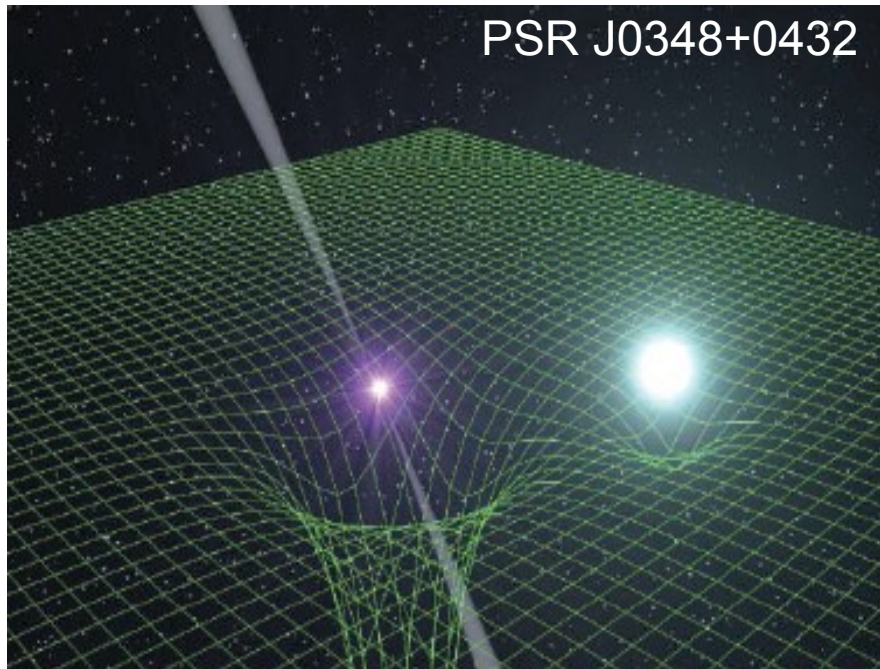


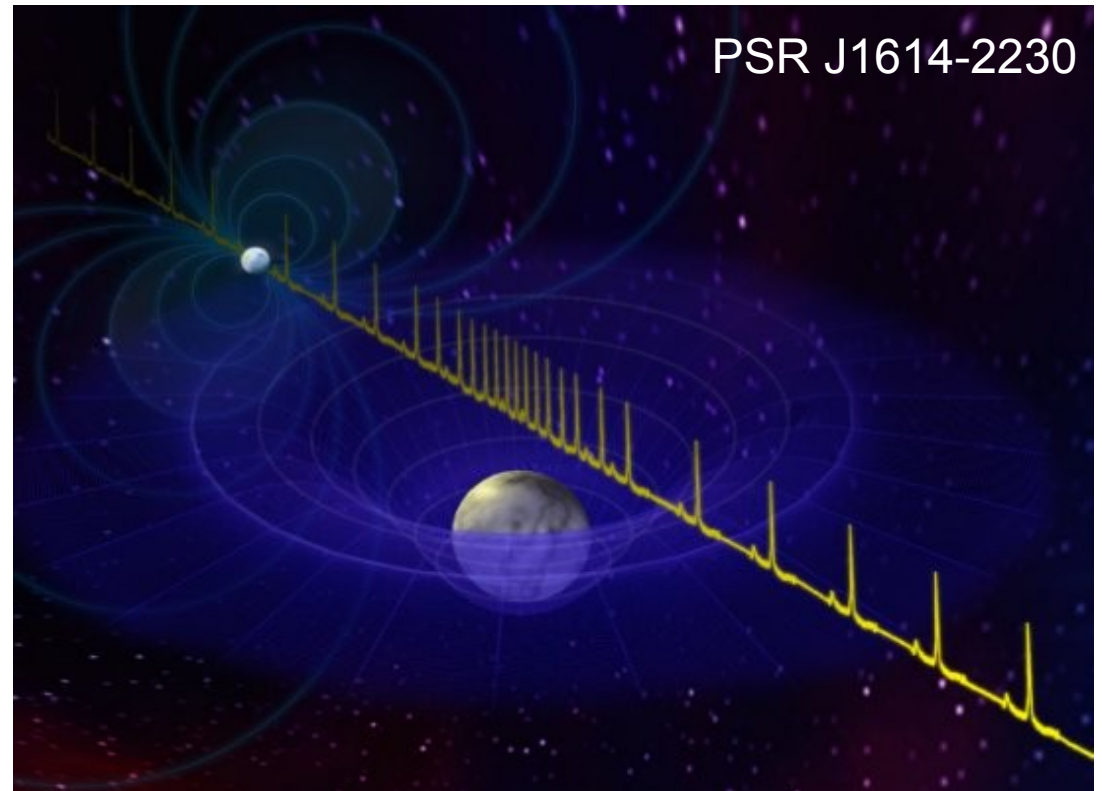
# High-mass twins as evidence for quark matter in compact stars

David Blaschke (University Wroclaw, JINR Dubna & MEPhI Moscow)



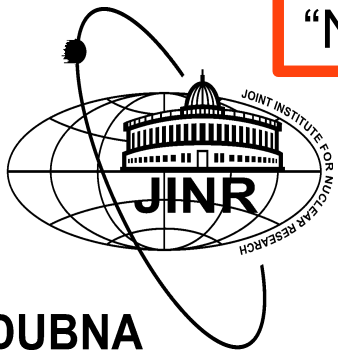
PSR J0348+0432

Antoniadis et al., Science 340 (2013) 448  
Demorest et al., Nature 467 (2010) 1081



PSR J1614-2230

“Nuclear physics, compact stars and mergers”, YITP Kyoto, 07.11.2016



DUBNA



# “The many faces of neutron stars” ...

phase transition = transition from a vase to a face

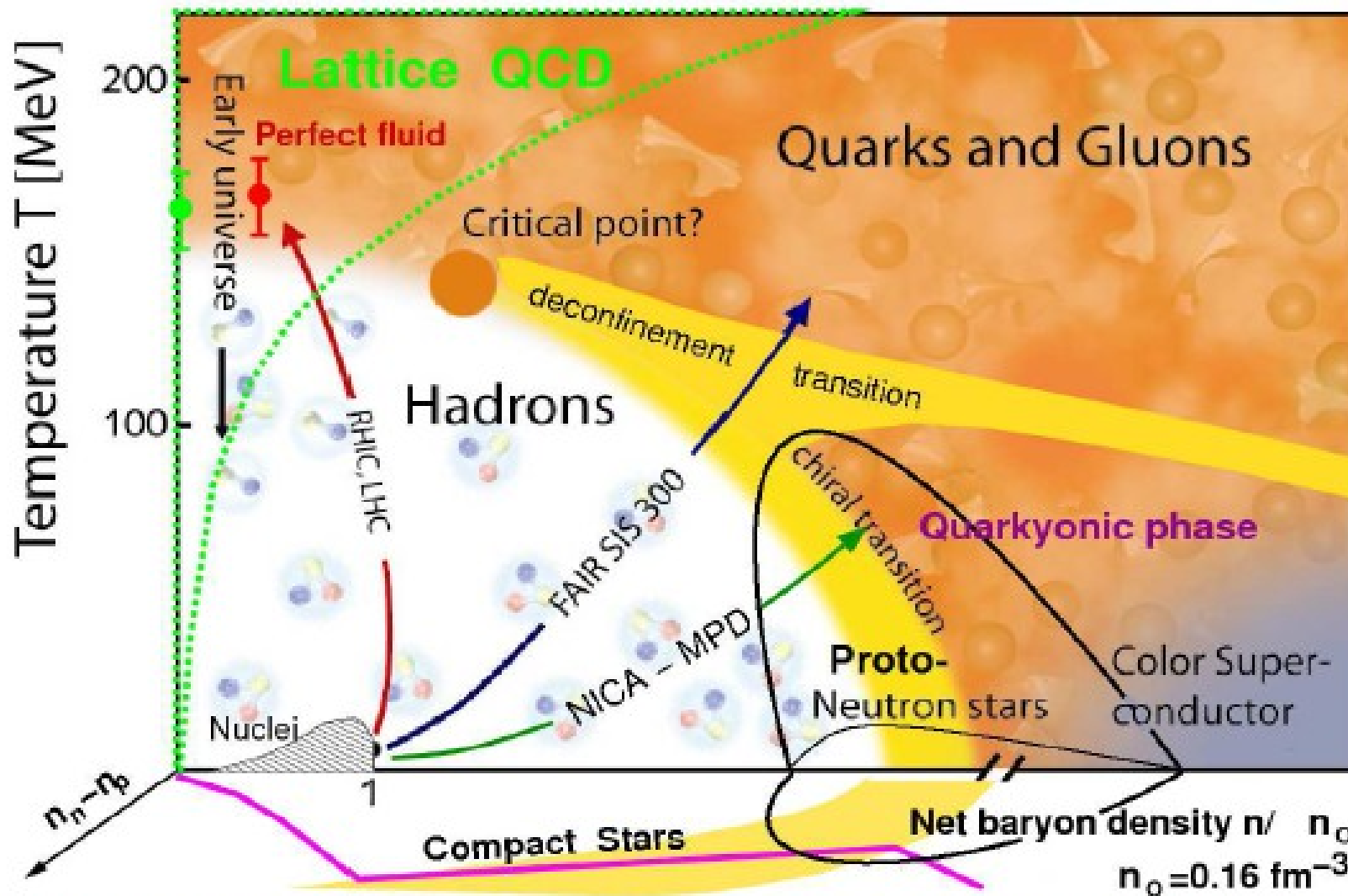


“holy grail”



face diagram  $\longrightarrow$

# Support a CEP in QCD phase diagram with Astrophysics?



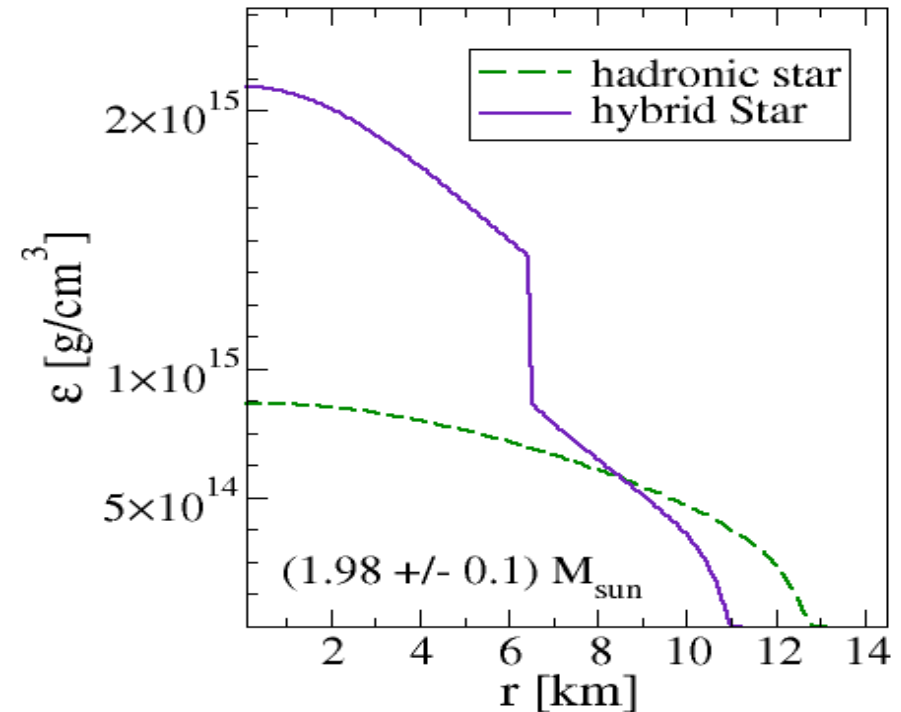
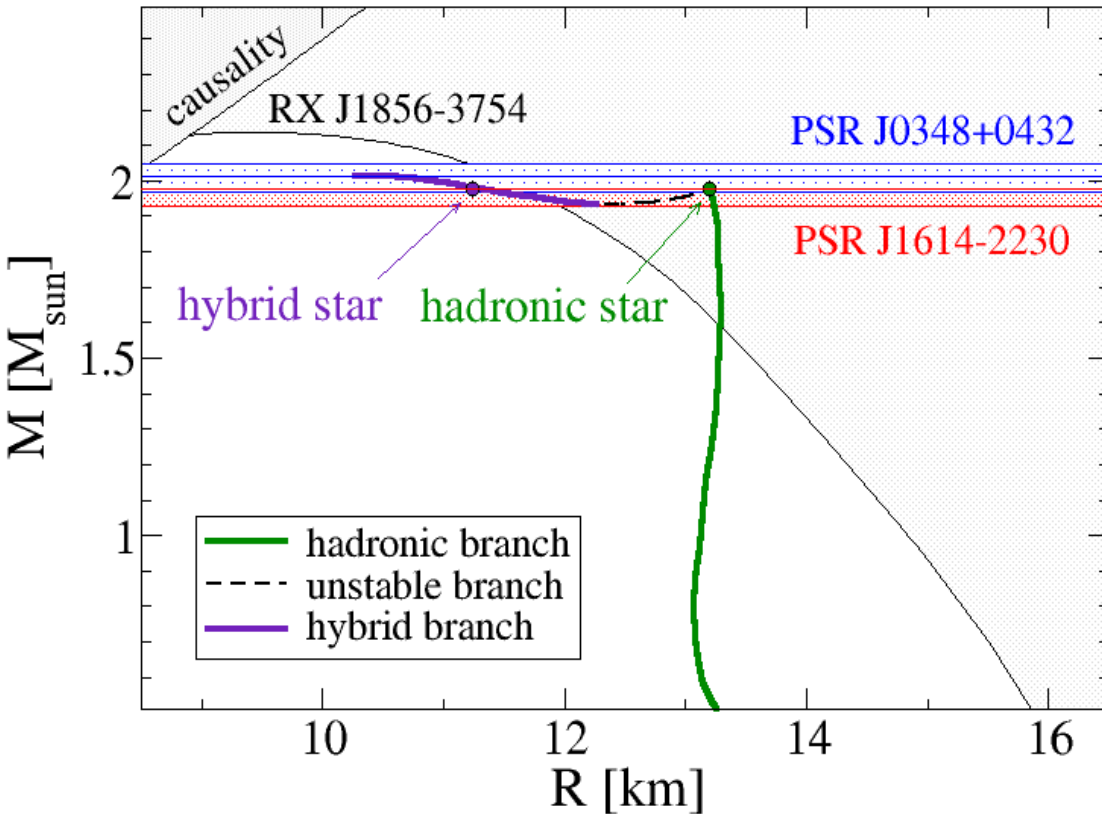
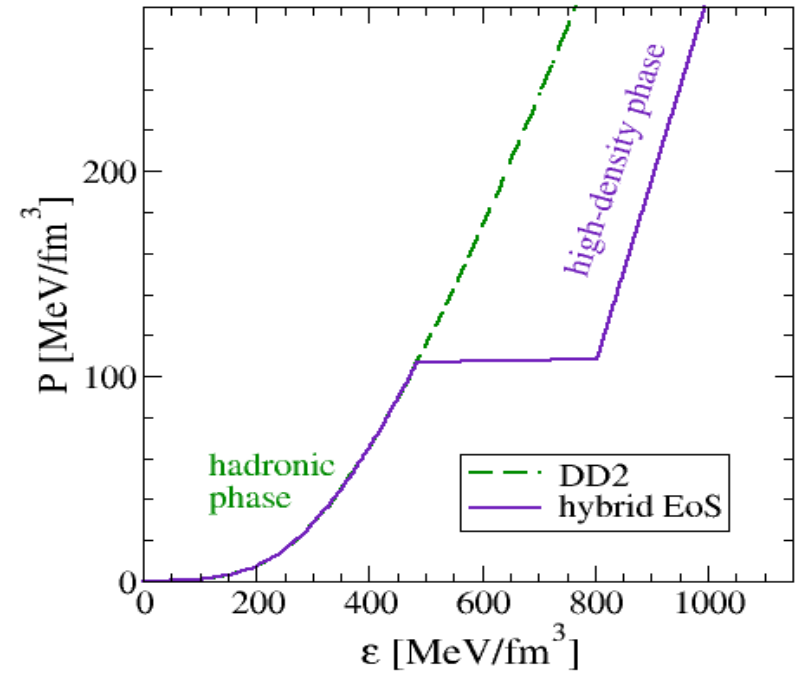
NICA White Paper, <http://theor.jinr.ru/twiki-cgi/view/NICA/WebHome>

Crossover at finite  $T$  (Lattice QCD) + First order at zero  $T$  (Astrophysics) = Critical endpoint exists!



# “Holy Grail” - High-Mass Twin Stars

Twins prove existence of **disconnected populations** (third family) in the M-R diagram  
 Consequence of a **first order phase transition**  
**Question:** Do twins prove the 1st order phase trans.?





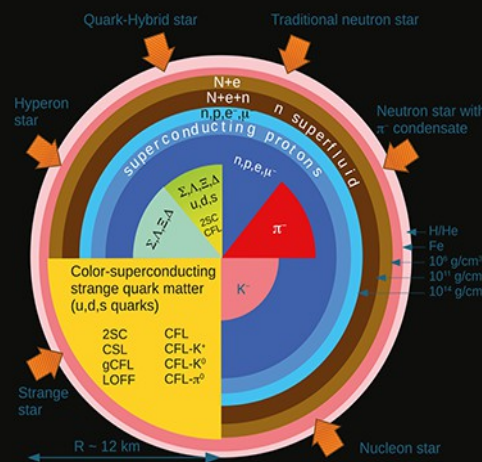
**Topical Issue on Exploring Strongly Interacting Matter at High Densities - NICA White Paper**  
 edited by David Blaschke, Jörg Aichelin, Elena Bratkovskaya, Volker Friese, Marek Gazdzicki, Jørgen Randrup, Oleg Rogachevsky, Oleg Teryaev, Viacheslav Toneev



From: Three stages of the NICA accelerator complex by V. D. Kekelidze et al.

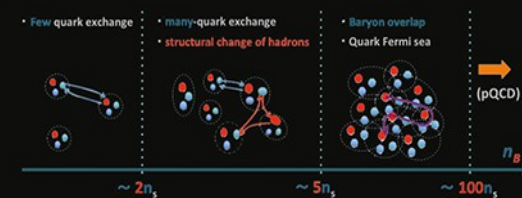


**Inside: Topical Issue on Exotic Matter in Neutron Stars**  
 edited by David Blaschke, Jürgen Schaffner-Bielich and Hans-Josef Schulze



From: Neutron star interiors: Theory and reality by J.R. Stone (left)

Phenomenological neutron star equations of state: 3-window modeling of QCD matter by T. Kojo (right)

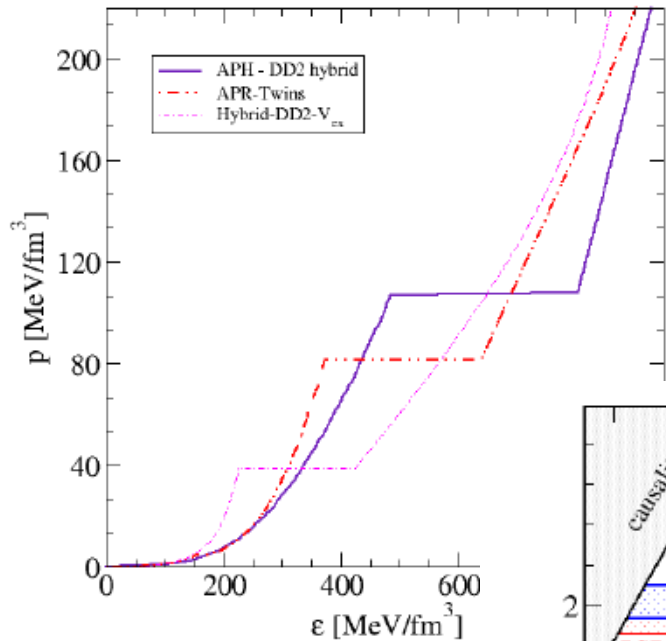


# NICA White Paper – selected topics ...

Many cross-relations with astrophysics of compact stars! High-mass twin stars prove CEP !

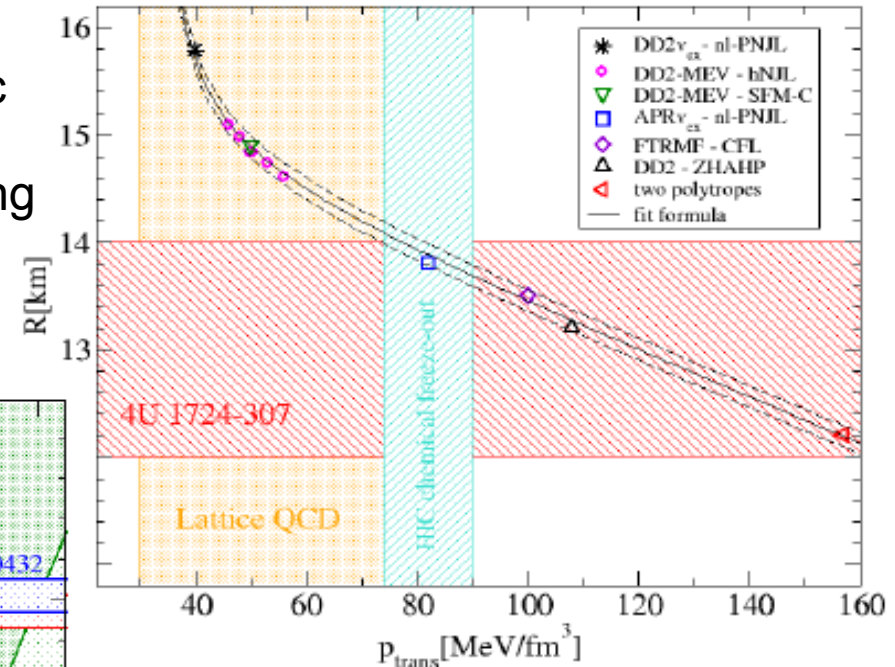
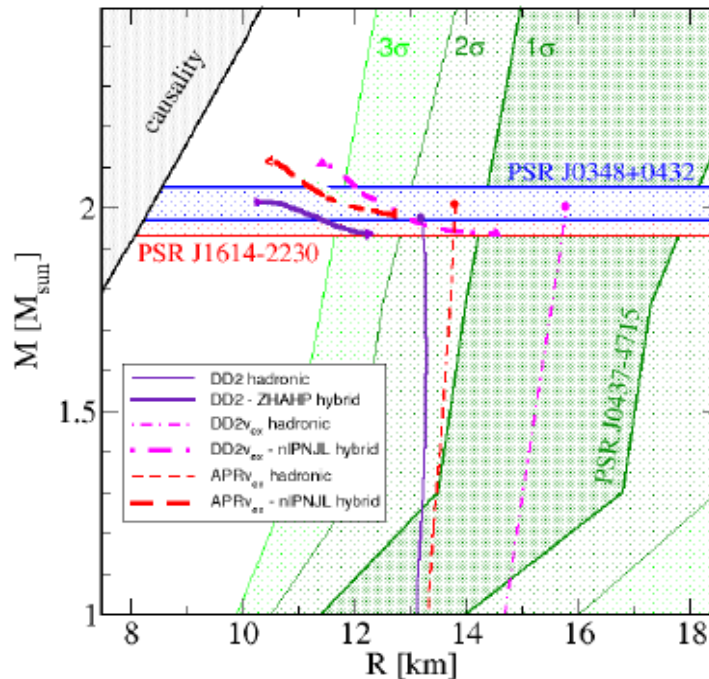
## #22 Neutron star mass limit at $2M_{\odot}$ supports the existence of a CEP

D. Alvarez-Castillo<sup>1,a</sup>, S. Benic<sup>2,b</sup>, D. Blaschke<sup>1,3,4</sup>, Sophia Han<sup>5,6</sup>, and S. Typel<sup>7</sup>



Endpoint of hadronic  
Neutron star config.  
At  $2M_{\text{sun}}$ , then strong  
Phase transition

Strong phase trans.  
High-mass twin stars



Universal transition pressure ?

Petran & Rafelski, PRC 88, 021901

$$P_{\text{trans}} = 82 \pm 8 \text{ MeV/fm}^3$$

# High-mass twins & quark matter in compact stars

David Blaschke (University Wroclaw, JINR Dubna & MEPhI Moscow)

0. Support for the QCD Critical EndPoint from Astrophysics?!

1. “Measuring” the cold Equation of States with Compact Stars

2. Microphysics of strong 1<sup>st</sup> order Phase Transitions

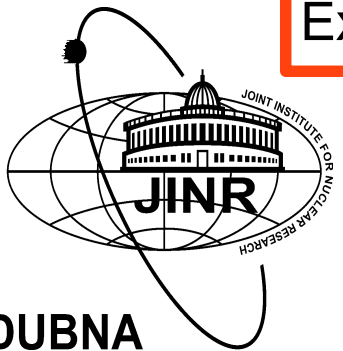
3. New Bayesian Analysis Scheme

4. Hybrid Star Matter @ NICA & FAIR



*The New is often the well-forgotten Old*

Exploring the QCD PhD through Energy Scans, INT Seattle, 06.10.2016



DUBNA





# Goal: Measure the cold EoS !

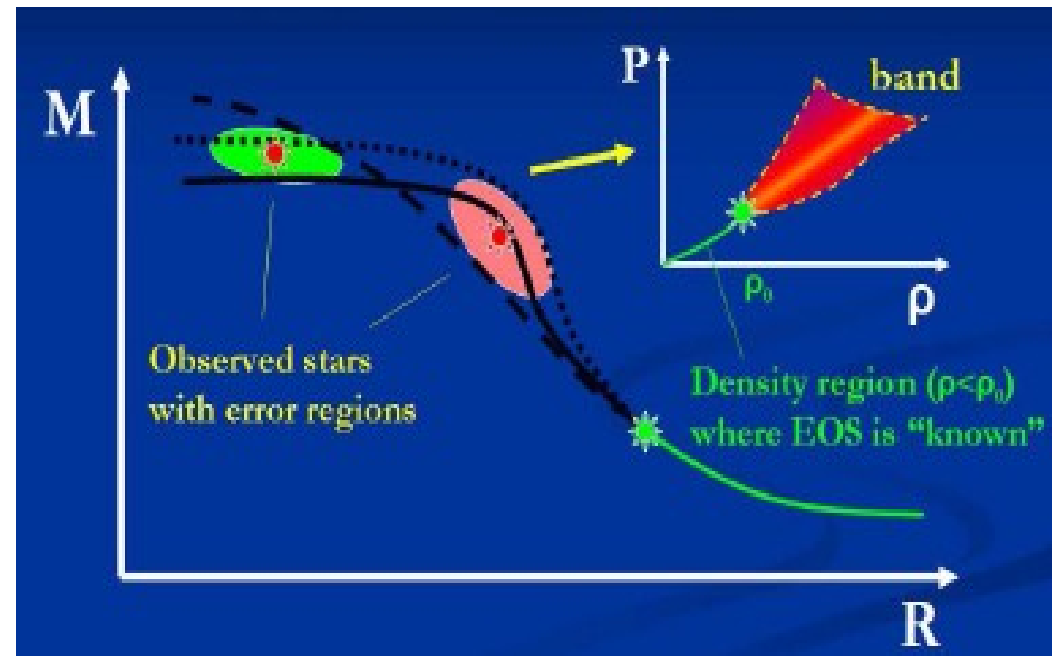
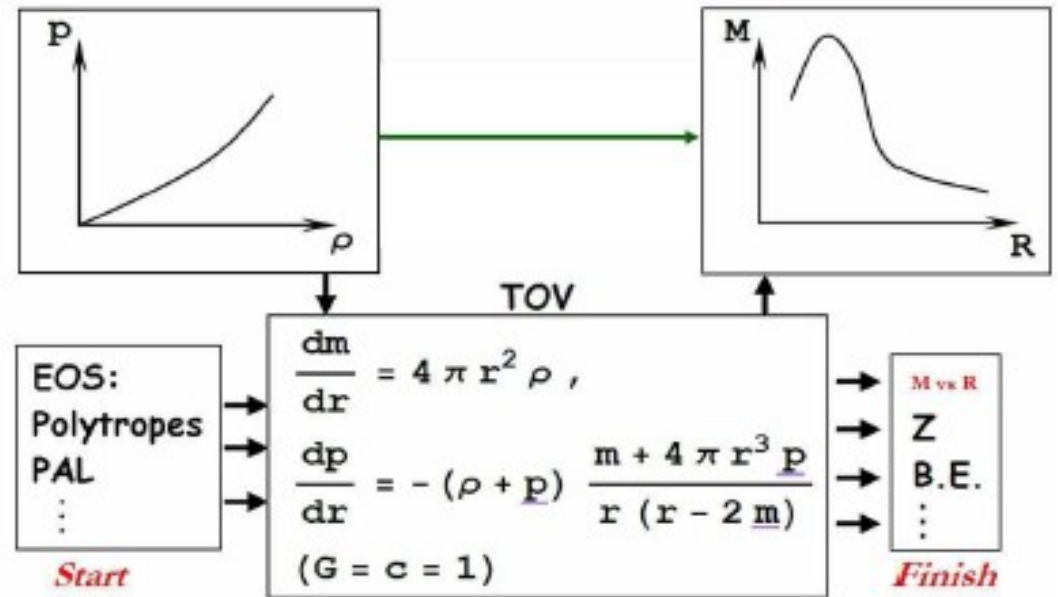
## Direct approach:

EoS is given as  $P(\rho)$   
 $\rightarrow$  solve the TOV Equation  
 to find  $M(R)$

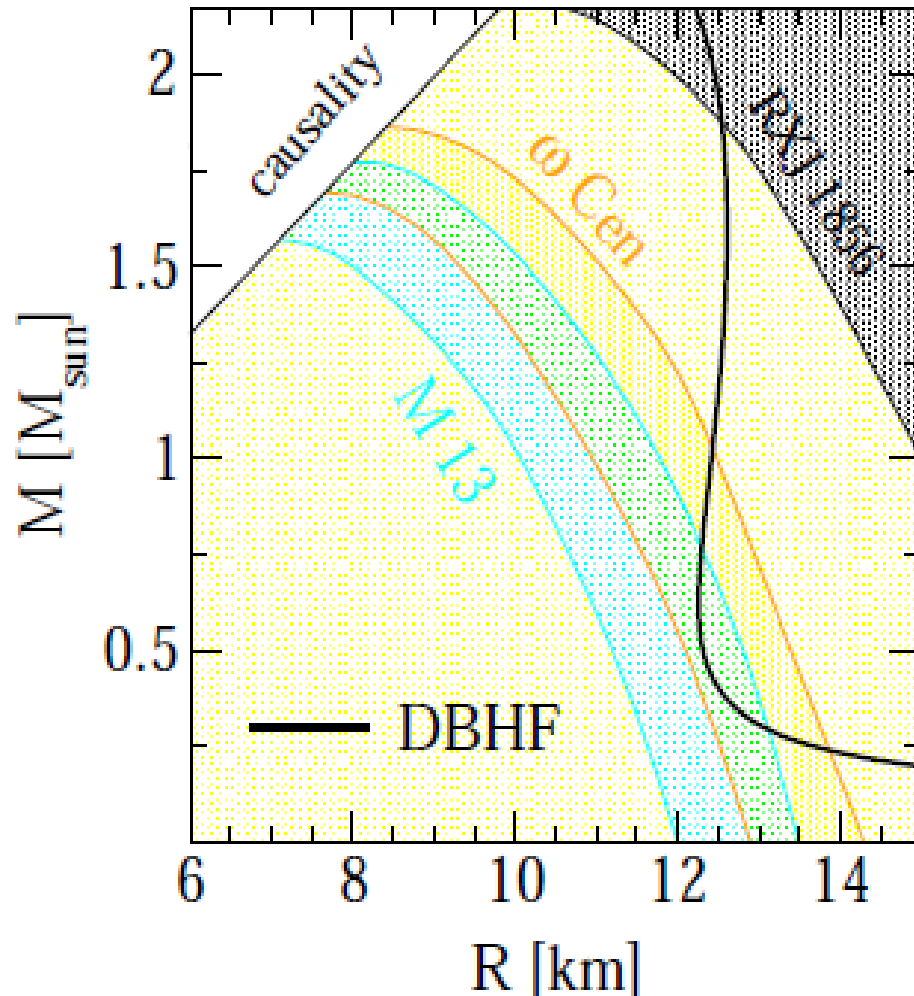
## Idea: Invert the approach

Given  $M(R) \rightarrow$  find the EoS

## Bayesian analysis



# Measure masses and radii of CS!



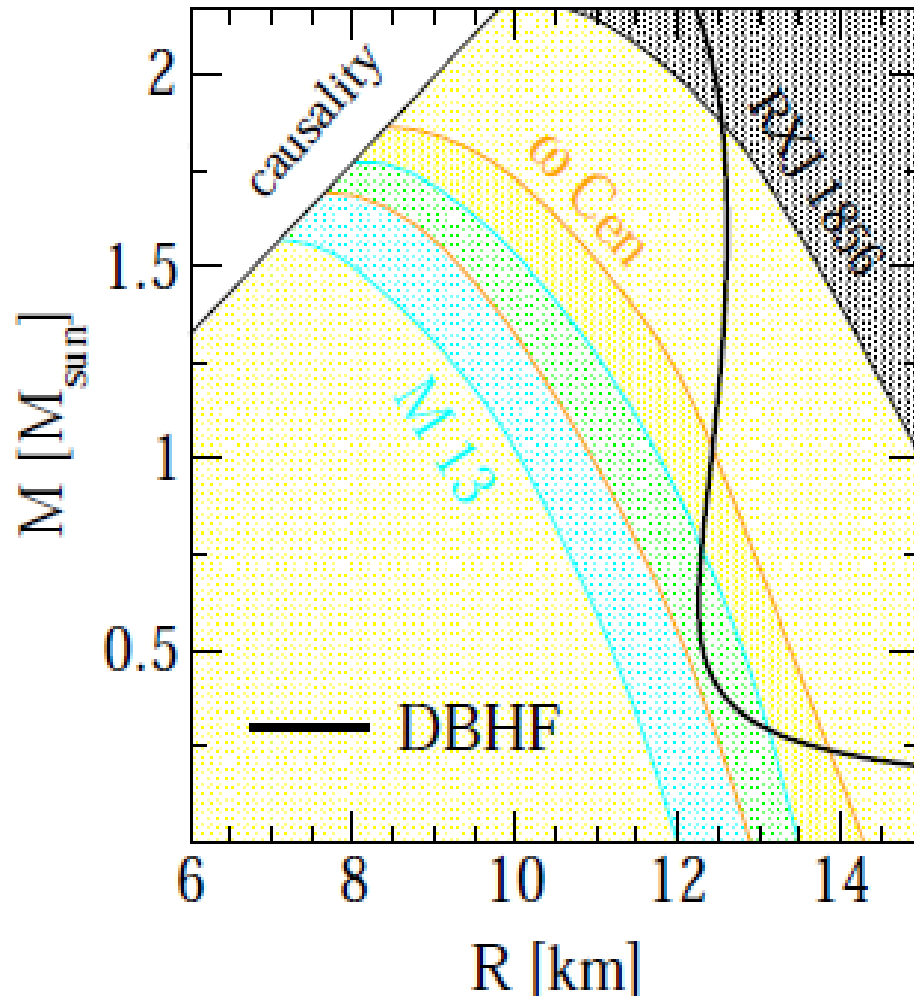
- Distance measured
  - Spectrum measured (ROSAT, XMM, Chandra)
  - Luminosity measured
- effective temperature  $T_{\infty}$   
 → photospheric radius

$$R_{\infty} = \frac{R}{\sqrt{1 - R_s/R}}, \quad R_s = 2GM$$

Object	$R_{\infty}$ [km]	Reference
RXJ 1856	16.8	Trümper et al. (2004)
$\omega$ Cen	$13.6 \pm 0.3$	Gendre et al. (2003)
M13	$12.8 \pm 0.4$	Gendre et al. (2004)

Lower limit from RXJ 1856 incompatible with  $\omega$  Cen and M13 ?

# Measure masses and radii of CS!



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Lower limit from RXJ 1856 incompatible with  $\omega$  Cen and M13 ?

... unless the latter sources emit X-rays from “hot spots” → lower limit on  $R$



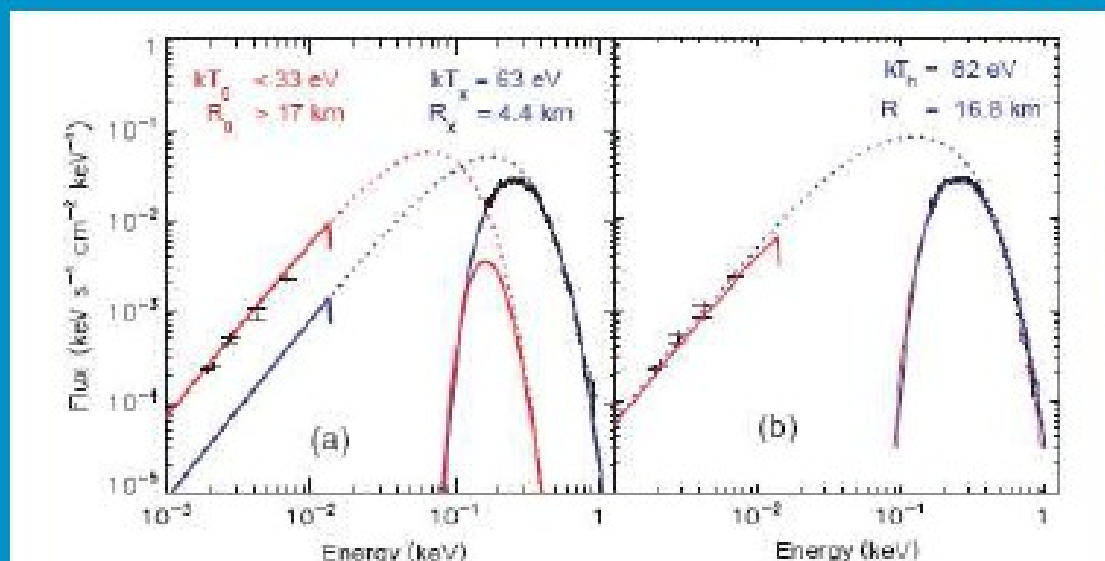
# The lesson learned from RX J1856

blackbody fits to the optical and X-ray spectra of RX J1856.5-3754 (Trümper, 2004)

radius determination  $\Rightarrow$  EoS  $\Rightarrow$  state of matter at high densities

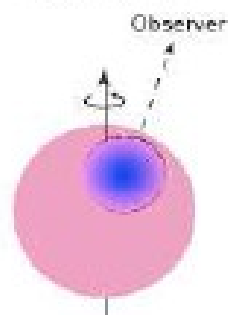
two-component model

model with continuous T-distribution



completely featureless X-ray spectrum:  
condensed surface?  
 $\Rightarrow$  strong B?

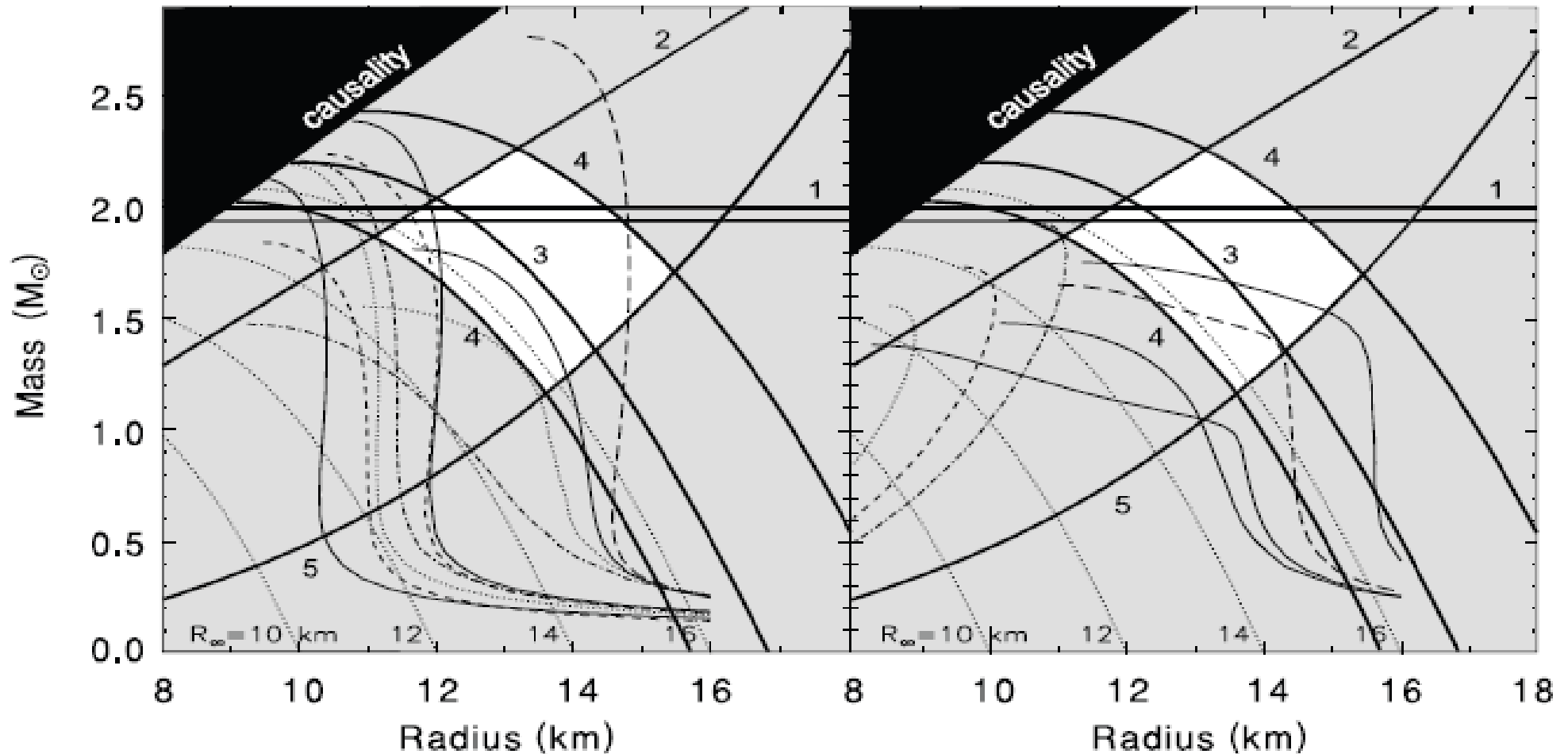
$$L_x = 5.4 \times 10^{30} \text{ erg s}^{-1}$$



pulsed fraction  $< 1\% \Rightarrow$   
line of sight  $\parallel$  rotation axis?

X-ray emitting region is a “hot spot”, J. Trümper et al., Nucl. Phys. Proc. Suppl. 132 (2004) 560

# Which constraints can be trusted ?



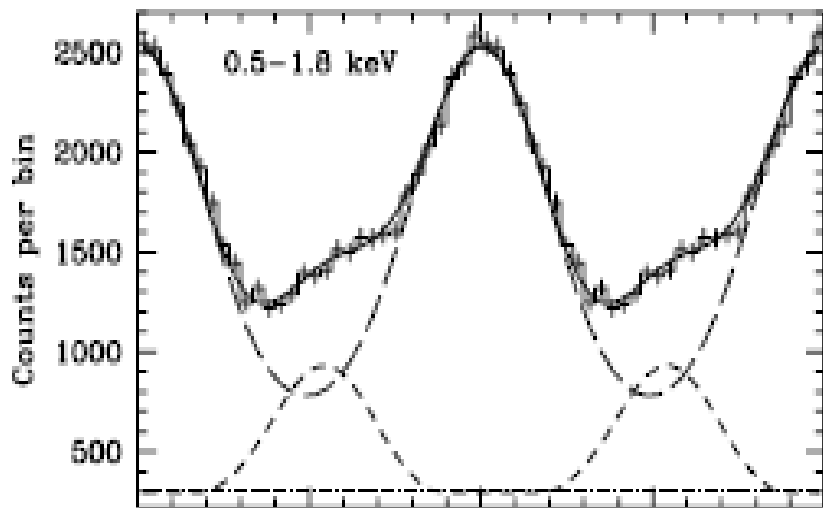
- 1 – Largest mass J1614 – 2230 (Demorest et al. 2010)
- 2 – Maximum gravity XTE 1814 – 338 (Bhattacharyya et al. (2005)
- 3 – Minimum radius RXJ 1856 – 3754 (Trumper et al. 2004)
- 4 – Radius, 90% confidence limits LMXB X7 in 47 Tuc (Heinke et al. 2006)
- 5 – Largest spin frequency J1748 – 2446 (Hessels et al. 2006)

# Which constraints can be trusted ?

Nearest millisecond pulsar PSR J0437 – 4715 revisited by XMM Newton

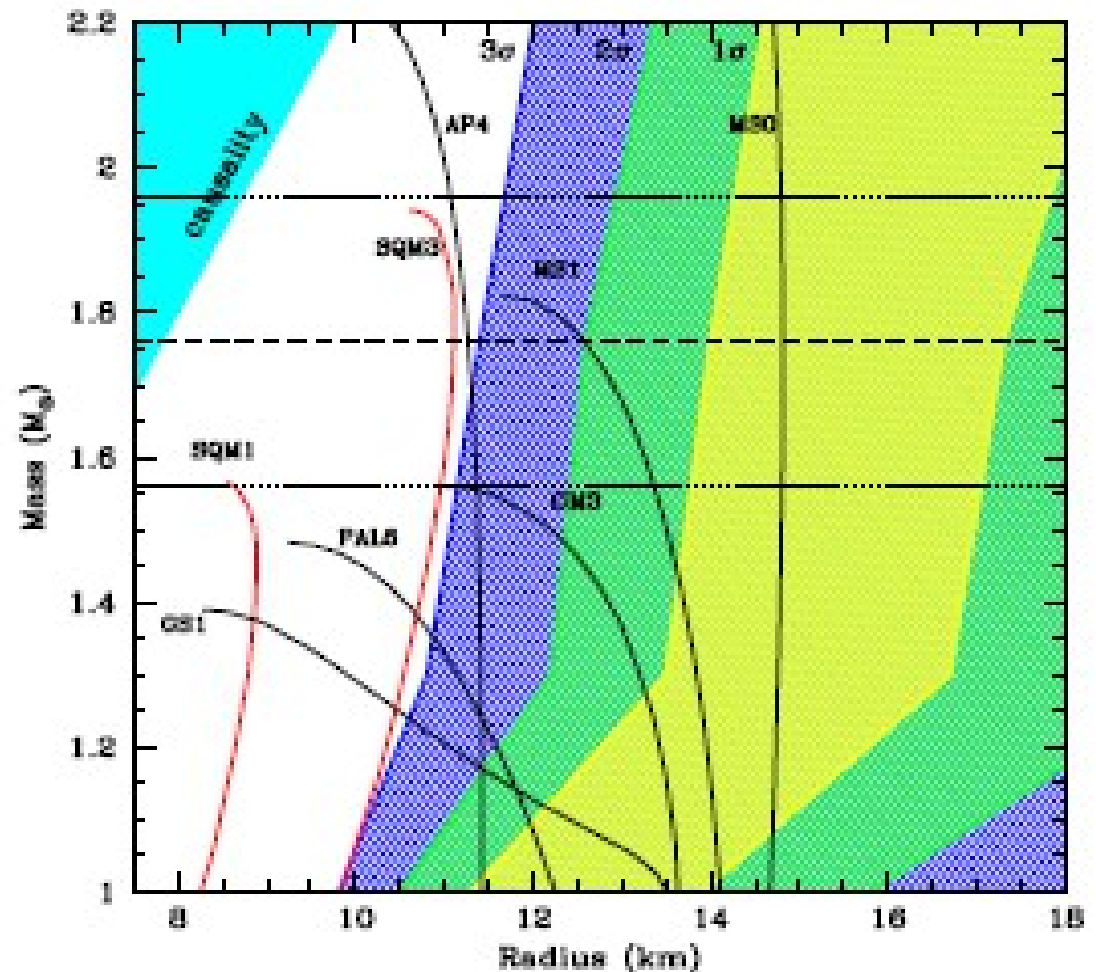
Distance:  $d = 156.3 \pm 1.3$  pc

Period:  $P = 5.76$  ms,  $\dot{P} = 10^{-20}$  s/s, field strength  $B = 3 \times 10^8$  G



Three thermal component fit  
 $R > 11.1$  km (at 3 sigma level)  
 $M = 1.76 M_{\text{sun}}$

S. Bogdanov, arxiv:1211.6113 (2012)

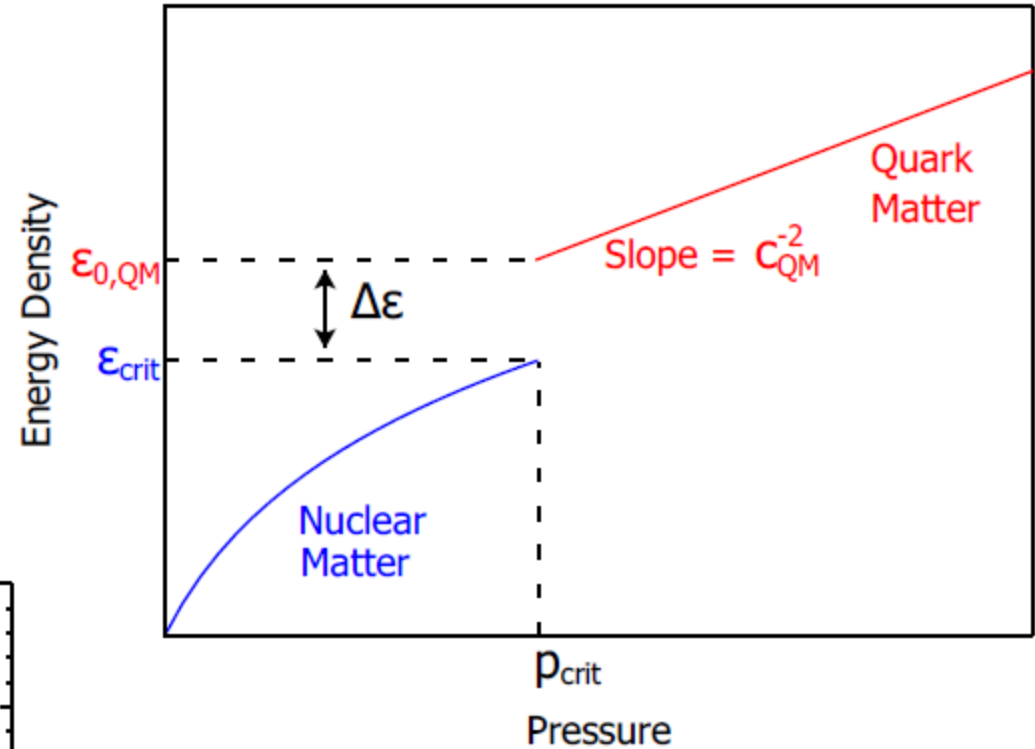
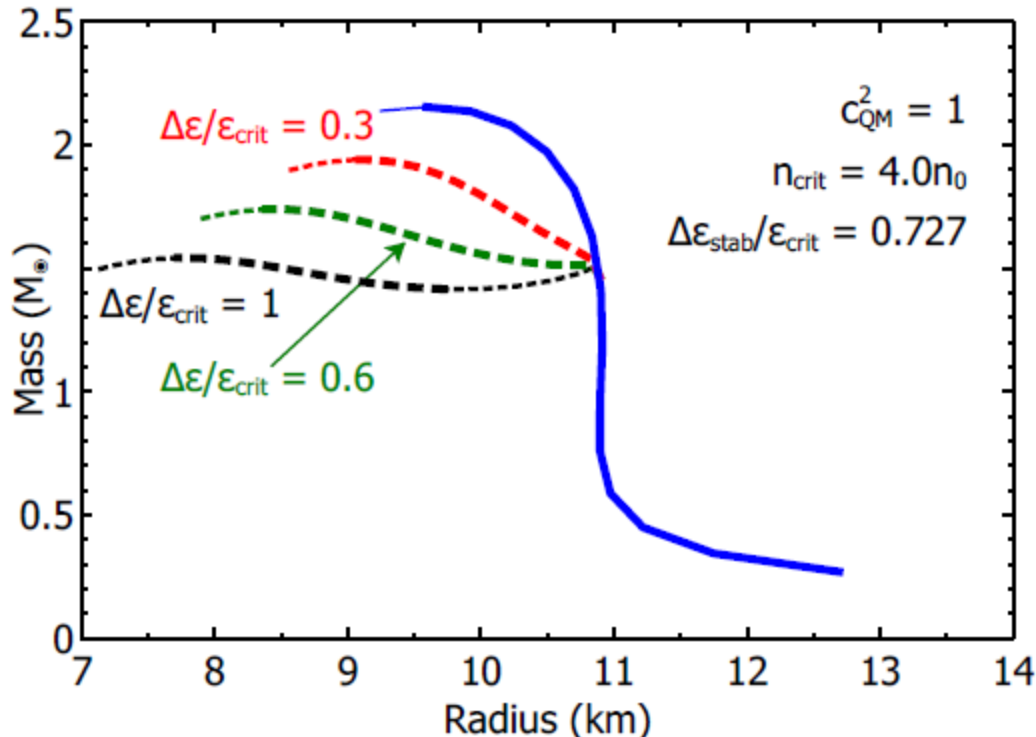




# Key fact: Mass “twins” $\leftrightarrow$ 1<sup>st</sup> order PT

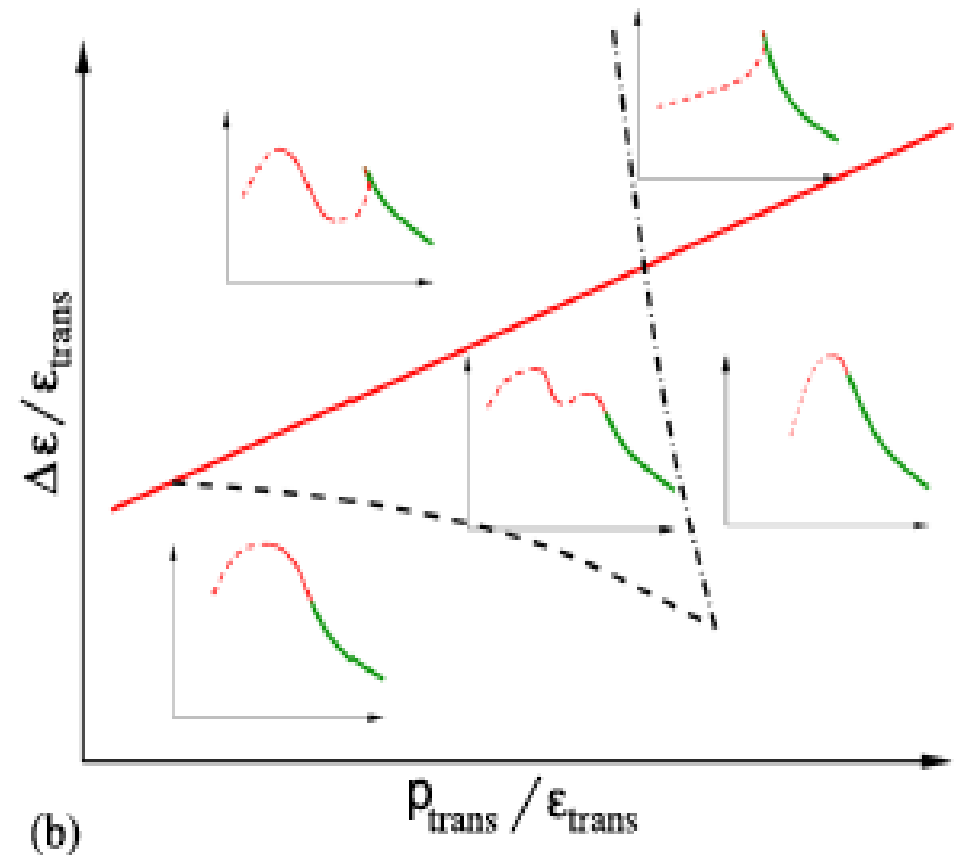
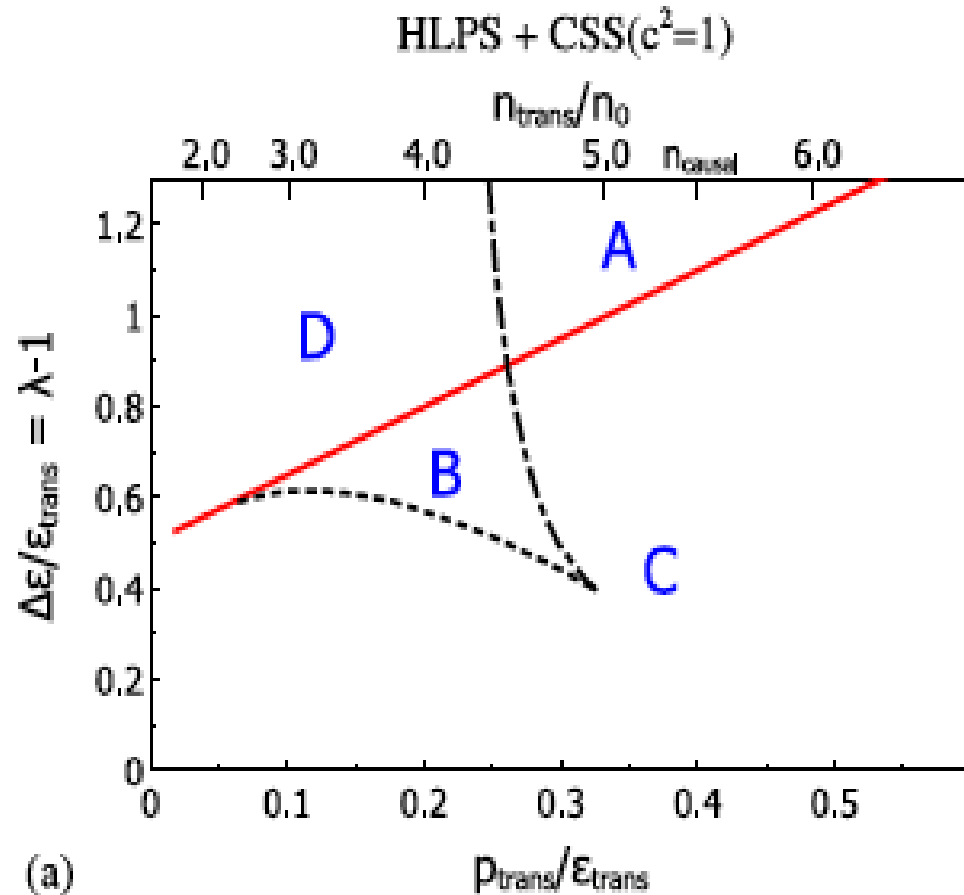
Alford, Han, Prakash, arxiv:1302.4732

First order PT can lead to a stable branch of hybrid stars with quark matter cores which, depending on the size of the “latent heat” (jump in energy density), can even be disconnected from the hadronic one by an unstable branch  $\rightarrow$  “**third family of CS**”.



Measuring two **disconnected populations** of compact stars in the M-R diagram would be the **detection of a first order phase transition** in compact star matter and thus the indirect proof for the existence of a **critical endpoint (CEP)** in the QCD phase diagram!

# Key fact: Mass “twins” $\leftrightarrow$ 1<sup>st</sup> order PT



Systematic Classification [Alford, Han, Prakash: PRD88, 083013 (2013)]

EoS  $P(\epsilon)$   $\leftrightarrow$  Compact star phenomenology  $M(R)$

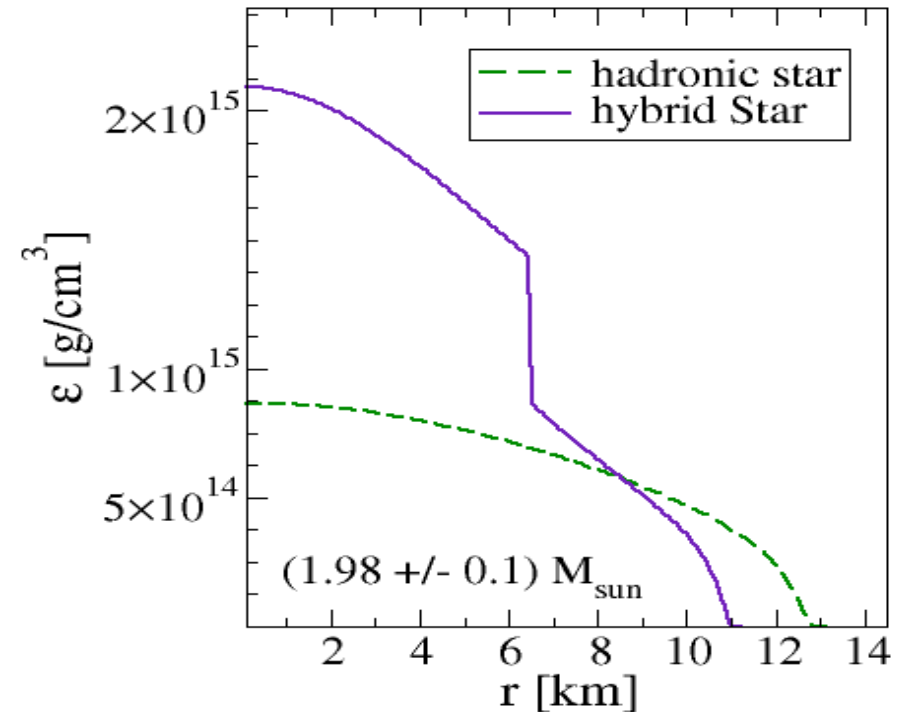
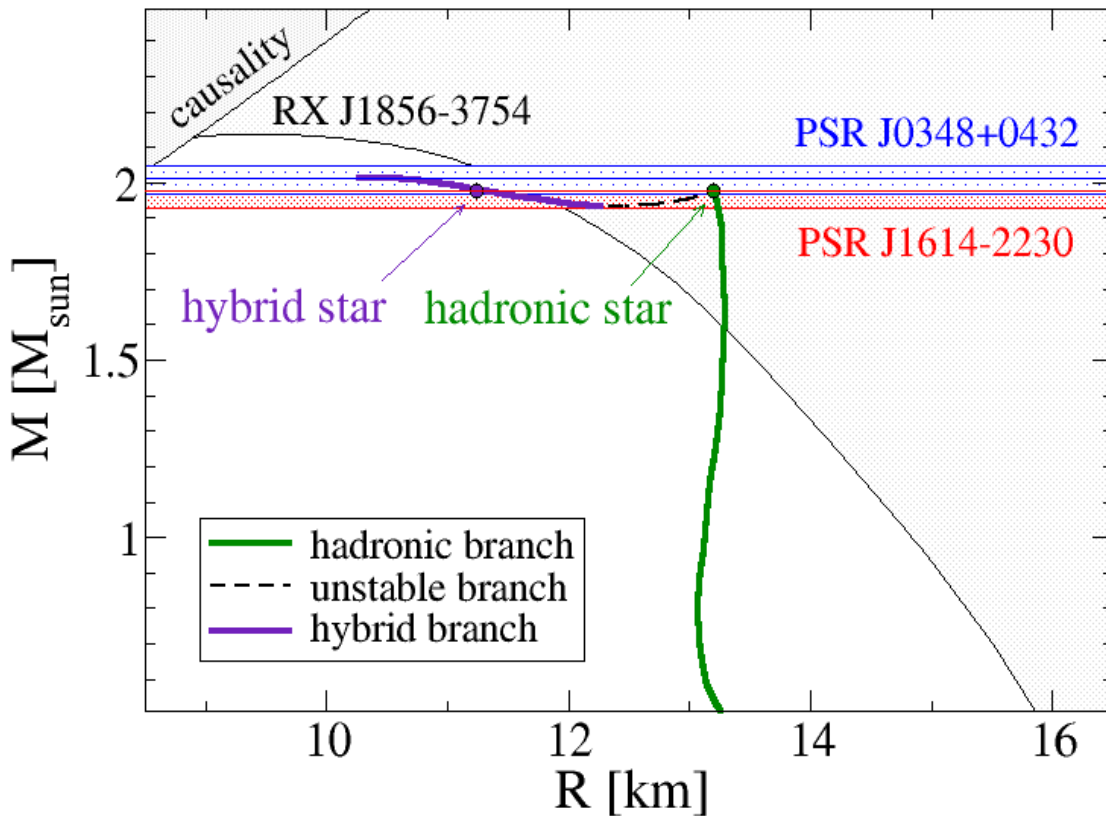
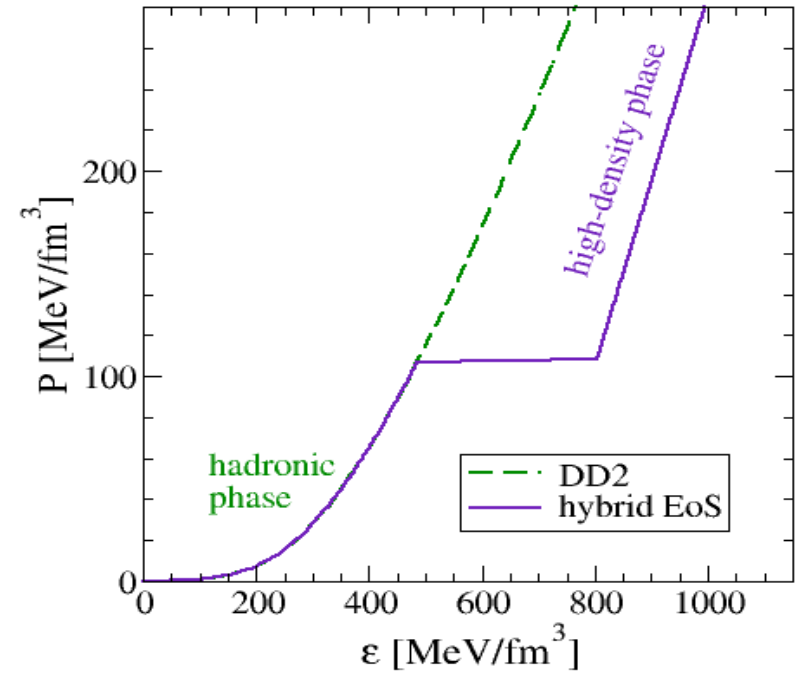
Most interesting and clear-cut cases: (D)isconnected and (B)oth – high-mass twins!

# “Holy Grail” - High-Mass Twin Stars

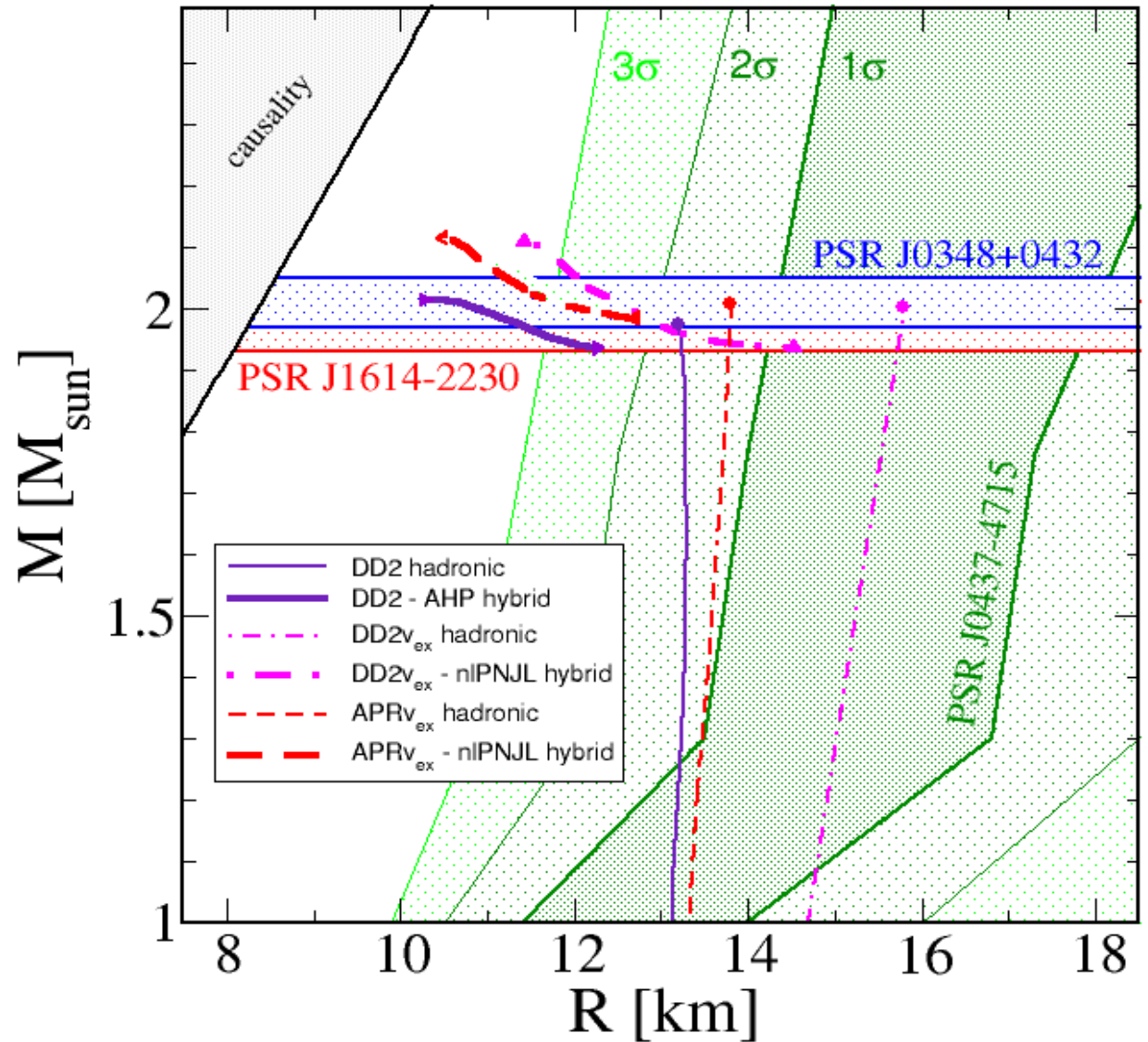
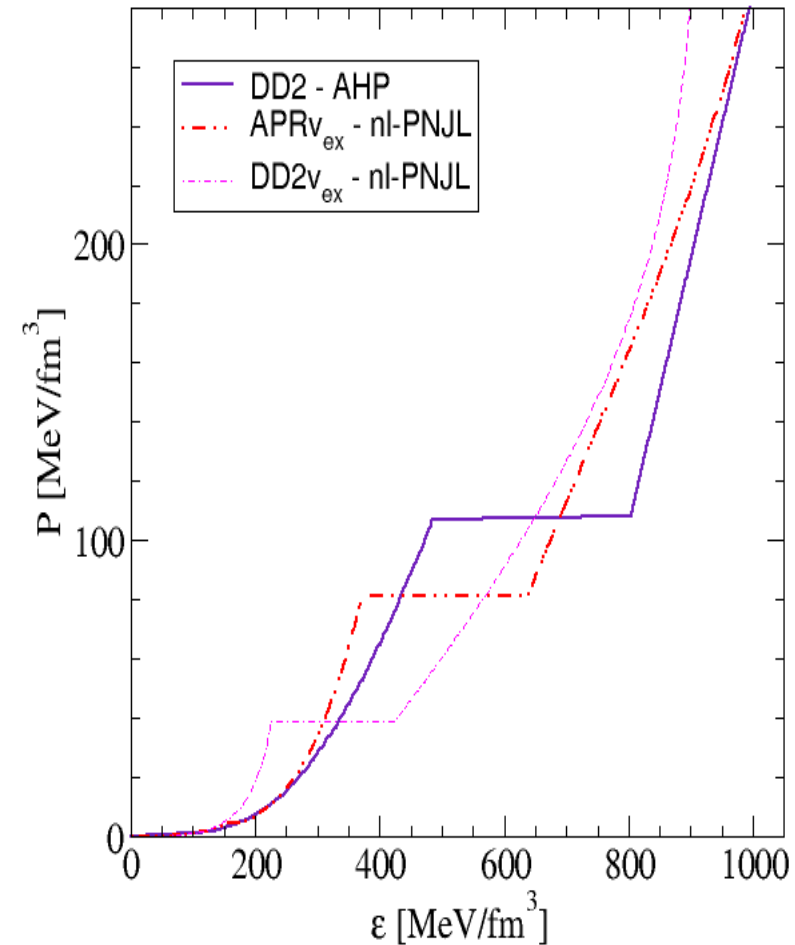
Twins prove existence of **disconnected populations** (third family) in the M-R diagram

Consequence of a **first order phase transition**

**Question:** Do twins prove the 1st order phase trans.?



# High mass twins: more examples !



## MESSAGE:

- excluded volume (quark Pauli blocking) important
  - high-density quark matter slightly stiffer  $\eta_{v=0.25}$
  - the scaled energy density jump (0.65) fulfills the twin condition of the schematic model by Alford et al. (2013)
- **Astronomers: Find disconnected star branches !!**

DB, Alvarez, Benic, arxiv:1310.3803  
Proceedings of CPOD 2013

## **2. Microphysical approach to strong 1<sup>st</sup> order PT**

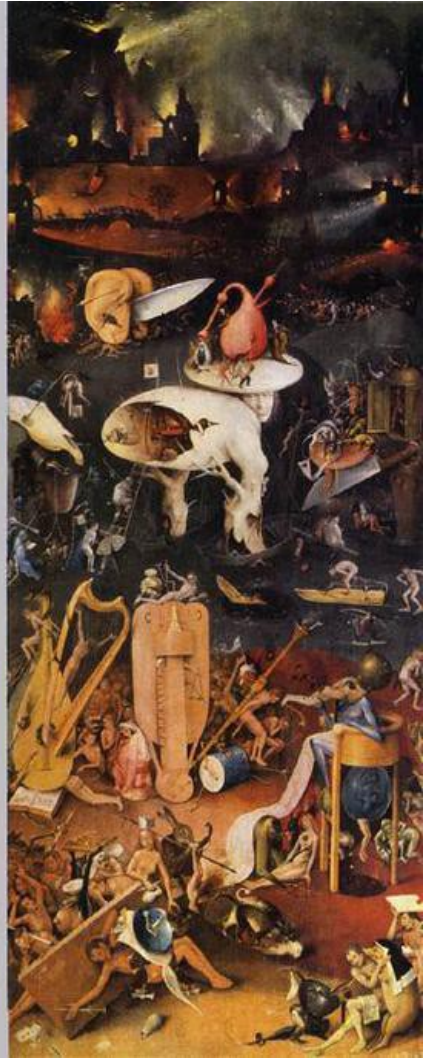


# 2. Story “Three-window picture of dense matter”



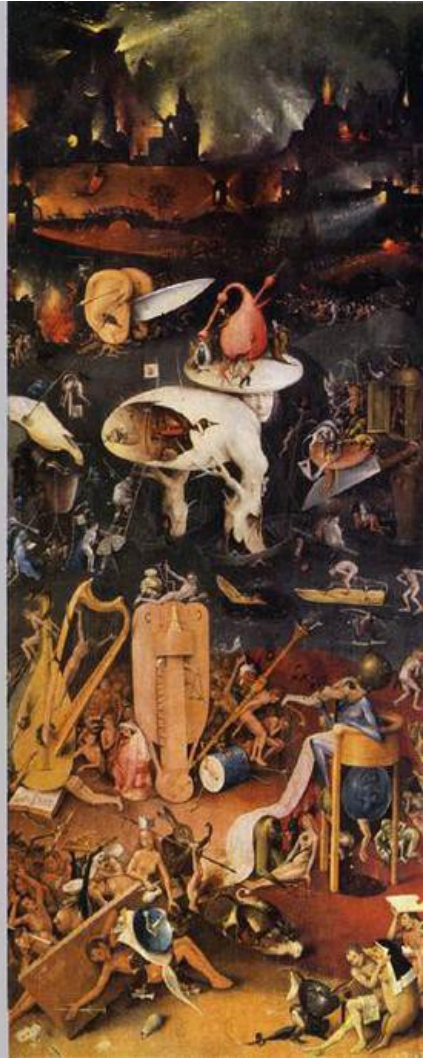
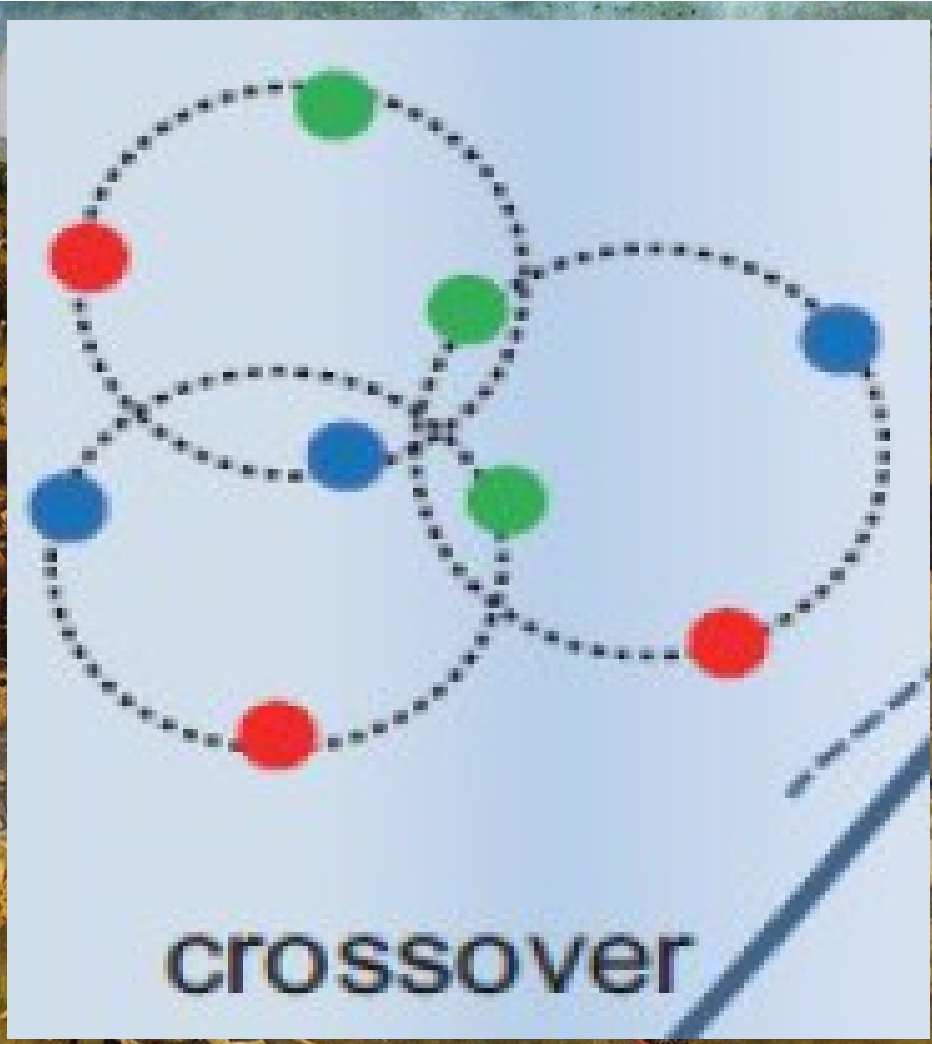


# 2. Story “Three-window picture of dense matter”



(C) WahooArt.com

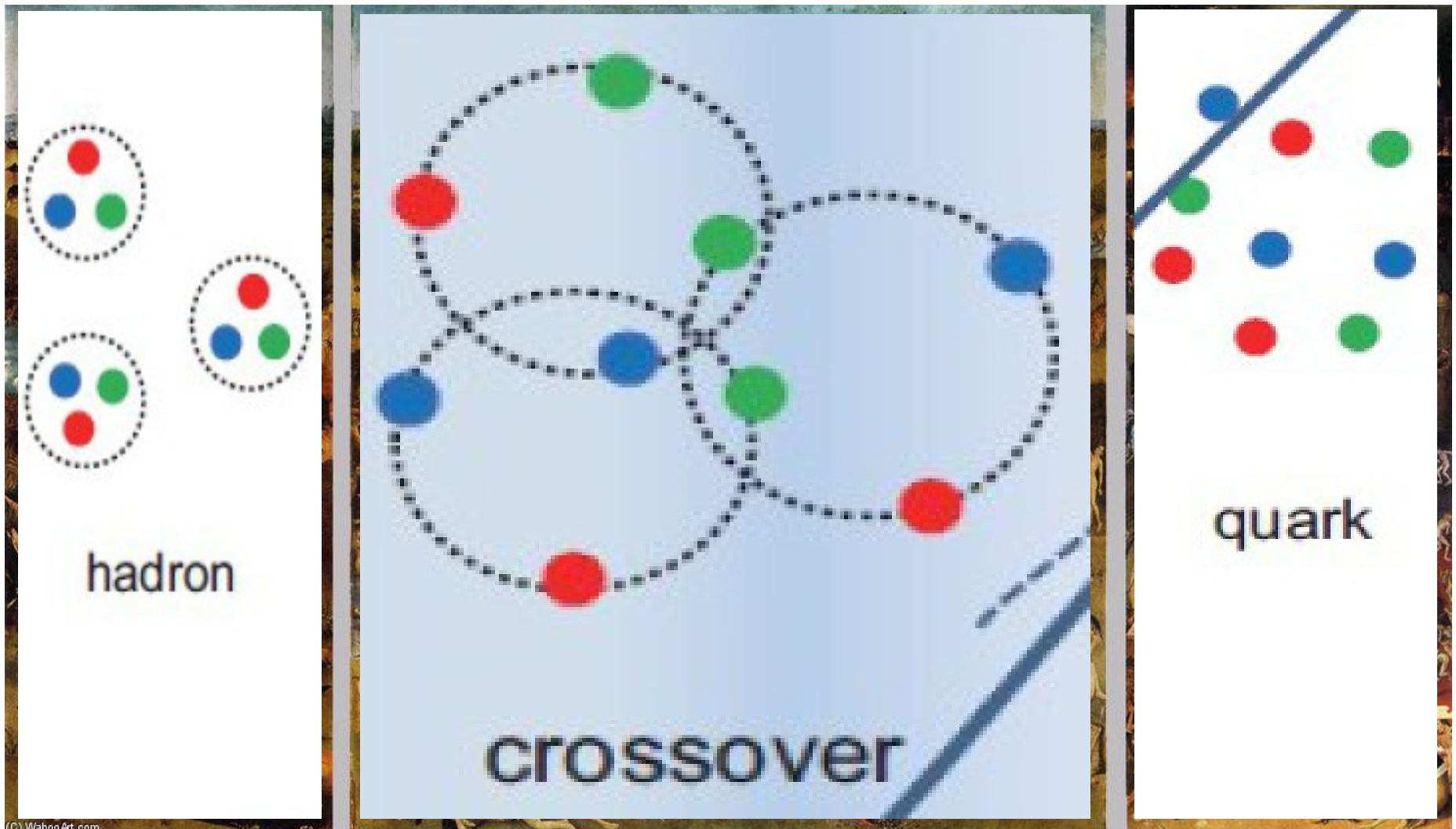
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(C) WahooArt.com



## 2. Story “Three-window picture of dense matter”





Yasuo Yamamoto

Teiji Kunihiro

D.B.

Tetsuo Hatsuda



# Hadron-Quark Crossover and Massive Hybrid Stars

Kota Masuda<sup>1,2,\*</sup>, Tetsuo Hatsuda<sup>2</sup>, and Tatsuyuki Takatsuka<sup>3†</sup>

<sup>1</sup>*Department of Physics, The University of Tokyo, Tokyo 113-0033, Japan*

<sup>2</sup>*Theoretical Research Division, Nishina Center, RIKEN, Wako 351-0198, Japan*

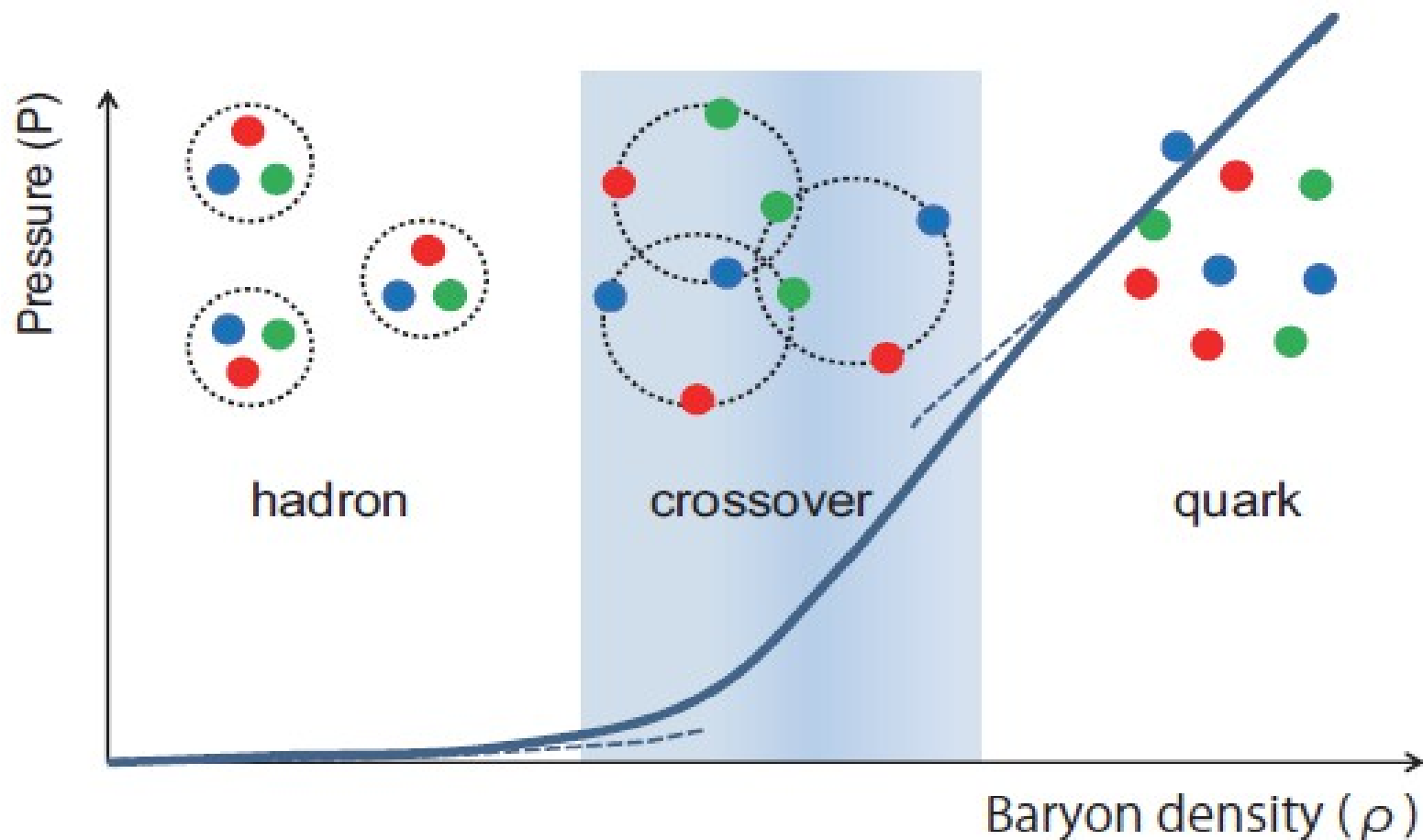
<sup>3</sup>*Iwate University, Morioka 020-8550, Japan*

\**E-mail: masuda@nt.phys.s.u-tokyo.ac.jp*

.....  
 On the basis of the percolation picture from the hadronic phase with hyperons to the quark phase with strangeness, we construct a new equation of state (EOS) with the pressure interpolated as a function of the baryon density. The maximum mass of neutron stars can exceed  $2M_{\odot}$  if the following two conditions are satisfied; (i) the crossover from the hadronic matter to the quark matter takes place at around three times the normal nuclear matter density, and (ii) the quark matter is strongly interacting in the crossover region. This is in contrast to the conventional approach assuming the first order phase transition in which the EOS becomes always soft due to the presence of the quark matter at high density. Although the choice of the hadronic EOS does not affect the above conclusion on the maximum mass, the three-body force among nucleons and hyperons plays an essential role for the onset of the hyperon mixing and the cooling of neutron stars.  
 .....

Subject Index      Neutron stars, Nuclear matter aspects in nuclear astrophysics, Hadrons and quarks in nuclear matter, Quark matter

# 1. Introduction



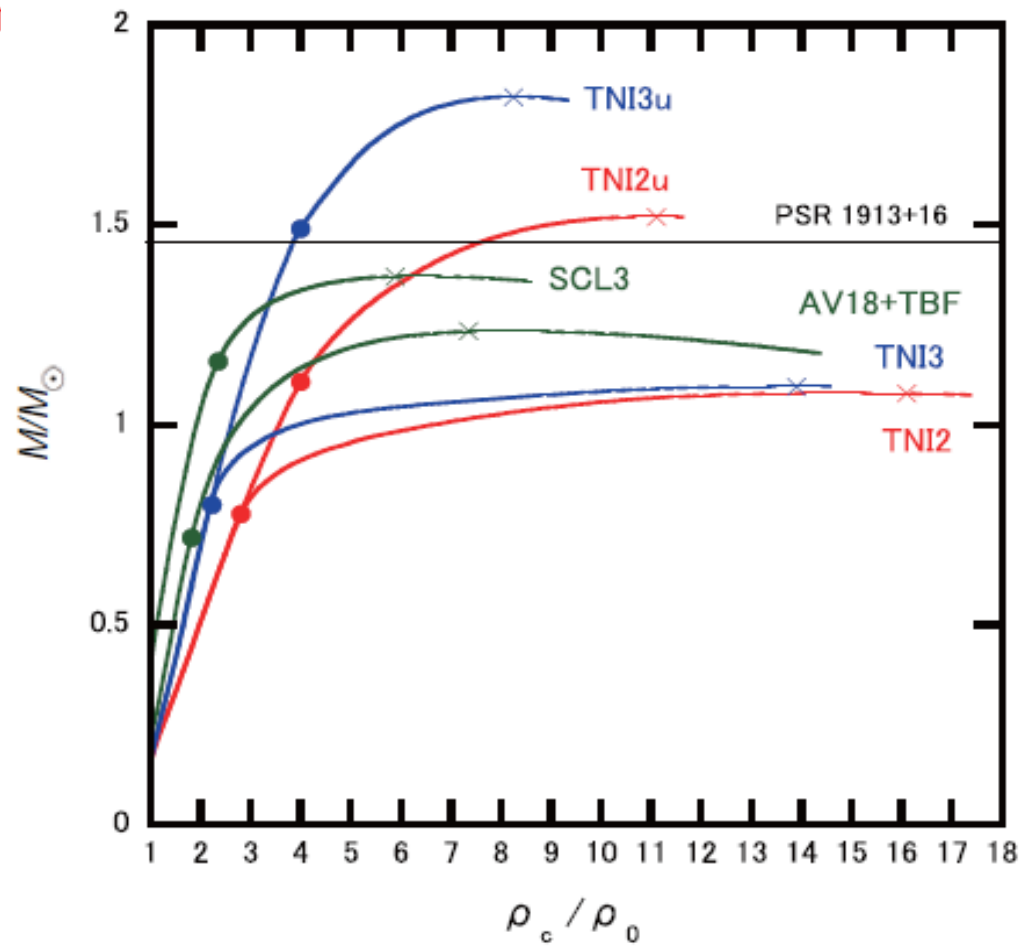
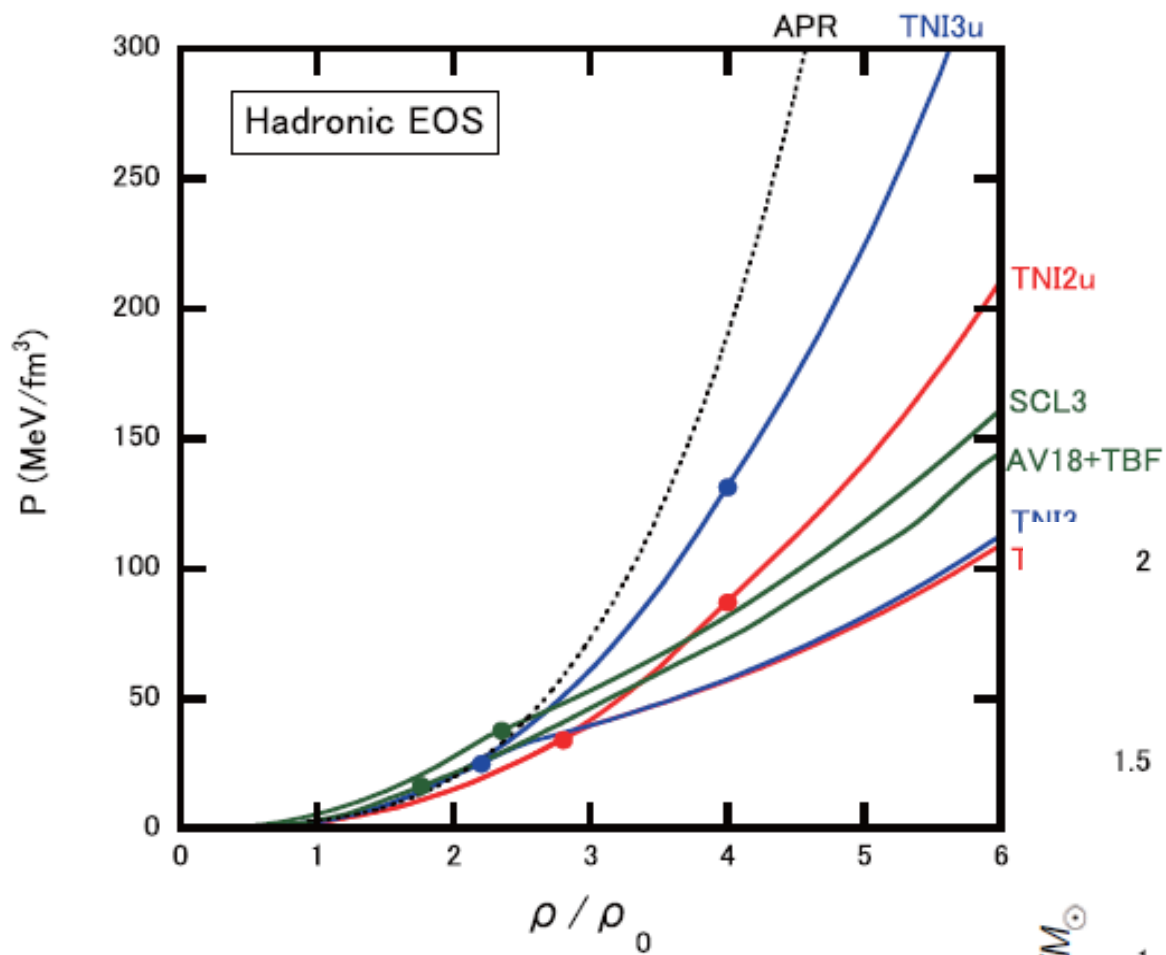
**Fig. 1** Schematic picture of the QCD pressure ( $P$ ) as a function of the baryon density ( $\rho$ ) under the assumption of the hadron-quark crossover. The crossover region where finite-size hadrons start to overlap and percolate is shown by the shaded area. The pressure calculated on the basis of the point-like hadrons (shown by the dashed line at low density) and that calculated on the basis of weakly interacting quarks (shown by the dashed line at high density) lose their validity in the crossover region, so that the naive use of the Gibbs conditions by extrapolating the dashed lines is not justified in general.



## 2. Hadronic EOS (H-EOS)

**Table 1** Properties of various hadronic EOSs with hyperons; TNI2, TNI3, TNI2u, TNI3u [33, 34], Paris+TBF, AV18+TBF [36–38] and SCL3 $\Lambda\Sigma$  [39].  $\kappa$  is the nuclear incompressibility and  $\rho_{\text{th}}$  is the threshold density of hyperon-mixing with  $\rho_0$  ( $=0.17/\text{fm}^3$ ) being the normal nuclear density.  $R$  and  $\rho_c$  denote the radius and central density for the maximum mass ( $M_{\text{max}}$ ) NS, respectively. The numbers in the parentheses are those without hyperons. \*s indicate that the numbers are read from the figures in [36].

EOS	TNI2	TNI3	TNI2u	TNI3u	Paris+TBF	AV18+TBF	SCL3 $\Lambda\Sigma$
$\kappa$ (MeV)	250	300	250	300	281	192	211
$\rho_{\text{th}}(\Lambda)/\rho_0$	2.95	2.45	4.01	4.01	2.9*	2.8*	2.24
$\rho_{\text{th}}(\Sigma^-)/\rho_0$	2.83	2.23	4.06	4.01	1.9*	1.8*	2.24
$M_{\text{max}}/M_{\odot}$	1.08 (1.62)	1.10 (1.88)	1.52	1.83	1.26 (2.06)	1.22 (2.00)	1.36 (1.65)
R(km)	7.70 (8.64)	8.28 (9.46)	8.43	9.55	10.46 (10.50)	10.46 (10.54)	11.42 (10.79)
$\rho_c/\rho_0$	16.10 (9.97)	13.90 (8.29)	11.06	8.26	7.35 (6.47)	7.35 (6.53)	6.09 (6.85)



### 3. Quark EOS (Q-EOS)

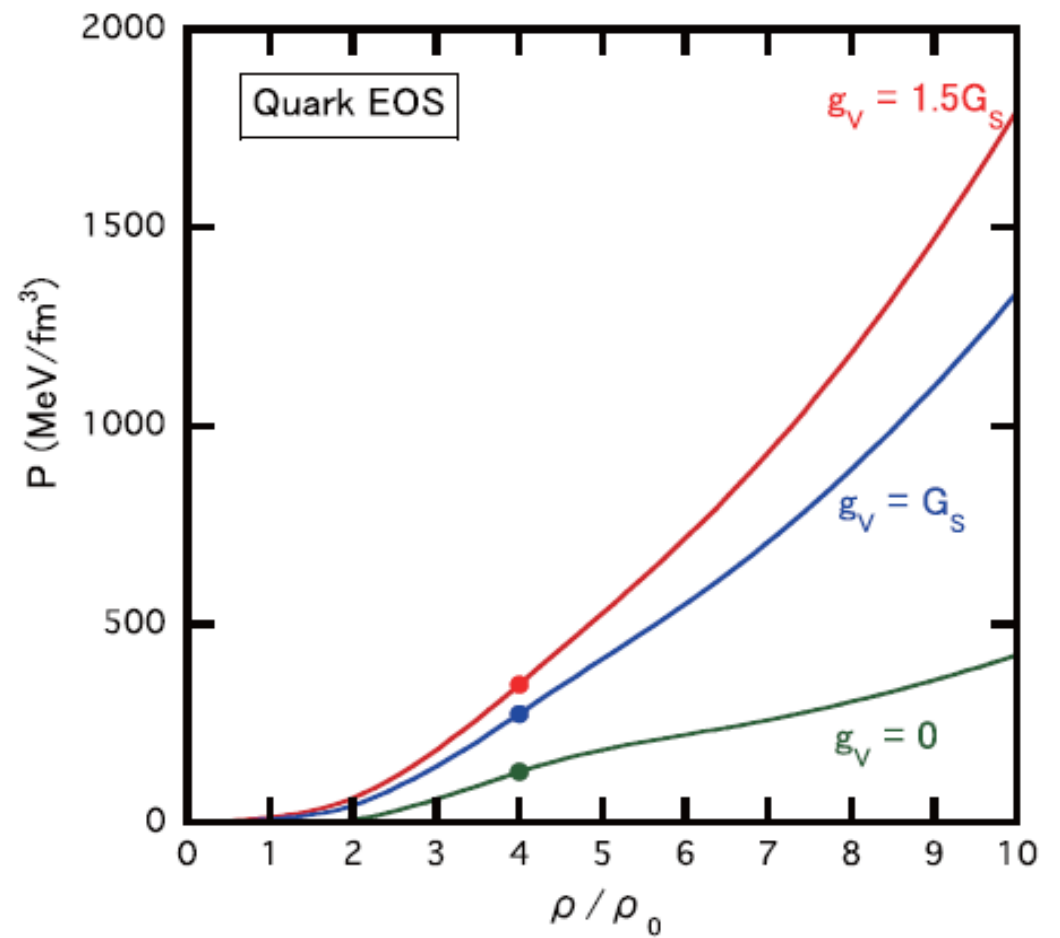
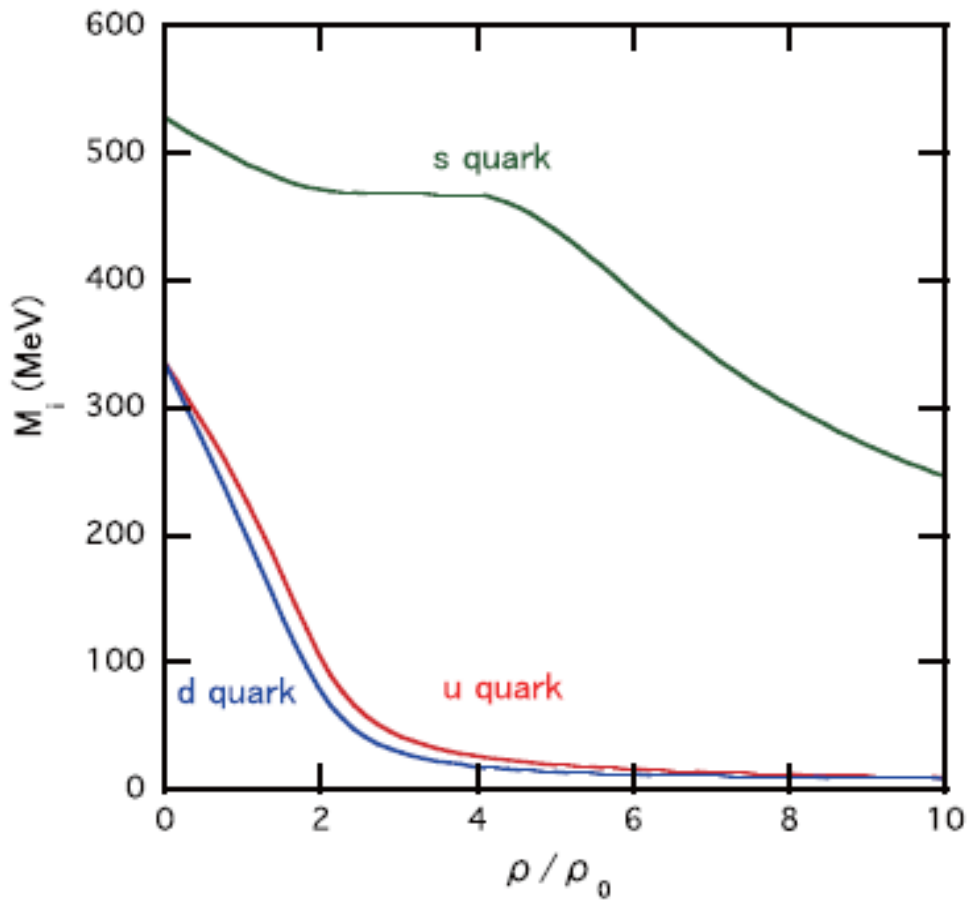
$$\mathcal{L}_{\text{NJL}} = \bar{q}(i\not{\partial} - m)q + \frac{1}{2}G_s \sum_{a=0}^8 [(\bar{q}\lambda^a q)^2 + (\bar{q}i\gamma_5\lambda^a q)^2] - G_D [\det\bar{q}(1 + \gamma_5)q + \text{h.c.}]$$

$$- \begin{cases} \frac{1}{2}g_v (\bar{q}\gamma^\mu q)^2 \\ \frac{1}{2}G_v \sum_{a=0}^8 [(\bar{q}\gamma^\mu\lambda^a q)^2 + (\bar{q}i\gamma^\mu\gamma_5\lambda^a q)^2] \end{cases}$$

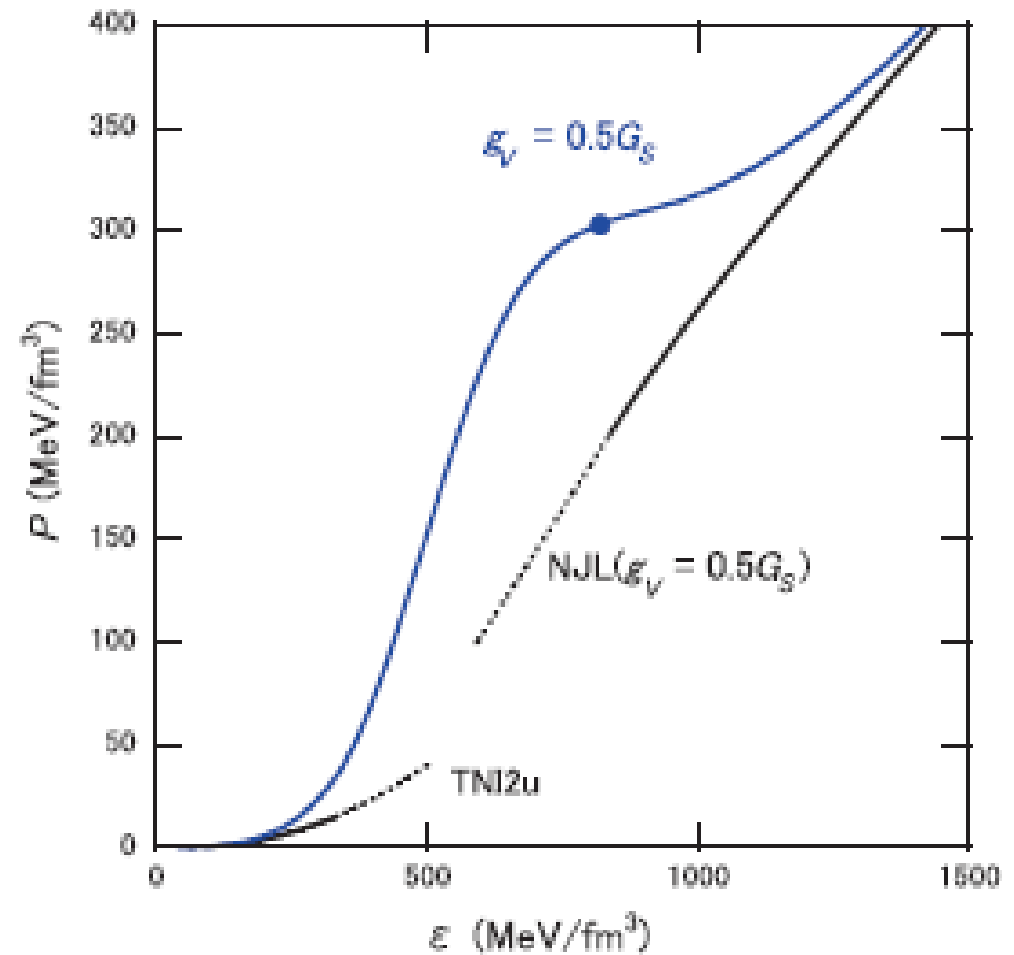
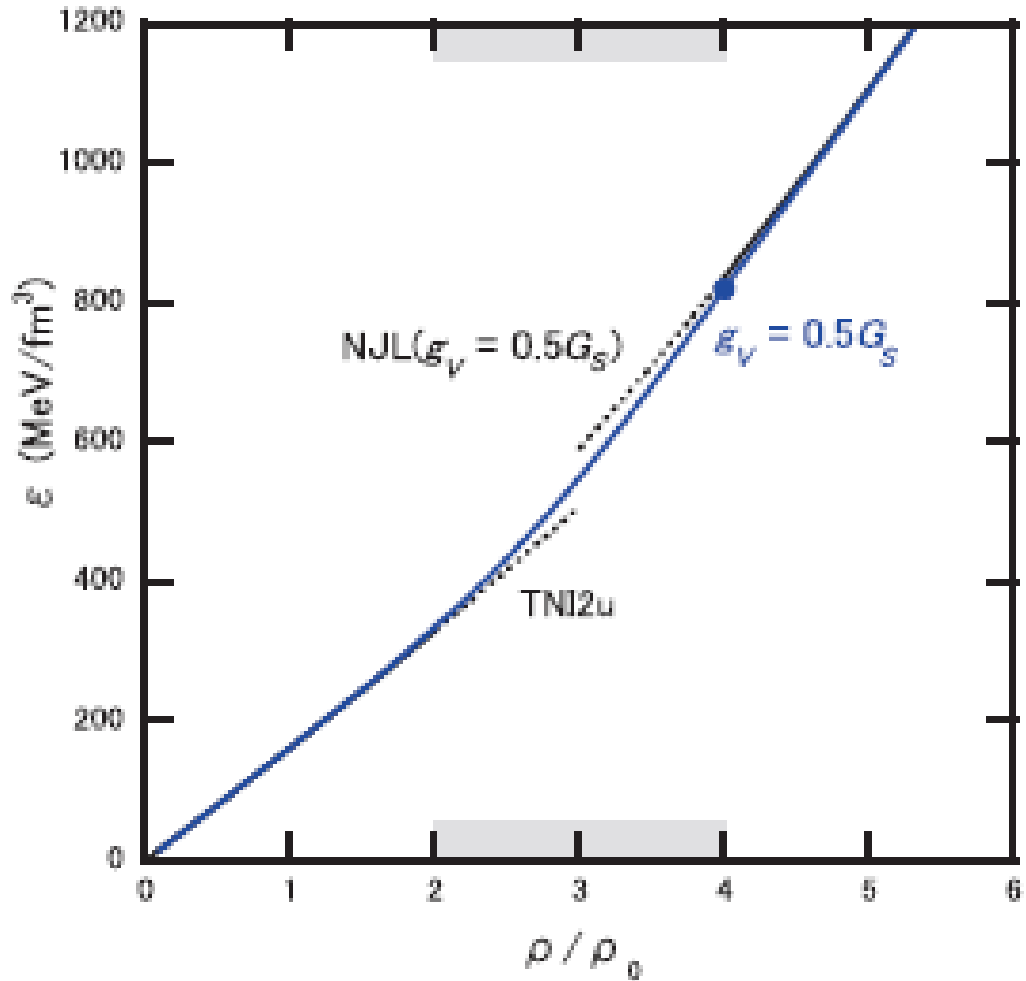
$$P(T, \mu_{u,d,s}) = T \sum_i \sum_\ell \int \frac{d^3p}{(2\pi)^3} \text{Tr} \ln \left( \frac{S_i^{-1}(i\omega_\ell, \mathbf{p})}{T} \right) - G_s \sum_i \sigma_i^2 - 4G_D \sigma_u \sigma_d \sigma_s + \begin{cases} \frac{1}{2}g_v (\sum_i n_i)^2 \\ \frac{1}{2}G_v \sum_i n_i^2 \end{cases}$$

$$S_i^{-1} = \not{p} - M_i - \gamma^0 \mu_i^{\text{eff}}, \quad \mu_i^{\text{eff}} \equiv \begin{cases} \mu_i - g_v \sum_j n_j \\ \mu_i - G_v n_i \end{cases}$$

	$\Lambda(\text{MeV})$	$G_s \Lambda^2$	$G_D \Lambda^5$	$m_{u,d}(\text{MeV})$	$m_s(\text{MeV})$
HK	631.4	3.67	9.29	5.5	135.7
RKH	602.3	3.67	12.36	5.5	140.7
LKW	750	3.64	8.9	3.6	87



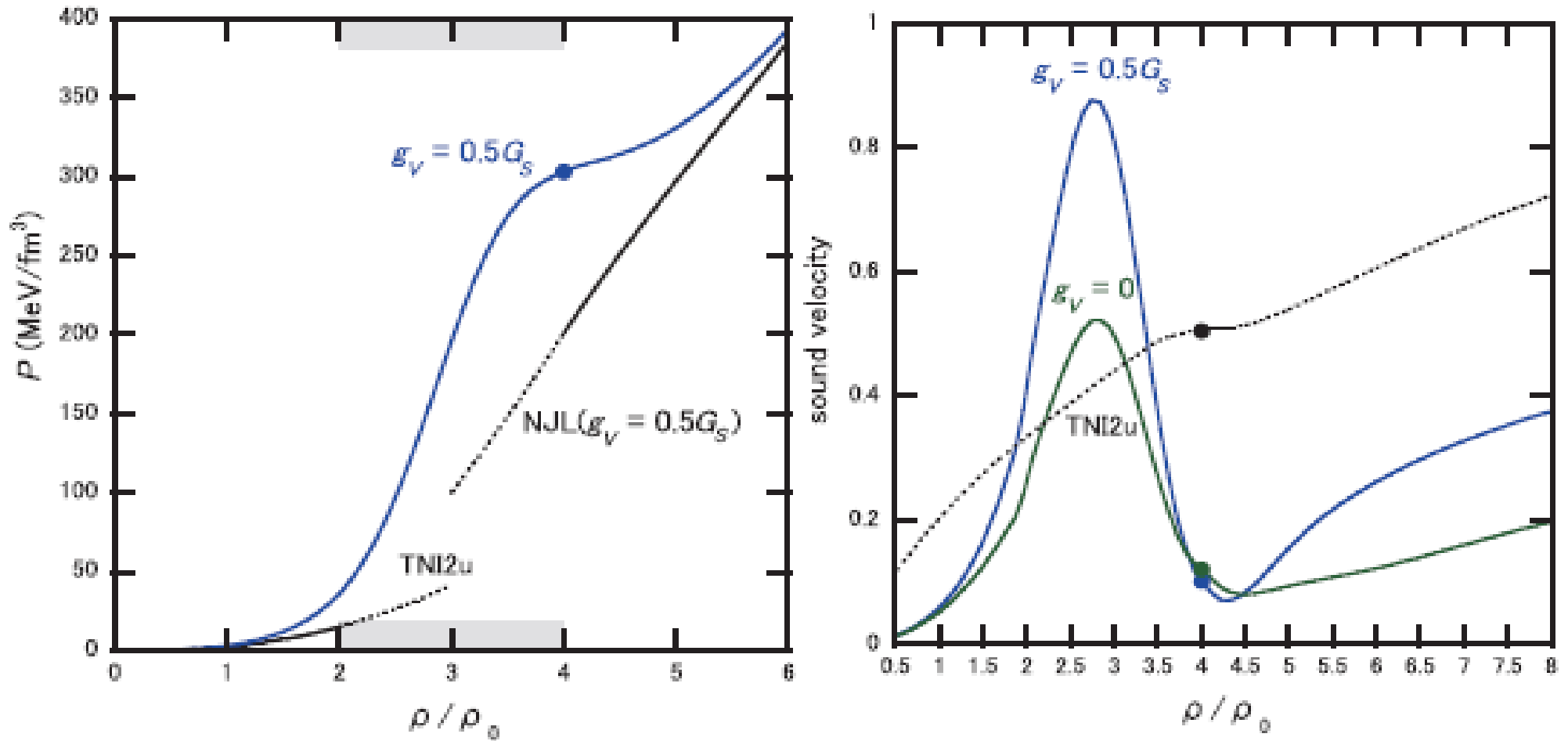
# Attn: Interpolation between energy densities $\epsilon(\rho)$ , not $P(\rho)$



Masuda, Hatsuda, Takatsuka, PTP 073D01 (2013); [arxiv:1212.6803v2]

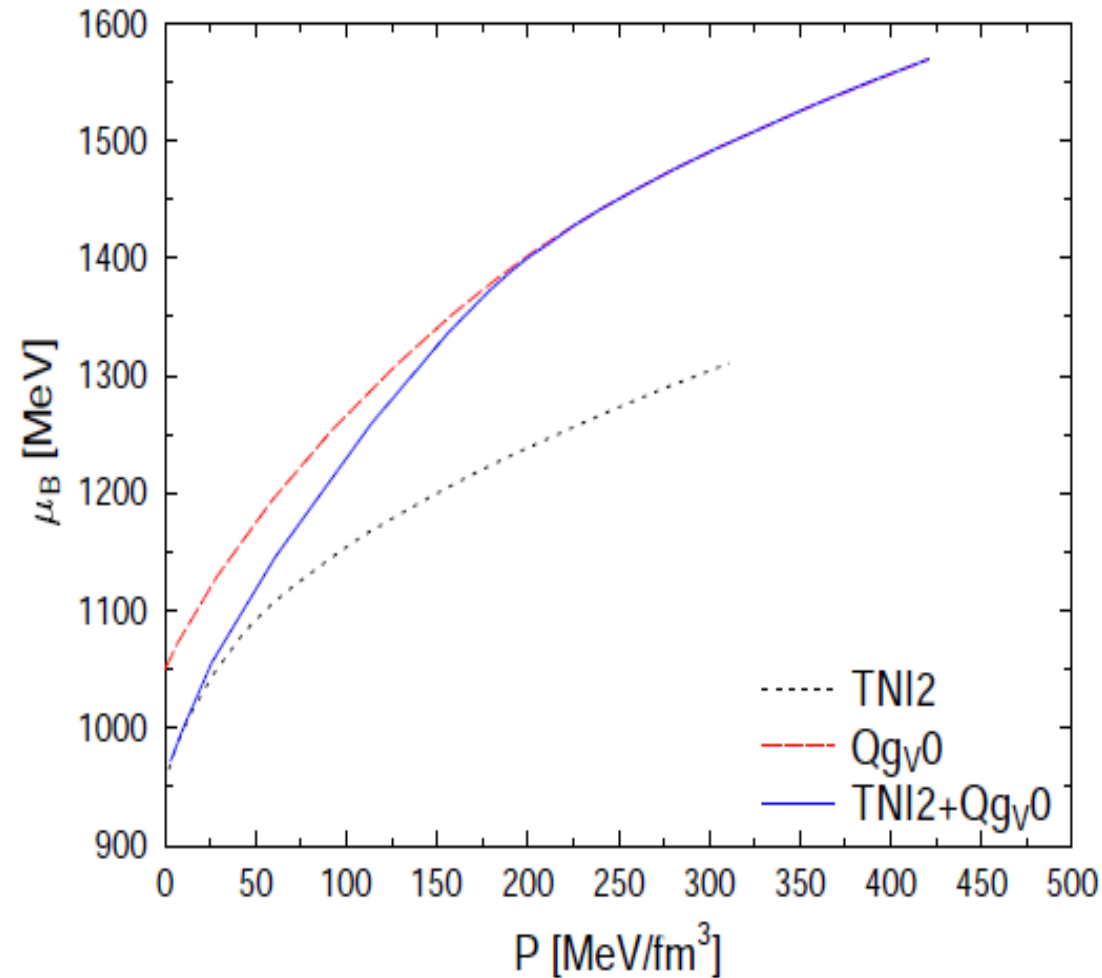
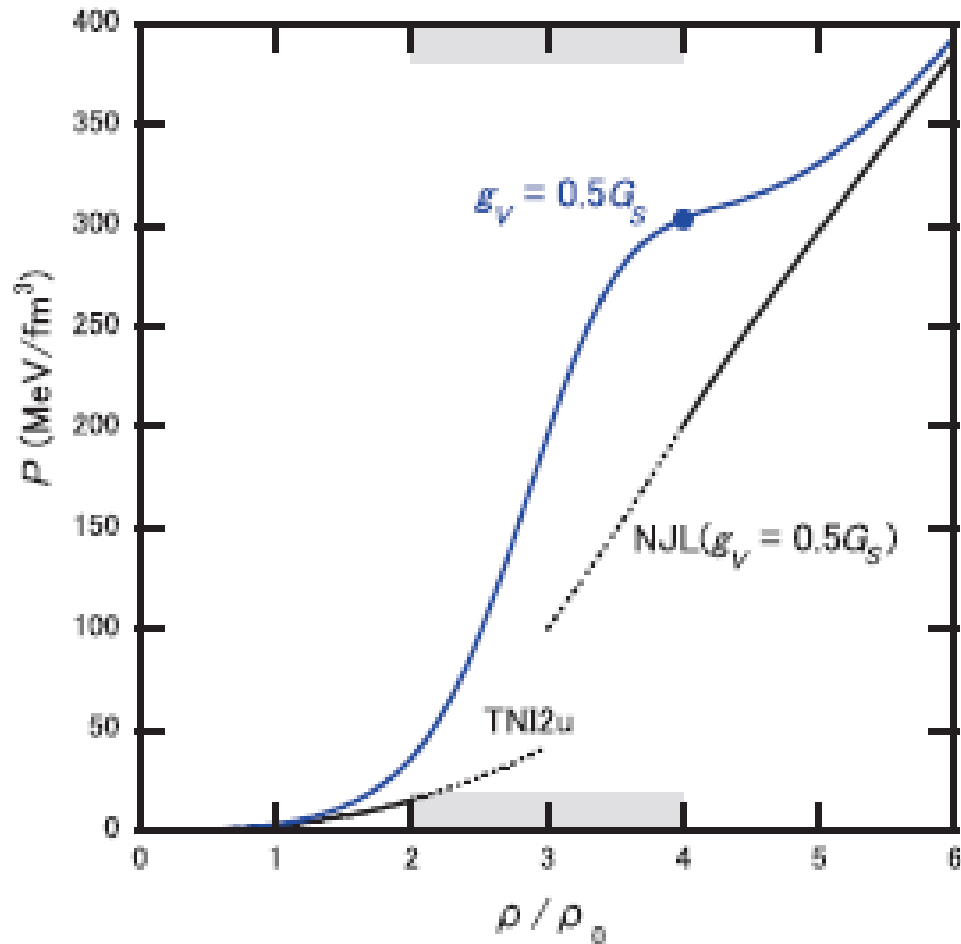


# Attn: Interpolation between energy densities $\epsilon(\rho)$ , not $P(\rho)$



**NOTE:** After a strong stiffening one observes the “dip” in the speed of sound which is typical for a phase transition and corresponds to the “plateau” in  $P(\rho)$

# Attn: Interpolation between energy densities $\epsilon(\rho)$ , not $P(\rho)$

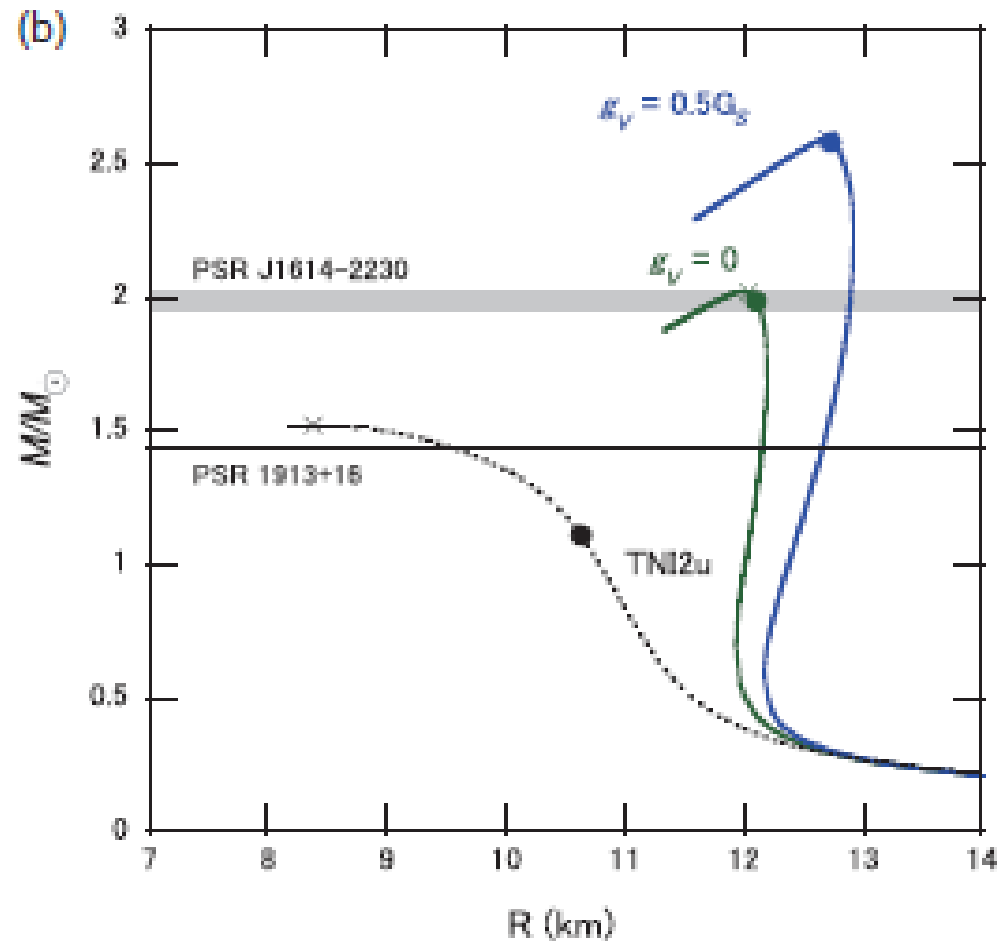
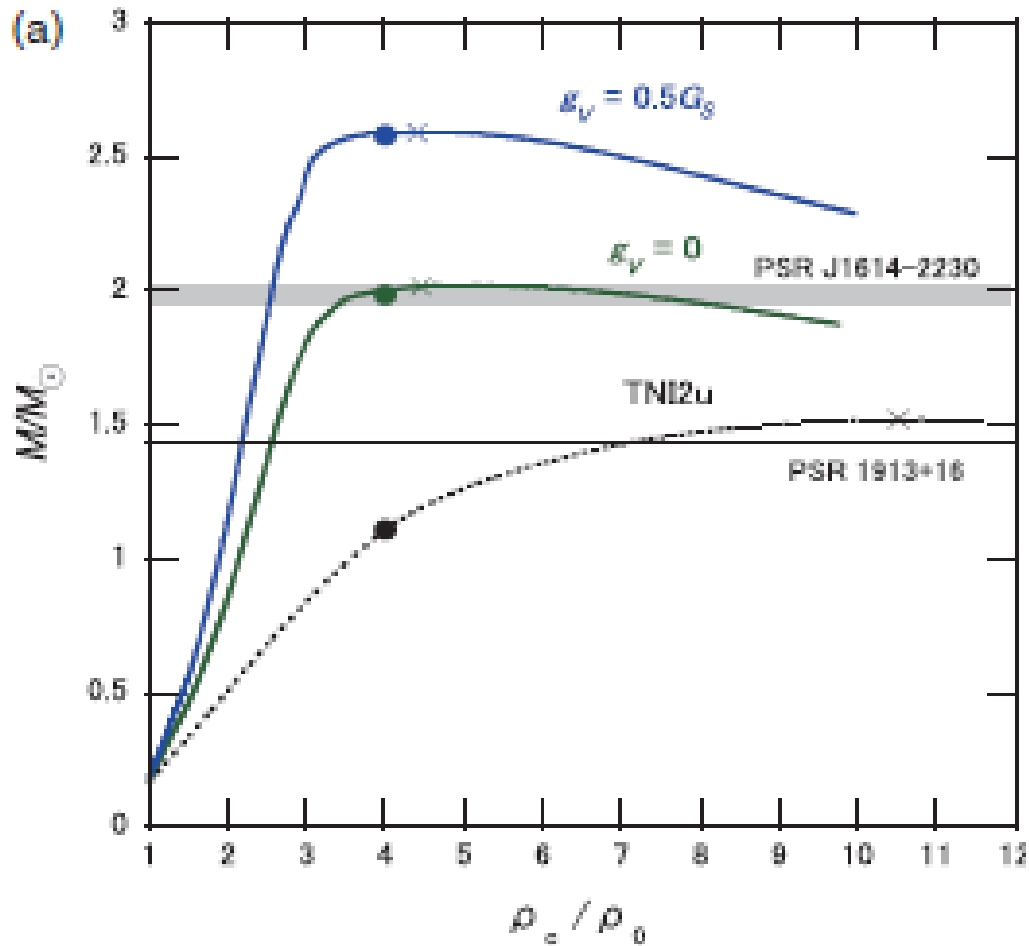


**NOTE:** This interpolation procedure in  $\epsilon(\rho)$  is not only thermodynamically consistent, but also a true interpolation, as can be seen from  $P(\mu)$  or its inversion  $\mu(P)$ .

Courtesy: Matthias Hempel, using data from arxiv:1212.6803v2

For hybrid star EoS with interpolation in  $P(\mu)$ , see arxiv:1302.6275; arxiv:1310.3803

# Attn: Interpolation between energy densities $\epsilon(\rho)$ , not $P(\rho)$



**Attention:**

Results with interpolation between energy densities  $\epsilon(\rho)$  are different from those with interpolation in pressures  $P(\rho)$

Which one is correct? ...

## 2. Microphysical approach to strong 1<sup>st</sup> order PT

**2. Another “three-window picture of dense matter”**

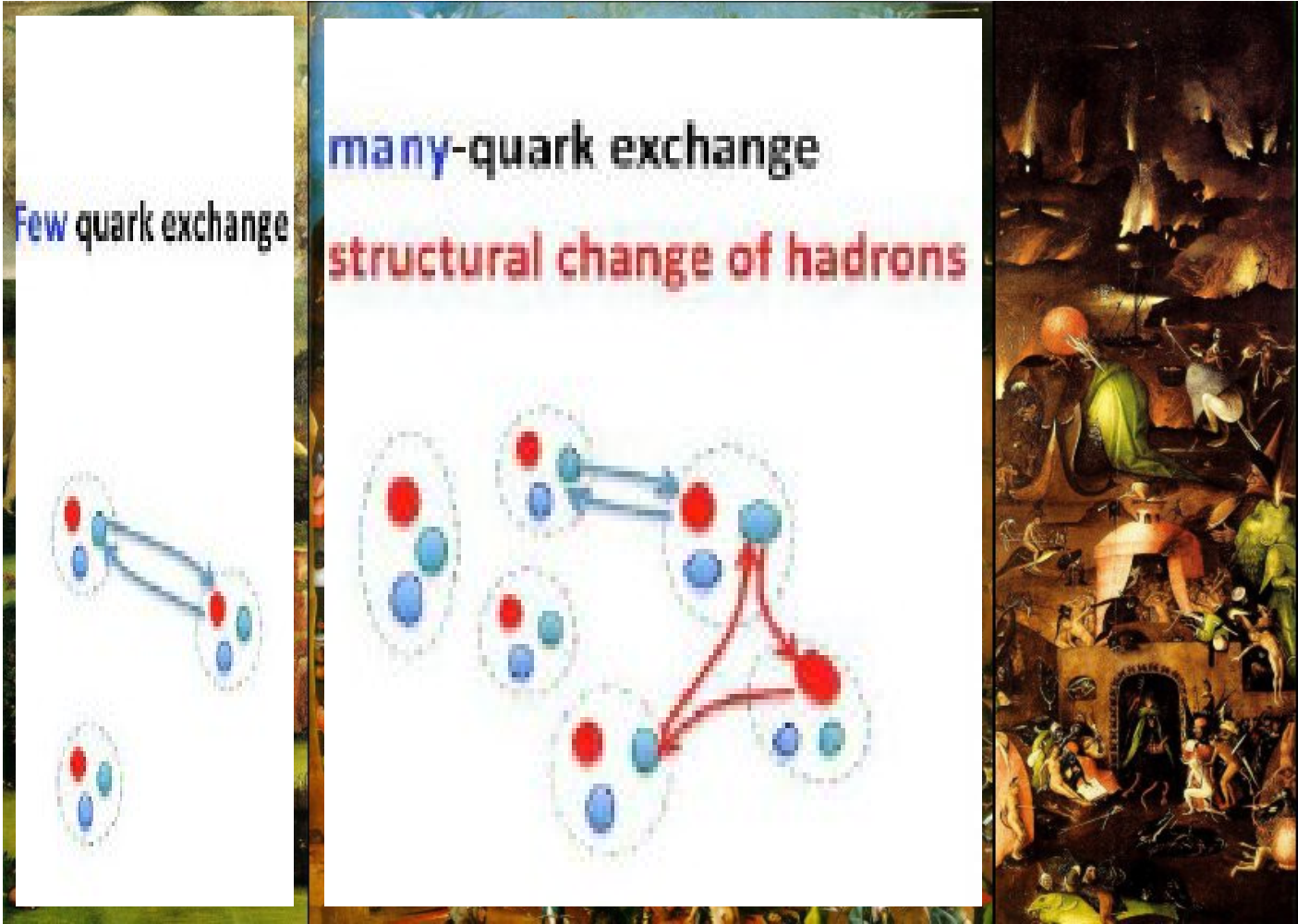




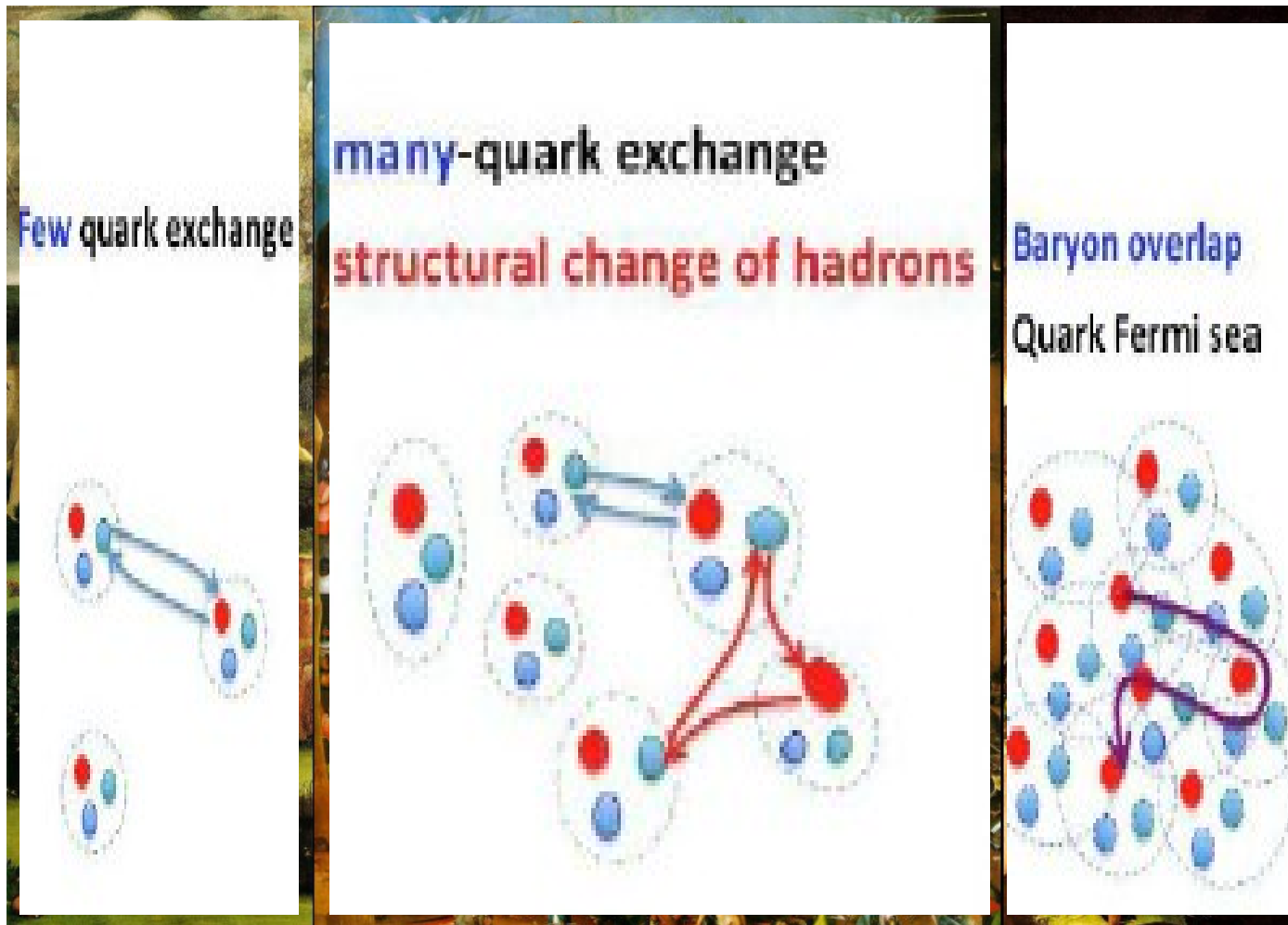
# 2. Another “three-window picture of dense matter”



# 2. Another “three-window picture of dense matter”



## 2. Another “three-window picture of dense matter”



Toru Kojo,  
EPJA 52,  
51 (2016)



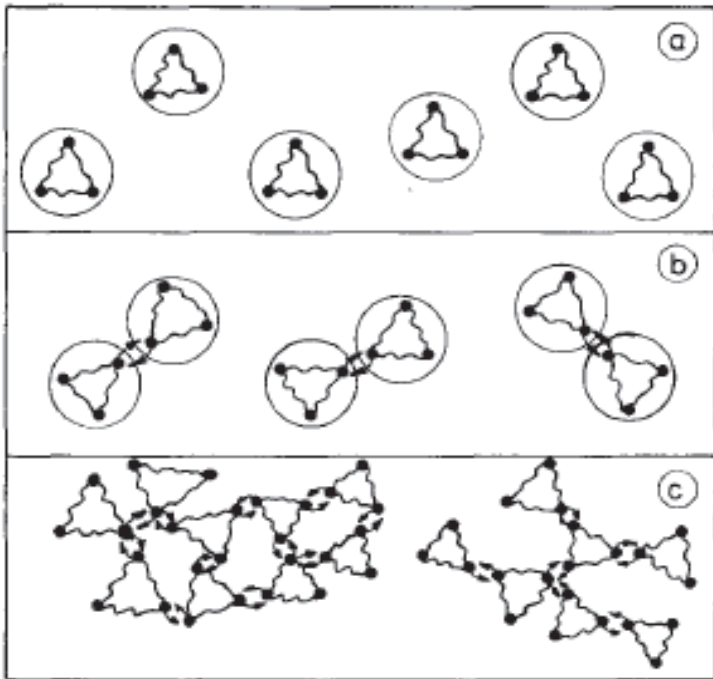


Toru Kojo

& D.B.

(@ CPOD 2016 in Wroclaw)

# 2.1. Pauli blocking among baryons

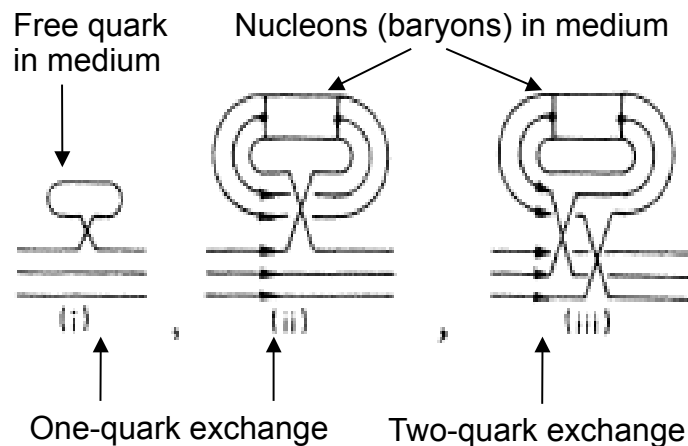


a) Low density: Fermi gas of nucleons (baryons)

b) ~ saturation: Quark exchange interaction and Pauli blocking among nucleons (baryons)

c) high density: Quark cluster matter (string-flip model ...)

Roepke & Schulz, Z. Phys. C 35, 379 (1987); Roepke, DB, Schulz, PRD 34, 3499 (1986)



Nucleon (baryon) self-energy --> Energy shift

$$\begin{aligned} \Delta E_{\nu P}^{\text{Pauli}} &= \sum_{123} |\psi_{\nu P}(123)|^2 [E(1) + E(2) + E(3) - E_{\nu P}^0] [f_{\alpha_1}(1) + f_{\alpha_2}(2) + f_{\alpha_3}(3)] \\ &+ \sum_{123} \sum_{456} \sum_{\nu P'} \psi_{\nu P}^*(123) \psi_{\nu P'}(456) f_3(E_{\nu P'}^0) \{ \delta_{36} \psi_{\nu P}(123) \psi_{\nu P'}^*(456) - \psi_{\nu P}(453) \psi_{\nu P'}^*(126) \} \\ &\quad \times [E(1) + E(2) + E(3) + E(4) + E(5) + E(6) - E_{\nu P}^0 - E_{\nu P'}^0] \\ &= \Delta E_{\nu P}^{\text{Pauli, free}} + \Delta E_{\nu P}^{\text{Pauli, bound}} \end{aligned}$$





PHYSICAL REVIEW D

VOLUME 34, NUMBER 11

1 DECEMBER 1986

## Pauli quenching effects in a simple string model of quark/nuclear matter

G. Röpke and D. Blaschke

*Department of Physics, Wilhelm-Pieck-Universität, 2500 Rostock, German Democratic Republic*

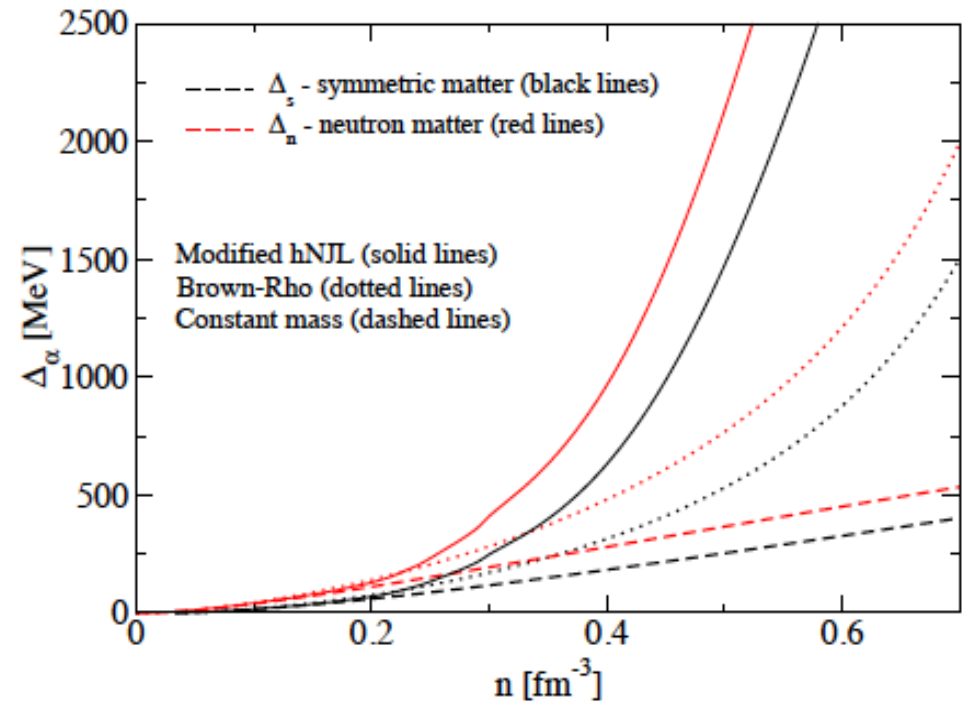
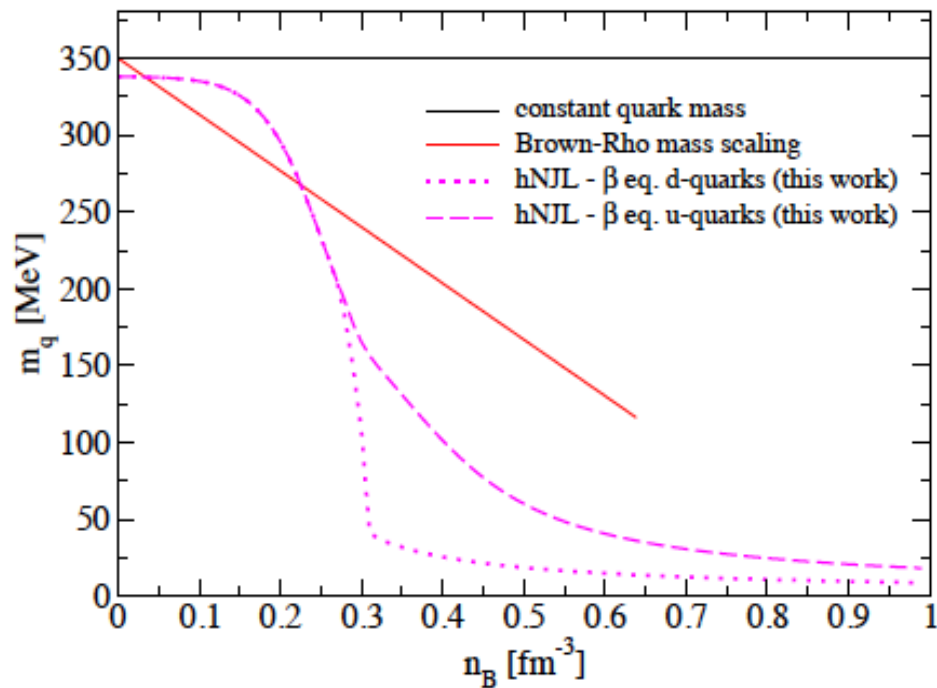
H. Schulz

*Central Institute for Nuclear Research, Rossendorf, 8051 Dresden, German Democratic Republic  
and The Niels Bohr Institute, 2100 Copenhagen, Denmark*

(Received 16 December 1985)

## 2.1. Pauli blocking among baryons – details

**New aspect: chiral restoration --> dropping quark mass**



**Increased baryon swelling at supersaturation densities:  
--> dramatic enhancement of the Pauli repulsion !!**

## 2.2. Pauli blocking among baryons – results

**New EoS: Joining RMF (Linear Walecka) for pointlike baryons with chiral Pauli blocking**

$$p = \frac{1}{4\pi^2} \sum_{\nu} \left[ -E_{\nu}^* m_{\nu}^{*2} p_{F\nu} + \frac{2}{3} E_{\nu}^* p_{F\nu}^3 + m_{\nu}^{*4} \log \left( \frac{E_{\nu}^* + p_{F\nu}}{m_{\nu}^*} \right) \right]$$

$$+ \frac{1}{2} \left( \frac{g_{\omega}}{m_{\omega}} \right)^2 n^2 - \frac{1}{2} \left( \frac{g_{\sigma}}{m_{\sigma}} \right)^2 n_s^2 + p_{ex};$$

$$\epsilon = \frac{1}{4\pi^2} \sum_{\nu} \left[ 2 E_{\nu}^{*3} p_{F\nu} - E_{\nu}^* m_{\nu}^{*2} p_{F\nu} - m_{\nu}^{*4} \log \left( \frac{E_{\nu}^* + p_{F\nu}}{m_{\nu}^*} \right) \right]$$

$$+ \frac{1}{2} \left( \frac{g_{\omega}}{m_{\omega}} \right)^2 n^2 + \frac{1}{2} \left( \frac{g_{\sigma}}{m_{\sigma}} \right)^2 n_s^2 + \epsilon_{ex},$$

$$\mu_{ex,\nu} = \Delta_{\nu}(n, x) = \Sigma_{\nu}(p_{F,\nu}; p_{Fn}, p_{Fp}),$$

$$\epsilon_{ex} = \sum_{\nu} \int_0^n dn' \{ x \Delta_p(n', x) + (1-x) \Delta_n(n', x) \},$$

$$p_{ex} = \sum_{\nu} \mu_{ex,\nu} n_{\nu} - \epsilon_{ex},$$

$$n_{s,\nu} = \frac{m_{\nu}^*}{\pi^2} \left[ E_{\nu}^* p_{F\nu} - m_{\nu}^{*2} \log \left( \frac{E_{\nu}^* + p_{F\nu}}{m_{\nu}^*} \right) \right],$$

$$E_{\nu}^* = \sqrt{m_{\nu}^{*2} + p_{F\nu}^2}$$

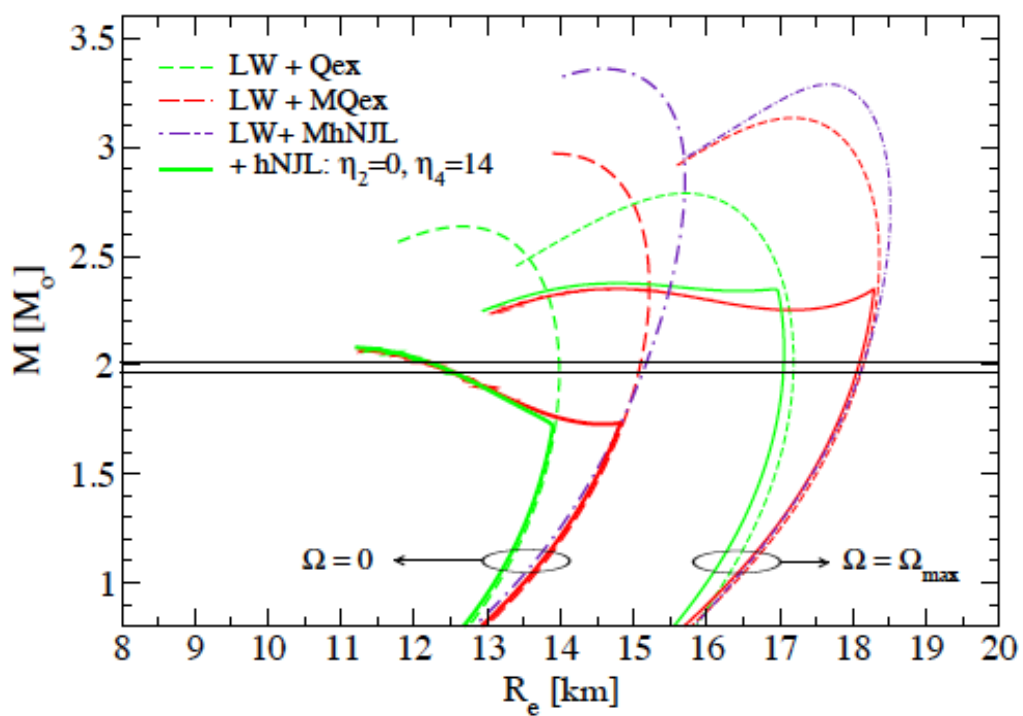
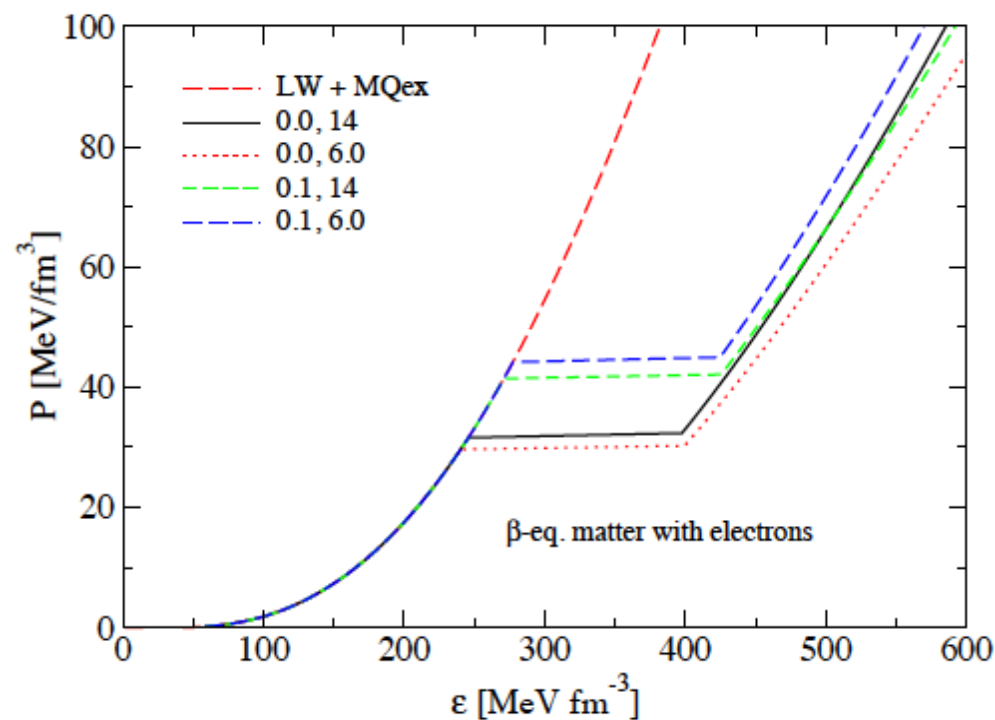
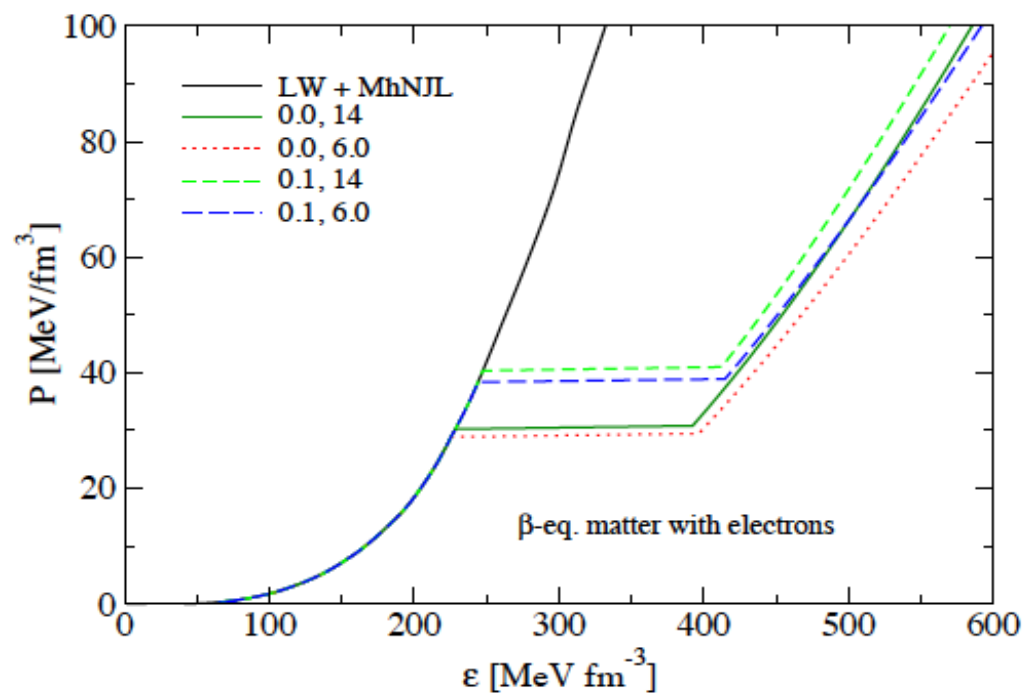
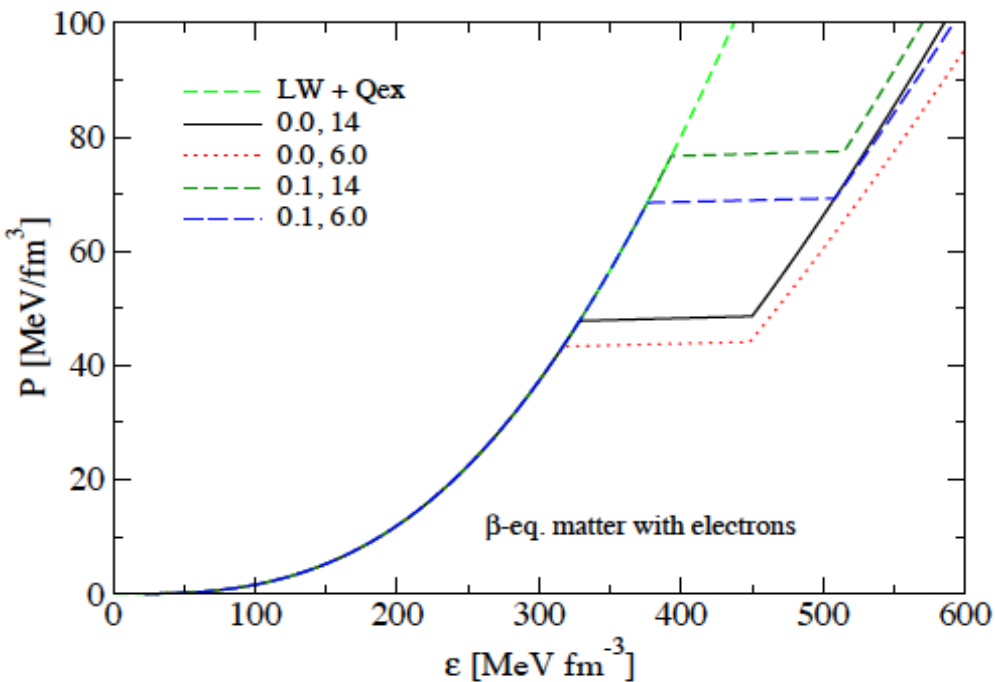
$$n_{\nu} = \frac{p_{F\nu}^3}{3\pi^2},$$

$$m_{\nu}^* = m_{\nu} - \left( \frac{g_{\sigma}}{m_{\sigma}} \right)^2 n_{s,\nu},$$

$$\mu_{\nu} = E_{\nu}^* + \left( \frac{g_{\omega}}{m_{\omega}} \right)^2 n_{\nu} + \mu_{ex,\nu}.$$



## 2.2. Pauli blocking among baryons – results





## 2.3. Pauli blocking among baryons – Summary

Pauli blocking selfenergy (cluster meanfield) calculable in potential models for baryon structure

Partial replacement of other short-range repulsion mechanisms (vector meson exchange)

Modern aspects:

- onset of chiral symmetry restoration enhances nucleon swelling and Pauli blocking at high  $n$
- quark exchange among baryons  $\rightarrow$  six-quark wavefunction  $\rightarrow$  “bag melting”  $\rightarrow$  deconfinement

Chiral stiffening of nuclear matter  $\rightarrow$  reduces onset density for deconfinement

Hybrid EoS:

Convenient generalization of RMF models,

Take care: eventually aspects of quark exchange already in density dependent vertices!

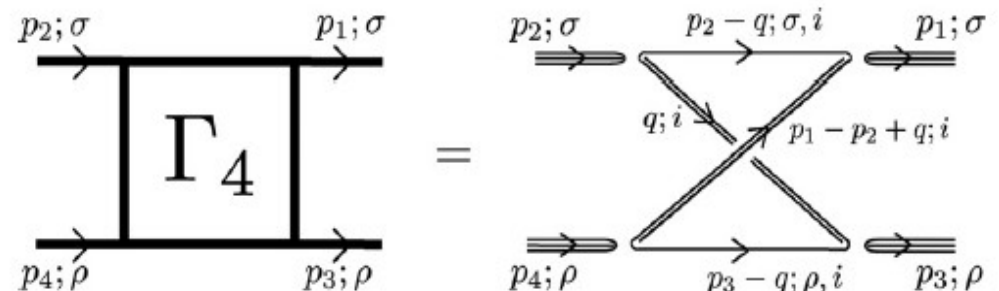
Other baryons:

- hyperons
- deltas

Again calculable, partially done in nonrelativistic quark exchange models, chiral effects not yet!

Relativistic generalization:

Box diagrams of quark-diquark model ...





Jean-Paul Blaizot

Kenji Maeda

D.B.

Helmholtz International Summer School  
“Dense QCD Phases in Heavy-Ion Collisions”  
Dubna, August 21 – September 4, 2010

## 2.4. Pauli blocking effect → Excluded volume

Well known from modeling dissociation of clusters in the supernova EoS:

- excluded volume: Lattimer-Swesty (1991), Shen-Toki-Oyamatsu-Sumiyoshi (1996), ...
- Pauli blocking: Roepke-Grigo-Sumiyoshi-Shen (2003), Typel et al. PRC 81 (2010)
- excl. Vol. vs. Pauli blocking: Hempel, Schaffner-Bielich, Typel, Roepke PRC 84 (2011)

**Here:** nucleons as quark clusters with finite size --> excluded volume effect !

Available volume fraction:  $\Phi = V_{av}/V = 1 - v \sum_{i=n,p} n_i$ ,  $v = \frac{1}{2} \frac{4\pi}{3} (2r_{nuc})^3 = 4V_{nuc}$

Equations of state for T=0 nuclear matter:  $p_{tot}(\mu_n, \mu_p) = \frac{1}{\Phi} \sum_{i=n,p} p_i + p_{mes}$ ,

$$\mathcal{E}_{tot}(\mu_n, \mu_p) = -p_{tot} + \sum_{i=n,p} \mu_i n_i$$

$$p_i = \frac{1}{4} (E_i n_i - m_i^* n_i^{(s)}),$$

$$n_i = \frac{\Phi}{3\pi^3} k_i^3,$$

$$n_i^{(s)} = \frac{\Phi m_i^*}{2\pi^2} \left[ E_i k_i - (m_i^*)^2 \ln \frac{k_i + E_i}{m_i^*} \right],$$

$$E_i = \sqrt{k_i^2 + (m_i^*)^2} = \mu_i - V_i - \frac{v}{\Phi} \sum_{j=p,n} p_j,$$

Effective mass:  $m_i^* = m_i - S_i$ .

Scalar meanfield:  $S_i \sim n_i^{(s)}$

Vector meanfield:  $V_i \sim n_i$



## 2.5. Stiff quark matter at high densities

S. Benic, Eur. Phys. J. A 50, 111 (2014)

$$\mathcal{L} = \bar{q}(i\not{\partial} - m)q + \mu_q \bar{q}\gamma^0 q + \mathcal{L}_4 + \mathcal{L}_8, \quad \mathcal{L}_4 = \frac{g_{20}}{\Lambda^2} [(\bar{q}q)^2 + (\bar{q}i\gamma_5\tau q)^2] - \frac{g_{02}}{\Lambda^2} (\bar{q}\gamma_\mu q)^2,$$
$$\mathcal{L}_8 = \frac{g_{40}}{\Lambda^8} [(\bar{q}q)^2 + (\bar{q}i\gamma_5\tau q)^2]^2 - \frac{g_{04}}{\Lambda^8} (\bar{q}\gamma_\mu q)^4 - \frac{g_{22}}{\Lambda^8} (\bar{q}\gamma_\mu q)^2 [(\bar{q}q)^2 + (\bar{q}i\gamma_5\tau q)^2]$$

**Meanfield approximation:**  $\mathcal{L}_{\text{MF}} = \bar{q}(i\not{\partial} - M)q + \tilde{\mu}_q \bar{q}\gamma^0 q - U,$

$$M = m + 2\frac{g_{20}}{\Lambda^2} \langle \bar{q}q \rangle + 4\frac{g_{40}}{\Lambda^8} \langle \bar{q}q \rangle^3 - 2\frac{g_{22}}{\Lambda^8} \langle \bar{q}q \rangle \langle q^\dagger q \rangle^2,$$

$$\tilde{\mu}_q = \mu_q - 2\frac{g_{02}}{\Lambda^2} \langle q^\dagger q \rangle - 4\frac{g_{04}}{\Lambda^8} \langle q^\dagger q \rangle^3 - 2\frac{g_{22}}{\Lambda^8} \langle \bar{q}q \rangle^2 \langle q^\dagger q \rangle,$$

$$U = \frac{g_{20}}{\Lambda^2} \langle \bar{q}q \rangle^2 + 3\frac{g_{40}}{\Lambda^8} \langle \bar{q}q \rangle^4 - 3\frac{g_{22}}{\Lambda^8} \langle \bar{q}q \rangle^2 \langle q^\dagger q \rangle^2 - \frac{g_{02}}{\Lambda^2} \langle q^\dagger q \rangle^2 - 3\frac{g_{04}}{\Lambda^8} \langle q^\dagger q \rangle^4.$$

**Thermodynamic Potential:**

$$\Omega = U - 2N_f N_c \int \frac{d^3 p}{(2\pi)^3} \left\{ E + T \log[1 + e^{-\beta(E - \tilde{\mu}_q)}] + T \log[1 + e^{-\beta(E + \tilde{\mu}_q)}] \right\} + \Omega_0$$



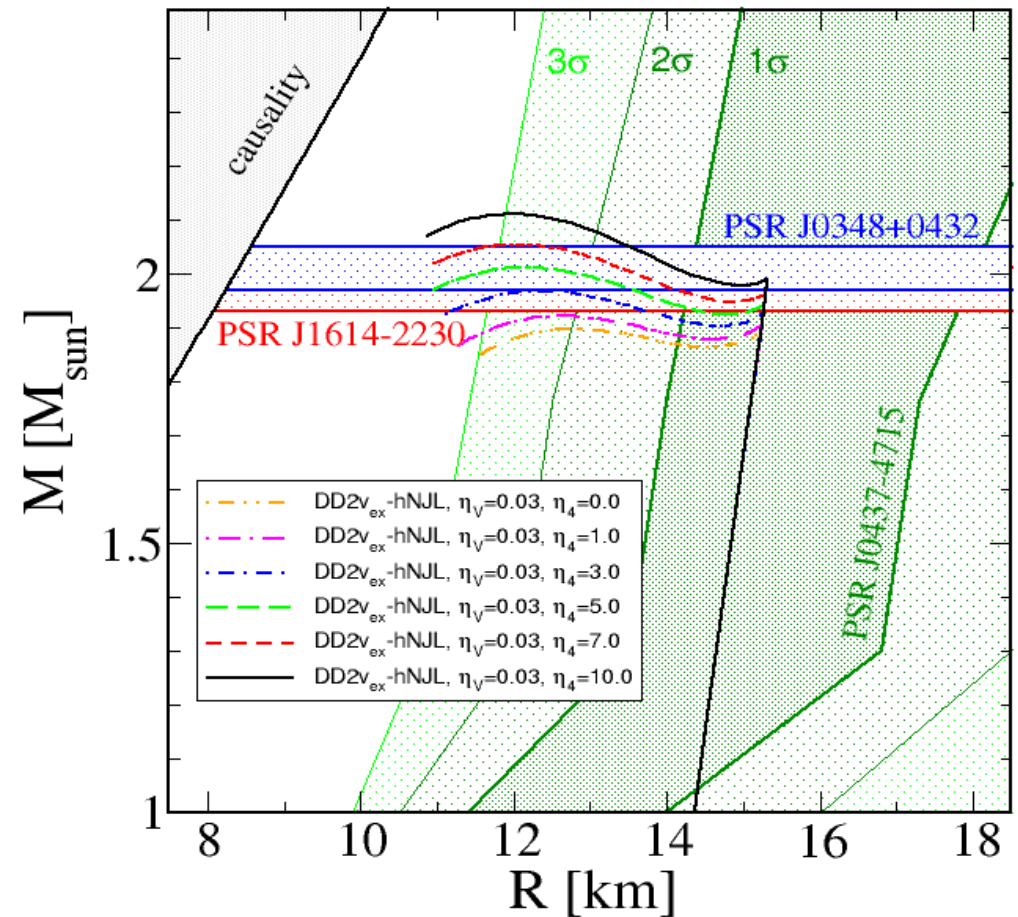
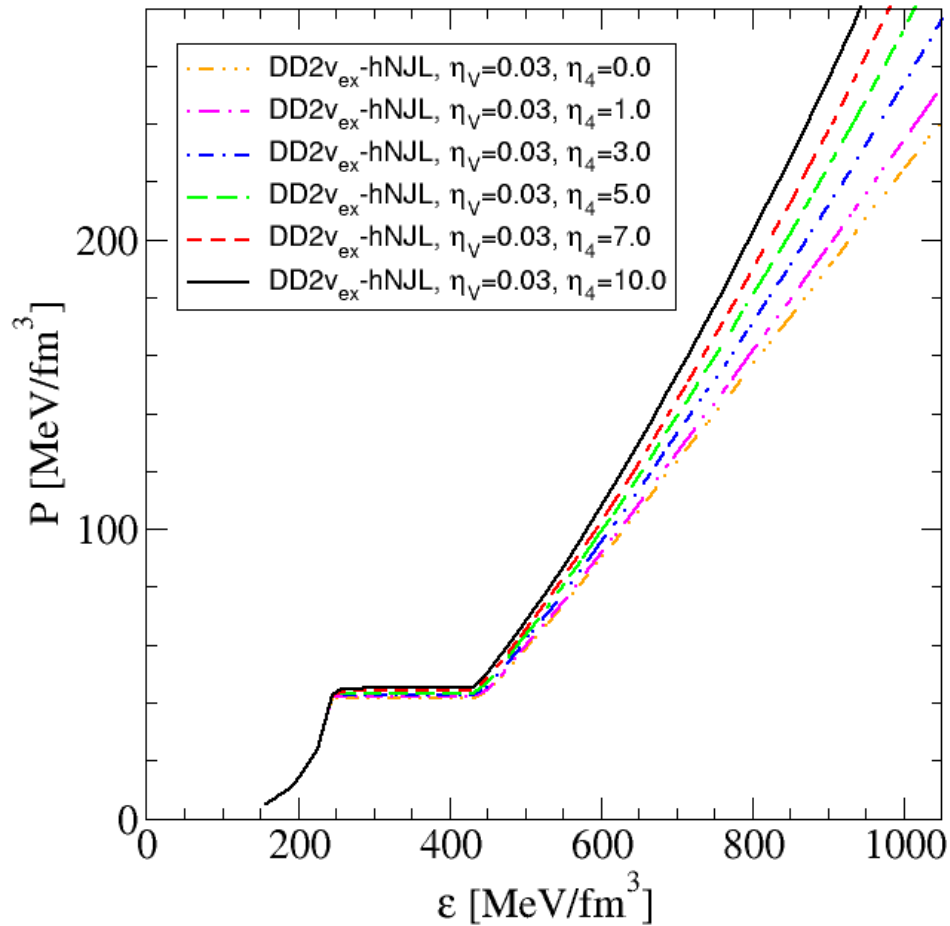


D.B. & Sanjin Benic in Tokyo



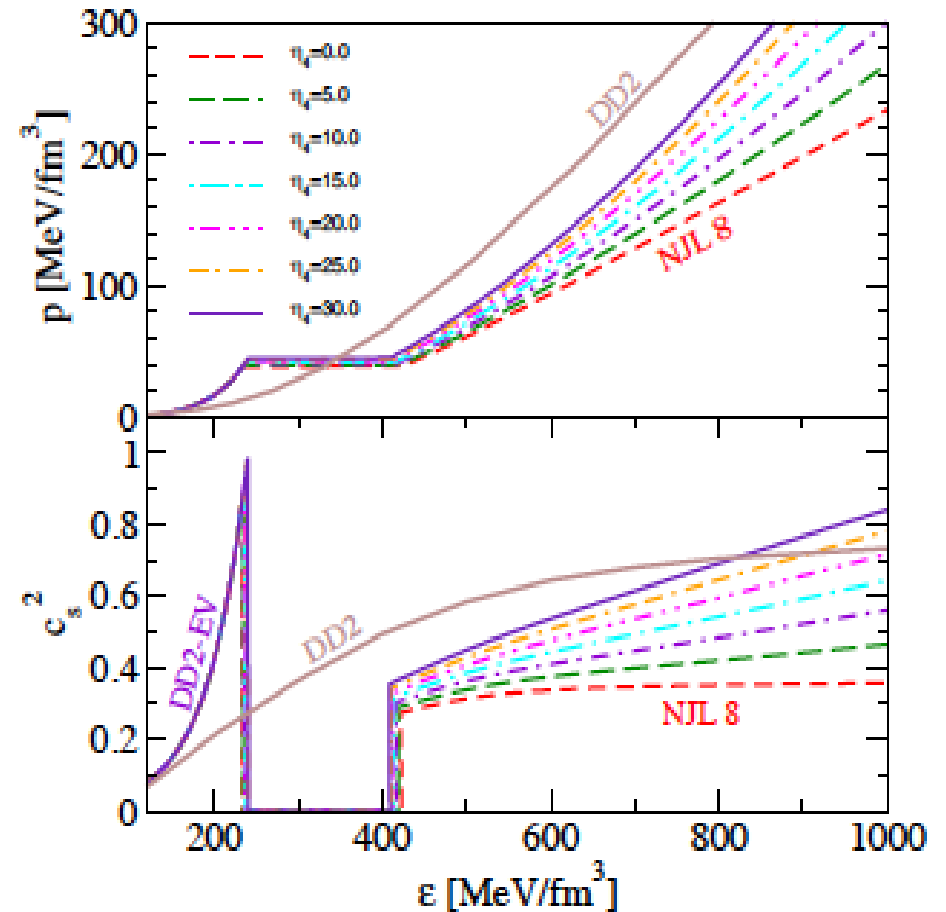
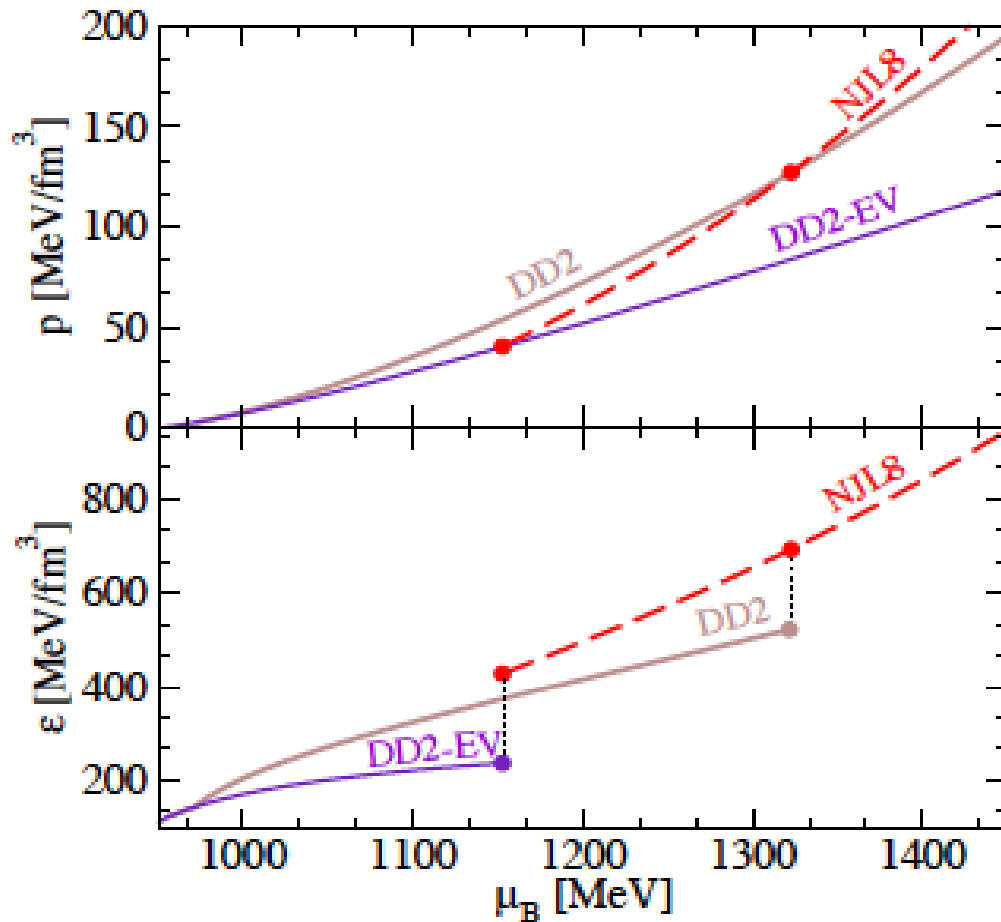
# Result: high-mass twins $\leftrightarrow$ 1st order PT

S. Benic, D. Blaschke, D. Alvarez-Castillo, T. Fischer, S. Typel, arxiv:1411.2856



Hybrid EoS supports  $M$ - $R$  sequences with high-mass twin compact stars

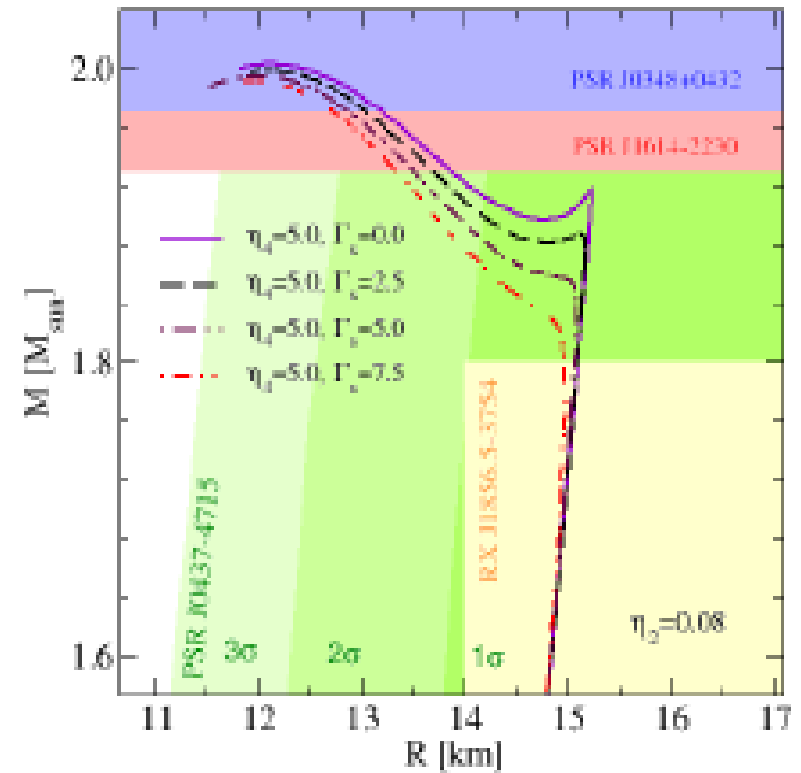
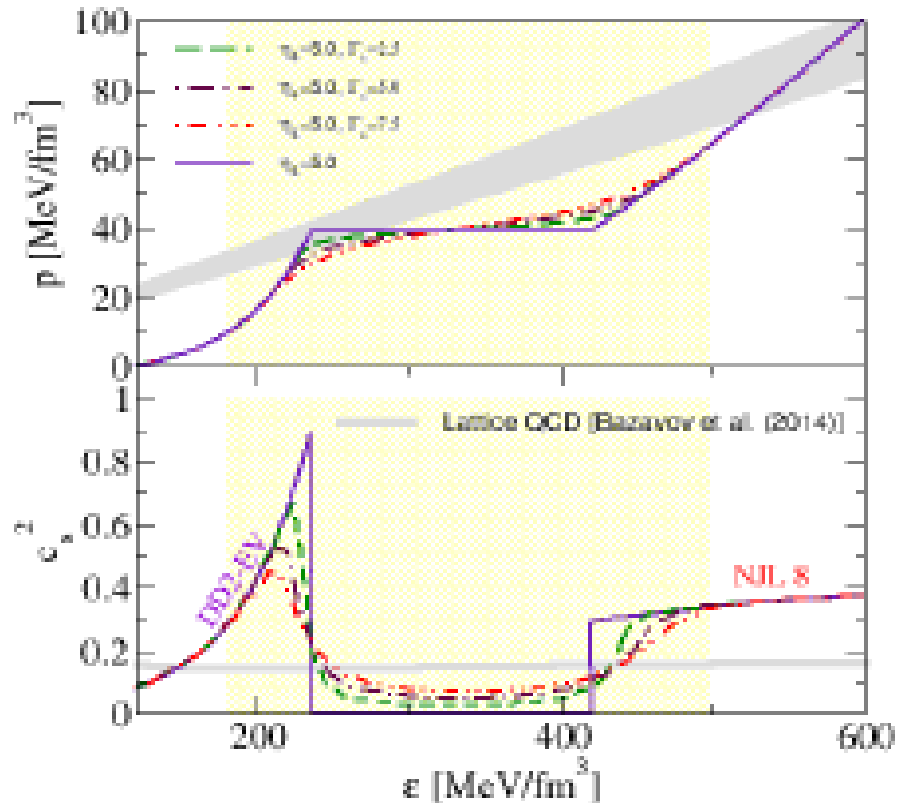
## 2.5. Stiff quark matter at high densities



Here: Stiffening of dense hadronic matter by excluded volume in density-dependent RMF

## 2.5. Stiff quark matter at high densities

Estimate effects of structures in the phase transition region (“pasta”)



High-mass Twins relatively robust against “smoothing” the Maxwell transition construction

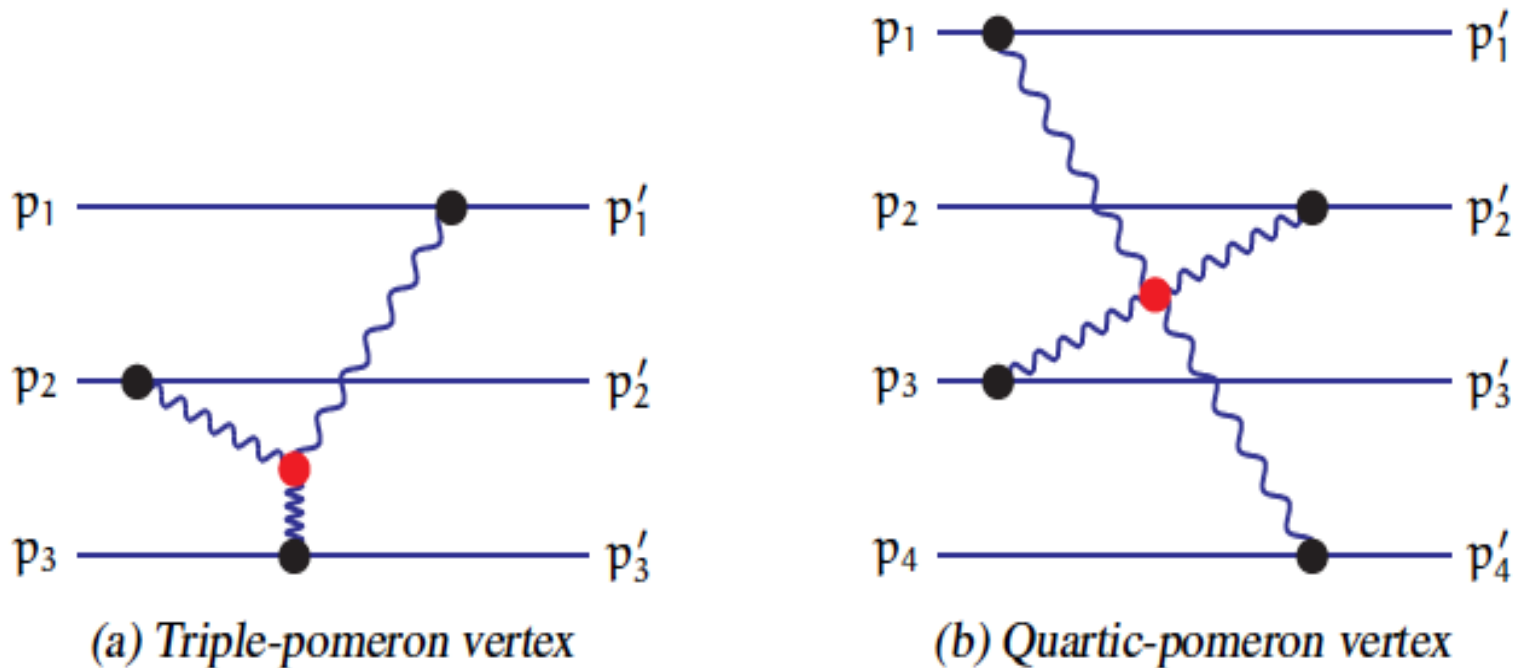
D. Alvarez-Castillo, D.B., arxiv:1412.8463; Phys. Part. Nucl. 46 (2015) 846



## 2.5. Support for universal stiffening of hadron and quark matter at high densities

Universal repulsion from multi-pomeron exchanges in baryon-baryon scattering.  
(Th. A. Rijken, private communication)

Y. Yamamoto, T. Furumoto, N. Yasutake, Th.A. Rijken; EPJA 52 (3) (2016)



The pomerons (wavy lines) couple to different quarks (solid lines) in quark matter (as in the hNJL Lagrangian) or to quarks in different baryons in nuclear matter (giving rise to repulsive 3- and 4- body interactions).



A. Illarionov H.-J. Schulze D.B.  
I. Bombaci ??  
T. Tatsumi N. Yasutake R. Lastowiecki  
(YITP Kyoto, 2010)



Tom Rijken in Kyoto (2016)

# NICA White Paper – selected topics ...

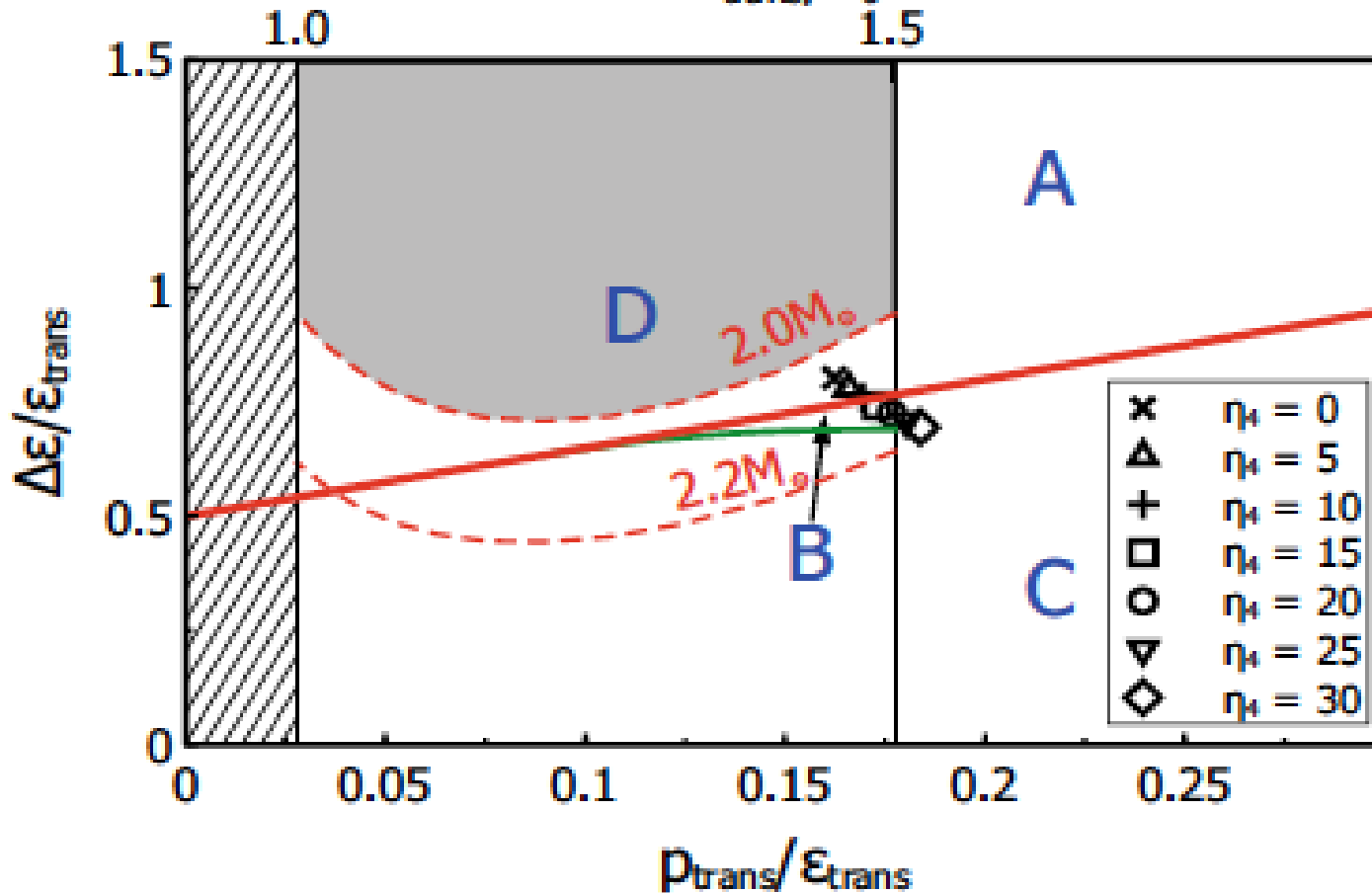
Many cross-relations with astrophysics of compact stars! High-mass twin stars prove CEP !

## #22 Neutron star mass limit at $2M_{\odot}$ supports the existence of a CEP

D. Alvarez-Castillo<sup>1,a</sup>, S. Benic<sup>2,b</sup>, D. Blaschke<sup>1,3,4</sup>, Sophia Han<sup>5,6</sup>, and S. Typel<sup>7</sup>

$$\text{DD2-EV} + c_{\text{QM}}^2 = 0.4$$

$$n_{\text{trans}}/n_0$$



The red line:

$$\frac{\Delta\varepsilon}{\varepsilon_{\text{trans}}} = \frac{1}{2} + \frac{3}{2} \frac{p_{\text{trans}}}{\varepsilon_{\text{trans}}}$$

Based on Hybrid EoS

S. Benic et al. (2015),  
A&A **577**, A40

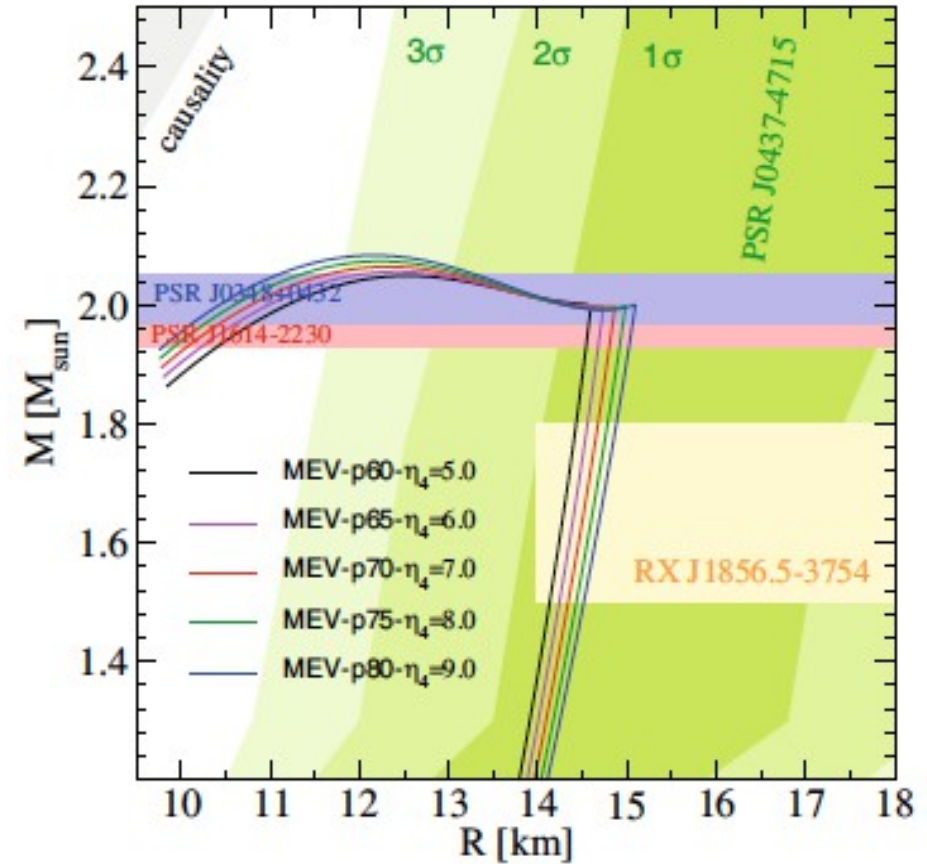
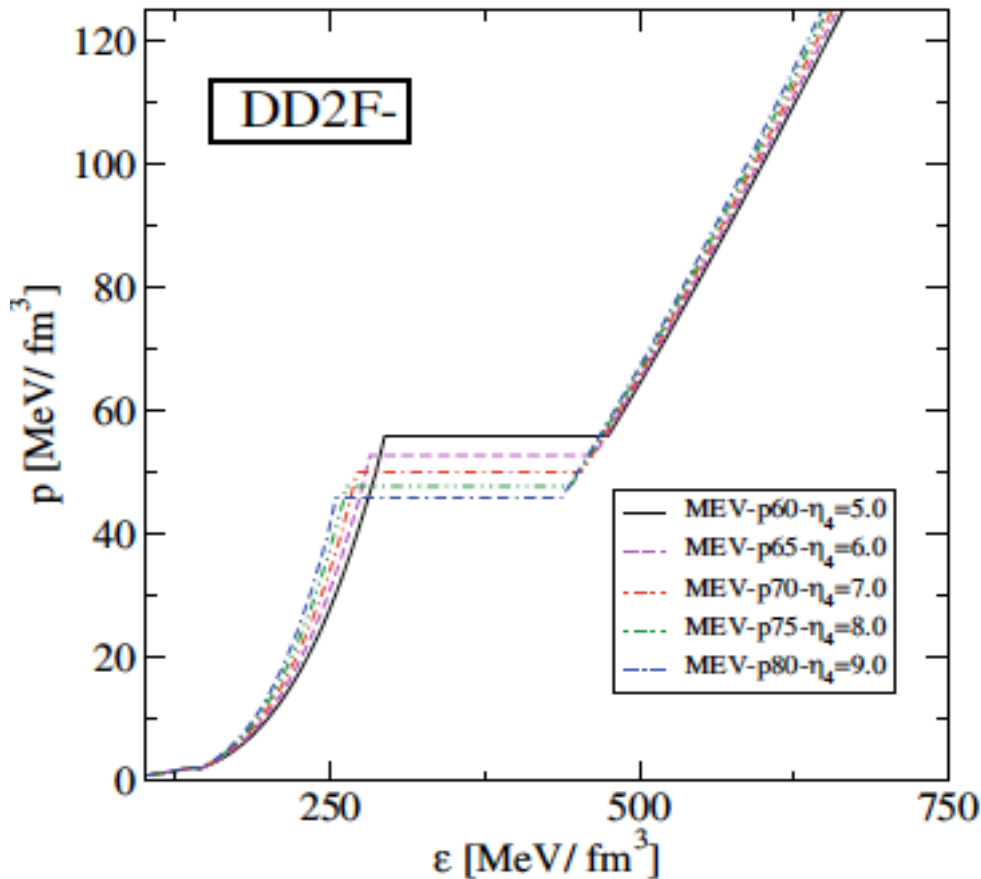


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Many cross-relations with astrophysics of compact stars! High-mass twin stars prove CEP !

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Vary stiffness of hadronic and quark EoS so that onset of instability at  $2 M_{\text{sun}}$ .

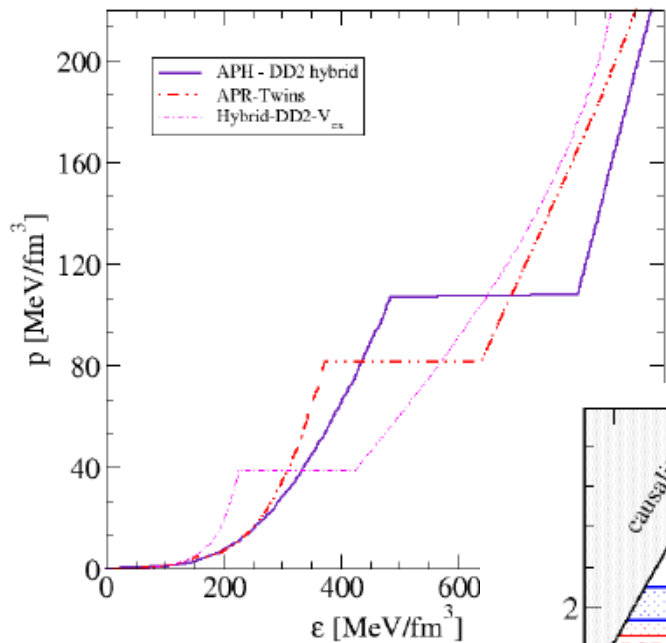


# NICA White Paper – selected topics ...

Many cross-relations with astrophysics of compact stars! High-mass twin stars prove CEP !

## #22 Neutron star mass limit at $2M_{\odot}$ supports the existence of a CEP

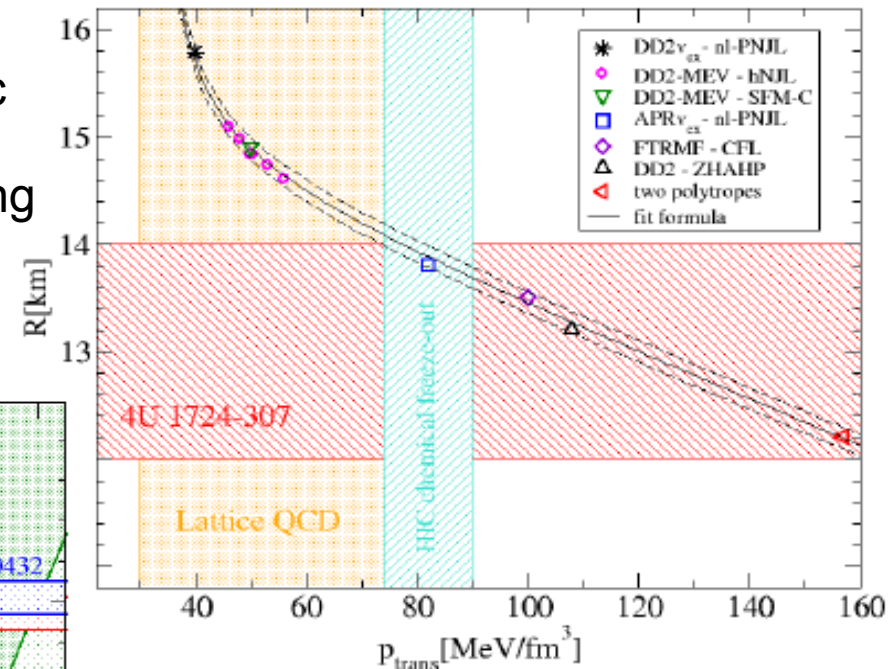
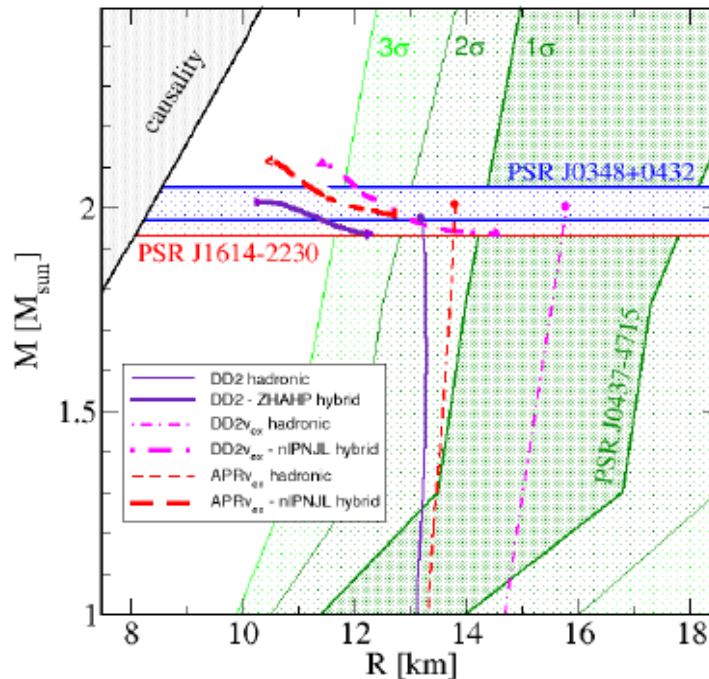
D. Alvarez-Castillo<sup>1,a</sup>, S. Benic<sup>2,b</sup>, D. Blaschke<sup>1,3,4</sup>, Sophia Han<sup>5,6</sup>, and S. Typel<sup>7</sup>



Endpoint of hadronic  
Neutron star config.  
At  $2M_{\text{sun}}$ , then strong  
Phase transition

Strong phase trans.

High-mass twin stars



Universal transition pressure ?

Petran & Rafelski, PRC 88, 021901

$$P_{\text{trans}} = 82 \pm 8 \text{ MeV/fm}^3$$



# Physics is fun ...



Hovik Grigorian

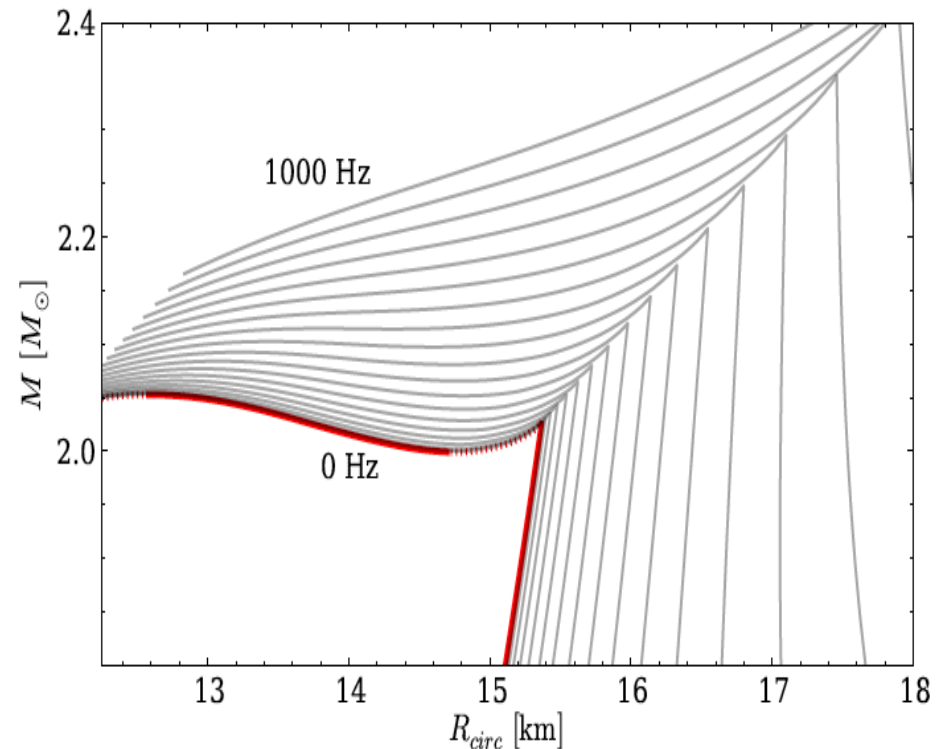
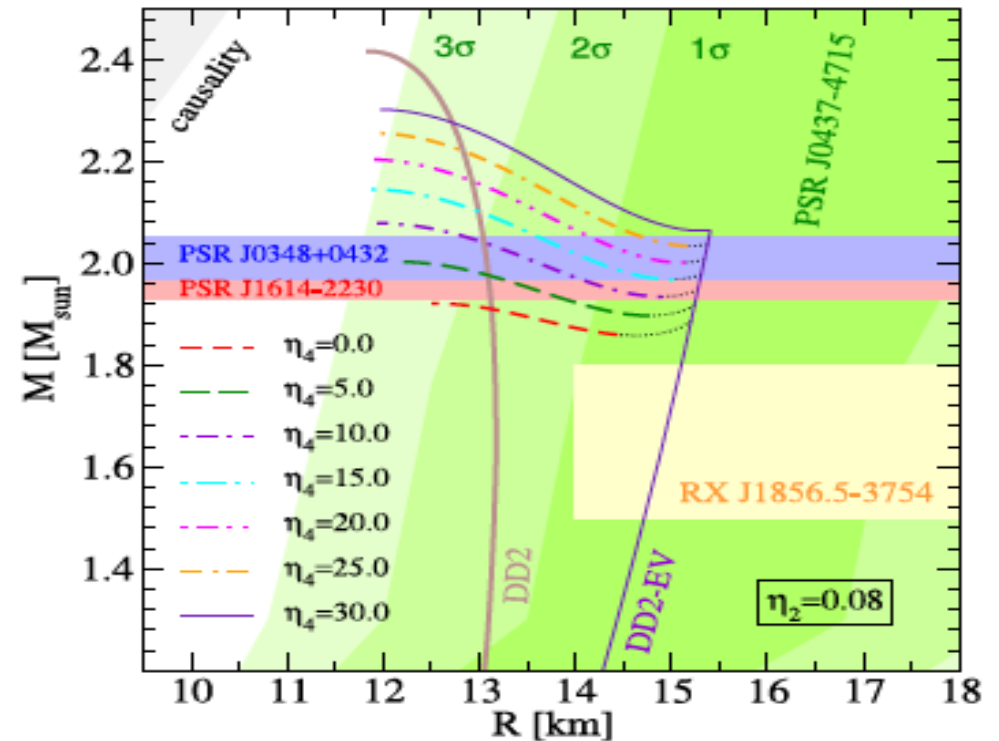
David Edwin Alvarez-Castillo



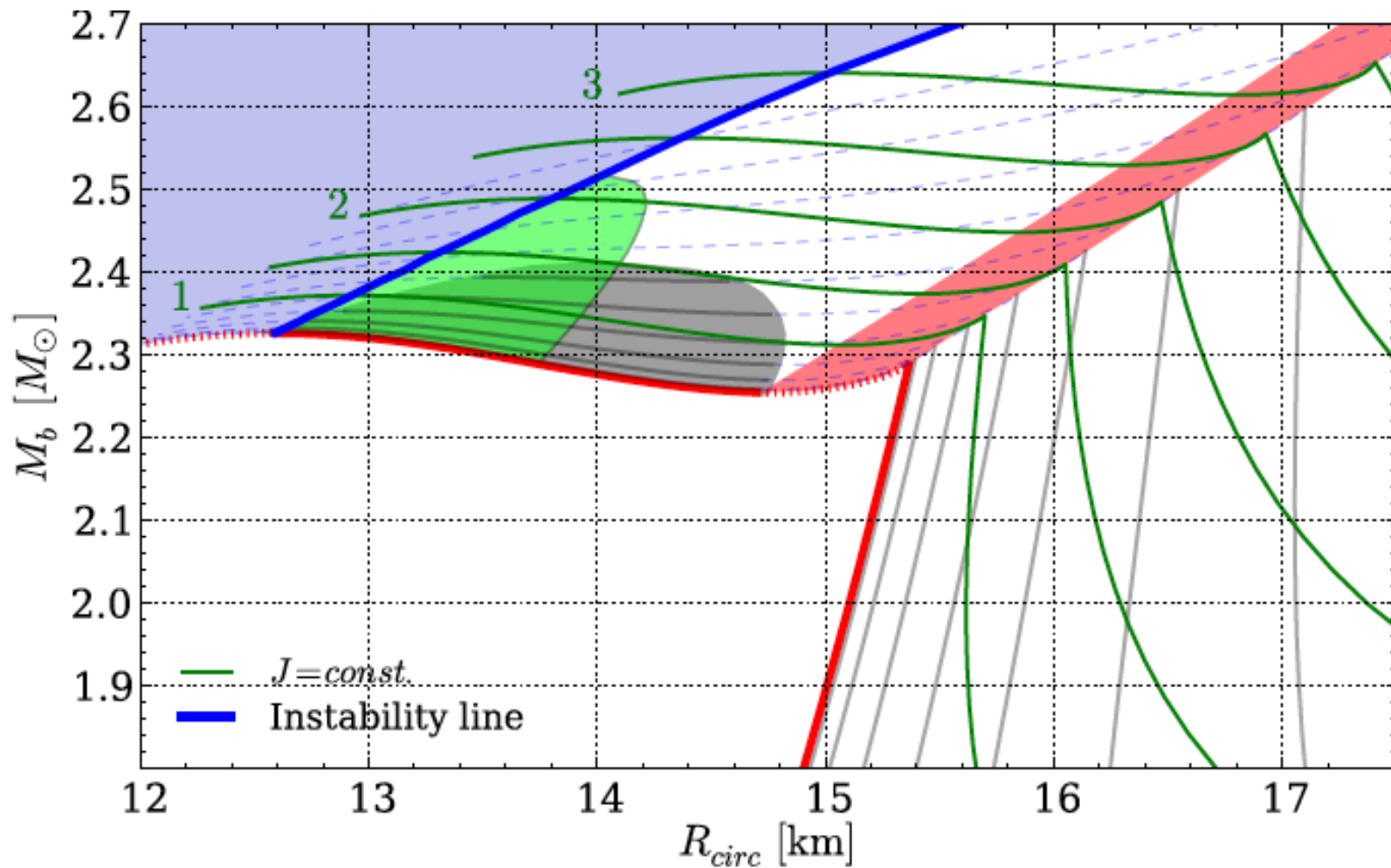
## 2.6. Rotation

- existence of 2  $M_{\text{sun}}$  pulsars and possibility of high-mass twins raises question for their inner structure: (Q)uark or (N)ucleon core ??
    - > degenerate solutions
    - > transition from N to Q branch
  - PSR J1614-2230 is millisecond pulsar, period  $P = 3.41$  ms, consider rotation !
  - transitions N --> Q must be considered for rotating configurations:
    - > how fast can they be?
- (angular momentum  $J$  and baryon mass should be conserved simultaneously)
- similar scenario as fast radio bursts (Falcke-Rezzolla, 2013) or braking index (Glendenning-Pei-Weber, 1997)

M. Bejger, D.B. et al., arxiv:1608.07049



## 2.6. Rotation and stability



**Red region** - strong phase-transition instability,  
**Blue region** - unstable w.r.t axisymmetric oscillations,  
Grey region - no back-bending,  
**Green region** - stable twin branch reached after the mini-collapse from the tip of  $J = const.$  curve, along  $M_b = const.$



## 2.6. Rotation - summary

This type of instability EOS provides a "natural" explanation for:

- ★ Lack of back-bending in radiopulsar timing,
- ★ Spin frequency cut-off at some moderate (but  $>716$  Hz) frequency,
- ★ Falcke & Rezzolla Fast Radio Burst (FRB) engine
  - ★ catastrophic mini-collapse to the second branch (or to a black hole),
  - ★ massive rearrangement of the magnetic field  $\rightarrow$  energy emission.

Astrophysical predictions:

- ★ Way to constraint on  $M_b$ ,  $J$ ,  $I$ , core EOS etc.,
- ★ Specific shape of NS-BH mass function (no mass gap?)
- $\rightarrow$  population of massive, low B-field NSs (radio-dead?),
- $\rightarrow$  population of massive, high B-field NSs (collapse enhances the field?),
- ★ Characteristic burst-like signature in GW emission during the mini-collapse.





Tobias Fischer (Uni Wroclaw)

Morgane Fortin

Michal Bejger (CAMK Warsaw)

### **3. New Bayesian Analysis scheme**

# Measure the cold EoS by Bayesian TOV!

## Bayesian TOV analysis:

Steiner, Lattimer, Brown, ApJ 722 (2010) 33

Most Probable Values for Masses and Radii for Neutron Stars Constrained to Lie on One Mass Versus Radius Curve

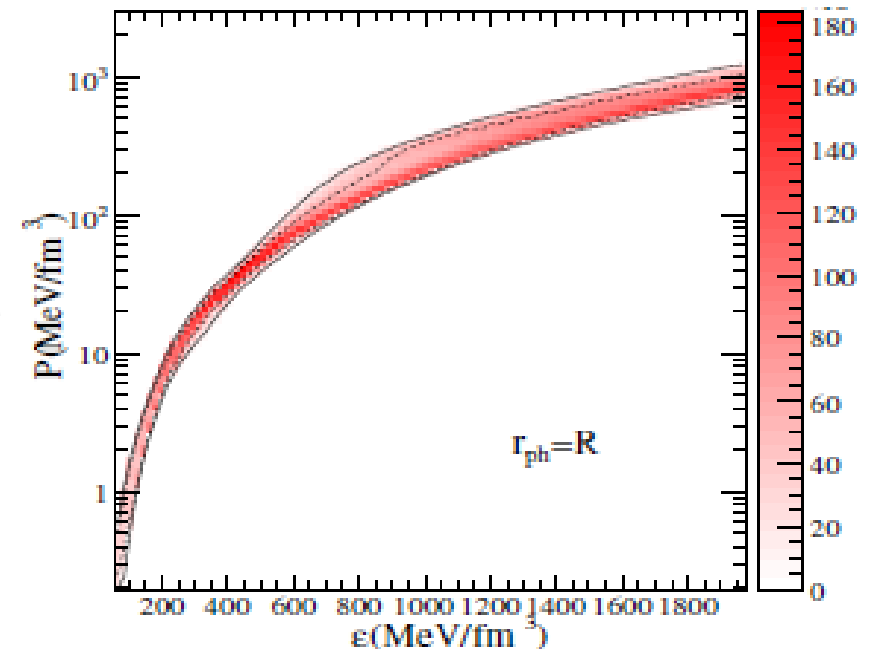
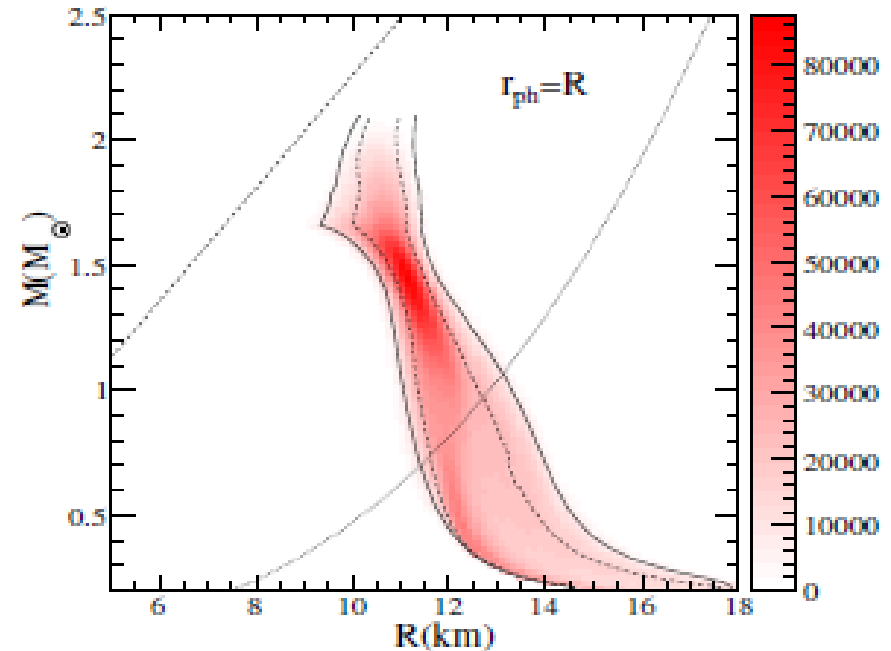
Object	$r_{\text{ph}} = R$		$r_{\text{ph}} \gg R$	
	$M (M_{\odot})$	$R \text{ (km)}$	$M (M_{\odot})$	$R \text{ (km)}$
4U 1608–522	$1.52^{+0.22}_{-0.18}$	$11.04^{+0.53}_{-1.50}$	$1.64^{+0.34}_{-0.41}$	$11.82^{+0.42}_{-0.89}$
EXO 1745–248	$1.55^{+0.12}_{-0.36}$	$10.91^{+0.86}_{-0.65}$	$1.34^{+0.450}_{-0.28}$	$11.82^{+0.47}_{-0.72}$
4U 1820–30	$1.57^{+0.13}_{-0.15}$	$10.91^{+0.39}_{-0.92}$	$1.57^{+0.37}_{-0.31}$	$11.82^{+0.42}_{-0.82}$
M13	$1.48^{+0.21}_{-0.64}$	$11.04^{+1.00}_{-1.28}$	$0.901^{+0.28}_{-0.12}$	$12.21^{+0.18}_{-0.62}$
$\omega$ Cen	$1.43^{+0.26}_{-0.61}$	$11.18^{+1.14}_{-1.27}$	$0.994^{+0.51}_{-0.21}$	$12.09^{+0.27}_{-0.66}$
X7	$0.832^{+1.19}_{-0.051}$	$13.25^{+1.37}_{-3.50}$	$1.98^{+0.10}_{-0.36}$	$11.3^{+0.95}_{-1.03}$

## Caution:

If optical spectra are not measured, the observed X-ray spectrum may not come from the entire surface  
But from a hot spot at the magnetic pole!

J. Trumper, Prog. Part. Nucl. Phys. 66 (2011) 674

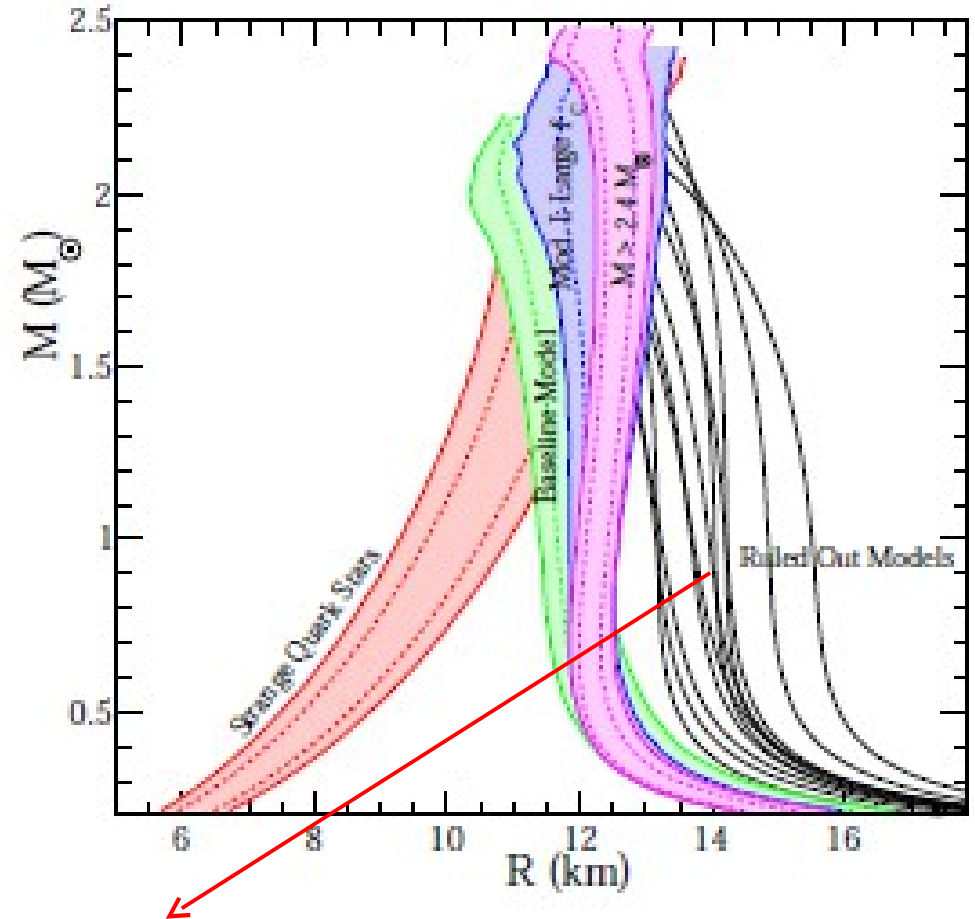
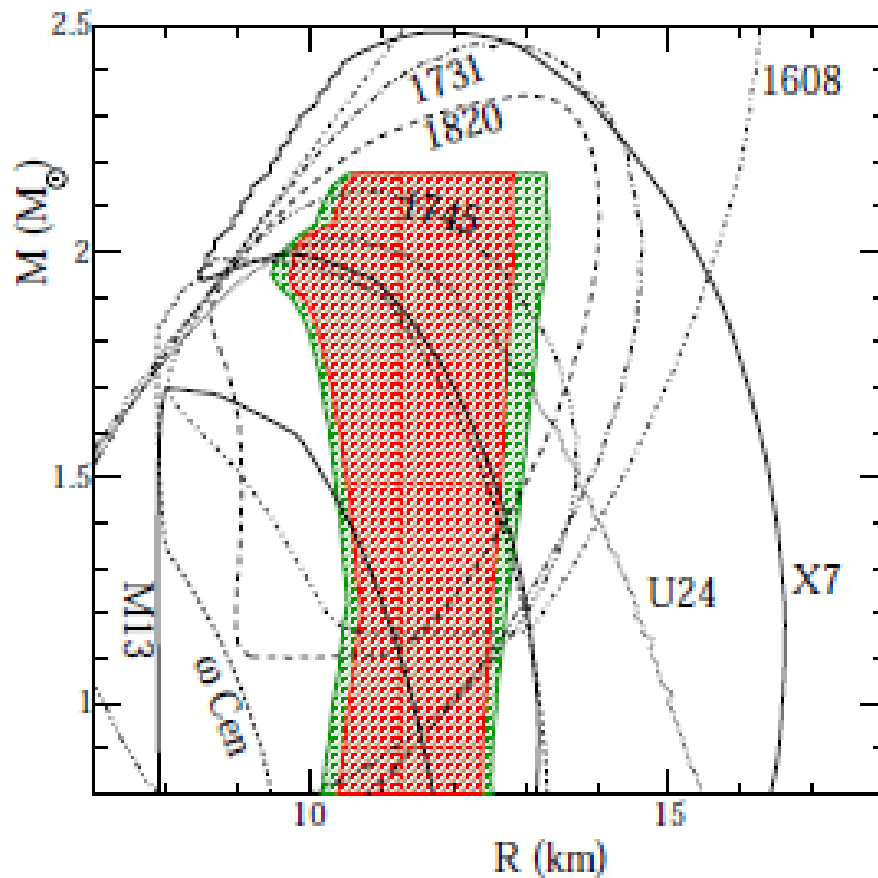
Such systematic errors are not accounted for in Steiner et al.  $\rightarrow$   $M(R)$  is a lower limit  $\rightarrow$  softer EoS





# Which constraints require caution ?

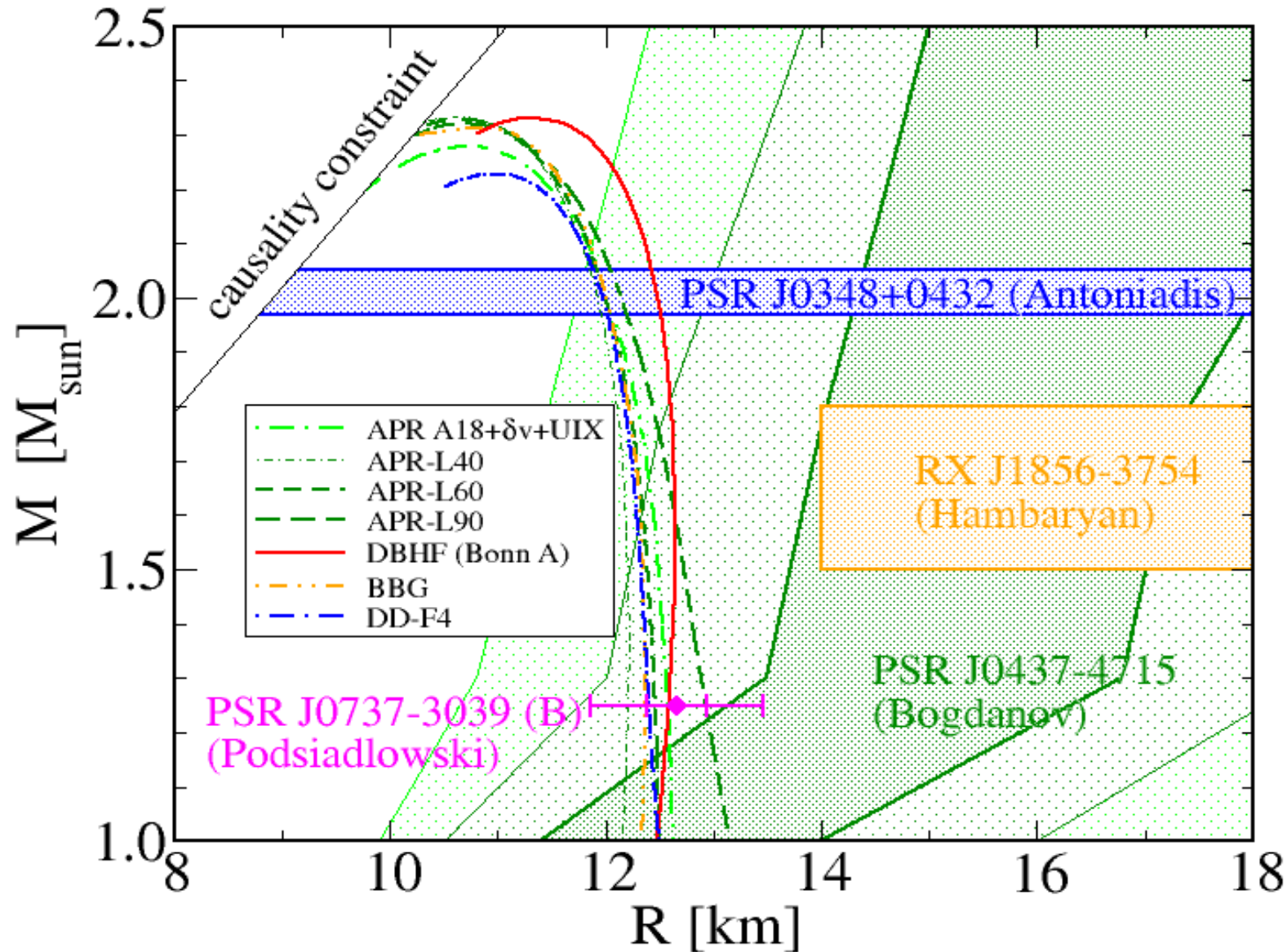
A. Steiner, J. Lattimer, E. Brown, ApJ Lett. 765 (2013) L5



**“Ruled out models” - too strong a conclusion!**

**$M(R)$  constraint is a lower limit, which is itself included in that from RX J1856, which is one of the best known sources.**

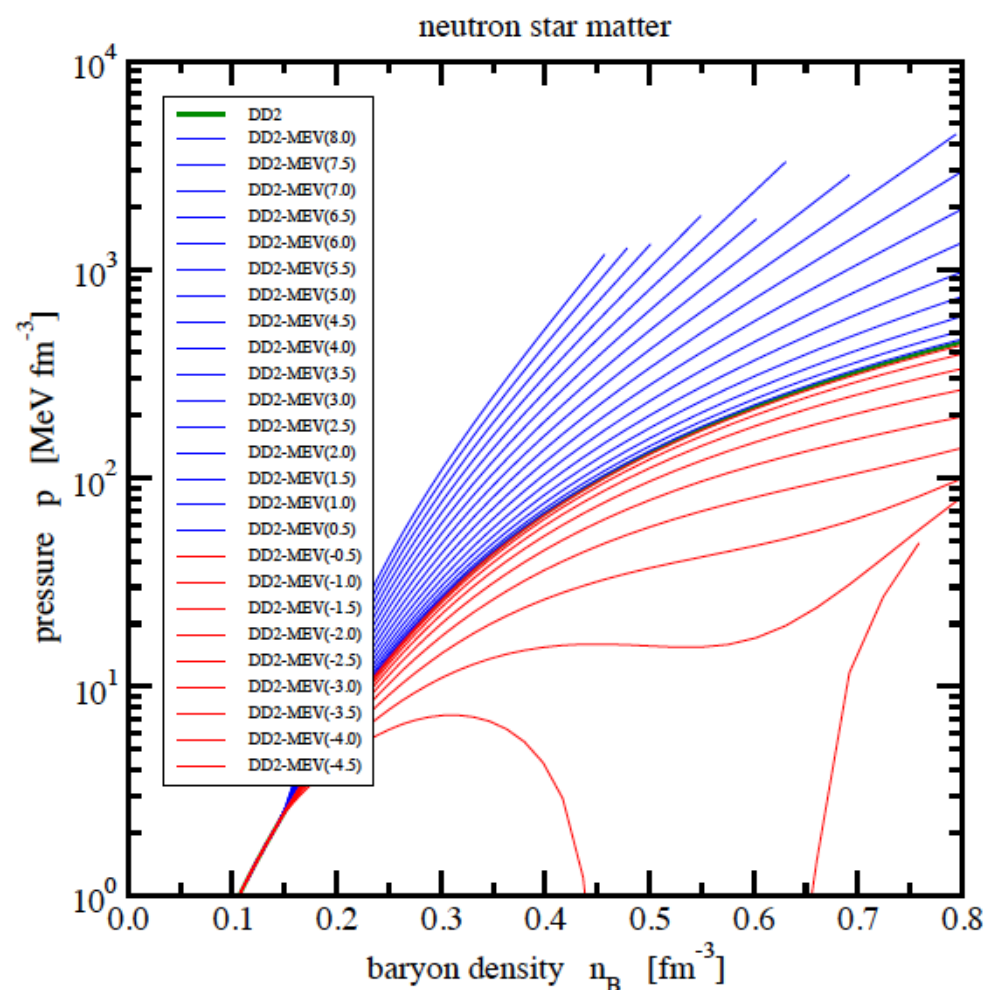
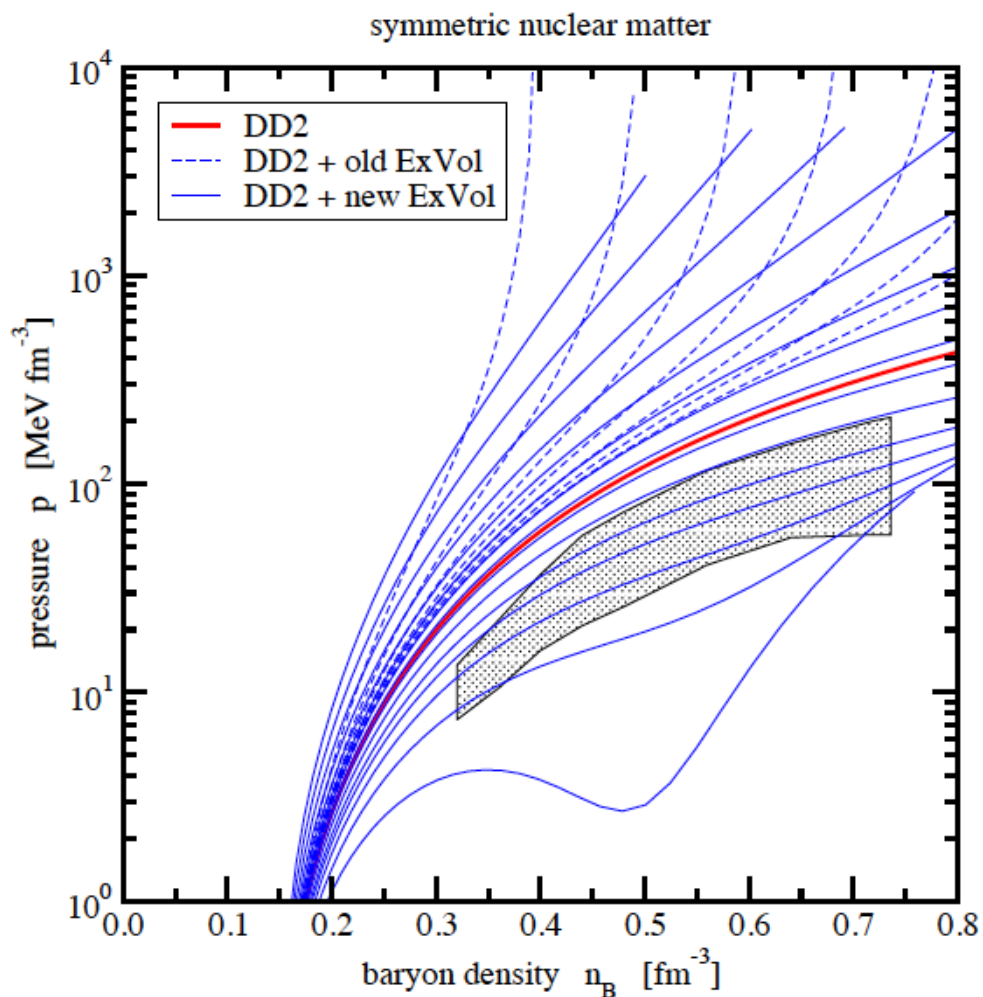
# Disjunct M-R constraints for Bayesian analysis !



## 3.1. Equation of state

- excluded volume corrections in the hadronic EoS,

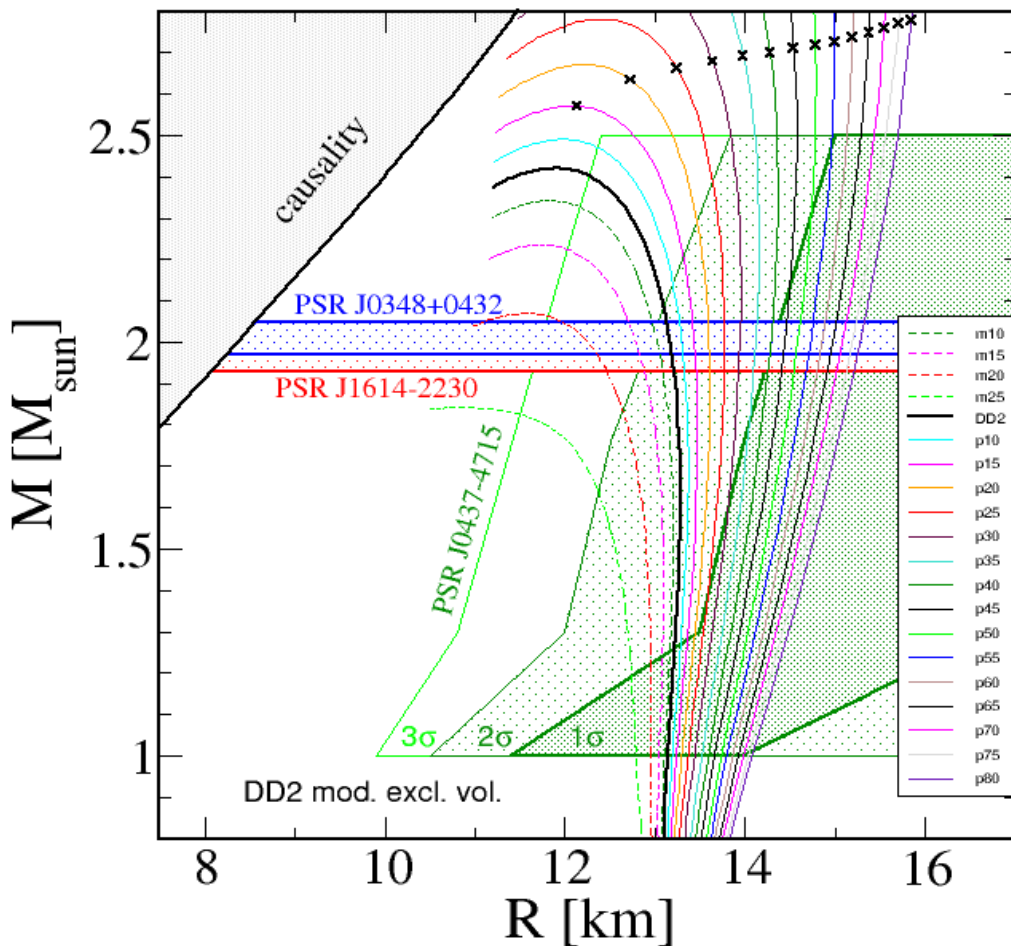
$$\Phi_N = \begin{cases} 1, & \text{if } n \leq n_{\text{sat}} \\ \exp[-v|v|(n - n_{\text{sat}})^2/2], & \text{if } n > n_{\text{sat}} \end{cases}$$



# 1. Equation of state <--> M-R relation

- excluded volume corrections in the hadronic EoS,

$$\Phi_N = \begin{cases} 1, & \text{if } n \leq n_{\text{sat}} \\ \exp[-v|v|(n - n_{\text{sat}})^2/2], & \text{if } n > n_{\text{sat}} \end{cases}$$



- crosses: violation of causality!

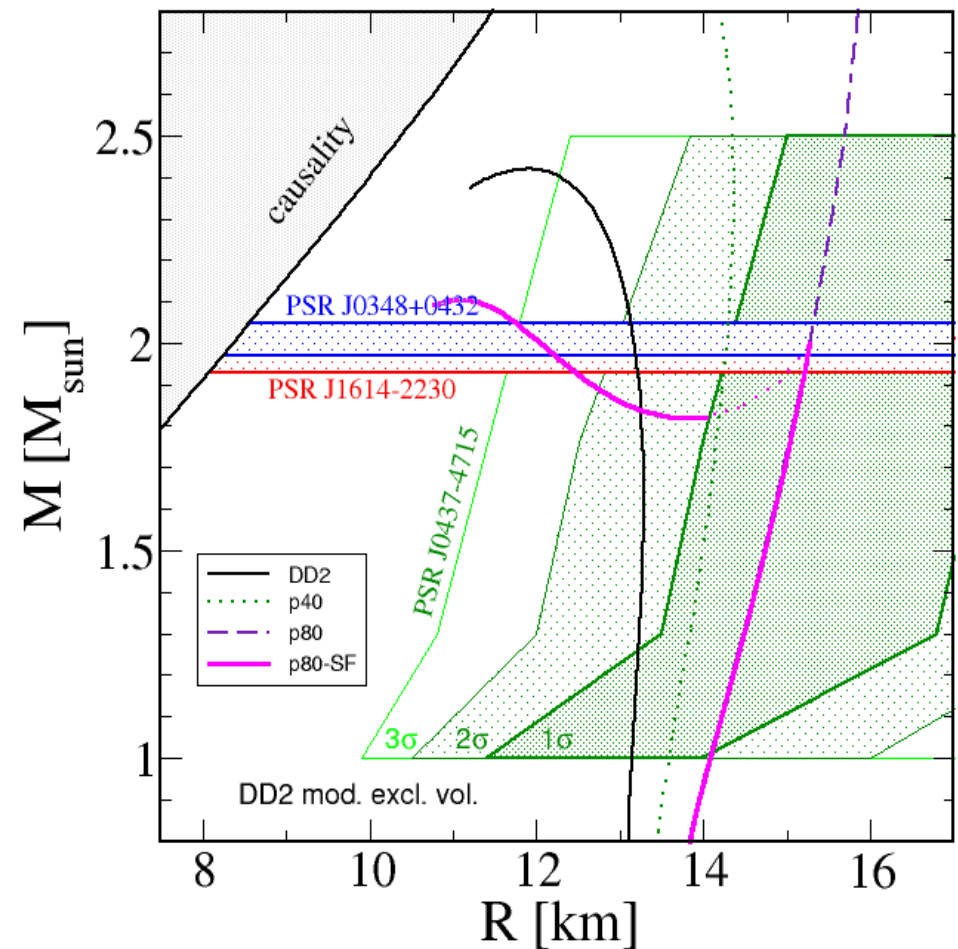
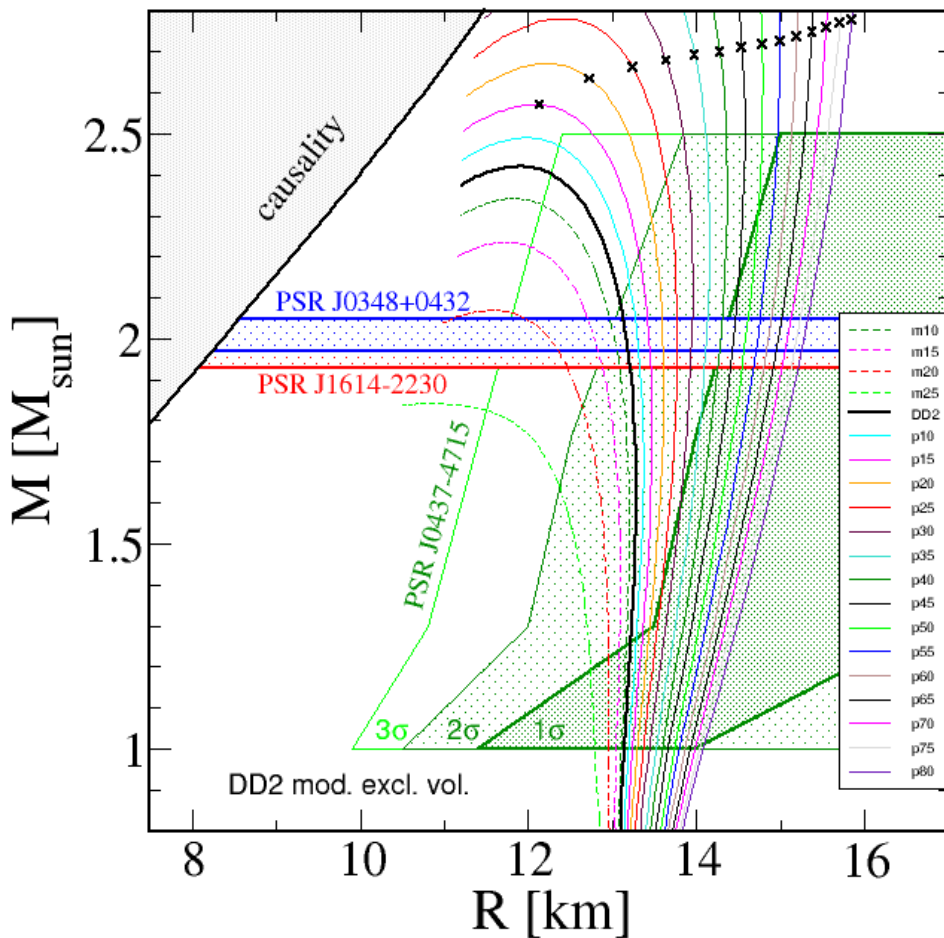
- consider phase transition to quark matter!



# 3.1. Equation of state <--> M-R relation

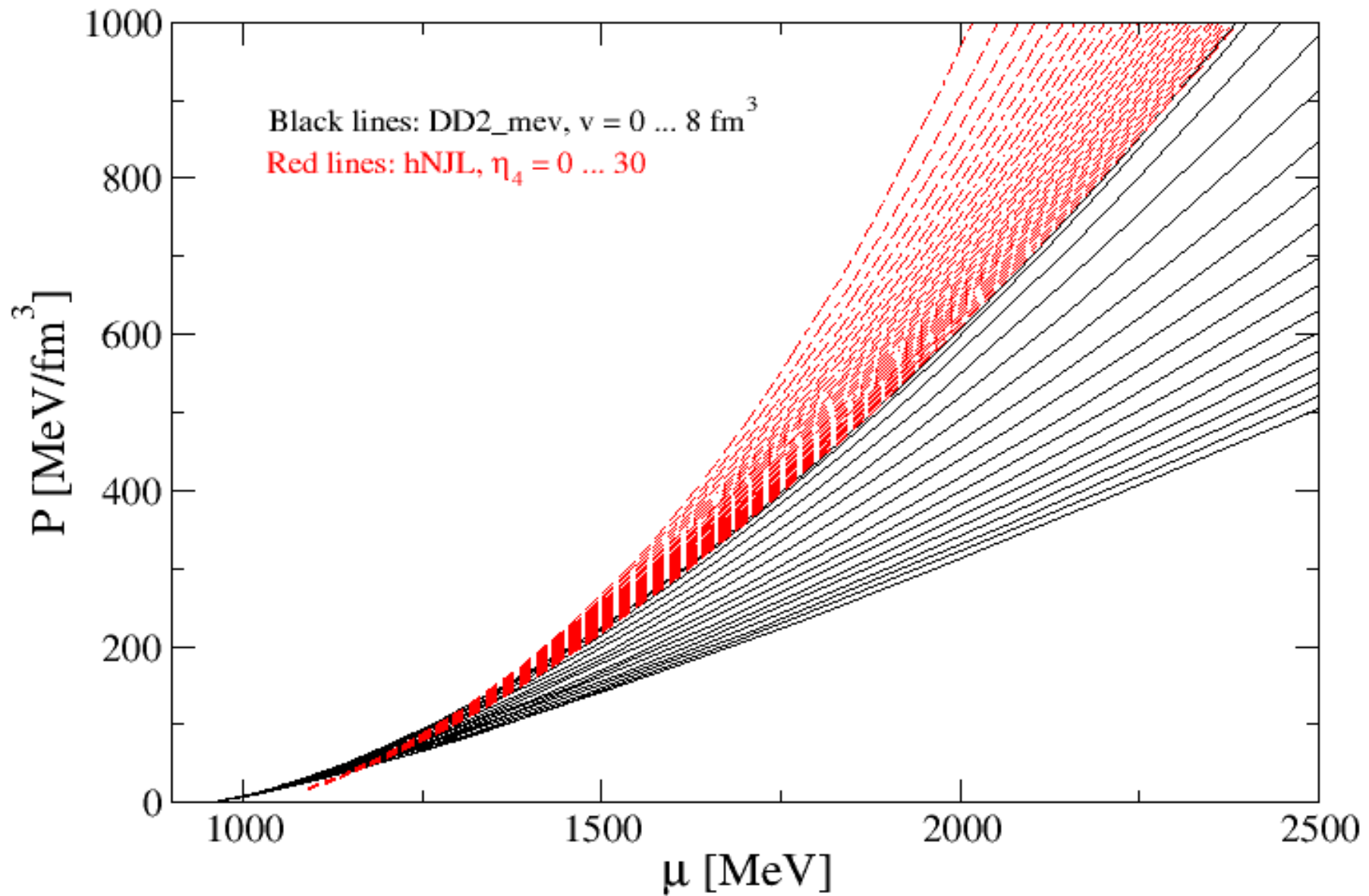
--> Phase transition removes problem with causality ...

--> Example: See Alvarez, Kaltenborn, Blaschke [arxiv:1511.05873] (magenta line)



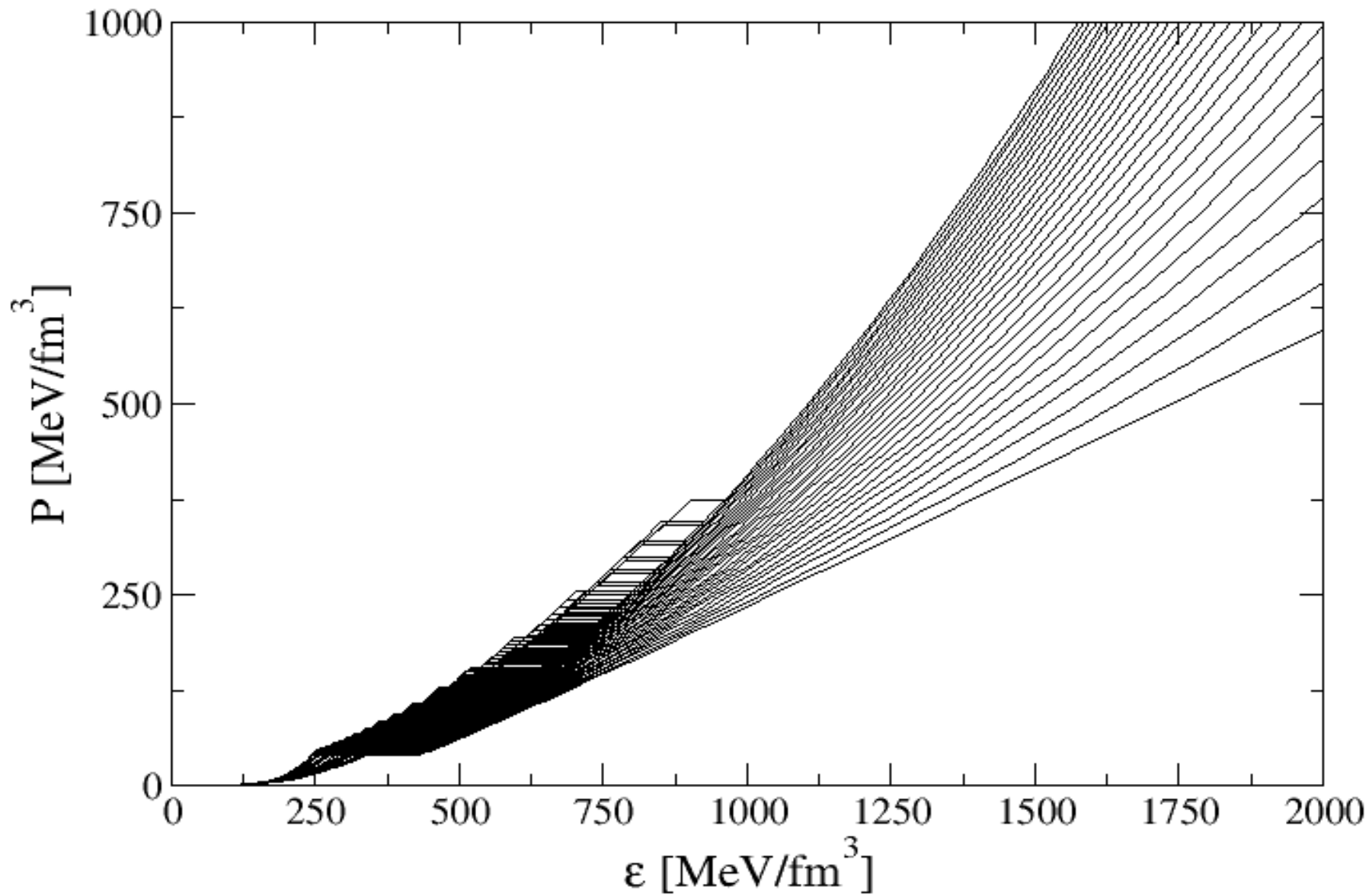
### 3.1. Equation of state

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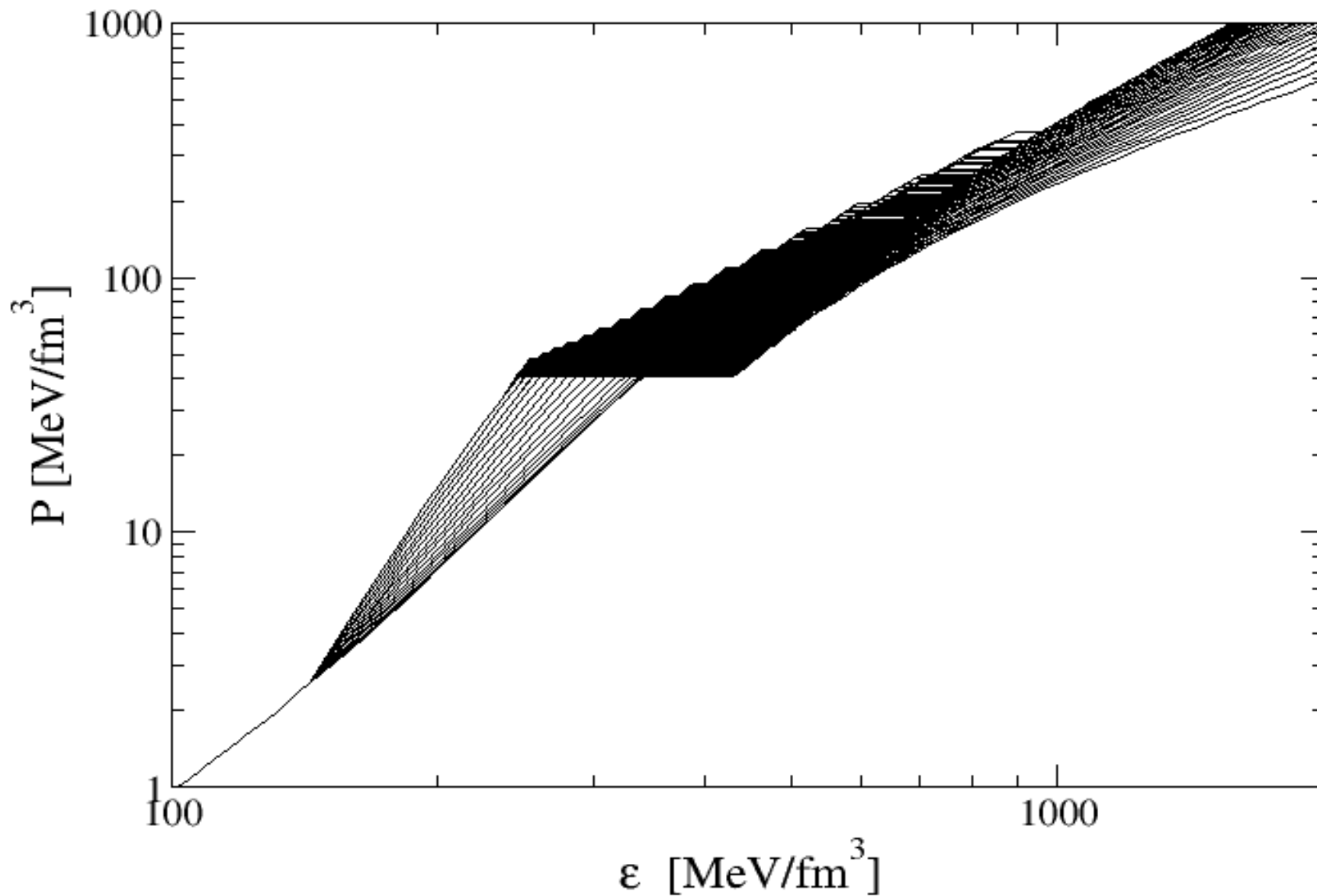
## 3.1. Equation of state

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## 3.1. Equation of state

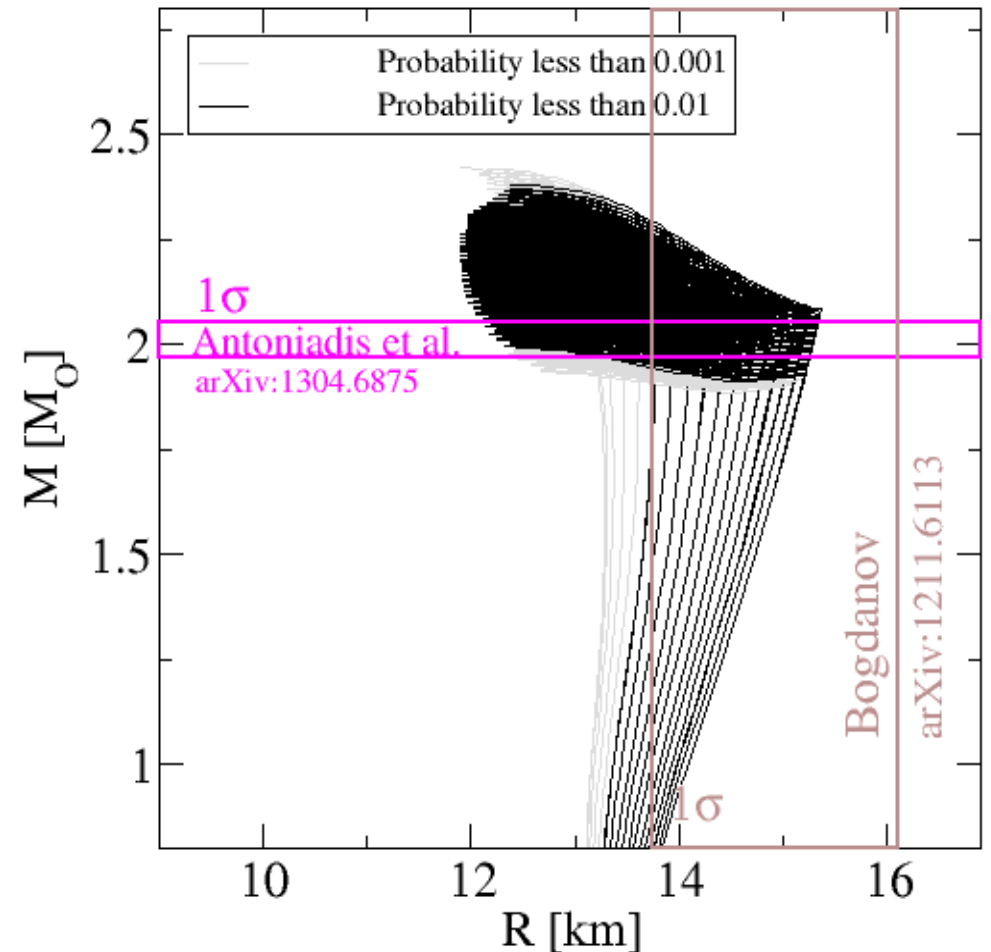
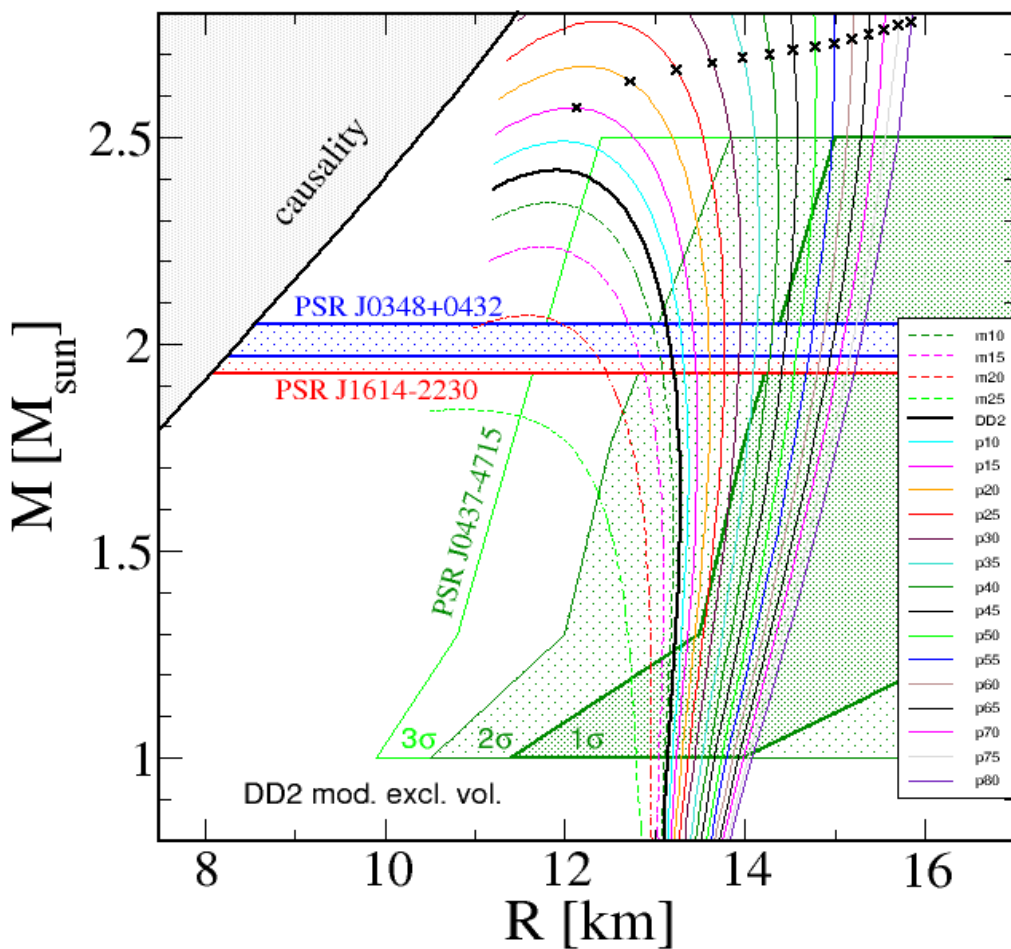
---





### 3.1. Equation of state <--> M-R relation

- excluded volume corrections in the hadronic EoS,
- multi-quark interactions for quark matter EoS.



## 3.2. New Bayesian analysis scheme

---

For the BA of the most probable EoS for given prior from (real or fictitious) observations, we start by defining a vector of free parameters  $\vec{\pi}(p, \eta_A)$ , which correspond to all the possible models with phase transition from nuclear to quark matter using the EoS described above. The way we sample these parameters is

$$\pi_i = \vec{\pi}(p(k), \eta_A(l)), \quad (2)$$

where  $i = 0, 1, \dots, N - 1$  with  $N = N_1 \times N_2$  such that  $i = N_2 \times k + l$  and  $k = 0, 1, \dots, N_1 - 1$ ,  $l = 0, 1, \dots, N_2 - 1$ , with  $N_1$  and  $N_2$  as the total number of parameters  $p_k$  and  $\eta_{(4)l}$  respectively. The goal is to find the set  $\pi_i$  corresponding to an EoS and thus a sequence of configurations which contains the most probable one based on the given constraints using BA. For initializing the BA we propose that *a priori* each vector of parameters  $\pi_i$  has the same probability:  $P(\pi_i) = 1/N$  for  $\forall i$ . We can calculate probability of  $\pi_i$  using Bayes' theorem [13]

$$P(\pi_i | E) = \frac{P(E | \pi_i) P(\pi_i)}{\sum_{j=0}^{N-1} P(E | \pi_j) P(\pi_j)}. \quad (3)$$

D. E. Alvarez-Castillo, A. Ayriyan, D. Blaschke and H. Grigorian, arXiv:1506.07755

D. Blaschke, H. Grigorian, D. E. Alvarez-Castillo and A. Ayriyan, J. Phys. Conf. Ser. **496**, 012002 (2014).

A. Ayriyan, D. E. Alvarez-Castillo, D. Blaschke, H. Grigorian and M. Sokolowski, Phys. Part. Nucl. **46**, 854 (2015).



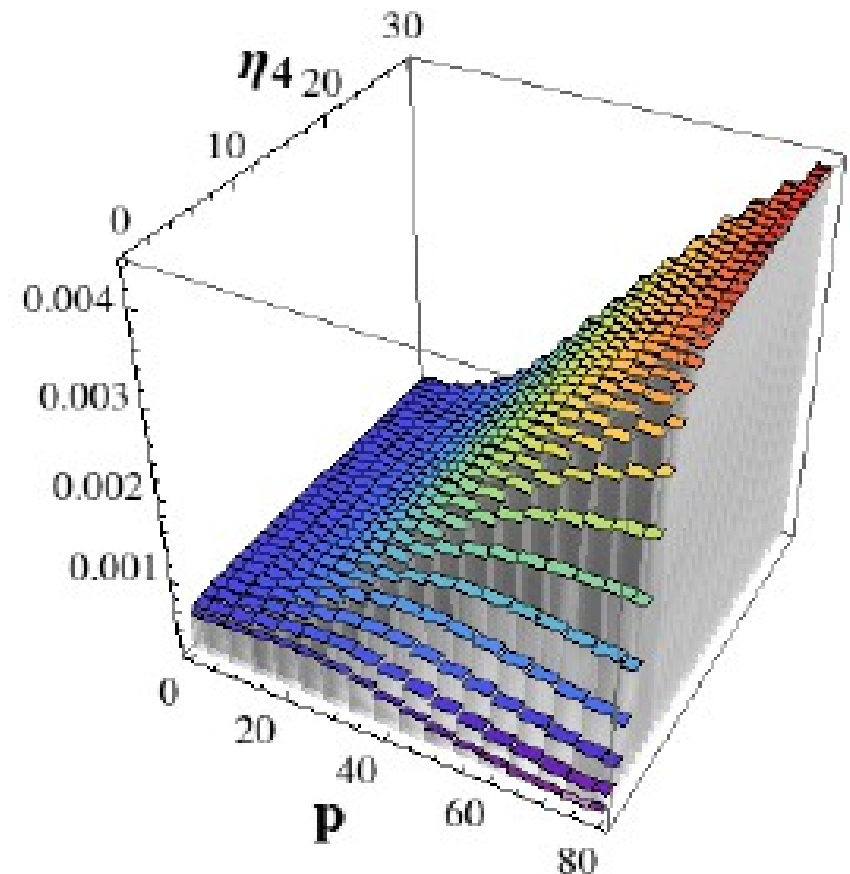
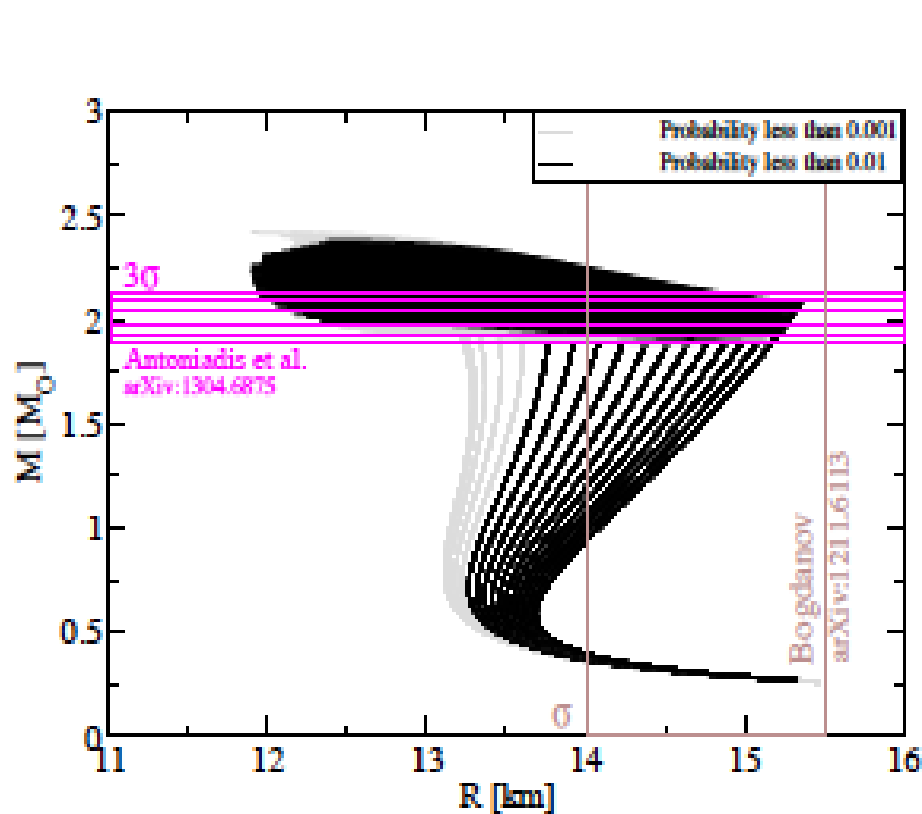


Hovik Grigorian David Alvarez-Castillo D.B. Alexander Ayriyan & wife

## 3.2. New Bayesian analysis scheme

case A

- a maximum mass constraint from PSR J0348+0432 [10],
- a radius constraint from the nearest millisecond pulsar and PSR J0437-4715 [12],





# Phase transition? Measure different radii at 2Mo !

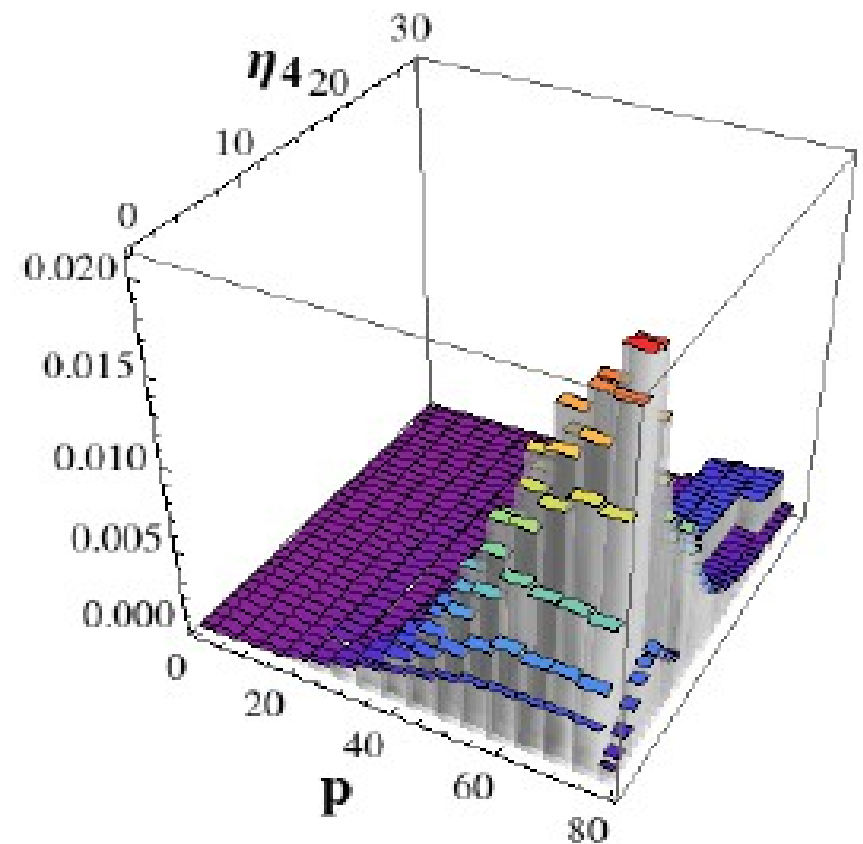
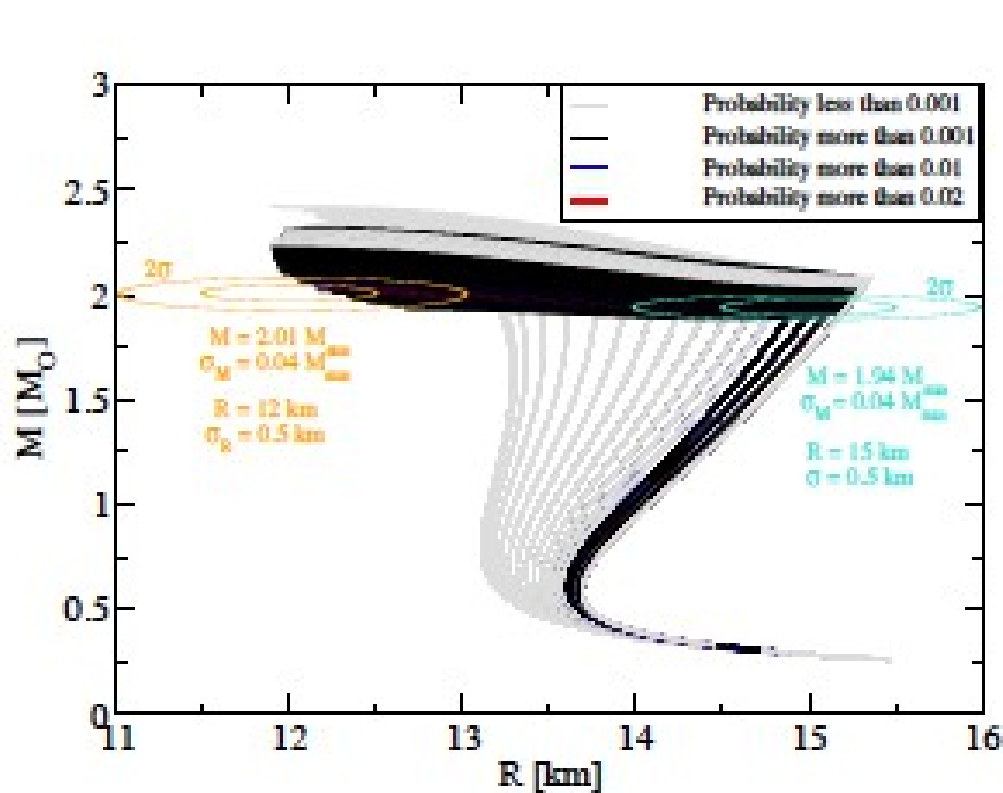
“Now let us travel into future. It is year **2017**, some new, reliable NS radius measurement methods are discovered and were used to find the size of two most massive pulsars, which still are PSR J0348+0432 and PSR J1614-2230. **The community was shocked** when received the results of observations: one radius is  $13 \pm 0.5$  km, while the other is  $11 \pm 0.5$  km!”

– *Michał Sokołowski*, Master Thesis, 2014

## 3.2. New Bayesian analysis scheme

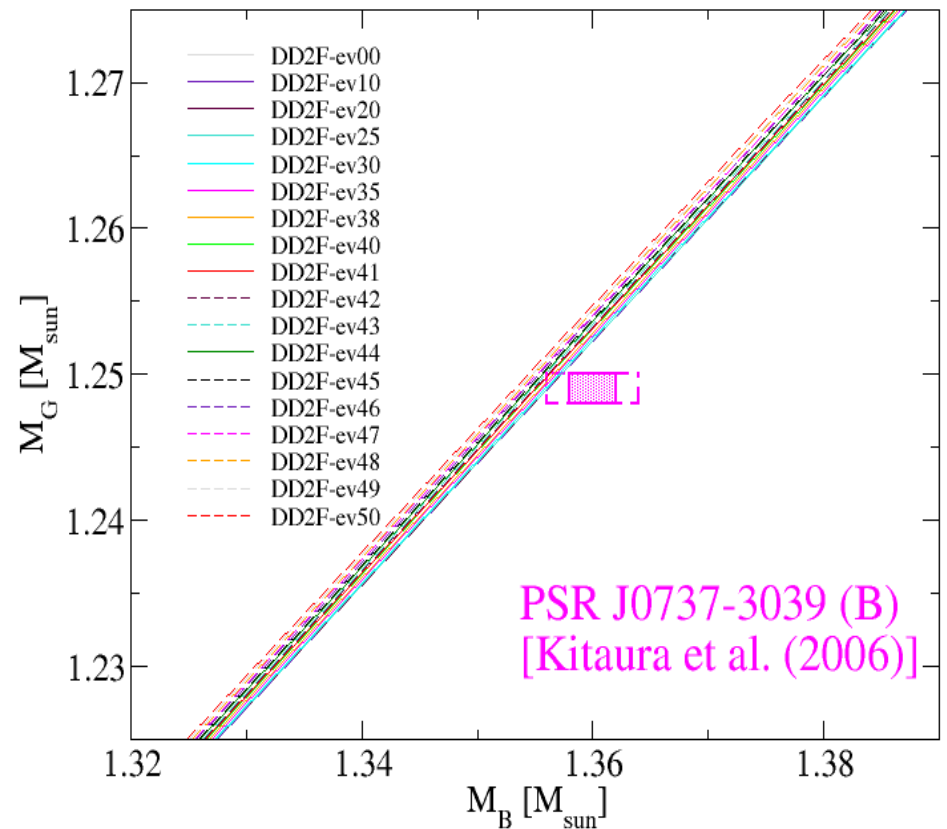
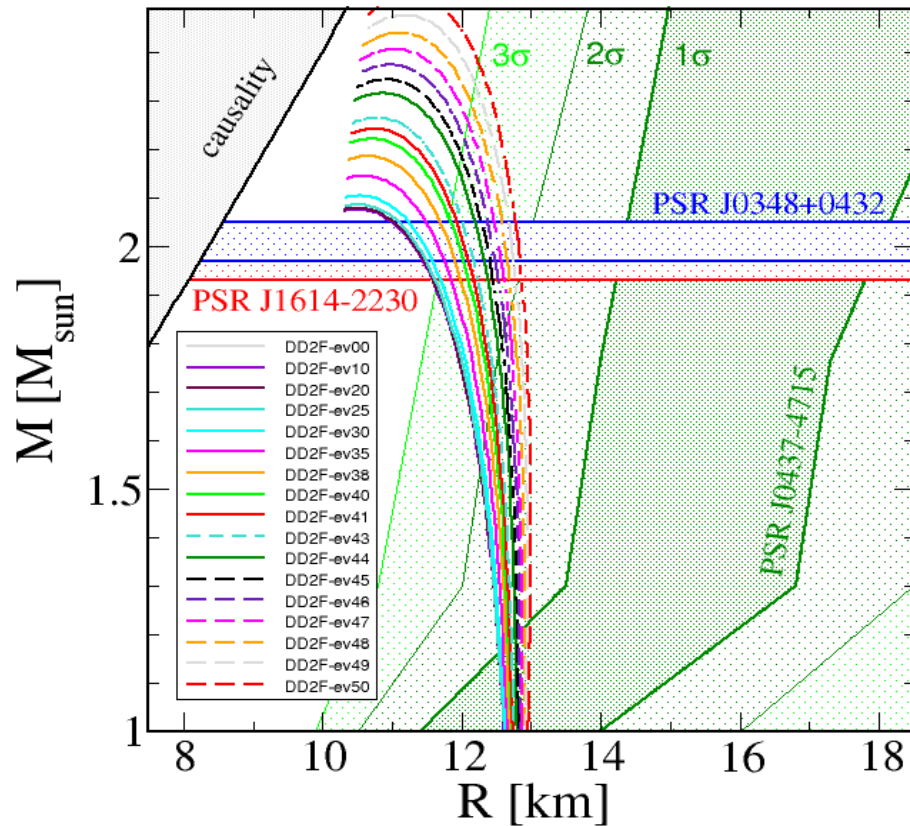
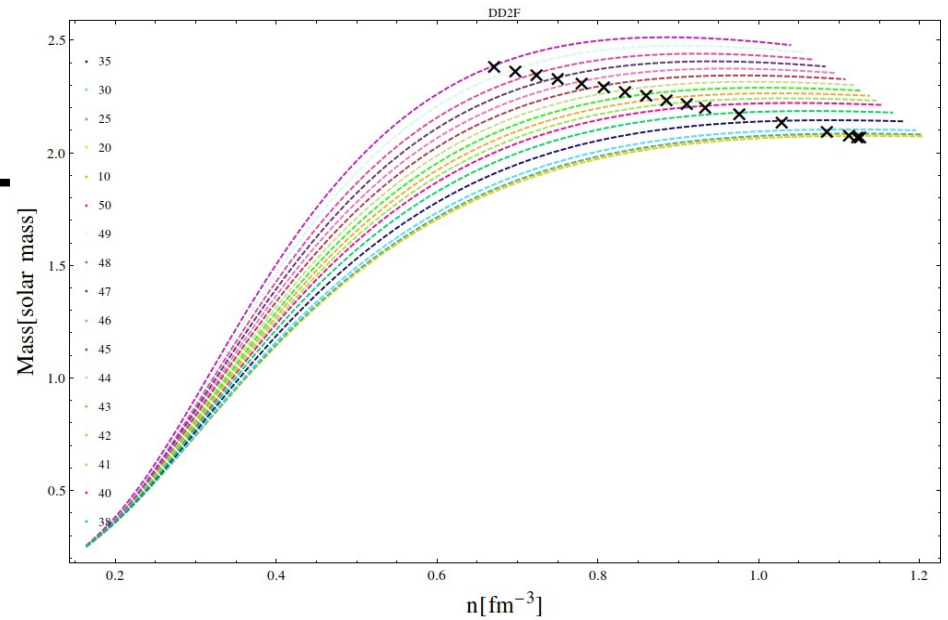
case B

- a radius measurement of  $R = 12 \pm 0.5$  km for PSR J0348+0432 with its known mass,
- a radius measurement of  $R = 15 \pm 0.5$  km for PSR J1614-2230 with its known mass,



## 3.3. Outlook

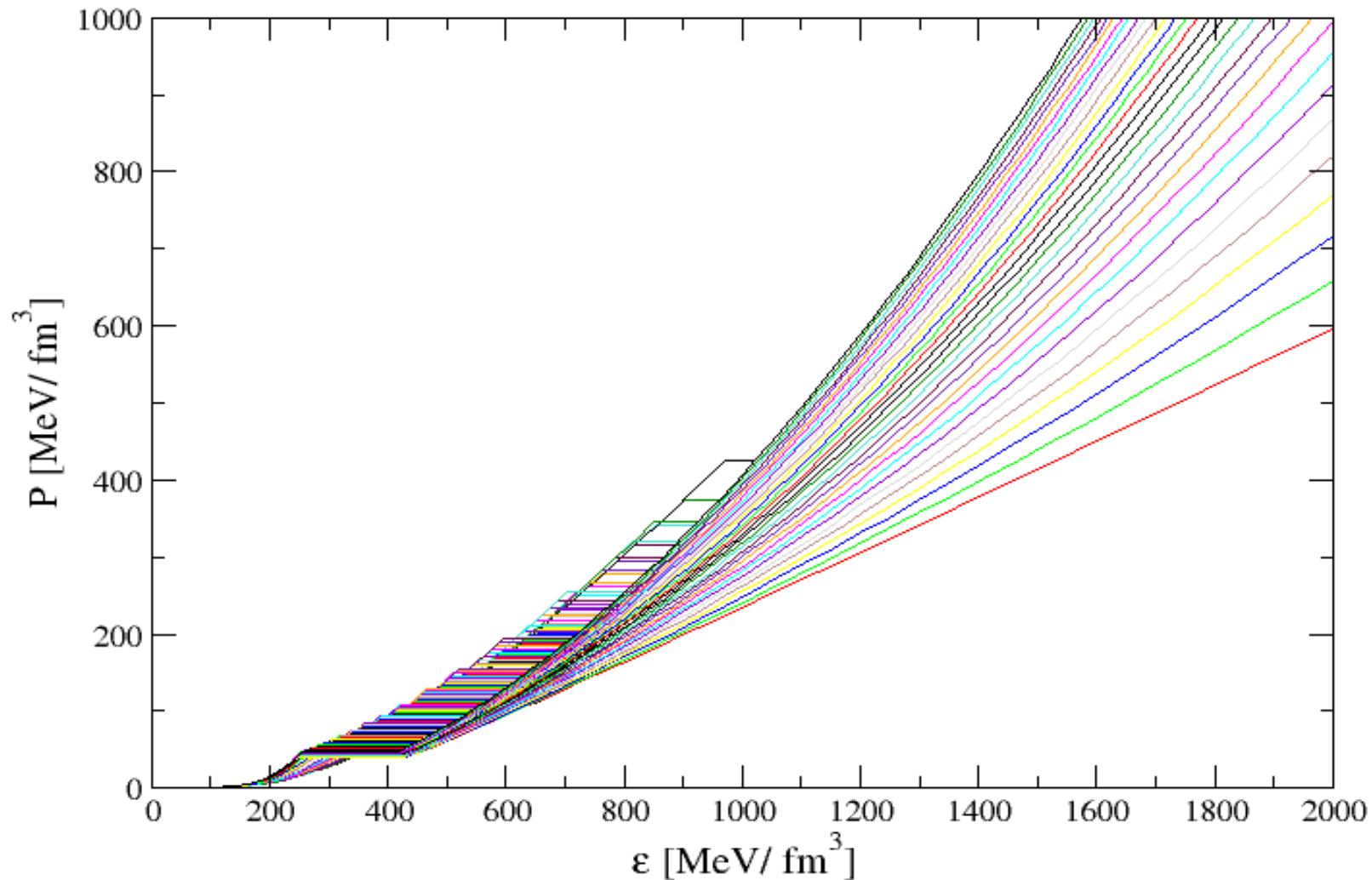
- hadronic EoS too stiff:
  - tension with Danielewicz' flow constraint
  - violates Podsiadlowski's  $M-M_B$  relation
- DD2 --> DD2F\_ev – Stefan Typel



### 3.3. Outlook: Extended Bayesian analysis scheme

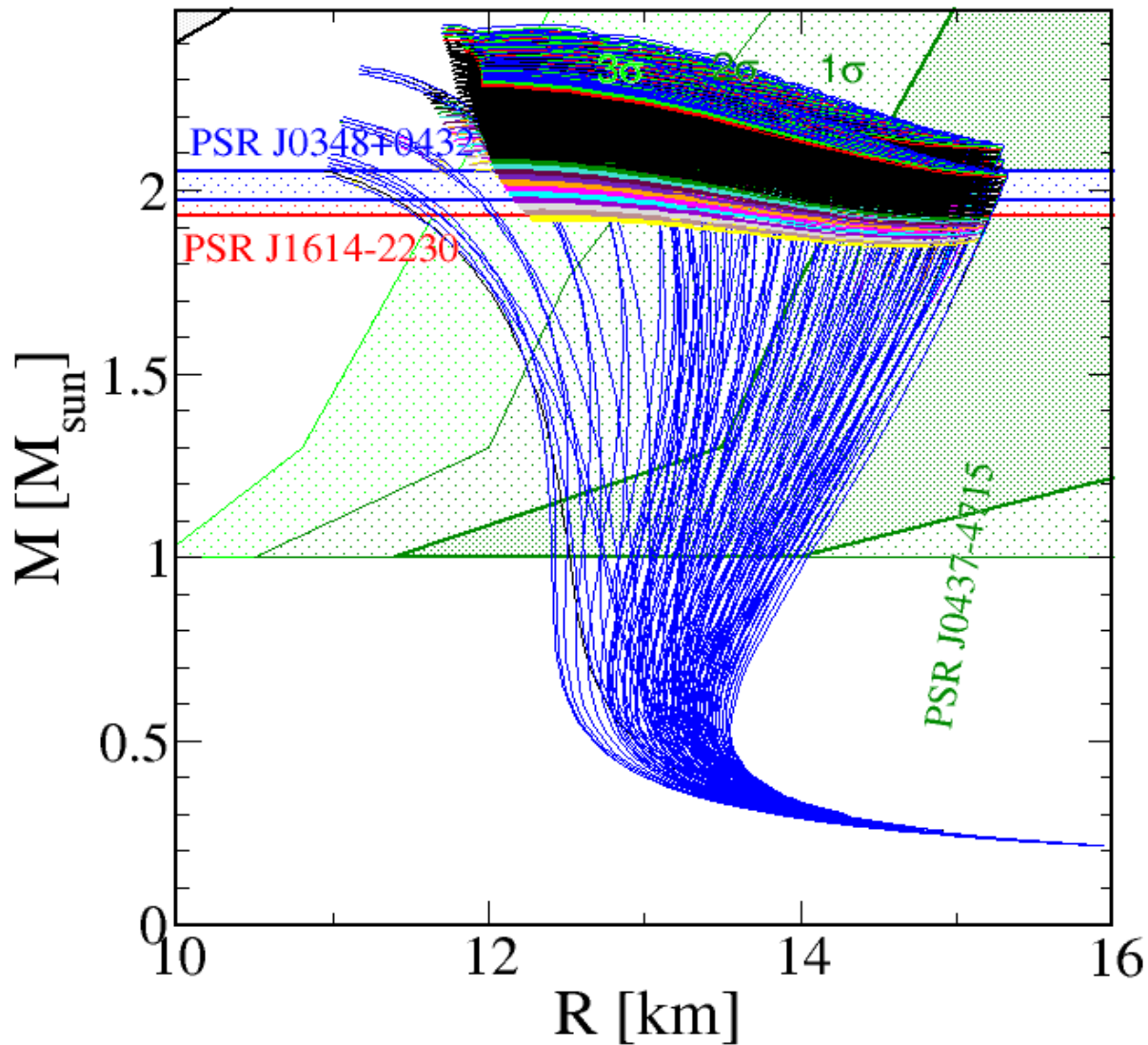
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EoS extension:  $(DD2 + DD2F) \times (E_s^m + E_s + E_s^p)$   
 $= DD2m + DD2 + DD2p + DD2Fm + DD2F + DD2Fp$





### 3.3. Outlook: Extended Bayesian analysis scheme



Hadronic EoS classes  
(varying  $v_{ex}$ ):

- DD2m
- + DD2
- + DD2p
- + DD2Fm
- + DD2F
- + DD2Fp

Quark matter EoS  
(varying  $\eta_4$ ):

hNJL

D. Alvarez-Castillo et al.,  
Eur. Phys. J. A52, 69 (2016)

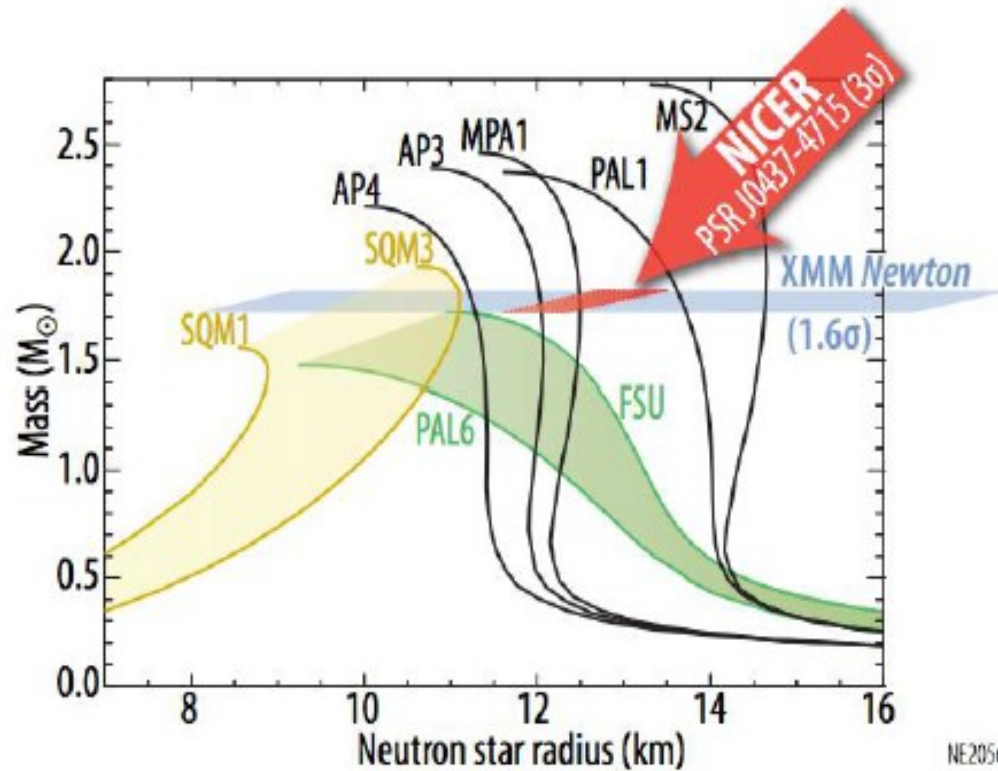
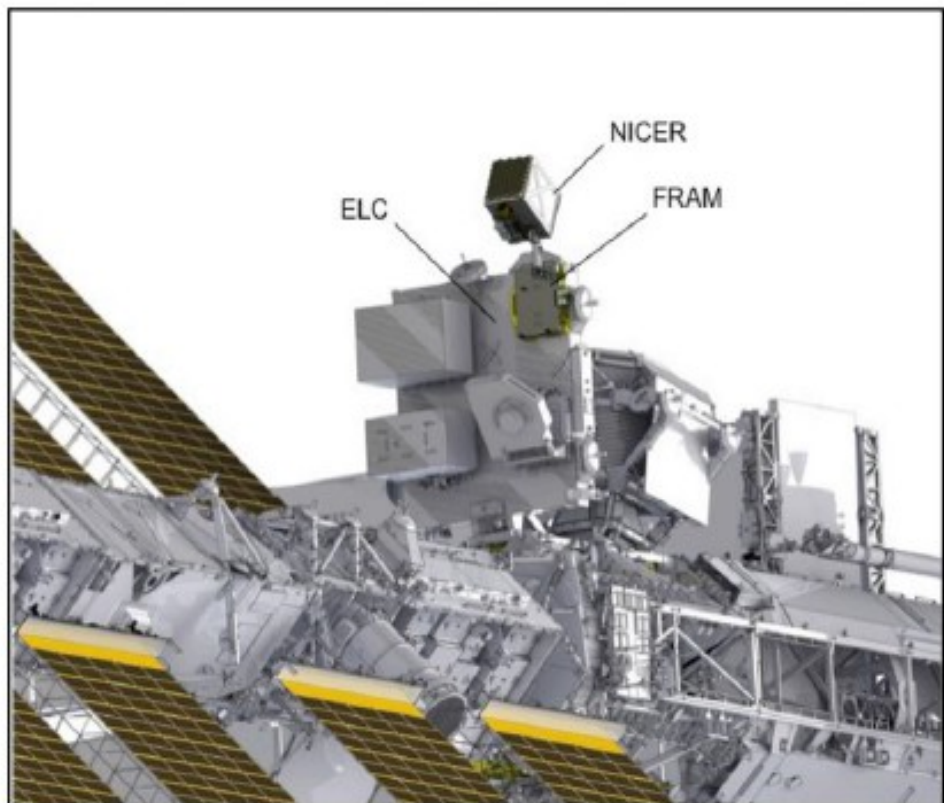
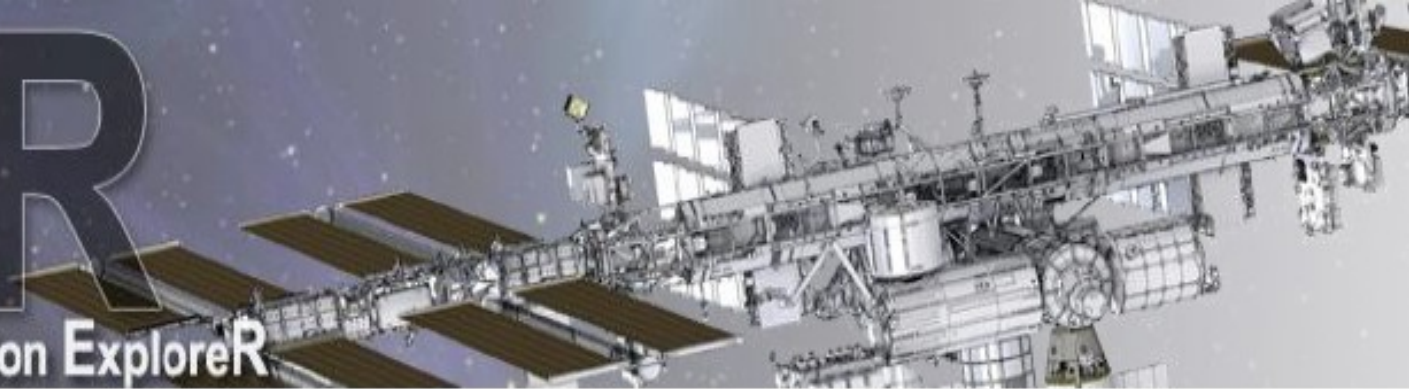
# Perspectives for new Instruments?



THE FUTURE: SKA - SQUARE KILOMETER ARRAY

# NICER

Neutron star Interior Composition Explorer



## NICER 2017

Gendreau, K. C., Arzoumanian, Z., & Okajima, T. 2012, Proc. SPIE, 8443, 844313

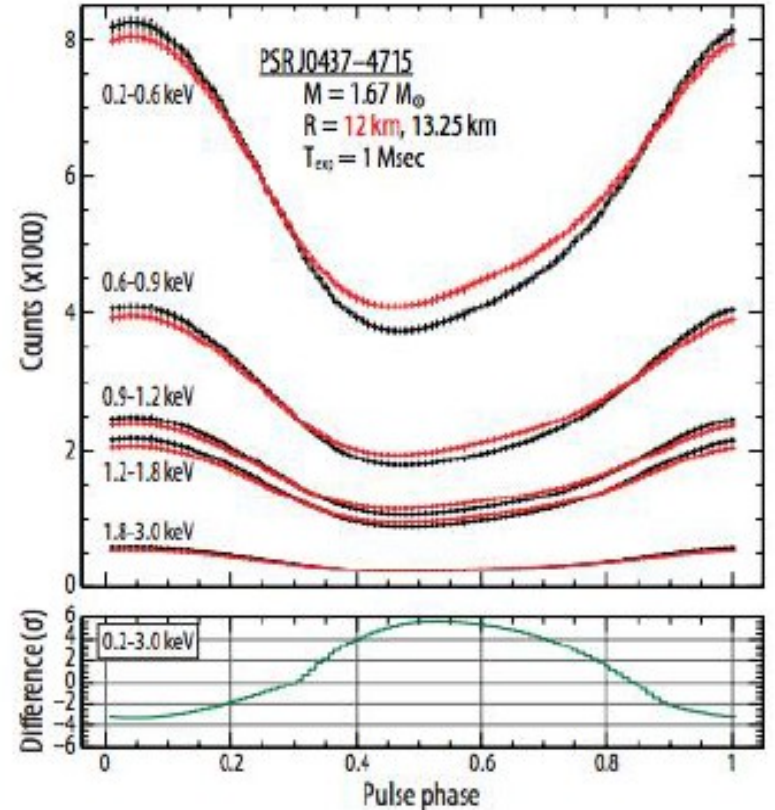
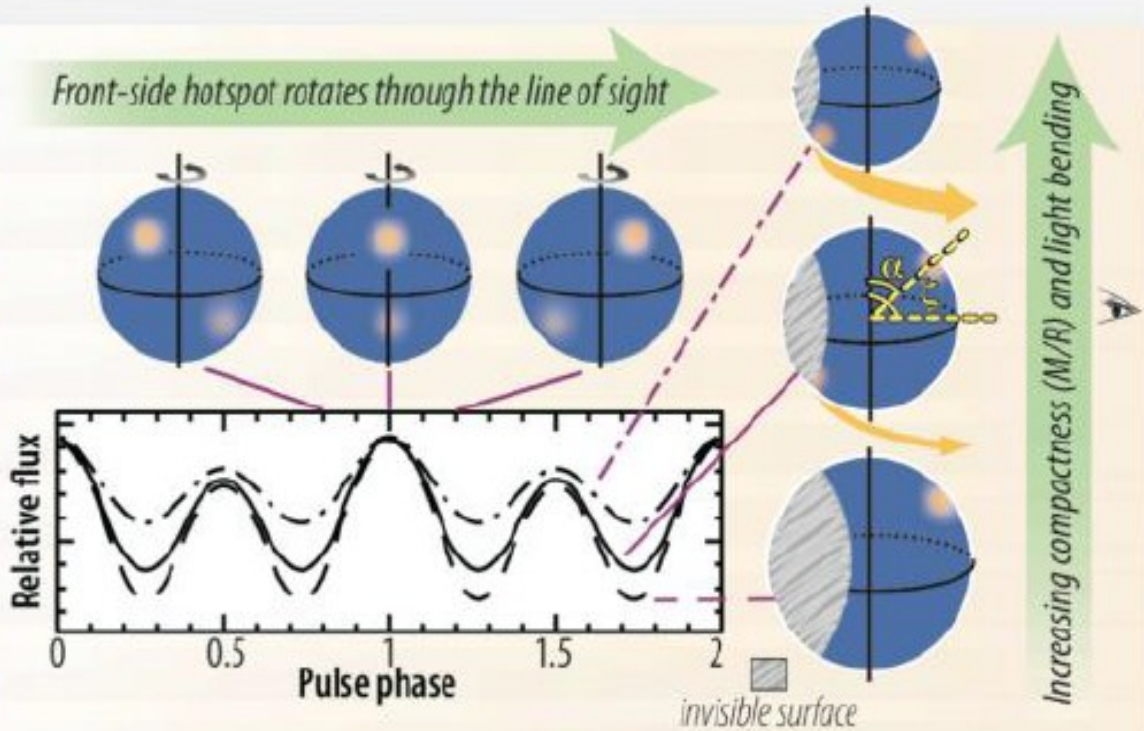


# NICER

Neutron star Interior Composition Explorer



Thermal Lightcurve Model



## Hot Spots



## **4. Hybrid star matter at NICA and FAIR**

# Hydrodynamic modelling for NICA / FAIR

**More complicated for lower energies:**

- baryon stopping effects,
- finite baryon chemical potential,
- EoS unknown from first principles

We want to simulate the effects of, and ultimately discriminate different EoS/PT types

The model has to be coupled to a detector response code to simulate detector events

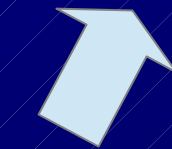


Initial state



3-fluid hydro,  
(Yu. Ivanov)

hydrodynamic evolution



adapt the procedure  
from existing hybrid model  
(Iu. Karpenko)

particlization

hadronic  
corona

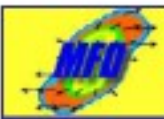


(optionally) cascade:  
PHSD, UrQMD, etc  
(E. Bratkovskaya,  
H. Petersen)

detector  
response



GEANT  
MPD, BM @N  
(O. Rogachevsky,  
V. Voronyuk, et al.)



# 3-Fluid Dynamics

Baryon Stopping

JINR, 24.08.10

Model

Rapidity Density

Fit

Reduced curvature

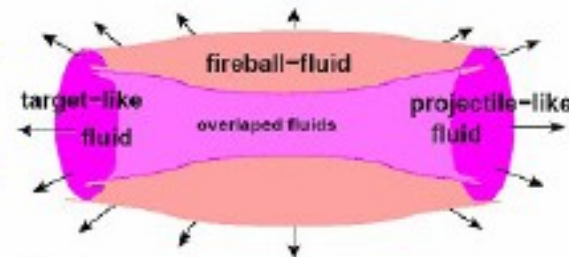
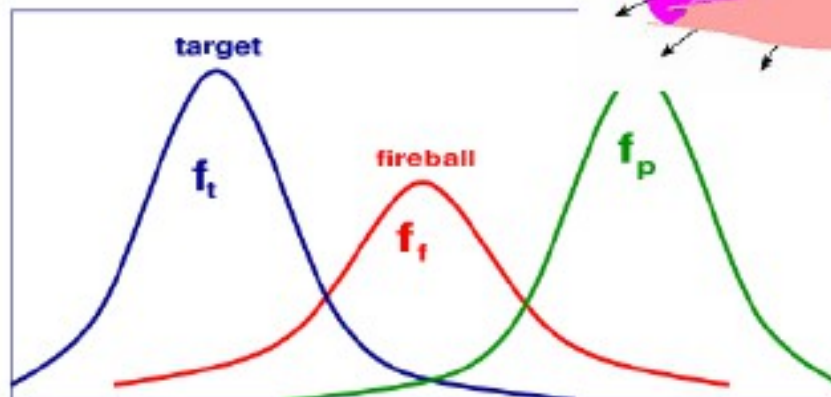
Trajectories

Crossover

Summary

Produced particles populate mid-rapidity  
 $\Rightarrow$  **fireball** fluid

distribution function



momentum along beam

**Target-like fluid:**

$$\partial_\mu J_t^\mu = 0$$

Leading particles carry bar. charge

$$\partial_\mu T_t^{\mu\nu} = -F_{tp}^\nu + F_{ft}^\nu$$

exchange/emission

**Projectile-like fluid:**

$$\partial_\mu J_p^\mu = 0,$$

$$\partial_\mu T_p^{\mu\nu} = -F_{pt}^\nu + F_{fp}^\nu$$

**Fireball fluid:**

$$J_f^\mu = 0,$$

Baryon-free fluid

$$\partial_\mu T_f^{\mu\nu} = F_{pt}^\nu + F_{tp}^\nu - F_{fp}^\nu - F_{ft}^\nu$$

Source term      Exchange

The **source term** is delayed due to a formation time  $\tau \sim 1 \text{ fm}/c$

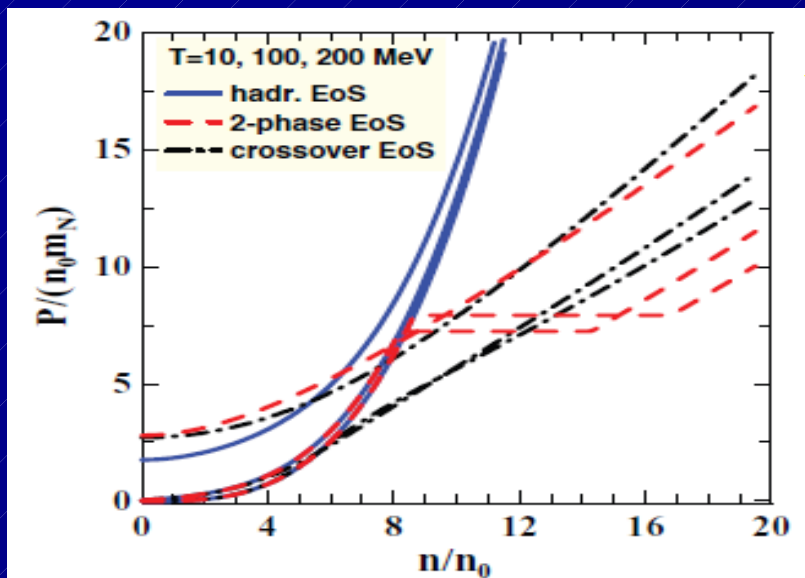
**Total energy-momentum conservation:**

$$\partial_\mu (T_p^{\mu\nu} + T_t^{\mu\nu} + T_f^{\mu\nu}) = 0$$

<http://theory.gsi.de/~ivanov/mfd/>

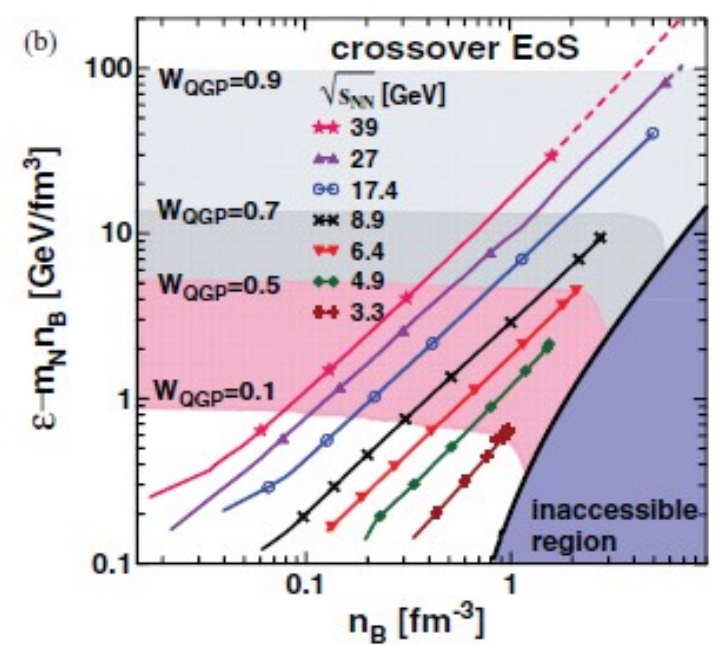
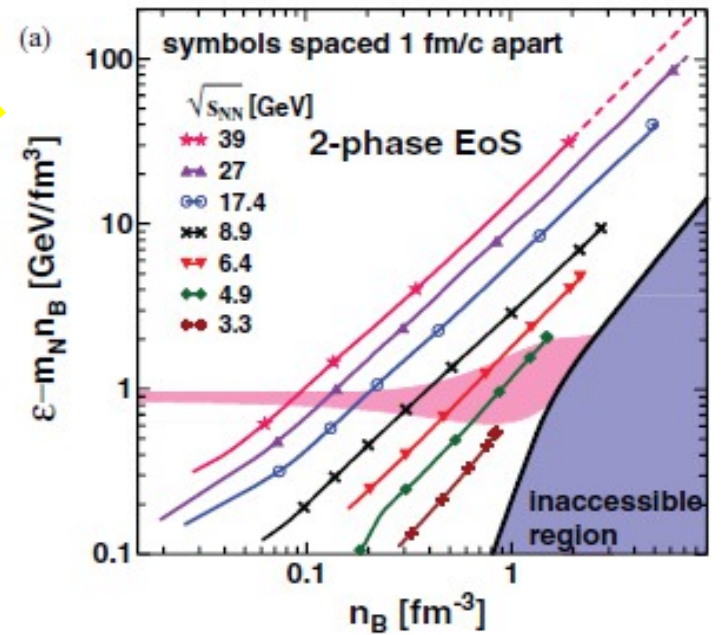
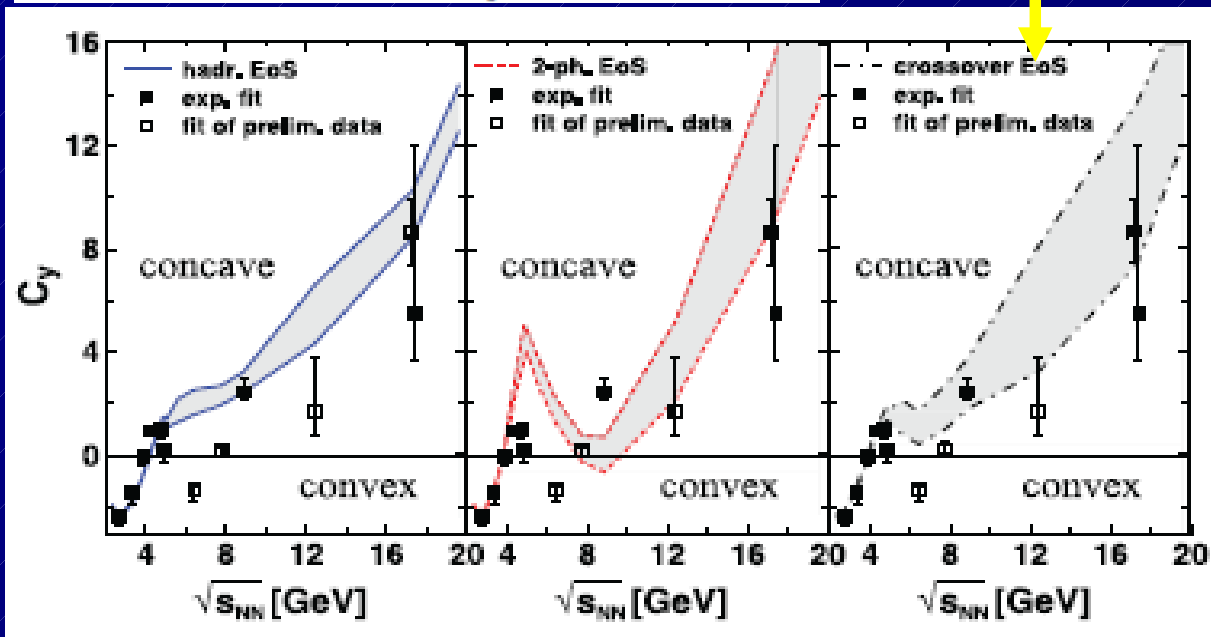
# Net proton rapidity distribution – test case for a 1<sup>st</sup> order PT signal

**Theory:** Yu.B. Ivanov, Phys. Rev. C 87, 064904 (2013)



**EoS**

3-fluid hydro  
Evolution ...



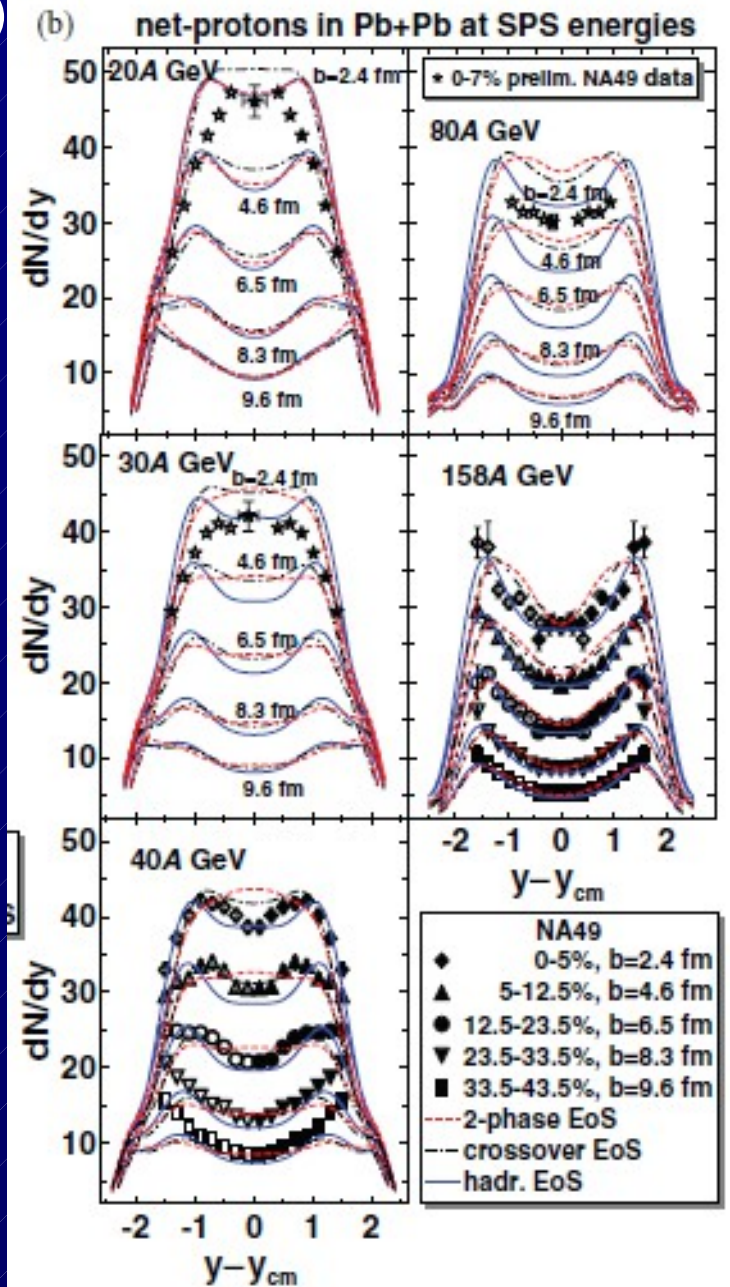
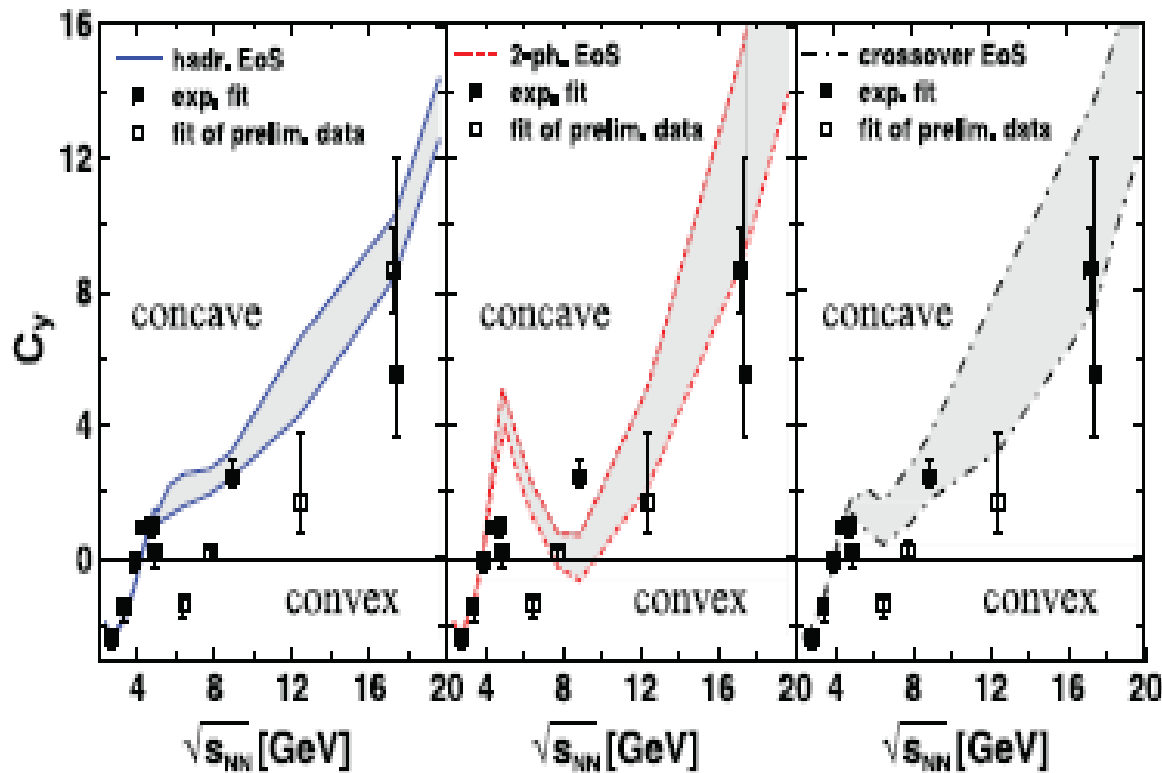


# Net proton rapidity distribution – test case for a 1<sup>st</sup> order PT signal

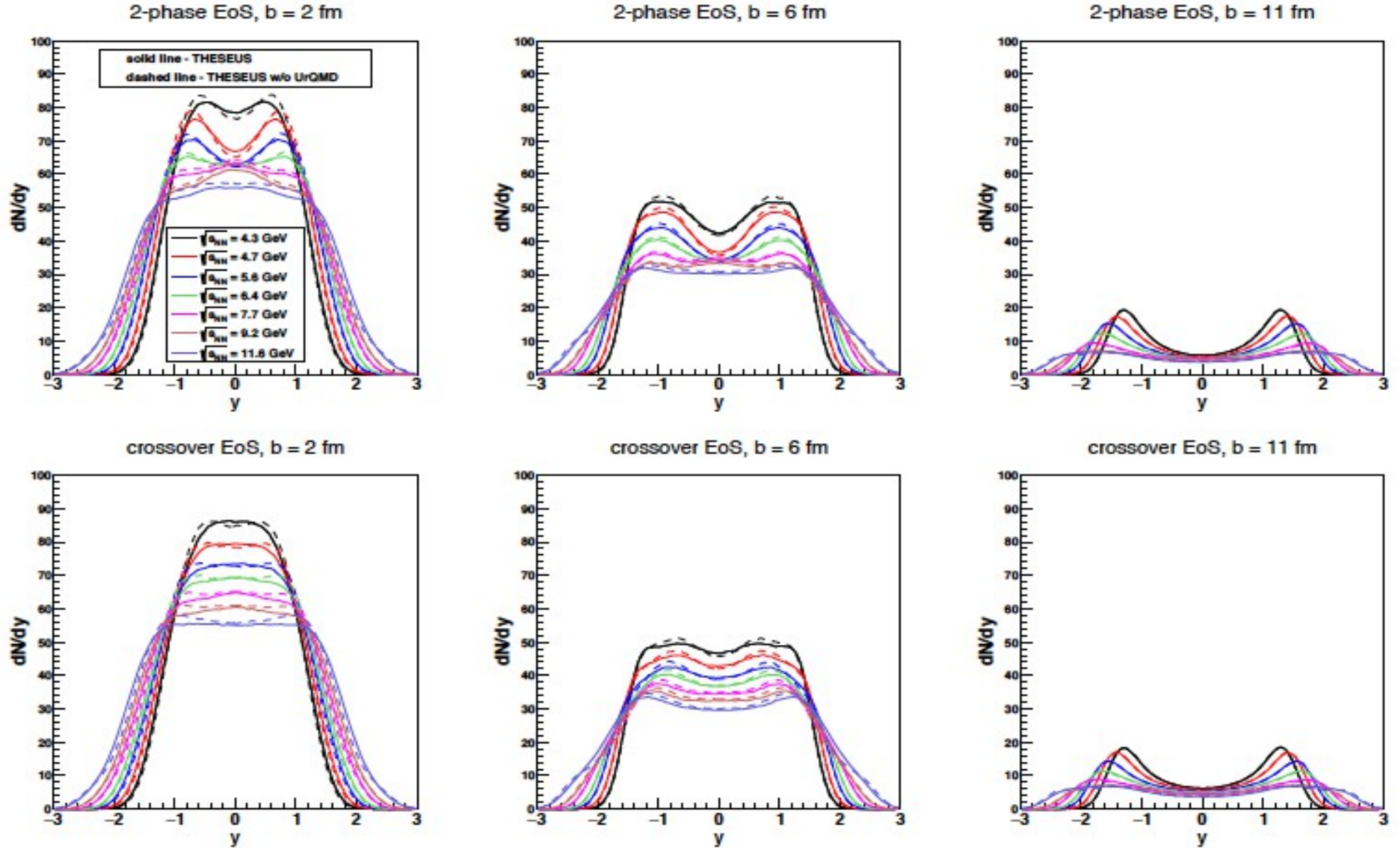
**Theory:** Yu.B. Ivanov, Phys. Rev. C 87, 064904 (2013)

$$C_y = \left( y_{c.m.}^3 \frac{d^3 N}{dy^3} \right)_{y=y_{c.m.}} / \left( y_{c.m.} \frac{dN}{dy} \right)_{y=y_{c.m.}}$$

$$= (y_{c.m.}/w_s)^2 (\sinh^2 y_s - w_s \cosh y_s).$$



# Baryon stopping signal for first order phase transition ?





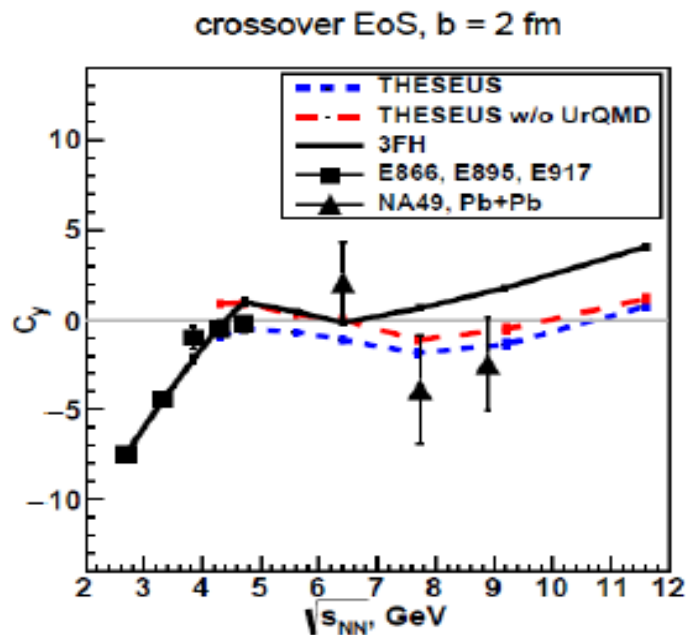
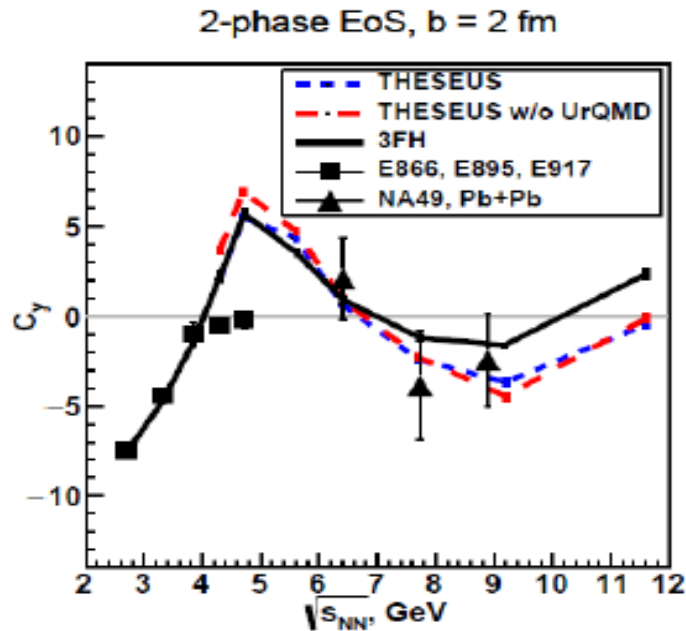


Marcus Bleicher (FIAS, Uni Frankfurt)

D.B. (Uni Wroclaw)



# Baryon stopping signal for first order phase transition ?



Energy scan of the curvature  $C_y$  of the net proton Rapidity distribution at central rapidity  $y_{cm}$

$$C_y = y_{cm}^2 (d^3 N_{net-p} / dy^3) / (dN_{net-p} / dy)$$

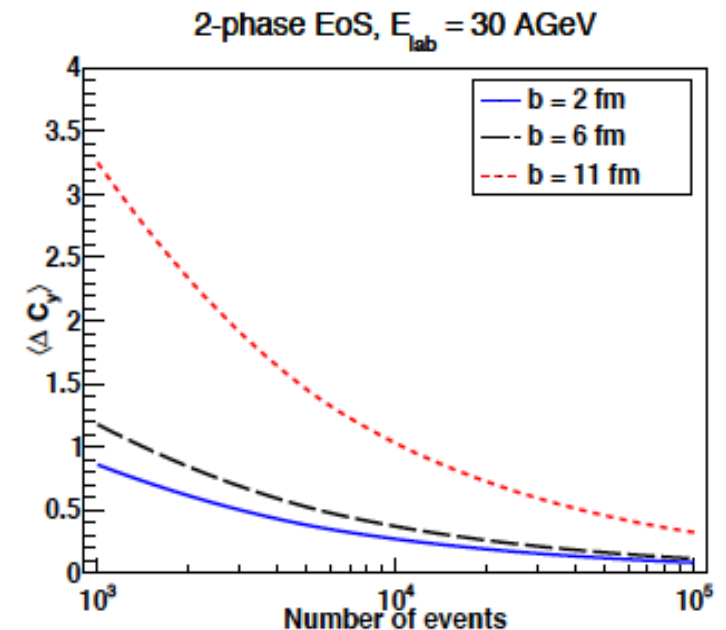
Reduced curvature calculated for a parabolic fit

$$\tilde{P}_2(y) = ay^2 + by + c \longrightarrow C_y = y_{beam}^2 2a/c$$

Statistical error

$$\Delta C_y = \frac{2y_{beam}^2}{c} \sqrt{(\Delta a)^2 + \frac{a^2}{c^2} (\Delta c)^2},$$

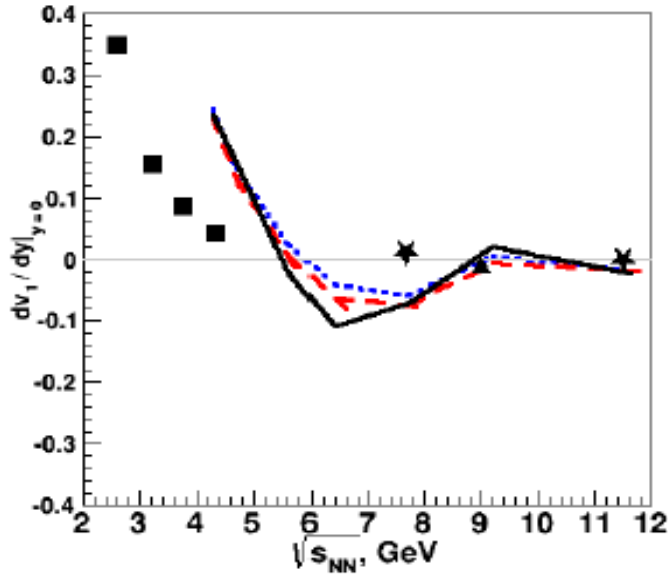
depends on number of events



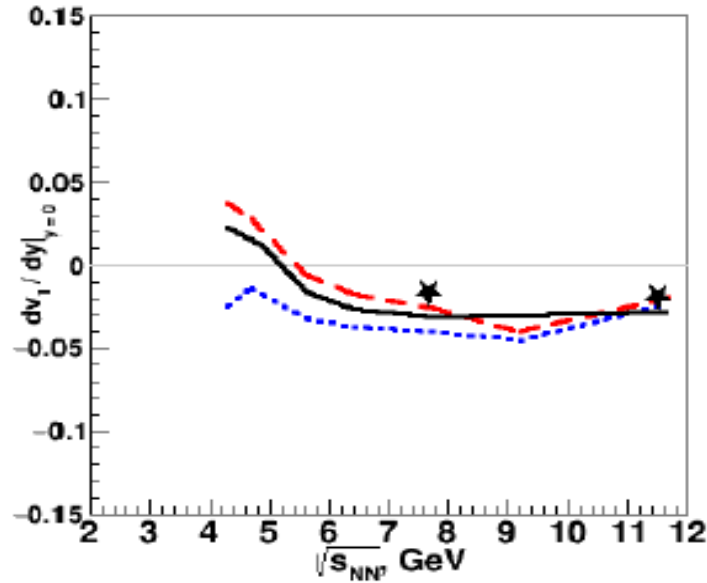


# THESEUS: Particlization of 3-fluid Hydro + UrQMD casc.

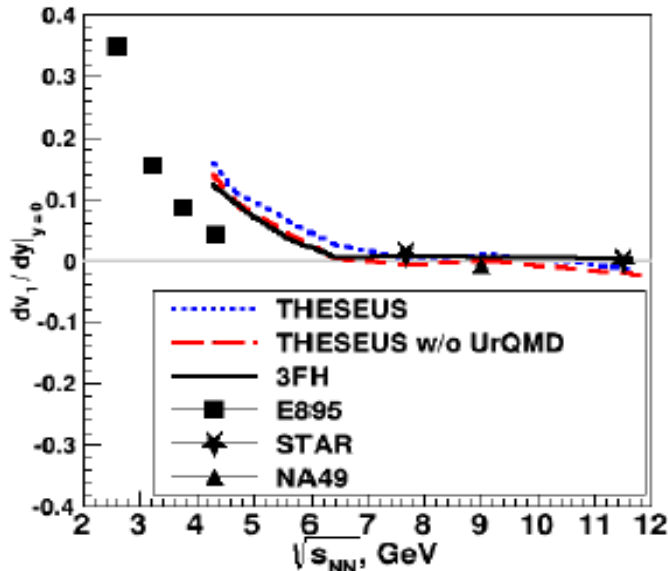
2-phase EoS,  $b = 6$  fm



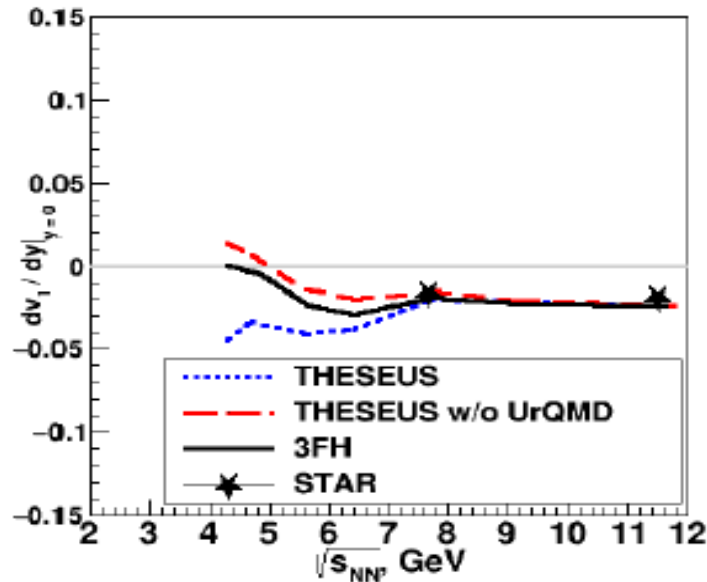
2-phase EoS,  $b = 6$  fm



crossover EoS,  $b = 6$  fm



crossover EoS,  $b = 6$  fm



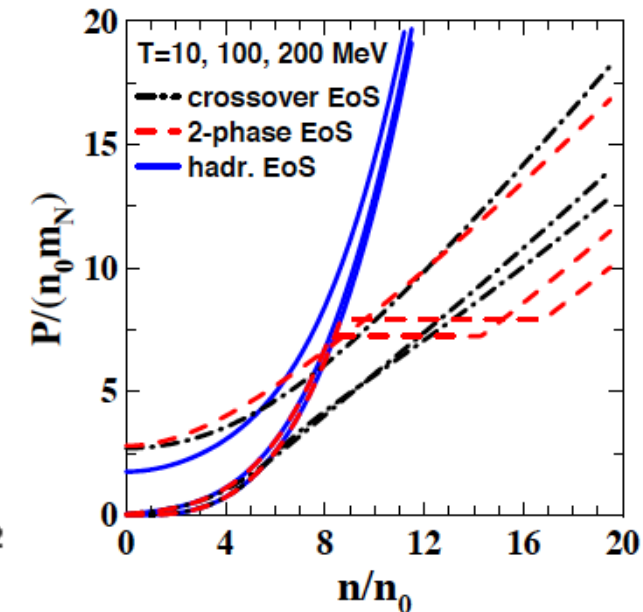
## Slope of directed flow

Protons (left)  
Pions (right)

First-order PT (upper)  
Crossover (lower)

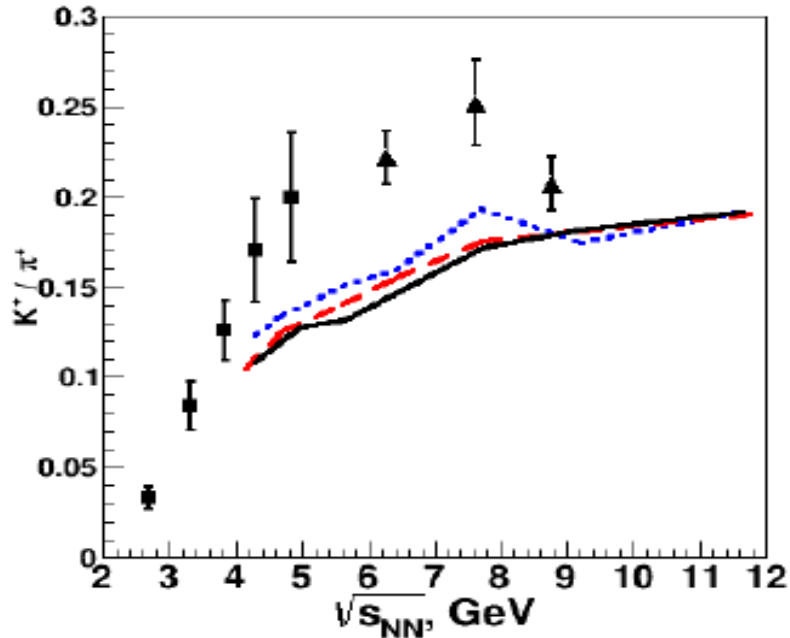
$\sqrt{s_{NN}}$ [GeV]	4.3	4.7	5.6	6.4	7.7	9.2	11.6
$E_{lab}$ [A GeV]	8	10	15	20	30	43	70

arxiv:1608.00965



# What about $K^+/\pi^+$ (Marek's horn) in THESEUS ?

2-phase EoS,  $b = 2$  fm



THESEUS simulation reproduces 3FH result, Thus it has the same discrepancy with experiment

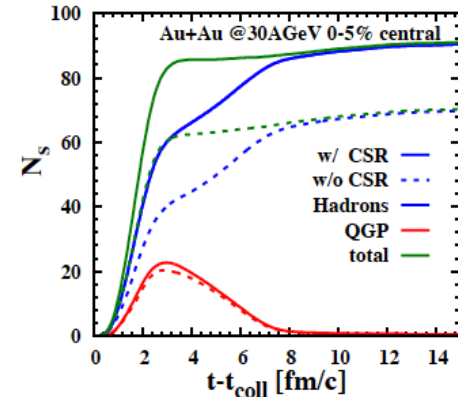
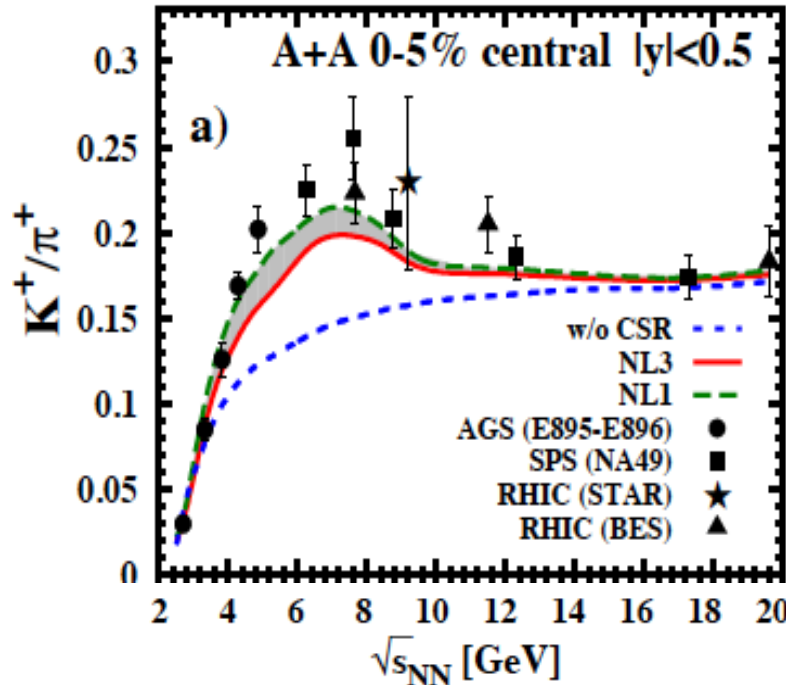
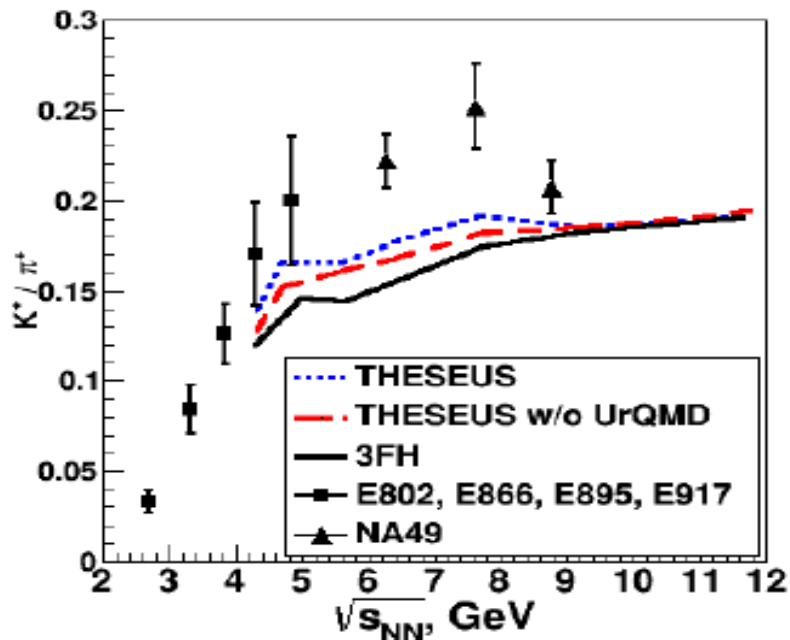
--> some key element still missing in the program

P. Batyuk, D.B., M. Bleicher et al.,  
Phys. Rev. C 94, 044093; arxiv:1608.00965

## Recent new development in PHSD

Chiral symmetry restoration in HIC at intermediate ...  
A. Palmese et al., arxiv: 1607.04073

crossover EoS,  $b = 2$  fm



Strange particle number increase by CSR



# Mott dissociation of $\pi$ and $K$ in hot, dense quark matter

D. Blaschke, A. Dubinin, A. Radzhabov, A. Wergieluk, arxiv:1608.05383



Andrey Radzhabov in front of the University of Wrocław

## PNJL model for $N_f=2+1$ quark matter with $\pi$ and $K$

$$\mathcal{L} = \bar{q} (i\gamma^\mu D_\mu + \hat{m}_0) q + G_S \sum_{a=0}^8 \left[ (\bar{q}\lambda^a q)^2 + (\bar{q}i\gamma_5\lambda^a q)^2 \right] - \mathcal{U}(\Phi[A], \bar{\Phi}[A]; T)$$

$$\Pi_{ff'}^{M^a}(q_0, \mathbf{q}) = 2N_c T \sum_n \int \frac{d^3p}{(2\pi)^3} \text{tr}_D \left[ S_f(p_n, \mathbf{p}) \Gamma_{ff'}^{M^a} S_{f'}(p_n + q_0, \mathbf{p} + \mathbf{q}) \Gamma_{ff'}^{M^a} \right]$$

$$\Gamma_{ff'}^{P^a} = i\gamma_5 T_{ff'}^a, \quad \Gamma_{ff'}^{S^a} = T_{ff'}^a, \quad T_{ff'}^a = \begin{cases} (\lambda_3)_{ff'}, \\ (\lambda_1 \pm i\lambda_2)_{ff'} / \sqrt{2}, \\ (\lambda_4 \pm i\lambda_5)_{ff'} / \sqrt{2}, \\ (\lambda_6 \pm i\lambda_7)_{ff'} / \sqrt{2}, \end{cases}$$

$$P^a = \pi^0, \pi^\pm, K^\pm, K^0, \bar{K}^0$$

$$\Pi_{ff'}^{P^a, S^a}(q_0 + i\eta, \mathbf{0}) = 4 \left\{ I_1^f(T, \mu_f) + I_1^{f'}(T, \mu_{f'}) \mp [(q_0 + \mu_{ff'})^2 - (m_f \mp m_{f'})^2] I_2^{ff'}(z, T, \mu_{ff'}) \right\}$$

$$I_1^f(T, \mu_f) = \frac{N_c}{4\pi^2} \int_0^\Lambda \frac{dp p^2}{E_f} \left( n_f^- - n_f^+ \right),$$

$$I_2^{ff'}(z, T, \mu_{ff'}) = \frac{N_c}{4\pi^2} \int_0^\Lambda \frac{dp p^2}{E_f E_{f'}} \left[ \frac{E_{f'}}{(z - E_f - \mu_{ff'})^2 - E_{f'}^2} n_f^- - \frac{E_{f'}}{(z + E_f - \mu_{ff'})^2 - E_{f'}^2} n_f^+ + \frac{E_f}{(z + E_{f'} - \mu_{ff'})^2 - E_f^2} n_{f'}^- - \frac{E_f}{(z - E_{f'} - \mu_{ff'})^2 - E_f^2} n_{f'}^+ \right]$$



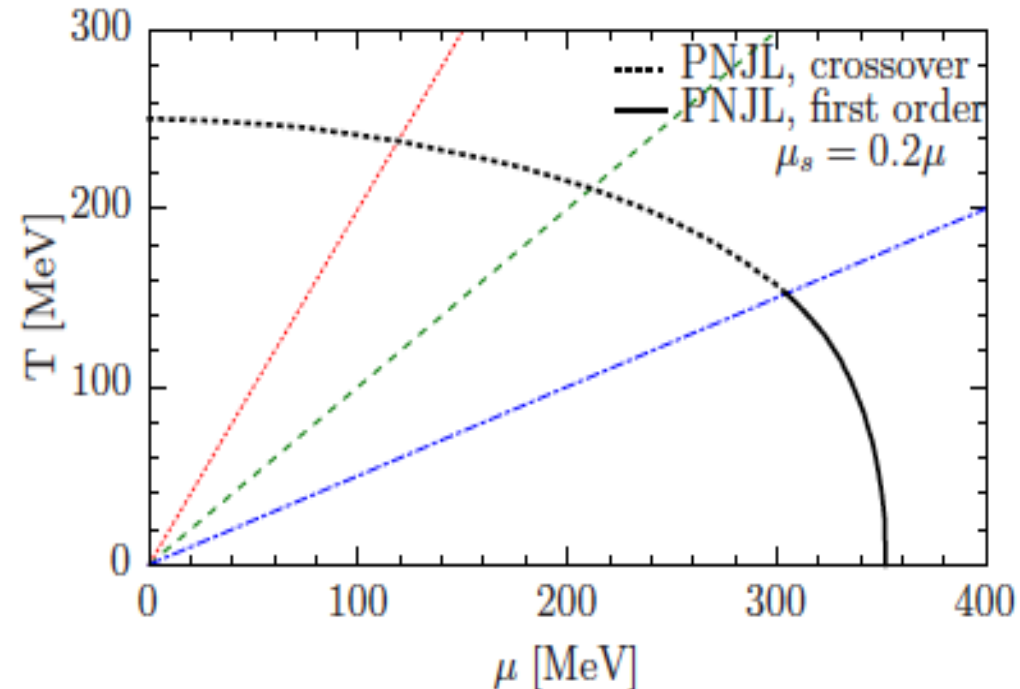
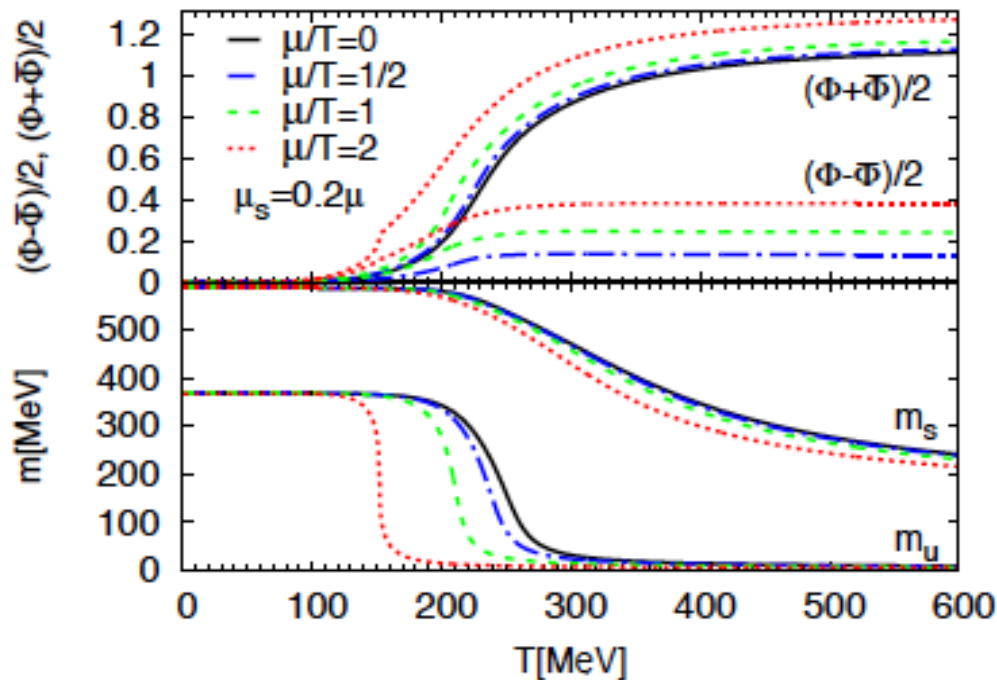
# PNJL model for $N_f=2+1$ quark matter with $\pi$ and K

$$m_f = m_{0,f} + 16 m_f G_S I_1^f(T, \mu), \quad \mathcal{P}_{ff'}^{M^a}(M_{M^a} + i\eta, \mathbf{0}) = 1 - 2G_S \Pi_{ff'}^{M^a}(M_{M^a} + i\eta, \mathbf{0}) = 0.$$

$$P_f = -\frac{(m_f - m_{0,f})^2}{8G} + \frac{N_c}{\pi^2} \int_0^\Lambda dp p^2 E_f + \frac{N_c}{3\pi^2} \int_0^\infty \frac{dp p^4}{E_f} [f_\Phi^+(E_f) + f_\Phi^-(E_f)]$$

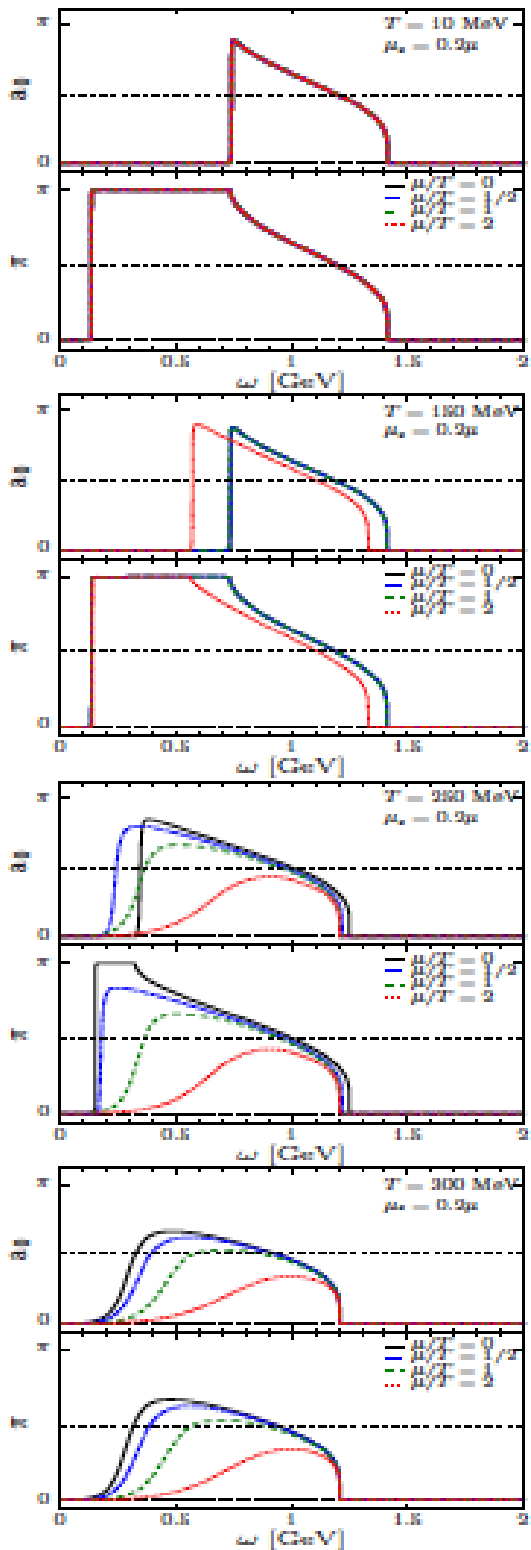
$$P_M = d_M \int \frac{d^3q}{(2\pi)^3} \int_0^\infty \frac{d\omega}{2\pi} \left\{ g(\omega - \mu_M) + g(\omega + \mu_M) \right\} \delta_M(\omega, \mathbf{q})$$

$$\delta_M(\omega, \mathbf{q}) = -\arctan \left\{ \frac{\text{Im} \left( \mathcal{P}_{ff'}^M(\omega - i\eta, \mathbf{q}) \right)}{\text{Re} \left( \mathcal{P}_{ff'}^M(\omega + i\eta, \mathbf{q}) \right)} \right\}$$



# Mott dissociation of pions and kaons in the Beth-Uhlenbeck approach ...

D.B., A. Dubinin, A. Radzhabov, A. Wergieluk, arxiv:1608.05383  
 D.B., M. Buballa, A. Dubinin, G. Ropke, D. Zablocki, Ann. Phys. (2014)



Thermodynamics of resonances (M) via phase shifts

$$P_M = d_M \int \frac{d^3q}{(2\pi)^3} \int_0^\infty \frac{ds}{4\pi} \frac{1}{\sqrt{s+q^2}} \left\{ g(\sqrt{s+q^2} - \mu_M) \right\} \delta_M(\sqrt{s}; T, \mu)$$

Polyakov-loop Nambu – Jona-Lasinio modell

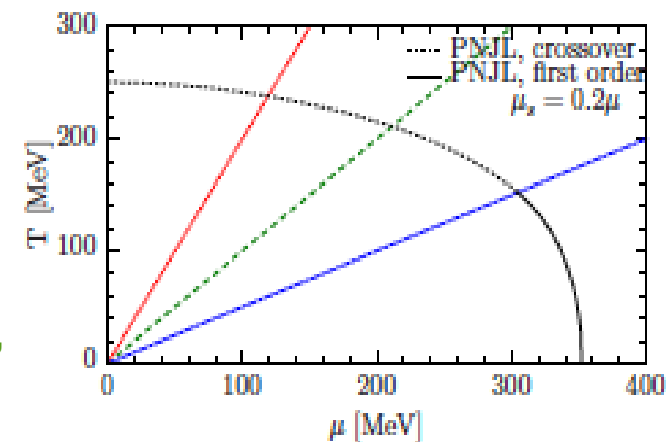
$$\Pi_{ff}^{M^*}(q_0, \mathbf{q}) = 2N_c T \sum_n \int \frac{d^3p}{(2\pi)^3} \text{tr} D \left[ S_f(p_n, \mathbf{p}) \Gamma_{ff}^{M^*} S_f(p_n + q_0, \mathbf{p} + \mathbf{q}) \Gamma_{ff}^{M^*} \right],$$

$$\mathcal{P}_{ff}^{M^*}(M_{M^*} + i\eta, \mathbf{0}) = 1 - 2G_S \Pi_{ff}^{M^*}(M_{M^*} + i\eta, \mathbf{0})$$

$$\delta_M(\omega, \mathbf{q}) = -\arctan \left\{ \frac{\text{Im} \left( \mathcal{P}_{ff}^M(\omega - i\eta, \mathbf{q}) \right)}{\text{Re} \left( \mathcal{P}_{ff}^M(\omega + i\eta, \mathbf{q}) \right)} \right\}$$

Evaluation along trajectories  
 $\mu/T = \text{const}$  in the phase diagram:

- Pion and a0 as partner states,
- Chiral symmetry restoration,
- Mott dissociation of bound states,
- Levinson theorem



# Mott dissociation of pions and kaons in the Beth-Uhlenbeck approach ...

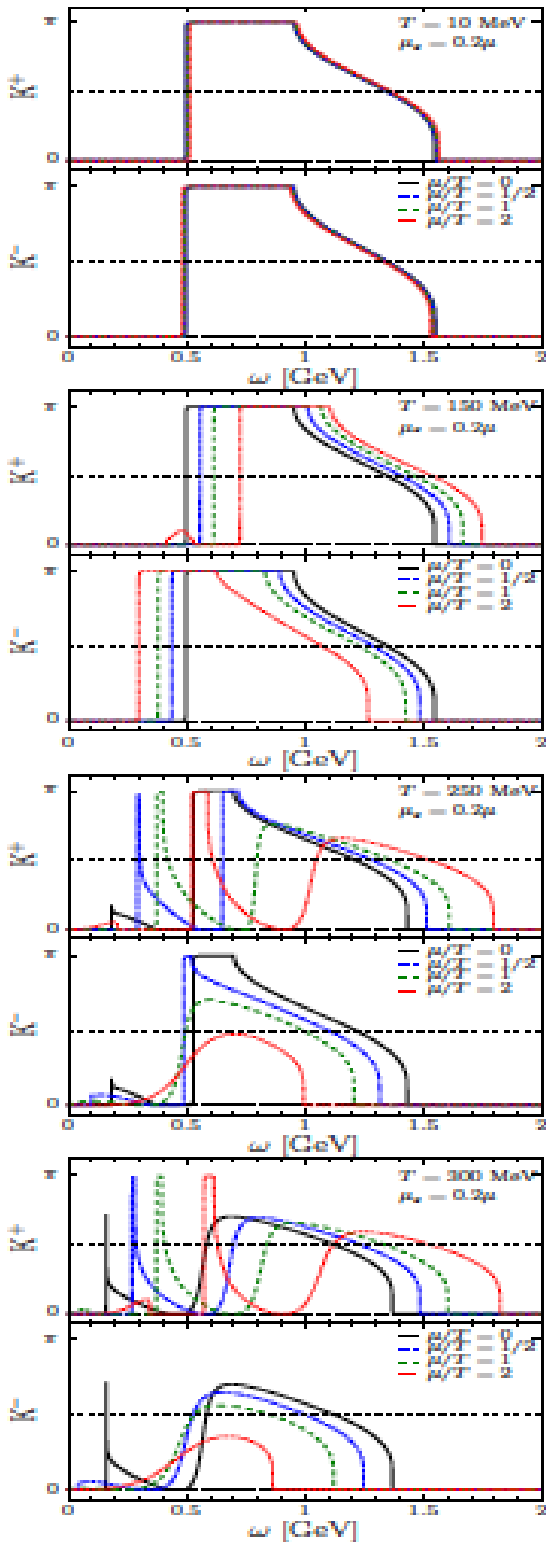
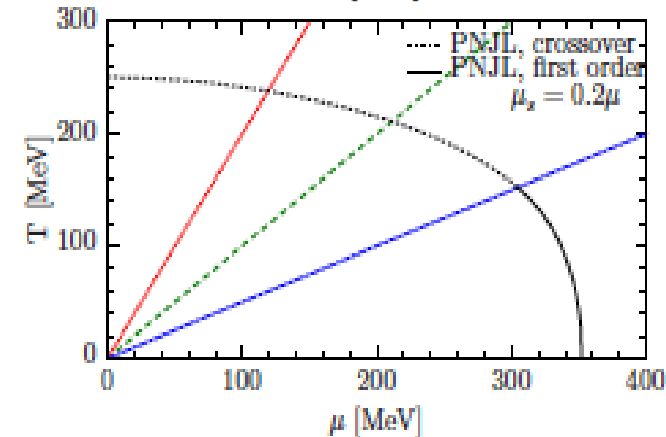
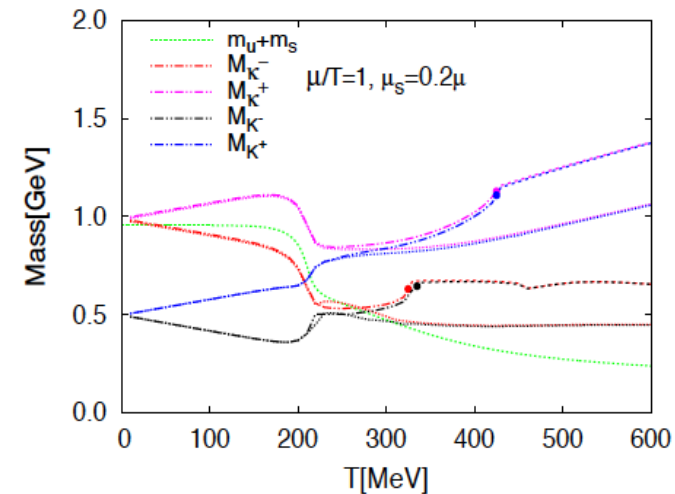
D.B., A. Dubinin, A. Radzhabov, A. Wergieluk, arxiv:1608.05383

Polarization loop in Polyakov-loop Nambu – Jona-Lasinio model

$$\Pi_{ff'}^{P^a, S^a}(q_0 + i\eta, \mathbf{0}) = 4 \{ I_1^f(T, \mu_f) + I_1^{f'}(T, \mu_{f'}) \mp [(q_0 + \mu_{ff'})^2 - (m_f \mp m_{f'})^2] I_2^{ff'}(z, T, \mu_{ff'}) \}$$

$$I_1^f(T, \mu_f) = \frac{N_c}{4\pi^2} \int_0^\Lambda \frac{dp p^2}{E_f} (n_f^- - n_f^+),$$

$$I_2^{ff'}(z, T, \mu_{ff'}) = \frac{N_c}{4\pi^2} \int_0^\Lambda \frac{dp p^2}{E_f E_{f'}} \left[ \frac{E_{f'}}{(z - E_f - \mu_{ff'})^2 - E_{f'}^2} n_f^- - \frac{E_{f'}}{(z + E_f - \mu_{ff'})^2 - E_{f'}^2} n_f^+ + \frac{E_f}{(z + E_{f'} - \mu_{ff'})^2 - E_f^2} n_{f'}^- - \frac{E_f}{(z - E_{f'} - \mu_{ff'})^2 - E_f^2} n_{f'}^+ \right]$$

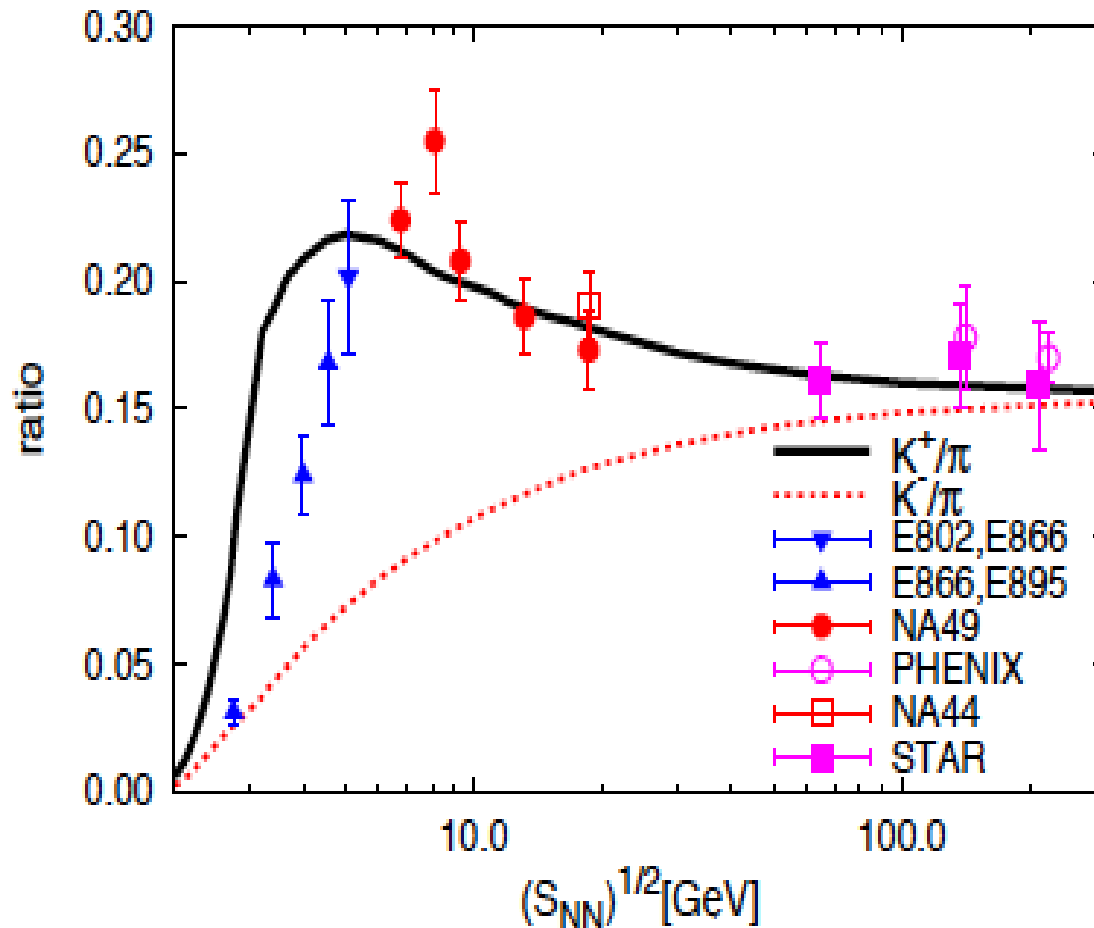


Anomalous low-mass mode for K+ in the dense medium !!

# Mott dissociation of pions and kaons in Beth-Uhlenbeck: Explanation of the “horn” effect for $K^+/\pi^+$ in HIC?

Ratio of yields in BU approach  
defined via phase shifts:

$$\frac{n_{K^\pm}}{n_{\pi^\pm}} = \frac{\int dM \int d^3p (M/E) g_{K^\pm}(E) [1 + g_{K^\pm}(E)] \delta_{K^\pm}(M)}{\int dM \int d^3p (M/E) g_{\pi^\pm}(E) [1 + g_{\pi^\pm}(E)] \delta_{\pi^\pm}(M)}$$



Evaluation along the freeze-out  
Curve parametrized by Cleymans et al.

- enhancement for  $K^+$  due to anomalous in-medium bound state mode
- no such enhancement for  $K^-$  or pions
- explore the effect in thermal statistical models and in THESEUS ...



31<sup>st</sup> Max Born Symposium “Three days of Critical Behaviour in QCD”, Wroclaw, June 2013

Tobias Fischer Misha Stephanov  
Pok Man Lo Edwin Laermann



Agnieszka Wergieluk

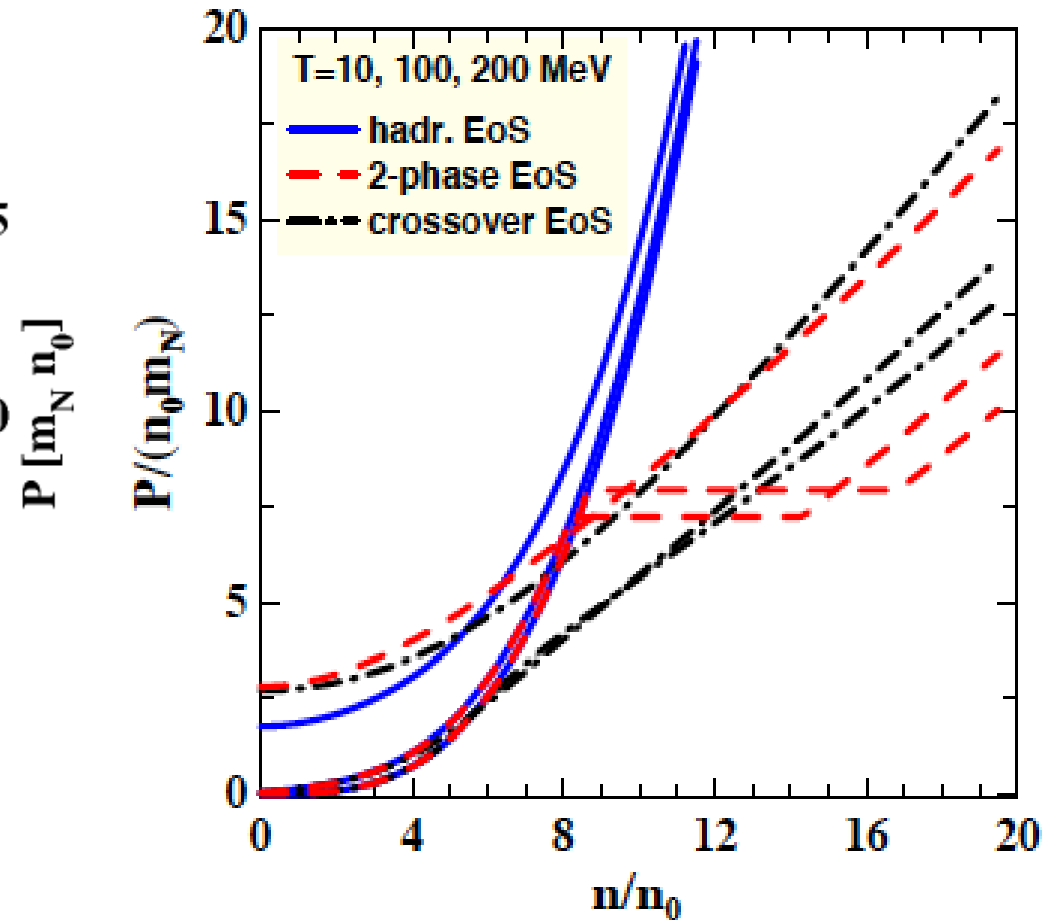
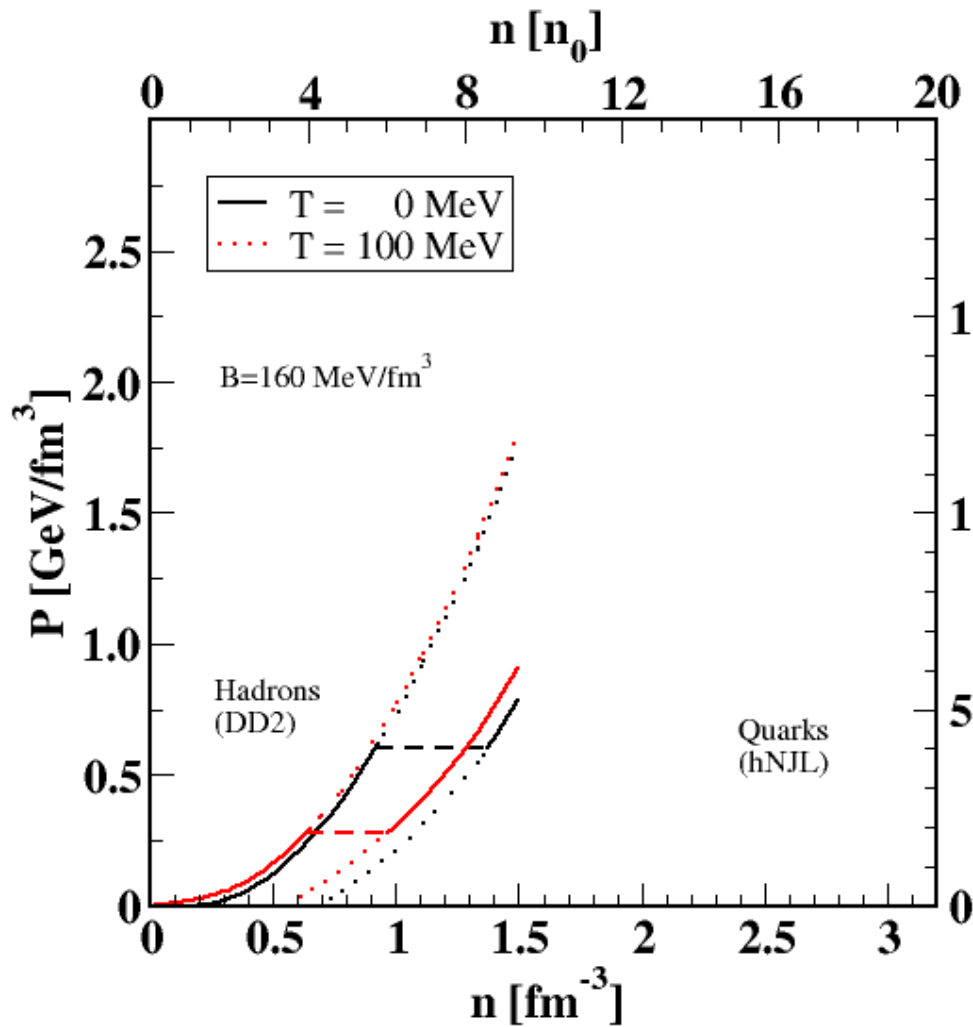
Aleksandr Dubinin

Larry McLerran

## **Further developments:**

- New 2-phase EoS (D.B. et al. @ UWr)**
- MPD Detector simulation (Rogachevsky et al. @ JINR)**
- Bayesian EoS analyses (UWr - JINR – MEPhI collab.)**

# Comparison 2-phase EoS



N.-U. Bastian, D. Blaschke (S. Benic, S. Typel),  
In progress (2016)

A. Khvorostukhin et al. EPJC 48 (2006) 531  
Yu. Ivanov, D. Blaschke, arxiv:1504.03992



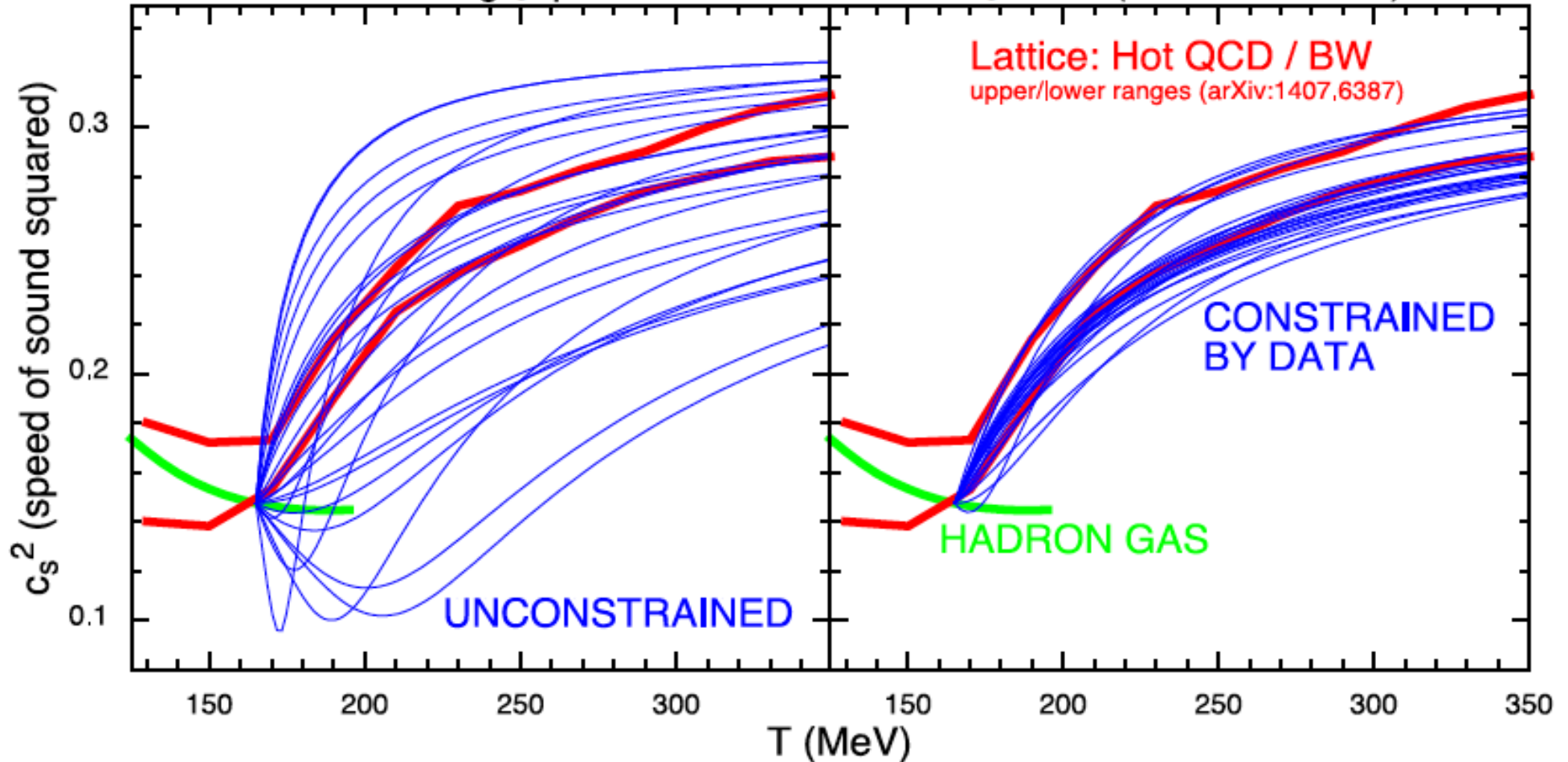
A man with a white beard and a dark suit is looking towards the camera. He is positioned in the bottom right corner of the frame. The background is a large, illuminated diorama of a prehistoric river scene. The scene features several large, gnarled tree trunks with thick, buttressed roots. The water is a vibrant blue-green color, and numerous fish are swimming in it. The fish are primarily blue and black, with some having a reddish-orange stripe. The lighting is dramatic, with strong highlights and deep shadows, creating a sense of depth and immersion. The overall atmosphere is that of a museum exhibit or a large-scale model.

# The QCD Phase Diagram Revisited ...



# Bayesian EoS analysis with HIC data works @ LHC - RHIC

S. Pratt, E. Sangaline, P. Sorensen, H. Wang Phys. Rev. Lett. (2015),



QCD theory+modeling **and constant experimental guidance** now give us a detailed picture of the evolution of nucleus-nucleus collisions

Emergent properties of QCD matter now experimentally accessible



Brookhaven National Laboratory  
CERN  
Aug 14-15, 2008  
Jürgen Bandrup  
LBNL

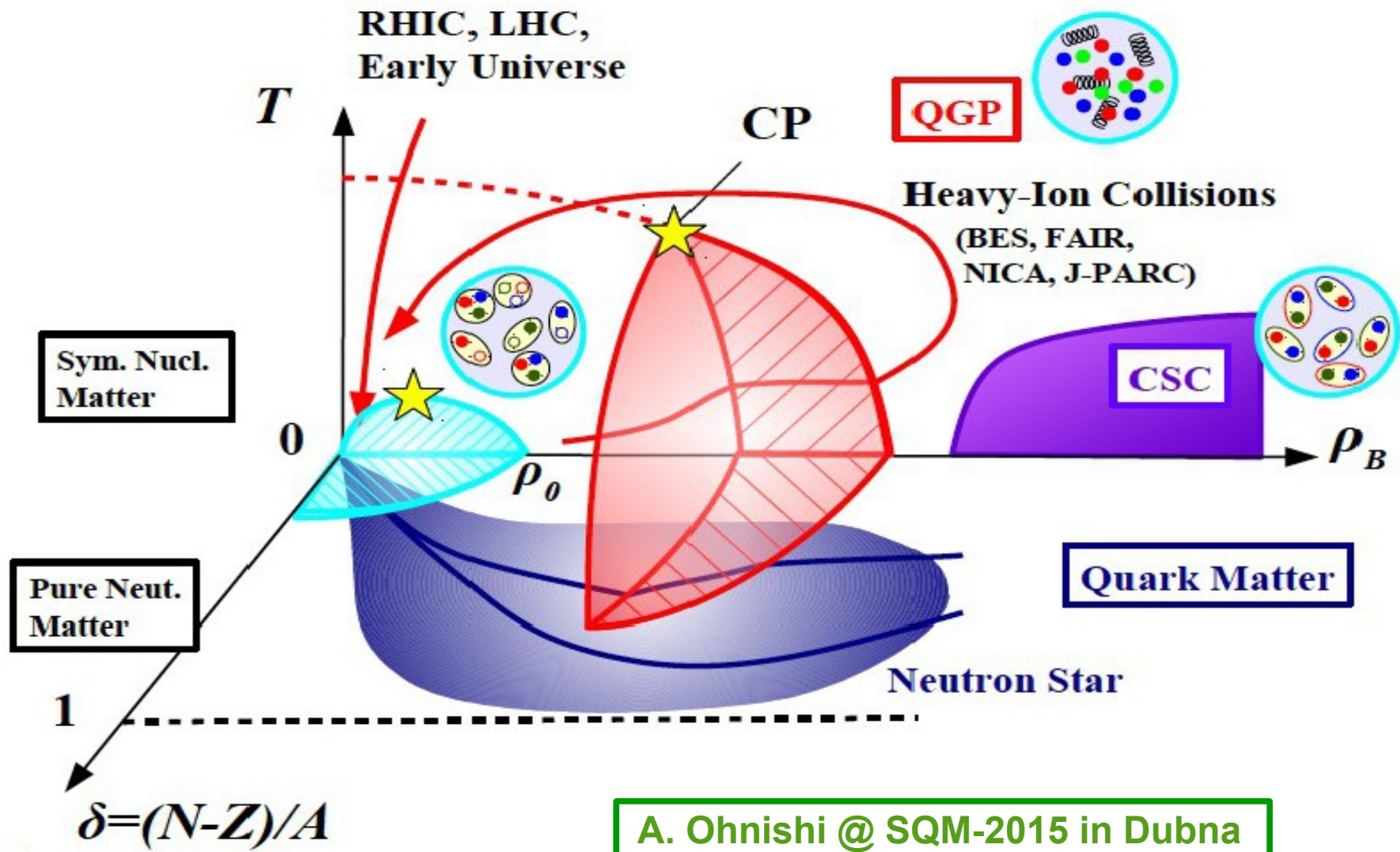
Brookhaven National Laboratory  
CERN  
Aug 14-15, 2008  
David Weyhle  
LBNL

Brookhaven National Laboratory  
CERN  
Aug 14-15, 2008  
Akira Hoshida  
LBNL

**BROOKHAVEN**  
NATIONAL LABORATORY



# Support a CEP in QCD phase diagram with Astrophysics?



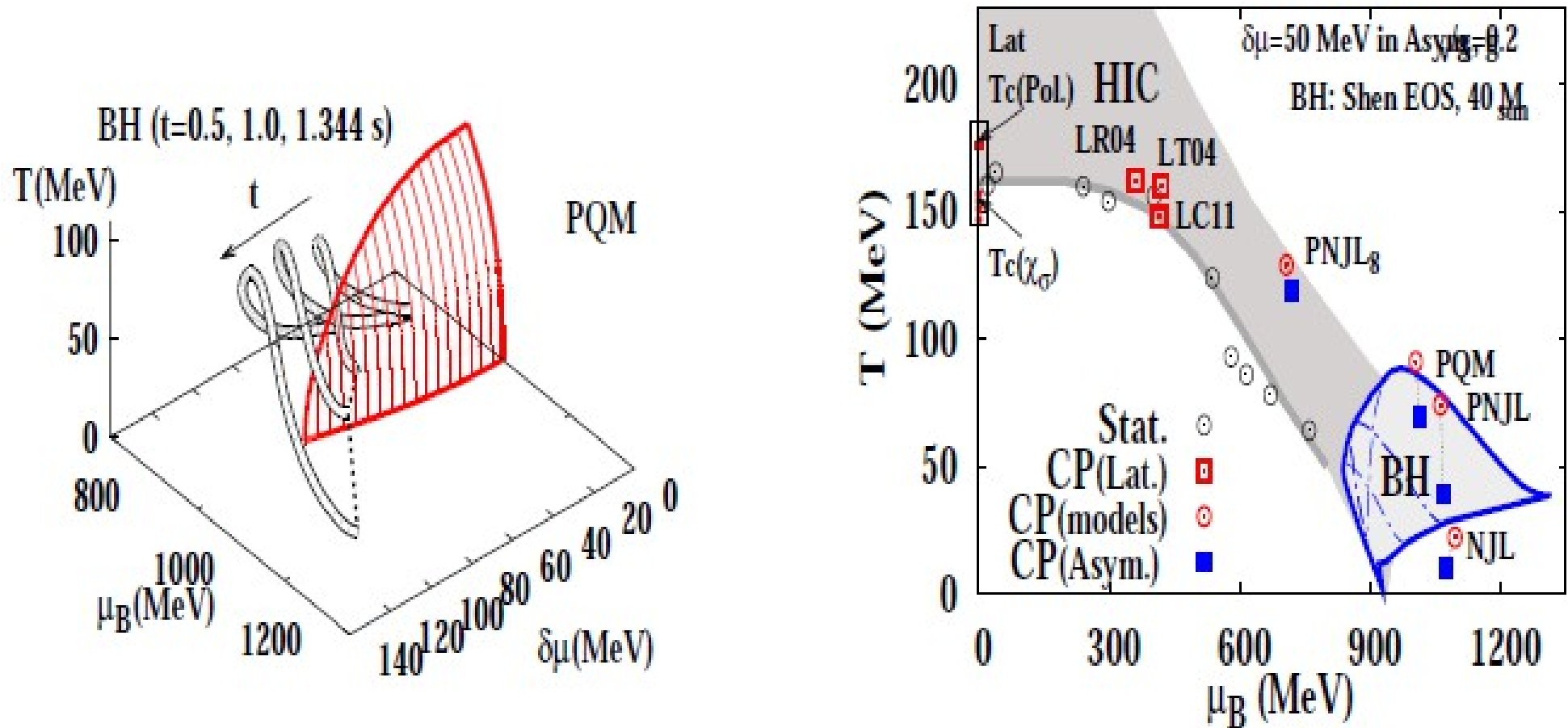
A. Ohnishi @ SQM-2015 in Dubna

Crossover at finite  $T$  (Lattice QCD) + First order at zero  $T$  (Astrophysics) = Critical endpoint exists!



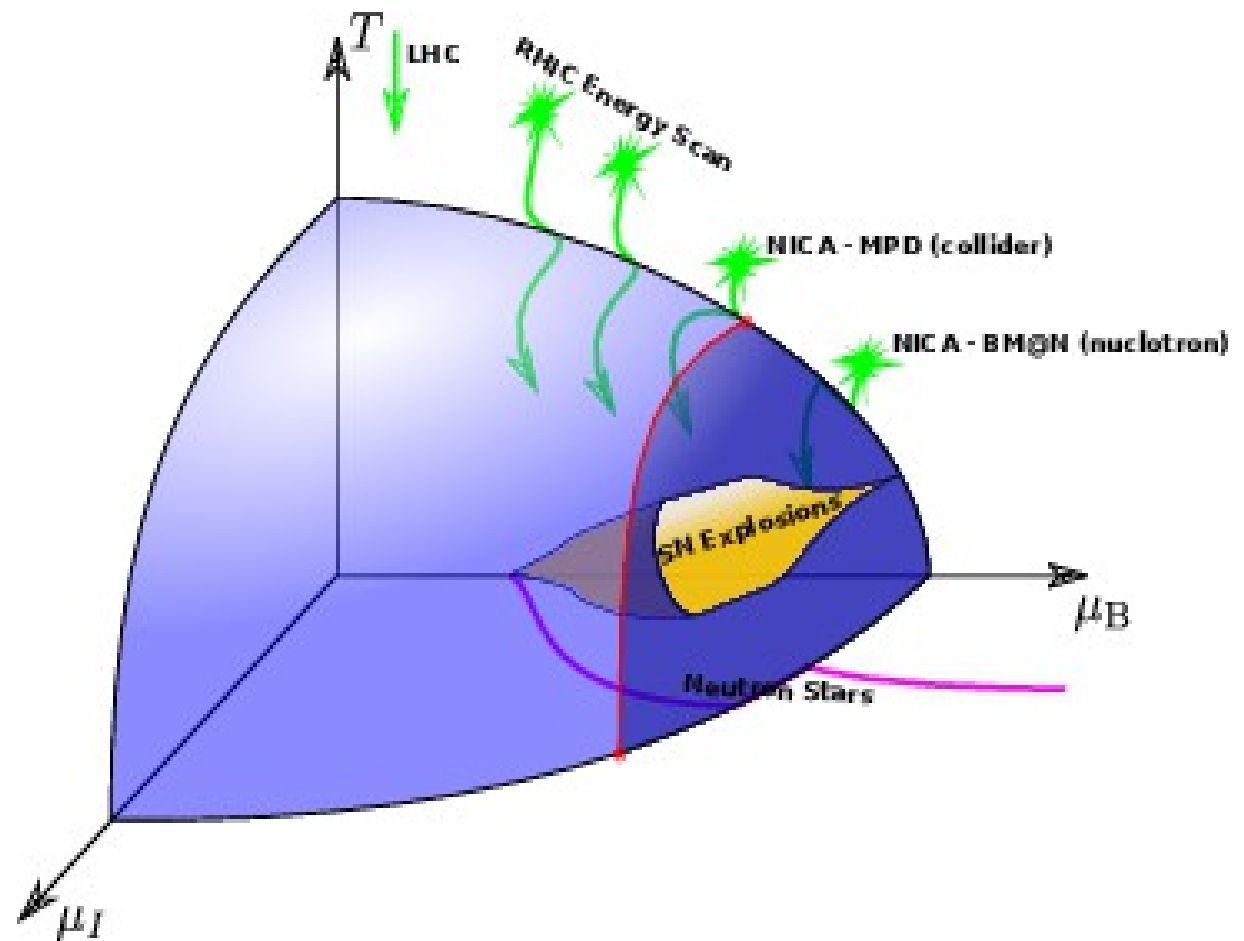
# How to probe the line of CEP's in Astrophysics?

→ by sweeping (“flyby”) the critical line in SN collapse and BH formation





Akira Ohnishi D.B. Ernst-Michael Ilgenfritz @ Volga beach in Dubna



## Conclusion:

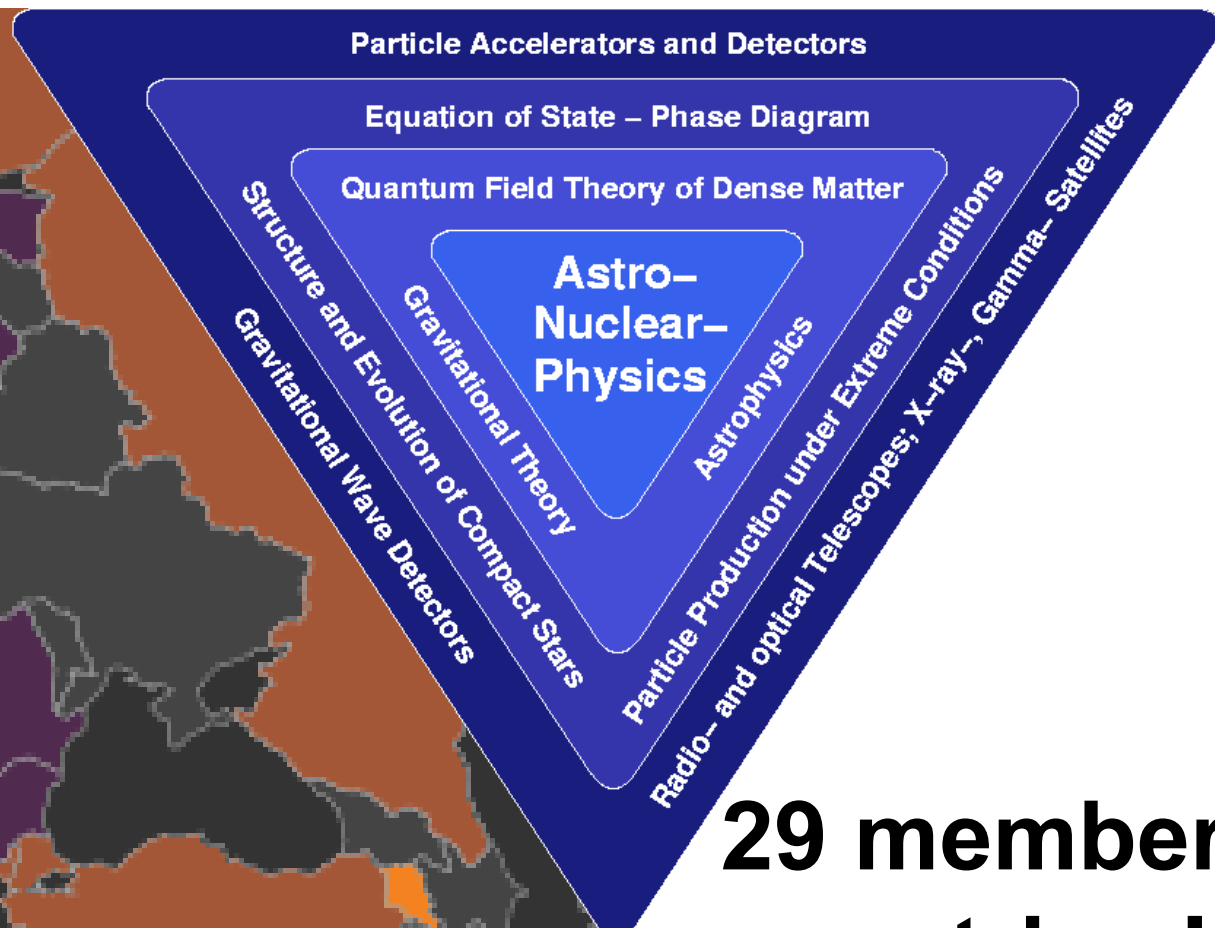
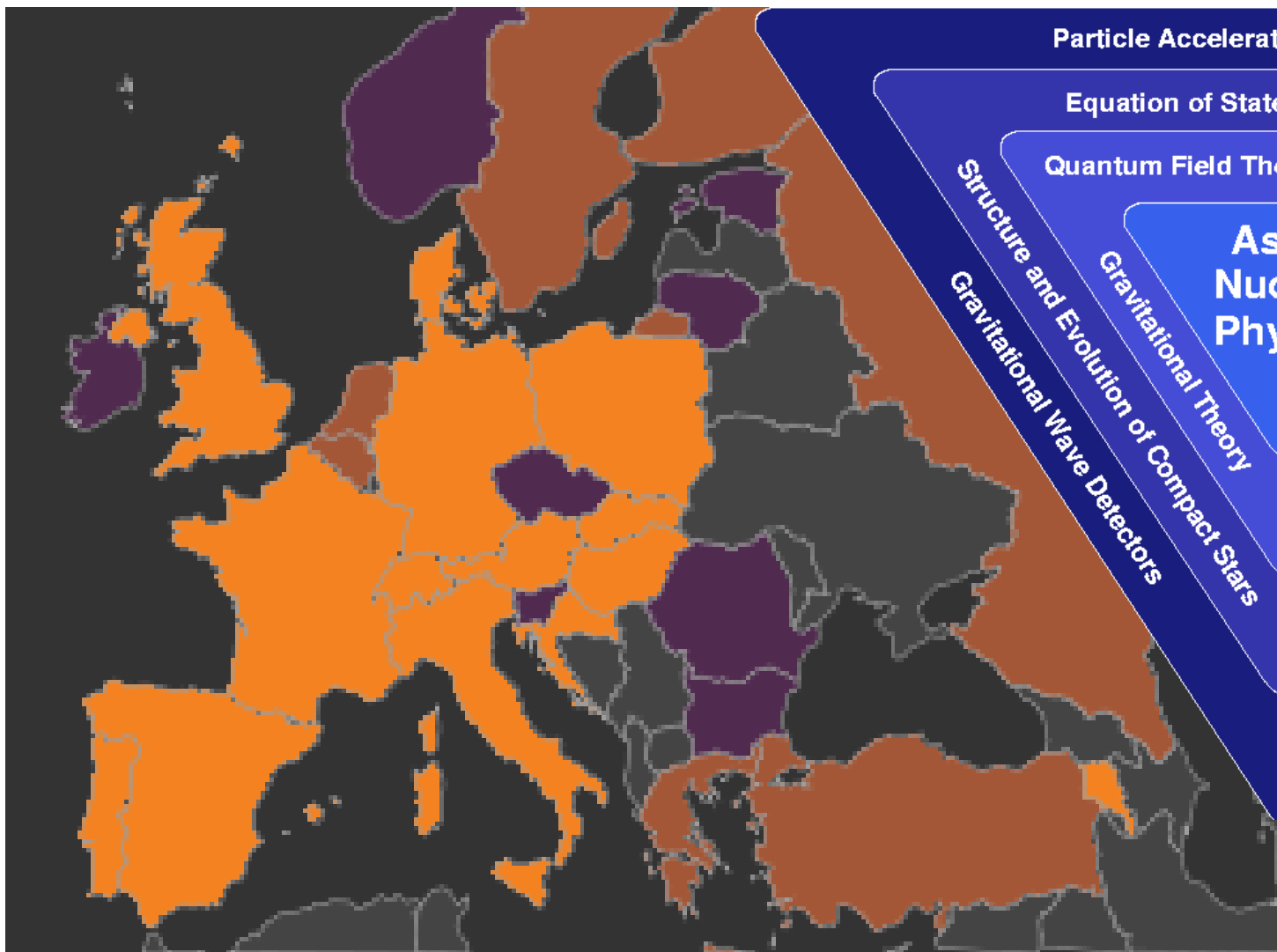
**Critical endpoint search in the QCD phase diagram with Heavy-Ion Collisions goes well together with Compact Star Astrophysics**





**Conclusion:**

**Critical endpoint search in the QCD phase diagram with Heavy-Ion Collisions goes well together with Compact Star Astrophysics**

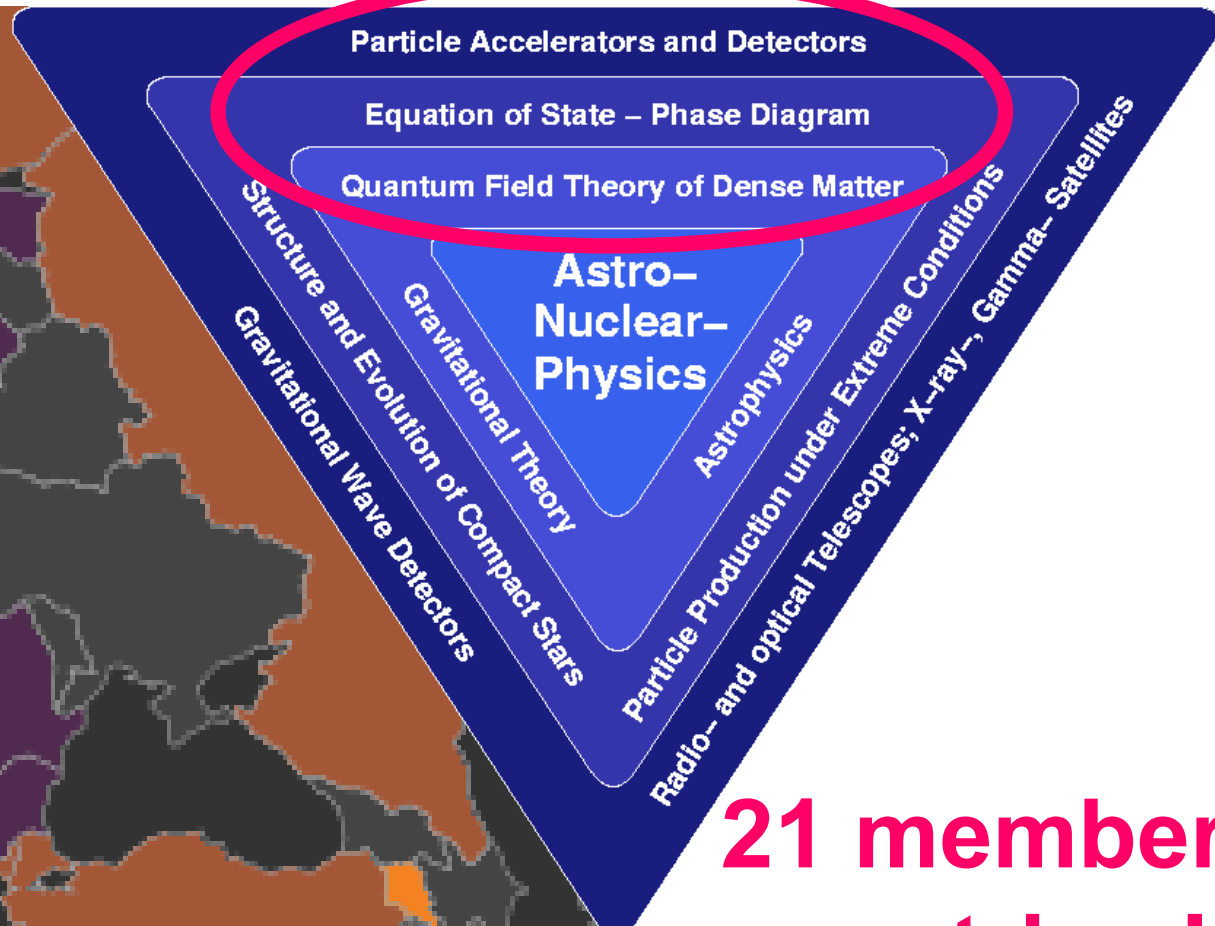
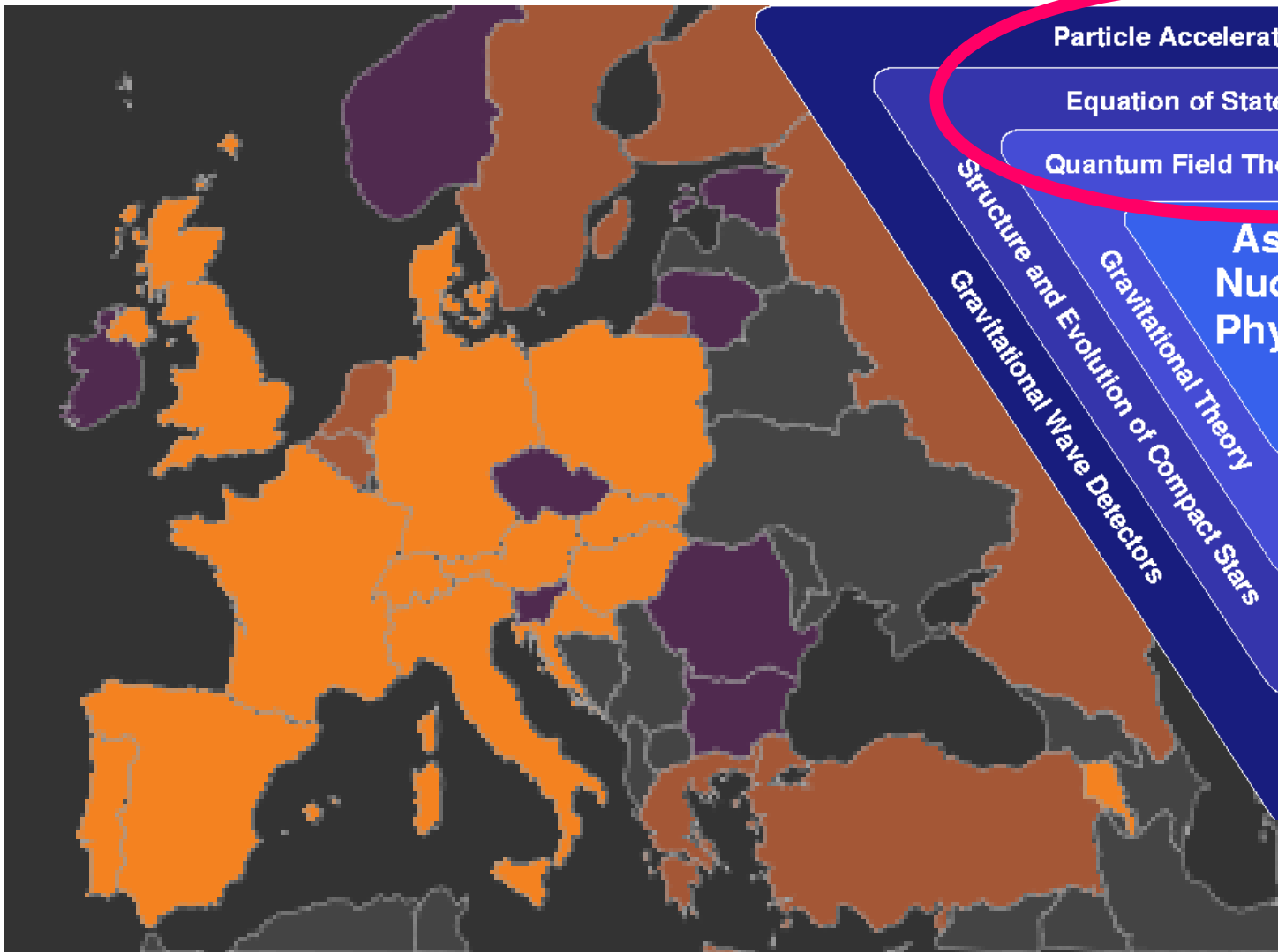


**29 member  
countries !!  
(MP1304)**

**New**



**Kick-off: Brussels, November 25, 2013**



**21 member countries !  
(CA15213)**

“Theory of **H**ot Matter in **R**elativistic Heavy-Ion Collisions”

**New: THOR !**



**Kick-off: Brussels, October 17, 2016**





International Conference “Critical Point and Onset of Deconfinement”  
University of Wroclaw, May 29 – June 4, 2016



**Backup slides**

## 4. Hadron-Quark crossover

As discussed in §1, treating the point-like hadron as an independent degree of freedom loses its validity as the baryon density approaches to the percolation region. In other words, the system cannot be described neither by an extrapolation of the hadronic EOS from the low-density side nor by an extrapolation of the quark EOS from the high-density side. Under such situation, it does not make much sense to apply the Gibbs criterion of two phases I and II,  $P_I(T_c, \mu_c) = P_{II}(T_c, \mu_c)$  since  $P_I$  and  $P_{II}$  are not reliable in the transition region.

we will consider a phenomenological “interpolation” between the H-EOS and Q-EOS as a first step. Such an interpolation is certainly not unique, but we adopt a simplest

$$P(\rho) = P_H(\rho)f_-(\rho) + P_Q(\rho)f_+(\rho),$$
$$f_{\pm}(\rho) = \frac{1}{2} \left( 1 \pm \tanh \left( \frac{\rho - \bar{\rho}}{\Gamma} \right) \right),$$

where  $P_H$  and  $P_Q$  are the pressure in the hadronic matter and that in the quark matter,

One should not confuse Eq.(7) with the pressure in the mixed phase associated with the first-order phase transition in which  $f_{\pm}$  is considered to the volume fraction of each phase. In our crossover picture, the system is always uniform and  $f_-$  ( $f_+$ ) should be interpreted as the degree of reliability of H-EOS (Q-EOS) at given baryon density.

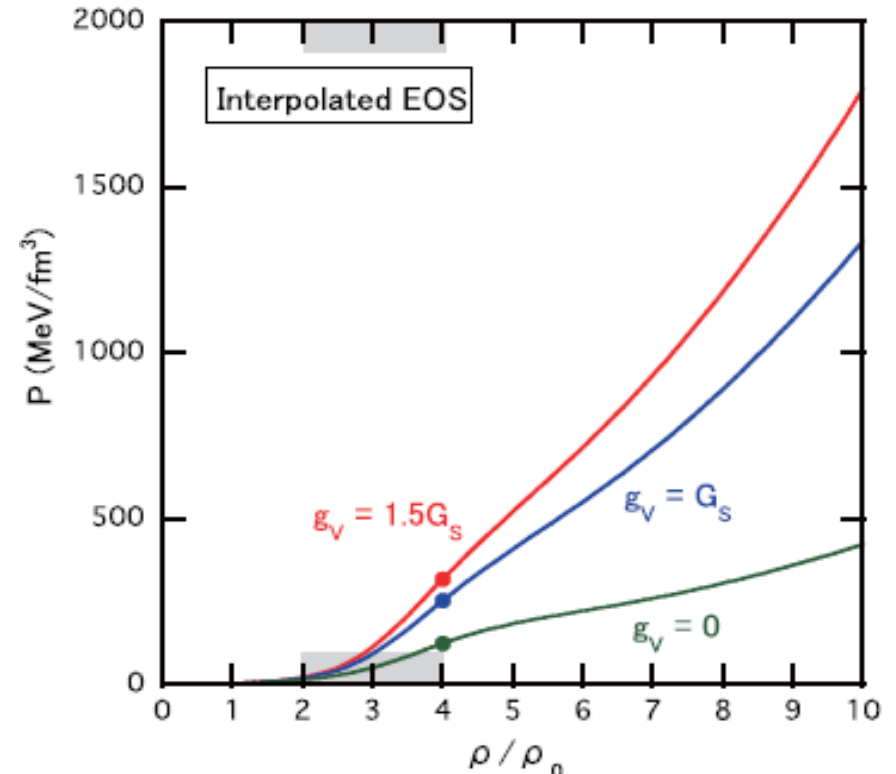
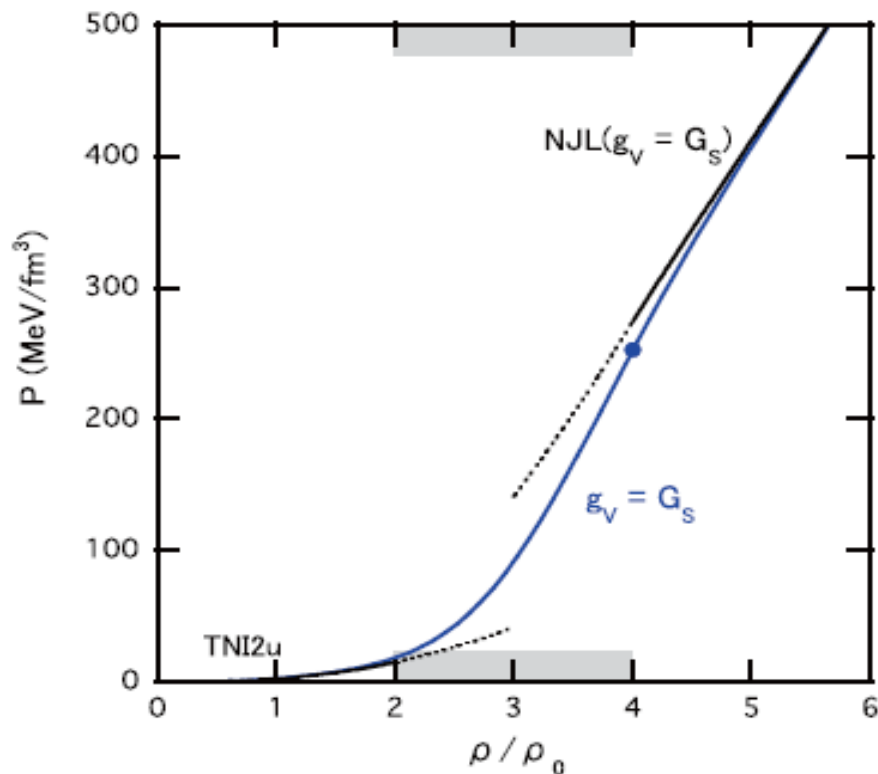


To calculate the energy density  $\varepsilon$  as a function of  $\rho$  in thermodynamically consistent way, we integrate the thermodynamical relation,  $P = \rho^2 \partial(\varepsilon/\rho)/\partial\rho$  and obtain

$$\varepsilon(\rho) = \varepsilon_H(\rho)f_-(\rho) + \varepsilon_Q(\rho)f_+(\rho) + \Delta\varepsilon$$

$$\Delta\varepsilon = \rho \int_{\bar{\rho}}^{\rho} (\varepsilon_H(\rho') - \varepsilon_Q(\rho')) \frac{g(\rho')}{\rho'} d\rho'$$

with  $g(\rho) = \frac{2}{\Gamma}(e^X + e^{-X})^{-2}$  and  $X = (\rho - \bar{\rho})/\Gamma$ . Here  $\varepsilon_H$  ( $\varepsilon_Q$ ) is the energy density obtained from H-EOS (Q-EOS).  $\Delta\varepsilon$  is an extra term which guarantees the thermodynamic consistency.



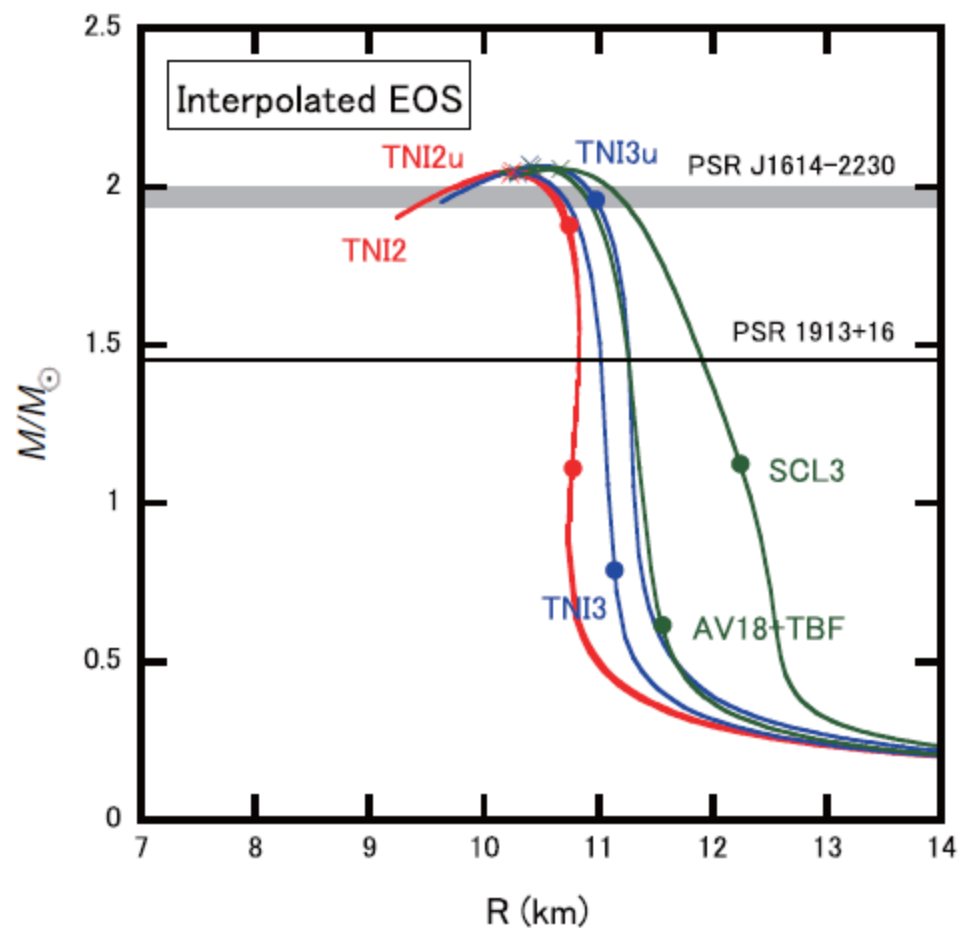
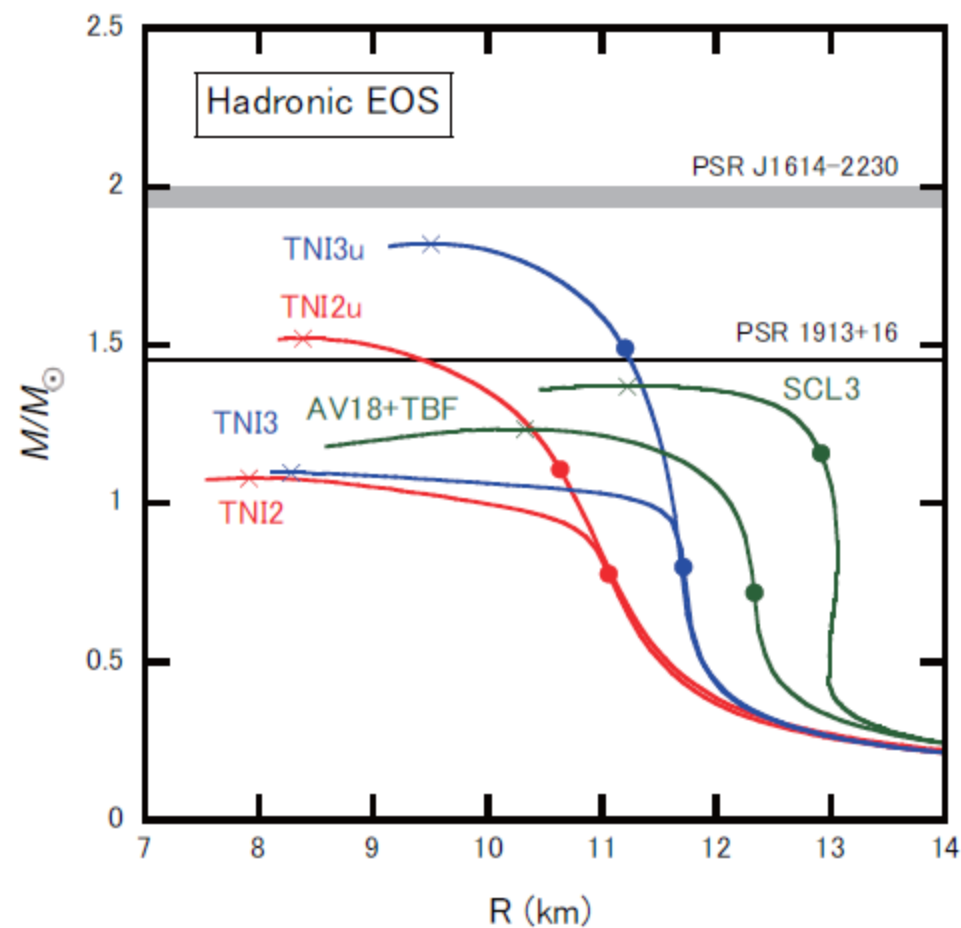
## 5. Numerical results and discussions

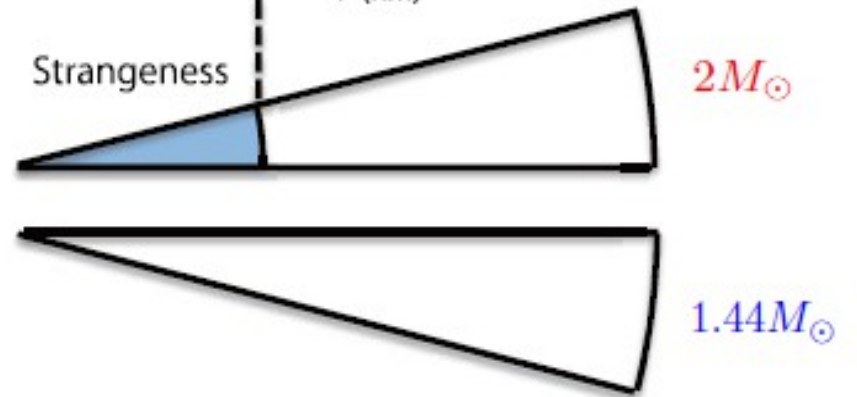
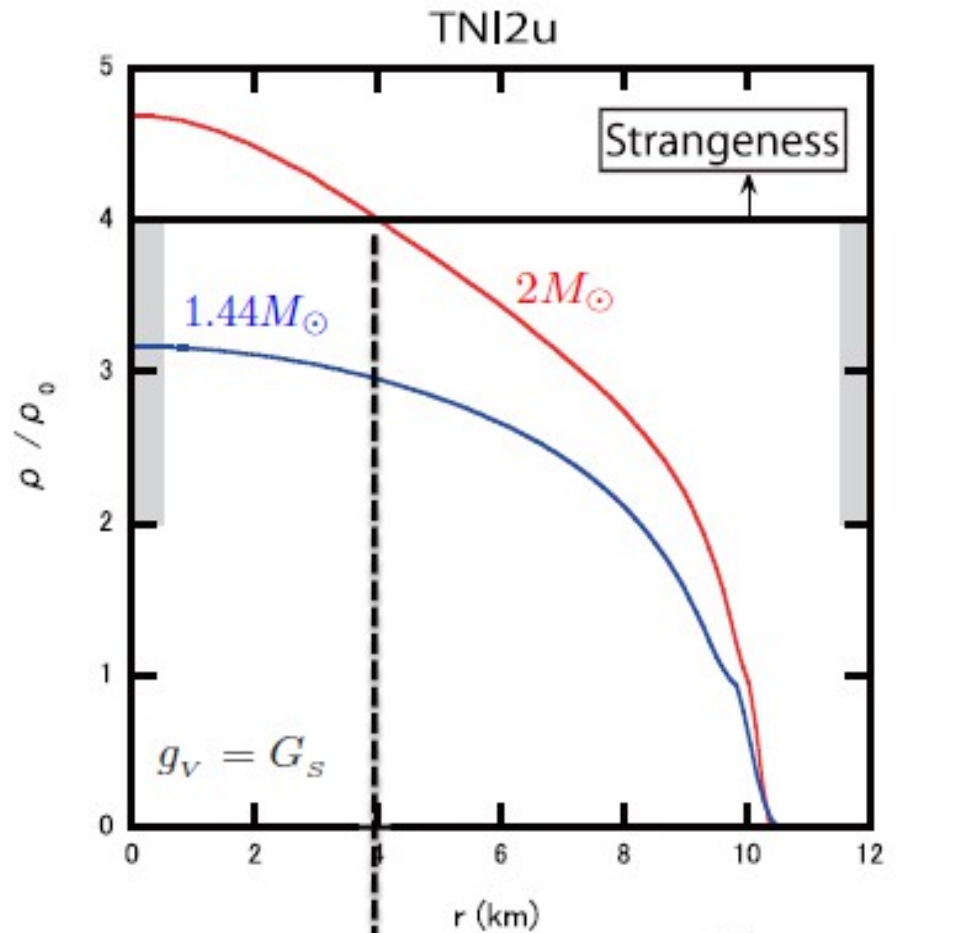
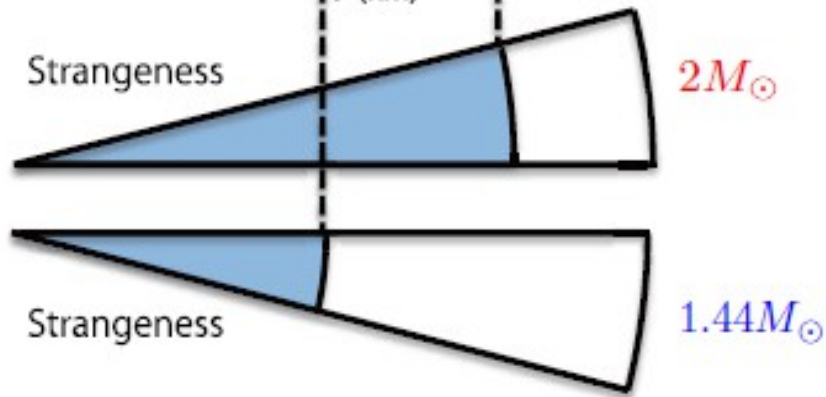
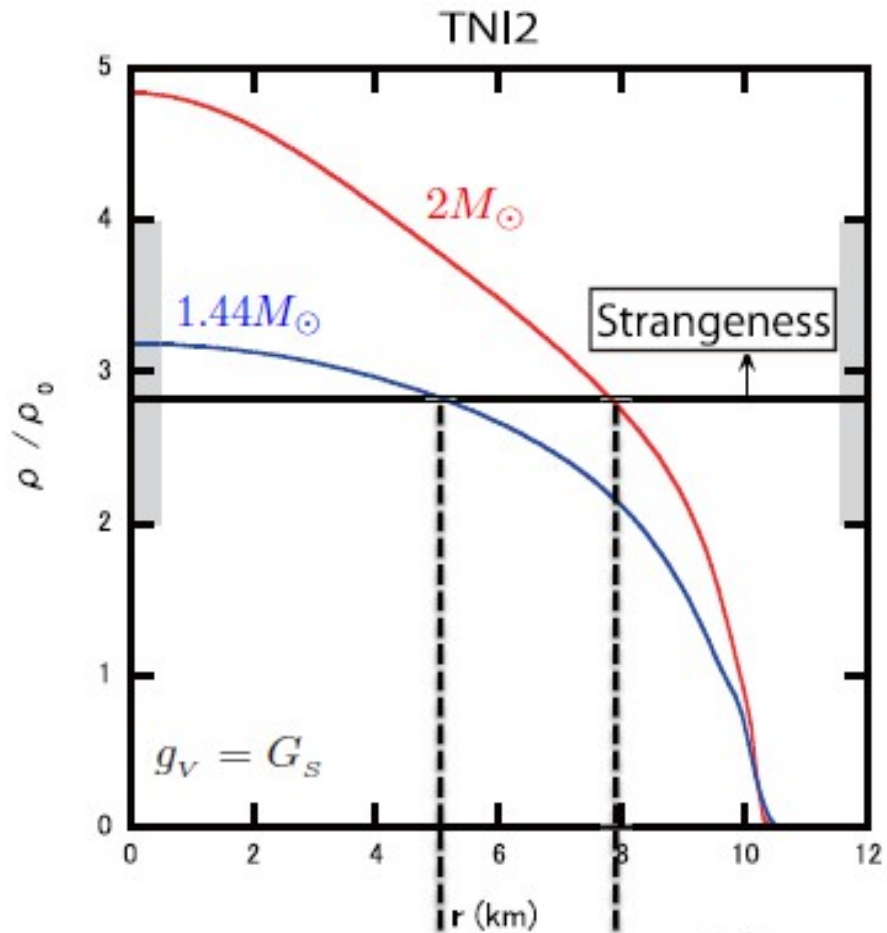
### 5.1. Massive hybrid star with strangeness

We now solve the following Tolman-Oppenheimer-Volkov (TOV) equation to obtain  $M$ - $R$  relationship by using the EOSs with and without the hadron-quark crossover:

$$\frac{dP}{dr} = -\frac{G}{r^2} (M(r) + 4\pi Pr^3) (\varepsilon + P) (1 - 2GM(r)/r)^{-1},$$

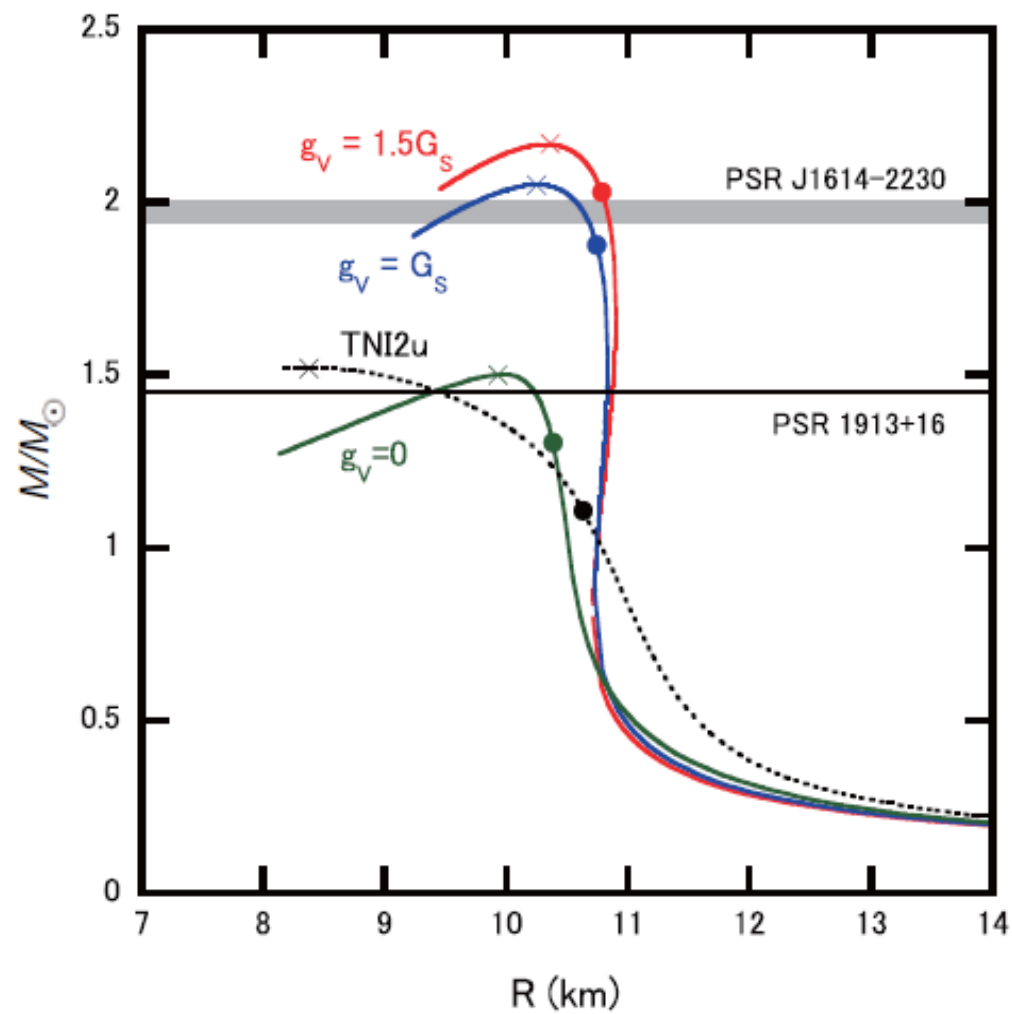
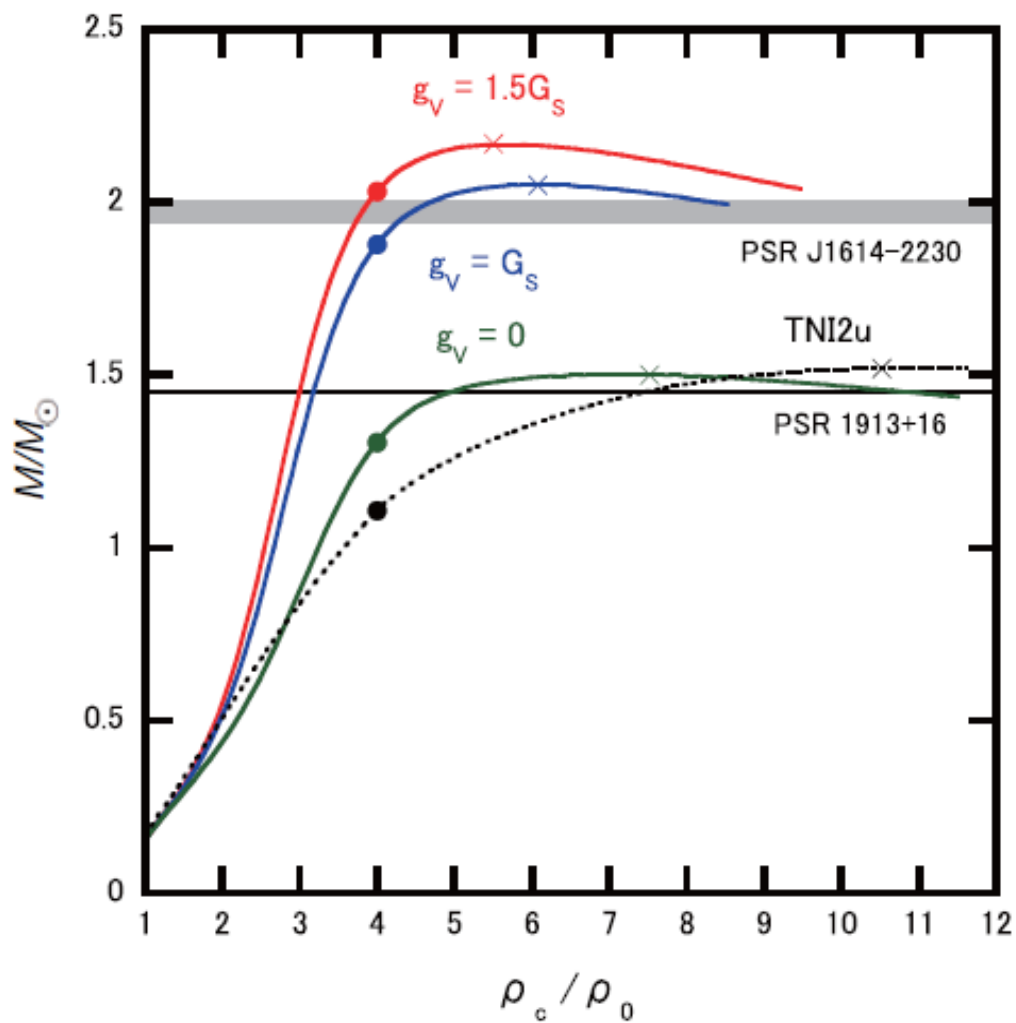
$$M(r) = \int_0^r 4\pi r'^2 \varepsilon(r') dr',$$





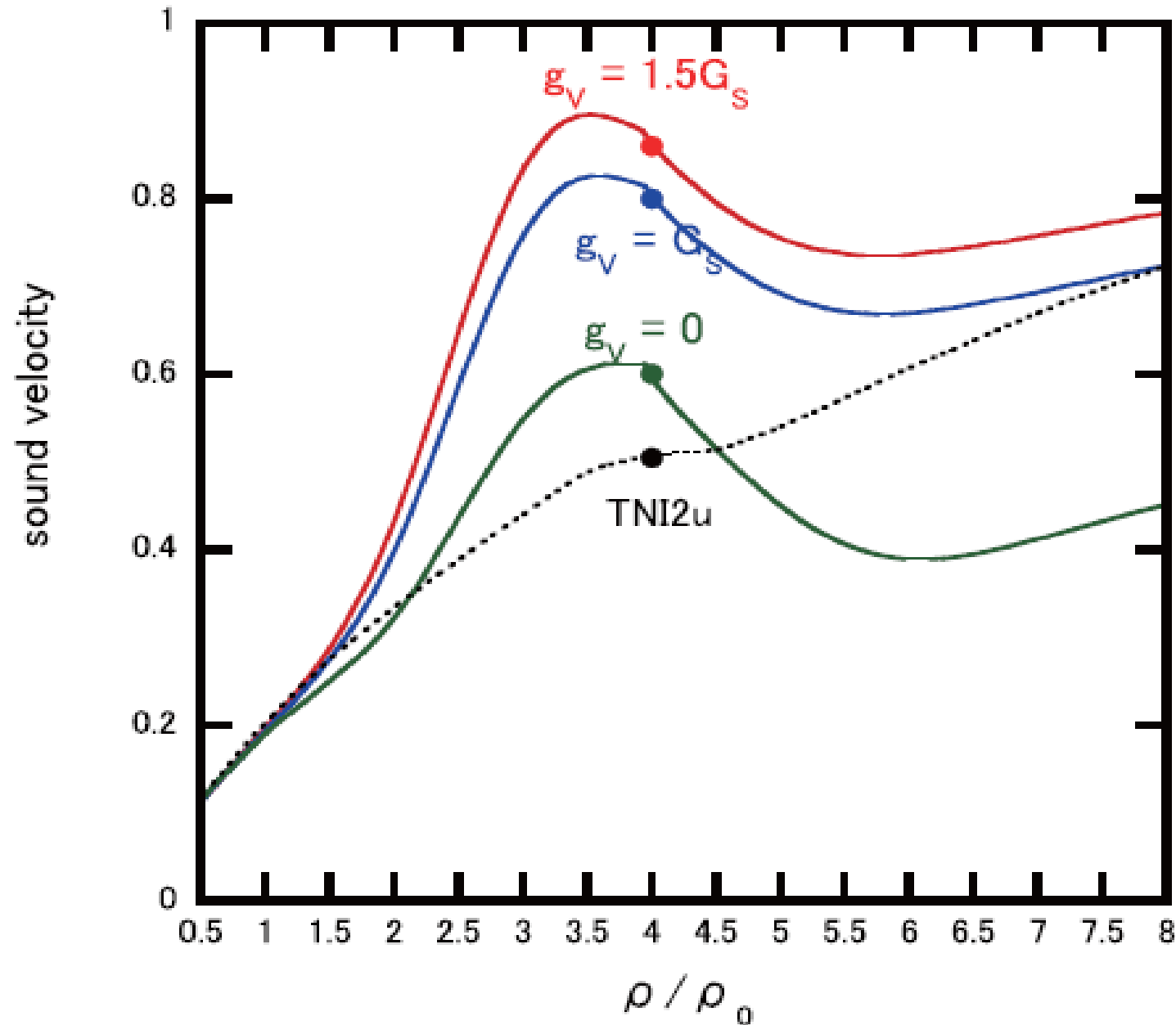


## 5.2. Dependence on Q-EOS



#### 5.4. Sound velocity of interpolated EOS

One of the measures to quantify the stiffness of EOS is the sound velocity  $v_s = \sqrt{dP/d\varepsilon}$ .



## 6. Summary and concluding remarks

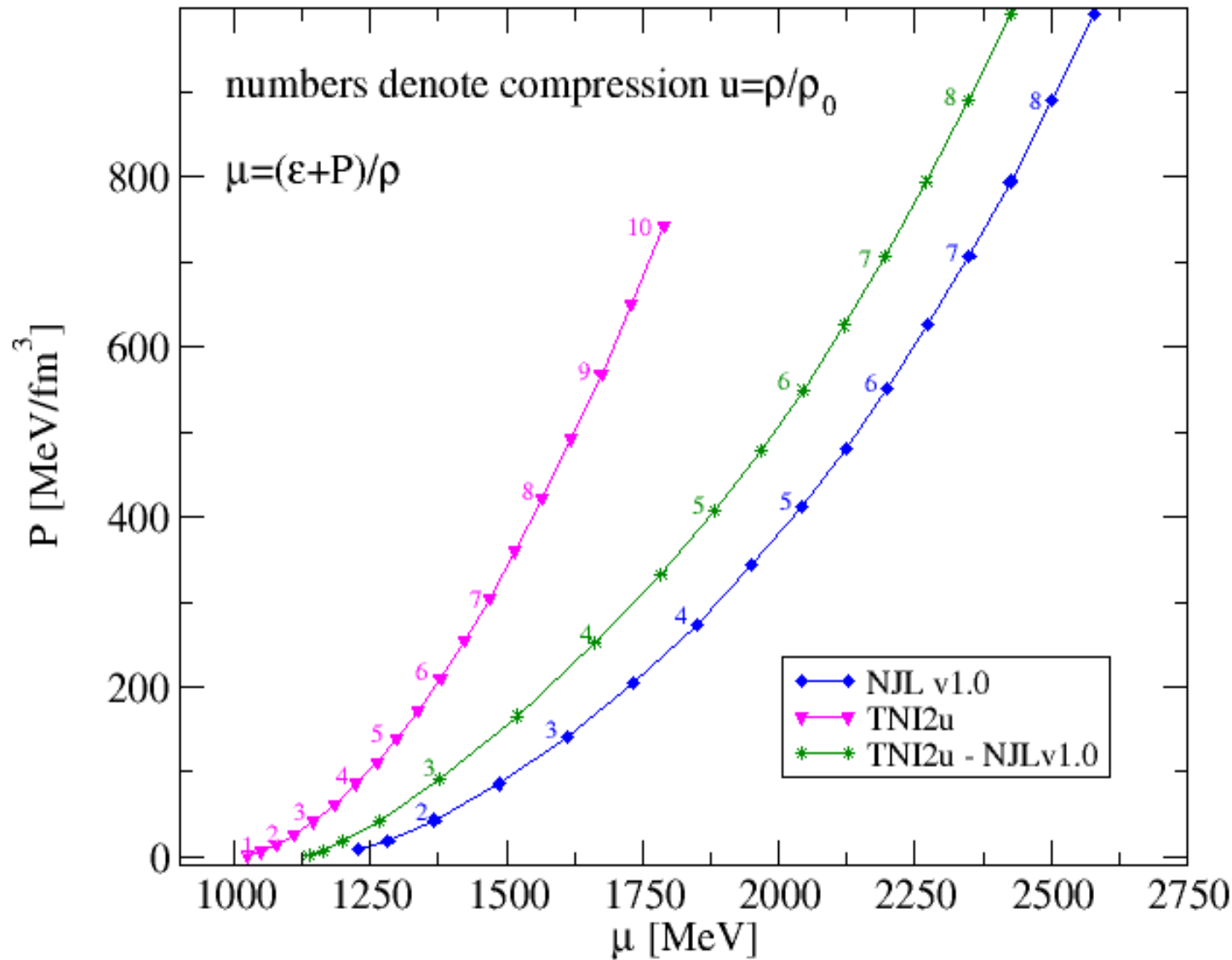
We have constructed an EOS by the interpolation between the H-EOS at lower densities and the Q-EOS at higher densities, and found that the hybrid stars could have  $M_{\text{max}} \sim 2M_{\odot}$ , compatible with the observation. This conclusion is in contrast to the conventional EOS for hybrid stars derived through the Gibbs construction in which the resultant EOS becomes always softer than hadronic EOS and thereby leads to smaller  $M_{\text{max}}$ .

The idea of rapid stiffening of the EOS starting from  $2\rho_0$  opens a possibility that the experimental nuclear incompressibility  $\kappa = (240 \pm 20)\text{MeV}$  at  $\rho \sim \rho_0$  is compatible with the existence of massive neutron stars. Also, the idea may well be checked by independent laboratory experiments with medium-energy heavy-ion collisions.

Finally, we remark that the crossover region may contain richer non-perturbative phases such as color superconductivity, inhomogeneous structures and so on [1]. How these structures as well as the associated cooling processes affect the results of the present paper would be an interesting future problem to be examined.



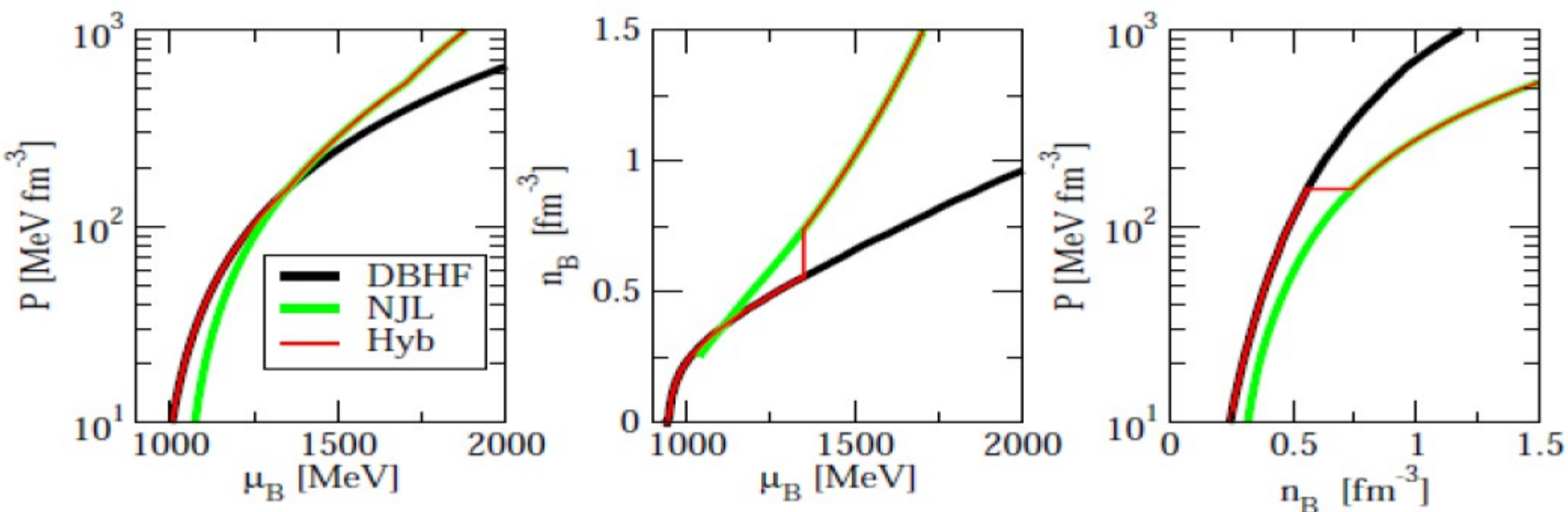
# Pressure vs. chem. Potential for H-EoS, Q-EoS and hybrid EoS with “Crossover”



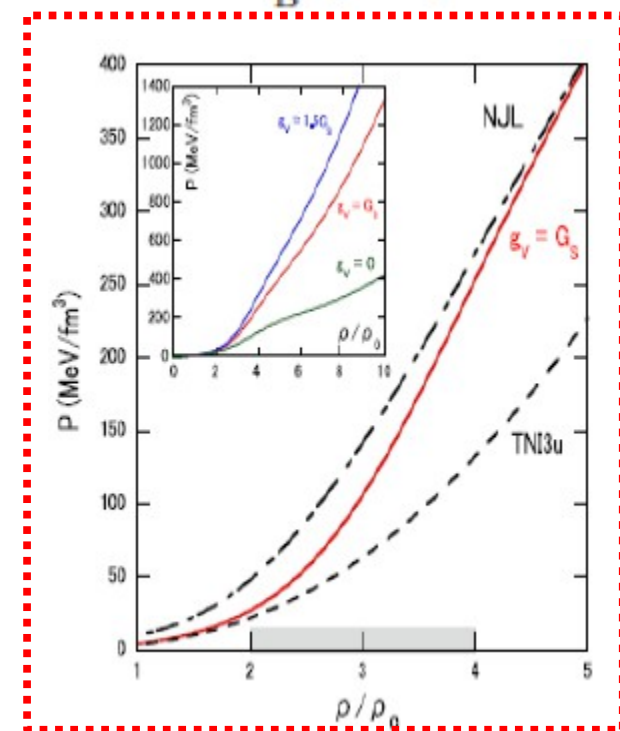
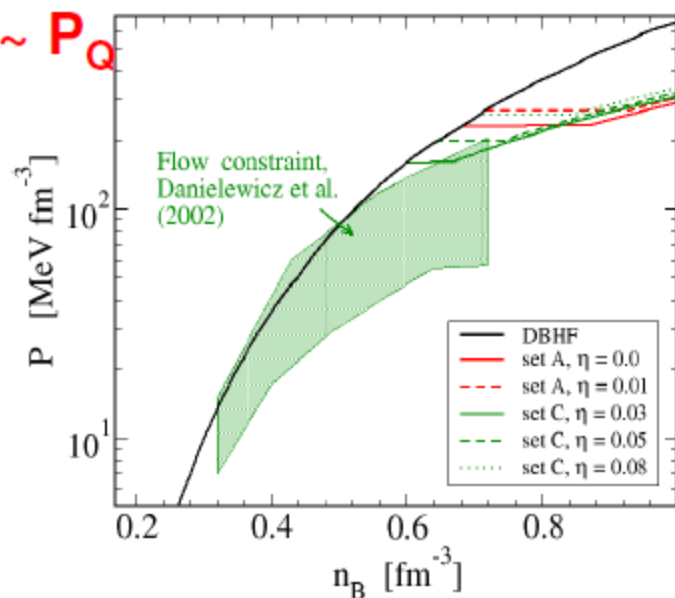
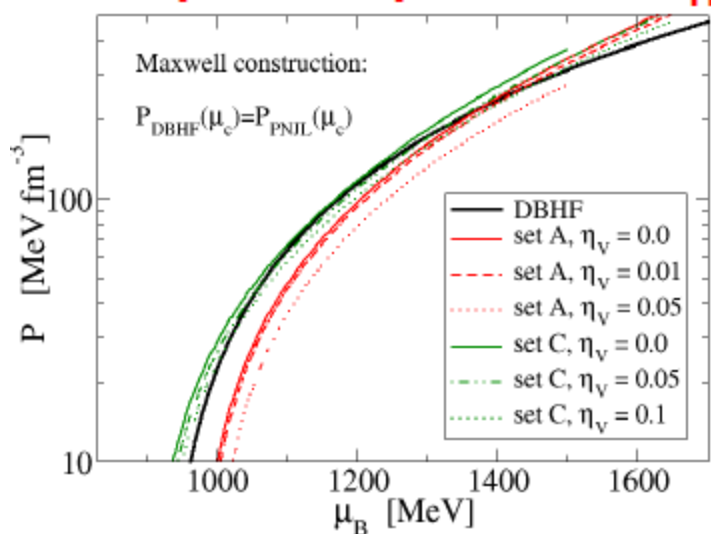
**PROBLEM:**  
The interpolation in  $P(\rho)$  is not a “crossover”, since thermodynamic consistency requires a shift  $\Delta\epsilon$  which isolates the resulting hybrid EoS (green) from the hadronic (magenta) and quark (blue) asymptotes.

Figure prepared with data from arxiv:1212.6803v1

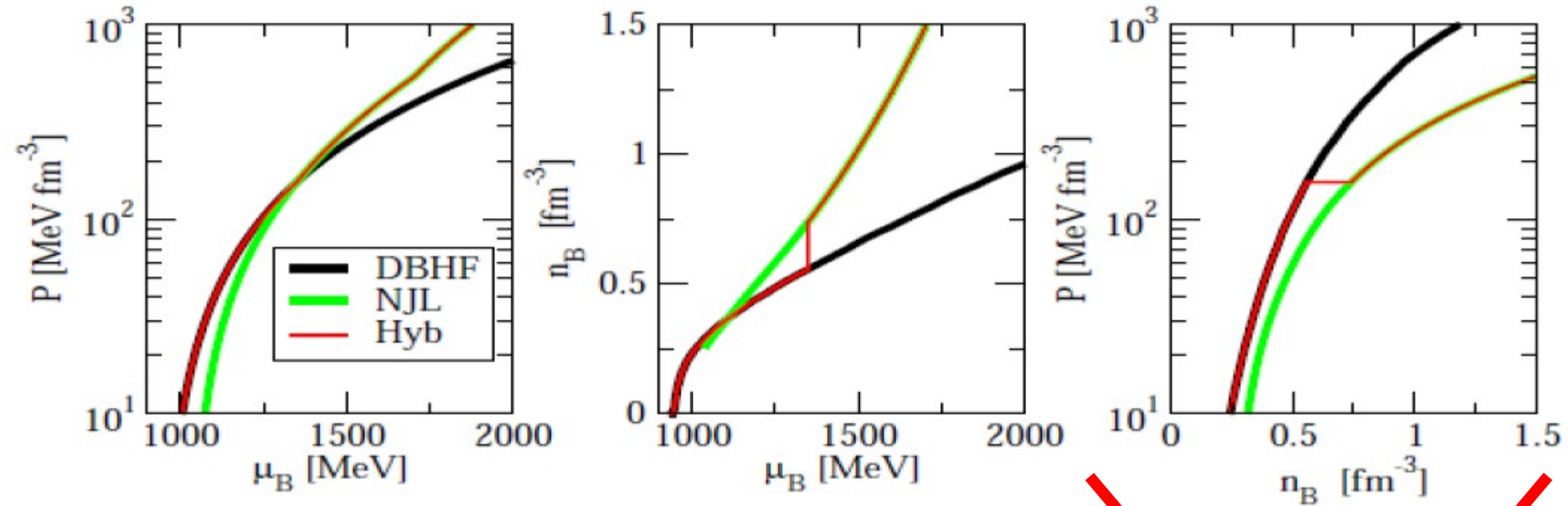
# Traditional: Pressure vs. chem. Potential for H-EoS, Q-EoS and hybrid EoS



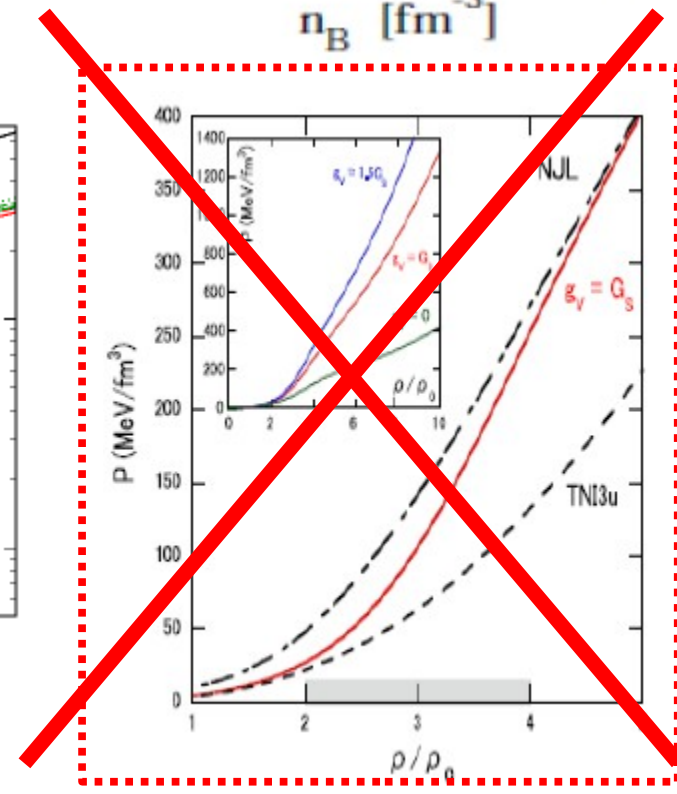
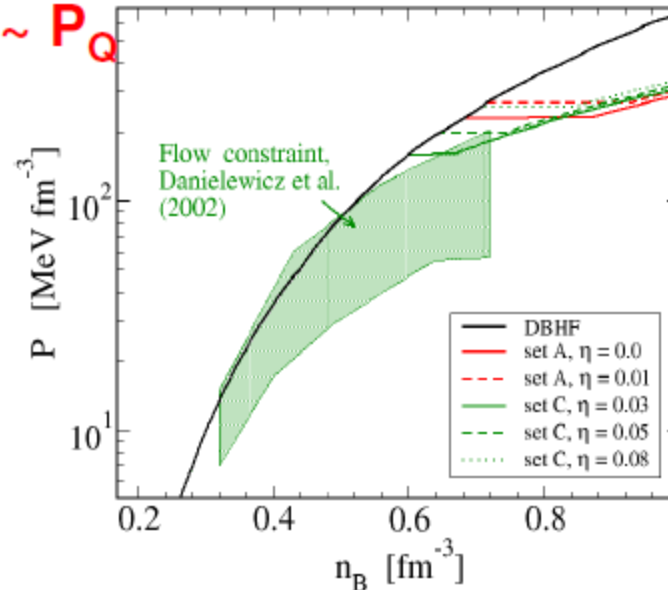
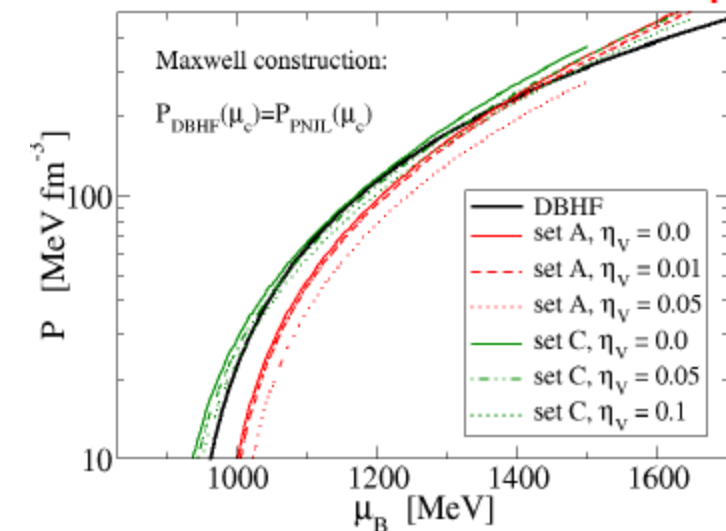
## Masquerade problem: $P_H \sim P_Q$



# Traditional: Pressure vs. chem. Potential for H-EoS, Q-EoS and hybrid EoS



## Masquerade problem: $P_H \sim P_Q$





# Maximum mass of neutron stars with a quark core

G.F. Burgio<sup>a</sup>, M. Baldo<sup>a</sup>, P.K. Sahu<sup>a</sup>, A.B. Santra<sup>b</sup>, H.-J. Schulze<sup>c</sup>

<sup>a</sup> *Istituto Nazionale di Fisica Nucleare, Sezione di Catania, Corso Italia 57, I-95129 Catania, Italy*

<sup>b</sup> *Nuclear Physics Division, Bhabha Atomic Research Center, Mumbai 400 085, India*

<sup>c</sup> *Departament d'Estructura i Constituents de la Matèria, Universitat de Barcelona, Av. Diagonal 647, E-08028 Barcelona, Spain*

Received 22 April 2001; received in revised form 28 September 2001; accepted 16 December 2001

Editor: J.-P. Blaizot

PHYSICAL REVIEW C 66, 025802 (2002)

## Hadron-quark phase transition in dense matter and neutron stars

G. F. Burgio,<sup>1</sup> M. Baldo,<sup>1</sup> P. K. Sahu,<sup>2</sup> and H.-J. Schulze<sup>1</sup>

## HADRONIC PHASE

$$\epsilon = \frac{1}{2} m_\omega^2 \bar{\omega}_0^2 + \frac{1}{2} m_\rho^2 (\bar{\rho}_0^3)^2 + \frac{1}{2} m_\sigma^2 \bar{\sigma}^2 + \frac{1}{3} b m_N (g_{\sigma N} \bar{\sigma})^3$$

$$+ \frac{1}{4} c (g_{\sigma N} \bar{\sigma})^4 + \sum_i \epsilon_{\text{FG}}(\bar{m}_i, \bar{\mu}_i) + \sum_l \epsilon_{\text{FG}}(m_l, \mu_l),$$

$$P = \frac{1}{2} m_\omega^2 \bar{\omega}_0^2 + \frac{1}{2} m_\rho^2 (\bar{\rho}_0^3)^2 - \frac{1}{2} m_\sigma^2 \bar{\sigma}^2 - \frac{1}{3} b m_N (g_{\sigma N} \bar{\sigma})^3$$

$$- \frac{1}{4} c (g_{\sigma N} \bar{\sigma})^4 + \sum_i P_{\text{FG}}(\bar{m}_i, \bar{\mu}_i) + \sum_l P_{\text{FG}}(m_l, \mu_l)$$

## QUARK PHASE

$$\epsilon_Q = \sum_q (\Omega_q + \mu_q \rho_q) + B, \quad P_Q = - \sum_q \Omega_q - B,$$

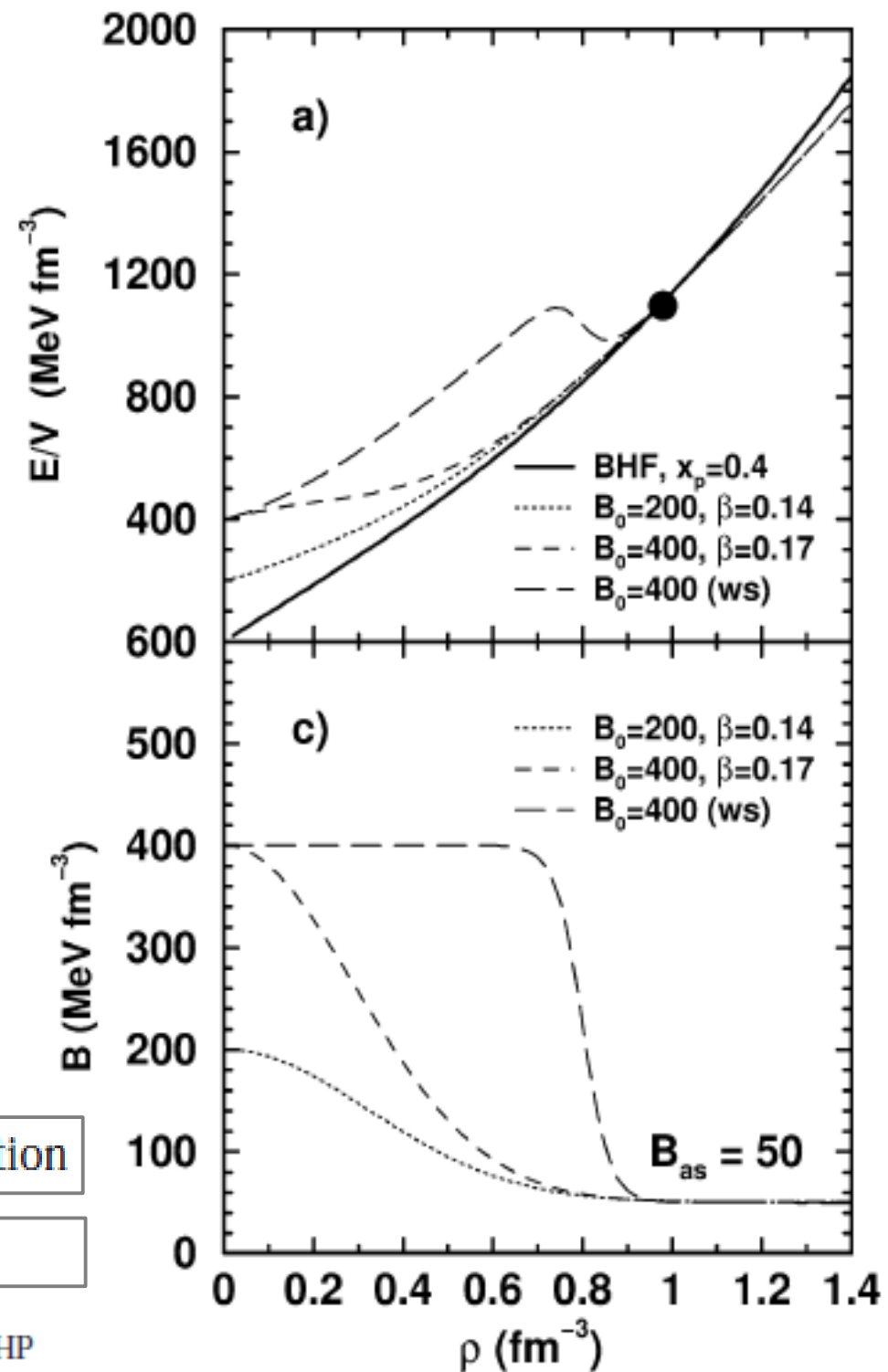
$$B(\rho) = B_{\text{as}} + (B_0 - B_{\text{as}}) \left[ 1 + \exp\left(\frac{\rho - \bar{\rho}}{\rho_d}\right) \right]^{-1}$$

Phase transition in  $\beta$ -stable neutron star matter

$$P_{\text{HP}}(\mu_e, \mu_n) = P_{\text{QP}}(\mu_e, \mu_n) = P_{\text{MP}} \quad \boxed{\text{Gibbs condition}}$$

$$\chi \rho_c^{\text{QP}} + (1 - \chi) \rho_c^{\text{HP}} = 0. \quad \boxed{\text{Global charge conservation}}$$

$$\epsilon_{\text{MP}} = \chi \epsilon_{\text{QP}} + (1 - \chi) \epsilon_{\text{HP}}, \quad \rho_{\text{MP}} = \chi \rho_{\text{QP}} + (1 - \chi) \rho_{\text{HP}}$$



Gibbs Phase transition  $\rightarrow$  Mixed phase, Softening the EoS; Quark Phase: stiff

