tau_3 S_\beta(p)\right]$$

$$p_0 = i(2m+1)\pi T + \mu, q_0 = i2n\pi T,$$



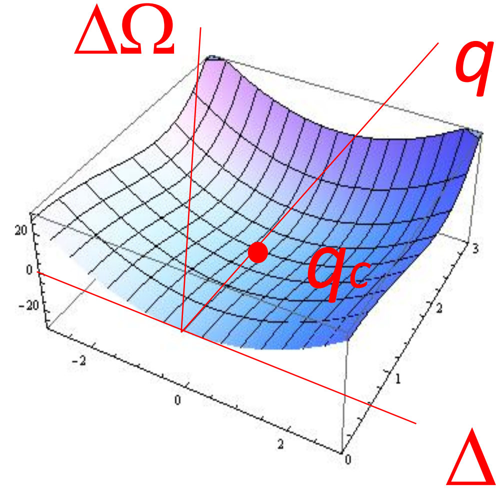




$$\phi \equiv -2G\bar{q}i\gamma_5\tau_3 q$$

$$G_{ps}(\mathbf{q},0) \sim \frac{1}{1-2G\Pi^0_{ps}(\mathbf{q},0)} = \frac{1}{\tau + \gamma(|\mathbf{q}|^2 - q_c^2)^2}$$ Near A

$\tau = 0, |\mathbf{q}| = q_c$:Thouless criterion n-th order vertex function



$$\overline{\Gamma}^{(n)}(\mathbf{q}_1,\mathbf{q}_2,\dots,\mathbf{q}_n)=(2\pi)^{3n}\left.\frac{\delta^n\Omega}{\delta\Phi(-\mathbf{q}_1)\delta\Phi(-\mathbf{q}_2)\dots\delta\Phi(-\mathbf{q}_n)}\right|_{\Phi=0}$$

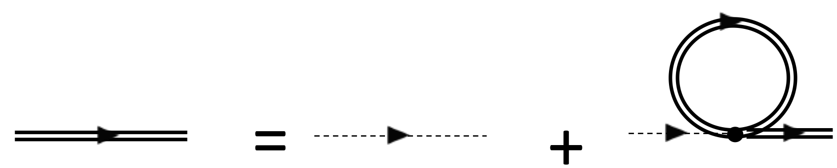
which includes the fluctuation effects given by ξ_a,

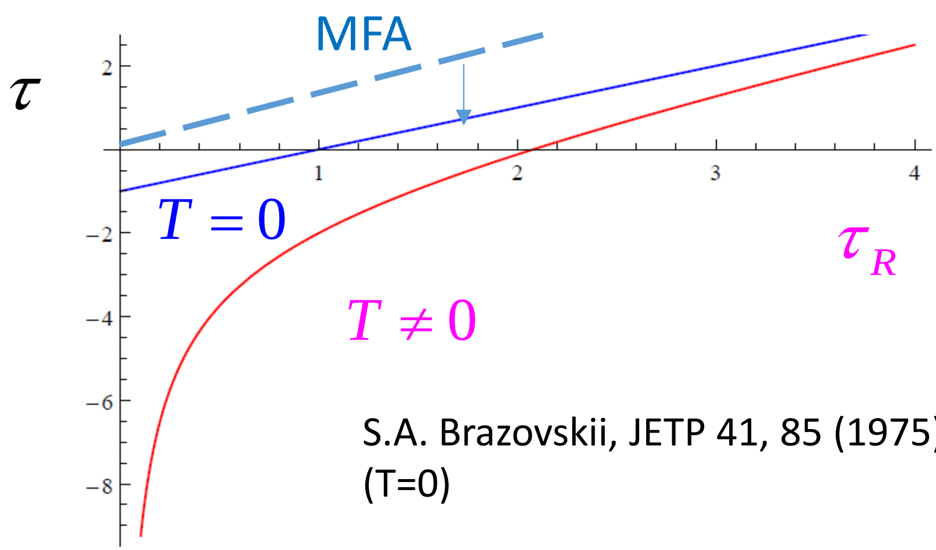
$\rightarrow \Gamma^{(n)}(\mathbf{q}_1,\mathbf{q}_2,\dots,\mathbf{q}_n)$ within MFA.

$$\Delta\Omega=\int d^3x\left[\frac{1}{2!}\overline{\Gamma}^{(2)}|\Phi|^2+\frac{1}{4!}\overline{\Gamma}^{(4)}|\Phi|^4+.\frac{1}{6!}\overline{\Gamma}^{(6)}|\Phi|^6..\right]$$

Second-order vertex function

$$\overline{\Gamma}^{(2)}(\mathbf{q}_1,\mathbf{q}_2)\propto\delta(\mathbf{q}_1+\mathbf{q}_2)\left(G^R_{ps}(i\omega_n,\mathbf{q}_1)\right)^{-1},$$





S.A. Brazovskii, JETP 41, 85 (1975)
(T=0)

Dyson eq. gives

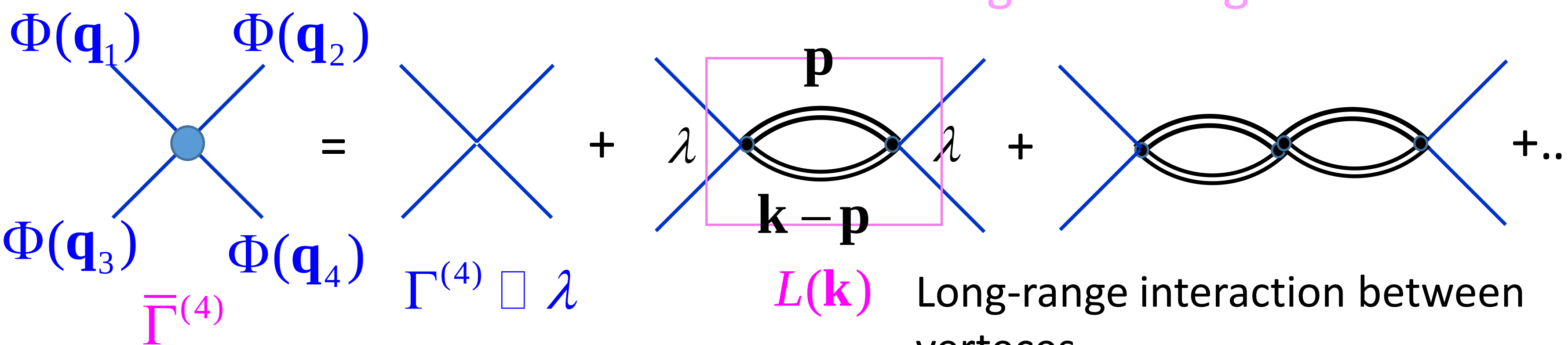
$$\tau=\tau_R-\frac{\lambda}{2}T\sum_{\omega_n}\int\frac{d^3p}{(2\pi)^3}G^R_{ps}(p,i\omega_n)$$

$$\square\ \tau_R-\frac{\lambda T q_c}{8\pi\gamma^{1/2}\tau_R^{1/2}},\quad T\neq 0$$

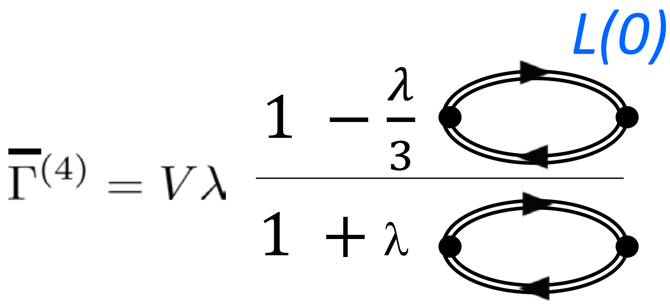
$$\square\ \tau_R-\frac{\lambda\Lambda^3}{96\alpha\pi^{5/2}\gamma^{3/2}q_c^3},\quad T=0$$

Fourth-order vertex function

Dangerous diagrams



Long-range interaction between verteces



S.A. Brazovskii, Sov.Phys.JETP 41,85 (1975) (T=0).
A.M. Dyugaev, JETP Lett. 22, 83(1975) (T=0)

$$L(0) = \frac{\lambda}{2} T \lim_{k\to 0} \sum_{\omega_n} \int \frac{d^3p}{(2\pi)^3} G^R_{ps}(\mathbf{p}, i\omega_n) G^R_{ps}(\mathbf{k}-\mathbf{p}, -i\omega_n)$$

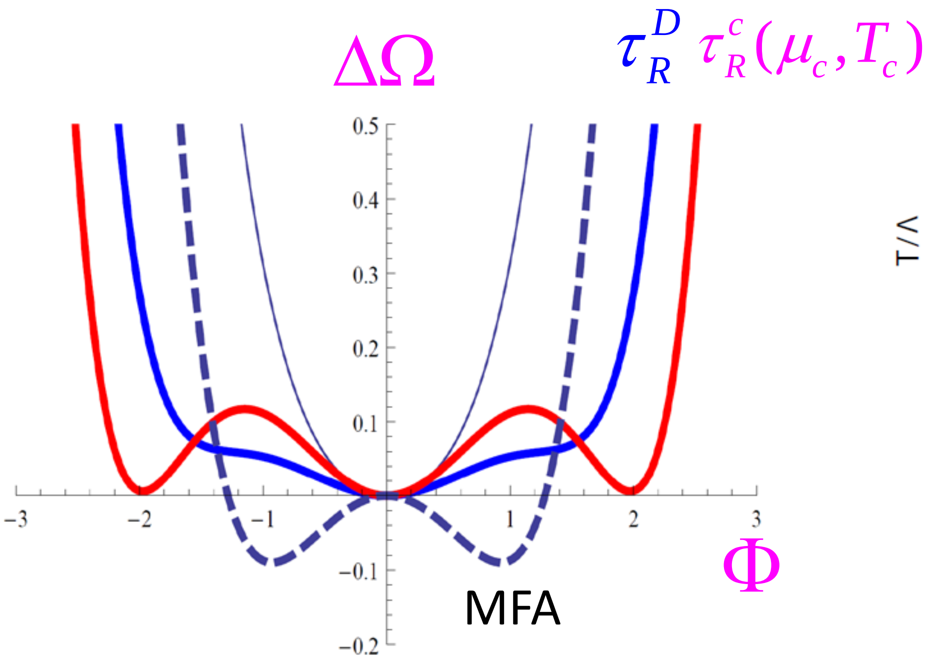
□ $\frac{\lambda T q_c}{16\pi\gamma^{1/2}} \frac{1}{\tau_R^{3/2}}, \quad T \neq 0$

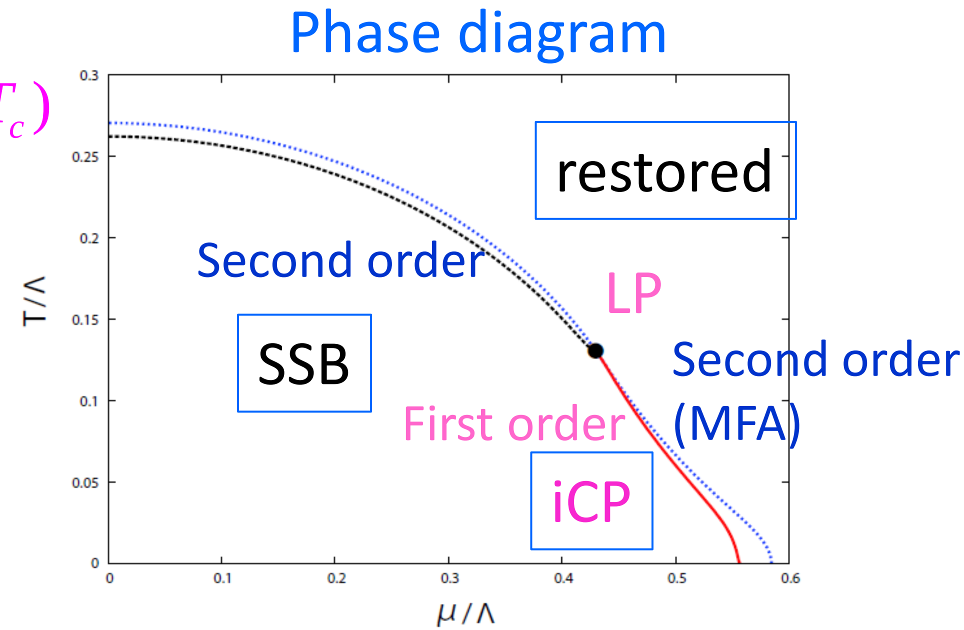
□ $\frac{\lambda q_c}{8a_1\pi^2\gamma^{1/2}} \frac{1}{\tau_R^{1/2}}, \quad T = 0$

$\bar{\Gamma}^{(4)}$ changes the sign as $\tau_R \to 0$, which signals the *first order phase transition*.

To summarize

$\overline{\Gamma}^{(6)} > 0$ (positive definite)





Fluctuation induced first order phase transition

Our results should be supported by
the renormalization group argument a la Shankar.

(P.C. Hohenberg and J.B. Swift, PRE 52, 1828 (1995))

V. Anomalies of the thermodynamic quantities

Usual second-order phase transitions

Susceptibilities (second derivatives of Ω)

Ex) Specific heat, number-susceptibility

$$C_v = -T\frac{\partial^2\Omega}{\partial T^2} \propto \left(T - T_c\right)^{-1/2}, \chi = -\frac{\partial^2\Omega}{\partial \mu^2}$$

How about the fluctuation-induced first-order phase transition?

Anomalies in the first derivatives

$$S = -\frac{\partial\Omega}{\partial T}, N = -\frac{\partial\Omega}{\partial \mu} \qquad \Omega - \Omega_f = T\sum_q\sum_{\omega_n}\log\left(1 - 2G\bar{\Pi}^0_{ps}(q,\omega_n)\right)$$

eg entropy density

$$s = s_f - \frac{q_c T}{2\pi\gamma^{1/2}}\tau_R^{-1/2} \longrightarrow$$

Implications for relativistic HI collisions?

s_f: entropy density for free quarks

VI Summary and concluding remarks

- Inhomogeneous chiral phase (iCP) may be realized in the QCD phase diagram due to the *nesting* effect of the Fermi surface.

- The phase of the condensate leads to a topological effect in the presence of the magnetic field
  ex)
  (i) Hybrid condensate
  (ii) Spontaneous magnetization

- Some astrophysical implications such as
  Cooling mechanism,
  Origin of the strong magnetic field in magnetars

T.T., T. Muto, PRD 89, 103005 (2014)

- We have studied the fluctuation effects on the inhomogeneous chiral transition.

  Fluctuation induced first-order phase transition

- Peculiar dispersion of the fluctuations is very important.

  Singular behavior on the sphere $|\mathbf{q}| = q_c$

- The effects of quantum and thermal fluctuations are figured out;

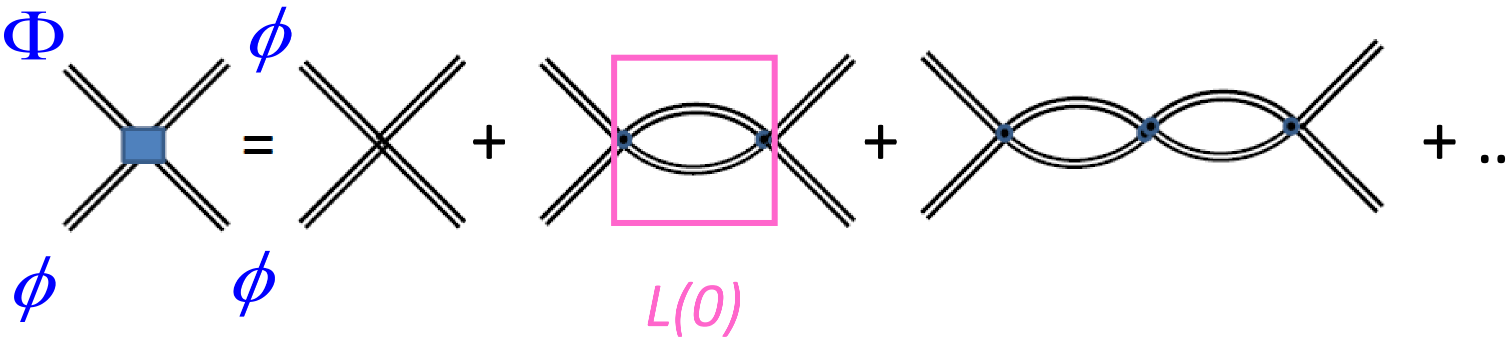
  Thermal fluctuations are more drastic than the quantum ones due to the dimensional reduction.

- Anomalous effect can be seen in the first derivatives of the thermodynamic potential, besides the discontinuous jump.

- Common features for inhomogeneous phase transitions.

  Application for the FFLO state should be interesting.

Dangerous diagrams=long range interaction between ϕ^2



$$L(0)=\frac{\lambda}{2}T\sum_{\nu_n}\int\frac{d^3p}{(2\pi)^3}G^R_{ps}(\mathbf{p},i\nu_n)G^R_{ps}(-\mathbf{p},-i\nu_n)$$

$$=\frac{\lambda T}{4\pi^2}\int_0^\infty ds\left[\frac{1}{2}\sqrt{\frac{\pi}{(4\gamma q_c^2)^3}}s^{-1/2}\tau_R^{-1/2}+\sqrt{\frac{\pi q_c^2}{4\gamma}}s^{1/2}\tau_R^{-3/2}\right]e^{-s}\coth\left(\frac{a_1\pi Ts}{\tau_R}\right)$$

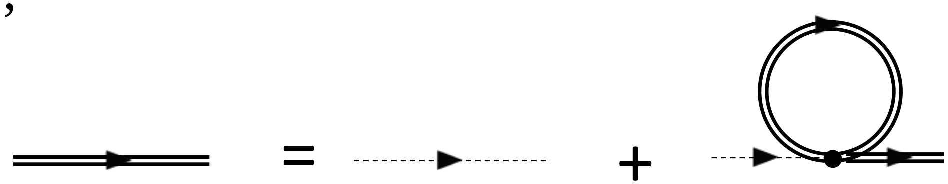
$$□\ \frac{\lambda T q_c}{16\pi\gamma^{1/2}}\frac{1}{\tau_R^{3/2}},\quad T\neq 0$$

$$□\ \frac{\lambda q_c}{8a_1\pi^2\gamma^{1/2}}\frac{1}{\tau_R^{1/2}},\quad T=0$$

S.A. Brazovskii, Sov.Phys.JETP 41,85 (1975)
(T=0).
A.M. Dyugaev, JETP Lett. 22, 83(1975)
(T=0)

$$G^R_{ps}(p, i\nu_n) = \frac{1}{\tau_R + \gamma(|\mathbf{p}|^2 - q_c^2)^2 + a_1|\nu_n|},$$

Dyson eq. gives

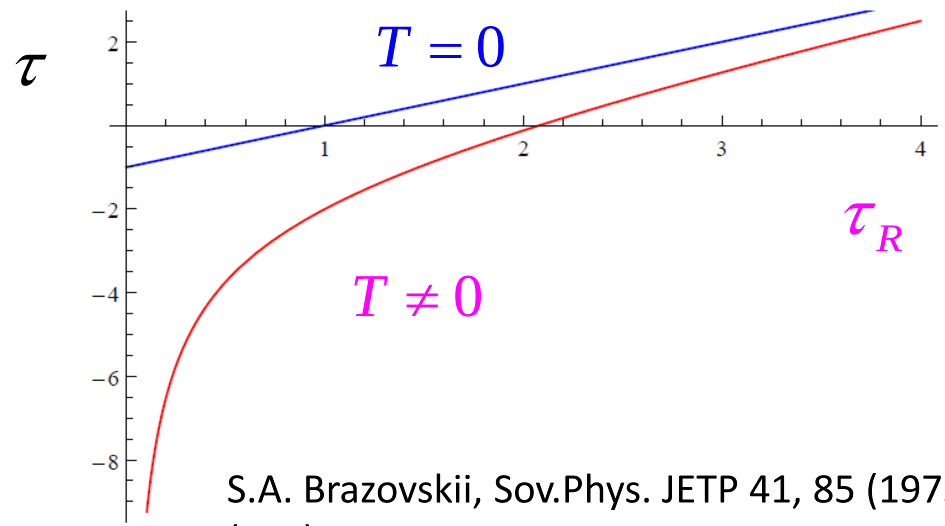


$$\tau = \tau_R - \frac{\lambda}{2} T \sum_{\nu_n} \int \frac{d^3p}{(2\pi)^3} G^R_{ps}(p, i\nu_n)$$

$$= \tau_R - \frac{\lambda T}{4\pi^2} \int_{\tau_R/\Lambda^2}^{\infty} ds \left[\frac{\tau_R^{1/2}}{2} \sqrt{\frac{\pi}{(4\gamma q_c^2 s)^3}} + \tau_R^{-1/2} \sqrt{\frac{\pi q_c^2}{4\gamma s}} \right] e^{-s} \coth\left(\frac{a_1 \pi T s}{\tau_R} \right)$$

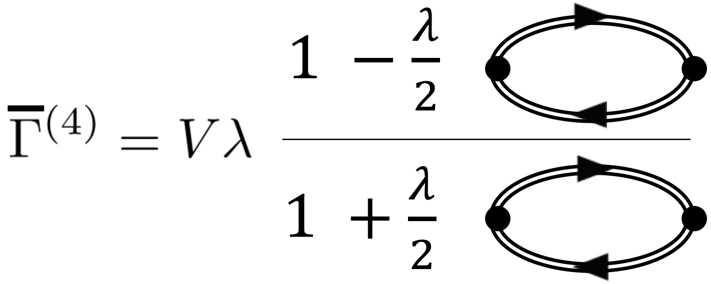
□ $\tau_R - \dfrac{\lambda T q_c}{8\pi\gamma^{1/2}\tau_R^{1/2}}, \quad T \neq 0$

□ $\tau_R - \dfrac{\lambda\Lambda^3}{96 a_1 \pi^{5/2} \gamma^{3/2} q_c^3}, \quad T = 0$

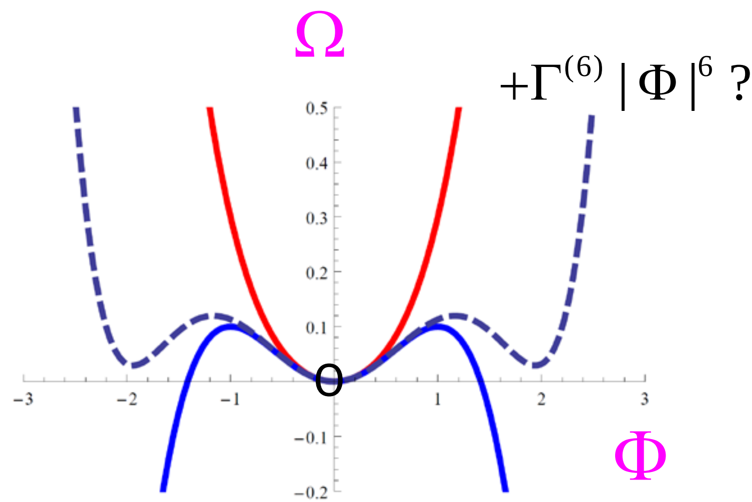


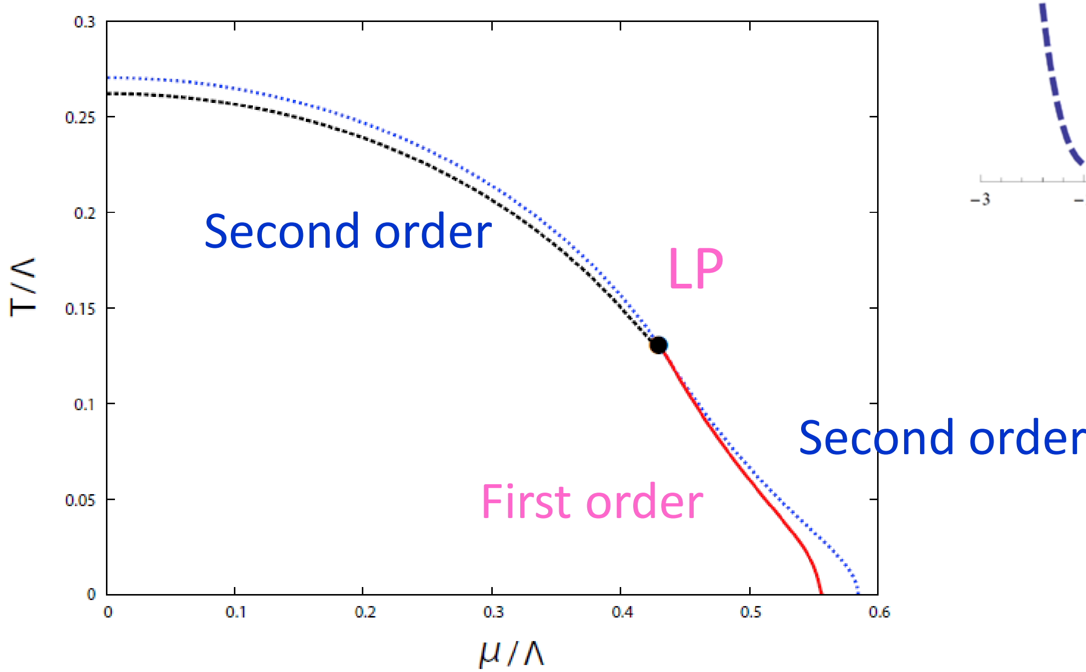
S.A. Brazovskii, Sov.Phys. JETP 41, 85 (1975)
(T=0)

We can prove similar behavior in the FFLO case.



Change of the sign of $\overline{\Gamma}^{(4)}$ as $\tau_R(\tau) \to 0$.





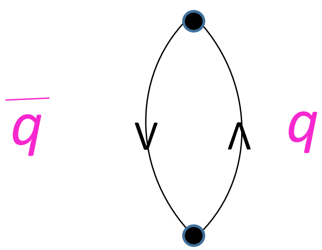
Fluctuation induced first order p.t.

II Brazovskii and Dyugaev effect

We discuss the properties of the phase transition to the inhomogeneous phases.

Pion condensation, liquid crystal,
FFLO state of superconductivity, diblock copolymer,...

Chiral pair fluctuations in the chiral-restored phase



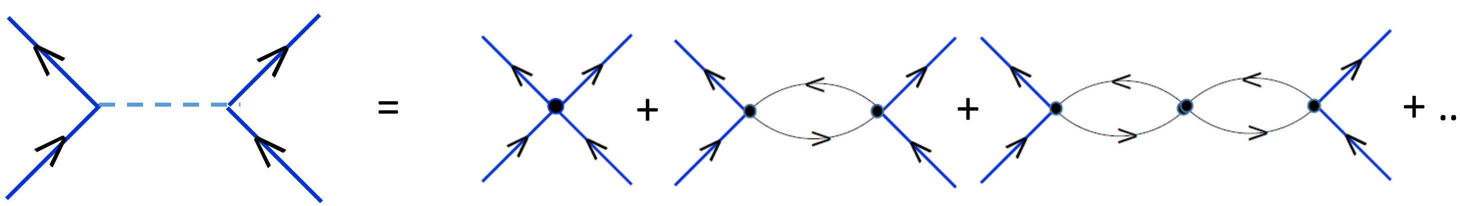
e.g. Pseudoscalar channel

(Lindhard function)

$$\Pi^0_{ps}(\mathbf{q}, i\nu_n) = -T\sum_m \sum_{\mathbf{p}} \mathrm{tr}\left[i\gamma_5\tau_3 S_\beta(p+q) i\gamma_5\tau_3 S_\beta(p) \right]$$

$$p_0 = i(2m+1)\pi T + \mu, q_0 = i2n\pi T,$$

$G_{ps}(q, i\omega_n)$



$\phi \equiv -2G\bar{q}i\gamma_5\tau_3 q$

Fluctuation effects:

Stability of the one-dimensional structure by the Nambu-Goldstone excitations ϕ

T.-G. Lee et al, PRD92,034024(2015)
Y. Hidaka et al., PRD92,(2015)

Anisotropic dispersion

$$\omega^2 = ak_z^2 + bk_\perp^4$$

Correlation function at large distance:

$$\left\langle \phi(z)\phi^*(0) \right\rangle \to \frac{1}{2}\Delta^2 \cos(qz)\left(\frac{z}{z_0}\right)^{-T/T_0}, z_0 = 2q/\Lambda^2$$

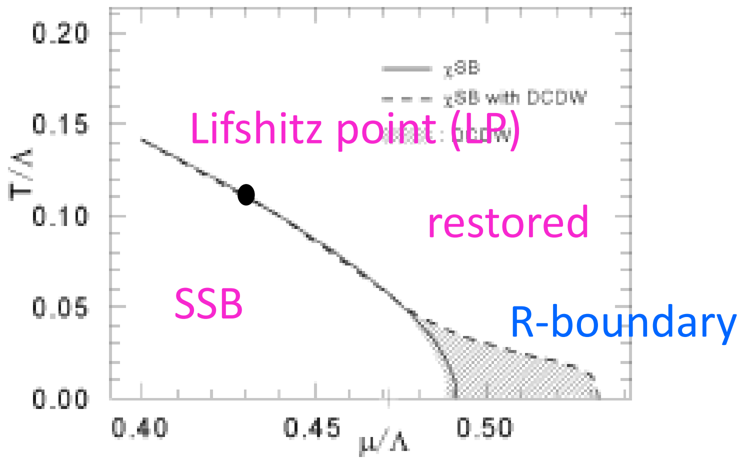
$$\left\langle \phi(\mathbf{r}_\perp)\phi^*(0) \right\rangle \to \frac{1}{2}\Delta^2 \left(\frac{r_\perp}{r_0}\right)^{-2T/T_0}, r_0 = \Lambda^{-1}$$

Quasi-long-range order (QLRO)

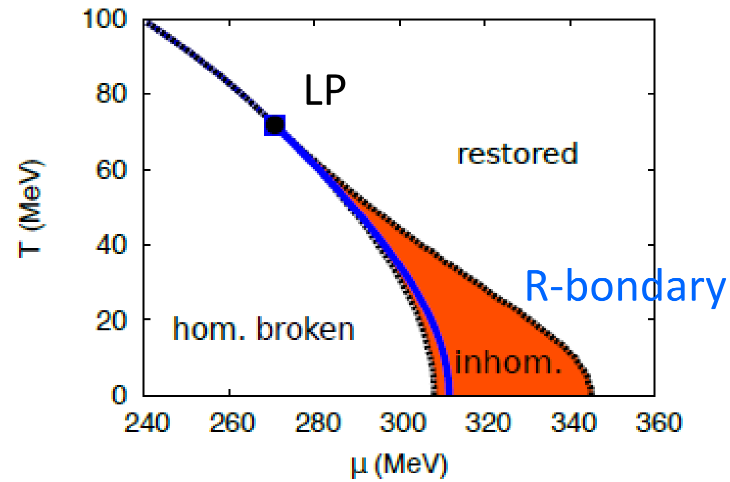
Note that LRO exists at T=0

Phase diagram of iCP's within the mean-field approximation (MFA)

DCDW



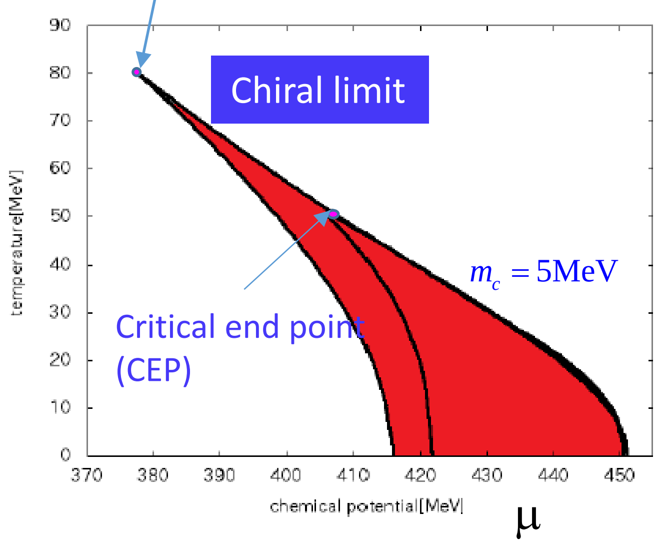
E.Nakano,T.T. Phys.Rev. D **71** 114116.



Real kink crystal (RKC) $\Delta(z), \theta = 0$

(D.Nickel, PRL 103(2009) 072301; PR**D** 80(2009) 074025.)

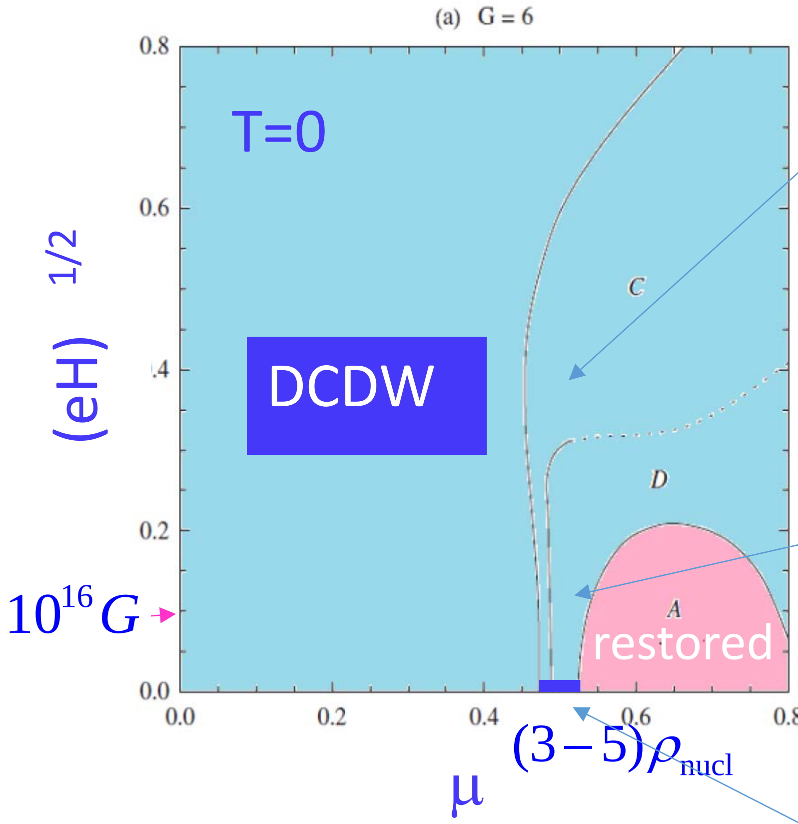
Tricritical point=Lifshitz point

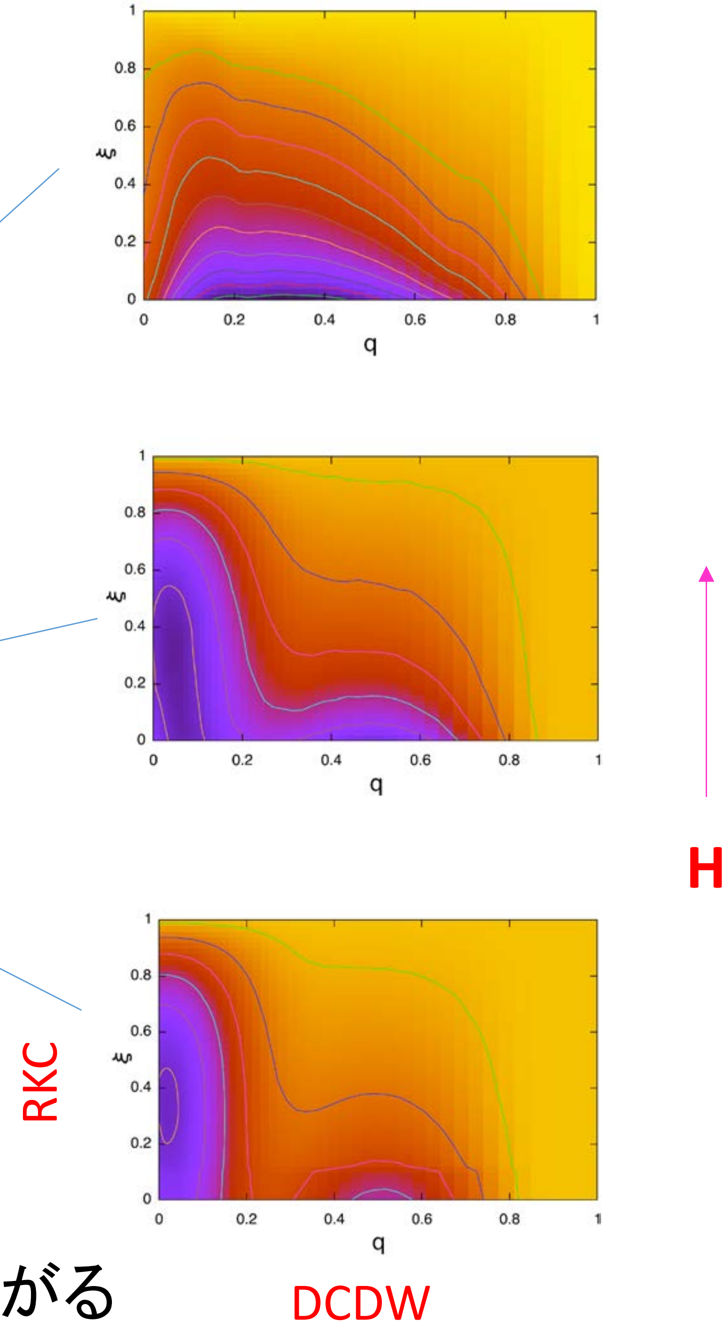


S. Karasawa and T.T., arXiv:1307.6448.

相図

DCDWとRKCの競合 (K. Nishiyama et al. 2014)



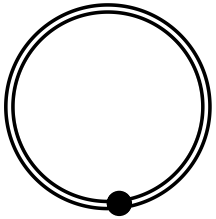


[Frolov et al,
(2010)]

磁場が存在するとDCDW相が大きく広がる

$$G_{ps}^{R}(\mathbf{p}, i\omega_n) = \frac{1}{\tau_R + \gamma(|\mathbf{p}|^2 - q_c^2)^2 + \alpha|\omega_n|}$$

$(p, i\omega_n)$



We can see how the difference is generated for quantum fluctuations and thermal fluctuations.

- All the Matsubara frequencies contribute to the integral at T=0

$$T\sum_{\omega_n}\int\frac{d^3p}{(2\pi)^3} \to \int\frac{d^4p}{(2\pi)^4} \qquad (\mathrm{Im}\,\overline{\Pi}_{ps}^{0} \neq 0 \text{ is important})$$

- On the other hand, n=0 gives a leading contribution at finite temperature

$$T\sum_{\omega_n}\int\frac{d^3p}{(2\pi)^3} \to \int\frac{d^3p}{(2\pi)^3}$$

(dimensional reduction)

Momentum integral becomes more singular

Fluctuation effect becomes more drastic.

(cf Coleman-Mermin-Wagner's theorem)

Magnetic susceptibility

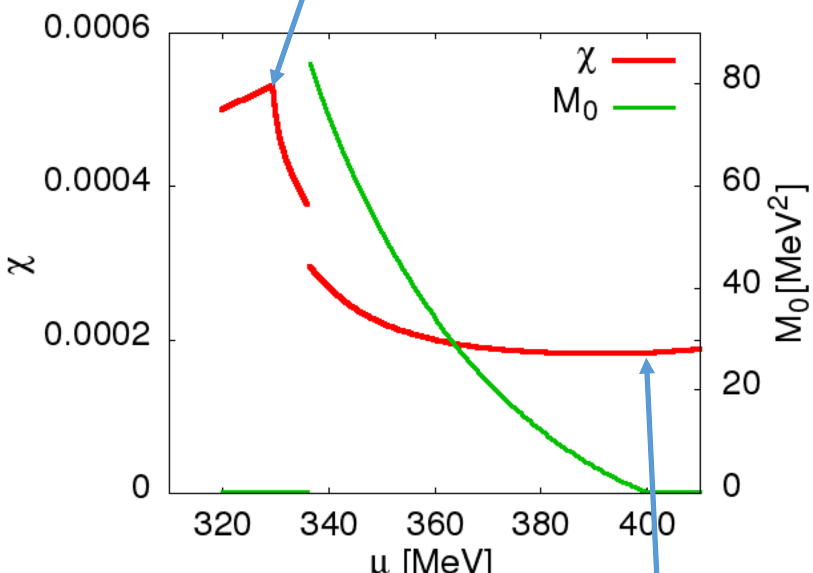
$$\chi \equiv \left.\frac{\partial M}{\partial B}\right|_{B=0} = -\left.\frac{\partial^2 \Omega_{\min}}{\partial B^2}\right|_{B=0}$$

※Normalization

$$\chi(\mu = 0, T = 0) = 0$$

T=0

Phase transition to the finite density

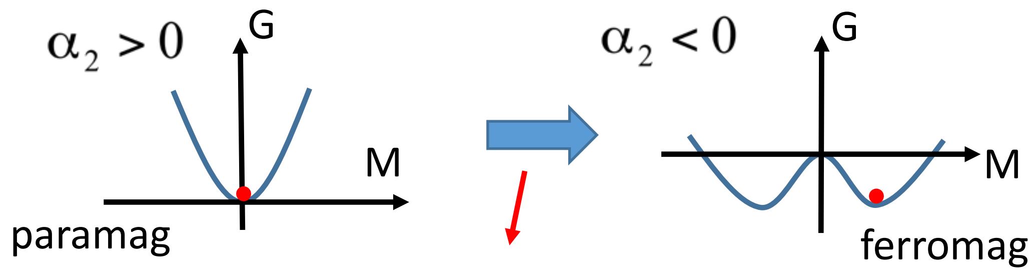


continuous at transition point

cf. Ising model

$$G = \frac{1}{2}\alpha_2 M^2 + \frac{1}{4}\alpha_4 M^4$$

$\alpha_2 > 0$ $\alpha_2 < 0$

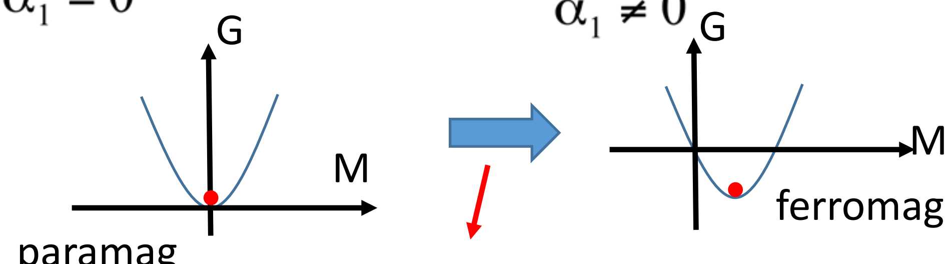


The curvature becomes 0. → χ diverges.

DCDW

$$G = \Omega_{\min} + BM\big|_{B=B(M)} = \alpha_1 M + \frac{1}{2}\alpha_2 M^2$$

$\alpha_1 = 0$ $\alpha_1 \neq 0$



χ does not diverge.