How to form a millisecond magnetar? Magnetic field amplification in protoneutron stars

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Plan of the talk

1. Introduction: Magnetic fields in core collapse supernovae

2. Can the magnetorotational instability grow? Linear analysis
   → Effects of neutrino radiation

3. How strong is the final magnetic field? Numerical simulations
   → Channel mode termination
   → Influence of buoyancy
   → Dependence on the magnetic Prandtl number
   → The dawn of global simulations

4. Conclusion & perspectives
Core collapse: formation of a neutron star

Introduction

Massive star

- Hydrogen
- Helium
- Oxygen
- Iron

- 600 millions km
- 3000 km
- 1.4 M$_{\text{sol}}$

Collapse of the iron core

- 1 sec
- 40 km

Neutrino emission

Explosion

- Iron
- NS
- Neutrino emission
- Stalled accretion shock
- ?

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A diversity of explosions

**Explosion kinetic energy:**
- Typical supernova: $10^{51}$ ergs
- Rare hypernova (& GRB): $10^{52}$ ergs
- Neutrino driven explosions?
  - e.g. Bruenn+14, Melson+15
- Millisecond magnetar?
  - e.g. Burrows+07, Takiwaki+09,11 Bucciantini+09, Metzger+11

**Total luminosity:**
- Typical supernova: $10^{49}$ ergs
- Superluminous supernovae: $10^{51}$ ergs
- Millisecond magnetar?
  - e.g. Woosley+10, Dessart+12, Nicholl+13, Inserra+13
Magnetic explosions?

Strong magnetic field: $B \sim 10^{15}$ G
+ fast rotation (period of few milliseconds)

$\Rightarrow$ powerful jet-driven explosions!

e.g. Sibata+06, Burrows+07, Dessart+08, Takiwaki+09,11, Winteler+12

But in 3D, jets can be unstable to kink instability
Moesta+2014

Open question:
Can magnetic explosions explain hypernovae?
Are millisecond magnetars powering superluminous supernovae?

Delayed energy injection by magnetar spin-down on timescale of weeks-months

=> very high luminosity

Light curves can be fitted by:
- strong dipole magnetic field:
  \( B \sim 10^{14}-10^{15} \, \text{G} \)
- fast rotation:
  \( P \sim 1-10 \, \text{ms} \)

Talk by Ken Chen last week
Magnetars:
- Anomalous X-ray pulsars (AXP)
- Soft gamma repeater (SGR)

Strong dipole magnetic field:
\[ B \sim 10^{14}-10^{15} \, \text{G} \]

Slow rotation:
\[ P \sim 1-10 \, \text{s} \]

Typical age:
\[ 10^4-10^5 \, \text{years} \]

Rotation at birth unknown: were some or all of them born as millisecond magnetars?
Introduction

Missing theoretical piece: magnetic field origin

Huge range of magnetic field strength:

→ Initially « weak » magnetic field: \( \lesssim 10^9 \, G \) ( ? )

→ After compression by the core-collapse: \( \lesssim 10^{12} - 10^{13} \, G \) ( ? )

→ Magnetar strength: \( \sim 10^{15} \, G \)

Amplification mechanism?

Magnetorotational instability (MRI)?
Similar to accretion disks
→ application to protoneutron stars

Convective dynamo?
Similar to solar & planetary dynamos
→ need of numerical simulations for neutron stars
In ideal MHD (i.e. no resistivity or viscosity) :

Condition for MRI growth \( \frac{d\Omega}{dr} < 0 \)

Growth rate : \( \sigma = \frac{q}{2} \Omega \)

with \( \Omega \propto r^{-q} \)

\[ \rightarrow \text{Fast growth for fast rotation} \]

Wavelength : \( \lambda \propto \frac{B}{\sqrt{\rho \Omega}} \)

\[ \rightarrow \text{Short wavelength for weak magnetic field} \]
Rotation frequency profile:
→ Differential rotation at radii > 10 km

Rotation frequency decreases with radius: => MRI unstable!

Obergaulinger et al (2009)
Main differences between proto-neutron stars and accretion disks:

→ Neutrinos: viscosity and drag
   → Prevent MRI growth ?

→ Buoyancy: radial entropy and composition gradients
   → Impact on magnetic field amplification by MRI ?

→ Geometry: spherical vs thin disk
   → Help global coherence ?
2. Can the magnetorotational instability (MRI) grow?

Effects of neutrino radiation
MRI growth

Effects of neutrino radiation: two regimes

- Diffusive regime: neutrino viscosity
- Optically thin regime: neutrino drag

Neutron star structure from a simulation by Hanke et al (2013)
MRI with neutrino viscosity

Dimensionless number: $E_{\nu} \equiv \frac{v_A^2}{\nu \Omega} \sim 0.02 \left( \frac{B}{10^{13} \, \text{G}} \right)^2 \left( \frac{\rho}{10^{13} \, \text{g.cm}^{-3}} \right)^{-1} \left( \frac{\Omega}{2000 \, \text{s}^{-1}} \right)^{-1} \left( \frac{\nu}{2 \times 10^{10} \, \text{cm}^2 \cdot \text{s}^{-1}} \right)^{-1}$

e.g. Pessah & Chan (2008)

MRI growth requires a minimum initial magnetic field strength of $> 10^{12} \, \text{G}$...
Neutrino drag: \(-\Gamma \mathbf{v}\), with damping rate: 
\[ \Gamma = \frac{4}{3} \frac{E_\nu}{l_\nu \rho c} \]

The MRI can grow near the PNS surface from any weak field strength!
MRI growth: different regimes

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MRI growth

Application to neutron star mergers

Guilet+2016

Hyaline

viscous

slow

drag


massive neutron star

torus

B field strength (G)

B field strength (G)

radius (km)

radius (km)

10^{15}

10^{14}

10^{13}

10^{12}

10^{11}

10^{10}

0

10

20

30

40

0

10

20

30

40

10^3

10^2

10^1

10^0

10^6

10^5

10^4

10^3

10^2

10^1

10^0

growth rate (s^{-1})

wavelength (cm)
3. How strong is the final magnetic field?

→ Channel mode termination

→ Influence of buoyancy

→ Dependence on the magnetic Prandtl number

→ The dawn of global simulations
Numerical simulations: local models

- Small box: ~km size at a radius $r \sim 20$-$40$ km
- Differential rotation
  $\Rightarrow$ shearing periodic boundary conditions
- Entropy/composition gradients
- Different numerical methods: spectral or finite volume
- Fully compressible or quasi-incompressible approximation

Obergaulinger+2009, Masada+2012, Guilet+2015, Rembiasz+2016a,b
Channel mode termination by parasitic instabilities

Rembiasz et al. 2016a&b
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Brünt-Väisälä frequency:

$$N^2 \equiv -\frac{g}{\rho} \left[ \frac{\partial \rho}{\partial S} \bigg|_{P,Y_e} \frac{dS}{dr} + \frac{\partial \rho}{\partial Y_e} \bigg|_{P,S} \frac{dY_e}{dr} \right]$$

Linear analysis of MRI with buoyancy:
stable buoyancy can stabilise the MRI

But: thermal diffusion allows the growth

Linear MRI growth with buoyancy

Confirms linear analysis:
thermal diffusion by neutrinos allows fast MRI growth
Impact of stratification on the MRI

MRI & buoyancy

unstable buoyancy

stable stratification

color: azimuthal magnetic field

Magnetic energy units of (10^{15} G)^2

Guilet & Müller (2015)

buoyancy parameter
3. How strong is the final magnetic field?
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Neutrino viscosity: \( \nu \sim 10^{10} \text{ cm}^2 \text{s}^{-1} \)

Resistivity: \( \eta \sim 10^{-3} \text{ cm}^2 \text{s}^{-1} \)

Magnetic Prandtl number: \( \mathcal{P}_m \equiv \frac{\nu}{\eta} \sim 10^{13} \)

Previous simulations used: \( \mathcal{P}_m = 4 \)

Behaviour at very large magnetic Prandtl number?
Dependence on the magnetic Prandtl number

Magnetic energy
units of \((10^{15} \text{ G})^2\)

\[ E_{\text{mag}} \propto P_m \]

Behaviour at very large magnetic Prandtl number?
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Local in vertical direction
global in horizontal direction

Good vertical resolution
but low horizontal resolution
Moesta+2015: first simulation with large-scale magnetic field generation... but started with magnetar strength dipolar field.
Global simulations

A simplified full-sphere MRI simulation

Preliminary simulations of a simplified model of full neutron star
→ incompressible approximation
→ start with a small-scale field of $\sim5 \times 10^{14} \text{ G}$
1. The MRI can grow in two different regimes:
   - Viscous regime deep inside the proto-neutron star:
     → requires a minimum initial magnetic field of $\sim 10^{12}$ G
   - Neutrino drag near the surface of the proto-neutron star

2. Final magnetic field strength:
   - Stratification matters: sub-magnetar strength in stable regions
   - Strong dependence on the magnetic Prandtl number
     → MRI may be more efficient than simulations suggest!
   - Generation of a dipolar magnetic field is still an open question
Still a long way to go: from the small to the large scales

Step 1: local MRI model
Step 2: global simulations
Step 3: hypernova & GRB jet

~ 1-5 km
~ 10-50 km
~ $10^5$-$10^6$ km

High Pm regime?
Neutrino drag regime?
Magnetic field geometry?
MRI vs convective dynamo
Explosion diversity?
Energy, jet properties, nucleosynthesis, luminosity etc..

Thanks!