Collective Neutrino Oscillations in SNe

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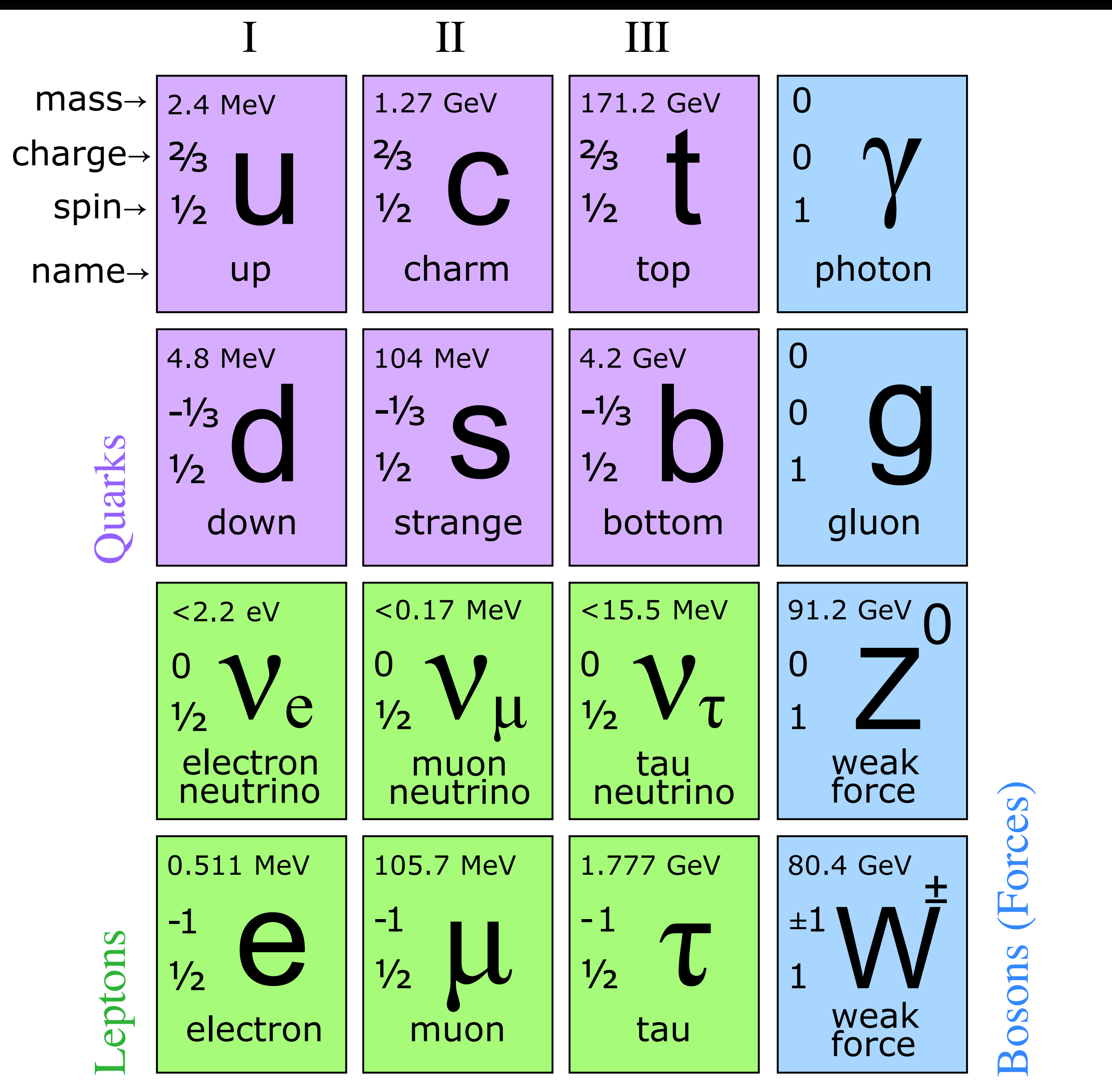
U.S. DEPARTMENT OF ENERGY | Office of Science

Outline

- **Introduction**
- Numerical models and results
- Recent progress and challenges
- Summary

Standard Model

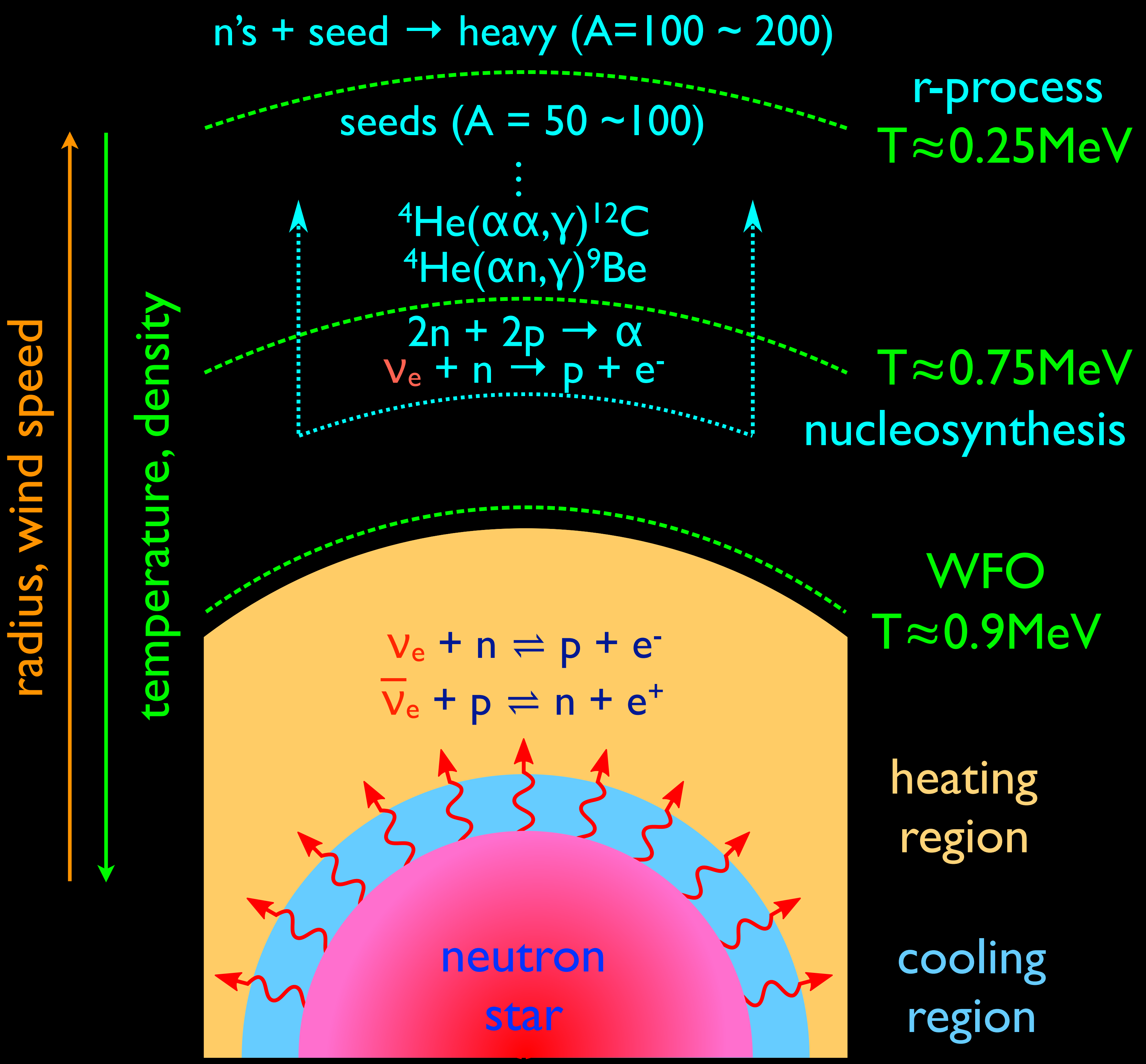


Wikimedia: Standard Model of Elementary Particles

Neutrinos in Standard Model:

- Three flavors
- No mass
- No electric charge, interacting weakly

Neutrinos in Supernovae



- ~10^{53} ergs, 10^{58} neutrinos in ~10 seconds
- All neutrino species, 10~30 MeV
- Dominate energetics
- Influence nucleosynthesis
- Probe into SNe

Vacuum Oscillations

neutrino mass eigenstates ≠ weak interaction states

$$|\nu_1\rangle = \cos\theta_{\rm v}|\nu_e\rangle + \sin\theta_{\rm v}|\nu_\mu\rangle \quad \text{with mass } m_1$$

$$|\nu_2\rangle = -\sin\theta_{\rm v}|\nu_e\rangle + \cos\theta_{\rm v}|\nu_\mu\rangle \quad \text{with mass } m_2$$

vacuum mixing angle

initially $|\psi(x=0)\rangle = |\nu_e\rangle$

$$P_{\nu_e\nu_e}(x) \equiv |\langle\nu_e|\psi(x)\rangle|^2 = 1 - \sin^2 2\theta_{\rm v} \sin^2\left(\frac{\delta m^2 x}{4E_\nu}\right)$$

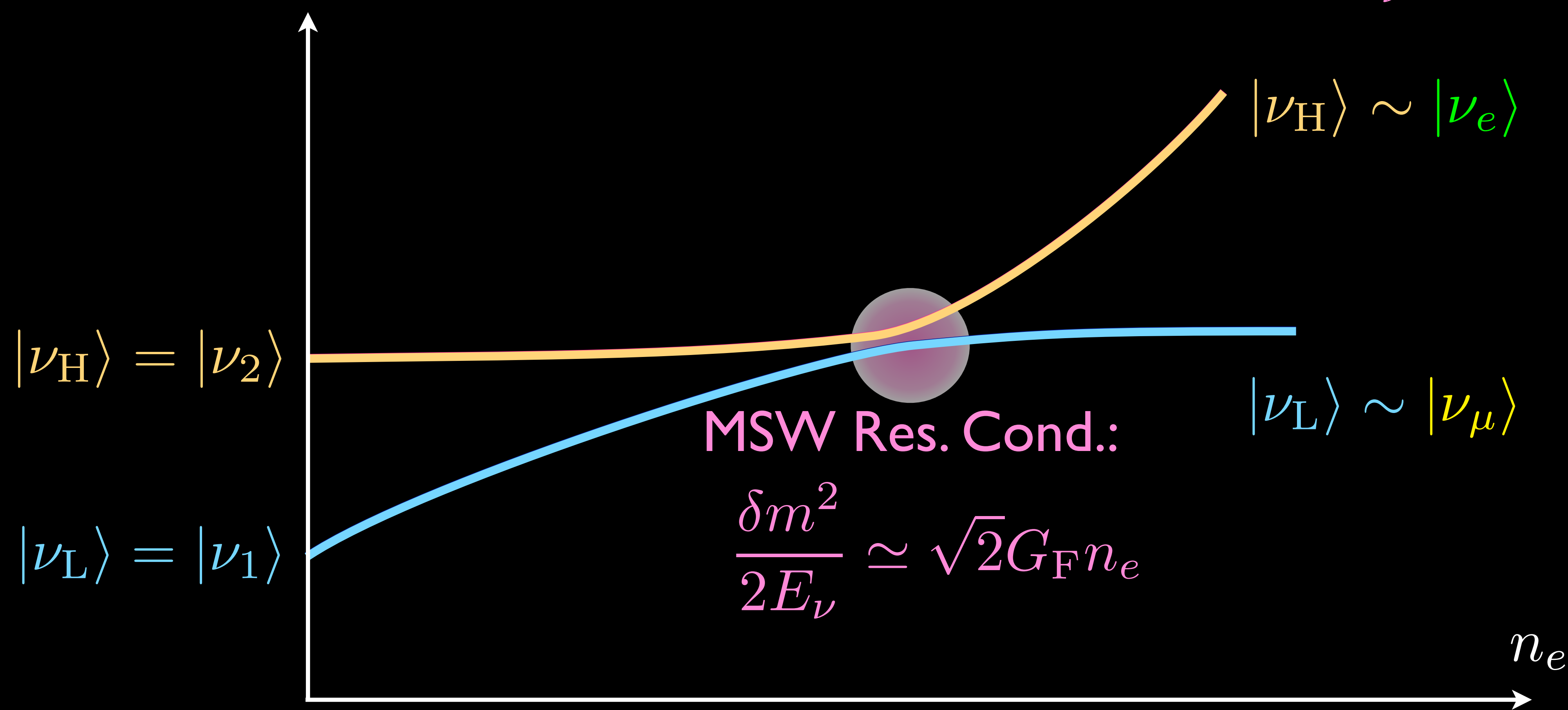
neutrino survival probability

Matter Effect

electron number density

$$\mathrm{i}\frac{\mathrm{d}}{\mathrm{d}x}\begin{bmatrix}\langle\nu_e|\psi_\nu\rangle\\ \langle\nu_\mu|\psi_\nu\rangle\end{bmatrix}=\frac{1}{2}\begin{bmatrix}2\sqrt{2}G_{\mathrm{F}}n_e-\omega\cos 2\theta_{\mathrm{v}} & \omega\sin 2\theta_{\mathrm{v}}\\ \omega\sin 2\theta_{\mathrm{v}} & \omega\cos 2\theta_{\mathrm{v}}\end{bmatrix}\begin{bmatrix}\langle\nu_e|\psi_\nu\rangle\\ \langle\nu_\mu|\psi_\nu\rangle\end{bmatrix}$$

vac. osc. freq. $\omega=\frac{\delta m^2}{2E_\nu}$



Mikheyev, Smirnov (1985)

Three Flavor Mixing

WEAK FLAVOR STATES

VACUUM MASS EIGENSTATES

$$\begin{pmatrix} |\nu_e\rangle \\ |\nu_\mu\rangle \\ |\nu_\tau\rangle \end{pmatrix} = \begin{pmatrix} c_{12}c_{13} & c_{13}s_{12} & s_{13} \\ -c_{23}s_{12}e^{\mathrm{i}\phi} - c_{12}s_{13}s_{23} & c_{12}c_{23}e^{\mathrm{i}\phi} - s_{12}s_{13}s_{23} & c_{13}s_{23} \\ s_{23}s_{12}e^{\mathrm{i}\phi} - c_{12}c_{23}s_{13} & -c_{12}s_{23}e^{\mathrm{i}\phi} - c_{23}s_{12}s_{13} & c_{13}c_{23} \end{pmatrix}^* \begin{pmatrix} |\nu_1\rangle \\ |\nu_2\rangle \\ |\nu_3\rangle \end{pmatrix}$$

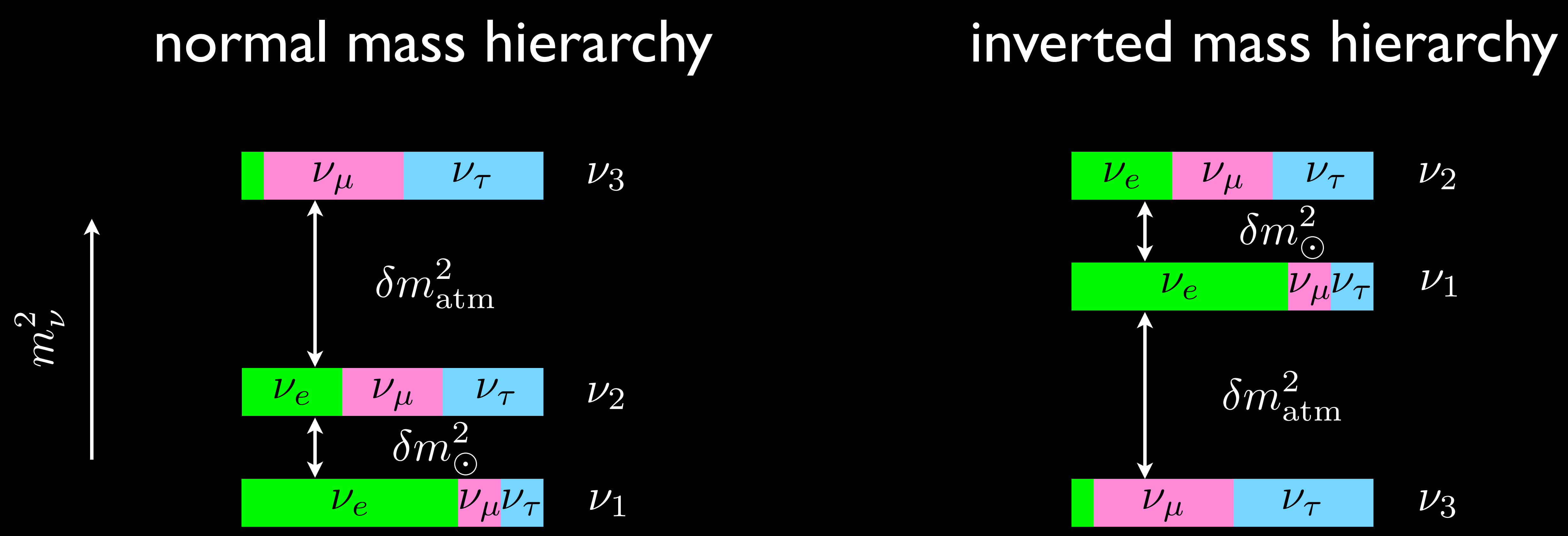
$$\delta m^2_{12} \simeq \delta m^2_\odot \simeq 7\text{–}8 \times 10^{-5}\mathrm{eV}^2, \quad \theta_{12} \simeq \theta_\odot \simeq 0.6$$

$$|\delta m^2_{23}| \simeq \delta m^2_{\mathrm{atm}} \simeq 2\text{–}3 \times 10^{-3}\mathrm{eV}^2, \quad \theta_{23} \simeq \theta_{\mathrm{atm}} \simeq \frac{\pi}{4}$$

$$|\delta m^2_{13}| \simeq |\delta m^2_{23}| \simeq 2\text{–}3 \times 10^{-3}\mathrm{eV}^2, \quad \theta_{13} \simeq 0.15$$

ϕ is unknown ⟵ CP VIOLATION PHASE

Mass Hierarchy



Density Matrix

Pure State:

$$|\psi\rangle \implies \hat{\rho} = |\psi\rangle\langle\psi|$$

$$\text{Example: } |\nu_e\rangle \implies \rho = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$$

Mixed State:

$$\rho \propto \begin{bmatrix} n_{\nu_e} & 0 \\ 0 & n_{\nu_x} \end{bmatrix}$$

In Dense Medium

$$(\partial_t + \hat{\mathbf{v}} \cdot \boldsymbol{\nabla})\rho = -\mathrm{i}[\mathsf{H},\ \rho]$$

mass matrix

electron density

$$\mathsf{H} = \frac{\mathsf{M}^2}{2E} + \sqrt{2} G_{\mathrm{F}}\, \mathrm{diag}[n_e, 0, 0] + \mathsf{H}_{\nu\nu}$$

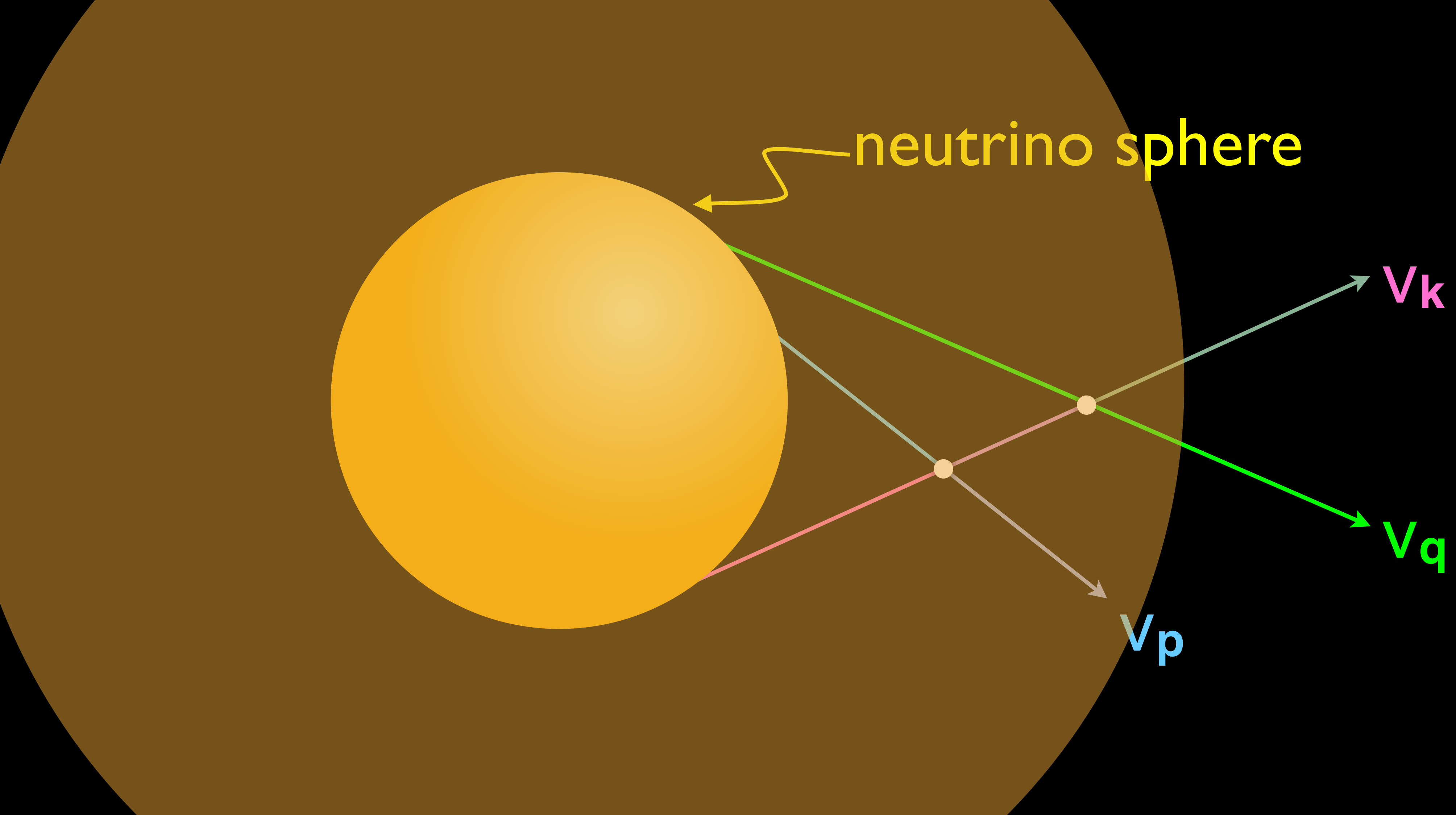
neutrino energy

v-v forward scattering
(self-coupling)

$$\mathsf{H}_{\nu\nu} = \sqrt{2} G_{\mathrm{F}} \int \mathrm{d}^3\mathbf{p}' (1 - \hat{\mathbf{v}} \cdot \hat{\mathbf{v}}')(\rho_{\mathbf{p}'} - \bar{\rho}_{\mathbf{p}'})$$

Oscillations in SN

$$\mathsf{H} = \frac{\mathsf{M}^2}{2E} + \sqrt{2}G_{\mathrm{F}}\,\mathrm{diag}[n_e, 0, 0] + \mathsf{H}_{\nu\nu}$$



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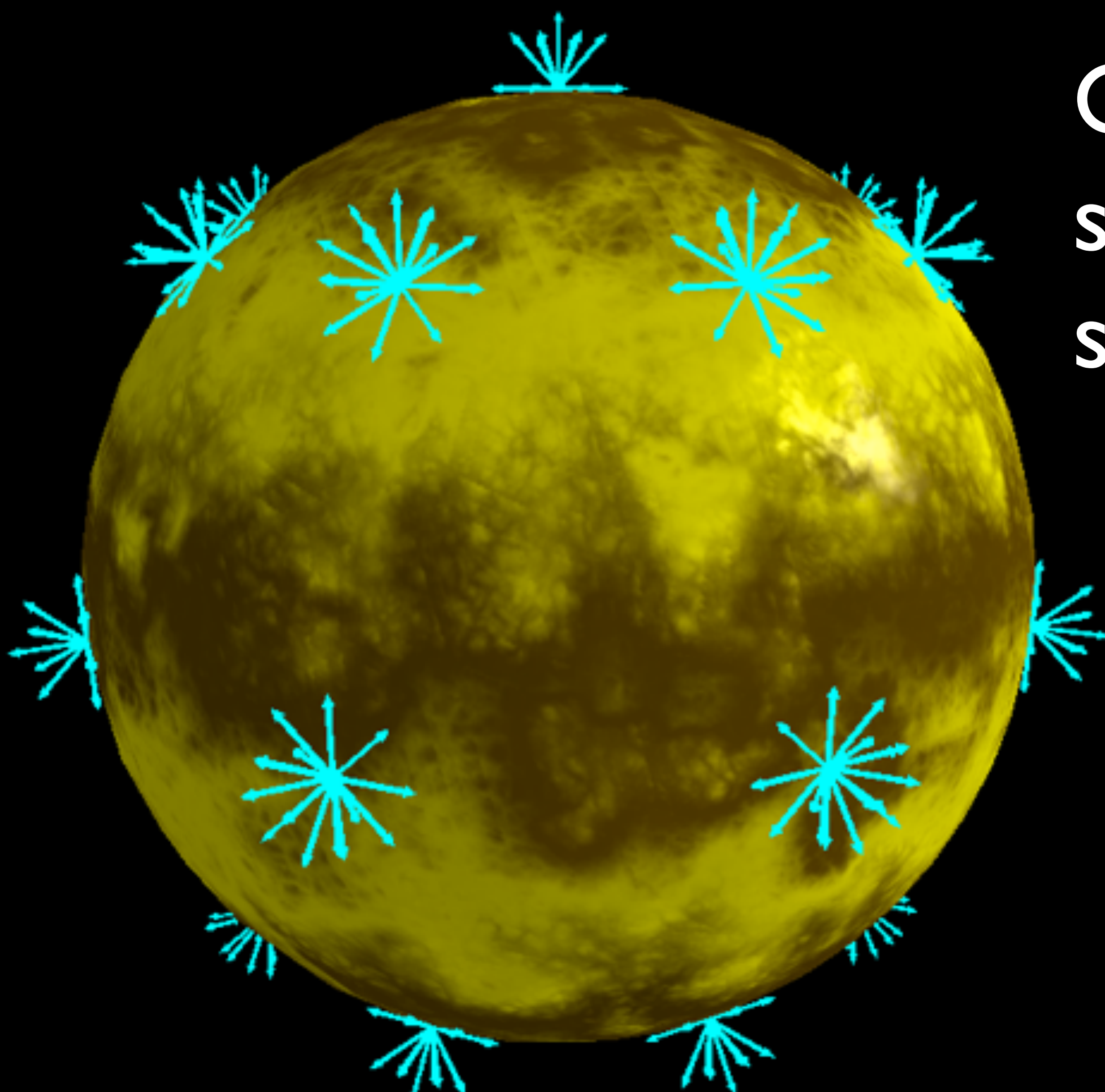
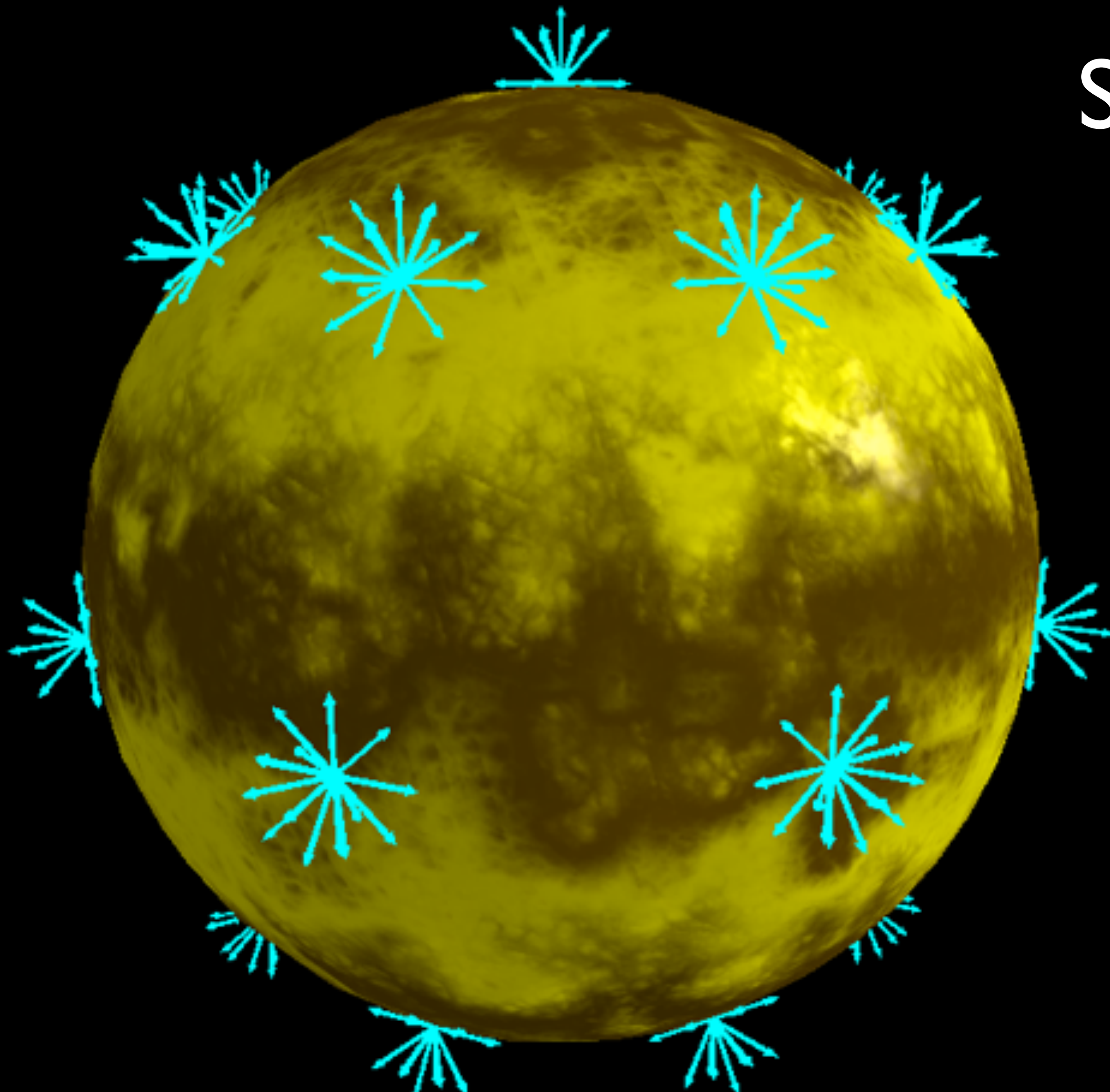
Numerical Models



Coherent forward scattering outside neutrino sphere

$$\rho(t; r, \Theta, \Phi; E, \vartheta, \varphi)$$

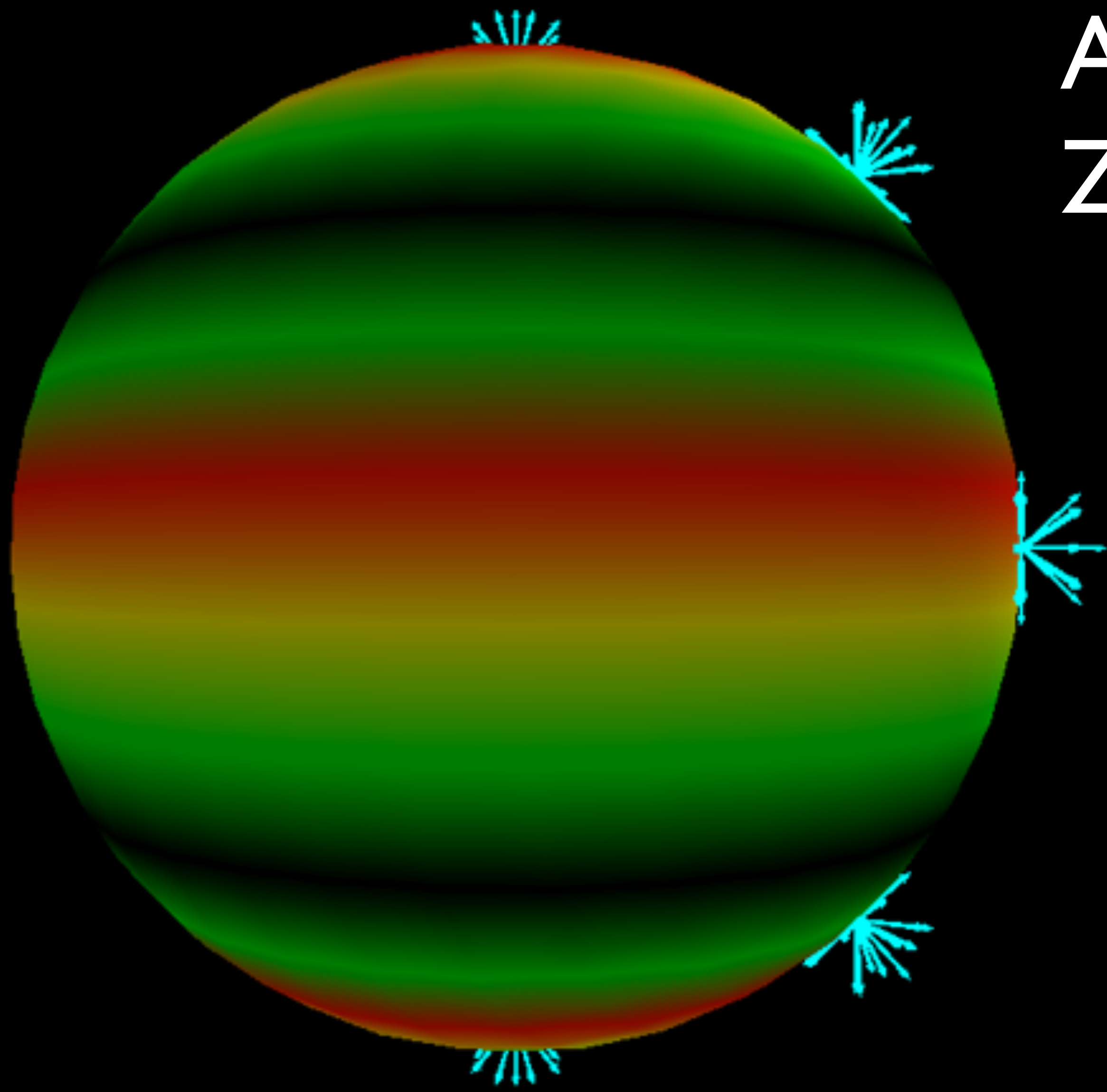
Numerical Models



Stationary emission

$$\rho(r, \Theta, \Phi; E, \vartheta, \varphi)$$

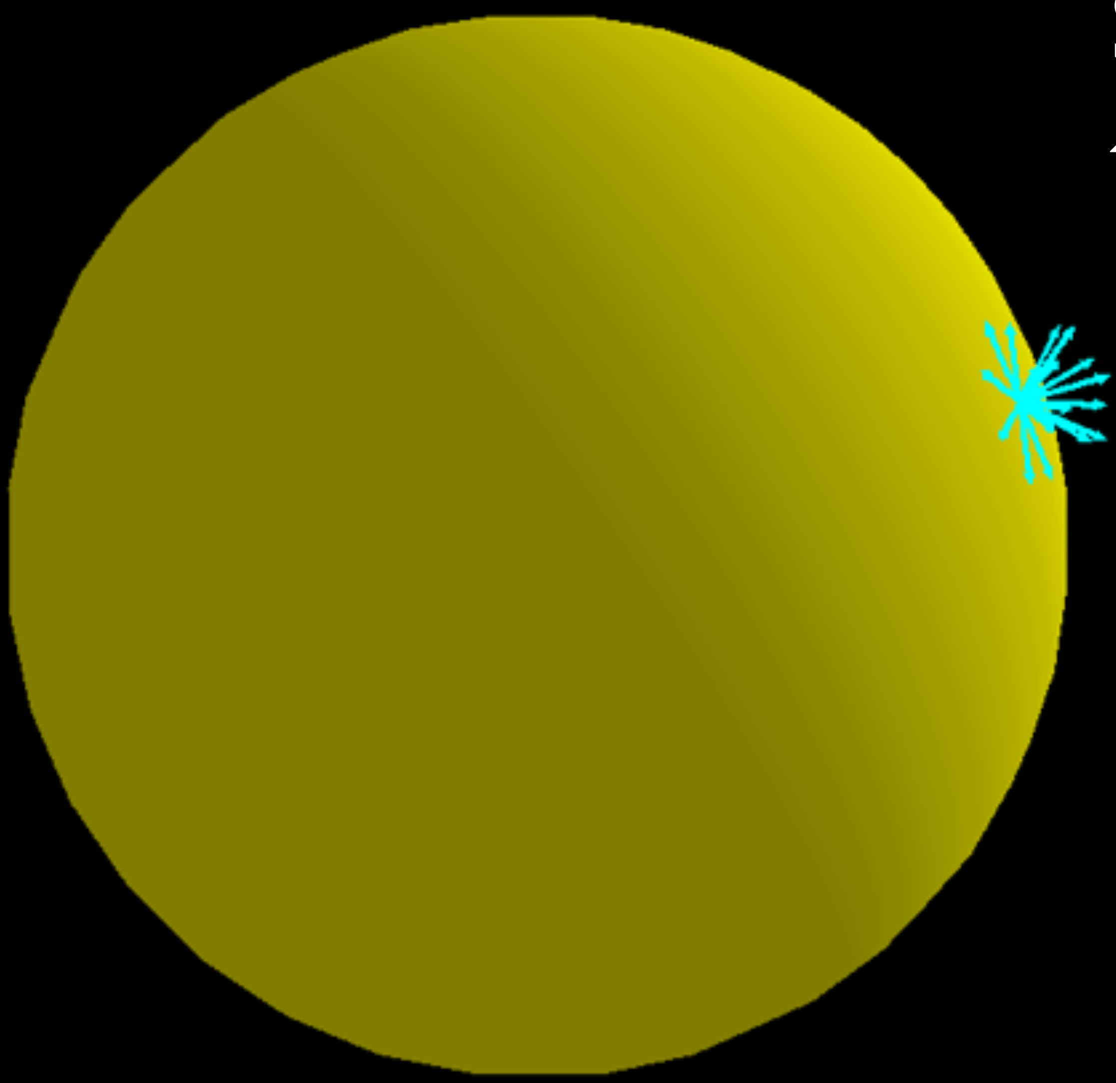
Numerical Models



Axial symmetry around the Z axis

$$\rho(r, \Theta; E, \vartheta, \varphi)$$

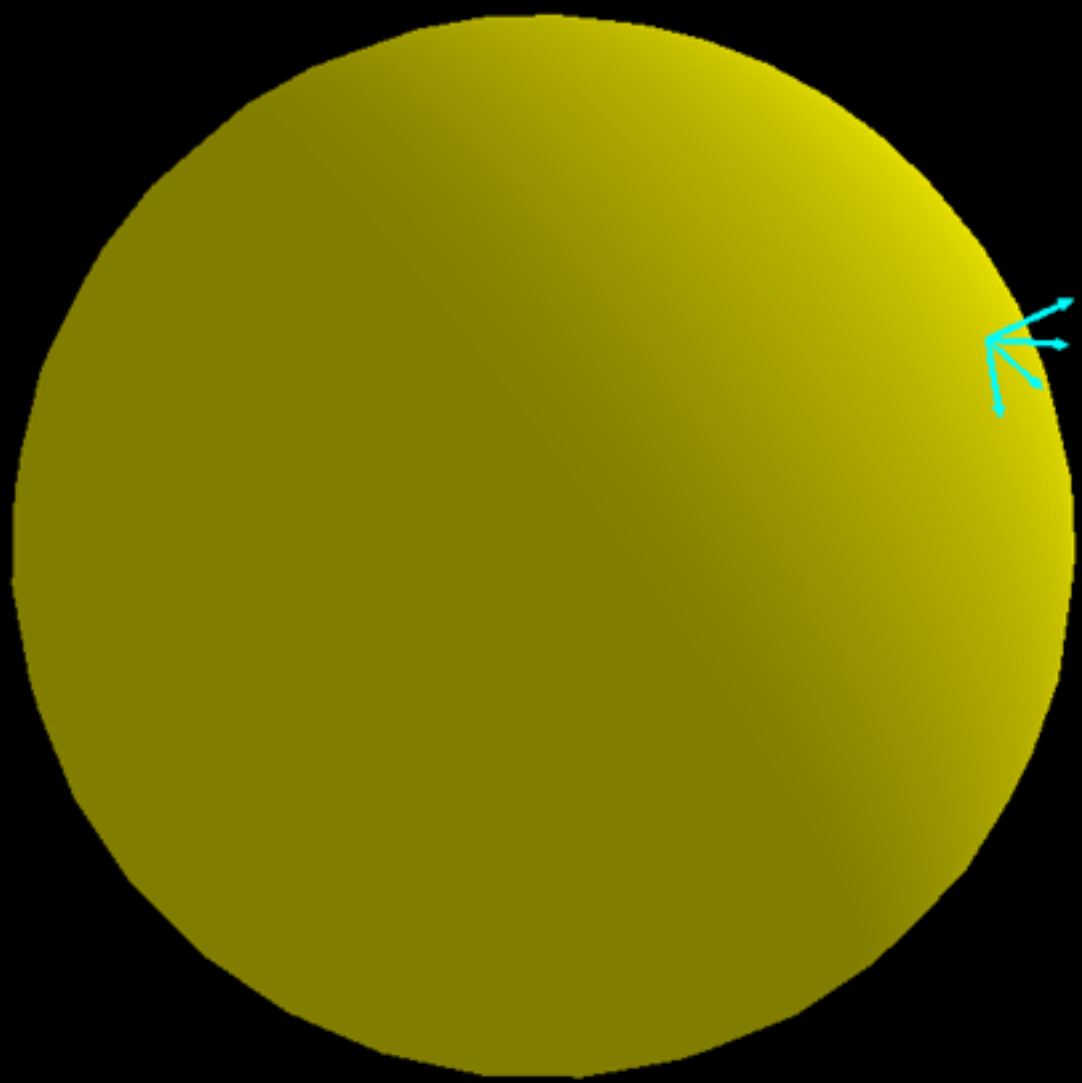
Numerical Models



Spherical symmetry about the center (inconsistent?)

$$\rho(r; E, \vartheta, \varphi)$$

Numerical Models

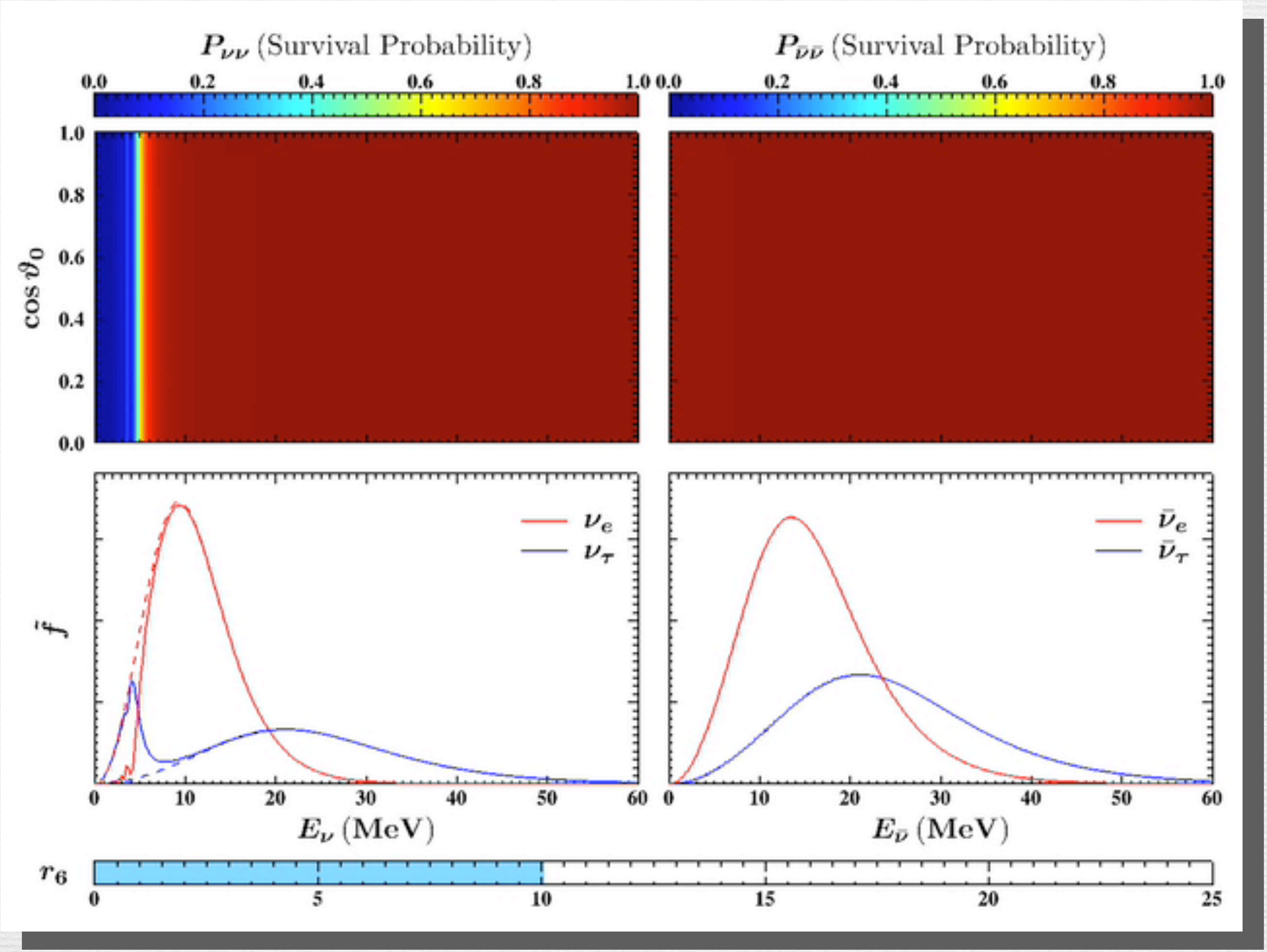


Azimuthal symmetry around any radial direction

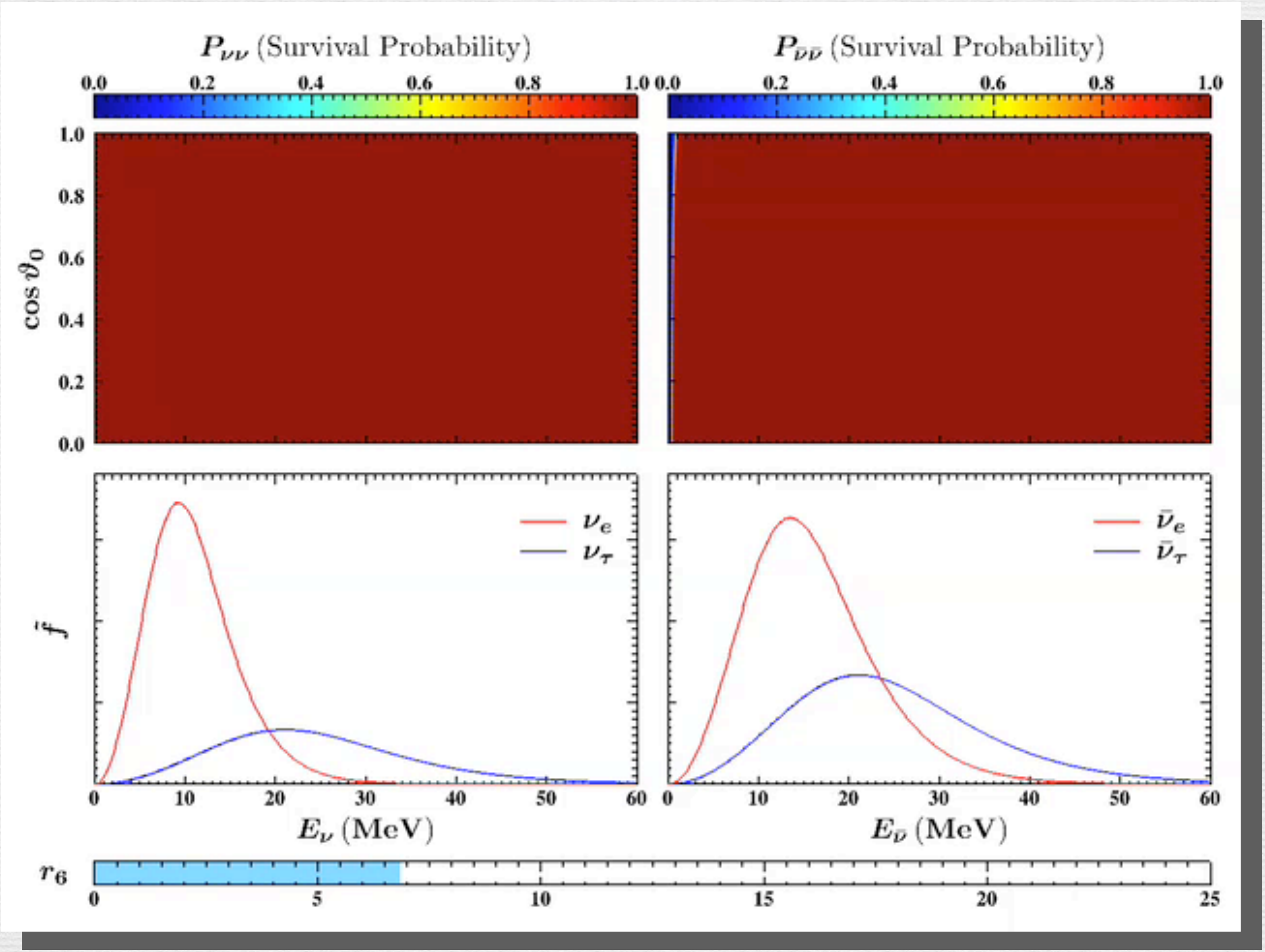
$$\rho(r; E, \vartheta)$$

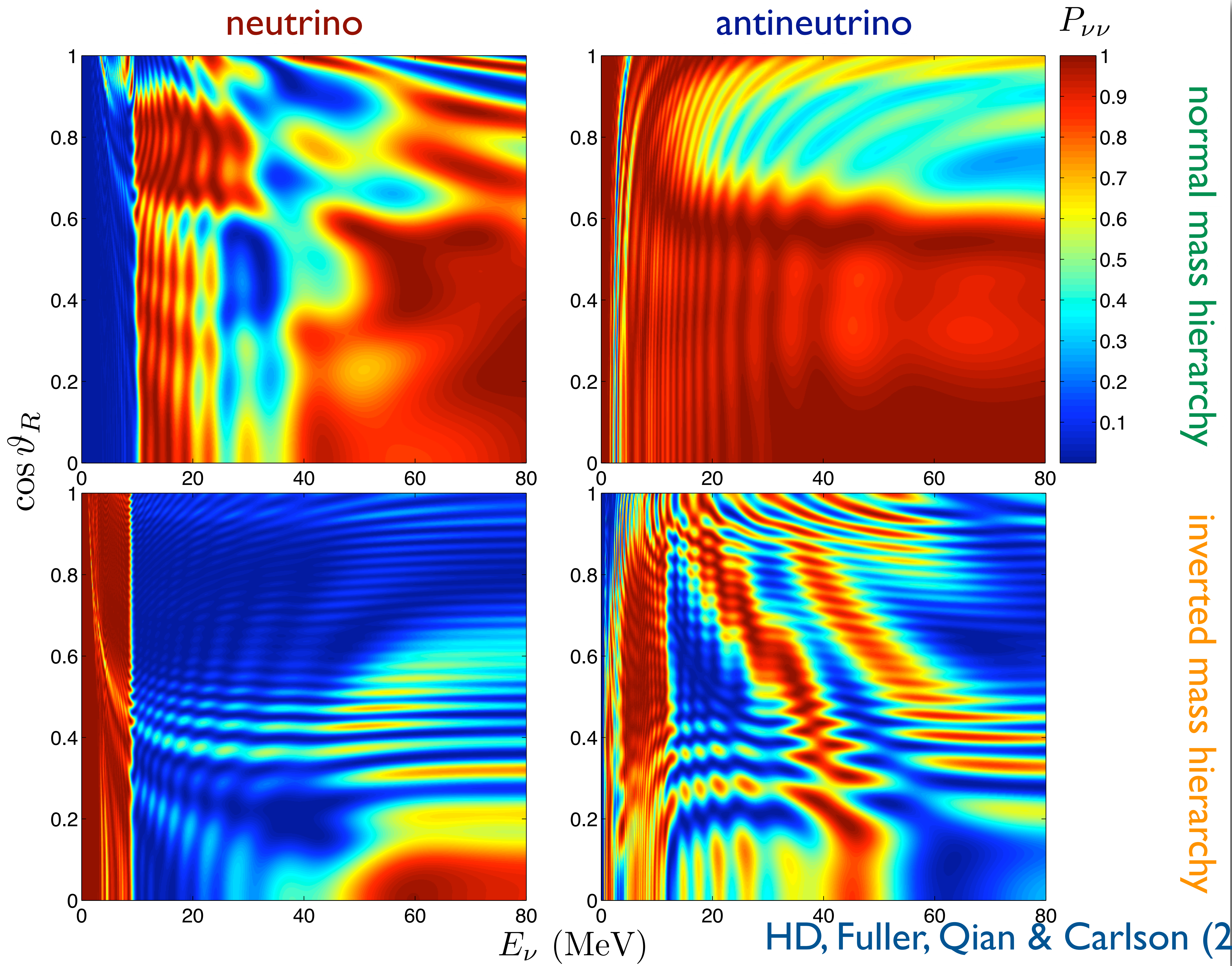
Bulb model

$$\delta m^2 = 3\times10^{-3}\,\mathrm{eV}^2 \simeq \delta m^2_{\rm atm},\ \theta_{\rm v} = 0.1,\ L_\nu = 0$$



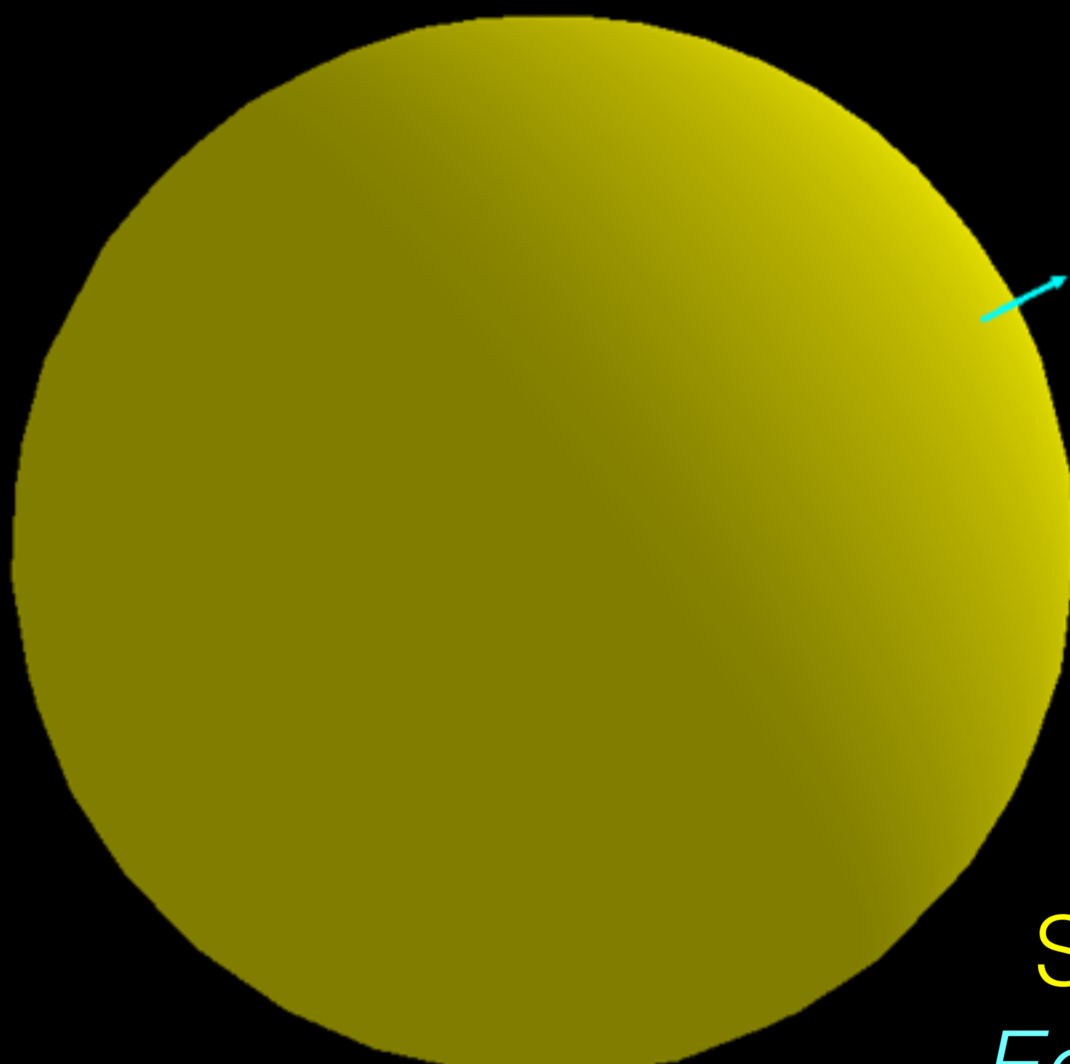
$$\delta m^2 = -3\times 10^{-3}\,\mathrm{eV}^2 \simeq \delta m^2_{\rm atm},\ \theta_{\rm v} = 0.1,\ L_\nu = 10^{51}\,\mathrm{erg/s}$$





HD, Fuller, Qian & Carlson (2006)

Numerical Models

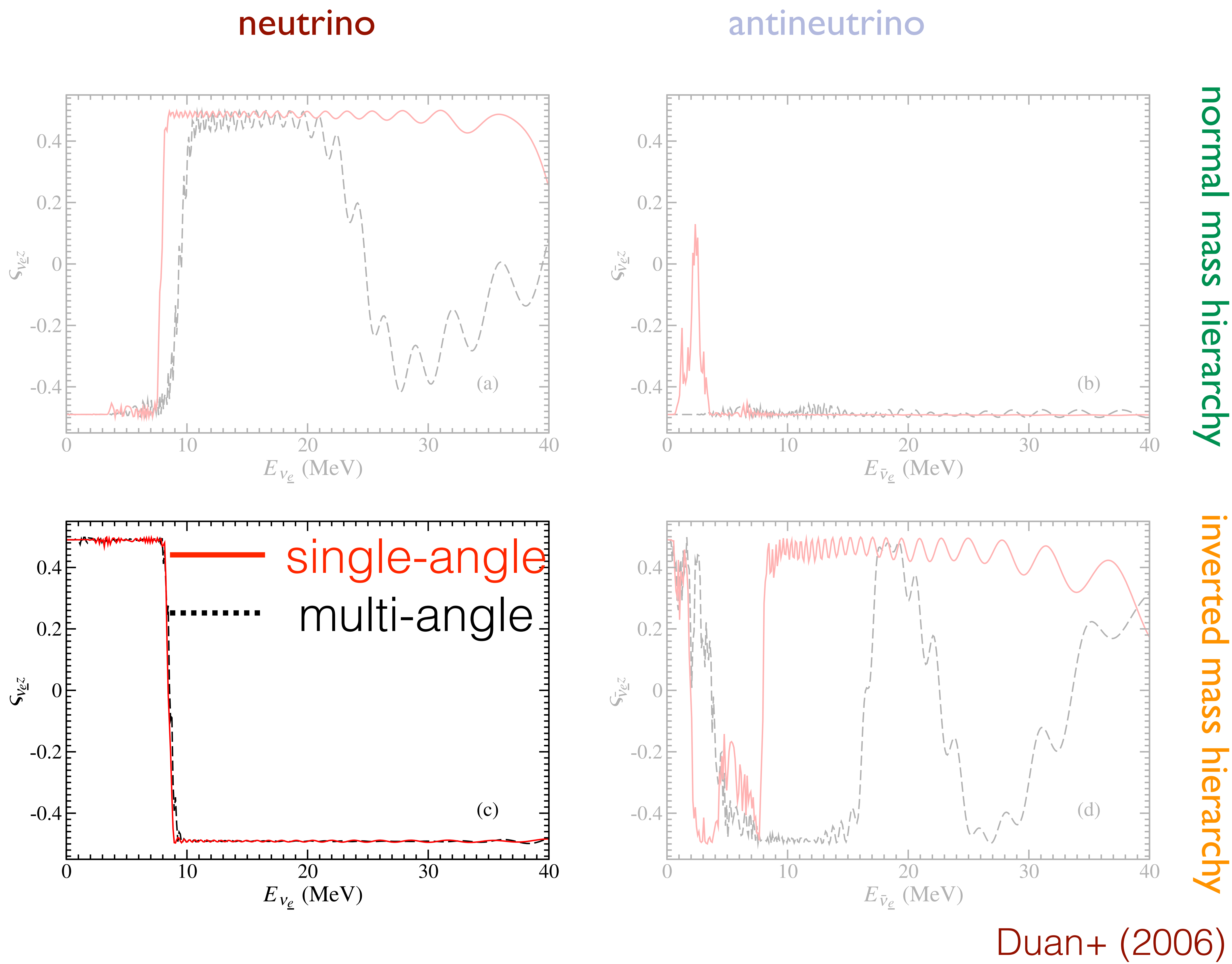


Trajectory independent neutrino flavor evolution

$$\rho(r; E)$$

Single-angle model

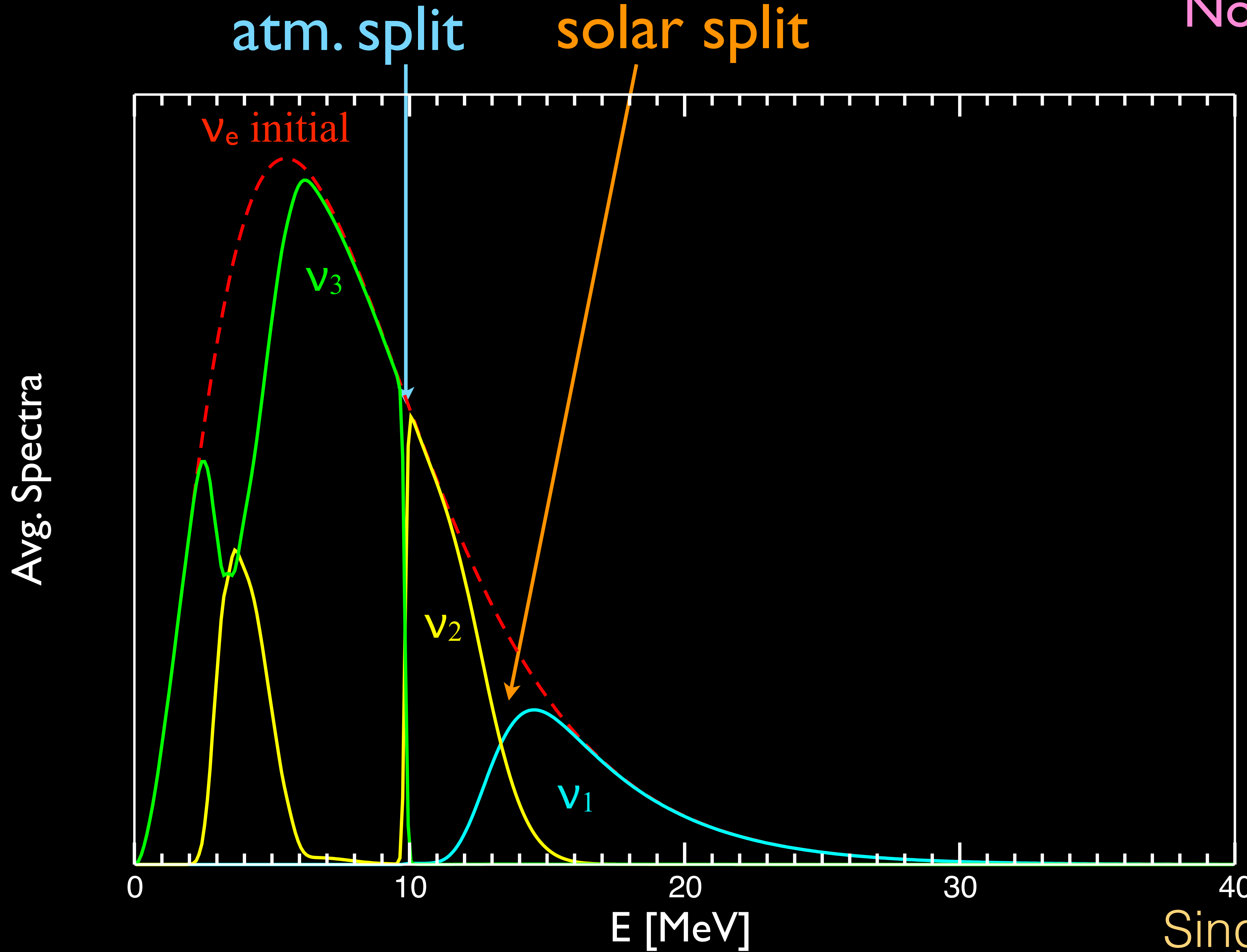
Equivalent to the expansion of a homogeneous, isotropic gas



neutrino
antineutrino
normal mass hierarchy
inverted mass hierarchy
single-angle
multi-angle
(a)
(b)
(c)
(d)
E_{ν_e} (MeV)
$E_{\bar{\nu}_e}$ (MeV)
$\varsigma_{\nu_e z}$
$\varsigma_{\bar{\nu}_e z}$
Duan+ (2006)

Neutronization Burst

Normal Mass Hierarchy



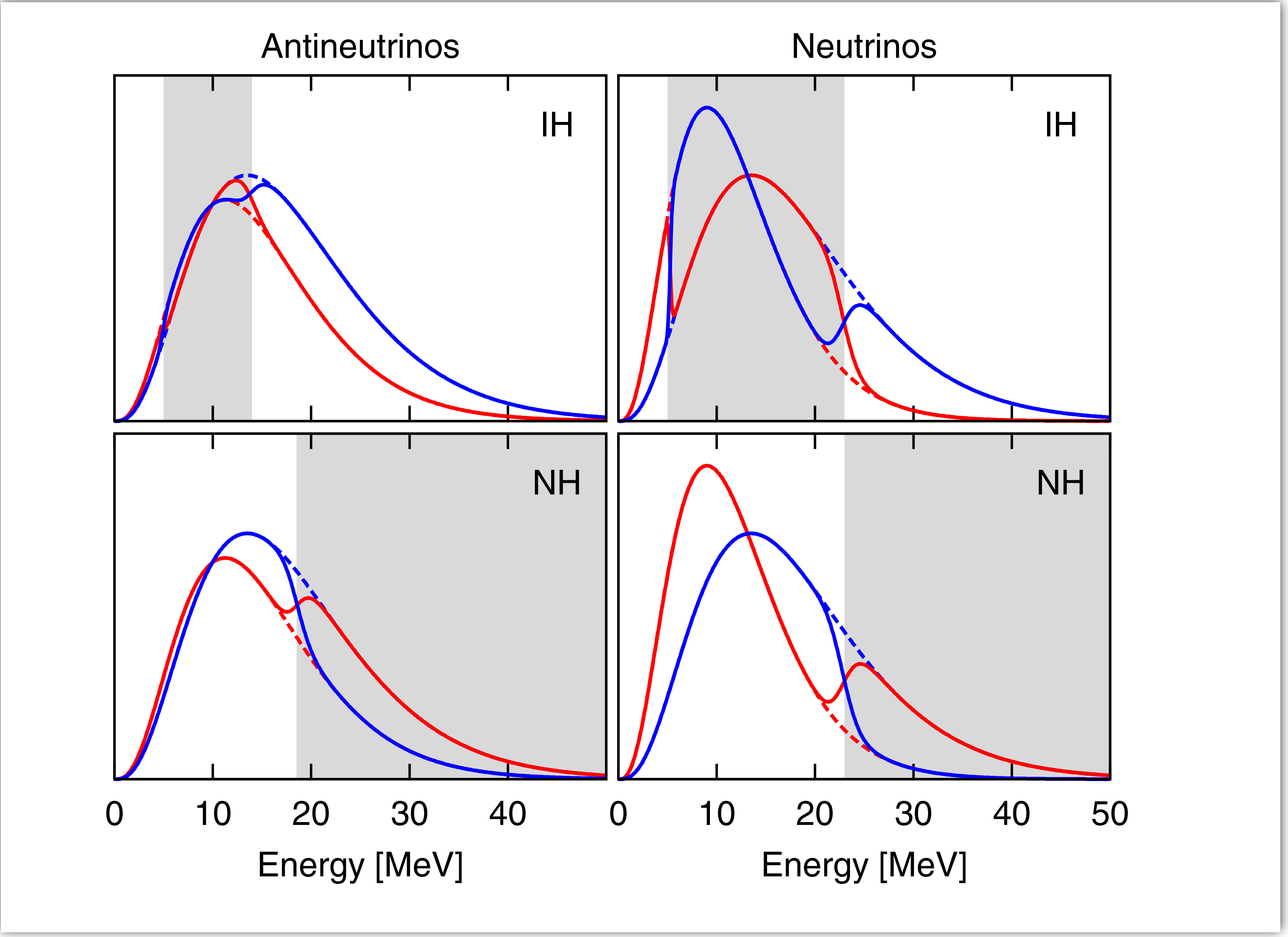
O-Ne-Mg Core-Collapse
Neutronization Pulse

$L_{\nu_e} = 10^{53}\,\mathrm{erg/s}$

$\langle E_{\nu_e} \rangle = 11\,\mathrm{MeV}$

r = 5000 km

Single Angle: Duan+ (2007)
Multi-Angle: Cherry+ (2010)



Dasgupta et al (2009)

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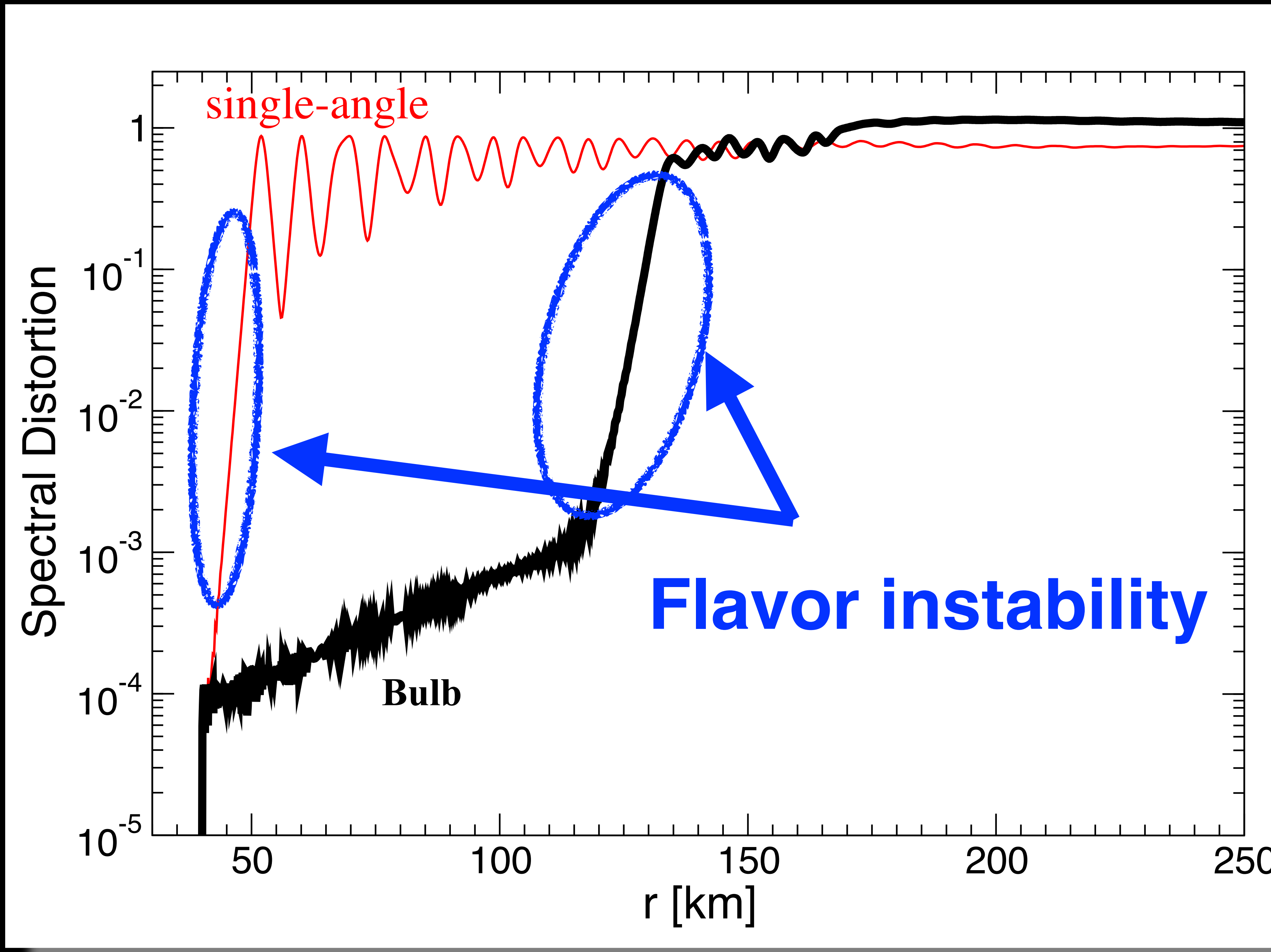
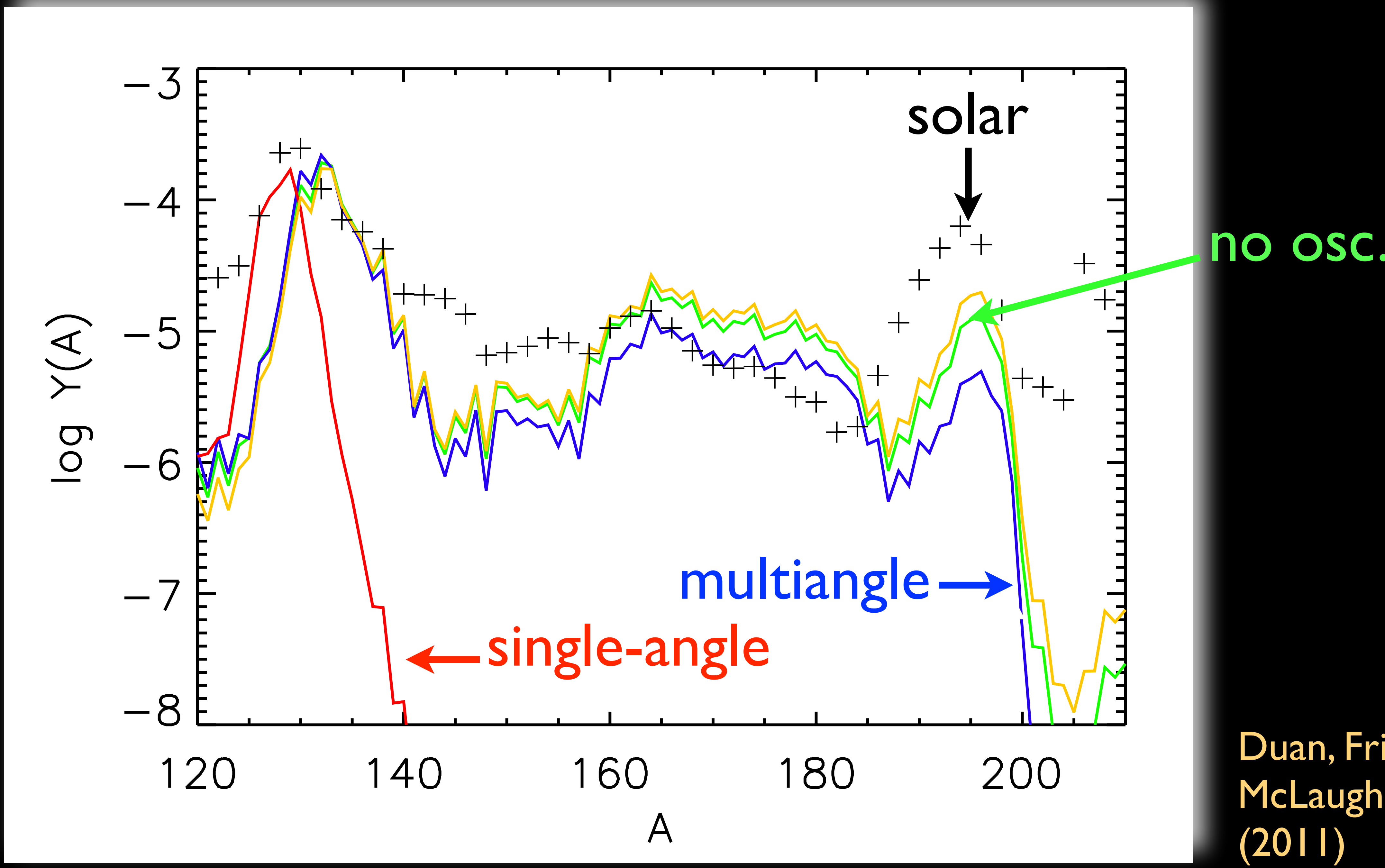
Dimension matters



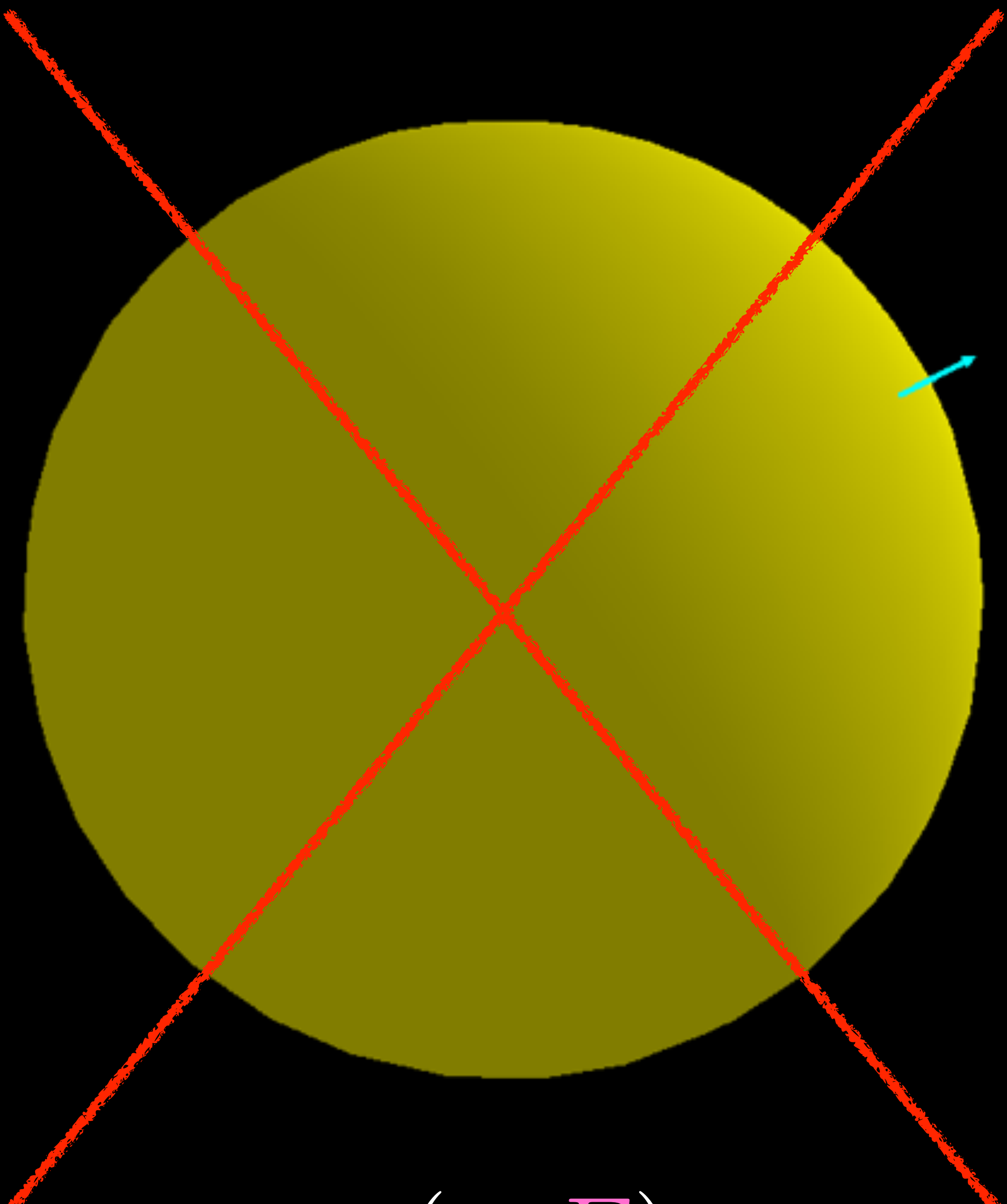
Duan & Friedland (2010)

Nucleosynthesis

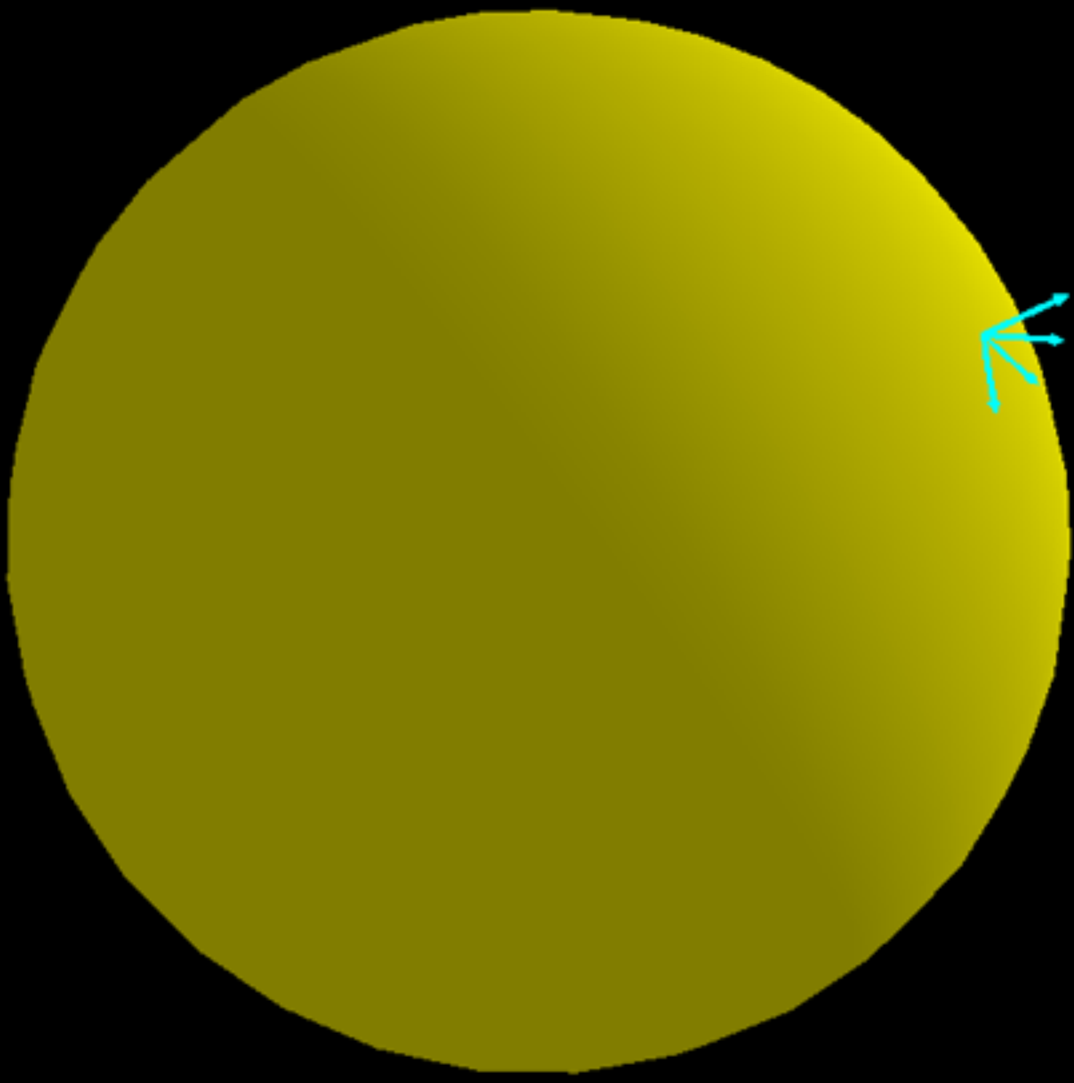


Duan, Friedland, McLaughlin & Surman (2011)

Trajectory Dependence



$\rho(r; E)$



$\rho(r; E, \vartheta)$

Directional Symmetry

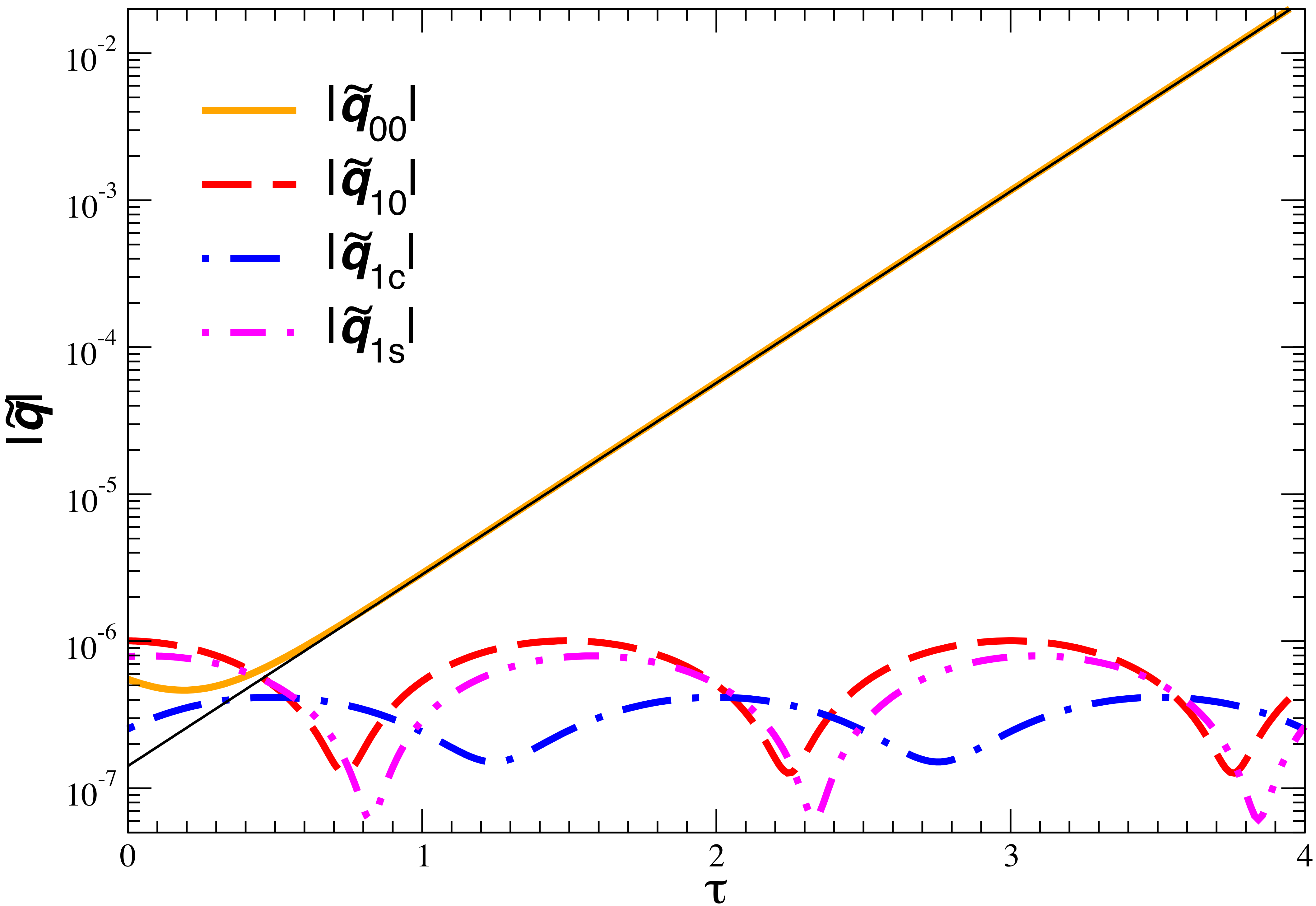
$$\mathsf{H}_{\nu\nu} = \sqrt{2}G_{\mathrm{F}} \int \mathrm{d}^3\mathbf{p}'(1 - \hat{\mathbf{v}} \cdot \hat{\mathbf{v}}')(\rho_{\mathbf{p}'} - \bar{\rho}_{\mathbf{p}'})$$

$$(1 - \hat{\mathbf{v}} \cdot \hat{\mathbf{v}}') = 4\pi \left[Y_{0,0}(\hat{\mathbf{v}}) Y^*_{0,0}(\hat{\mathbf{v}}') - \frac{1}{3} \sum_{m=0,\pm 1} Y_{1,m}(\hat{\mathbf{v}}) Y^*_{1,m}(\hat{\mathbf{v}}') \right]$$

- Monopole (l=0) and dipole (l=1) modes are unstable in opposite neutrino mass hierarchies.
- Unstable dipole (l=1) modes break the directional symmetry.

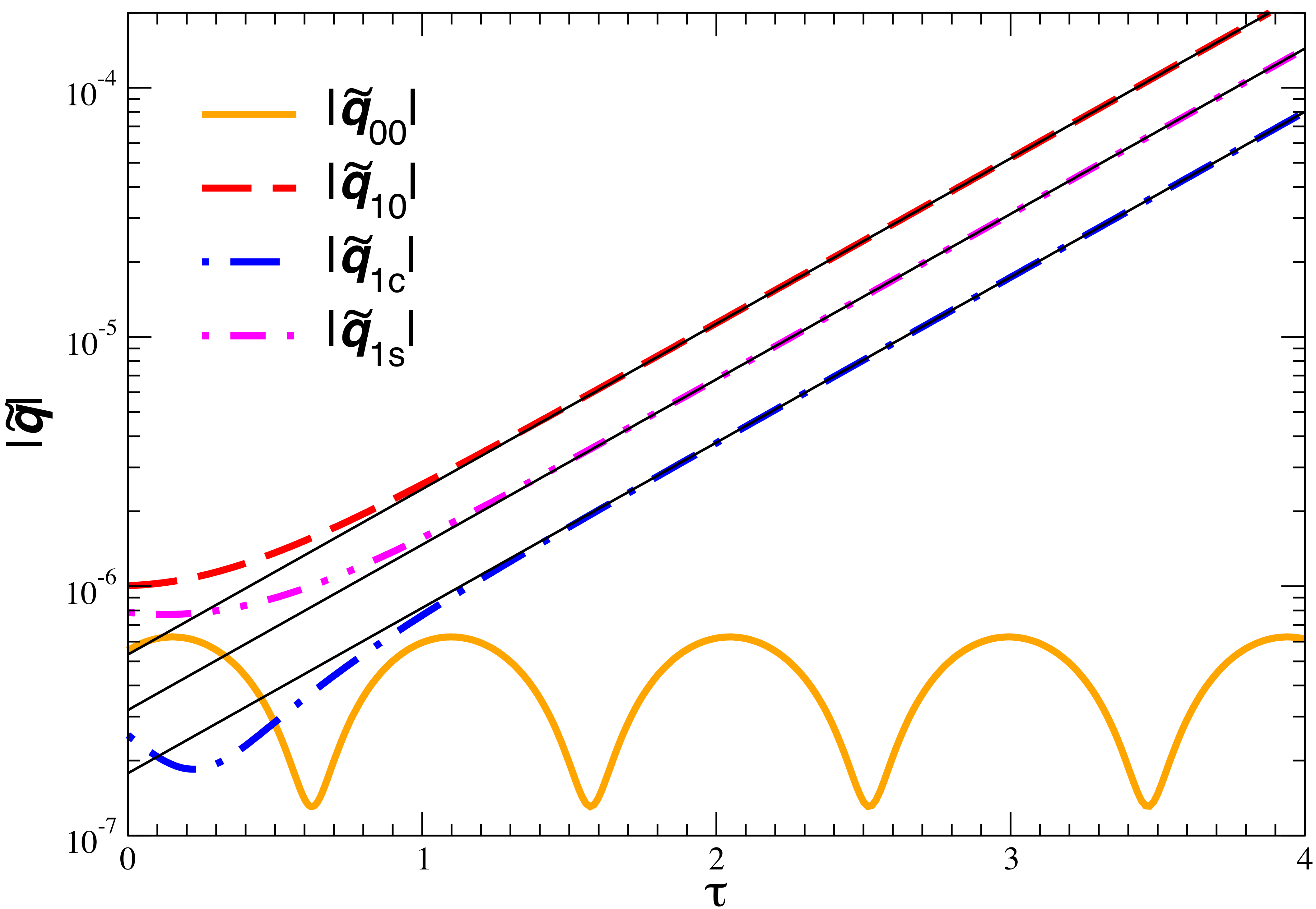
Duan (2013)

Inverted Hierarchy

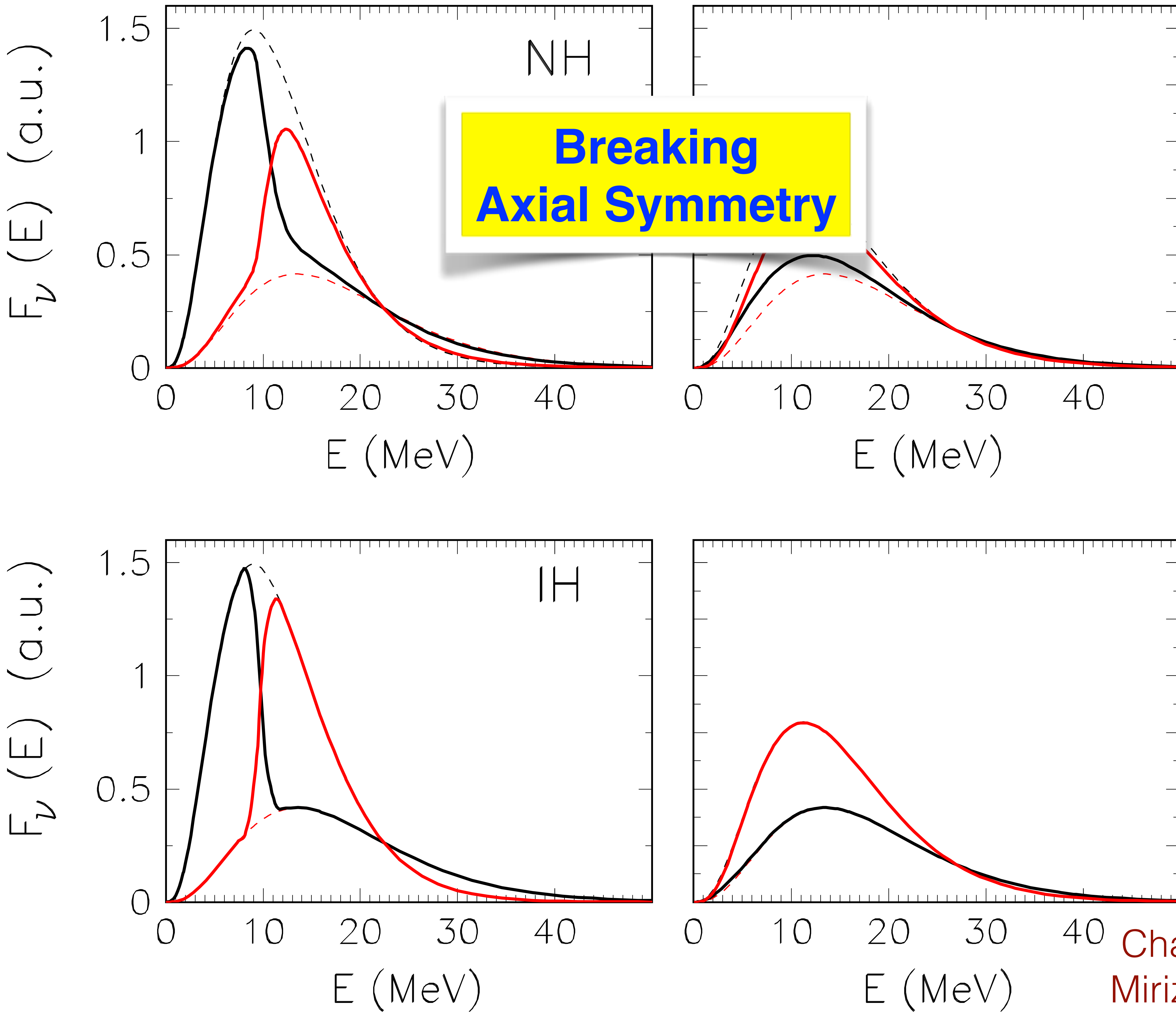


Duan (2013)

Normal Hierarchy



Duan (2013)



Chakraborty, Mirizzi (2013)

$\propto L/r^4$

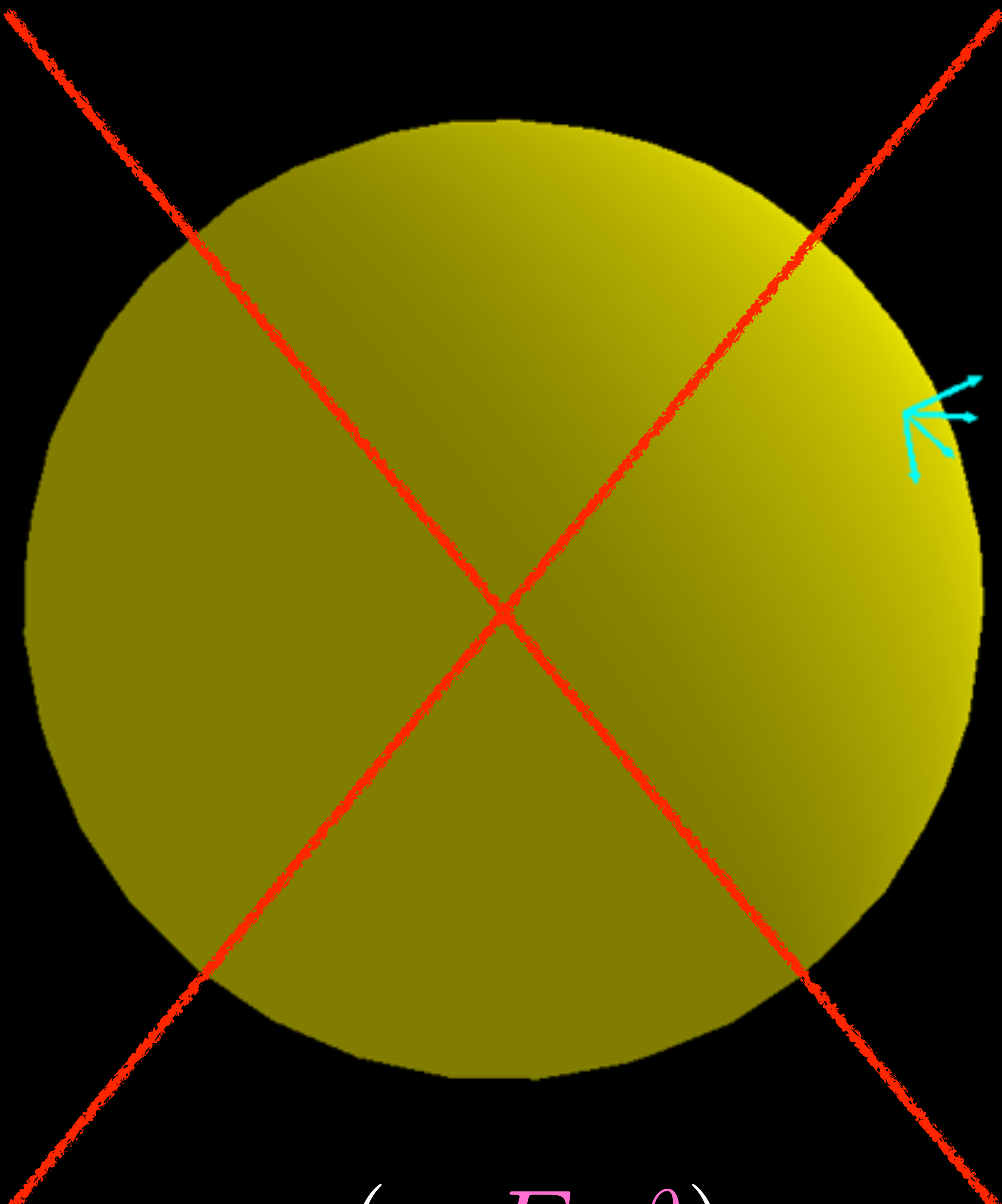
Matter Suppression

$\propto \rho/r^2$

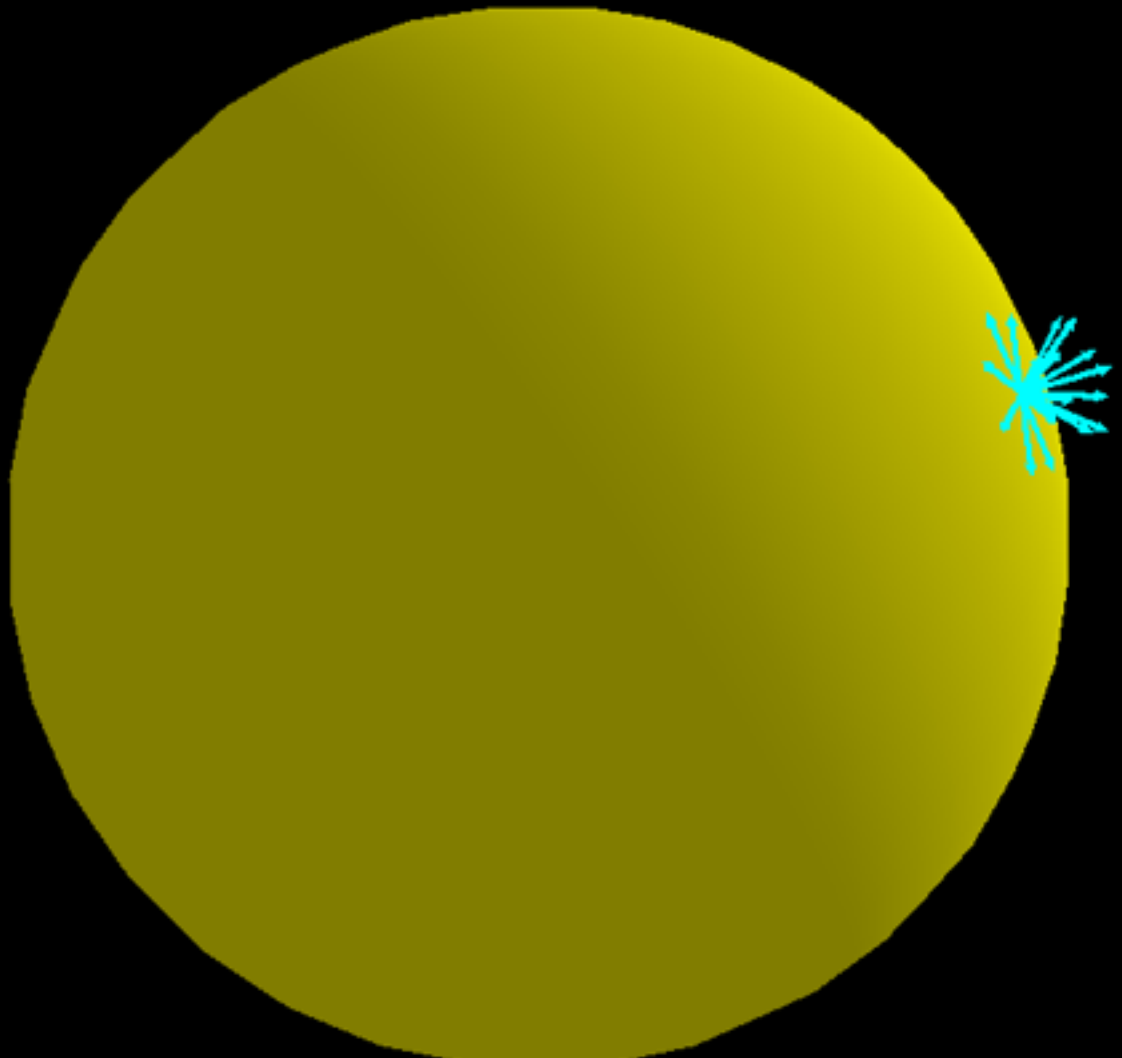
Self Suppression

Raffelt+ (2013)

Directional Symmetry



$\rho(r; E, \vartheta)$

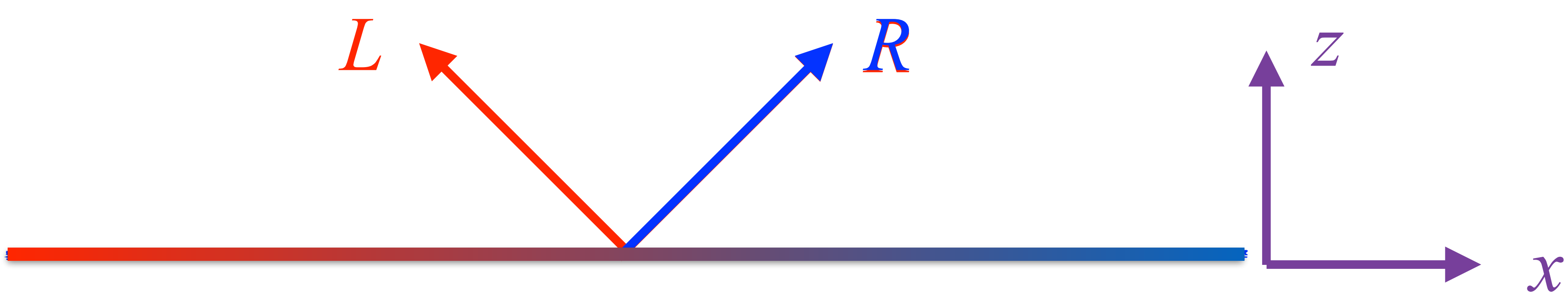


$\rho(r; E, \vartheta, \varphi)$

Line Model

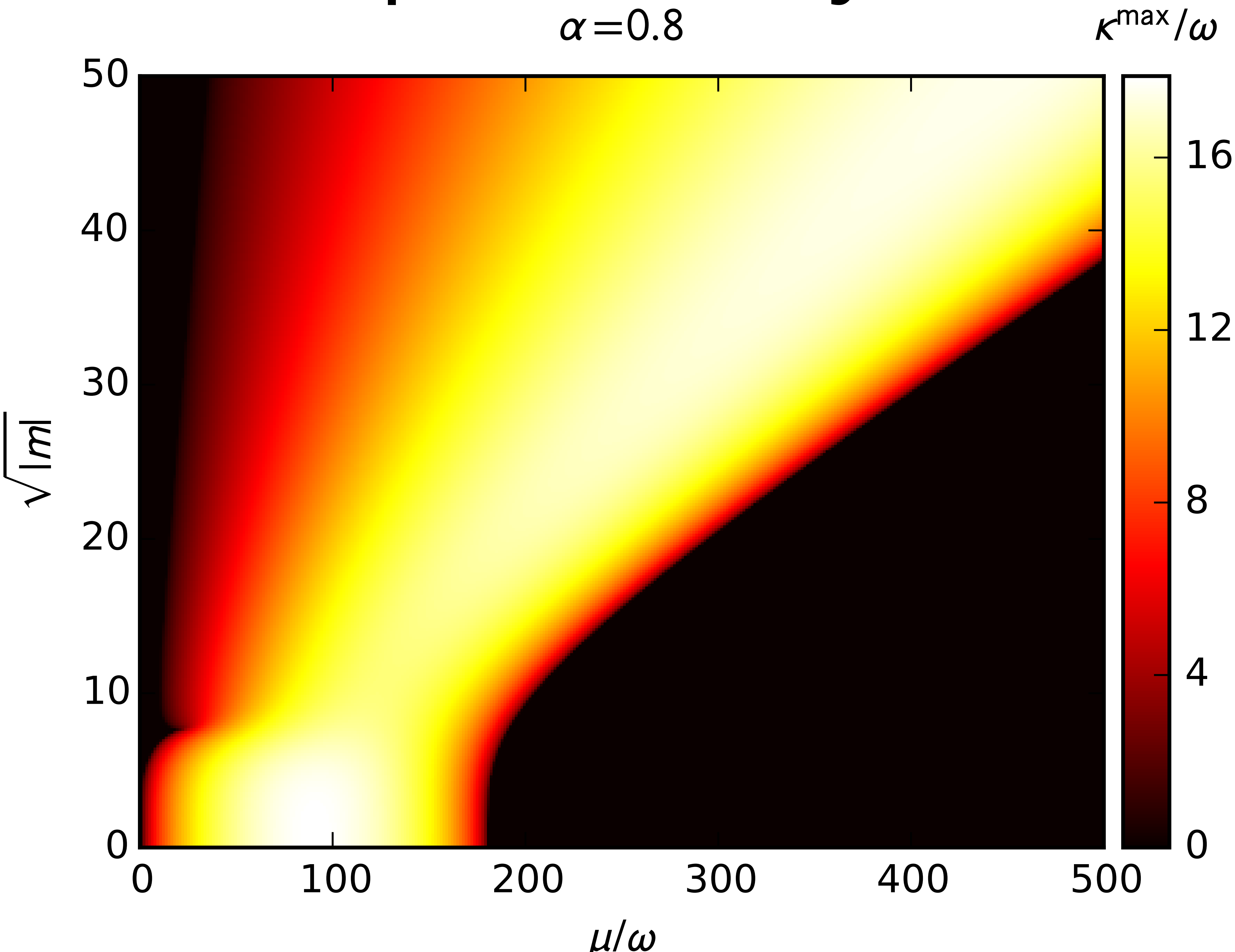
- ~~x translation symmetry~~
- ~~left-right symmetry~~

$$\rho_m(z) = \frac{1}{L}\int_0^L \rho(x,z)^{-2m\pi \mathrm{i} x/L}\,\mathrm{d}x$$



Duan & Shalgar (2015)

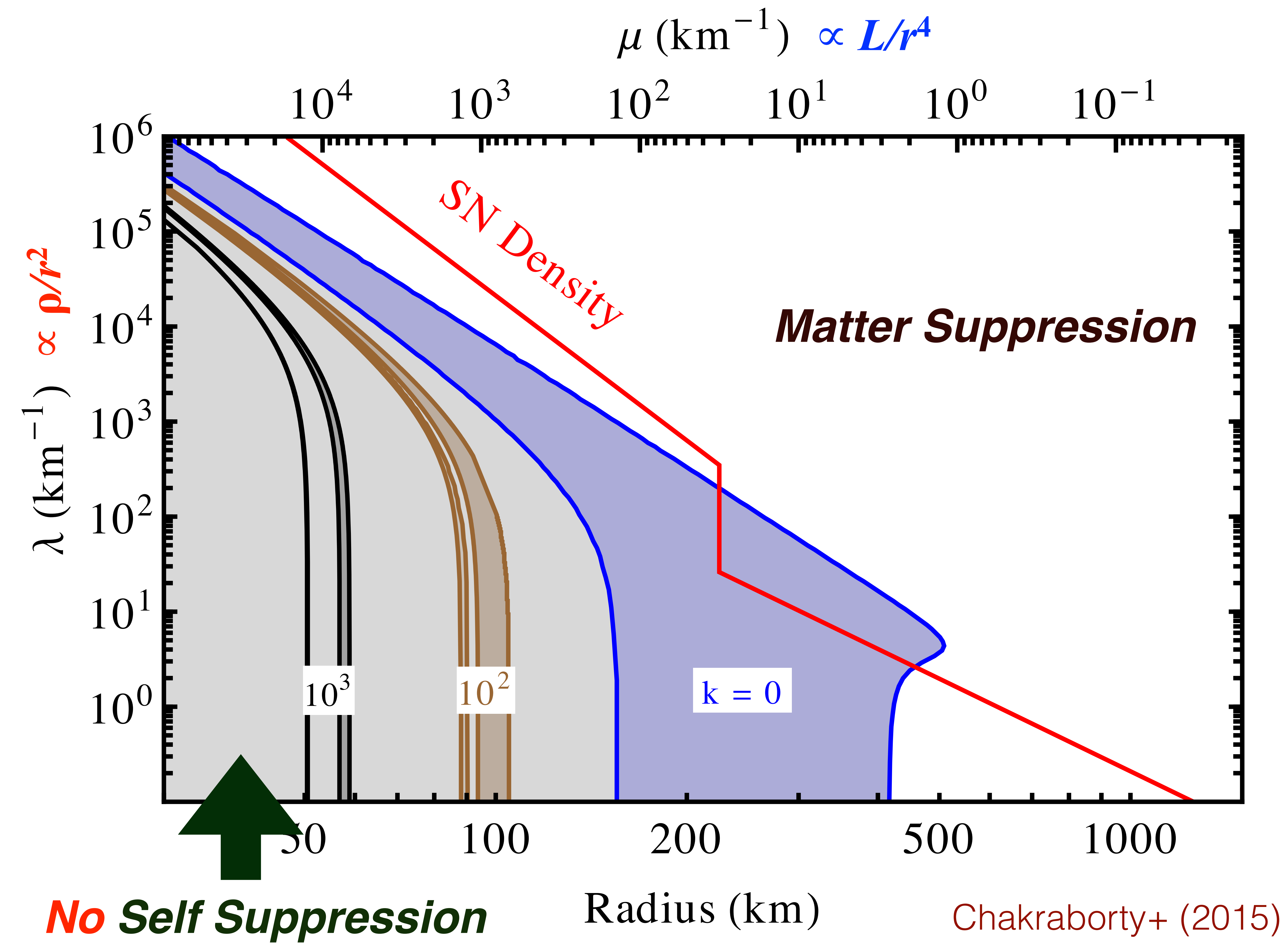
Spatial Symmetry



$\alpha = n_{\bar{\nu}}/n_{\nu}$

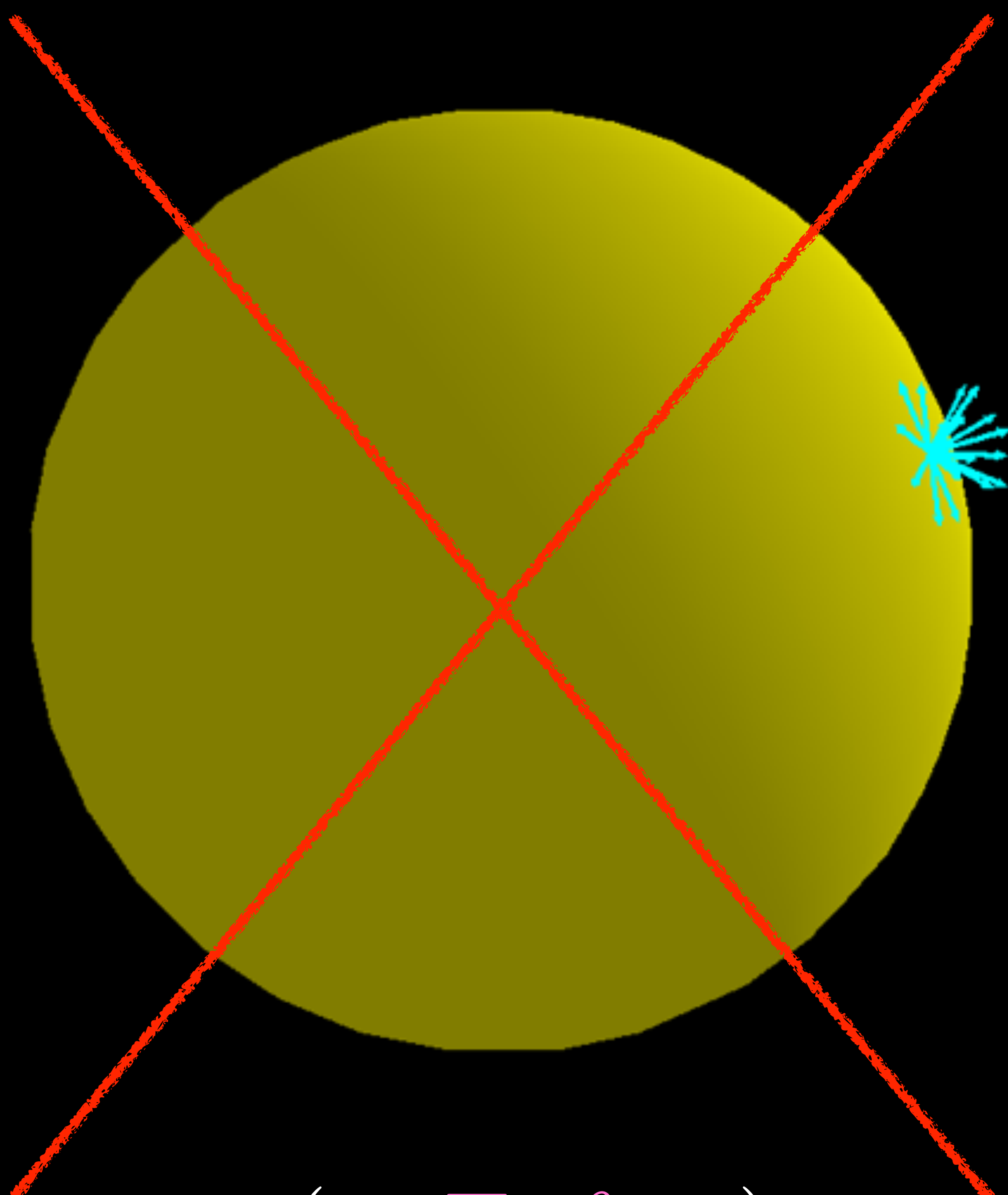
$\mu \propto G_{\mathrm{F}} n_{\nu}$

Duan & Shalgar (2015)

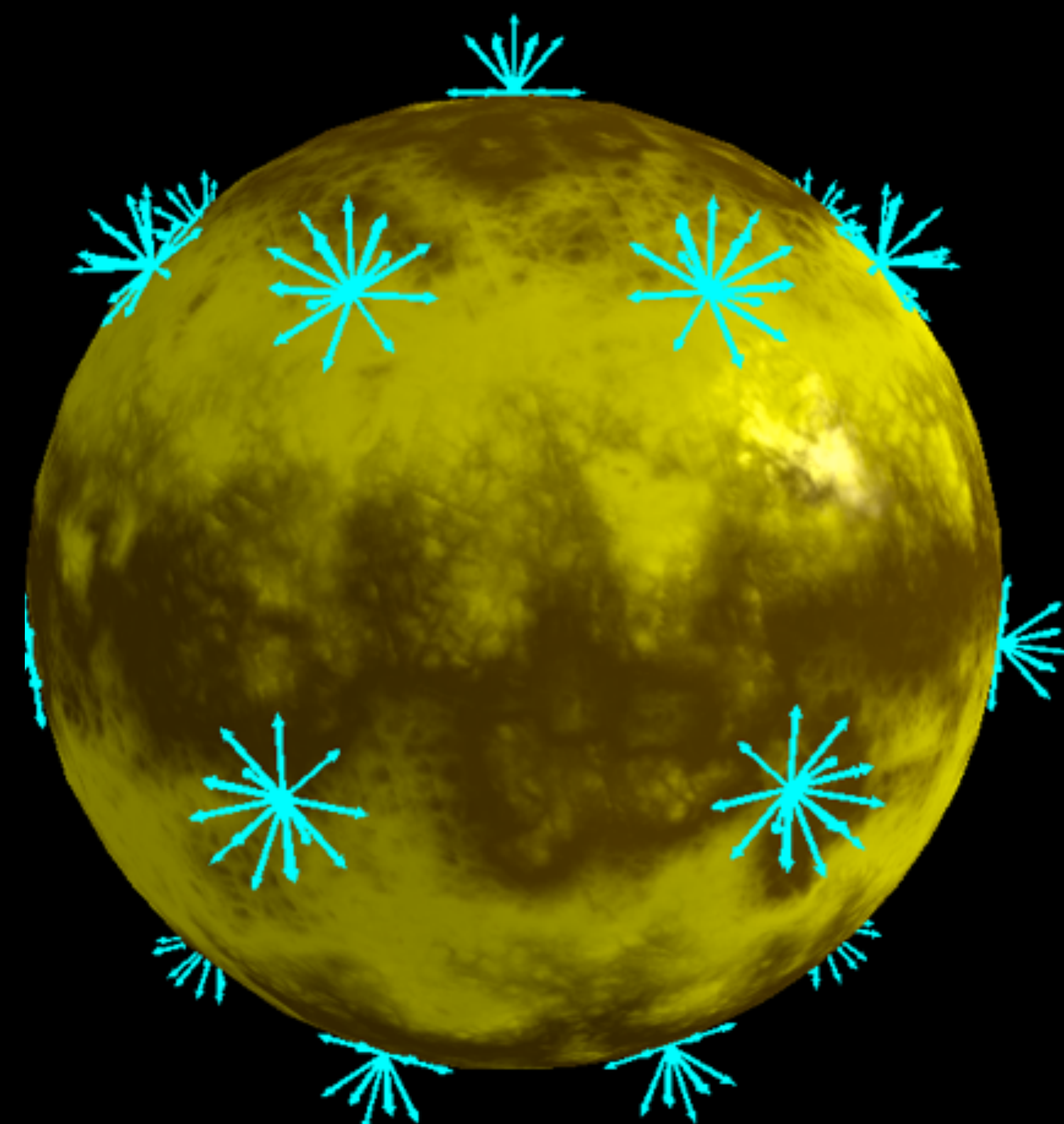


μ (km⁻¹) ∝ L/r⁴
10⁴
10³
10²
10¹
10⁰
10⁻¹
10⁶
10⁵
10⁴
10³
10²
10¹
10⁰
λ (km⁻¹) ∝ ρ/r²
SN Density
Matter Suppression
10³
10²
k = 0
50
100
200
500
1000
Radius (km)
No Self Suppression
Chakraborty+ (2015)

Spatial Symmetry

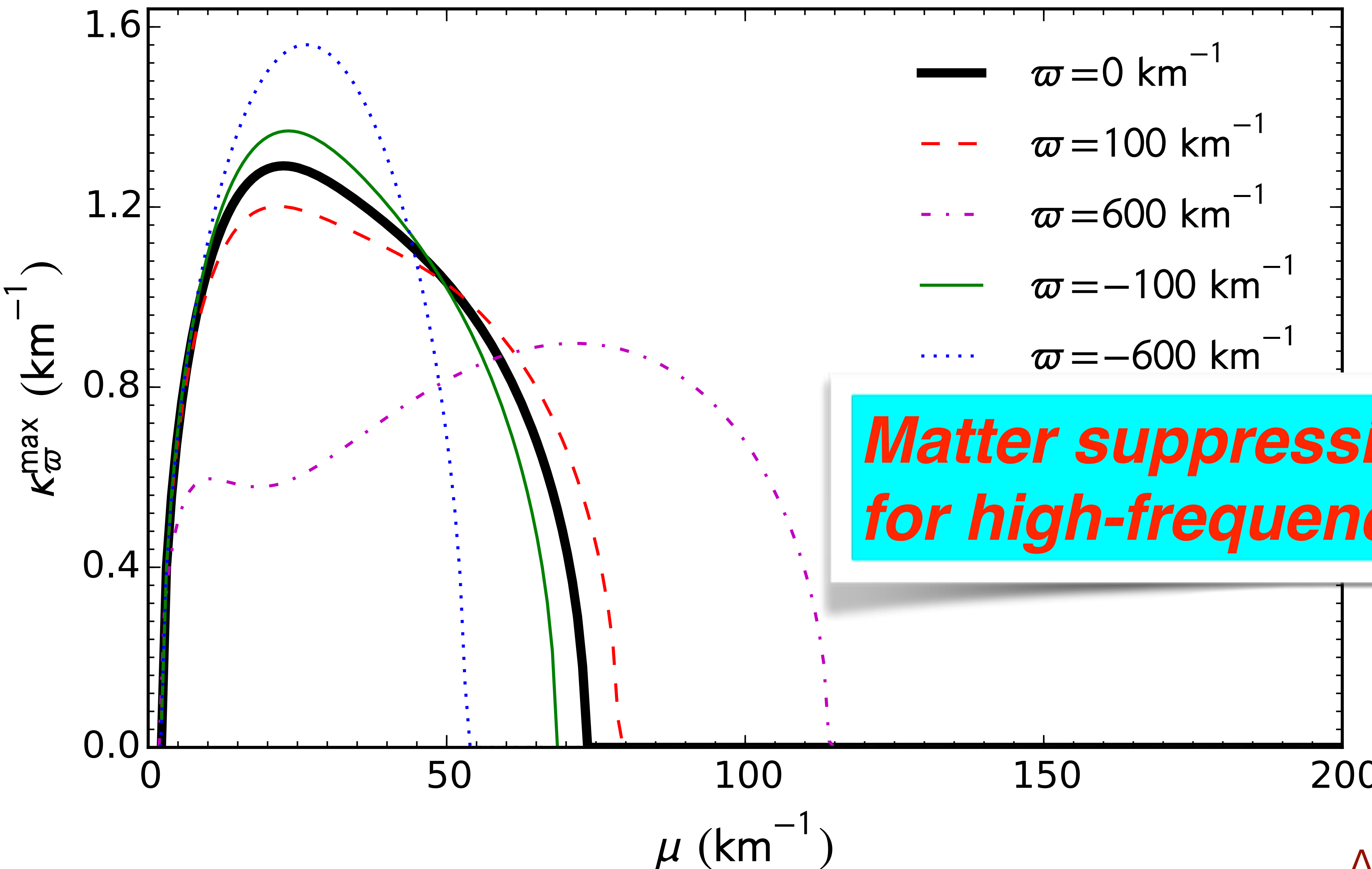


$\rho(r; E, \vartheta, \varphi)$



$\rho(r, \Theta, \Phi; E, \vartheta, \varphi)$

Temporal Symmetry

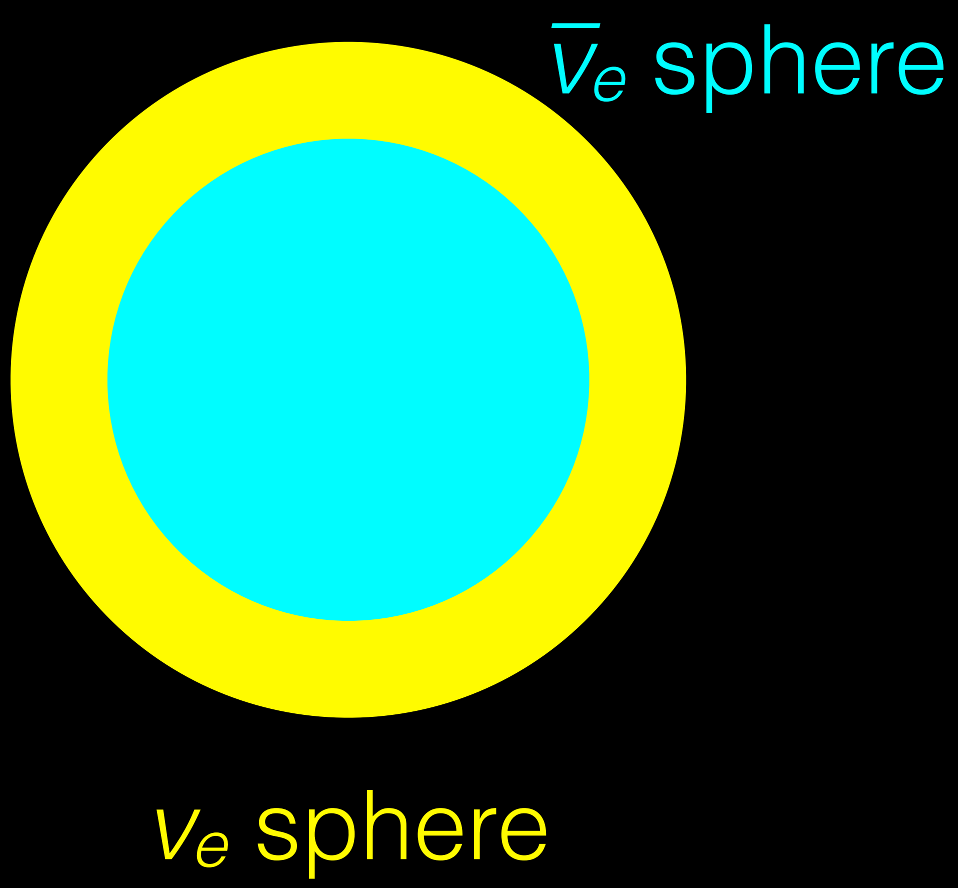


Matter suppression is relieved for high-frequency modes

Abbar & Duan (2015)
Dasgupta & Mirizzi (2015)

Fast Neutrino Oscillations

- Usually flavor instabilities grow at rates comparable to vacuum oscillation frequency.
- Fast oscillations grow at rates comparable to ($G_F\, n_\nu$).
- Fast oscillations can occur because of different angular distributions of ν_e and anti-ν_e.
- Can fast oscillations occur within the proto-neutron star?



Sawyer (2015)
Chakraborty+ (2016)

Summary

- Neutrinos are important in SNe (dynamics, nucleosynthesis, new probe).
- Neutrino oscillations are also important because they change fluxes in different flavors.
- The dense neutrino medium surrounding the nascent neutron star can oscillate collectively (**Lecture 1**).
- Neutrino oscillations can be qualitatively different in different models.

Summary

- Assumptions of the bulb model:
  - Axial symmetry (in momentum space).
  - Spherical symmetry (in real space).
  - Stationary assumption (time translation symmetry).
  - Same neutrino sphere (or angular distribution) for all flavors.

Summary

- Recent progress (**Lecture 2**):
  - ~~Axial symmetry (in momentum space)~~ -> oscillations in both neutrino mass hierarchies.
  - ~~Spherical symmetry (in real space)~~ -> relief in self-suppression.
  - ~~Stationary assumption (time translation symmetry)~~ -> relief in matter suppression.
  - ~~Same neutrino sphere (or angular distribution) for all flavors~~ -> fast oscillations.