Equation of State Dependence of Gravitational Waves from Core-Collapse Supernovae

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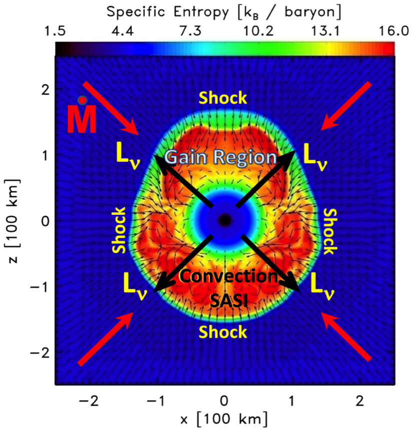
Outline

1. Overview of CCSNe and their gravitational waves
2. Results from 1824 simulations
3. Neutrino Transport in Supernovae and NS Mergers

Rotating Core-Collapse

Core-Collapse Supernovae



(Ott 2009)

SN-GRB Association

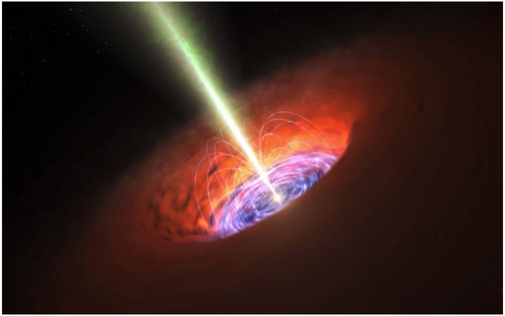
- Hypernovae
- Coincident GRB + SN Ic/bl
- Young star-forming regions

Interior rotation is still poorly understood.

Long GRBs: Explosion Mechanisms

Collapsar Mechanism

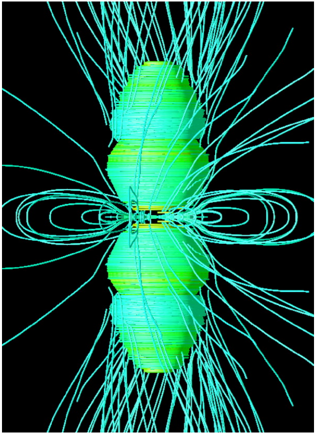
Collapse to a Black Hole



(ESO / L. Calçada)

Magnetorotational Mechanism

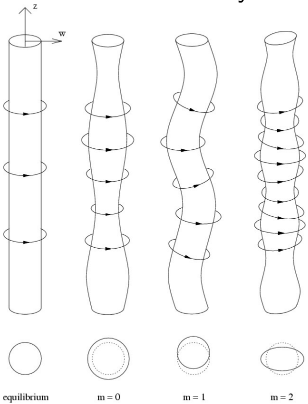
Rapidly-spinning proto-magnetar



(Burrows et al. 2007)

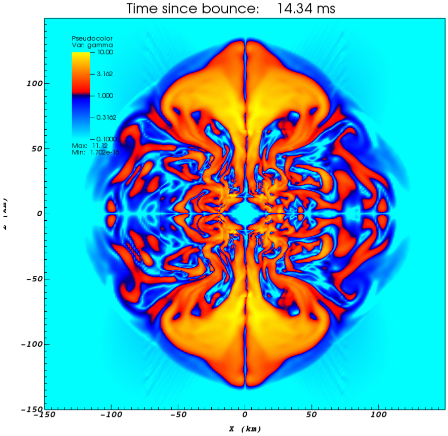
Jet Instability

Kink Instability



(Braithwaite 2006)

(growth rate) × time

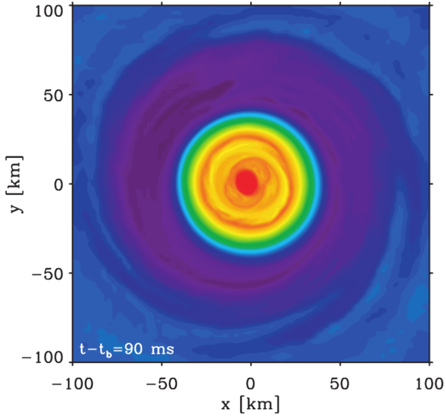


- **growth rate:** $\gamma_{\max} = \frac{1}{2a}\frac{Bz}{B_\phi}$

Gravitational Waves from Core Collapse

$$h \approx \frac{2G}{c^4 d}\ddot{I}$$ (Finn & Evans 1990)

- High-β dynamical instability
- Low-β secular instability
- Post-bounce convection / SASI
- r-mode instability
- Asymmetric energy distribution
- Rotating collapse and bounce
- ...



Ott et al. 2006

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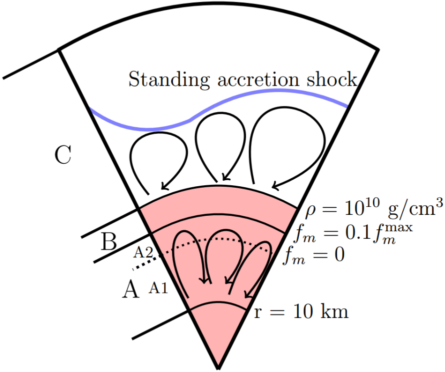
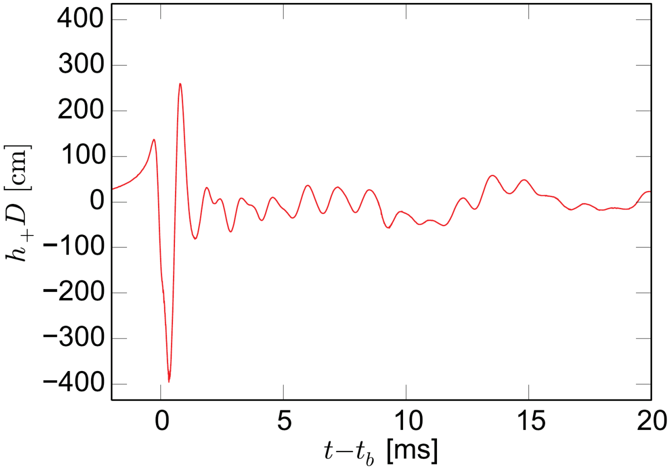


Andresen et al. 2016

Gravitational Waves from **Rapidly Rotating** Core Collapse



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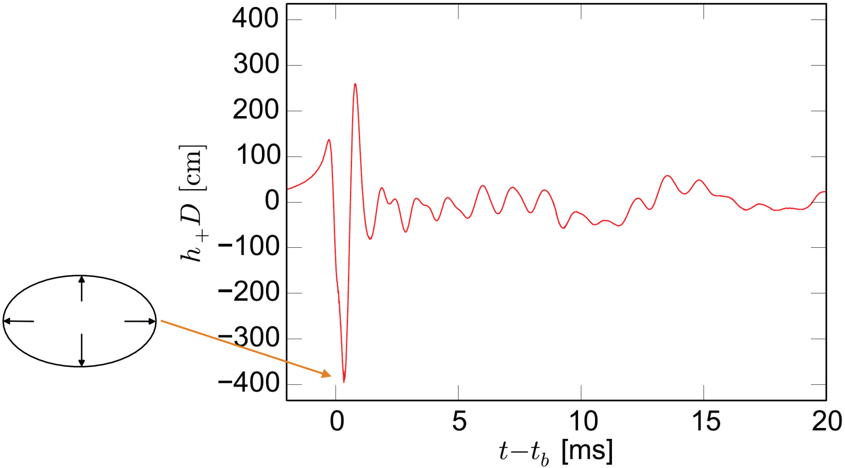

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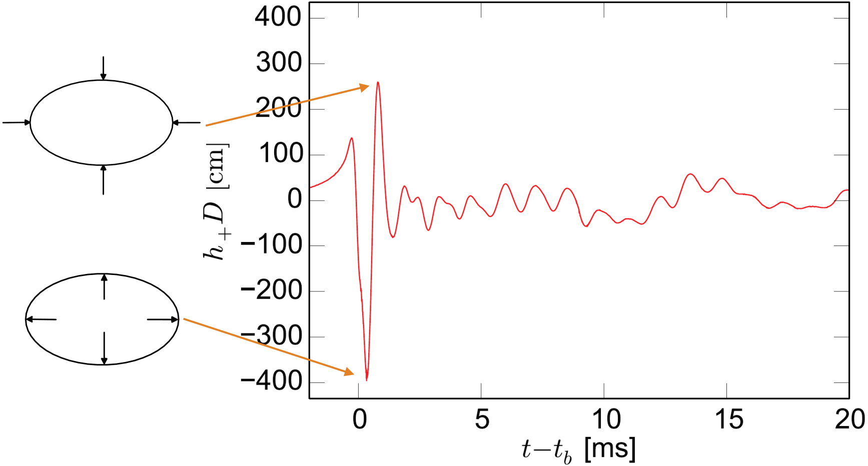

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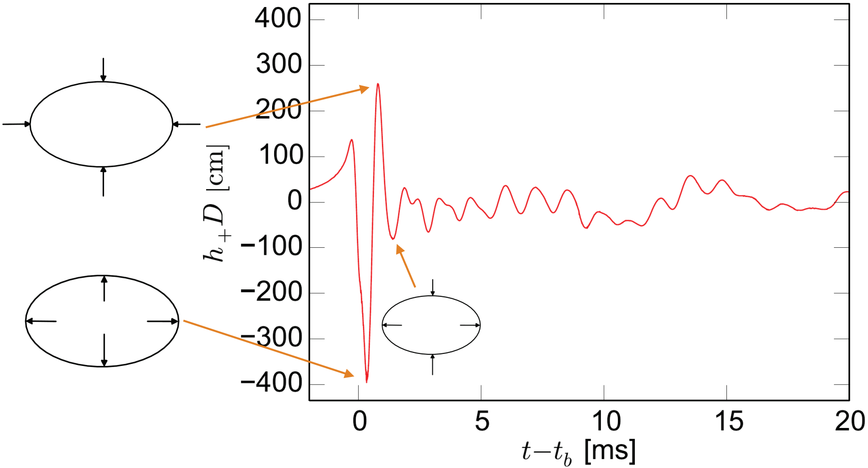

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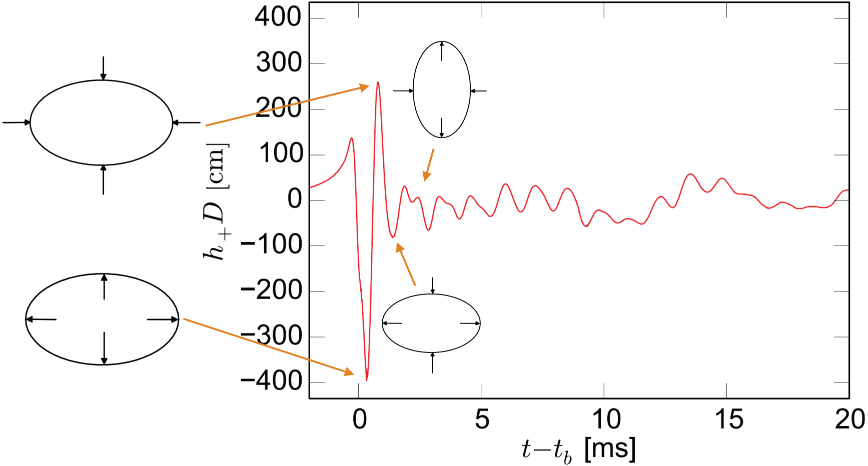

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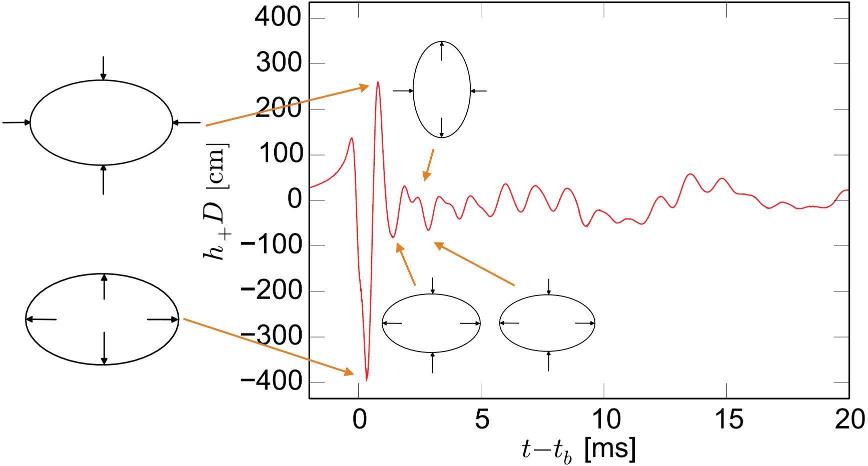

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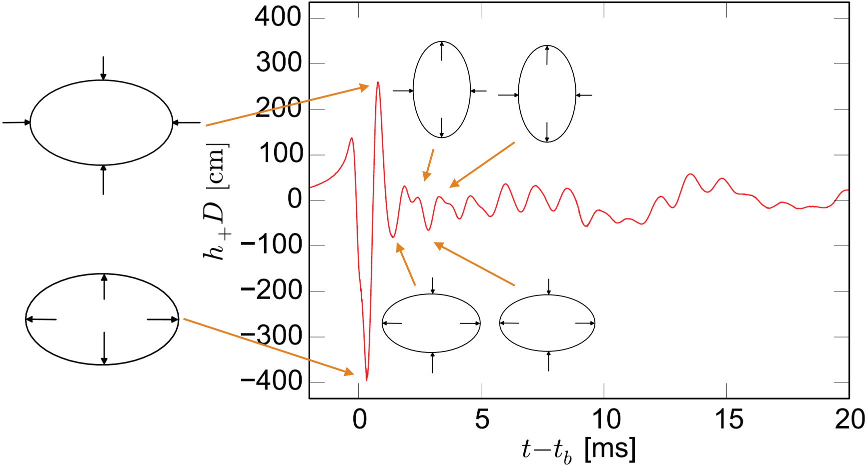
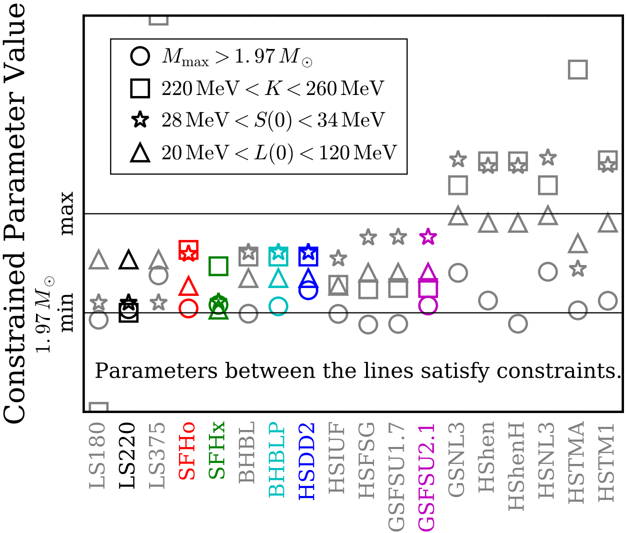


18 Equations of State

$$E(x,\beta) = -E_0 + \frac{K}{18}x^2 + K'x^3 + ... + S_2(x)\beta^2 + S_3(x)\beta^3 + ...$$

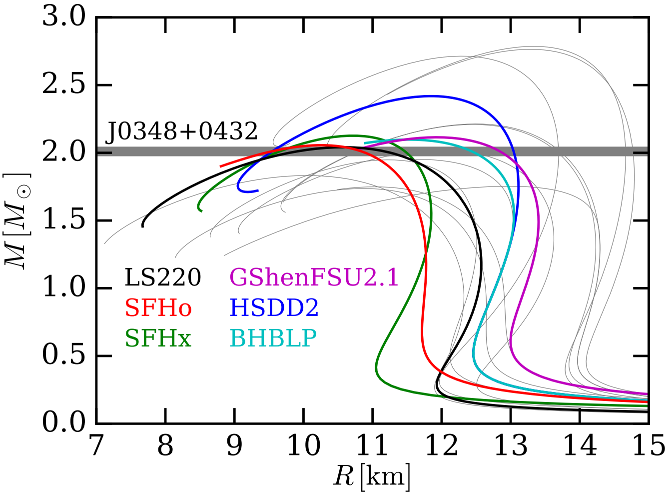


$$S_2(x) = J + \frac{L}{3}x + ...$$

$$x = \frac{n - n_s}{n_s}$$

$$\beta = 2(0.5 - Y_e)$$

18 Equations of State



Parameter Study Methods

1824 Simulations

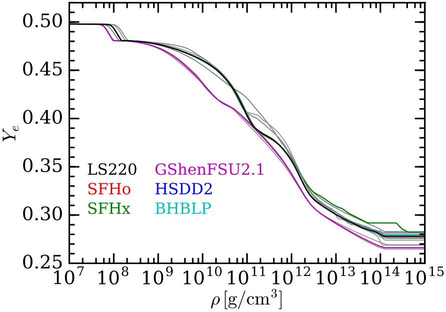
18 equations of state, 98 rotation profiles

Deleptonization (GR1D)

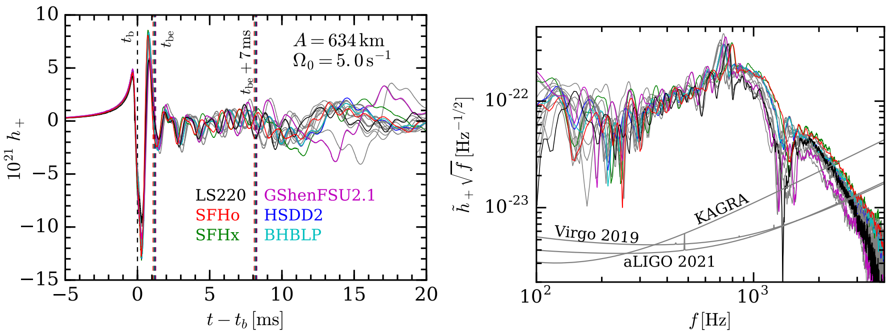
- Spherically symmetric GRHD
- M1 neutrino transport
  (O'Connor 2015)

2D Simulations (CoCoNuT)

- Conformally flat GRHD
- Neutrino Leakage
  (Dimmelmeier+02,05)



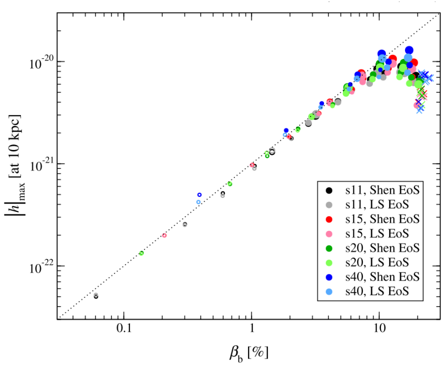
GW Observables

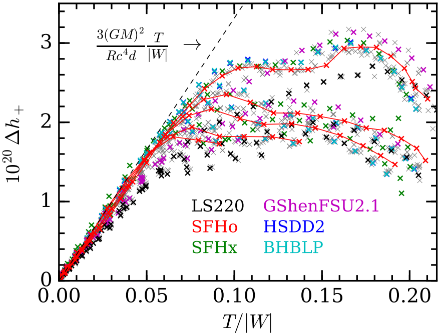


Bounce signal Δh_+ in time domain

Peak frequency f_{peak} in frequency domain.

Bounce Amplitude

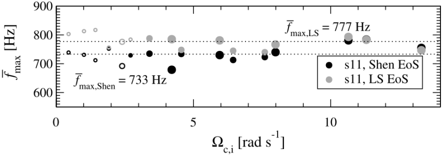




(Dimmelmeier et al. 2008)

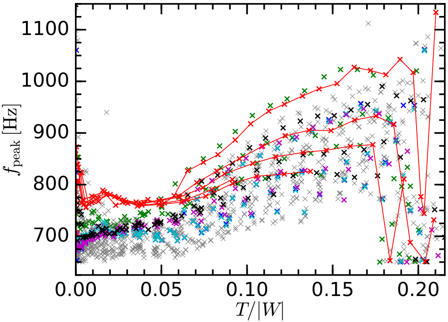
EOS and rotation influence $M_{\rm IC,b}$.

Rotation increases deformation.

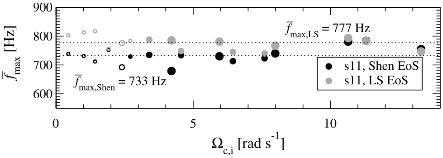


(Dimmelmeier+08)

$150\,\mathrm{Hz}$ variation due to EOS!



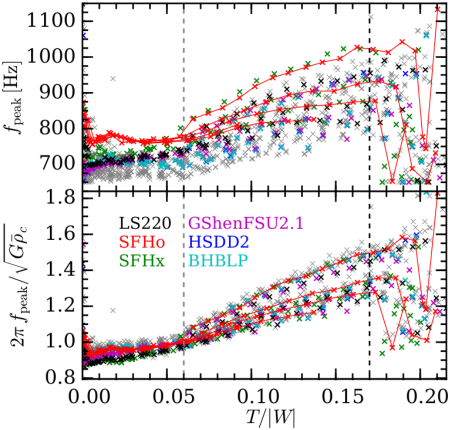
Peak Frequency



(Dimmelmeier+08)

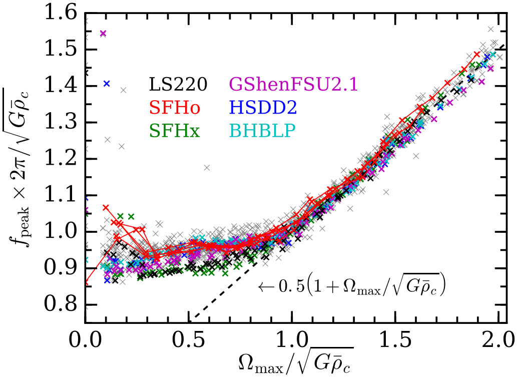
$150\,\mathrm{Hz}$ variation due to EOS

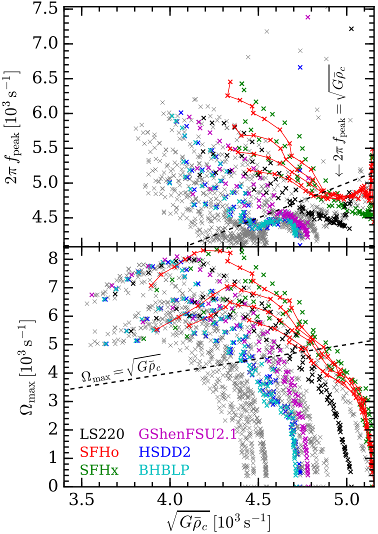
Explained by the **dynamical time**.



Effects of rotation are *independent* of the effects of the EOS.

Now, let's measure rotation differently.





Inertial effects increase frequency and confine modes to poles.

Peak Frequency

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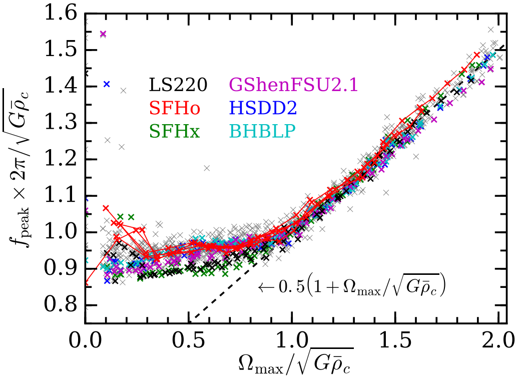
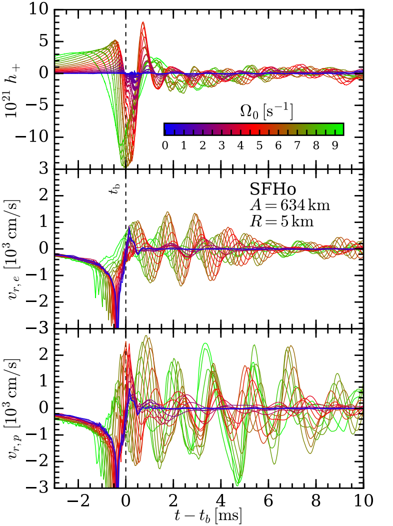
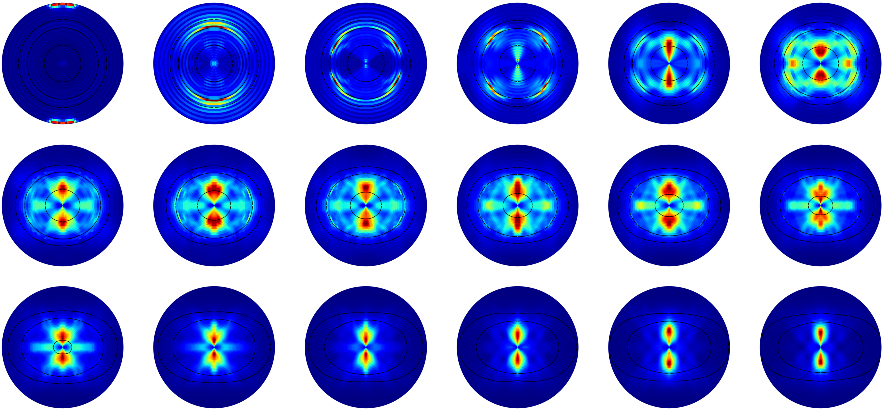




Inertial effects increase frequency and confine modes to poles.

Inertial Mode Confinement



High rotation rates suppress equatorial fluctuations.

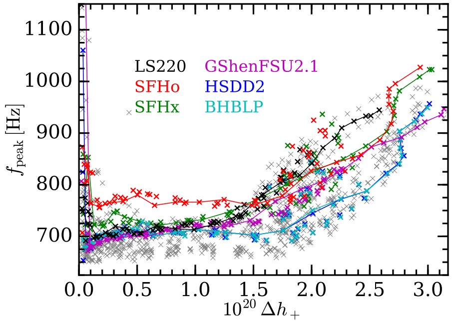
Correlations

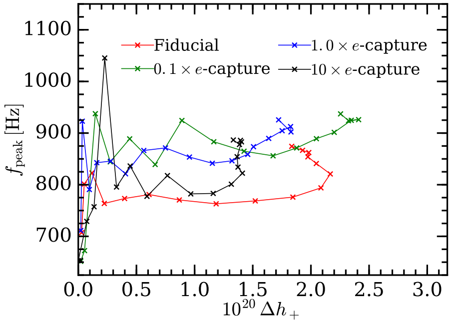
$$\mathcal{C}_{AB} = \frac{\sum(\frac{A-\bar{A}}{s_A})(\frac{B-\bar{B}}{s_B})}{N-1}$$

Lower triangle: $\Omega_{\rm max} < \sqrt{G\bar{\rho}_c}$; upper triangle: $\Omega_{\rm max} \geq \sqrt{G\bar{\rho}_c}$

	$R_{11,\rm eq,b}$	$M_{\rm IC,b}$	$j_{\rm IC,b}$	$T/\lvert W\rvert$	$\tilde{\Omega}_{\rm max}$	$\Omega_{\rm max}$	Δh_+	$\tilde{f}_{\rm peak}$	$f_{\rm peak}$	Ω_0	$Y_{e,\rm c,b}$	L	J	K	$R_{1.4}$	$M_{\rm max}$
$R_{11,\rm eq,b}$		0.93	0.97	0.97	0.96	0.95	0.84	0.96	0.89	0.97	0.08	0.09	0.06	0.02	0.07	0.01
$M_{\rm IC,b}$	0.63		0.98	0.94	0.95	0.91	0.90	0.94	0.81	0.95	0.22	0.04	0.20	0.02	0.11	0.01
$j_{\rm IC,b}$	0.66	0.85		0.98	0.98	0.95	0.87	0.97	0.85	0.99	0.09	0.03	0.10	0.02	0.06	0.01
$T/\lvert W\rvert$	0.67	0.80	0.98		0.99	0.98	0.85	0.98	0.91	1.00	0.00	0.03	0.03	0.04	0.04	0.03
$\tilde{\Omega}_{\rm max}$	0.64	0.79	0.99	0.97		0.97	0.88	0.99	0.88	0.99	0.05	0.04	0.06	0.01	0.05	0.01
$\Omega_{\rm max}$	0.65	0.76	0.98	0.96	0.99		0.86	0.96	0.96	0.97	0.02	0.14	0.12	0.09	0.15	0.07
Δh_+	0.70	0.83	0.98	0.99	0.95	0.95		0.88	0.80	0.87	0.27	0.10	0.09	0.09	0.02	0.07
$\tilde{f}_{\rm peak}$	0.01	0.06	0.01	0.02	0.05	0.06	0.04		0.90	0.99	0.05	0.02	0.03	0.05	0.03	0.03
$f_{\rm peak}$	0.01	0.18	0.17	0.11	0.18	0.15	0.10	0.86		0.89	0.02	0.32	0.31	0.10	0.33	0.07
Ω_0	0.65	0.79	0.99	0.98	1.00	0.99	0.96	0.04	0.17		0.00	0.02	0.02	0.02	0.02	0.02
$Y_{e,\rm c,b}$	0.29	0.47	0.05	0.03	0.02	0.03	0.06	0.06	0.04	0.02		0.24	0.29	0.01	0.02	0.27
L	0.42	0.05	0.01	0.02	0.01	0.07	0.04	0.03	0.37	0.00	0.20		0.77	0.43	0.88	0.40
J	0.29	0.34	0.04	0.04	0.00	0.08	0.01	0.08	0.31	0.01	0.35	0.76		0.21	0.81	0.28
K	0.08	0.07	0.01	0.03	0.01	0.06	0.03	0.03	0.29	0.02	0.02	0.42	0.21		0.57	0.52
$R_{1.4}$	0.31	0.19	0.02	0.04	0.00	0.09	0.03	0.05	0.41	0.01	0.04	0.88	0.80	0.57		0.54
$M_{\rm max}$	0.00	0.02	0.01	0.02	0.00	0.03	0.03	0.01	0.19	0.01	0.26	0.39	0.28	0.53	0.54	

Can We Constrain the EOS?





Probably not. BUT we understand how uncertainties in nuclear physics propagate to GW predictions.

Reliable predictions require detailed treatment of pre-bounce **neutrino transport** and detailed **electron capture rates**.

An Aside on Reliable Neutrino Transport

3D

GRMHD

EOS

Nuclei

ν-Radiaton

Boltzmann Equation

$$\boxed{\frac{\partial f}{\partial t} + \vec{\Omega} \cdot \vec{\nabla} f = C(f)}$$

$$\vec{\Omega} = \text{direction}$$

$$C(f) = \text{collisional terms}$$

$$f(\vec{x}, \vec{\Omega}, E_\nu, t) = N_\nu/\text{str}/\text{Hz}/\text{cm}^3$$

7-dimensional!

Simulation State of the Art

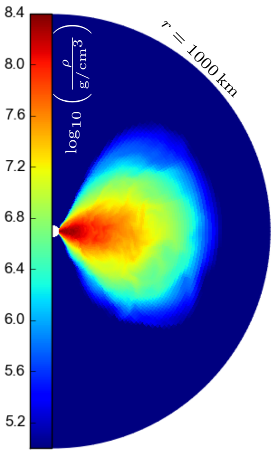
3D	Low resolution
GRMHD	Artificially large B fields
EOS	Loosely constrained
Nuclei	Extrapolated reaction rates, post-processed
ν-**Radiaton**	Local two-moment

Rapidly rotating core collapse provides an ideal testbed.

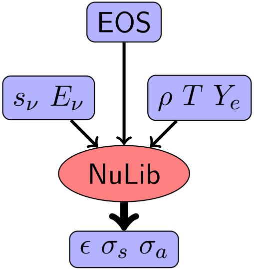
Sedonu + NuLib

1. Take fluid snapshot
2. Emit
3. Propagate
4. Scatter and Absorb
5. Calculate rates

Open Source!



Data: Metzger & Fernández (2014)



Neutrino Transport: Monte Carlo

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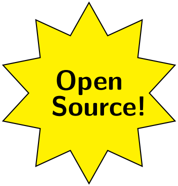

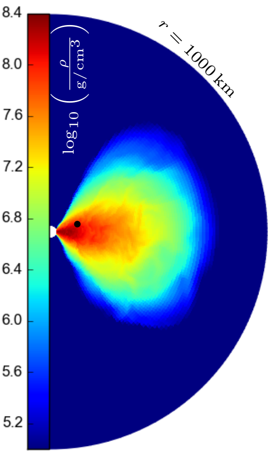

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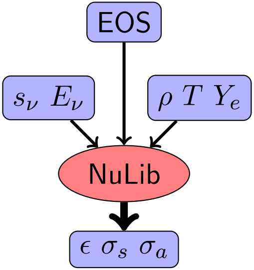

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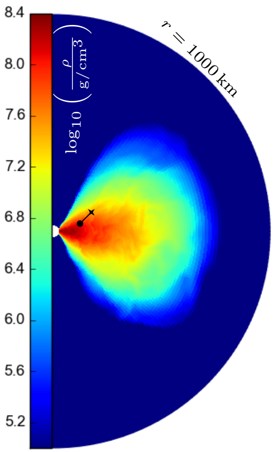

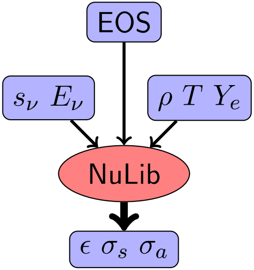

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Neutrino Transport: Monte Carlo

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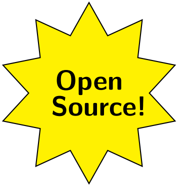

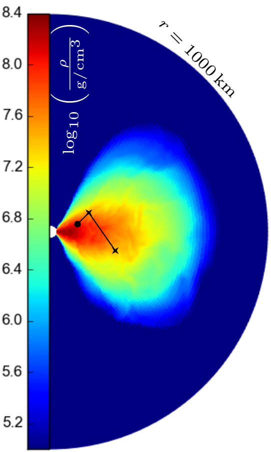

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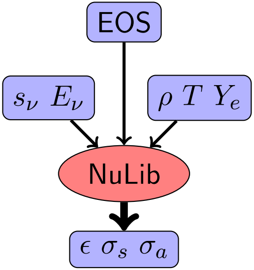

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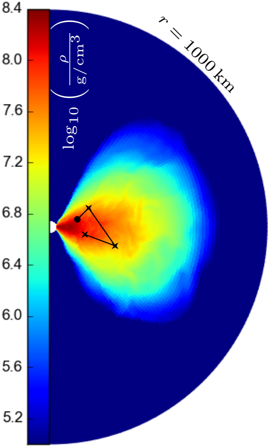

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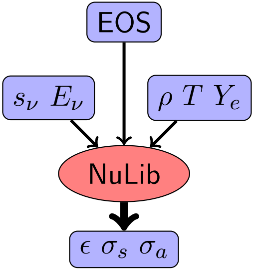

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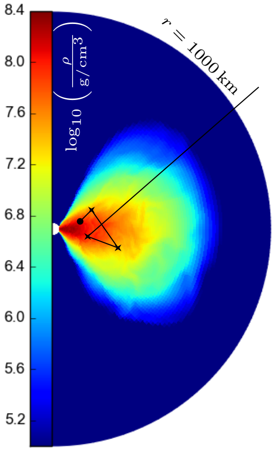

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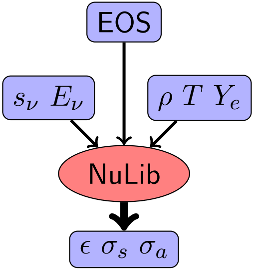

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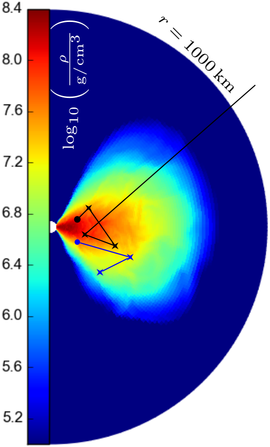

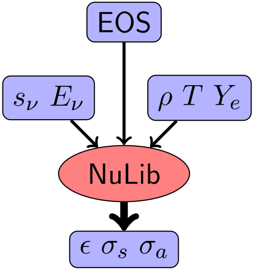

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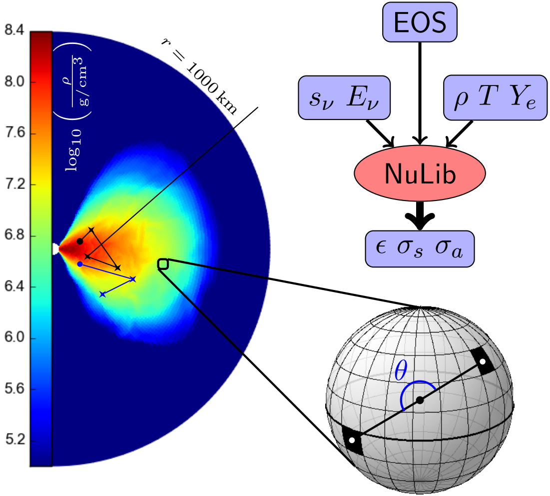
Open Source!

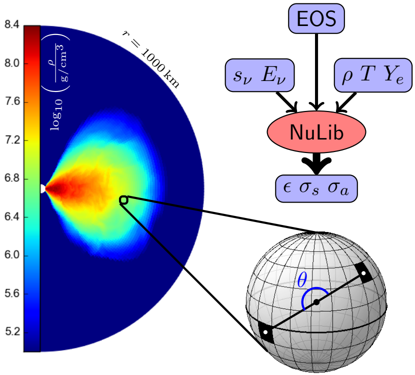


Data: Metzger & Fernández (2014)

Neutrino Pair Annihilation

$$Q_{\text{ann}} \sim \int dE_\nu \int dE_{\bar{\nu}} \int d\Omega_\nu \int d\Omega_{\bar{\nu}} \times I\bar{I}(E+\bar{E})(1-\cos(\theta))^2$$

Strong angular dependence



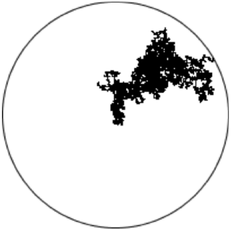
Data: Metzger & Fernández (2014)

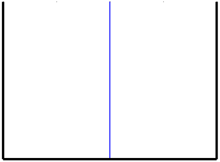
Random Walk Approximation

random walk

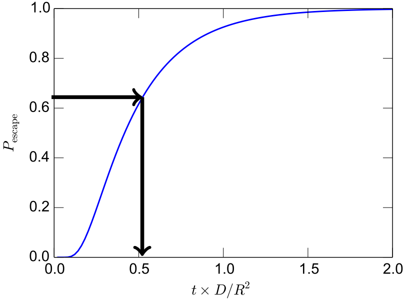
⇓

diffusion

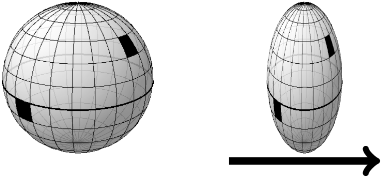




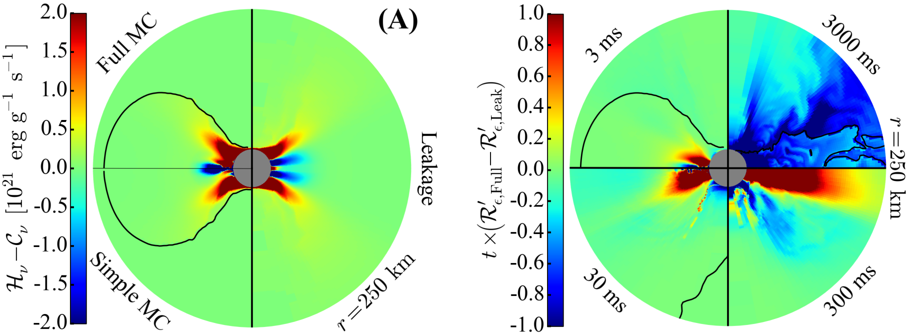
1) Use analytic solution to find travel time.



2) Transform out of the comoving frame.

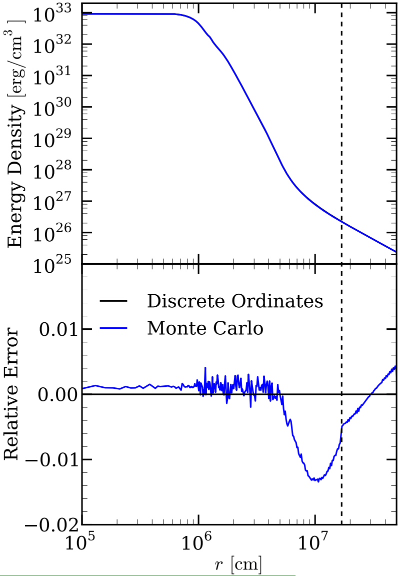


Comparison to Leakage



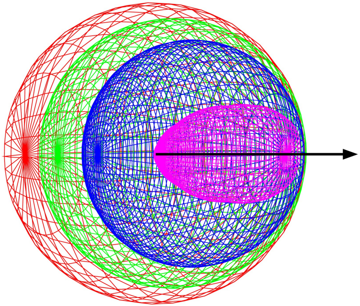
MC transport would likely significantly affect disk thermodynamics.

Monte Carlo vs Discrete Ordinates

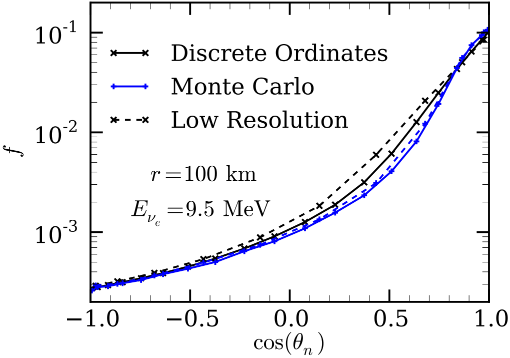


- Good agreement in 1D.
- MC is **noisy**, but converges with increasing particle count.

Monte Carlo vs Discrete Ordinates



(Hiroki Nagakura)



- DO is **diffusive**, but converges with increasing resolution.

The Next Generation: Two Moment Transport

1. Rewrite Boltzmann equation in terms of angular moments of f

$$J := \nu^3 \int d\Omega f(\nu, \Omega, x^\mu)$$
$$H^\alpha := \nu^3 \int d\Omega f(\nu, \Omega, x^\mu) l^\alpha$$
$$L^{\alpha\beta} := \nu^3 \int d\Omega f(\nu, \Omega, x^\mu) l^\alpha l^\beta$$
$$N^{\alpha\beta\gamma} := \nu^3 \int d\Omega f(\nu, \Omega, x^\mu) l^\alpha l^\beta l^\gamma$$
$$\ldots\text{etc}$$

$$\frac{dJ}{dt} = S_0(J, H^\alpha, L^{\alpha\beta})$$
$$\frac{dH^\alpha}{dt} = S_1(J, H^\alpha, L^{\alpha\beta}, N^{\alpha\beta\gamma})$$
$$\ldots\text{etc}$$

(Thorne 1981, Shibata 2011)

2. Make a guess for $L^{\alpha\beta}(J, H^\alpha)$ and $N^{\alpha\beta\gamma}(J, H^\alpha)$
3. Evolve J and H^α *only*

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2. Make a **good** guess for $L^{\alpha\beta}$ and $N^{\alpha\beta\gamma}$
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A Simple Closure Relation

Pressure Tensor:

$$P^{ij} = \frac{3\chi-1}{2} P^{ij}_{\text{thin}} + \frac{3(1-\chi)}{2} P^{ij}_{\text{thick}}$$

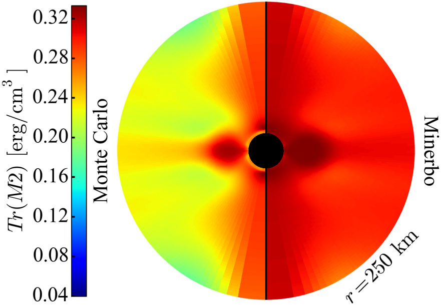
Closure Relation:

$$\chi(\omega) = \frac{1}{3} + \omega^2 \frac{6-2\omega+6\omega^2}{15}$$

Eddington Factor:

$$\omega = \frac{F_\alpha F^\alpha}{E^2}$$

(Minerbo 1978)
(Foucart et al. 2015)



Take Away

- We **quantify uncertainties** in GW observables due to nuclear physics.
- **Universal relations** obeyed by all EOS and rotation profiles.
- Sedonu enables **detailed transport comparisons**.
- **Robust** predictions are within reach.

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Equation of State Details

Parameter limits (Steiner et al. 2013):

- Incompressibility: K=220-260 MeV
- Symmetry Energy: J=28-34MeV
- Derivative of Symmetry Energy: L=20-120MeV
- Maximum NS Mass: M=1.93-2.01

Interaction Models

- Liquid Drop
- Relativistic Mean Field Theory

Nuclei

- Single Nucleus Approximation
- Hartree-Fock
- Statistical

PDF Biasing

Problem: too much computational power is going where it is not needed.

Solution: Tweak PDF and weight simultaneously to decrease $\langle E^2_\text{deposit}\rangle$ while keeping $\langle E_\text{deposit}\rangle$ unchanged.

$$\begin{aligned}\langle E_\text{deposit}\rangle &= \int_0^\infty E_\nu \sigma_a x p(x) dx \\ &= \int_0^\infty (wE_\nu)\sigma_a x \frac{p(x)}{w} dx\end{aligned}$$

Signal to Noise Ratio