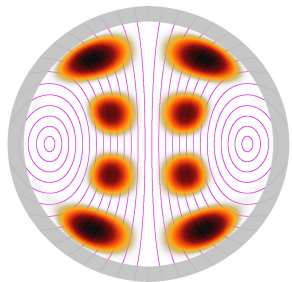
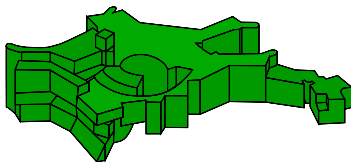
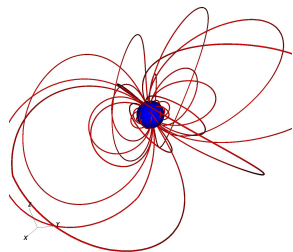


# What can magnetar QPOs tell us about the NS EoS

Michael Gabler



E. Müller,  
P. Cerdá-Durán, T. Font,  
N. Stergioulas



Coco2Casa



European Research Council  
Established by the European Commission







## 1 Introduction

- Constraints on NS properties
- Magnetar quasi-periodic oscillations (QPOs)
  - Observations
  - Elastic oscillations
  - Alfvén oscillations

## 2 Superfluid magneto-elastic QPOs inside the magnetar

- Superfluid effects
- Different regimes and damping of crust modes
- Different constraints
  - Outbreak
  - High-frequency QPOs
  - Identifying observed frequencies

## 3 Constraining neutron star properties with QPOs

## 4 Conclusions

# Neutron stars (NS) as unique laboratory

- Ultra-dense matter: interior of NS widely unknown

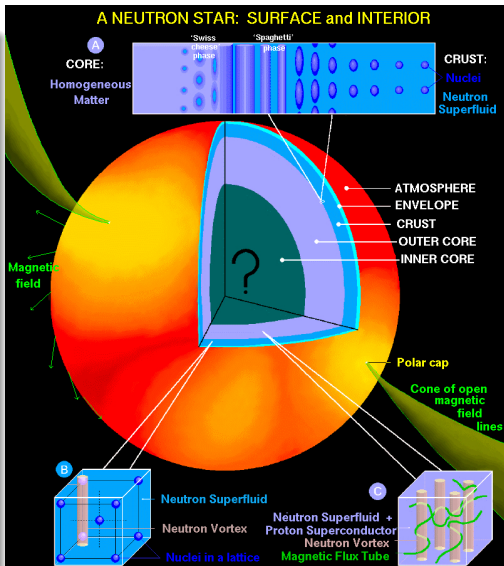
- ▶ Nuclear matter ???
- ▶ Hyperons ???
- ▶ Quarks ???

expected properties:

- ▶ Superfluidity
- ▶ Superconductivity
- ▶ Color superconductivity
- ▶ ...

- Exotic physics in strong magnetic fields ( $B \gtrsim B_{\text{QED}}$ ):

- ▶ Appearance of chain molecules
- ▶ One-photon pair creation
- ▶ Photon splitting
- ▶ ...



# Ways to constrain the NS equation of state

## Radius and compactness constraints

- Gravitational redshift of spectral lines:  $z = 1 - (1 - 2GM/Rc^2)^{-1/2}$
- X-ray burst (PRE)
- Burst oscillation pulses (distorted by GR effects depending on compactness)

## Gravitational Waves

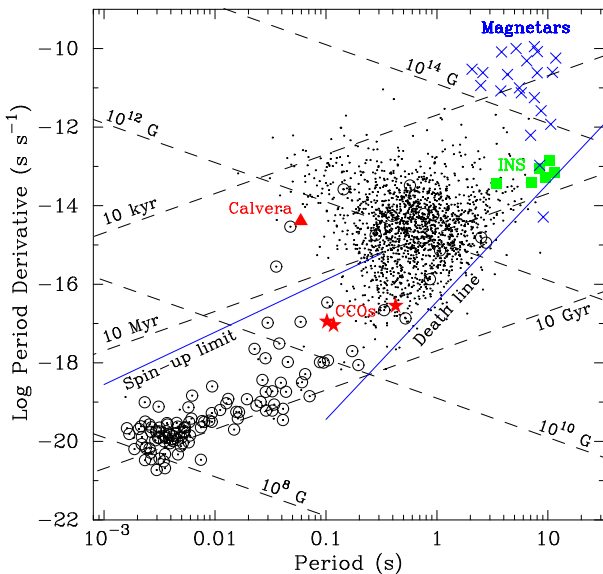
- Tidal interaction during NS merger
- Oscillations of hyper-massive NS after merger
- Magnetar giant flares

## Problems

GW wave detectors not sensitive enough,  
uncertainties in distances and atmospheric models,

...

# Different classes of NS



(Halpern et al. 2013)

## Accreting Pulsars

(circled dots)

- accretion spins up NS
- maximum spin period observed 716Hz

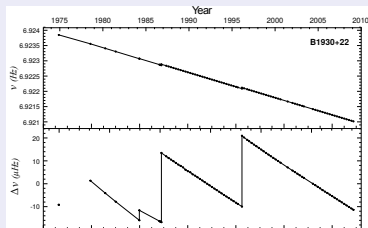
## Isolated Neutron stars

(green squares)

- Thermal emission from surface
- model dependent:
  - ▶ distance?
  - ▶ absorption?
  - ▶ atmosphere?

## Pulsar glitches

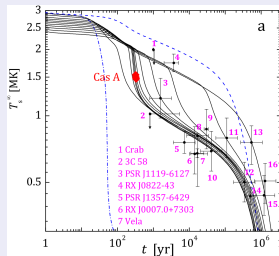
- Pulsars spin down continuously
- Occasionally sudden jumps: 'glitches'
- Neutron superfluid vortices couple to crust through pinning only
- Once  $\Delta\Omega_{\text{SF-crust}}$  too large, SF vortices unpin collectively  $\Rightarrow$  glitch



Espinoza et al. 2011

## Cassiopeia A cooling

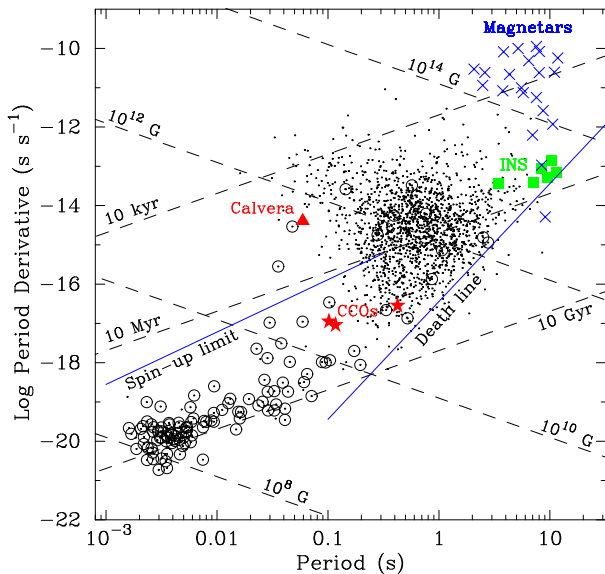
- Neutron stars cool by  $\nu$  and photon emission
- Cooling curve of Cas A shows steep decrease in  $T_S^\infty$
- Good fit only for models that include superconducting protons and superfluid neutrons



Shternin et al. 2011



# Spin period - spin down $P\dot{P}$ diagram:



(Halpern et al. 2013)

## Observations

- High  $P\dot{P}$
- High X-ray luminosity
- Repeated  $\gamma$ -bursts
- 3 giant flares

## Magnetar

- Magnetic dipole spin down:  $B \sim 10^{15}$  G
- Strong  $B$  powers X-ray luminosity
- Magnetic field evolution can account for bursts and power giant flares luminosity

Magnetar giant flare ...



... or just a Takoyaki

# Observations - quasi-periodic oscillations (QPOs)

## Confirmed QPO frequencies

SGR 1806-20: 18, 26, 30, 92  
150, 625, 1840 Hz

SGR 1900+14: 28, 53, 84,  
155 Hz

(Israel et al. 2005, Strohmayer &  
Watts 2006, ... )

## QPOs in normal bursts

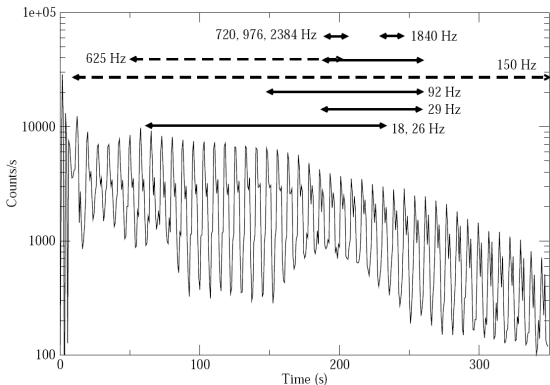
93, 127 and 260Hz

(Huppenkothen et al. 2014)

## Unconfirmed QPOs

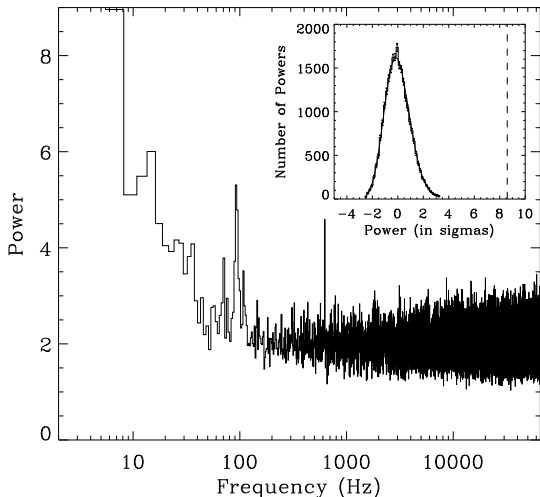
17, 21, 36, 59, and 116 Hz

(Hambaryan et al. 2011)



Strohmayer & Watts 2006

# Challenges in the detection of QPOs



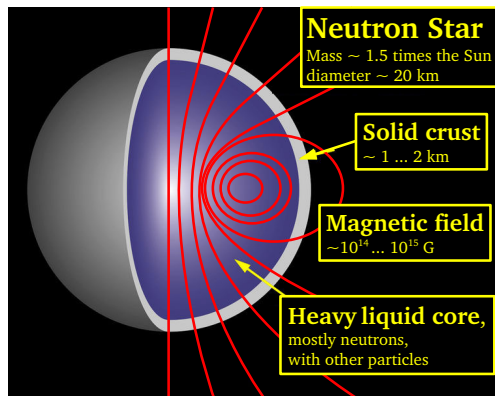
(Strohmayer & Watts 2006)

Need for more and better data! (Intermediate flares)

- Clear signals at  $f \sim 625$  Hz and 92 Hz
- Ok for  $f \gtrsim 30$  Hz
- Unclear for  $f < 30$  Hz
- **Only 3 giant flares**

- ⇒ More sophisticated techniques
- ⇒ Analysis of normal bursts
- **BUT** time variability of the burst is of the order of 100 ms
- Hard to distinguish intrinsic variability from QPO (in short bursts)

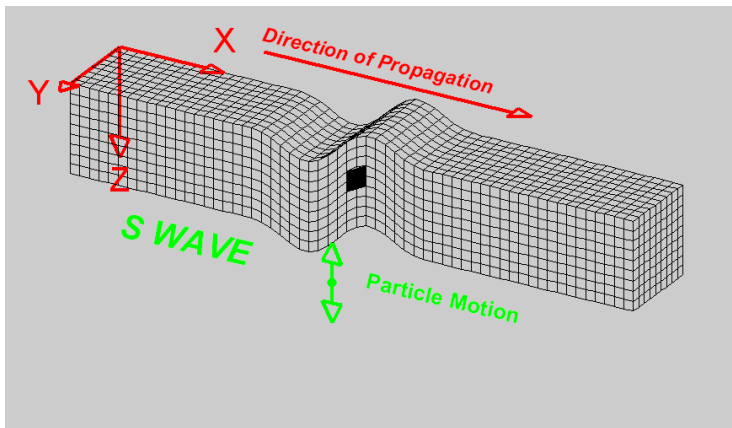
# Where do the QPOs come from? Are they Starquakes?



## Possible origin of the observed frequencies

- Discrete Shear modes (**crust**)?
- Alfvén oscillations at the turning points of a continuum (**core+crust**)?
- **Magnetospheric** oscillations?

# Shear waves



Larry Braile, Purdue University



# Torsional shear modes

Samuelsson & Andersson 2007

Observed frequency in Hz		Shear mode	
SGR 1806-20	SGR 1900+14	n	l
18		???	???
26		???	???
30	28	0	2
	53	0	4
92	84	0	6
150		0	10
	155	0	11
625		1	
1840		3	

(Shoemaker & Thorne 1983, Duncan 1998, Strohmayer & Watts 2005, Piro 2005, Sotani et al. 2007, Samuelsson & Andersson 2007, Steiner & Watts 2009, Deibel et al. 2014, Sotani et al. 2016, ...)

- No magnetic field
- Free slip / zero traction at crust core interface
- Relativistic estimates for  $f$ :  
 $\Rightarrow n = 0$ :

$$f^2 \sim \frac{\mu_S}{\rho} \frac{(l-1)(l+2)}{RR_c}$$

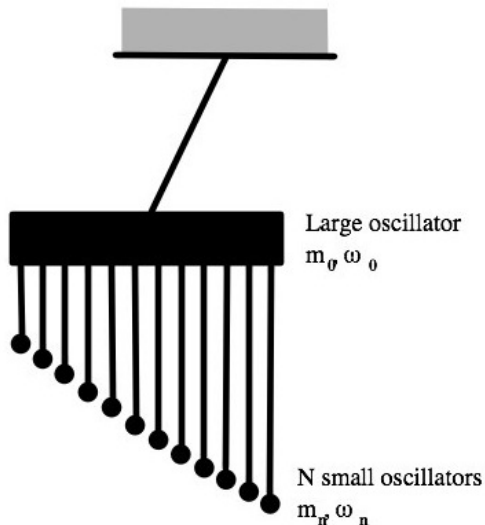
$R_c$  - radius of crust

$\Rightarrow n > 1$ :

$$f \sim \sqrt{\frac{\mu_S}{\rho}} \frac{n}{\Delta}$$

$\Delta$  - crust thickness

## A toy model for Alfvén continuum

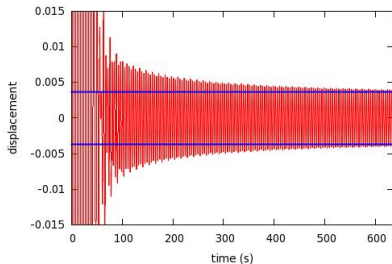
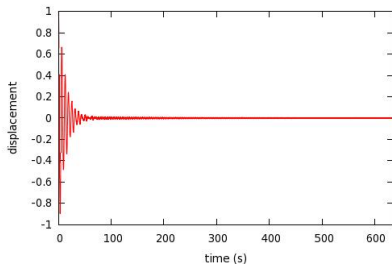


- Big pendulum with large mass
  - Many small oscillators
  - Each small pendulum has different  $f$
- ⇒ Quasi-continuum

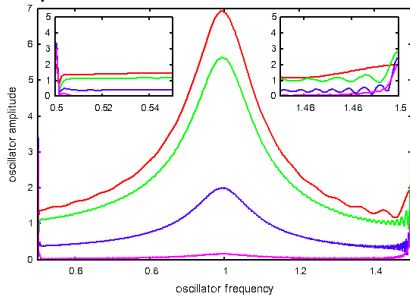
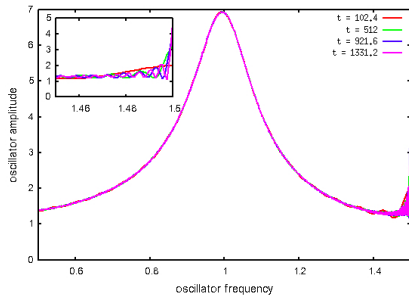
- Excite big pendulum
- ⇒ Strong damping of oscillation
- Excitation of small oscillators
  - Increasing oscillations at the edge of the continuum

# A toy model for Alfvén continuum

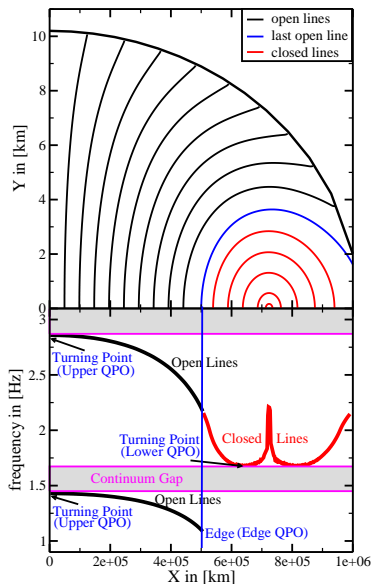
## Time evolution of displacement



## Oscillator amplitude



# The Alfvén continuum



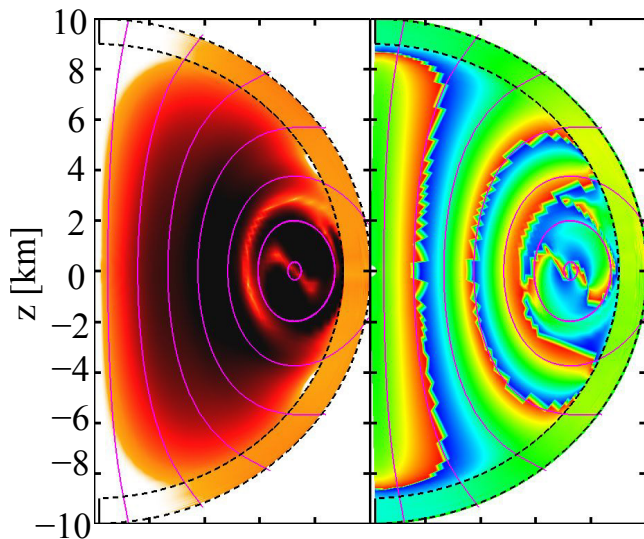
- Each field line has proper eigenfrequency (purely poloidal magnetic field + torsional oscillations)
- Field lines are coupled through:
  - (i) surface boundary conditions
  - (ii) the crust
- Long-lived QPOs exist at the turning-points or edges of the continuum
- Gaps between successive Alfvén overtones

(Levin 2006 & 2007, Sotani et al. 2007 & 2008, Cerdá-Durán et al. 2009, Colaiuda et al. 2009)

# Continuous phase

FFT amplitude

FFT phase



# Superfluid neutron star core - (Newtonian) two-fluid model

## Neutrons

$$\partial_t \rho_n + \nabla \cdot (\rho_n \mathbf{v}_n) = 0$$

$$(\partial_t + \mathbf{v}_n \nabla)(\mathbf{v}_n + \varepsilon_n \mathbf{w}_{pn}) + \nabla(\Phi + \mu_n) + \varepsilon_n w_k^{pn} \nabla v_n^k = 0$$

## Charged particles (protons)

$$\partial_t \rho_p + \nabla \cdot (\rho_p \mathbf{v}_p) = 0$$

$$(\partial_t + \mathbf{v}_p \nabla)(\mathbf{v}_p + \varepsilon_p \mathbf{w}_{np}) + \nabla(\Phi + \mu_p) + \varepsilon_p w_k^{np} \nabla v_p^k = \frac{(\nabla \times \mathbf{B}) \times \mathbf{B}}{4\pi \rho_p}$$

$$\mathbf{w}_{np} = -\mathbf{w}_{pn} = \mathbf{v}_n - \mathbf{v}_p, \quad \varepsilon \text{ entrainment parameter}$$



# Superfluid neutron star core - one-fluid approximation

Effective one fluid model (decoupling n from p):

$$\rho \rightarrow \rho_p \sim 0.05\rho$$

$$v_A^2 = \frac{B^2}{\rho} \rightarrow \frac{B^2}{\epsilon_* X_c \rho} = \frac{B^2}{\rho_p}$$

## Fundamental QPOs

Exist as before but with:

$$f_{\text{sf}} \sim \frac{1}{t_A} \sim \frac{v_A}{R} \sim \frac{B}{R\sqrt{\rho_p}} \sim \frac{B}{R\sqrt{0.05\rho}} \sim 5 \times f_n$$

To match observed QPOs non-superfluid:

$$2 \times 10^{15} \lesssim B \lesssim 10^{16} \text{ G}$$

superfluid

$$5 \times 10^{14} \lesssim B \lesssim \text{few} \times 10^{15} \text{ G}$$

# Magneto-elastic QPOs inside the magnetar

$$B^2 \ll \mu_S \quad B \sim 10^{13} \text{ G} \quad B \sim 10^{15} \text{ G} \quad B^2 \gg \mu_S$$



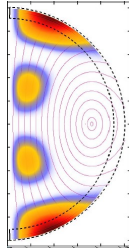
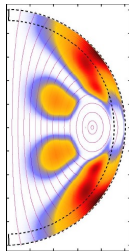
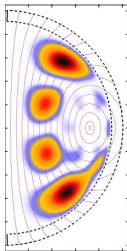
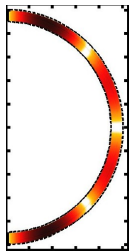
**predominantly  
shear modes**

**shear modes  
strongly  
damped**

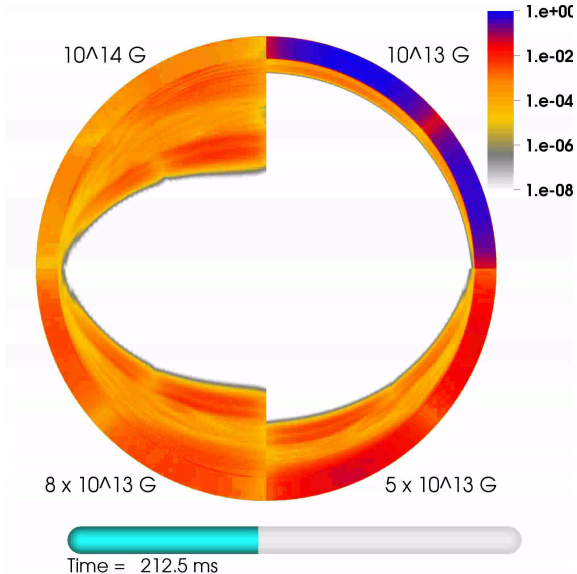
**magneto-elastic QPOs  
confined  
to core**

**reach  
surface**

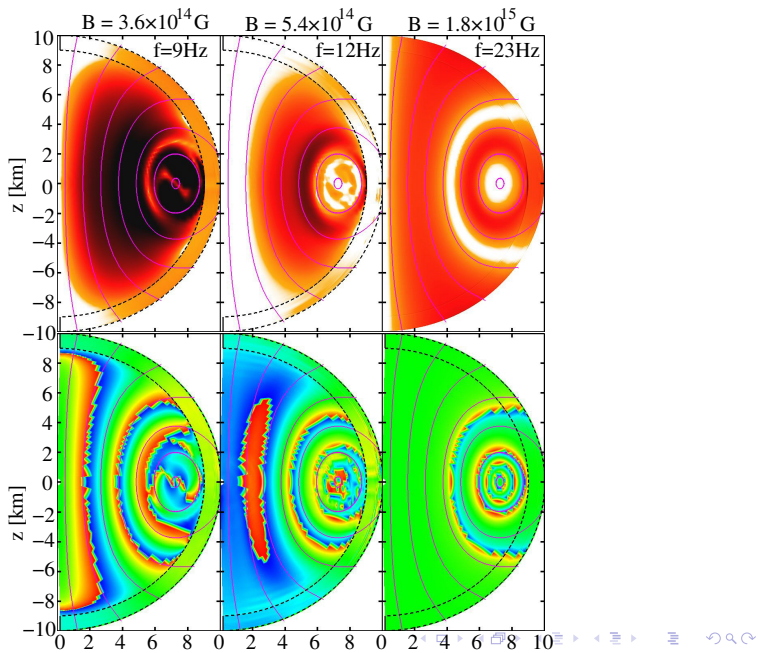
**predominantly  
Alfvén QPOs**



# Damping of crust modes

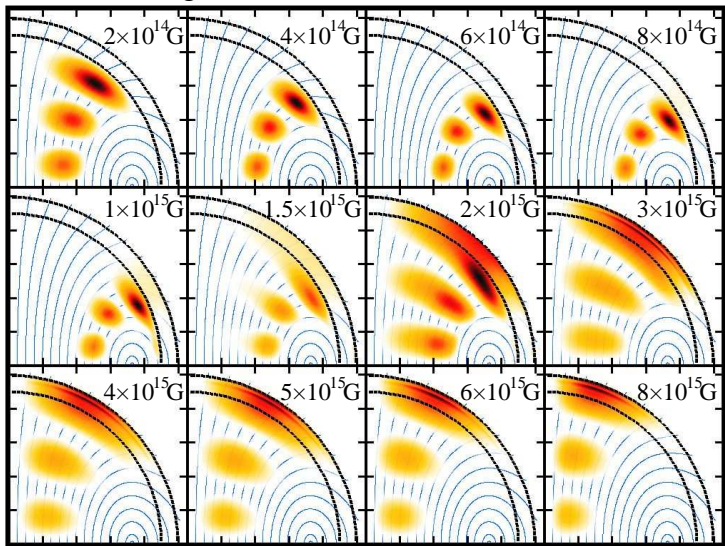


# Continuous vs constant phase



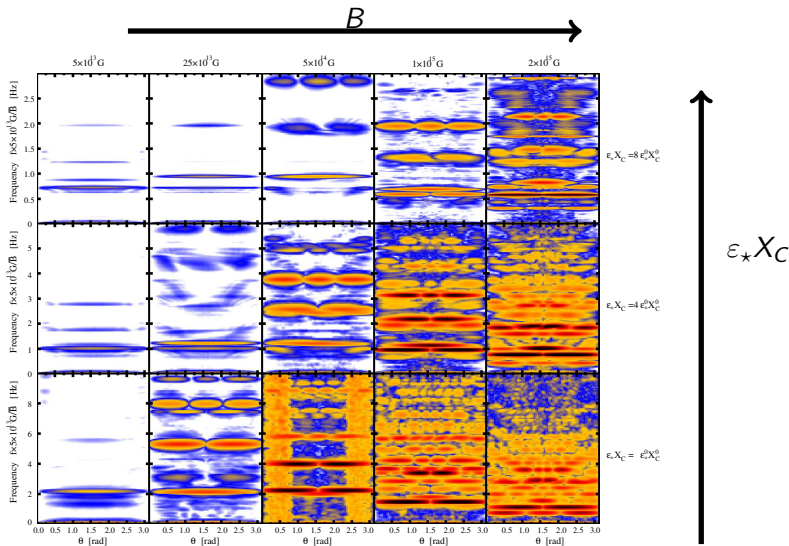
# Magneto-elastic QPOs break out to surface

Magnitude of the FFT at different  $B$



# Magneto-elastic QPOs break out to surface

Magnitude of the FFT at the surface





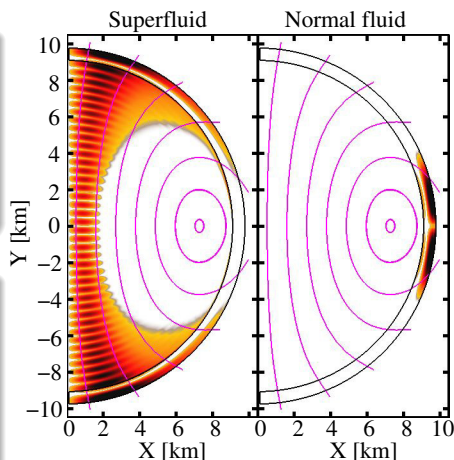
# High frequency QPOs

## Normal fluid

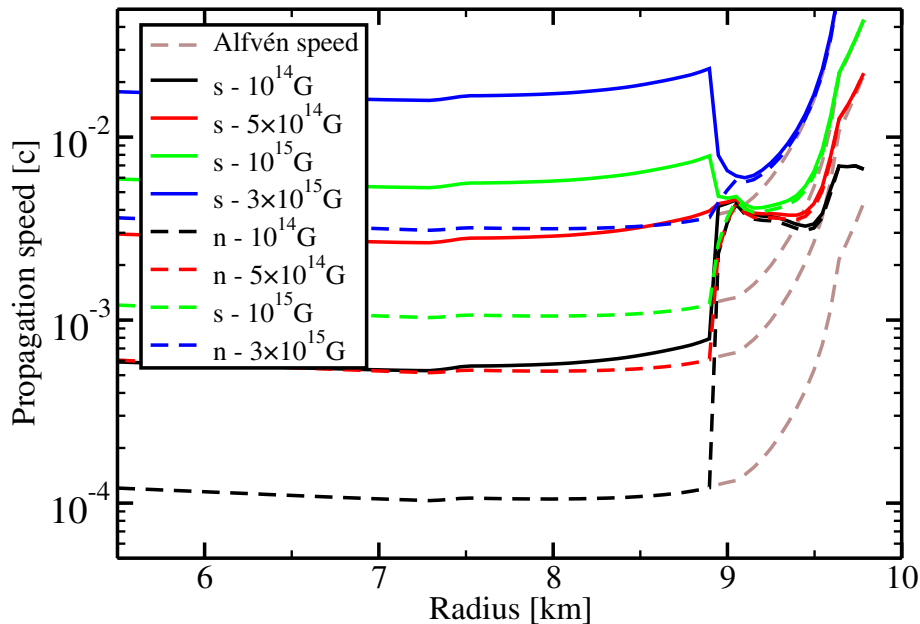
- $n = 1$  radial shear mode structure
- Localized close to equatorial plane
- $\hat{B} \perp \hat{r} \Rightarrow$  predominantly shear mode only in crust

## Superfluid

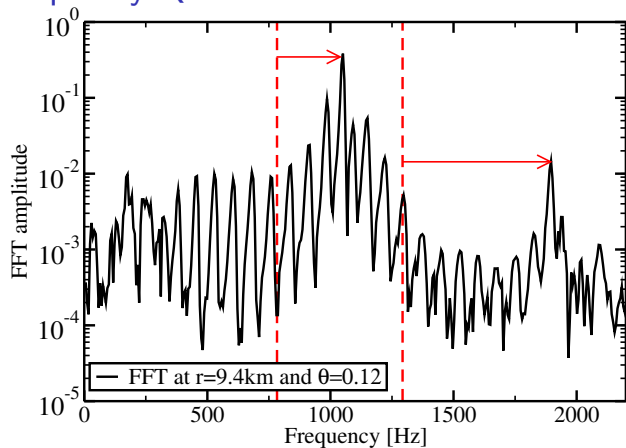
- $n = 1$  radial shear mode structure
- Close to pole
- Resonance with Alfvén overtone of core



## Propagation speeds



## High frequency QPOs



- Initial perturbation with crustal mode (red dashed lines)
- Resonantly excited magneto-elastic oscillation always at higher frequency
- $f_{2t_n} = f_{2t_n}^0 (1 + a_{2t_n} \bar{B}_{15})^{1/2}$

## Identifying observed frequencies

- Frequency ratio of low frequency magneto-elastic QPOs (odd, even) is roughly

$$1 : 2 : 3 : 4 : 5 : \dots$$

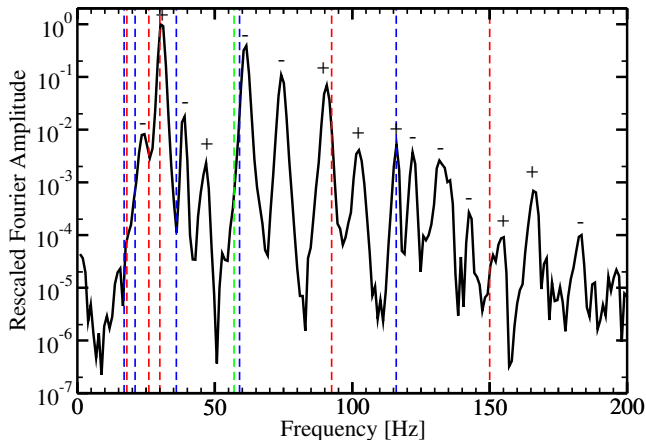
- Different magnetic field configurations gives more than one fundamental
- High frequency QPO as resonance of higher Alfvén overtone in core with  $n > 0$  crustal mode if core is superfluid

SGR 1806-20: (18), 26, 30, 92, 150, 625, 1840 Hz

SGR 1900+14: 28, 53, 84, 155 Hz

or 28, 53, 84, 155 Hz

## One particular example



- Match fundamental with 30Hz QPO
- Other observed QPOs  $f > 30$ Hz match nicely
- Problems for  $f < 30$ Hz

- APR+DH EoS
- $\bar{B} = 1.1 \times 10^{15}$  G
- $\varepsilon_* X_C = 0.046$
- $M = 1.4 M_\odot$
- $R = 12.26$  km

# Empirical relations

## Outbreak of oscillations

- Outbreak for  $\varepsilon_* X_c = 4 \times \varepsilon_*^0 X_c^0$  and  $\bar{B} = 10^{15}$  G
- All terms in equations contain either  $\sqrt{\mu}$  or  $(\varepsilon_* X_c)^{-1/2} \bar{B}_{14}$
- $\mu_{cc} \lesssim \mu_{cc}^{ref} (17.23 \varepsilon_* X_c)^{-1} \bar{B}_{14}^2$

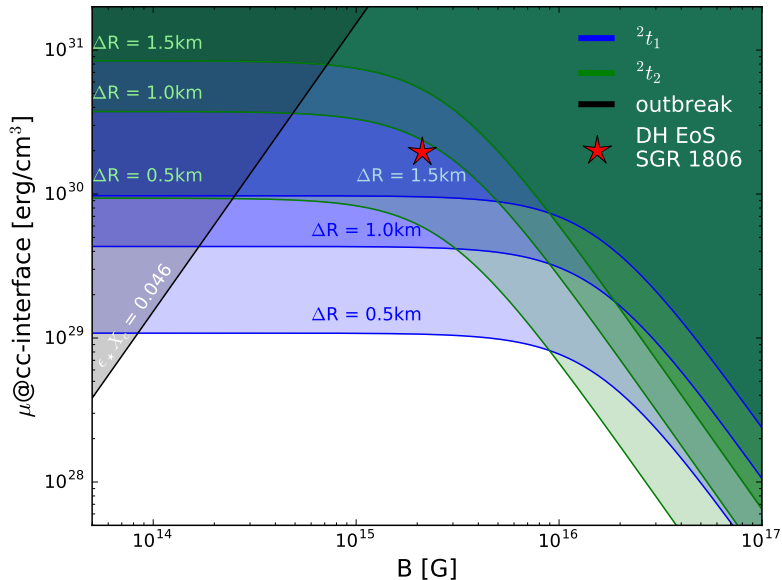
## High frequency QPO

- $f_{2t_n} = f_{2t_n}^0 (1 + a_{2t_n} \bar{B}_{15})^{1/2}$
- assuming  $v_s = \text{const.} \Rightarrow f_0 = \frac{v_s}{\Delta R} = \sqrt{\frac{\mu_{cc}}{\rho_{cc}}} \frac{1}{\Delta R} \Rightarrow \mu_{cc} \lesssim \frac{f_{\text{obs}}^2 \Delta R^2 \rho_{cc}}{1 + a_{2t_n} \bar{B}_{15}^2}$

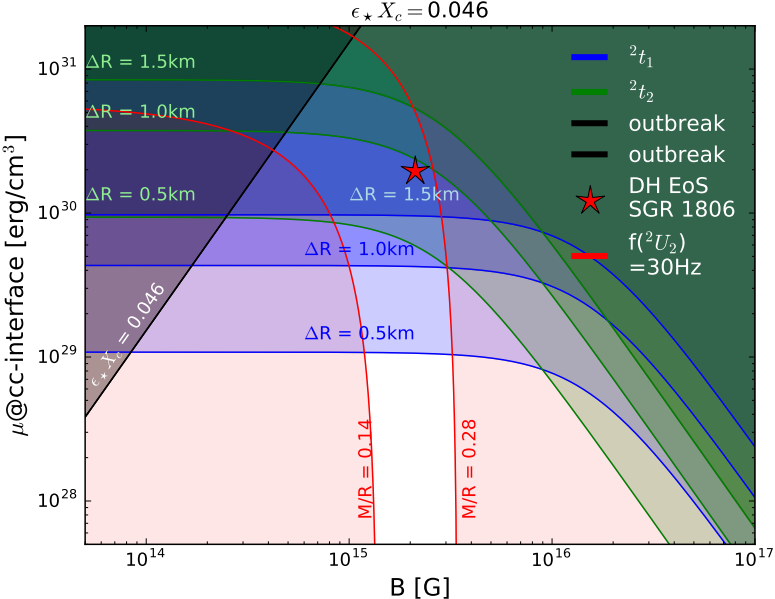
## Matching the frequency of ${}^2U_2$

$$f_{2U_2} [\text{Hz}] = \left( 2.8 \times (\varepsilon_* X_c)^{-0.55} \sqrt{\frac{\mu_{cc}}{\mu_{cc}^{ref}}} + 0.66 (\varepsilon_* X_c)^{-0.33} \bar{B} [10^{14} \text{G}] \right) \\ \times \left( \frac{1 - 4.58 M/R + 6.06 (M/R)^2}{1 - 4.58 (M/R)_{\text{ref}} + 6.06 (M/R)_{\text{ref}}^2} \right) \quad (\text{Sotani et al. 2008})$$

# Constraint on shear modulus at base of the crust

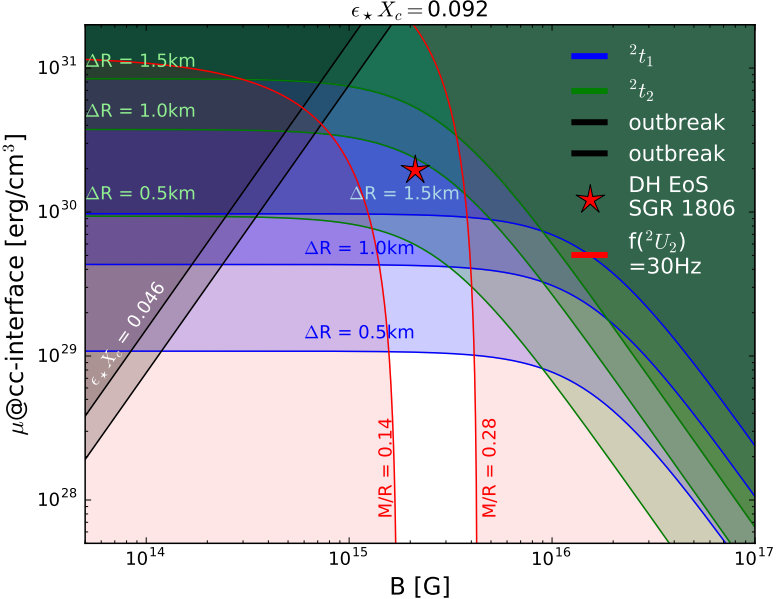


# Constraint on shear modulus at base of the crust

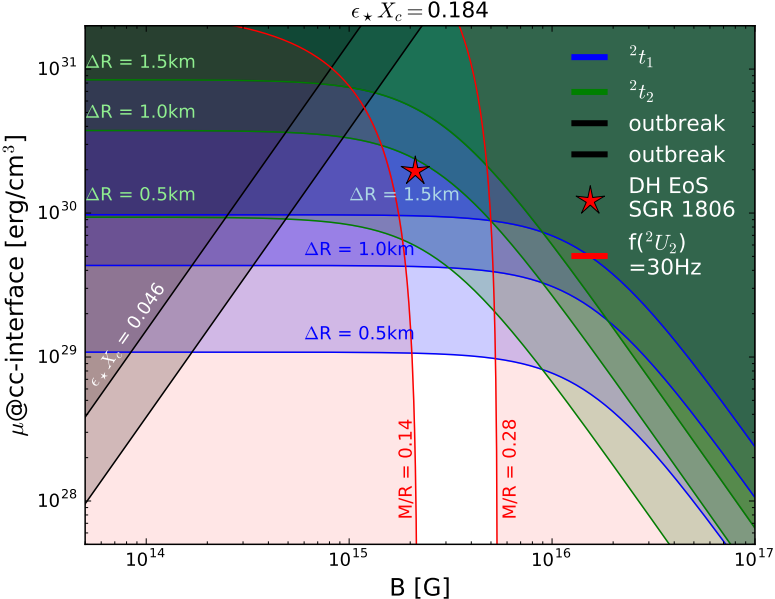




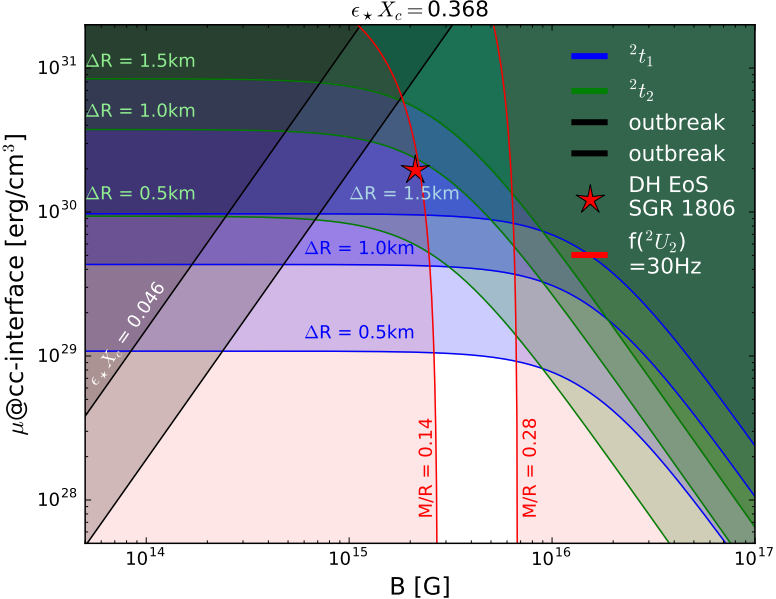
# Constraint on shear modulus at base of the crust



# Constraint on shear modulus at base of the crust

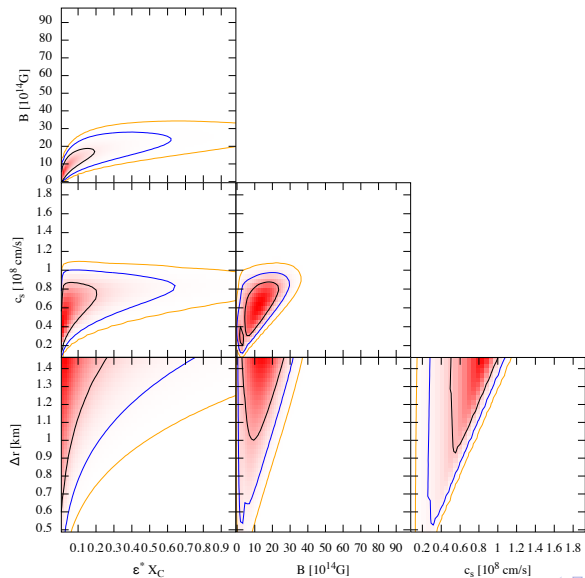


# Constraint on shear modulus at base of the crust

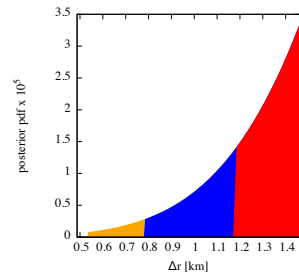
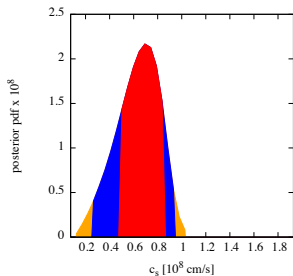
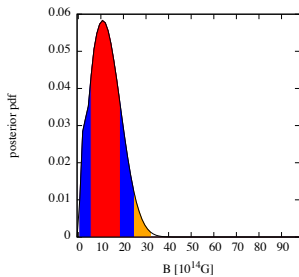
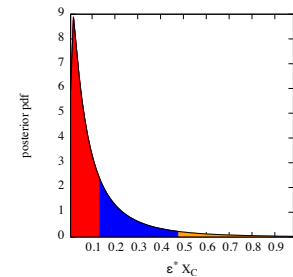


# Preliminary - Inferring constraints on the EOS

4 parameters:  $B$   $c_s$   $\Delta r$   $\epsilon_* X_c$



# Preliminary - Inferring constraints on the EOS



# Conclusions

- Crustal modes are damped too efficiently
- Magneto-elastic oscillations can explain magnetar QPOs
- Constraints on EoS from
  - ▶ Outbreak of oscillations
  - ▶ High frequency QPO
  - ▶ Matching of fundamental  ${}^2U_2$
- E.g. DH and  $\bar{B} = 2.1 \times 10^{15} \text{G}$  requires  $\varepsilon_{\star} X_c = 0.184$  and  $\Delta R \sim 2.0 \text{ km}$  (in conflict with theoretical models)
- Inference gives
  - ▶ B-field estimates in agreement with spin-down magnetic field strengths
  - ▶ Preference for superfluid core
  - ▶ low shear speeds ( $c_s \lesssim 10^8 \text{ cm/s}$ )



## Conclusions II - meeting reality

- Problem: degeneracy between EoS, superfluid properties, magnetic field strength and configuration
- ⇒ Solution: see JEROME's talk  
many simulations with different EoS and magnetic fields
- Problem:
  - ▶ Very limited observations
  - ▶ Robustness of observed pattern 1:3:5 ?
  - ▶ Dependence on B-strength ?
- ⇒ Solution: new satellites or new giant flares
- Further generalizations of model:
  - ▶ Coupled toroidal-poloidal oscillations
  - ▶ Superconductivity
  - ▶ Modulation mechanism for emission

# Complicated problem and many papers published

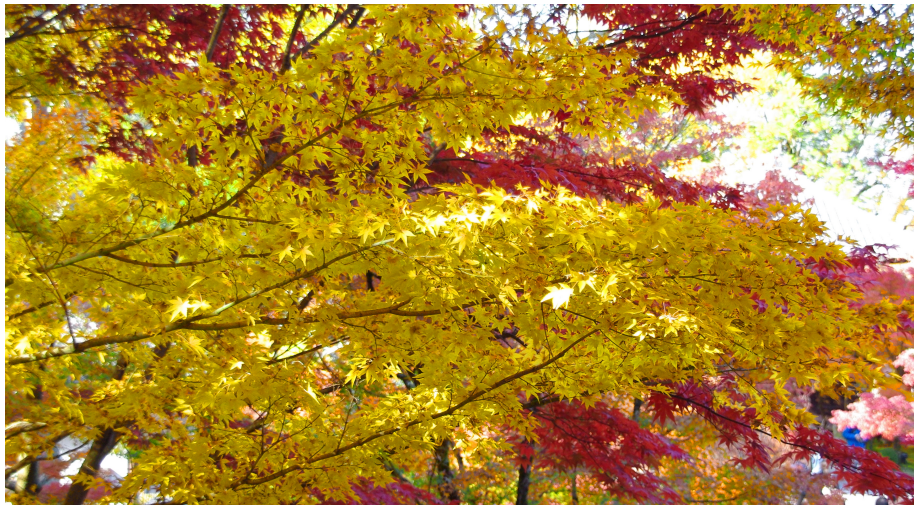




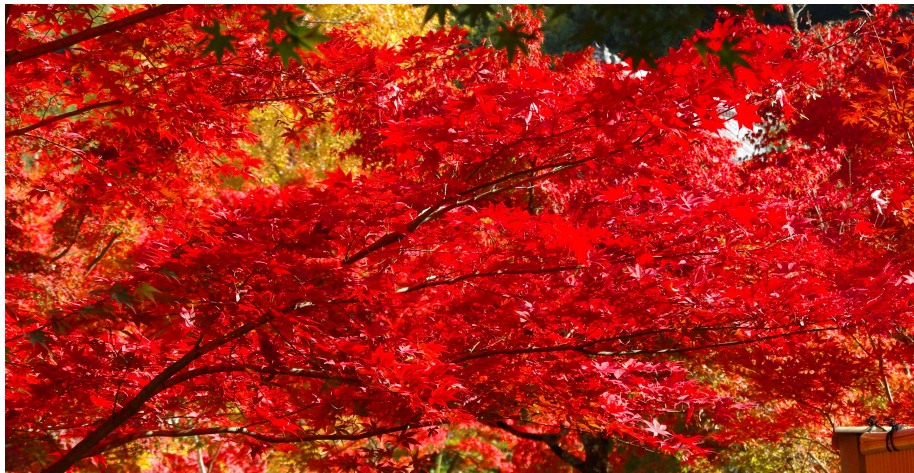
Some of you may become green with envy



It's only yellow press



You first become angry ... you are seeing red!

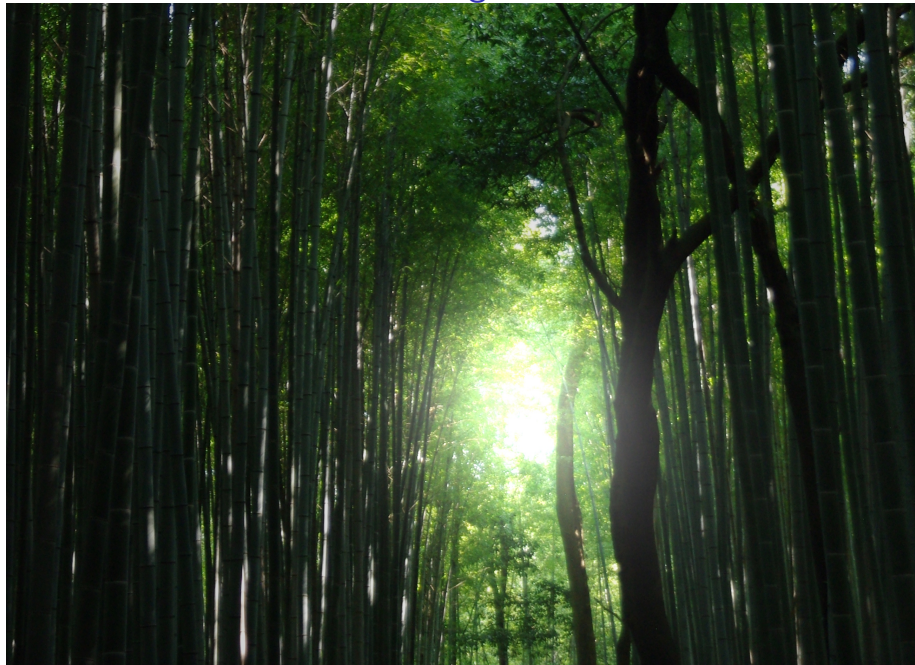


Be careful for not seeing the forest for the trees





But don't be afraid, there is light at the end of the tunnel



Doors will open



Sooner or later you will find epiphany



You will be able to walk on water



THANK YOU