Gravitational-wave memory observables and charges of the extended BMS algebra

David A. Nichols[1]

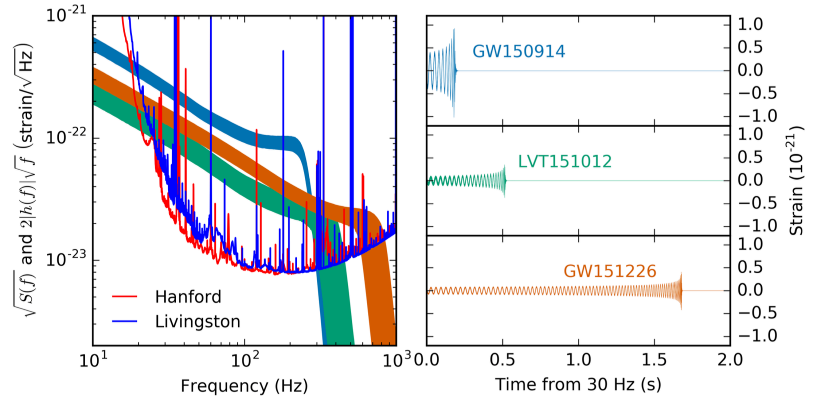
[1]Dept. of Astrophysics, IMAPP, Radboud University Nijmegen
In collaboration with Éanna É. Flanagan and Abraham I. Harte

Yukawa Institute for Theoretical Physics—NPCSM Meeting
10 November 2016

Outline of introduction and summary of results

- Gravitational-wave (GW) observations as probe of nonlinear and dynamical regime of general relativity (GR)
- GW memory as an example of nonlinear, dynamical GR
- Qualitative review of GW memory
- Description of asymptotic symmetries and charges
- New memories from new symmetries of gravitational scattering
- Summary of work on computations of charges ("conserved" quantities) and memory observables
- More details about calculations after introduction

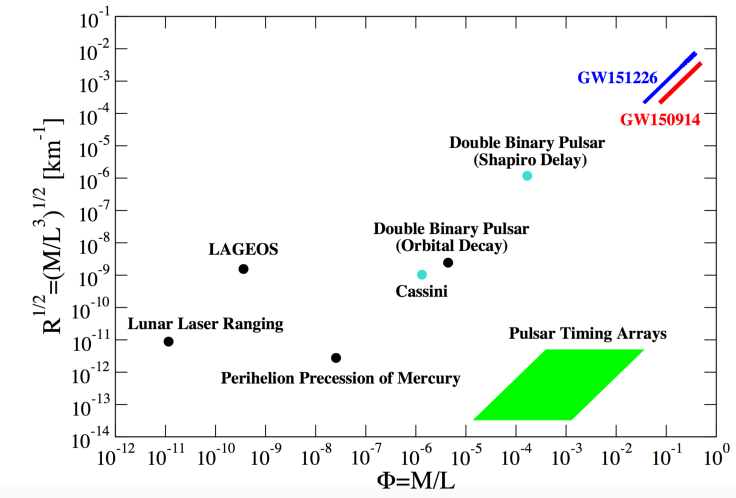
Gravitational-wave (GW) detections of binary black holes (BBHs)



LIGO Scientific Collaboration, arXiv:1606.04856

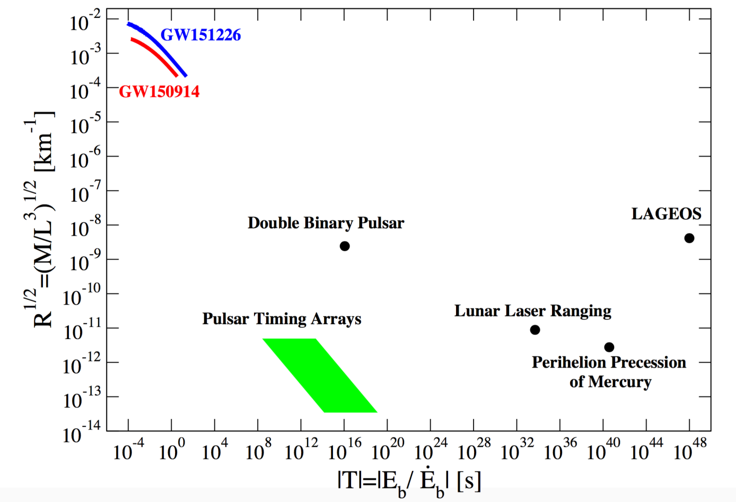
- GW150914: $> 5\sigma$, $m_1 \approx 36M_\odot$, $m_2 \approx 29M_\odot$, $D_L \approx 420$ Mpc
- GW151226: $> 5\sigma$, $m_1 \approx 14M_\odot$, $m_2 \approx 7.5M_\odot$, $D_L \approx 440$ Mpc
- LVT151012: $\sim 2\sigma$, $m_1 \approx 23M_\odot$, $m_2 \approx 13M_\odot$, $D_L \approx 1$ Gpc

Observational GR on new lengthscales. . .



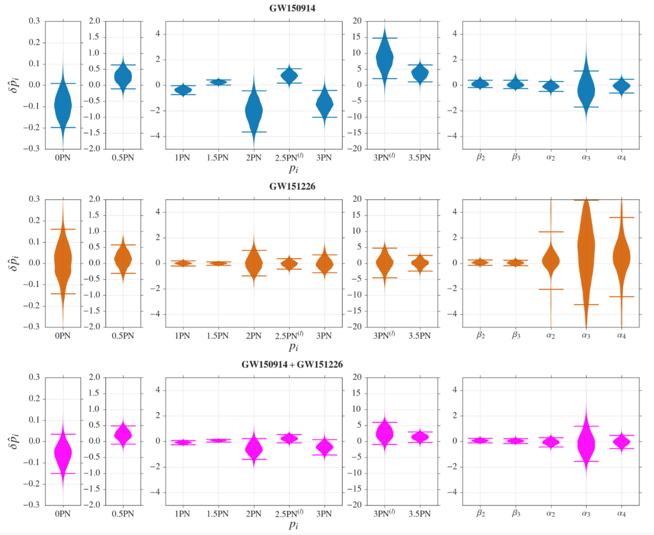
Yagi et al., arXiv:1603.08955

. . . and on new timescales



Yagi et al., arXiv:1603.08955

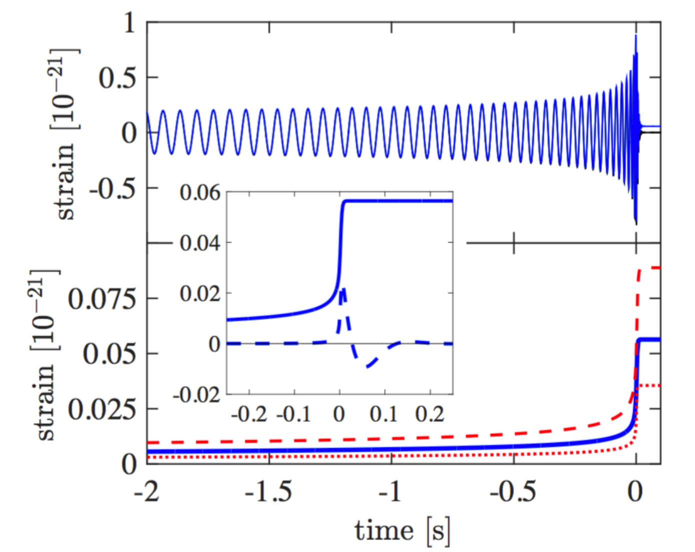
Tests of general relativity (GR) with BBHs



- $h(f) \sim Ae^{-i\Psi(f)}$ $\qquad$ $\Psi(f) \sim \sum_j (p_j^{\mathrm{GR}} + \delta p_j) f^{(j-5)/3}$

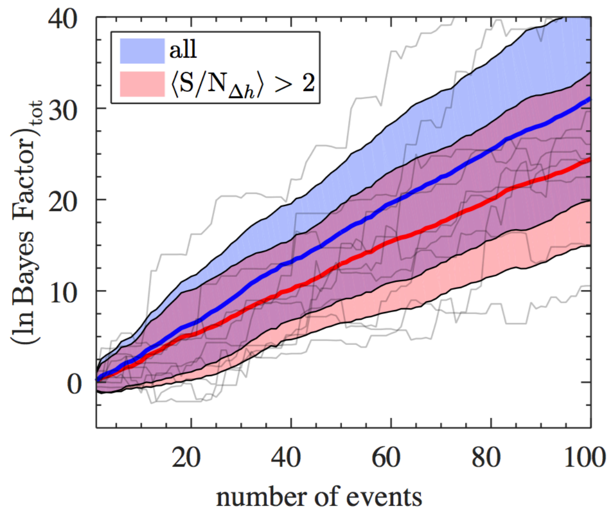
LIGO Scientific Collaboration, arXiv:1602.04856

Memory effect from GW150914 in LIGO



Lasky et al., arXiv:1605.01415

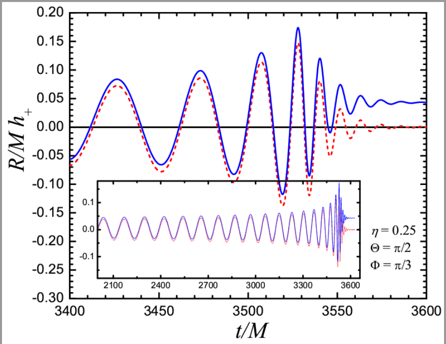
Building evidence for memory by stacking detections



Lasky et al., arXiv:1605.01415

Memory: Several perspectives on the phenomenon

Spacetime quantity



M. Favata, arXiv:0811.3451

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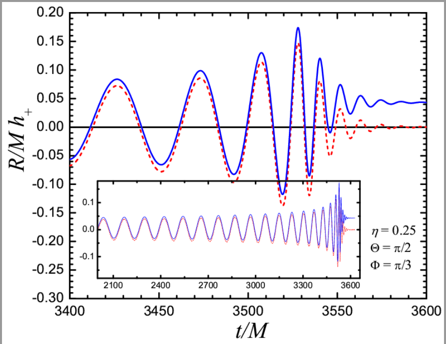

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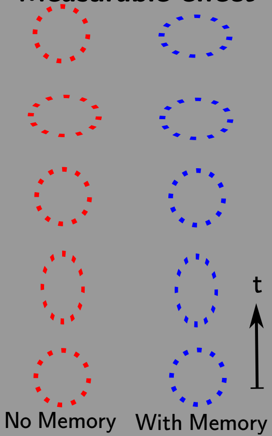
Measurable effect



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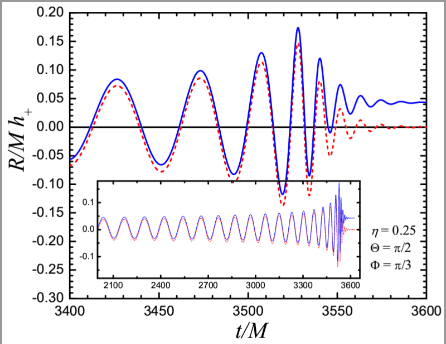

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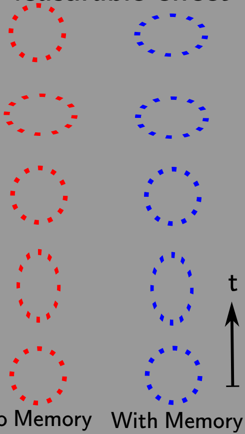
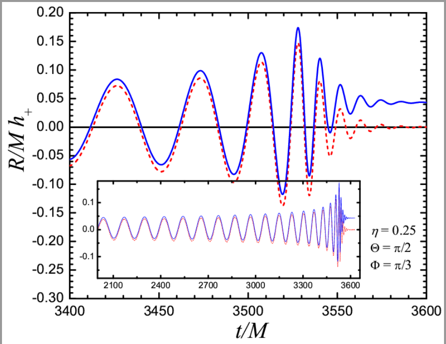


Sources

$$\text{``}\Delta h\text{''} \sim \overbrace{\Delta m}^{\text{Ordinary}} + \overbrace{r^2 \int du \left(T_{uu} + \underbrace{\mathcal{T}^{\text{GW}}_{uu}}_{\text{Nonlinear}} \right)}^{\text{Null}}$$

Linear

Spacetime quantity



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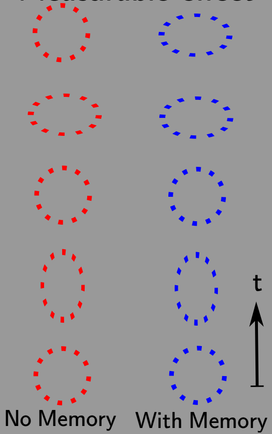


Sources

Null

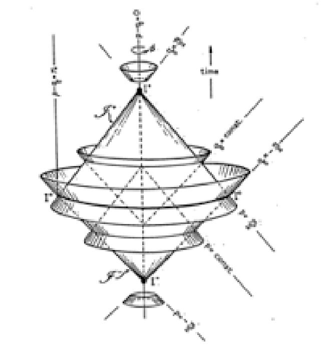
$$``\Delta h" \sim \underbrace{\overbrace{\Delta m}^{\text{Ordinary}} + r^2 \int du \Big(T_{uu}}_{\text{Linear}} + \underbrace{\mathcal{T}^{\text{GW}}_{uu}}_{\text{Nonlinear}} \Big)$$

Symmetries

BMS supertranslation

(next slide. . .)

Overview of asymptotic symmetries



R. Penrose, Les Houches, 1963

Memory related to supertranslation between early and late non-radiative frames

- Symmetry group of asymptotically flat spacetimes ($\mathscr{I}^+$) is the Bondi-Metzner-Sachs (BMS) group

  Bondi et al., 1962; Sachs, 1962

- BMS has semidirect form: Supertranslations (ST) $\rtimes$ Lorentz (Poincaré: Translations $\rtimes$ Lorentz)
- ST: infinite-dimensional, abelian, 4D translation subgroup; roughly "angle-dependent translations"
- Corresponding charges: 4-momentum, supermomentum, and relativistic angular momentum (spin and center of mass)

Memory effects and symmetries: Recent developments

Extended symmetry groups

- Barnich & Troessaert '09+: extend BMS algebra to include locally defined (but not globally defined) symmetries
- Extended algebra: ST $\rtimes$ Virasoro
- Virasoro called "superrotations" (SR) in the context of 4D asymptotically flat case
- Intuition for SR: contains Lorentz subalgebra; "angle-dependent rotations and boosts"
- Showed certain charges are finite and well defined

Physical relevance of extended symmetries

Digression on charges, memories, symmetries of gravitational scattering

- Strominger,+ '13+: Identify BMS subgroup of past ($\mathscr{I}^-$) and future ($\mathscr{I}^+$) null infinity in a class of spacetimes
- Supertranslation charges related: $\mathcal{Q}_\alpha^- = \mathcal{Q}_\alpha^+$
- $\mathcal{S}$ matrix satisfies: $\langle out|(\mathcal{Q}_\alpha^+ \mathcal{S} - \mathcal{S}\mathcal{Q}_\alpha^-)|in\rangle = 0$
- In particle basis: $\lim_{\omega\to 0} \mathcal{M}_{n+1} = S^{(0)}\mathcal{M}_n$ with $\mathcal{M}_n$ n-particle amplitude and $S^{(0)}$ related to memory effect
- "Triangle" of relations: soft theorem $\Leftrightarrow$ BMS symmetry $\Leftrightarrow$ memory effect

Similar types of relations between subleading soft theorem, extended BMS symmetry, and a new "spin" memory effect (Pasterski+, '15)

Overview of Results

1. Review asymptotic flatness, symmetries, and charges
   - Show how supermomentum charges are related to "ordinary" memory
2. Compute charges conjugate to superrotation symmetries in more general contexts than before
   - Find charges contain information about the total memory and the ordinary spin memory
3. Investigate the spin memory
   - Show it can be measured inertially, but not locally in space
4. Look for other intertial memory effects
   - Besides displacement effect, there are proper-time, rotation, and velocity memory effects
   - Relative displacement is the only effect that is locally measurable at $O(1/r)$

Outline of details

1. Asymptotically flat spacetimes, in brief
2. Charges ("conserved" quantities) of the BMS group
3. Memory effects and charges
4. Extended BMS algebra and its charges
5. Relation of extended charges and memory effects
6. Search for additional classical memory observables

Bondi-Sachs framework

Work in Bondi coordinates (u, r, θ^A):

$$\begin{aligned} ds^2 = &- du^2 - 2dudr + r^2 h_{AB} d\theta^A d\theta^B \\ &+ \frac{2m}{r} du^2 + rC_{AB} d\theta^A d\theta^B + D^B C_{AB} d\theta^A du + \ldots \end{aligned}$$

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- θ^A: coordinates on S^2 with 2-metric h_{AB} and covariant derivative operator D_A
- $m = m(u, \theta^A)$: Bondi mass aspect
- $C_{AB} = C_{AB}(u, \theta^C)$: shear tensor (symmetric trace-free)

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- $C_{AB} = C_{AB}(u, \theta^C)$: shear tensor (symmetric trace-free)
- $N_{AB} = \partial_u C_{AB}$: news tensor (vanishes when stationary)
- $N_A = N_A(u, \theta^B)$: Bondi angular-momentum aspect

Einstein equations and initial data

Einstein equations (evolution equations for $\dot{m} = \partial_u m$ and $\dot{N}_A$)

$$\dot{m} = 4\pi \hat{T}_{uu} - \frac{1}{8} N_{AB} N^{AB} + \frac{1}{4} D_A D_B N^{AB}$$

where $T_{uu} = \hat{T}_{uu}(u, \theta^A)/r^2$

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$$\begin{aligned}\dot{N}_A = & - 8\pi \hat{T}_{uA} + \pi D_A \partial_u \hat{T}_{rr} + D_A m + \frac{1}{4} D_B D_A D_C C^{BC} \\ & - \frac{1}{4} D_B D^B D^C C_{CA} + \frac{1}{4} D_B (N^{BC} C_{CA}) + \frac{1}{2} D_B N^{BC} C_{CA}.\end{aligned}$$

and $T_{uA} = \hat{T}_{uA}(u, \theta^B)/r^2$, $T_{rr} = \hat{T}_{rr}(u, \theta^A)/r^4$

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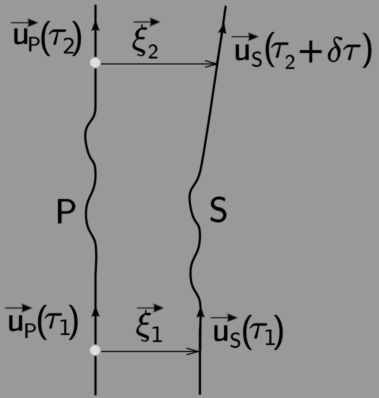
- At $u = u_0$, specify $m(u_0, \theta^C)$, $N_A(u_0, \theta^C)$, $C_{AB}(u_0, \theta^C)$
- News N_{AB} unconstrained; also certain components of T_{ab}

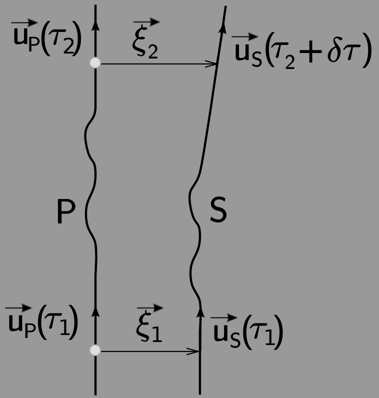
Nearby freely falling observers



$\vec{u}(\tau)$: 4-velocity
$\vec{\xi} = \xi^{\hat{i}}\vec{e}_{\hat{i}}(\tau)$: "separation"

- Primary geodesic observer, P: 4-velocity $\vec{u}_P(\tau)$
- Secondary geodesic observer, S: 4-velocity $\vec{u}_S(\tau)$
- At τ_1, P and S co-moving; S at location $\xi_1^{\hat{i}}$

Nearby freely falling observers



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- At τ_2, $\xi_2^{\hat{i}} = \xi_1^{\hat{i}} + \delta\xi^{\hat{i}}$
- $\delta\xi^{\hat{i}} = \int d\tau \int d\tau' R^{\hat{i}}{}_{\hat{0}\hat{j}\hat{0}}\xi^{\hat{j}}$
- Bondi coordinates: $\vec{u}_P = \vec{\partial}_u + O(r^{-1})$, $R^{\hat{i}}{}_{\hat{0}\hat{j}\hat{0}} = r^{-1}\ddot{C}_{\hat{A}\hat{B}}\delta^{\hat{i}\hat{A}}\delta^{\hat{B}}{}_{\hat{j}} + O(r^{-2})$
- $\boxed{\delta\xi_{\hat{A}} = r^{-1}\Delta C_{\hat{A}\hat{B}}\xi^{\hat{B}}} + O(r^{-2})$

BMS symmetries in Bondi coordinates

BMS transformations take the form

$$\bar{u} = [u + \alpha(\theta^A)]/\omega(\theta^A) \qquad \bar{\theta}^A = \bar{\theta}^A(\theta^B)$$

$\bar{\theta}^A(\theta^B)$: conformal transformation of 2-sphere (6-parameter group)
$\omega(\theta^A)$: required rescaling of u

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Infinitesimal BMS symmetries on $\mathscr{I}^+$ generated by $\vec{\zeta}$:

$$\vec{\zeta} = f(\theta^A)\vec{\partial}_u + Y^A(\theta^B)\vec{\partial}_A$$

$$f(\theta^A) = \alpha(\theta^A) + \frac{1}{2}uD_B Y^B(\theta^A), \quad 2D_{(A}Y_{B)} - D_C Y^C h_{AB} = 0$$

$\alpha \leftrightarrow$ ST; $Y^A \leftrightarrow$ Lorentz transformation ($\ell = 1$ vector spherical harmonic)

Transformation of Bondi-metric functions

C_{AB}, m, N_A transform nontrivially

$$\delta C_{AB} = fN_{AB} - (2D_A D_B - h_{AB} D^2) f - \frac{1}{2} D_C Y^C C_{AB} + \mathcal{L}_{\vec{Y}} C_{AB}$$

Transformation of Bondi-metric functions

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$$\begin{aligned}\delta m =& f\dot{m} + \frac{1}{4} N^{AB} D_A D_B f + \frac{1}{2} D_A f D_B N^{AB} + \frac{3}{2} m\psi + Y^A D_A m \\ &+ \frac{1}{8} C^{AB} D_A D_B \psi \\ \delta N_A =& 3mD_A f + \frac{1}{4} C_{AB} D^B D^2 f - \frac{3}{4} D_B f (D^B D_C C^C{}_A - D_A D_C C^{BC}) \\ &+ \frac{3}{8} D_A (C^{BC} D_B D_C f) + \frac{1}{4}(2D_A D_B f - h_{AB} D^2 f) D_C C^{BC} \\ &+ f\dot{N}_A \mathcal{L}_{\vec{Y}} N_A + \psi N_A + \ldots\end{aligned}$$

Certain BMS transformations can simplify C_{AB}, M, and N_A

Charges corresponding to asymptotic symmetries

- Charges not conserved at $\mathscr{I}^+$, because fluxes of matter and GW carry charges
- A variant of Noether's theorem exists to compute "conserved" quantities conjugate to asymptotic symmetries $\vec{\zeta}$

Wald & Zoupas, arXiv:gr-qc/9911095

- Charge $Q[\mathcal{C}, \vec{\zeta}]$ is linear in $\vec{\zeta}$ and depends on cut $\mathcal{C}$

$$Q[\mathcal{C}, \vec{\zeta}] = \int_{\mathcal{C}} \boldsymbol{\Xi}$$

for a 2-form $\boldsymbol{\Xi}$ (similar to Gauss' law)

- In stationary, vacuum, & Bondi coordinates

$$Q[\mathcal{C}, \vec{\zeta}] = \frac{1}{16\pi} \int_{\mathcal{C}} d^2\Omega \Big[4\alpha m - 2uY^A D_A m + 2Y^A N_A - \frac{1}{8} Y^A D_A (C_{BC} C^{BC}) - \frac{1}{2} Y^A C_{AB} D_C C^{BC} \Big]$$

Fluxes of charges

- Difference in charges is related to exact 3-form flux $\boldsymbol{d\Xi}$

$$Q[\mathcal{C}_2, \vec{\zeta}] - Q[\mathcal{C}_1, \vec{\zeta}] = \int_{\mathscr{I}^+_{2,1}} \boldsymbol{d\Xi}$$

  (similar to EM continuity equation)
- $\boldsymbol{d\Xi}$ is related to Noether current,
- For stationary solutions, $\boldsymbol{d\Xi} = 0$
- Flux has form

$$\int_{\mathscr{I}^+_{2,1}} \boldsymbol{d\Xi} = -\int_{\mathscr{I}^+_{2,1}} \left(\frac{1}{32\pi} N^{AB} \delta C_{AB} + \hat{T}_{ua} \zeta^a \right) du\, d^2\Omega \,.$$

- Using Einstein equations, can show consistency of charge and flux formulas

Charges in Bondi coordinates and in a "canonical" frame

- There exists a "canonical" frame $\mathcal{C}_c$ for stationary spacetimes:

$$m(\theta^A) = m_0 = \text{const.}, \quad C_{AB}(\theta^C) = 0\,,$$

$$N_A(\theta^B) = \text{magnetic parity}, \; \ell = 1$$

- Only nonzero charges

$$Q[\mathcal{C}_c, Y_{0,0}\vec{\partial}_u] = m_0 \qquad Q[\mathcal{C}_c, X^A_{1,m}\vec{\partial}_A] = N_{1,m}$$

- In essence, a vacuum, stationary, asymptotically flat spacetime is characterized by mass and spin to this order in $1/r$

Charges in a supertranslated frame

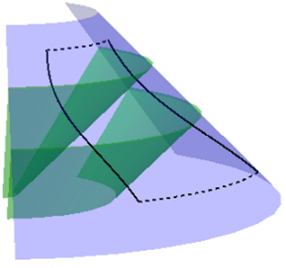
- Supertranslate by $\Delta\Phi(\theta^A)$ and linearize (for simplicity) from canonical frame

$$m(\theta^A) = m_0 = \text{const.}, \quad N_A(\theta^B) = N_A^{\ell=1} - \frac{3}{2} m_0 D_A \Delta\Phi \,,$$

$$C_{AB}(\theta^C) = \frac{1}{2}(2D_A D_B - h_{AB} D^2)\Delta\Phi$$

- Supermomentum are invariant under supertranslations
- Angular momentum invariant to linear order in $\Delta\Phi$, for $\Delta\Phi$ with $\ell > 1$
- Memory is not completely encoded in supermomentum charges in a stationary region

Memory effect, supermomenta, and nonradiative-to-nonradiative transitions



- Nonradiative transitions: Spacetimes with $N_{AB} \to 0$ as $u \to \pm\infty$
- Consider supermomentum charges in a "basis" of delta functions $\alpha(\theta^A) = 4\pi\delta(\theta^A - \theta'^A)$
- Supermomentum is just $m(u, \theta^A)$ in nonradiative region!

Flux formula $Q[\mathcal{C}_2, \alpha\vec{\partial}_u] - Q[\mathcal{C}_1, \alpha\vec{\partial}_u] = \int_{\mathscr{I}^+_{2,1}} \boldsymbol{d\Xi}$ is equivalent to integrating Einstein's equation

$$\Delta m = -4\pi\Delta\mathcal{E} + \mathcal{D}\Delta\Phi$$

where $\mathcal{D} = D^2(D^2+2)/8$ and $\Delta\mathcal{E} = \int du \left[\hat{T}_{uu} + N_{AB}N^{AB}/(32\pi)\right]$

Interpretation of memory effect

Define $\mathcal{P}$ as projector that removes $\ell = 0, 1$ harmonics; can solve for the memory

$$\Delta\Phi = \mathcal{D}^{-1}\mathcal{P}(\Delta m + 4\pi\Delta\mathcal{E})$$

1. $\Delta\Phi$: memory observable
2. Δm: supermomentum ("ordinary" memory)
3. $\Delta\mathcal{E}$: energy flux ("null" memory)

- Total memory not a charge; ordinary memory is though
- $\Delta\Phi$ is supertranslation to reach canonical frame as $u \to \infty$ from canonical frame as $u \to \infty$
- Total memory is observable; is there a charge for it?

Extended symmetry algebra of Barnich & Troessaert

- Extended BMS algebra of form ST $\rtimes$ Virasoro
- Virasoro: infinite-dimensional, called superrotations (SR), Lorentz subalgebra; roughly a generalization of boosts
- SR are the infinite number of singular solutions Y^A of

$$2D_{(A}Y_{B)} - D_C Y^C h_{AB} = 0$$

- Common basis uses stereographic coords $z = e^{i\phi} \cot \theta/2$

$$l_m = -z^{m+1}\partial_z \qquad \bar{l}_m = -\bar{z}^{m+1}\partial_{\bar{z}}$$

with and $m \in \mathbb{Z}$

Conjugate charges to SR

- Will use Wald-Zoupas procedure to compute charges
- Remains formally valid, but may encounter problems
- For convenience define

$$\hat{N}_A = N_A - u D_A m - \frac{1}{16} D_A (C_{BC} C^{BC}) - \frac{1}{4} C_{AB} D_C C^{BC}$$

- SR charge in terms of $\hat{N}_A$ is

$$Q[\mathcal{C}, \vec{Y}] = \frac{1}{8\pi} \int d^2\Omega \, Y^A \hat{N}_A$$

- Charge integrals are finite for any smooth $\hat{N}_A$
- Split $\hat{N}_A = D_A \phi + \epsilon_{AB} D^B \psi$ (electric & magnetic parity)
- Decomposition of charges $Q[\mathcal{C}, \vec{Y}] = Q_e[\mathcal{C}, \vec{Y}] + Q_b[\mathcal{C}, \vec{Y}]$

Charges in stationary vacuum cuts

- In canonical frame, $Q_e[\mathcal{C}_c, \vec{Y}] = Q_b[\mathcal{C}, \vec{Y}] = 0$
- In frame supertranslated by $\Delta\phi$ (linearized)

$$Q_e[\mathcal{C}, \vec{Y}] = -\frac{3m_0}{16\pi}\int_{\mathcal{C}} d^2\Omega Y^A D_A \Delta\Phi \qquad Q_b[\mathcal{C}, \vec{Y}] = 0$$

- Q_e charges contain (incomplete) information about C_{AB} (and thence the total memory)!
- In more general stationary frames, $Q_e \neq 0$ and $Q_b \neq 0$
- Nomenclature: Q_e will call "super-center-of-mass; Q_b will call "superspin"
- For Y^A Lorentz, Q_e is center of mass, Q_b is spin

Charge and flux relationship

- Also check if $\int \boldsymbol{d\Xi}$ is difference in charges for SR
- Find a discrepancy

$$\int_{\mathscr{I}^+_{2,1}} \boldsymbol{d\Xi} = Q[\mathcal{C}_2, \vec{Y}] - Q[\mathcal{C}_1, \vec{Y}] - \frac{1}{32\pi} \int_{\mathscr{I}^+_{2,1}} du\, d^2\Omega Y^A \epsilon_{AB} D^B \mathcal{D}\Psi$$

where $C_{AB} = (D_A D_B - h_{AB}/2D^2)\Phi + D_{(A}\epsilon_{B)C} D^C \Psi$

- Can resolve by modifying flux or adding a nonlocal field $\int du\, \Psi$ to the charge; not a formal derivation, though
- $\int du\, \Psi$ is closely related to new "spin memory" of Pasterski+

Spin memory, superspin, and nonradiative transitions

- Consider the change in the magnetic-parity part of $\hat{N}_A$, by taking the curl of its evolution equation

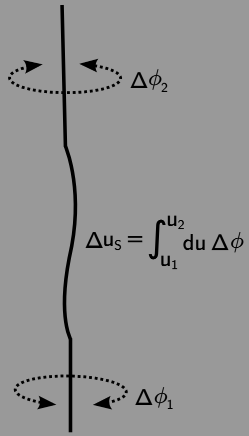
$$\Delta\epsilon^{AB} D_B \hat{N}_A = -8\pi\epsilon^{AB} D_B \Delta\mathcal{J}_A + D^2\mathcal{D}\int du\,\Psi$$

- $\Delta\mathcal{J}_A = \int du(\hat{T}_{uA} + \mathcal{T}_{uA})$ is the angular momentum per solid angle radiated in matter and GWs
- Can solve for spin memory

$$\int du\,\Psi = \mathcal{D}^{-1} D^{-2}\mathcal{P}\epsilon^{AB} D_B(\Delta\hat{N}_A + \Delta\mathcal{J}_A)$$

- $\int du\,\Psi$: total spin memory; $\Delta\mathcal{J}_A$: null part; $\Delta\hat{N}_A$: ordinary part
- Now turn to measurablility of this memory

Spin memory (integrated Sagnac) effect



$\Delta\phi$: Sagnac effect
Δu_S: integrated Sagnac effect

Proposal of Pasterski+ to measure spin memory:

- Sagnac effect $\Delta\phi$ vanishes for inertial observers
- Must measure with "BMS observers" who accelerate to stay fixed in Bondi coordinates (i.e., noninertial)
- Effect related to u integral of twist $\omega_{AB} = D_{[A}a_{B]}$ for $a_B = D^C C_{BC}$
- For an "infinitesimal" detector

$$\Delta u_S = 2 \int_{-\infty}^{\infty} du\, \mathcal{D}\Psi$$

However, constructing Bondi coordinates locally may not be possible

Spin memory measured by families of inertial observers

- Consider the u integral of the displacement memory $\delta\xi^A$

$$\int_{-\infty}^{\infty} du\,\delta\xi^A = \int_{-\infty}^{\infty} du \left[\frac{1}{2}(2D_A D_B - h_{AB}D^2)\Phi + D_{(A}\epsilon_{B)C}D^C\Psi\right]\xi^B$$

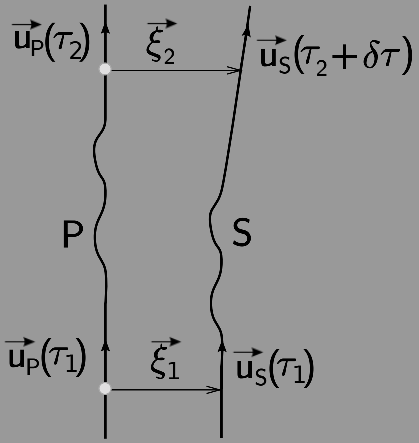
- $(2D_A D_B - h_{AB}D^2)\Phi$ is the displacement memory; the integral will go as u as $u \to \infty$
- Magnetic-parity part equivalent to Sagnac measurement of spin memory

$$\delta s_A = \int_{-\infty}^{\infty} du\, D_{(A}\epsilon_{B)C}D^C\Psi\xi^B$$

- Need inertial observers distributed around source to extract magnetic-parity part (again non-local).

Are there additional local memory observables besides displacement and spin memories?

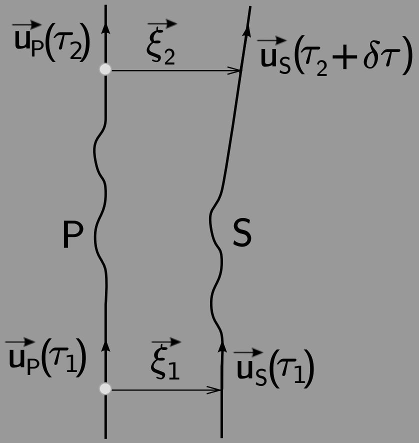
Nearby freely falling observers



- Primary geodesic observer, P: 4-velocity $\vec{u}_P(\tau)$
- Secondary geodesic observer, S: 4-velocity $\vec{u}_S(\tau)$
- At τ_1, P and S co-moving; S at location $\xi_1^{\hat{i}}$

$\vec{u}(\tau)$: 4-velocity
$\vec{\xi} = \xi^{\hat{i}}\vec{e}_{\hat{i}}(\tau)$: "separation"
(in Fermi coordinates)

Nearby freely falling observers



$\vec{u}(\tau)$: 4-velocity
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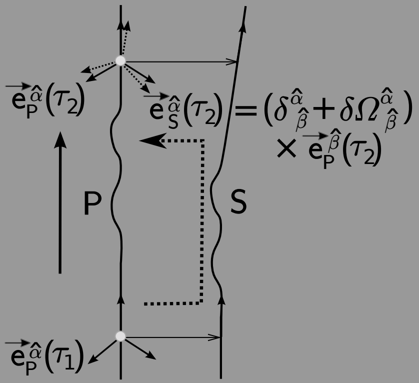
(in Fermi coordinates)

- Primary geodesic observer, P: 4-velocity $\vec{u}_P(\tau)$
- Secondary geodesic observer, S: 4-velocity $\vec{u}_S(\tau)$
- At τ_1, P and S co-moving; S at location $\xi_1^{\hat{i}}$
- At τ_2, P and S not co-moving; S at location $\xi_2^{\hat{i}} = \xi_1^{\hat{i}} + \delta\xi^{\hat{i}}$
- Proper time elapsed for P and S differ by $\delta\tau$
- $\delta\xi^{\hat{i}}$: measurable effect of memory; "displacement" memory.
- $\delta\tau$: a "proper-time" memory

Additional memory observables

- Two additional observable effects:
  1. $\delta\Omega^{\hat{i}}{}_{\hat{j}}$: Relative rotation of triad $\vec{e}_S^{\,\hat{i}}(\tau)$ with respect to inertial standards at P.
  2. $\delta\Omega^{\hat{i}}{}_{\hat{0}} \equiv \delta\dot{\xi}^{\hat{i}}$: Relative boost of S with respect to geodesic P
- 1 is a "rotation" memory and 2 is a "velocity" memory
- 1 is measurable in principle with inertial gyroscopes



$\vec{e}_P^{\,\hat{\alpha}}(\tau)$: $\perp$ tetrad of P
$\vec{e}_S^{\,\hat{\alpha}}(\tau)$: $\perp$ tetrad of S transported to P

Expressions for memory observables

Velocity and rotation memories

Covariant Riemann	3+1 Split of Riemann
$\delta\dot{\xi}^{\hat{i}}(\tau) = -\int_{\tau_1}^{\tau} d\tau' R^{\hat{i}}{}_{\hat{0}\hat{j}\hat{0}}(\tau')\xi^{\hat{j}}$	$\delta\dot{\xi}^{\hat{i}} = -\int_{\tau_1}^{\tau} d\tau' (\mathcal{E}^{\hat{i}}{}_{\hat{j}} - 4\pi T^{\hat{i}}{}_{\hat{j}})\xi^{\hat{j}} - \frac{4\pi}{3}\int_{\tau_1}^{\tau} d\tau' (2T^{\hat{k}}{}_{\hat{k}} + T_{\hat{0}\hat{0}})\xi^{\hat{i}}$
$\delta\Omega_{\hat{i}\hat{j}} = -\int_{\tau_1}^{\tau} d\tau' R_{\hat{i}\hat{j}\hat{0}\hat{k}}(\tau')\xi^{\hat{k}}$	$\delta\Omega_{\hat{i}\hat{j}} = \int_{\tau_1}^{\tau} d\tau' (8\pi T_{\hat{0}[\hat{i}}\xi_{\hat{j}]} - \epsilon_{\hat{i}\hat{j}\hat{k}}\mathcal{B}^{\hat{k}}{}_{\hat{n}}\xi^{\hat{n}})$

Recall $\mathcal{E}_{\hat{i}\hat{j}} = C_{\hat{0}\hat{i}\hat{0}\hat{j}}$ $\qquad \mathcal{B}_{\hat{i}\hat{j}} = *C_{\hat{0}\hat{i}\hat{0}\hat{j}}$

Expressions for memory observables

Velocity and rotation memories

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$\delta\Omega_{\hat{i}\hat{j}} = -\int_{\tau_1}^{\tau} d\tau' R_{\hat{i}\hat{j}\hat{0}\hat{k}}(\tau')\xi^{\hat{k}}$	$\delta\Omega_{\hat{i}\hat{j}} = \int_{\tau_1}^{\tau} d\tau' (8\pi T_{\hat{0}[\hat{i}}\xi_{\hat{j}]} - \epsilon_{\hat{i}\hat{j}\hat{k}}\mathcal{B}^{\hat{k}}{}_{\hat{n}}\xi^{\hat{n}})$

Recall $\mathcal{E}_{\hat{i}\hat{j}} = C_{\hat{0}\hat{i}\hat{0}\hat{j}}$ $\qquad$ $\mathcal{B}_{\hat{i}\hat{j}} = *C_{\hat{0}\hat{i}\hat{0}\hat{j}}$

Displacement and proper-time memories

$$\delta\xi^{\hat{i}}(\tau) = \int_{\tau_1}^{\tau} d\tau' \delta\dot{\xi}^{\hat{i}}(\tau') \qquad \delta\tau = -\frac{1}{2}\delta\dot{\xi}^{\hat{i}}\xi_{\hat{i}}$$

Asymptotic fall off of fields

In Bondi-type coordinates (u, r, θ^A), peeling implies

$$\begin{aligned}
\mathcal{E}_{rr} &= r^{-3} E^{(0)}_{rr} + O(r^{-4}) \\
\mathcal{E}_{r\hat{A}} &= r^{-2} E^{(0)}_{r\hat{A}} + O(r^{-3}) \\
\mathcal{E}^{(\mathrm{TF})}_{\hat{A}\hat{B}} &= r^{-1} E^{(0)}_{\hat{A}\hat{B}} + O(r^{-2})
\end{aligned}$$

Similar for $\mathcal{B}_{ij}$; finiteness and conservation of stress-energy implies

$$\begin{aligned}
T_{uu} &= r^{-2} T^{(0)}_{uu} + O(r^{-3}) \\
T_{u\hat{A}} &= r^{-3} T^{(0)}_{u\hat{A}} + O(r^{-4})
\end{aligned}$$

Other components of $T_{\mu\nu}$ fall off faster with r

Leading memory effects

- At $O(r^{-1})$, the leading memory effects have the form,

$$\delta\dot{\xi}^{\hat{A}}_{(0)} = -\int du\, E^{\hat{A}\hat{B}}_{(0)}\xi_{\hat{B}}$$

$$\delta\Omega^{(0)}_{\hat{r}\hat{A}} = -\int du\, \epsilon_{\hat{r}\hat{A}\hat{B}} B^{\hat{B}\hat{C}}_{(0)}\xi_{\hat{C}}$$

$$\delta\xi^{\hat{A}}_{(0)} = \int du\, \delta\dot{\xi}^{\hat{A}}_{(0)} \qquad \delta\tau_{(0)} = -\frac{1}{2}\delta\dot{\xi}^{\hat{A}}_{(0)}\xi_{\hat{A}}$$

- Now specialized to linearized gravity
- Solve for $\int du\, E^{\hat{A}\hat{B}}_{(0)}$, etc., in terms of $T_{\mu\nu}$ and $E^{(0)}_{rr}$ and $B^{(0)}_{rr}$ using the Bianchi identities $\nabla_d R_{abc}{}^d = 0$ Bieri & Garfinkle, arXiv:1312.6871
- Consider formal limit $u \to \pm\infty$

Sources of memory effects

- Using Bianchi identities, displacement memory is

$$\delta\xi^{\hat{A}}_{(0)} \neq 0$$

$$\int_{-\infty}^{\infty} du \int_{-\infty}^{u} du' E^{(0)}_{AB} = \frac{1}{2}(D_A D_B - 2h_{AB} D^2)\Phi^{(0)}$$

$$\frac{1}{2}(D^4 + 2D^2)\Phi^{(0)} = \Delta E^{(0)}_{rr} - 8\pi \int_{-\infty}^{\infty} du\, T^{(0)}_{uu}$$

- $D_A \leftrightarrow$ covariant derivative on S^2; h_{AB} metric on S^2
- From Bianchi identities, velocity, rotation, & proper-time memories determined by $\int_{-\infty}^{\infty} du\, E^{(0)}_{AB}$
- For asymptotic stationarity as $u \to \pm\infty$ require $\int_{-\infty}^{\infty} du\, E^{(0)}_{AB} = 0$ and all other leading memories vanish

$$\delta\tau_{(0)} = \delta\dot{\xi}^{(0)}_{\hat{A}} = \delta\Omega^{(0)}_{\hat{r}\hat{A}} = 0$$

Subleading memory effects

- At $O(r^{-2})$, all effects nonvanishing, but extremely weak!
- Velocity and proper-time memories:

$$\Delta\dot{\xi}^{(1)}_{\hat{r}} = -\int_{-\infty}^{\infty} du\, E^{(0)}_{\hat{r}\hat{A}}\xi^{A}$$

$$\Delta\dot{\xi}^{(1)}_{\hat{A}} = -\int_{-\infty}^{\infty} du\,(E^{(1)}_{\hat{A}\hat{B}}\xi^{\hat{B}} + E^{(0)}_{\hat{r}\hat{A}}\xi^{\hat{r}} - 4\pi T^{(0)}_{uu}\xi_{\hat{A}})$$

$$\delta\tau = -\frac{1}{2}(\delta\dot{\xi}^{\hat{r}}\xi_{\hat{r}} + \delta\dot{\xi}^{\hat{A}}\xi_{\hat{A}})$$

- Rotation memory:

$$\delta\Omega^{(1)}_{\hat{r}\hat{A}} = -\epsilon_{\hat{r}\hat{A}}{}^{\hat{B}}\int_{-\infty}^{\infty} du(B^{(1)}_{\hat{B}\hat{C}}\xi^{\hat{C}} + B^{(0)}_{\hat{r}\hat{B}}\xi^{\hat{r}})$$

$$\delta\Omega^{(1)}_{\hat{A}\hat{B}} = -\int du\,\epsilon_{\hat{A}\hat{B}}{}^{\hat{r}}B^{(0)}_{\hat{r}\hat{C}}\xi^{\hat{C}}$$

- From Bianchi identities, effects are determined by $\Delta E^{(0)}_{rr}$, $\int du\, T^{(0)}_{uu}$, and change in 4-momentum!

Conclusions

- GW memory is an observable effect, a prediction of GR, and a probe of the strong-field, dynamical part of the theory
- Memory also understood as transformation between the canonical frames
- Supermomentum charge corresponds to ordinary memory; super-CoM contains total memory
- Superspin charge corresponds to ordinary spin memory
- Relative displacement is the only effect that is locally measurable at $O(1/r)$
- Proper-time, rotation, and velocity effects are all $O(1/r^2)$
- The spin memory is a new $O(1/r)$ effect, but it involves a nonlocal measurement in space to observe