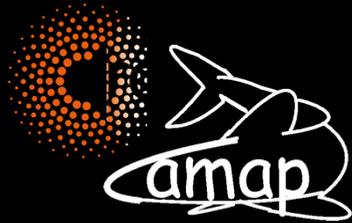


# Neutron star magnetospheres

**Pablo Cerdá-Durán**  
**University of Valencia**



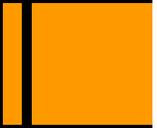
Collaborators:

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T. Akgün, J. Pons, J.A. Miralles (U. Alicante)  
M. Gabler, E. Müller (MPA)  
N. Stergioulas (U. Thessaloniki)



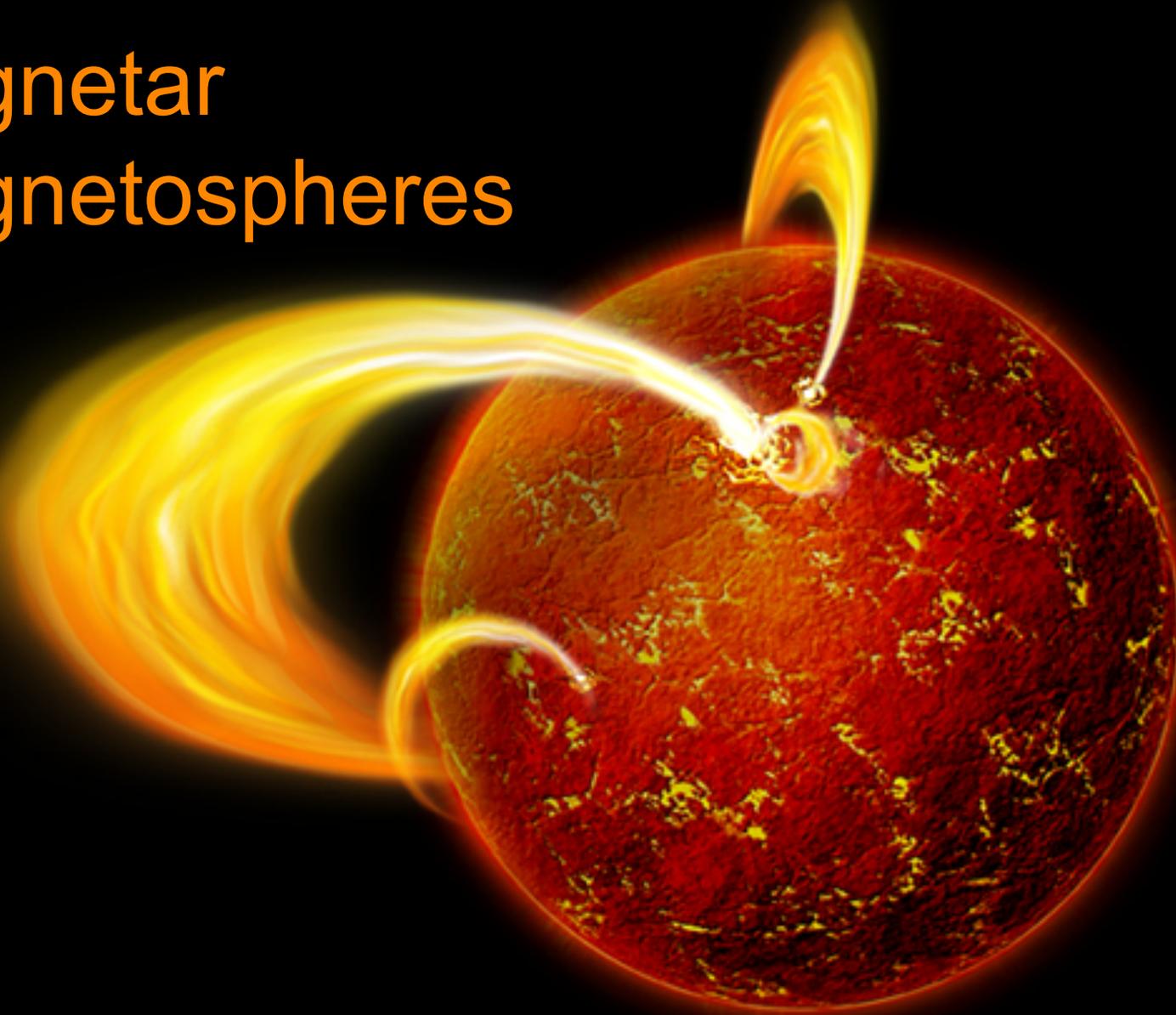
Kyoto, 16 November 2016

# Outline

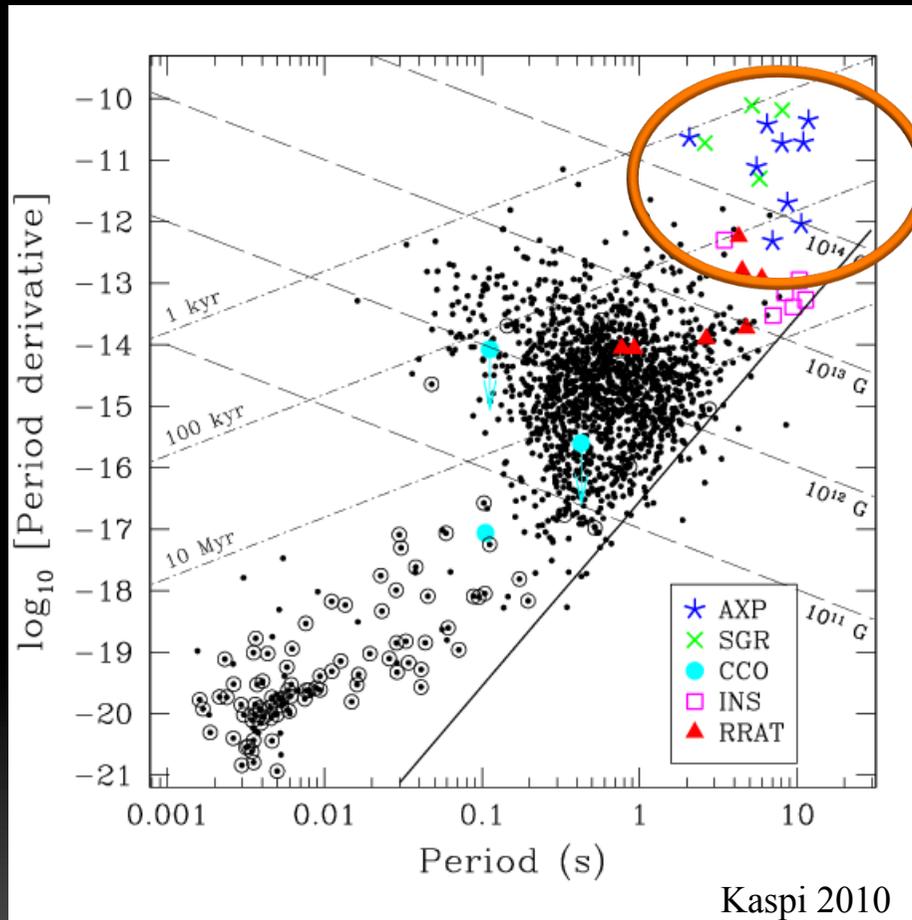


- Magnetar magnetospheres
  - Observations and models
  - Force-free twisted magnetospheres
  - Magnetosphere dynamics
- Supernova fallback and magnetic field burial

# Magnetar magnetospheres

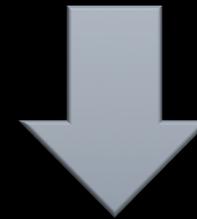


# What are magnetars?



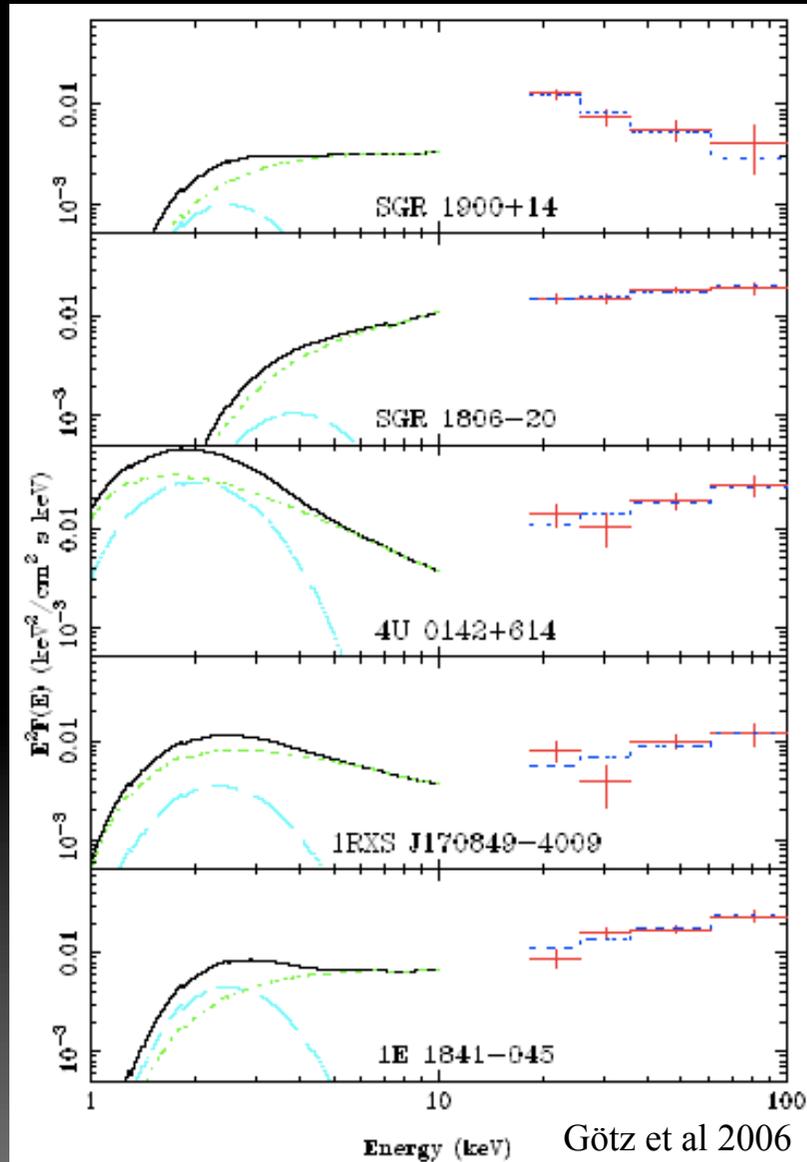
X-ray pulsars (no radio emission):

- Long rotation period: 2-12 s
- Rapid spin-down ( $10^3$ - $10^5$  y)



- Large inferred magnetic field  $10^{14}$ - $10^{15}$  G

# Quiescence spectrum



- X-ray luminosity  $\sim 10^{34}$ - $10^{36}$  erg/s
- Thermal black body ( $\sim 0.5$  keV)
- Soft X-ray tail (2-10 keV)
- Hard X-ray component (15-100 keV)



- Magnetically powered emission
- Part of the emission comes from the magnetosphere

# Magnetar magnetosphere

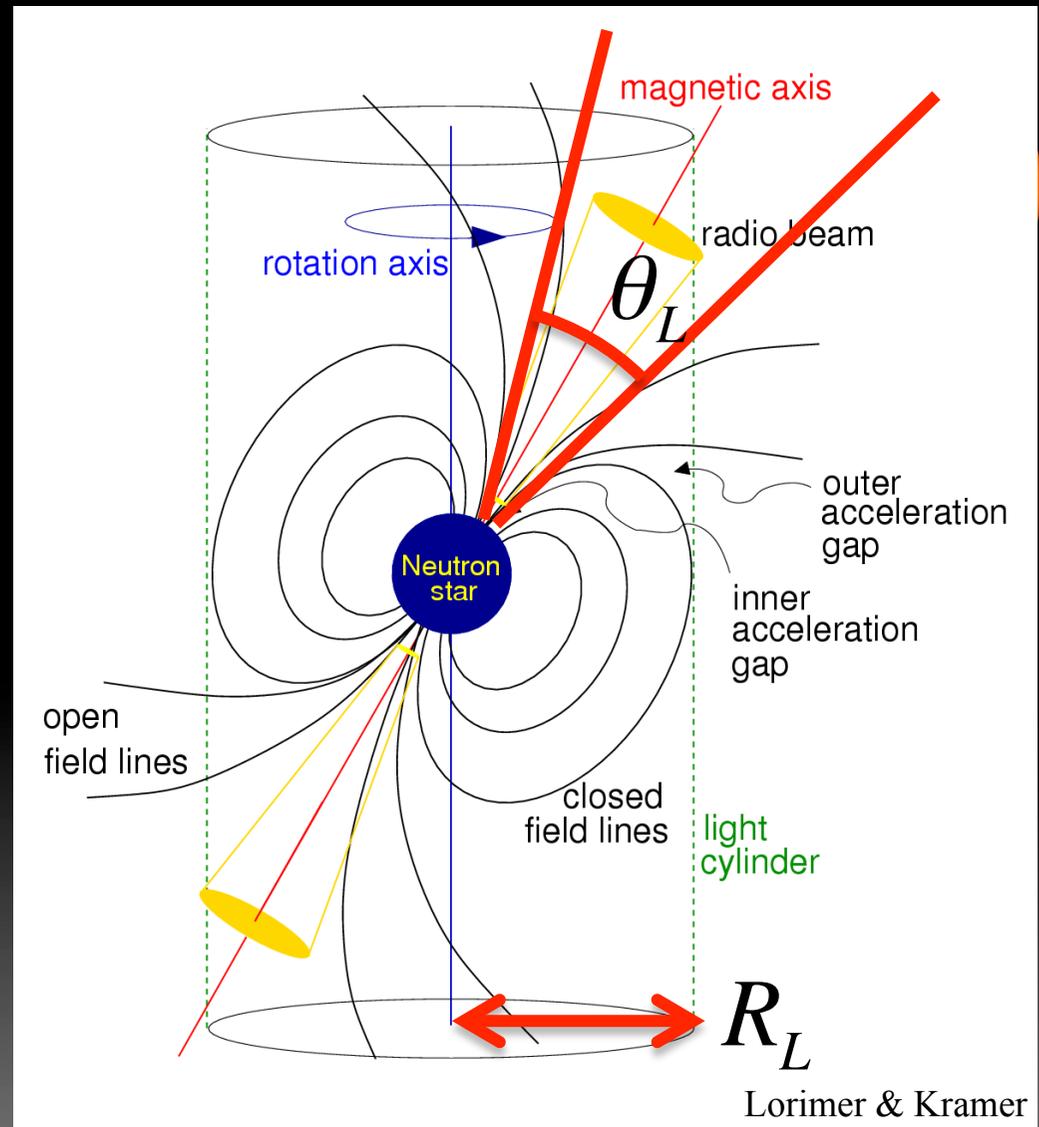
- Light cylinder:

$$R_L \approx 10^5 \left( \frac{P}{2s} \right) km$$

- Outside the light cylinder:
  - “open” field lines
  - Small bundle at the polar cap

$$\theta_L \approx \sqrt{R_* / R_L} \approx 0.5^\circ$$

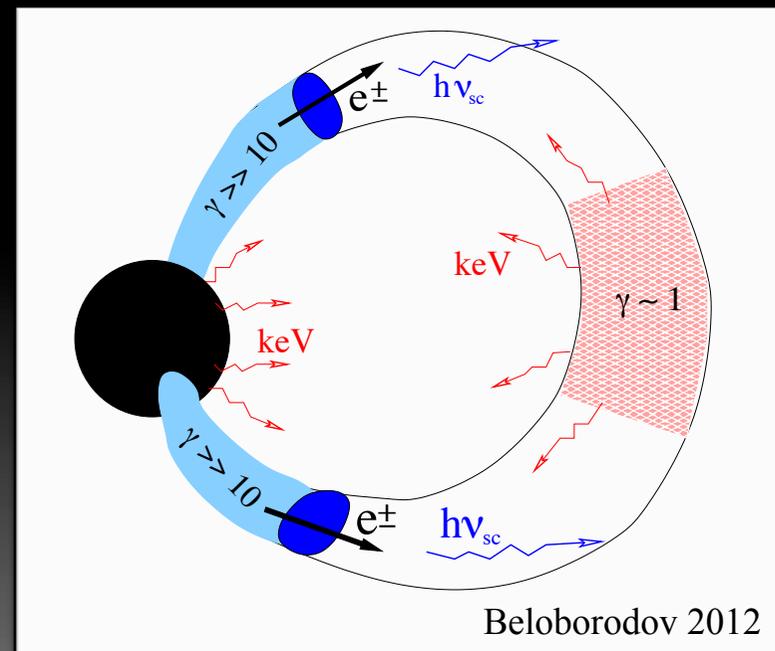
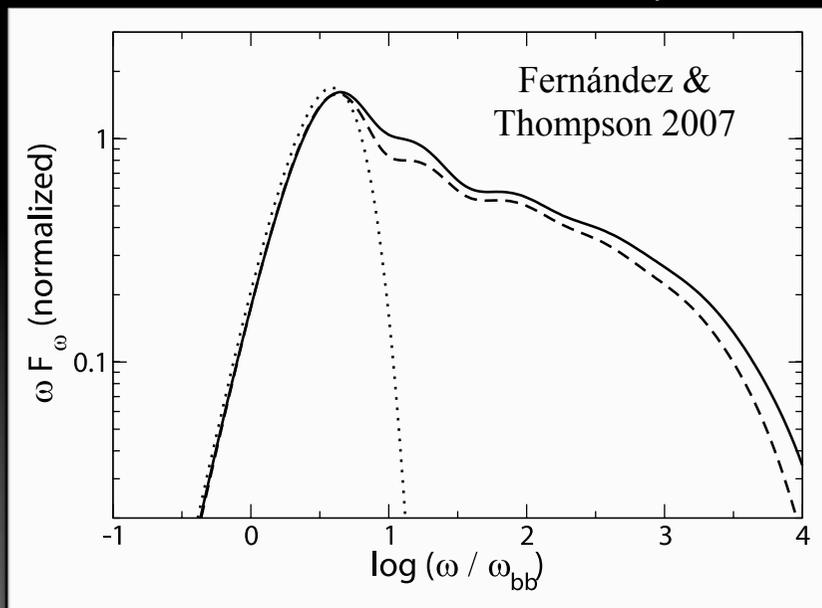
- Inside the light cylinder:
  - Closed field lines
  - Force-free magnetosphere  
Currents sustained by  $e^-e^+$  pair creation (Beloborodov & Thompson 2007)



# Emission mechanism

## Resonant cyclotron scattering (RCS) model

- Black body photons from the surface of the star
- Photons upscattered by currents in the magnetosphere (Lyutikov & Gavriil 2006, Fernández & Thompson 2007, Nobili et al 2008, Taverna et al 2015) → soft X-ray tail
- Accelerated pairs produce hard X-ray at  $\sim 100$  km (Thompson & Beloborodov 2005, Beloborodov & Thompson 2007, Beloborodov 2013, Chen & Beloborodov et al 2016 )



# Origin of magnetospheric currents

$$f = \rho_q E + J \times B \quad \text{Lorentz force density}$$

$$\text{Stationary solution: } 4\pi J = \nabla \times B \quad (\text{Ampère's law})$$

$$\text{Force-free: } f = 0$$

$$\text{Charge neutrality: } \rho_q = 0$$



$$J \times B = 0 \quad : \text{ Currents flow along field lines}$$

# Origin of magnetospheric currents

Axisymmetric force-free solutions

$$B = \nabla P \times e_\phi + T e_\phi \quad \left\{ \begin{array}{l} P : \text{poloidal function} \\ T : \text{toroidal function} \end{array} \right.$$

$$\nabla P \times \nabla T = 0 \quad \rightarrow \quad T(P)$$

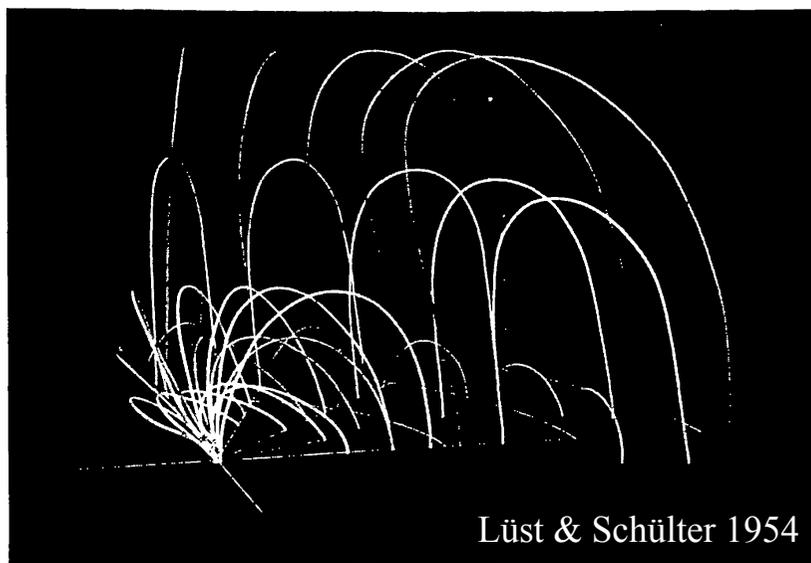
$$\Delta_{GS} P = -T(P)T'(P) \quad : \text{Grad-Shafranov equation}$$

$$4\pi J = T'(P)B \quad : \text{Twist} \quad \leftrightarrow \quad \text{magnetospheric currents}$$

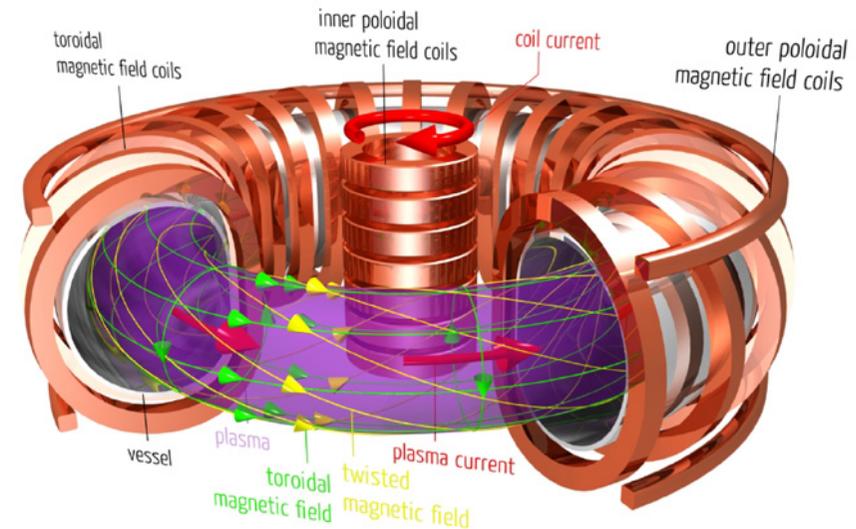
# Grad-Shafranov equation

- Lüst & Schülter 1954 (astrophysical context)
- Grad & Rubin 1958; Shafranov 1966 (plasma confinement)

Early force-free solution  
Of a twisted dipolar field

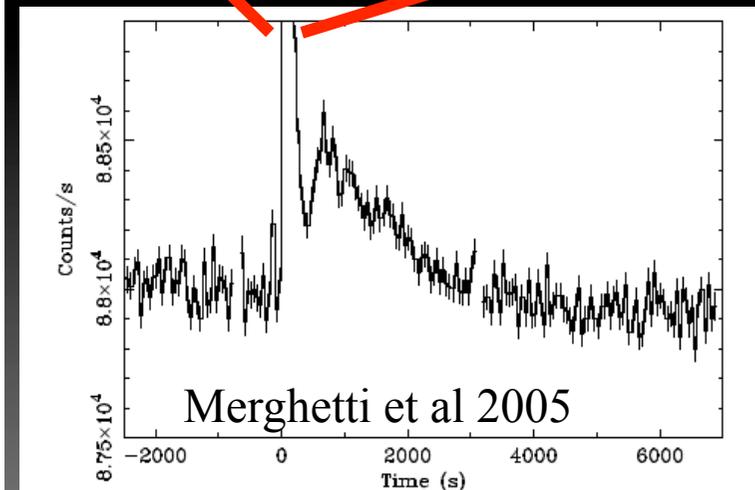
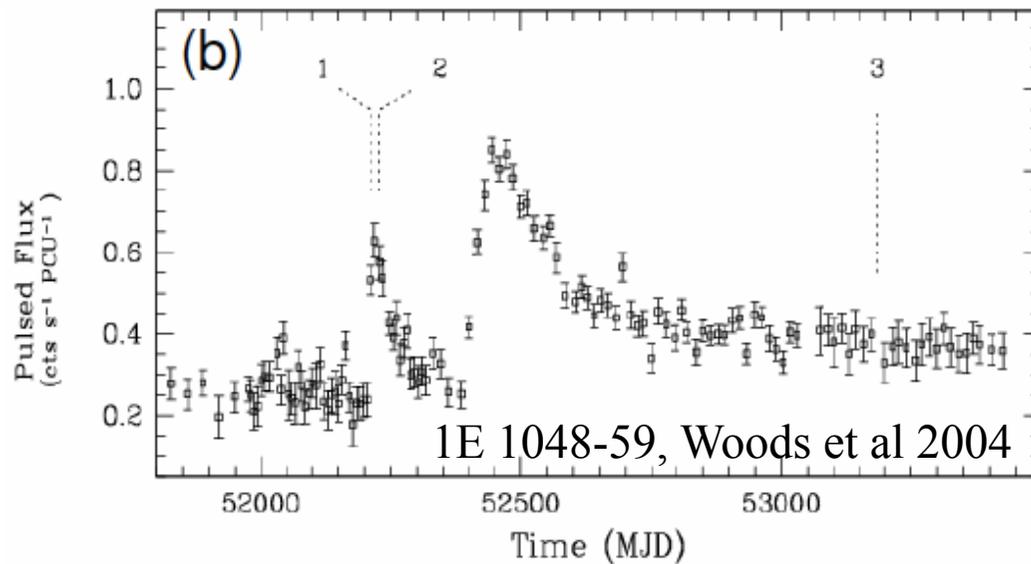
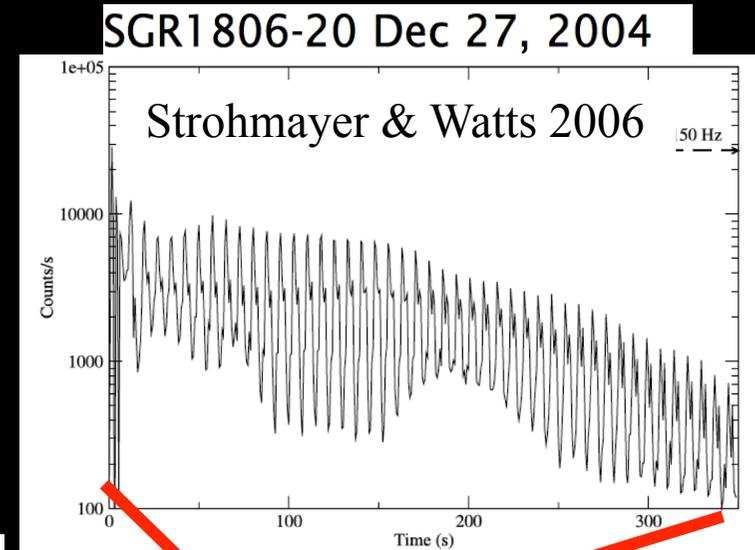


Tokamak fusion reactor



# Variability

- Repeated burst activity:  $10^{42}$  erg/s in 0.01-1 s
- Giant flares (3):
  - Initial spike:  $10^{44} - 10^{47}$  erg/s in 0.25-0.5 s
  - Pulsating tail:  $10^{44}$  erg/s in 200-400 s
- Long term variability: hours to years



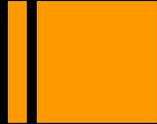
# Untwisting magnetospheres

- Twisted magnetospheres are not static → energy loses by radiation
- Magnetospheres untwist in secular time-scales (Beloborodov 2009, 2012, Chen & Beloborodov 2016)
- Pair plasma flowing along twisted field lines → hot spot at the surface
- Twisted field at  $\sim 100\text{km}$  → magnetar corona → hard X-ray component

## Model ingredients:

- Thermal emission from the surface
- Current distribution at the magnetosphere
  - **Force-free magnetic field configuration**
  - $e^-$  and  $e^+$  momentum/spatial distribution: multiplicity?
- Back-reaction:
  - Photon flux  $\leftrightarrow e^-$  and  $e^+$  distribution
  - Hot spots

# Burst models



Magneto-thermal evolution of the crust (Perna & Pons 2011)

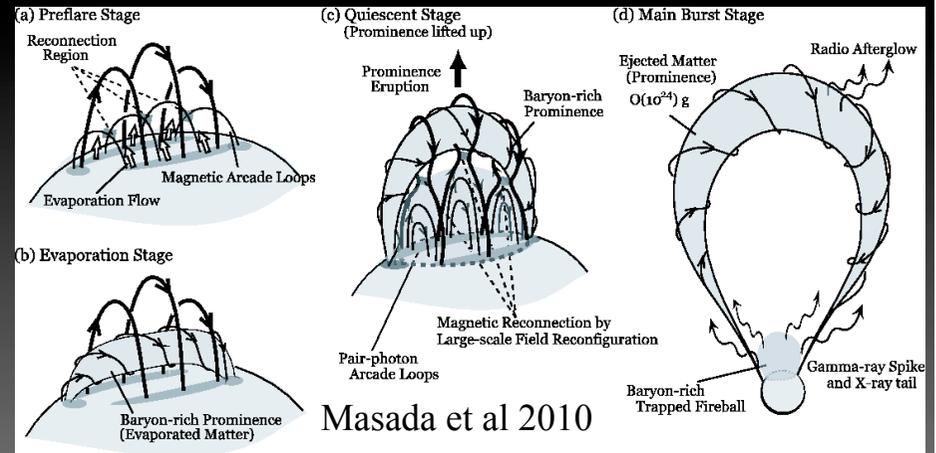
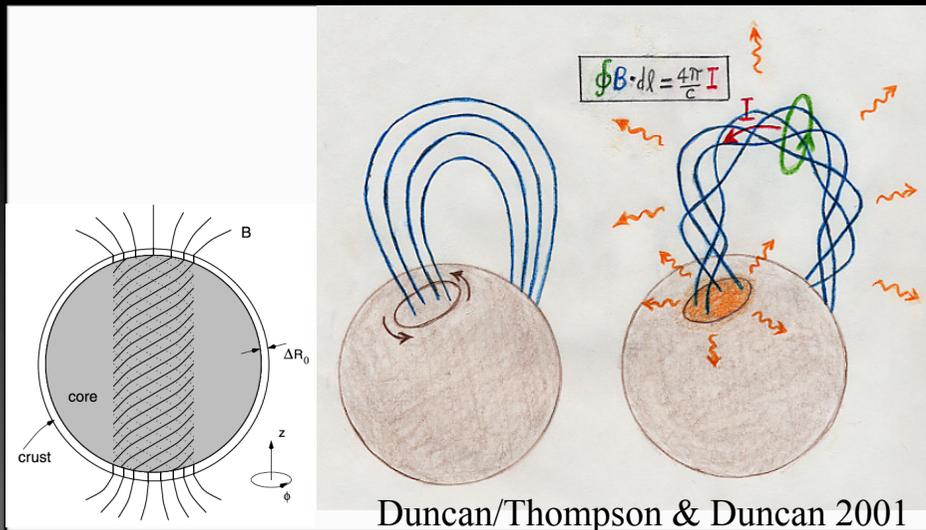
- Hall drift timescale  $\sim 10^3$ - $10^4$  yr
- Stress builds in the crust

Internal mechanism:

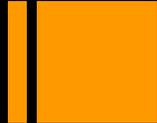
- Reach breaking strain  $\sim 0.1$  (Horowitz & Kadau 2009)
- “Crustquake” (Thompson & Duncan 1996)
- Mechanical failure may propagate too slow (Levin & Lyutikov 2012, Belobodorov & Levin 2014, Li et al 2016)

External mechanism:

- Stress build-up limited by plastic deformations
- Highly twisted magnetosphere leads to magnetic reconnection event
- Solar-like flare (Lyutikov 2006, Masada et al 2010, Lyutikov 2014)



# Maximum magnetospheric twist



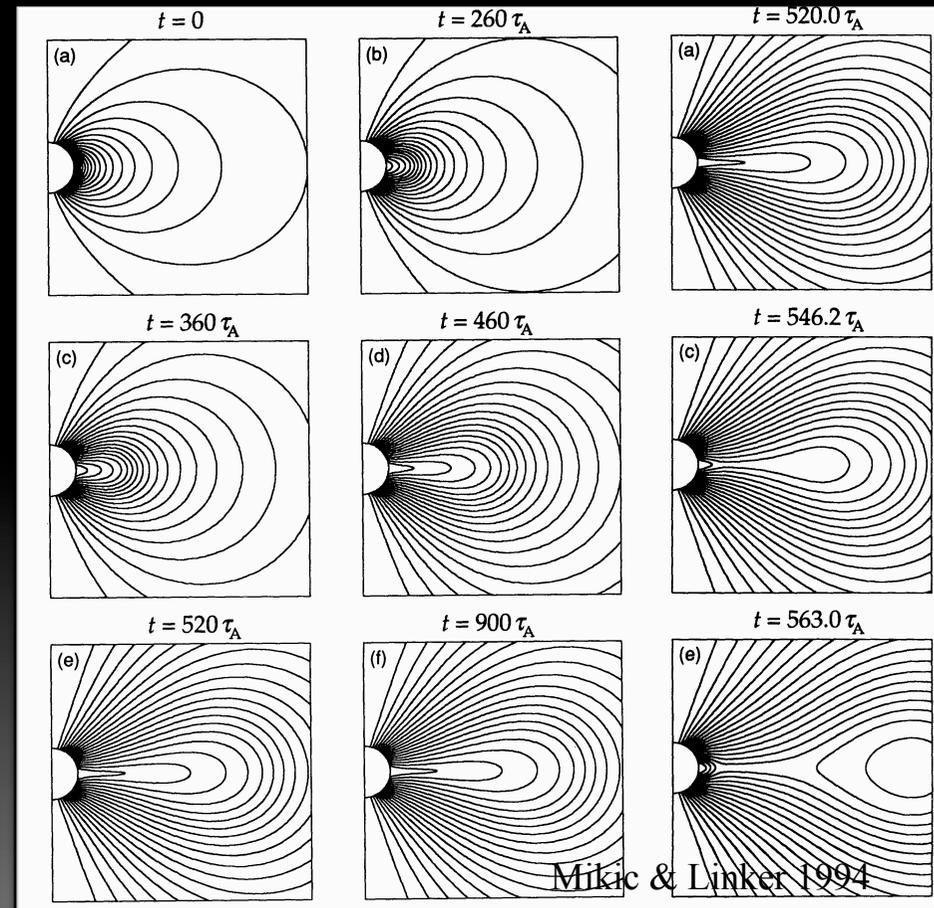
MHD dynamical calculations (Mikic & Linker 1994, Parfrey et al 2013)

- Maximum strain  $\sim$  maximum twist  $\sim 1 - 4$  rad
- Results sensitive to:

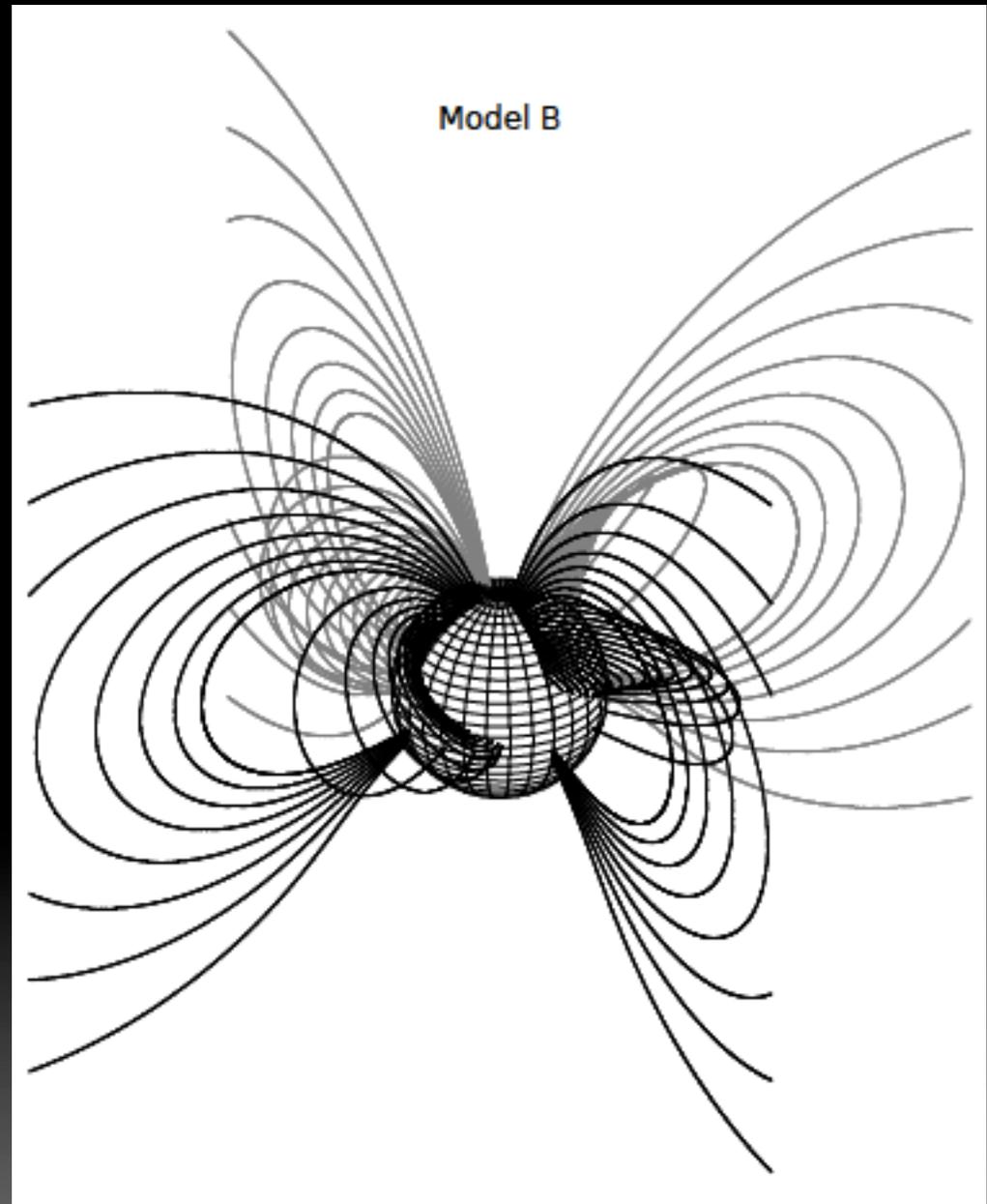
- How fast you twist the magnetosphere
- Resistivity
- Magnetic field configuration
- Twist profile



Can we learn something from force-free equilibrium models?



# Force-free twisted magnetospheres



# Force-free magnetospheres

Akgün, Miralles, Pons & CD, MNRAS, 462, 1894 (2016)

$$\Delta_{GS}P = -T(P)T'(P) \quad : \text{Grad-Shafranov equation}$$

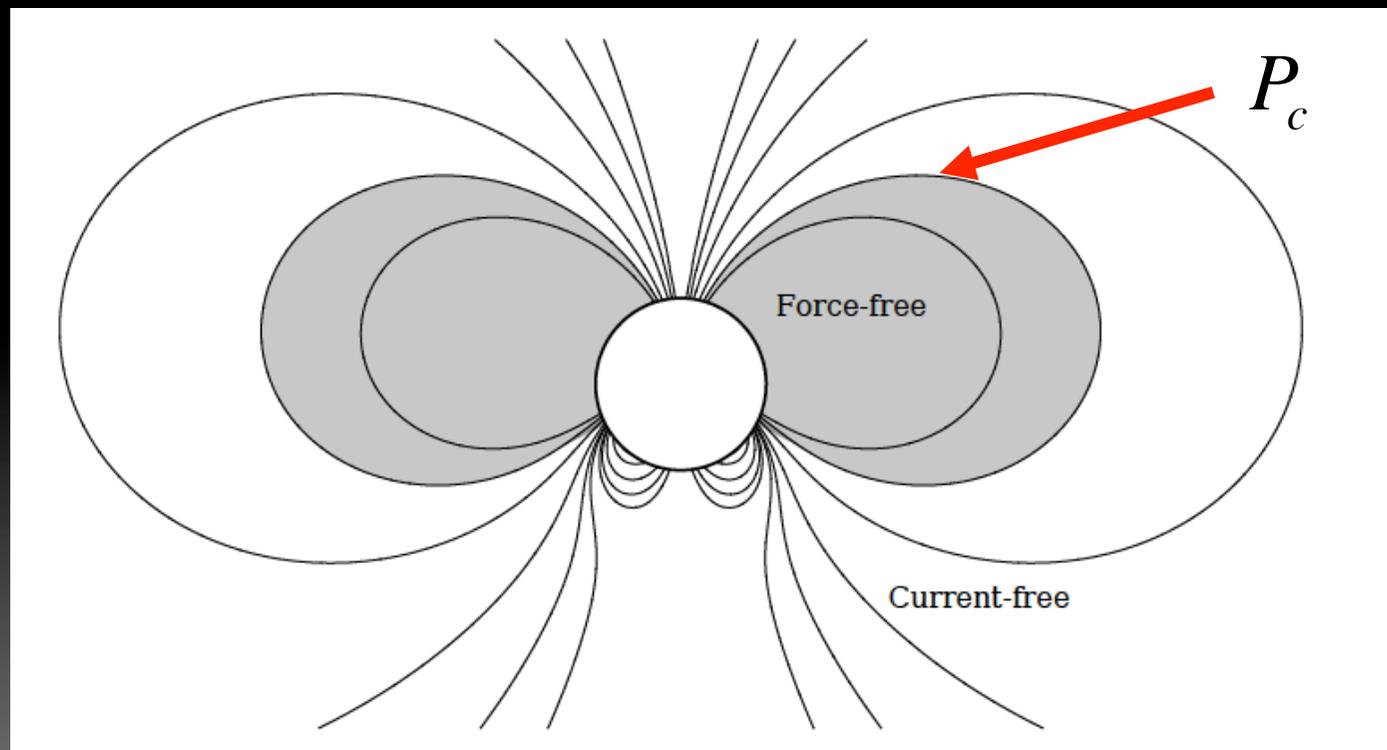
$$4\pi J = T'(P)B \quad : \text{Twist} \leftrightarrow \text{magnetospheric currents}$$

$T(P)$  : Toroidal function  $\rightarrow$  fixed by the field at the NS surface

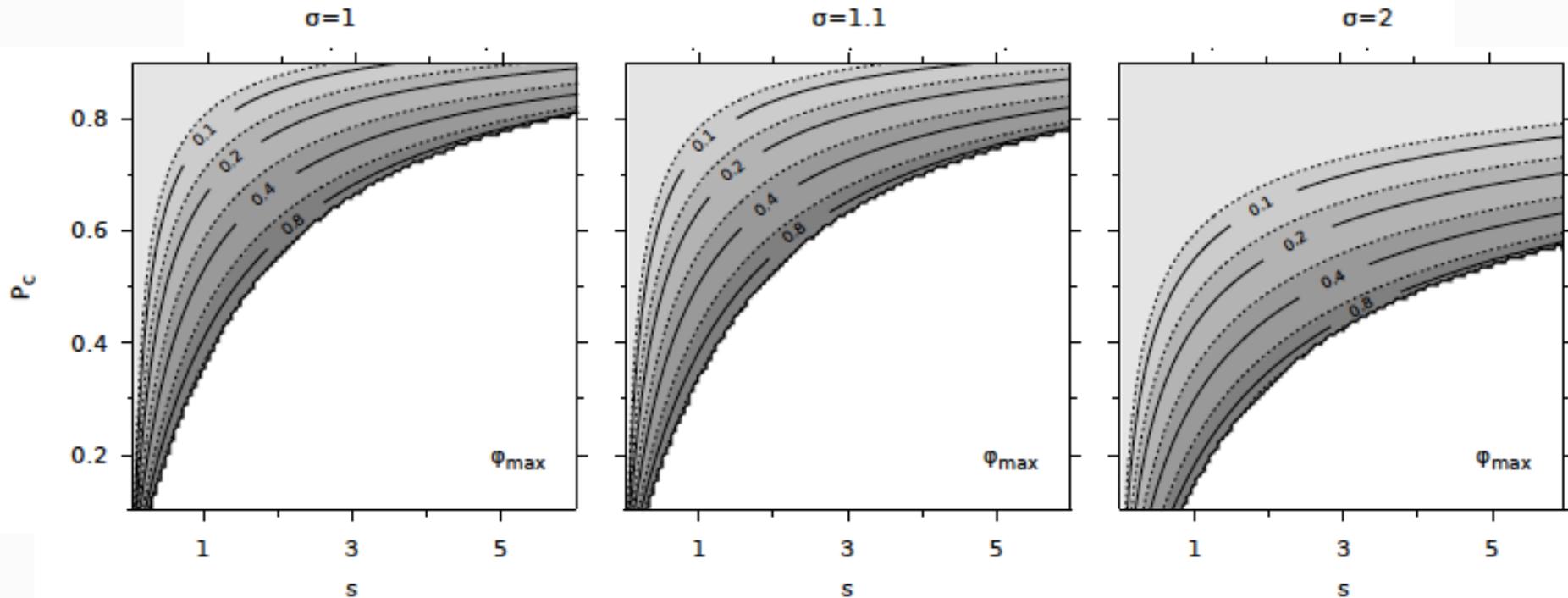
Non-linear elliptic equation  $\rightarrow$  iterative numerical method  
(needs initial guess)

# Toroidal function

$$T(P) = \begin{cases} s(P - P_c)^\sigma & \text{for } P \geq P_c, \\ 0 & \text{for } P < P_c. \end{cases}$$

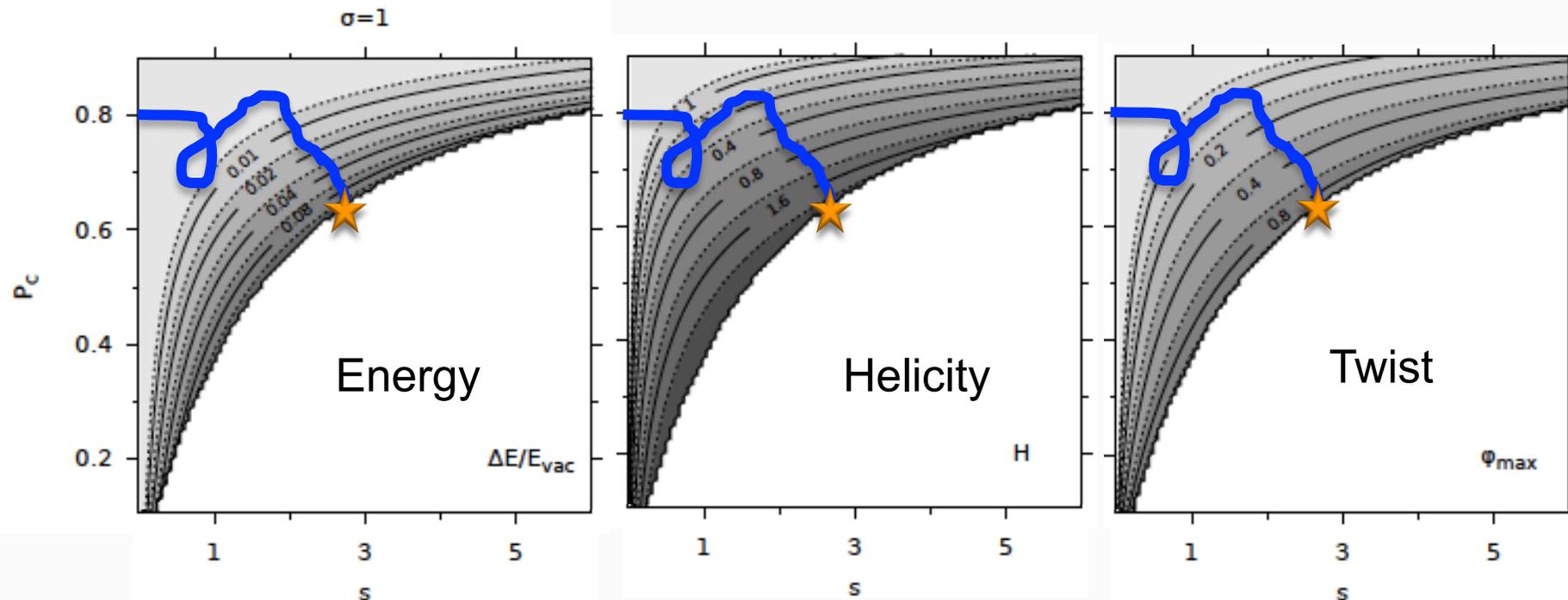


# Twist



- Solutions of the GS equation with twist larger than  $\sim 1$  cannot be found
- This limit is similar to dynamical simulations.
- **Is this limit related to the stability of the solution?**

# Applications



- More realistic magnetospheres to compute emission
- If we can reliably estimate maximum twist with this method...
  - Force-free configurations can be computed within seconds.
  - Can be coupled to magnetothermal evolutions.

# Uniqueness of the solution

$$\Delta_{GS}P = -T(P)T'(P) : \text{Grad-Shafranov equation}$$

- Current free (potential solutions):

$$\Delta_{GS}P = 0 + \text{boundary conditions} \rightarrow \text{Unique solution}$$

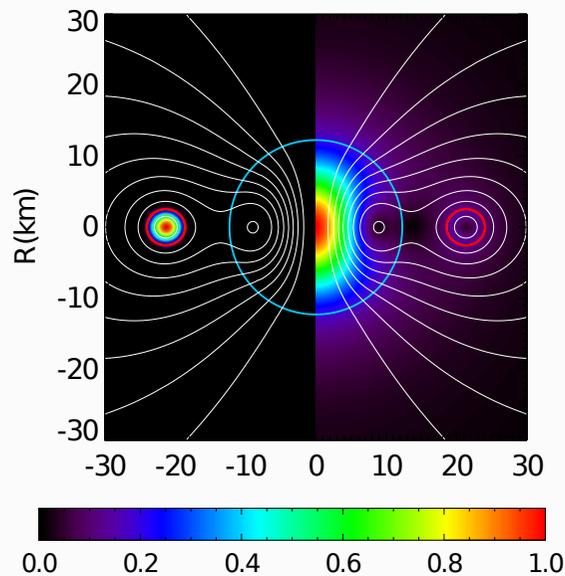
- Linear perturbations in  $T(P) \rightarrow$  Unique solution (see e.g. Gabler et al 2014): potential solution +  $T(P)$
- Bineau 1972 proved uniqueness for sufficiently small twist
- General case: it is not possible to use a maximum principle to prove uniqueness of the solution.



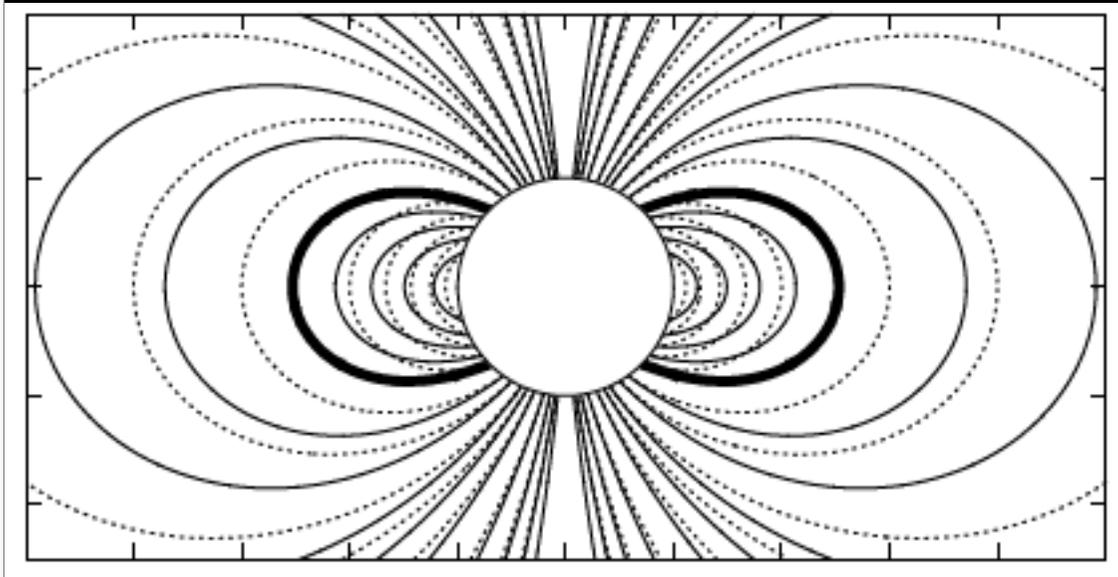
**Solution may not be unique above  
certain threshold twist**

# Uniqueness of the solution?

Pili et al 2015



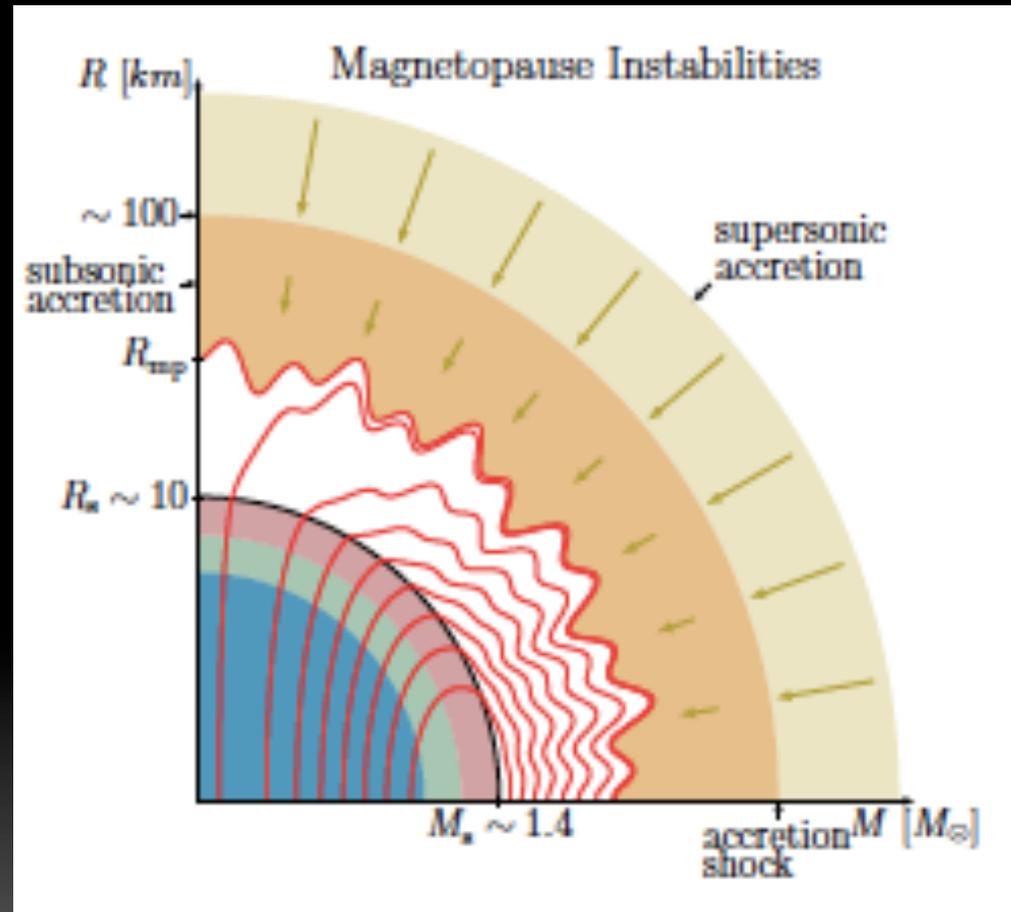
Akgün et al 2015



Discrepancy in force-free configurations for similar boundary conditions:

- Pili et al 2015 found different topologies of the magnetic field
- **Are we facing a problem with non-unique solution?**

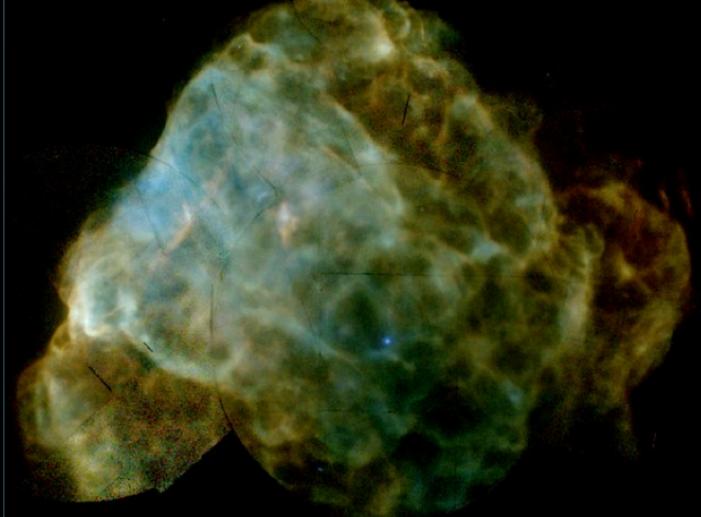
# Fallback and magnetic field burial



# Central compact objects (CCOs)

- Isolated NS with no radio emission
- Associated to supernova remnants
- Inferred magnetic field smaller than typical radio-pulsars
- Spind-down age  $\gg$  real age  $\rightarrow$  CCOs were born with present spin

Puppis A – Chandra/XMM



Katsuda+2010

**Table 1.** Central Compact Objects in Supernova Remnants. From left to right the columns show the CCO name, the age, the distance  $d$ , the period  $P$ , the inferred surface magnetic field,  $B_s$ , the bolometric luminosity in X-rays,  $L_{x,bol}$ , the name of the remnant, the characteristic age, and bibliographical references.

CCO	Age (kyr)	$d$ (kpc)	$P$ (s)	$B_s$ $10^{11}G$	$L_{x,bol}$ ( $erg\ s^{-1}$ )	SNR	$\tau_c$ (Myr)	References
J0822.0-4300	3.7	2.2	0.112	0.65	$6.5 \times 10^{33}$	Puppis A	190	1, 2
1E 1207.4-5209	7	2.2	0.424	2	$2.5 \times 10^{33}$	PKS 1209-51/52	310	2, 3, 4, 5, 6, 7
J185238.6 + 004020	7	7	0.105	0.61	$5.3 \times 10^{33}$	Kes 79	190	8, 9, 10, 11

**References:** (1) Hui & Becker (2006), (2) Gotthelf & Halpern (2013), (3) Zavlin et al. (2000), (4) Mereghetti et al. (2002), (5) Bignami et al. (2003), (6) De Luca et al. (2004), (7) Gotthelf & Halpern (2007), (8) Seward et al. (2003), (9) Gotthelf et al. (2005), (10) Halpern et al. (2003), (11) Halpern & Gotthelf (2010)

# CCO models

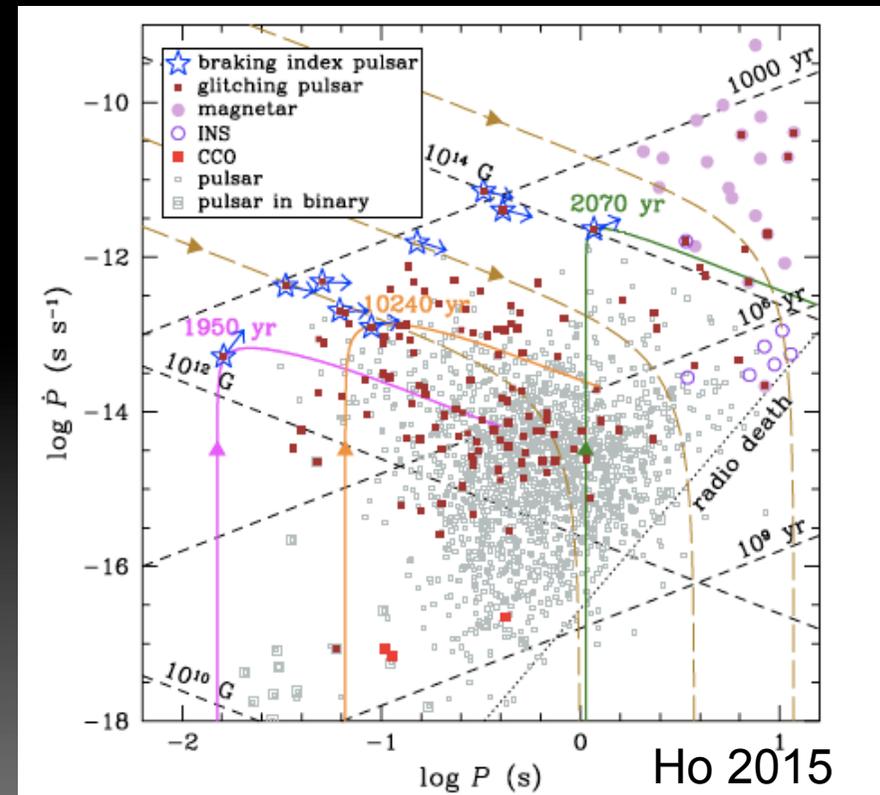
## Hidden magnetic field model

- Magnetic field buried by SN fallback
- Re-emergence of the magnetic field in  $1-10^7$  kyr  
(Young & Chanmugan 1995, Muslimov & Page 1995, Geppert et al. 1999, Shabaltas & Lai 2012, Ho 2011, Viganò & Pons 2012, Ho 2015).
- CCOs could be evolutionary linked to braking index pulsars (Ho 2015)

## “Anti-magnetar” model

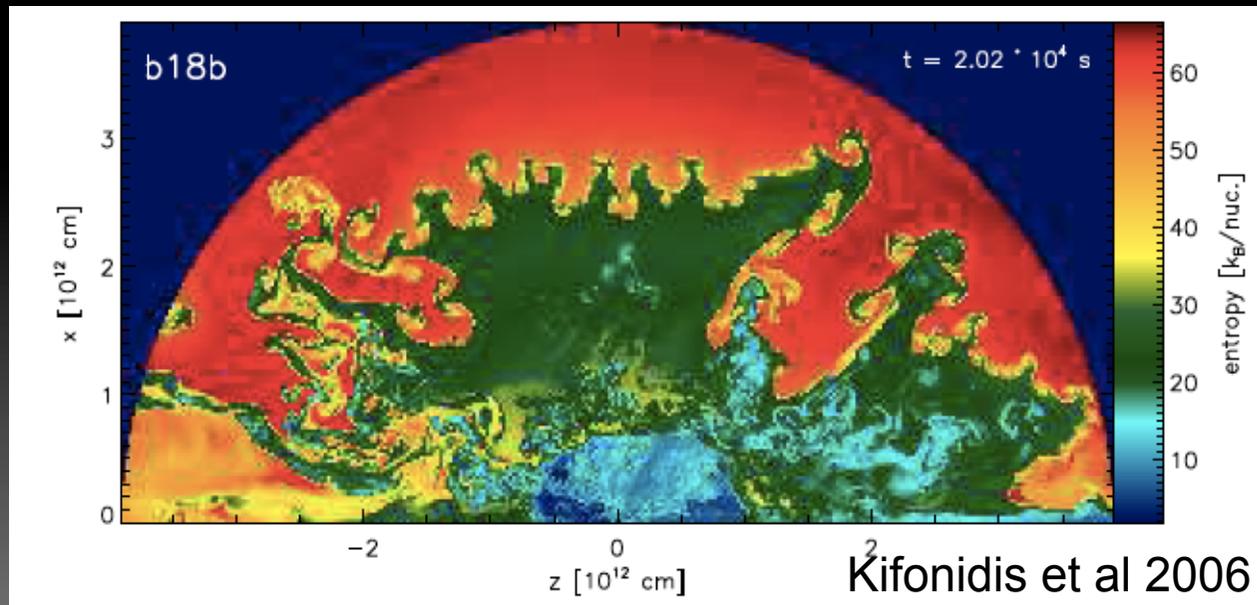
(Halpern et al 2007)

- Born with low magnetic field
- Slowly rotating progenitors
- Numerical simulations show non-rotating progenitors can produce pulsar-like magnetic fields (Endeve et al 2012, Obergaulinger et al 2014)

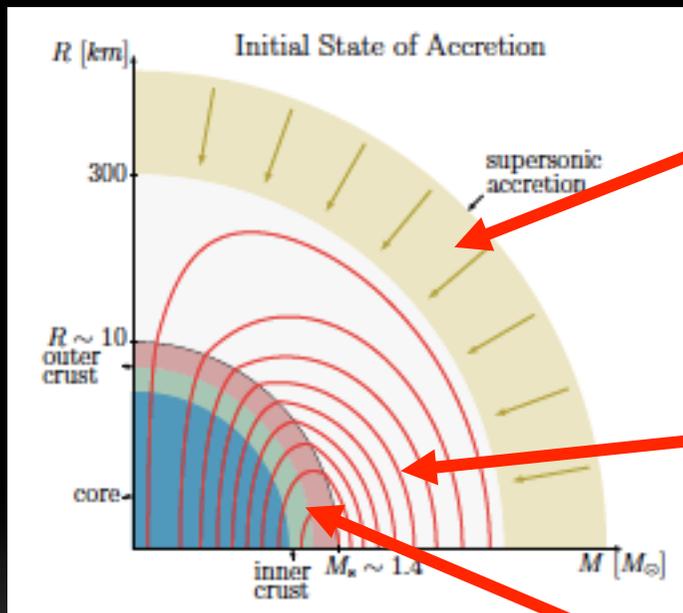


# Supernova fallback

- SN shock produces reverse shock at composition discontinuities (e.g. H-He transition)
- Some material falls back into the NS (Colgate 1971, Chevalier 1989)
- Amount of fallback material  $\sim 10^{-4} - 1 M_{\text{sun}}$   
(Woosley et al. 1995; Zhang et al. 2008; Ugliano et al. 2012, Ertl et al 2016)
- Accretion rate  $\sim t^{5/3} \rightarrow$  most of the matter accretes in  $10^3 - 10^4$  s



# Fallback into neutron star



## Fallback material

- Supersonic accretion
- Super-Eddington ( $>10^6$ )
- Adiabatic compression (no cooling)
- $s \sim 1-100$   $k_N/\text{nuc}$
- Basically unmagnetized

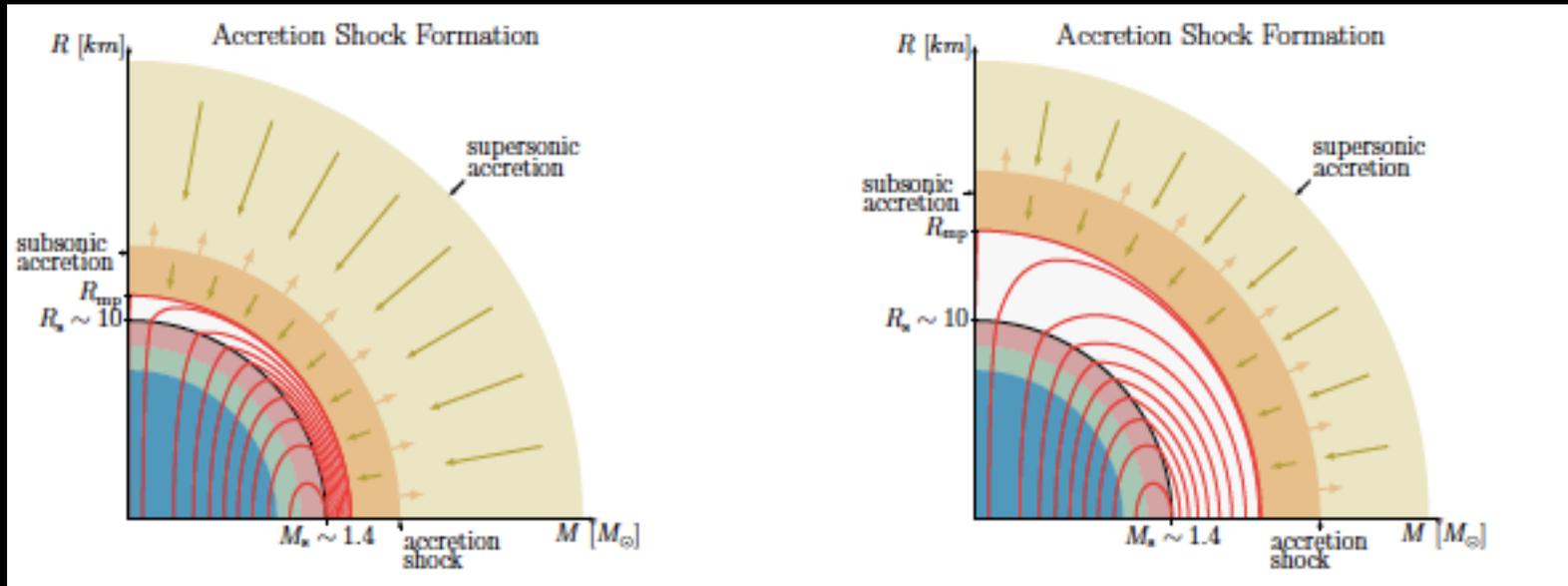
## Magnetically dominated magnetosphere

## NS $\sim 1$ hour after onset of explosion

- Cold NS
- Inner crust crystalized

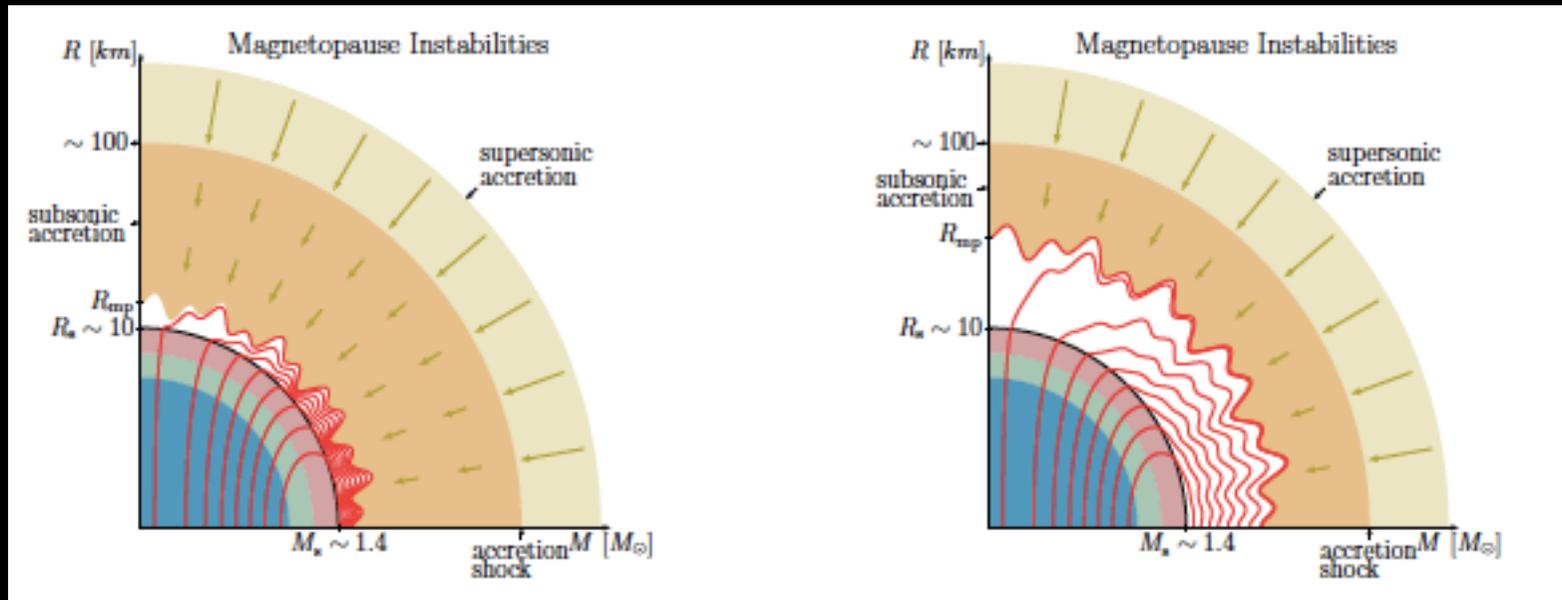
(Page et al 2004, Aguilera et al 2008)

# Accretion shock formation



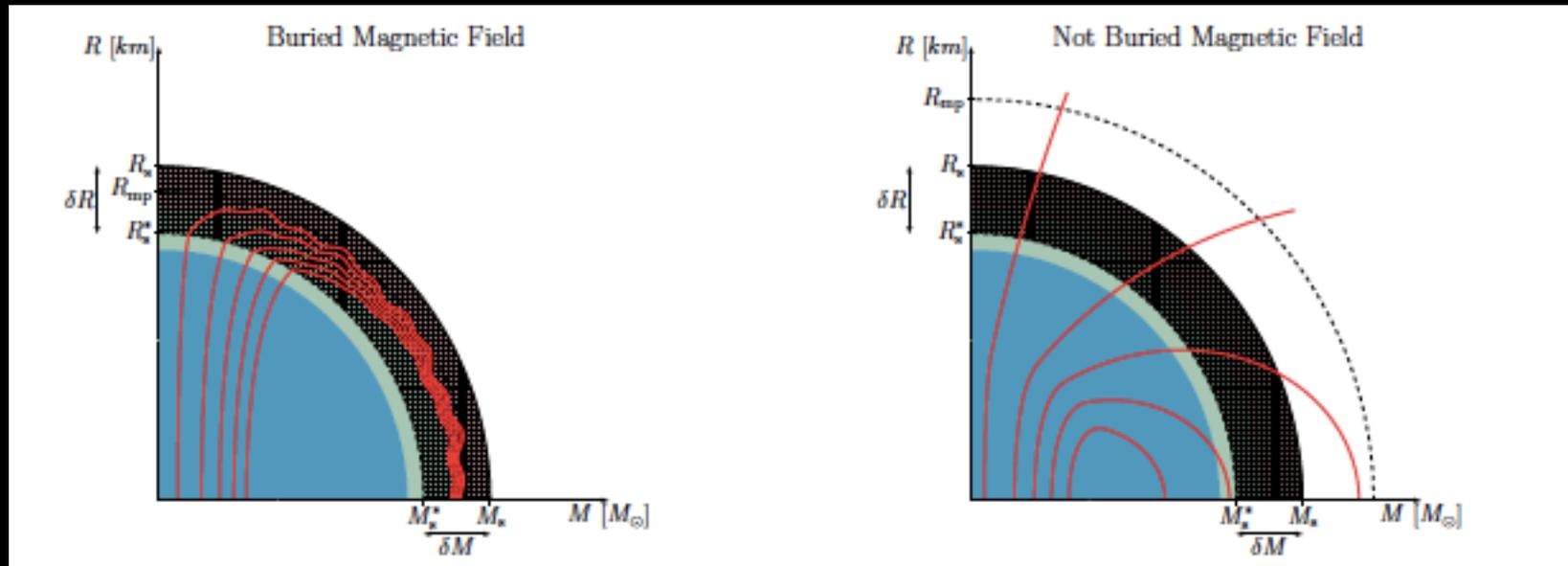
- Accretion shock is formed as the shock is slowed down by the NS surface or the compressed magnetosphere (Chevalier 1989)
- The shock stalls at about  $10^7$ - $10^8$  km (Houck et al 1991)

# Development of instabilities



- The compressed magnetosphere is supporting the fluid
- Magnetopause subject to interchange instabilities (Kruskal & Schwarzschild 1954, Arons & Lea 1976, Michel 1977)
- Mixing may allow accretion onto NS surface and dynamical reemergence of the magnetic field

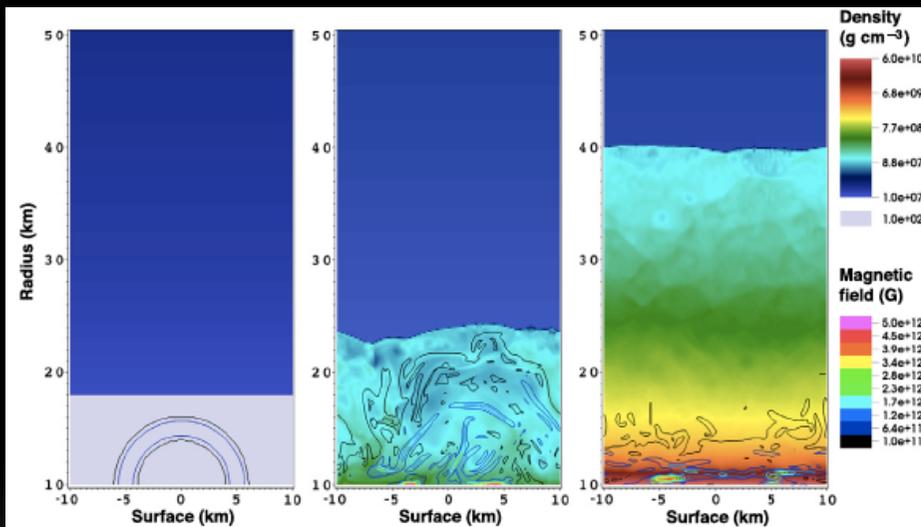
# End of the accretion phase



- High accretion / low B field
  - instability vertical scale  $\ll$  burial depth
  - Buried field
- Low accretion / high B field
  - Instability vertical scale  $\gg$  burial depth
  - Dynamical reemergence  $\rightarrow$  non buried field?

# Previous works

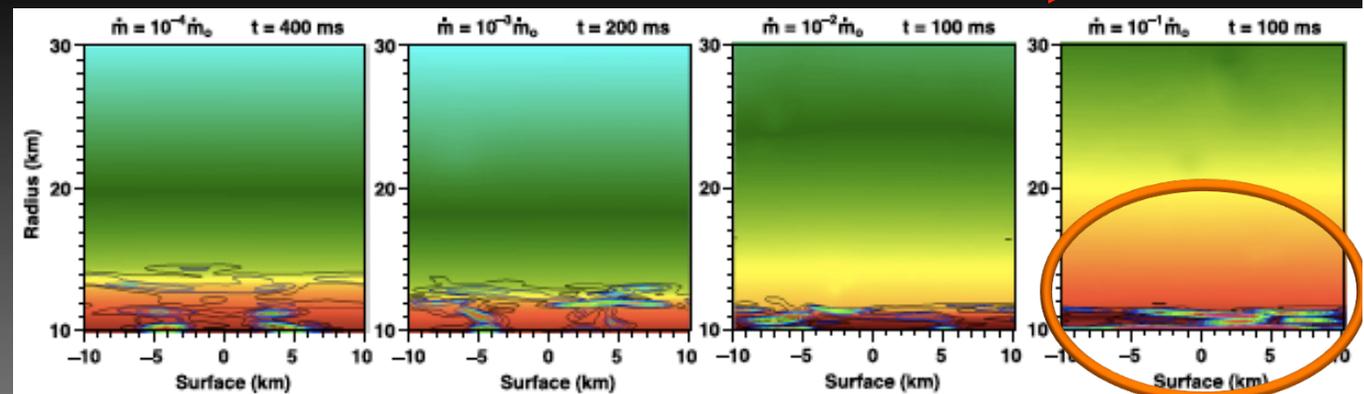
- Local MHD simulations
- Simplified geometries
- Difficult to resolve numerically all relevant regimes



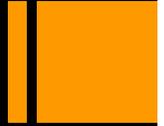
- Payne & Melatos (2004, 2007)
- Bernal et al. (2010, 2013);
- Mukherjee et al (2013a,b)

Increasing accretion rate

Bernal et al 2013



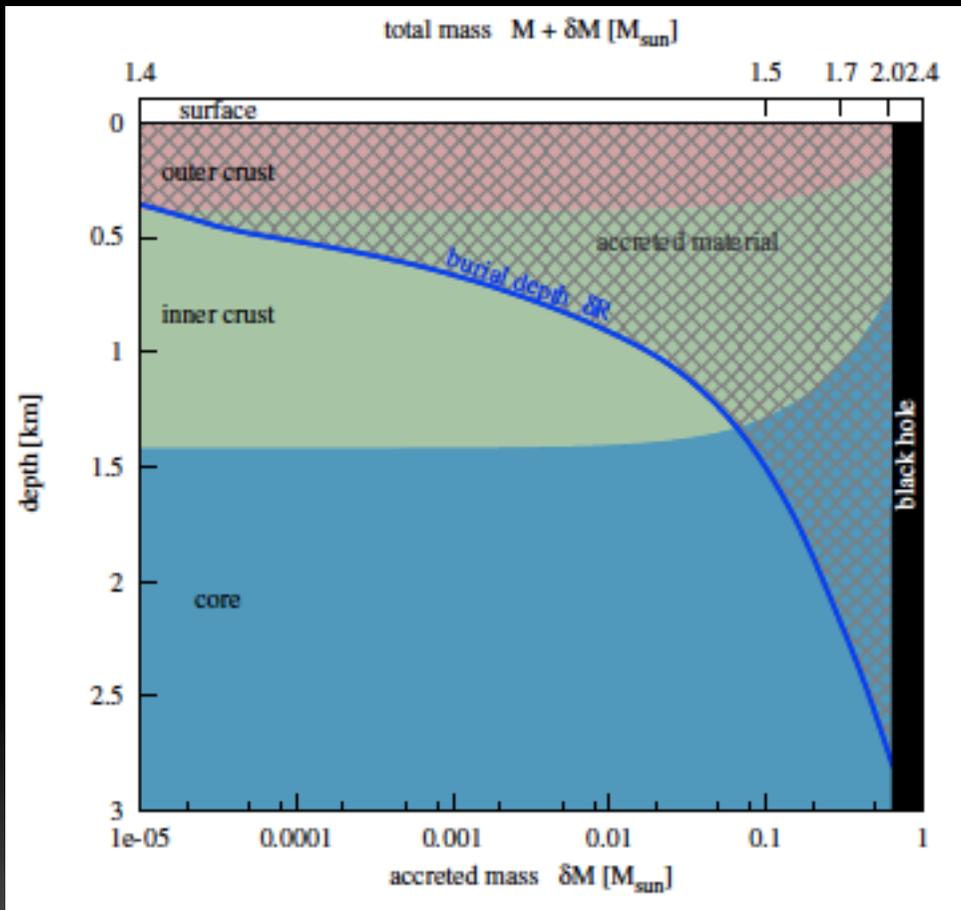
# Our work (Torres-Forné et al 2016)



instability vertical scale vs burial depth

- Simple model: easy to explore parameter space
- Covers different regimes with similar accuracy
- Burial condition do not depend on details of the instabilities
- Non-buried case may depend on details of the instabilities

# Burial depth

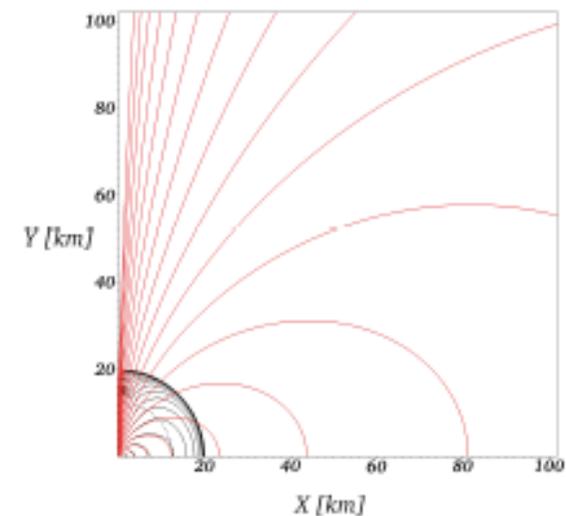
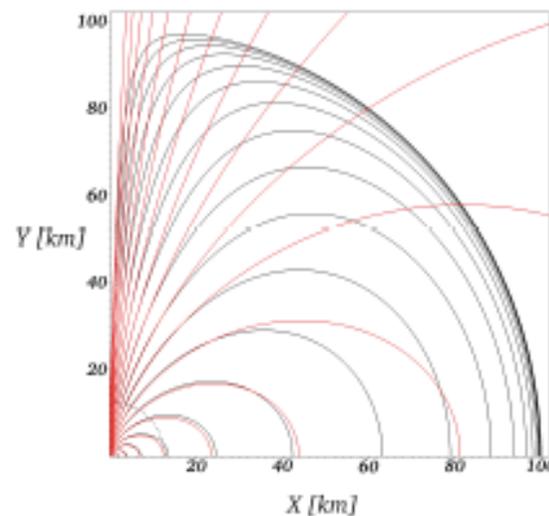
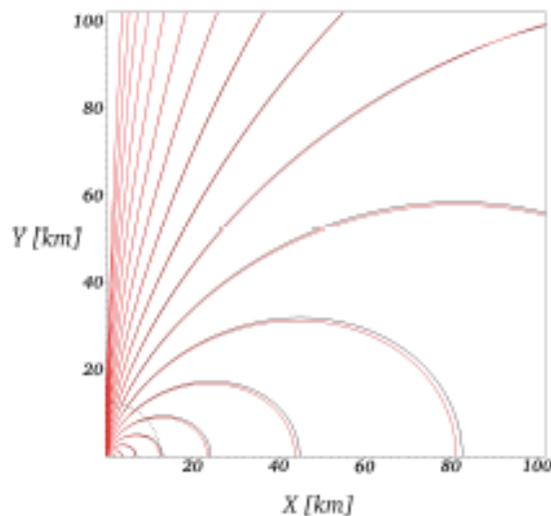


Depends on:

- Total accreted mass
- Equation of state
- Initial NS mass

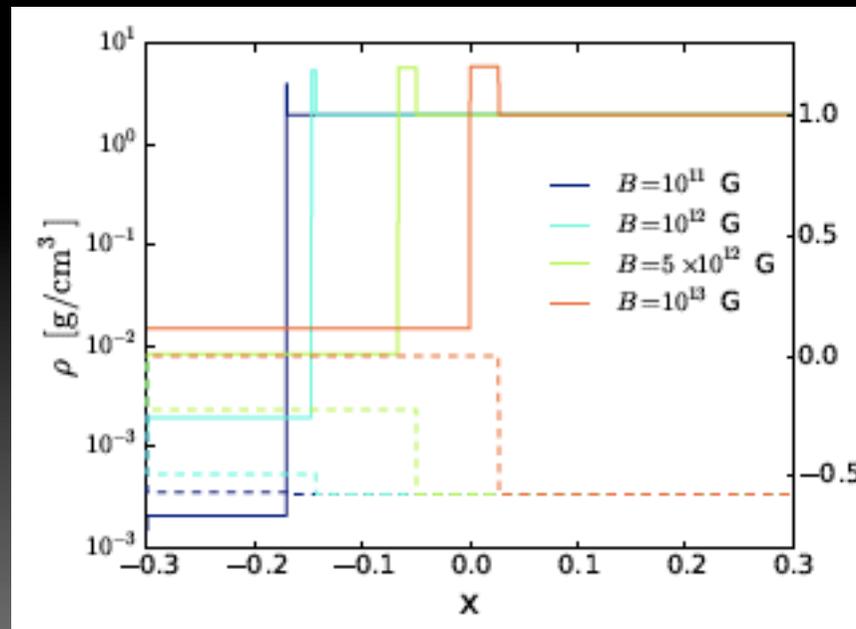
# Compressed magnetosphere

- Force-free magnetosphere (potential solution) compressed by spherically accreting matter (non-dynamic)
- We use different configurations for the NS field
- Magnetic pressure increases as magnetosphere is compressed
- Magnetic pressure is highest at the equator



# Magnetopause location

- Equilibrium point between ram pressure of infalling material and magnetic pressure
- We solve the MHD Riemann problem (Romero et al 2005) to find the equilibrium point (zero velocity contact discontinuity)
- Iteratively computed with magnetosphere
- Depends on: composition, entropy per baryon, accretion rate
- Helmholtz EoS

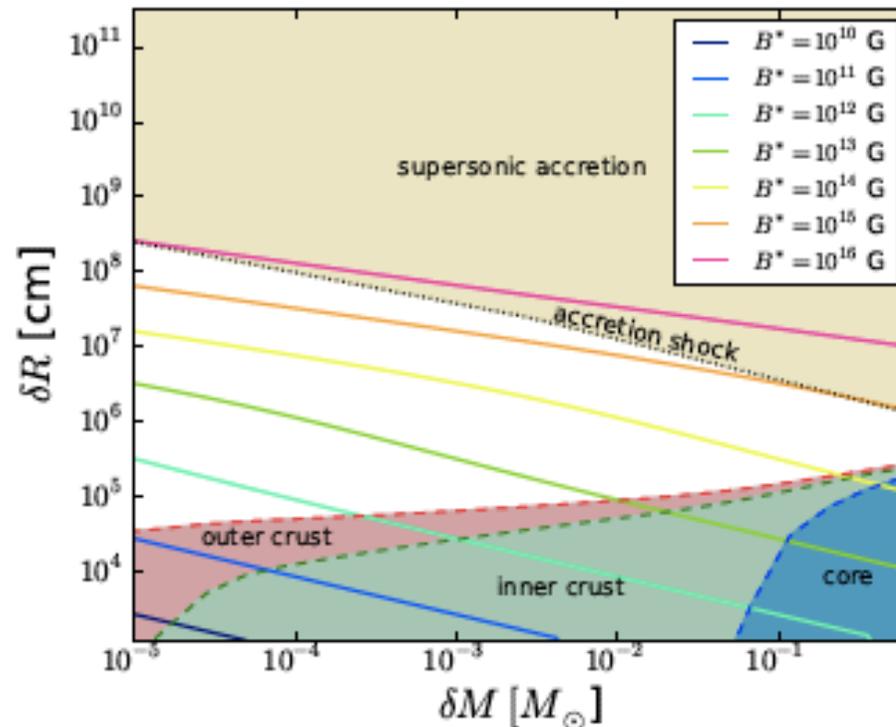


# Instability vertical scale

Interchange instability (Kruskal & Schwarzschild 1954):

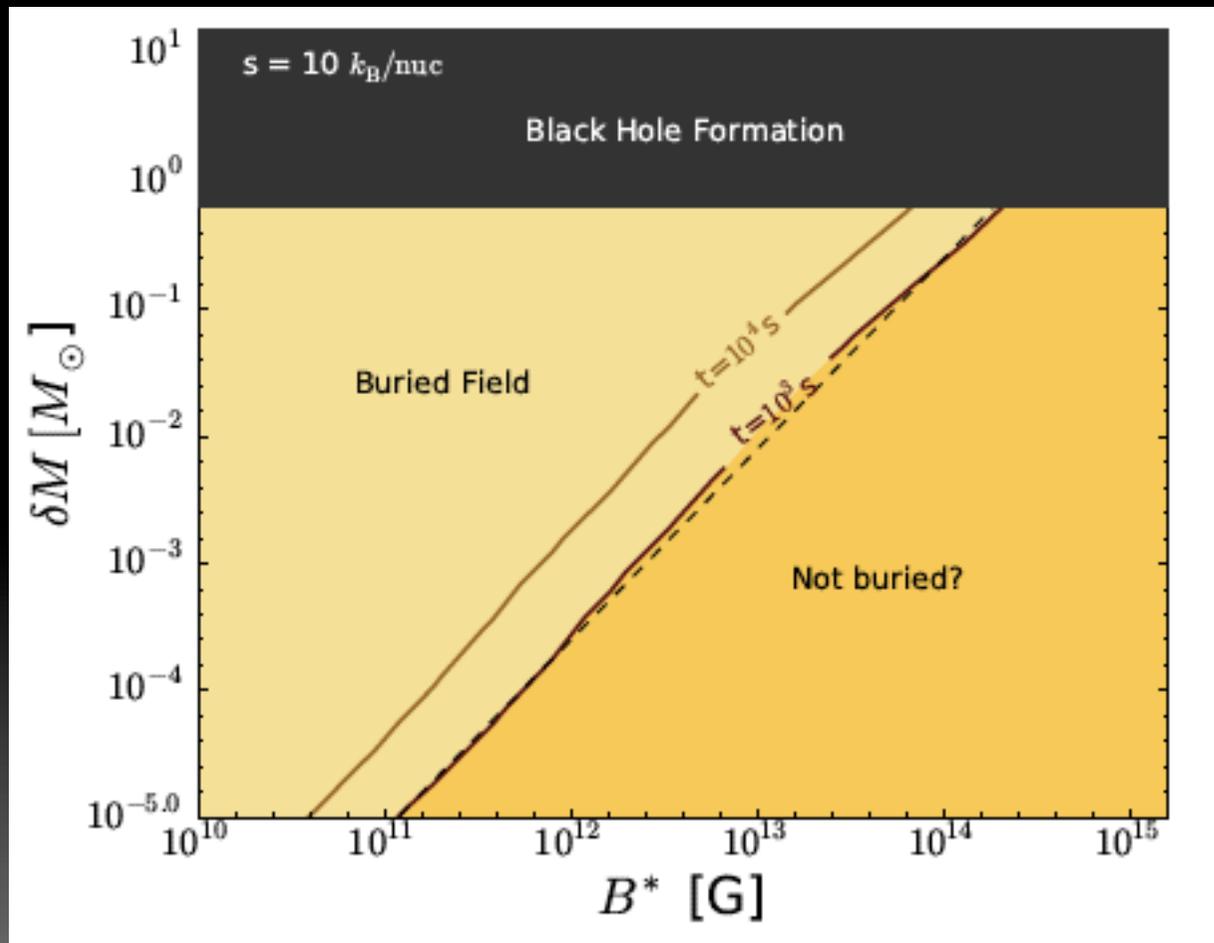
- All wavelengths are unstable
- Instability limited by the height of the magnetosphere

Magnetopause height over NS  $\sim$  instability vertical scale



# Exploring the parameter space

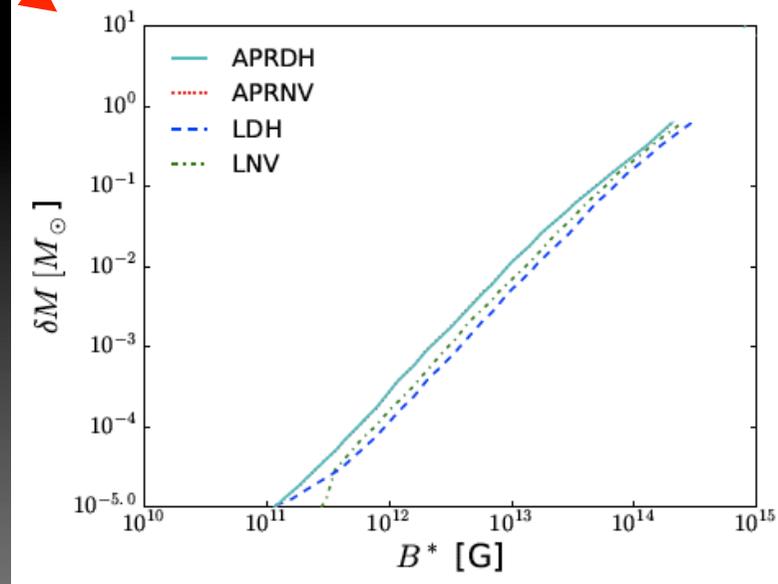
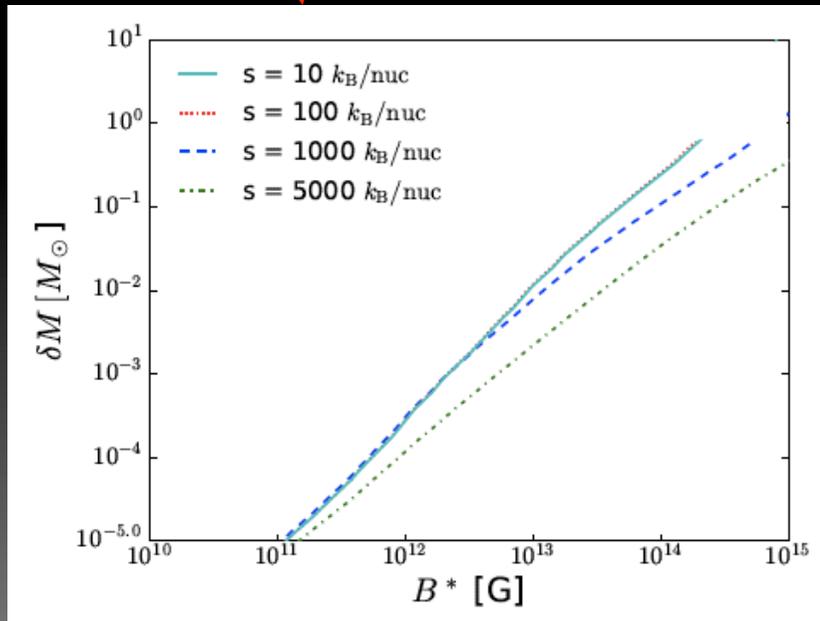
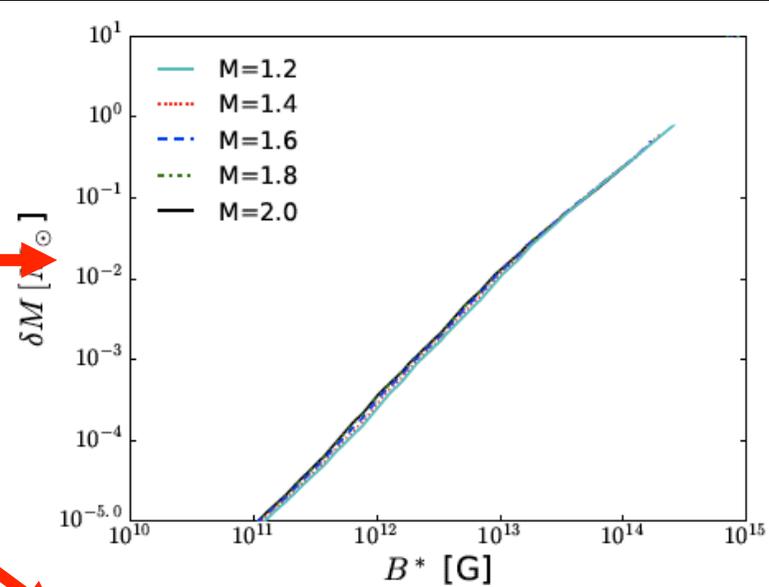
- Typical pulsar is easily buried by fallback
- Very difficult to bury magnetar fields



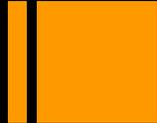
# Exploring the parameter space

Results not very sensitive to different parameters

- Mass
- NS EoS
- Entropy per baryon



# Conclusions



- Force-free magnetosphere models matching NS interior fields
  - emission mechanism
  - Magnetothermal evolution
  - Possible estimation of reconnection events
- Internal magnetar oscillations couple to the magnetosphere
  - QPO modulation mechanism
  - 1:3:5 frequencies  $\leftrightarrow$  odd/even symmetry
- CCOs: Buried magnetic field scenario is plausible