The 26th Nishinomiya-Yukawa Memorial International Workshop
"Novel Quantum States in Condensed Matter 2011 (NQS2011)".

Chirality, topology and magnetotransport in a chiral helimagnet

Jun Kishine
Kyushu Institute of Technology

[1] JK, Proskurin and Ovchinnikov, **Phys. Rev. Lett. 107**, 017205 (2011)
[2] JK, Ovchinnikov, and Proskurin, **Phys. Rev. B 82**, 064407 (2010)
[3] JK and Ovchinnikov, **Phys. Rev. B 81**, 134405(2010)
[4] JK and Ovchinnikov, **Phys. Rev. B 79**, 220405**(R)** (2009)
[5] Borisov, JK, Bostrem, and Ovchinnikov, **Phys. Rev. B 79**, 134436(2009)
[6] Bostrem, JK and Ovchinnikov, **Phys. Rev. B78**, 064425(2008)

Collaborators

Experiments

✓ Lorentz TEM

- Yoshihiko Togawa (Osaka Pref. Univ., Japan)
- Shigeo Mori (Osaka Pref. Univ. , Japan)
- Tsukasa Koyama (Osaka Pref. Univ. , Japan)
- Sadafumi Nishihara(Hiroshima Univ. , Japan)

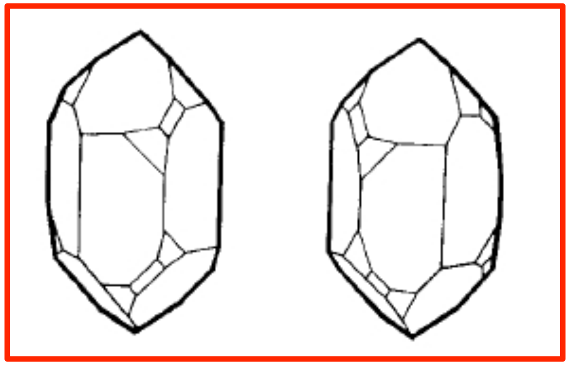
✓ Material synthesis and neutron diffraction

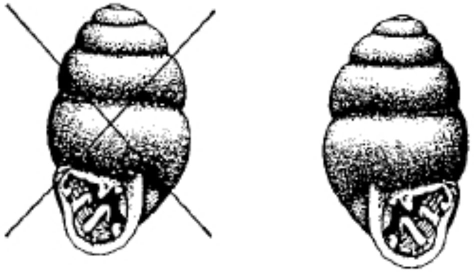
- Jun Akimitsu (Aoyama Gakuin Univ. , Japan)
- Yusuke Kousaka (Aoyama Gakuin Univ. , Japan)
- Katsuya Inoue (Hiroshima Univ. , Japan)

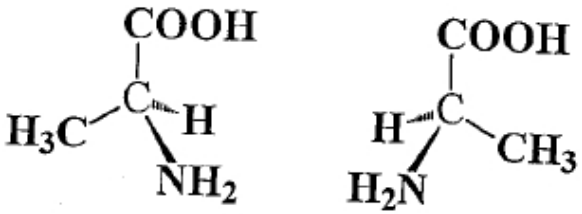
Theory

- Alexander Ovchinnikov(Ural Federal Univ., Russia)
- Igor Proskurin (Ural Federal Univ. , Russia)

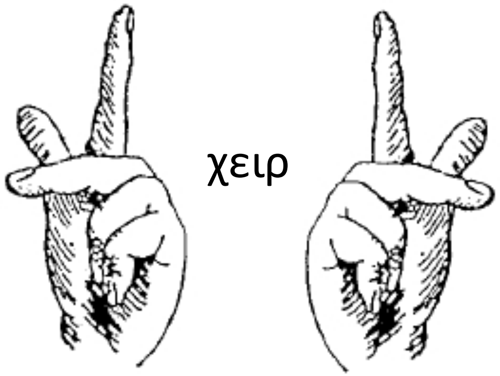
Chirality in nature



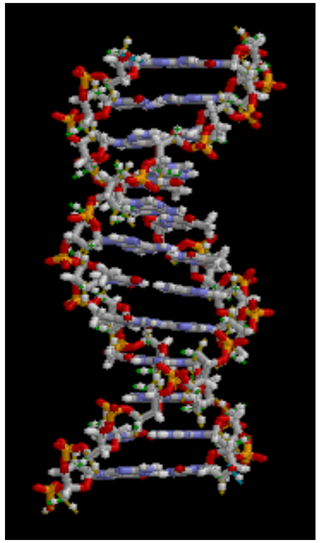




光学活性酸



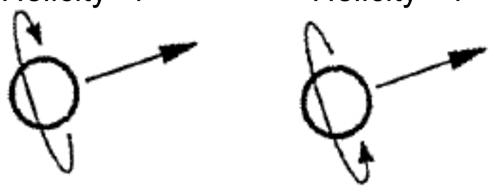
χειρ



DNA

Helicity -1

Helicity +1



Kitzerow and Bahr (eds.) Chirality in liquid crystals (Springer, 2001)

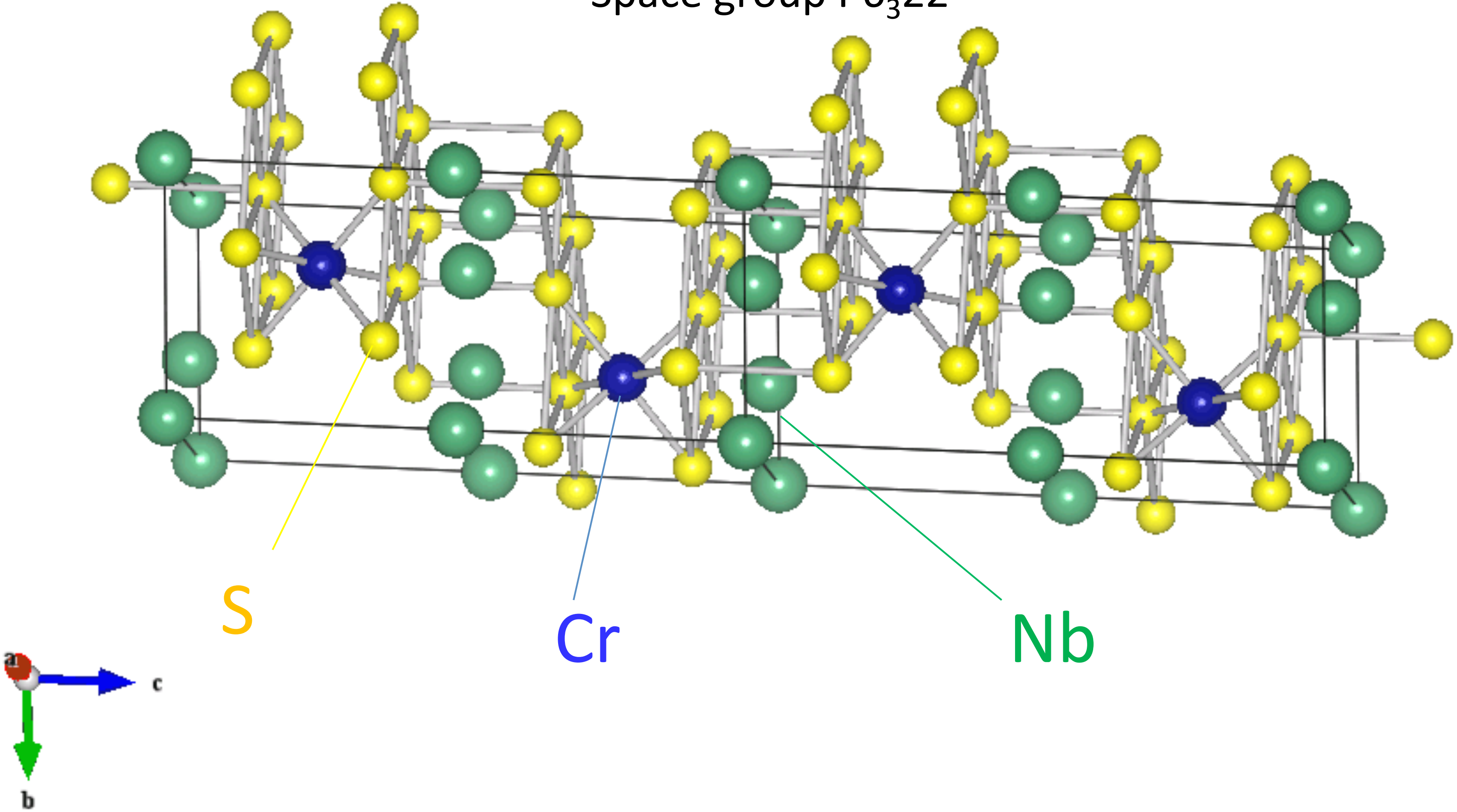
Lewis Carroll,
"Through the Looking-Glass and
What Alice Found There"

Macroscopic functions of a single crystal generally comes from

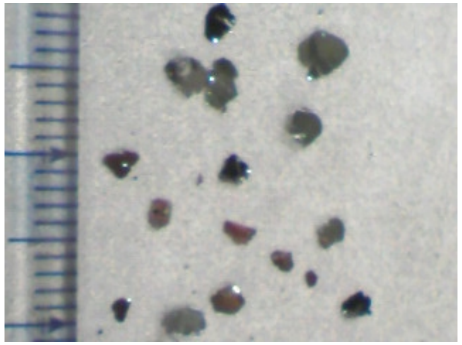
- ✓ ***Asymmetry***
- ✓ ***Non-linearity***
- ✓ ***Off-equilibrium***

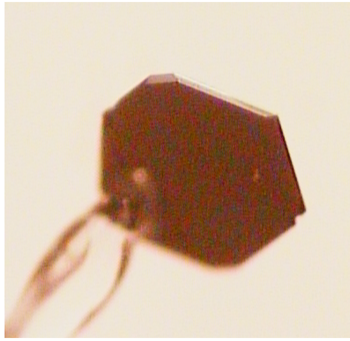
Our theory is motivated by Chiral Magnetic Crystal $Cr_{1/3}NbS_2$

Space group $P6_322$



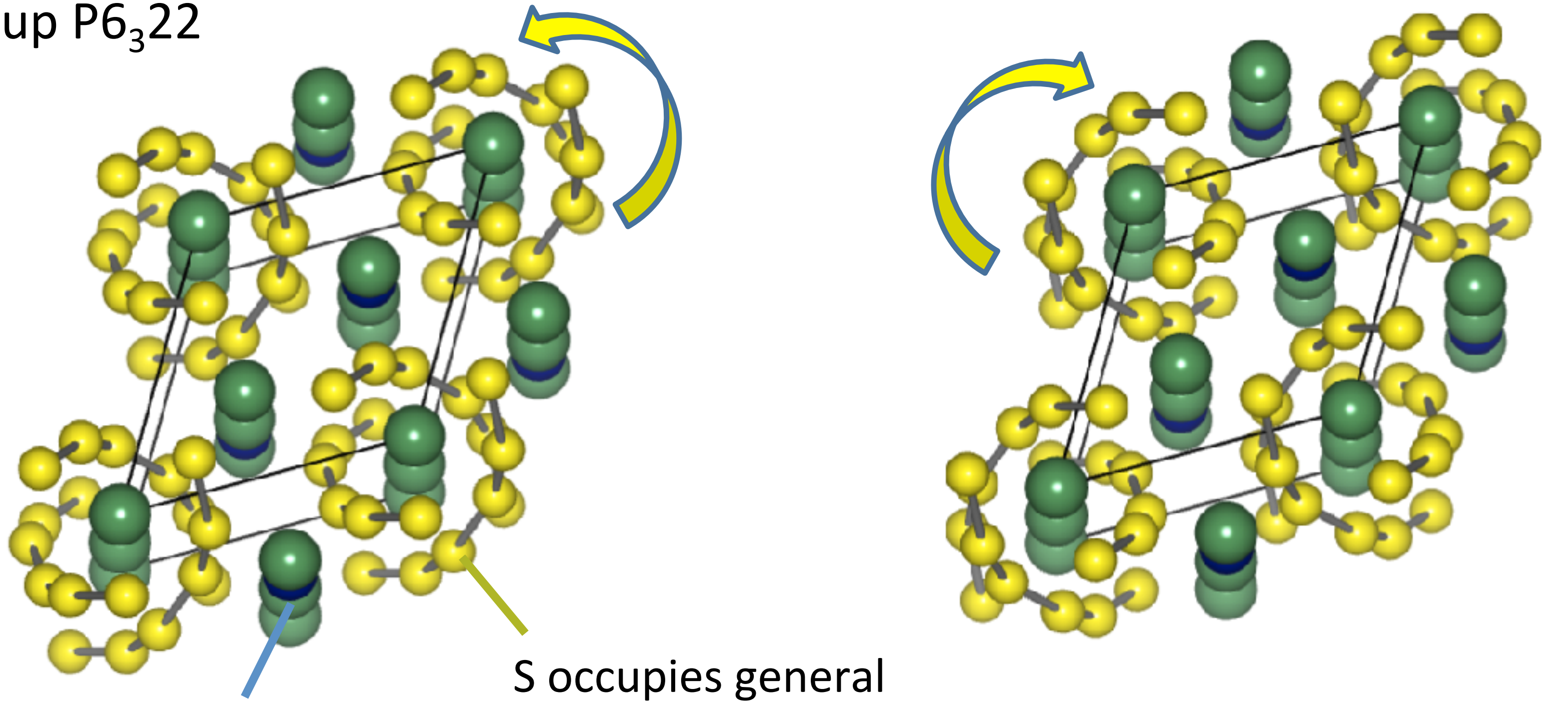
Chiral magnetic crystal $Cr_{1/3}NbS_2$





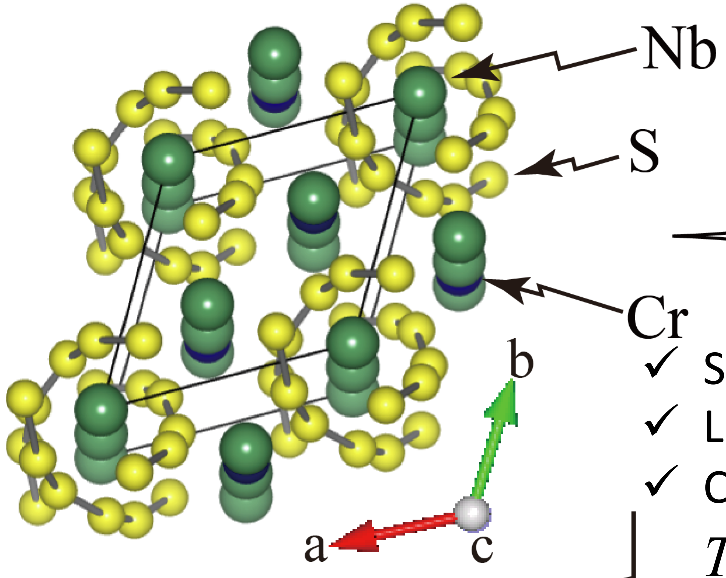
Yusuke Kousaka
and
Jun Akimitsu

Space group $P6_322$



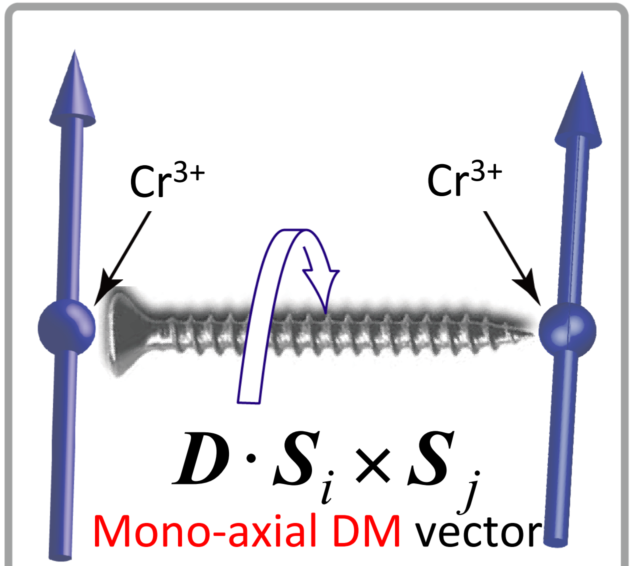
- Chiral symmetry breaking
- Spins see crystal chirality via spin-orbit coupling

Chiral helimagnetic order of Cr's S=3/2

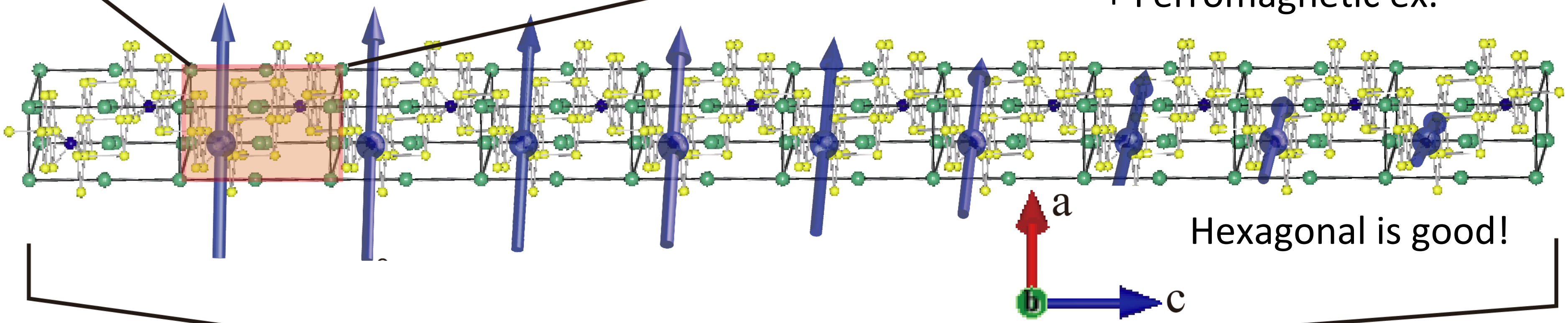


- ✓ Spin-orbit
- ✓ Lifshitz invariant
- ✓ Complex 1D IR

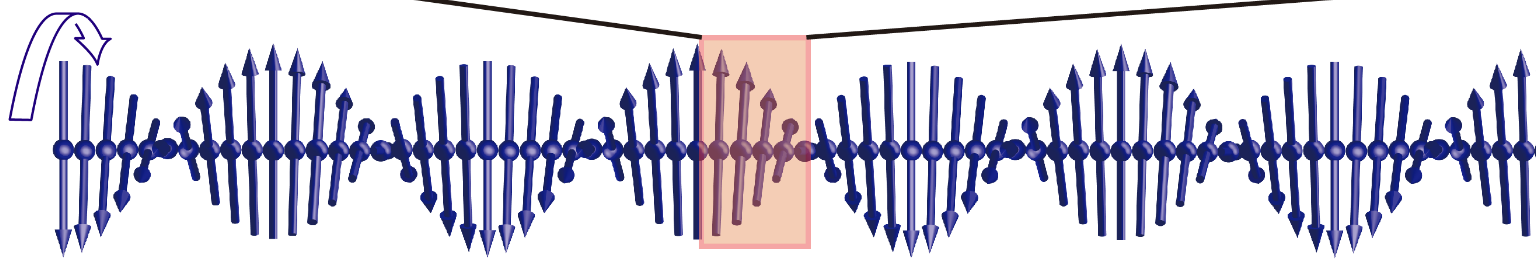
$T_c \approx 125\text{K}$



Mono-axial DM vector along chiral axis (c axis)
+ Ferromagnetic ex.



Hexagonal is good!

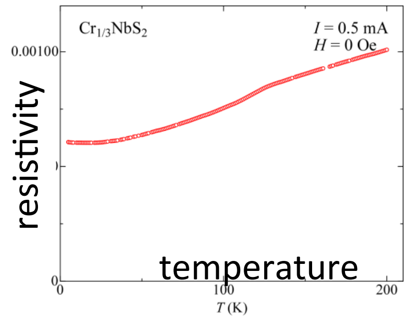


$$L_0 = \frac{2\pi}{Q_0} = \frac{a_0}{\arctan(D/J)} \simeq \frac{J}{D} a_0$$

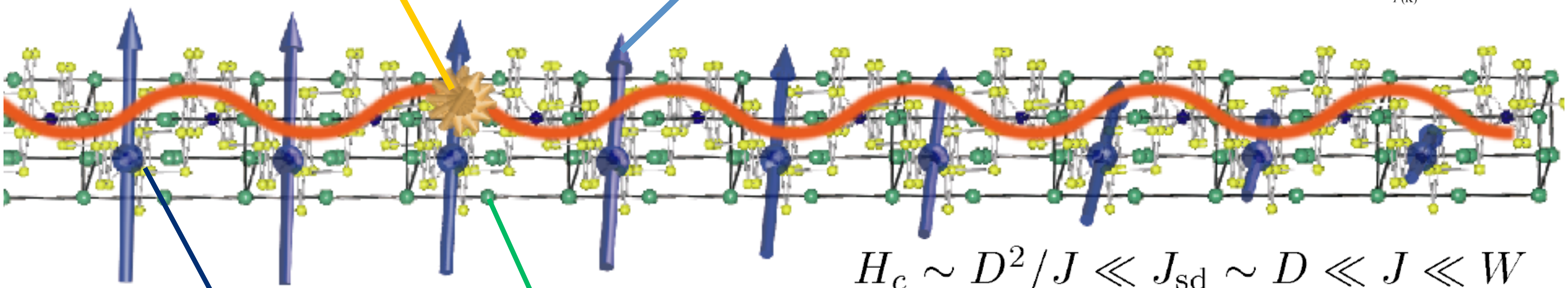
Chiral helimagnetic order
= Spin Phase Object

	Materials	Space Group
Metal	MnSi	$P2_13$
Metal	$Fe_{1-x}Co_xSi$	$P2_13$
Metal	FeGe	$P2_13$
Metal	$Cr_{1/3}NbS_2$	$P6_322$
Insulator	$CsCuCl_3$	$P6_122$
Insulator	CuB_2O_4	$I\bar{4}2d$

Coupling of localized and itinerant spins



Itinerant Quantum Spin and Localized Classical Spin



$$H_c \sim D^2/J \ll J_{\rm sd} \sim D \ll J \ll W$$

3d electron of Cr
⇒Localized S=3/2

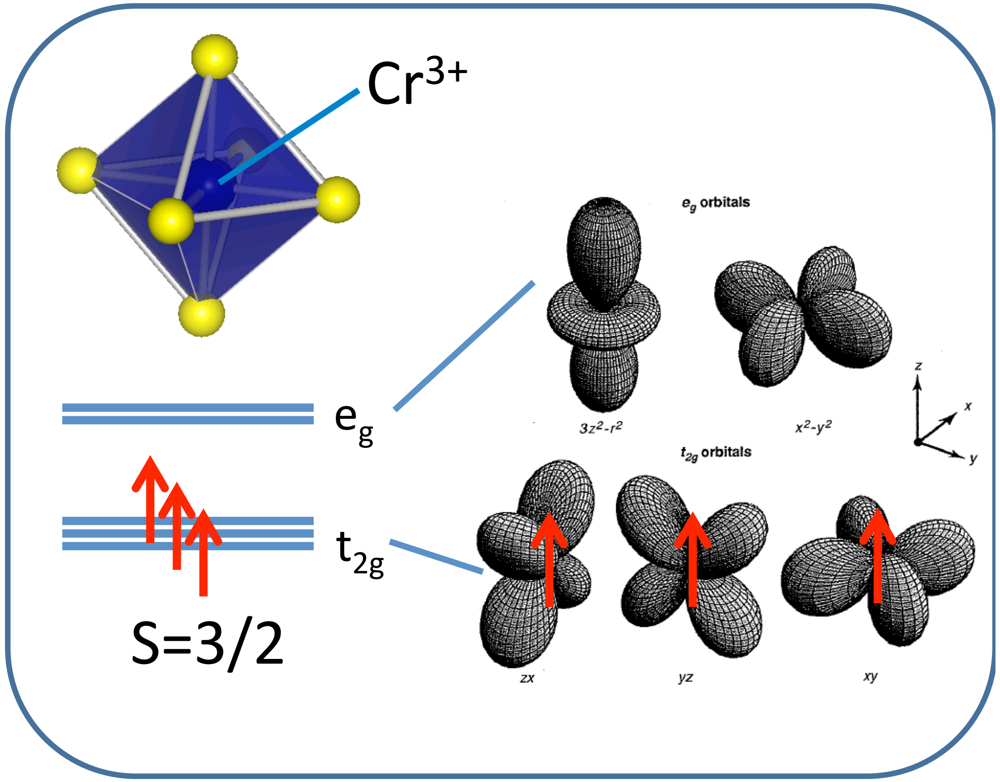
4d electron of Nb
⇒ Itinerant S=1/2

✓ Only t_{2g} orbitals occupied
 ⇒Low orbital symmetry
 ⇒Hybridization with Nb's 4d orbit difficult
 ⇒ High localizability of Cr's mag. mom.

✓ e_g orbitals unoccupied
 ⇒ excitations from t2g to eg
 ⇒orbital fluctuations active
 ⇒ DM interaction enhanced

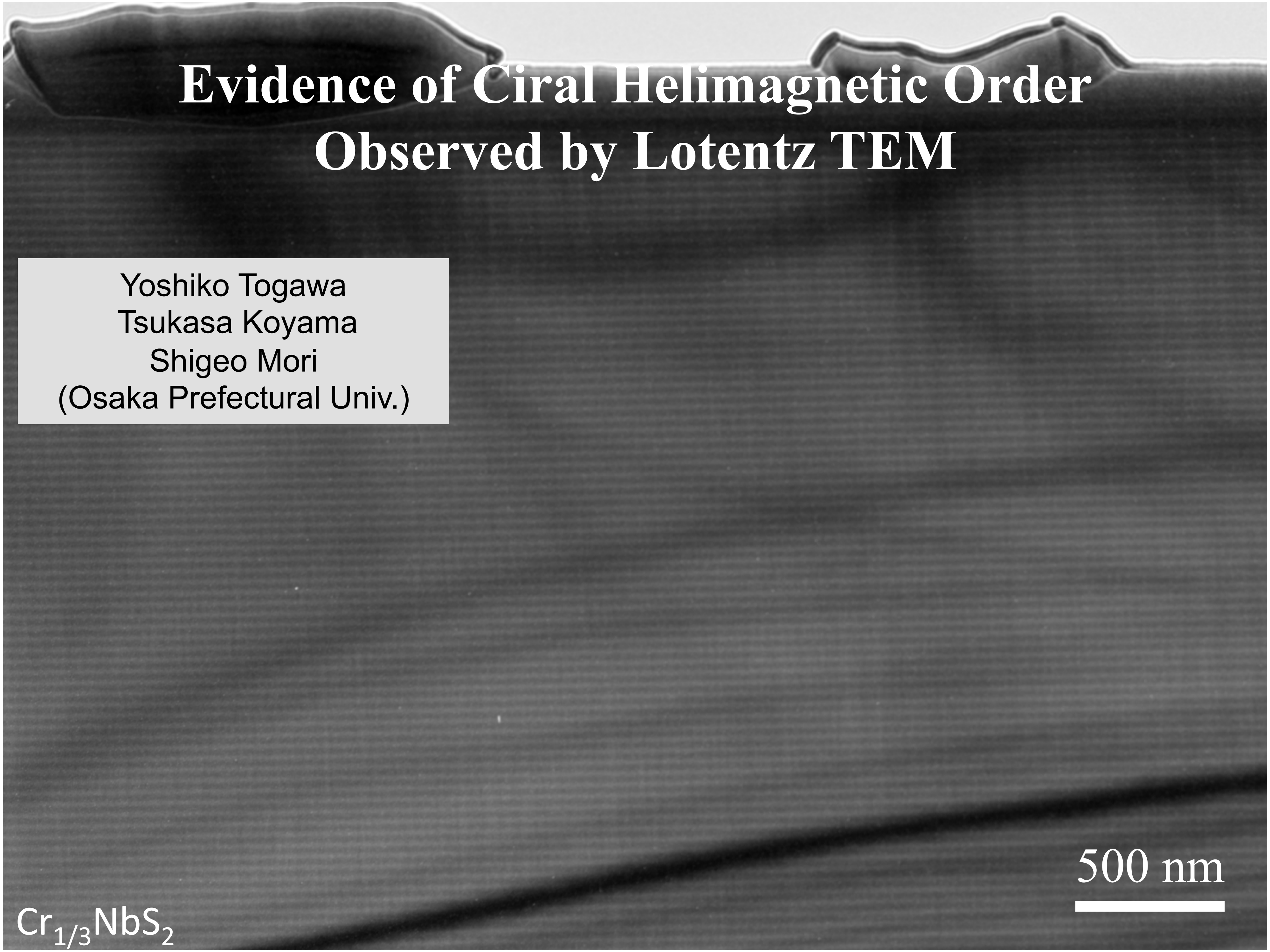
$Cr_{1/3}NbS_2$

- Chiral crystal ⇒DM
- highly localized S=3/2 and itinerant S=1/2 coupled
- conducting

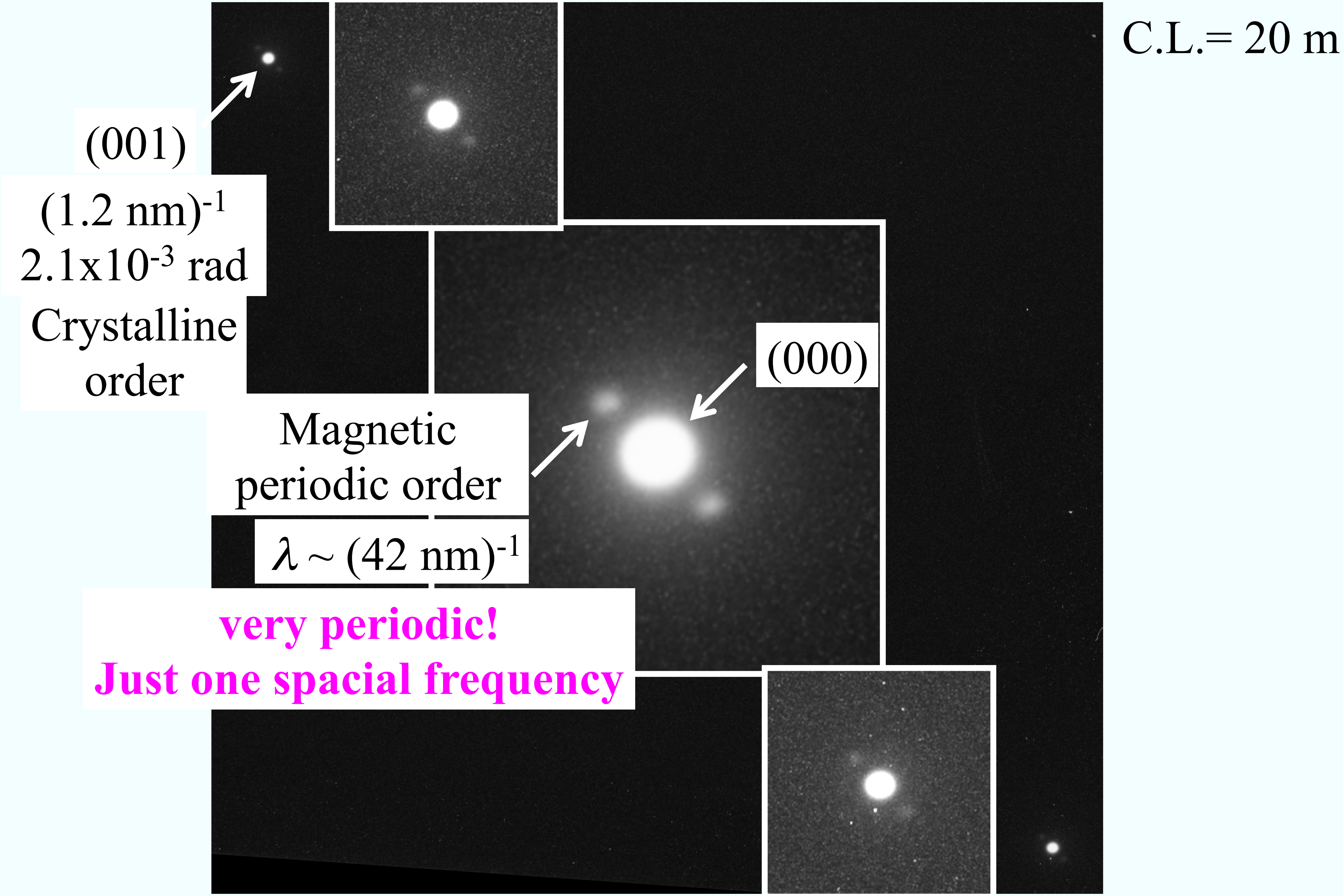


Evidence of Ciral Helimagnetic Order Observed by Lotentz TEM

Yoshiko Togawa
Tsukasa Koyama
Shigeo Mori
(Osaka Prefectural Univ.)

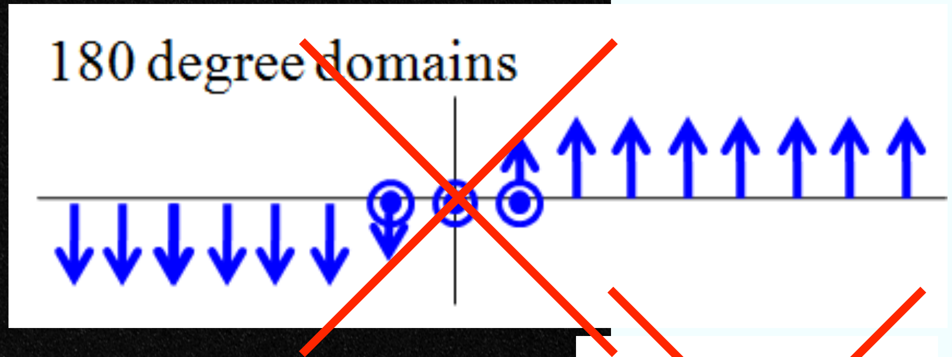


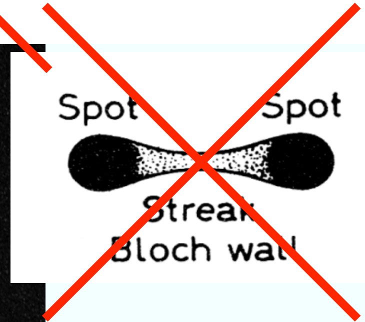
Small angle electron diffraction (SAED)

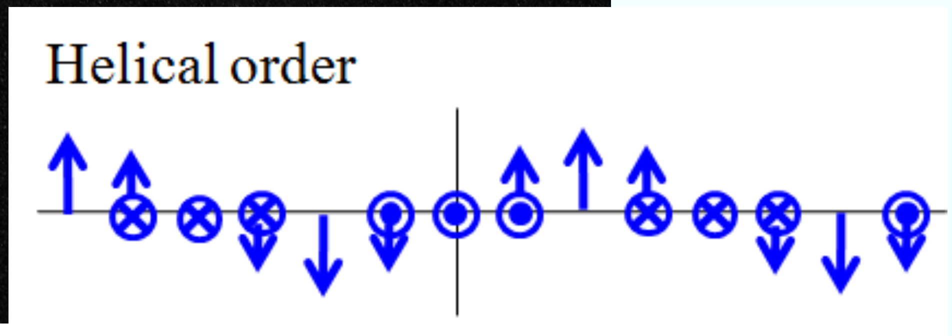


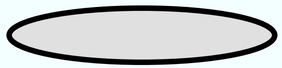
Magnetic small angle electron diffraction (MSAED)

C.L.= 300 m









Qualitatively: excellent!

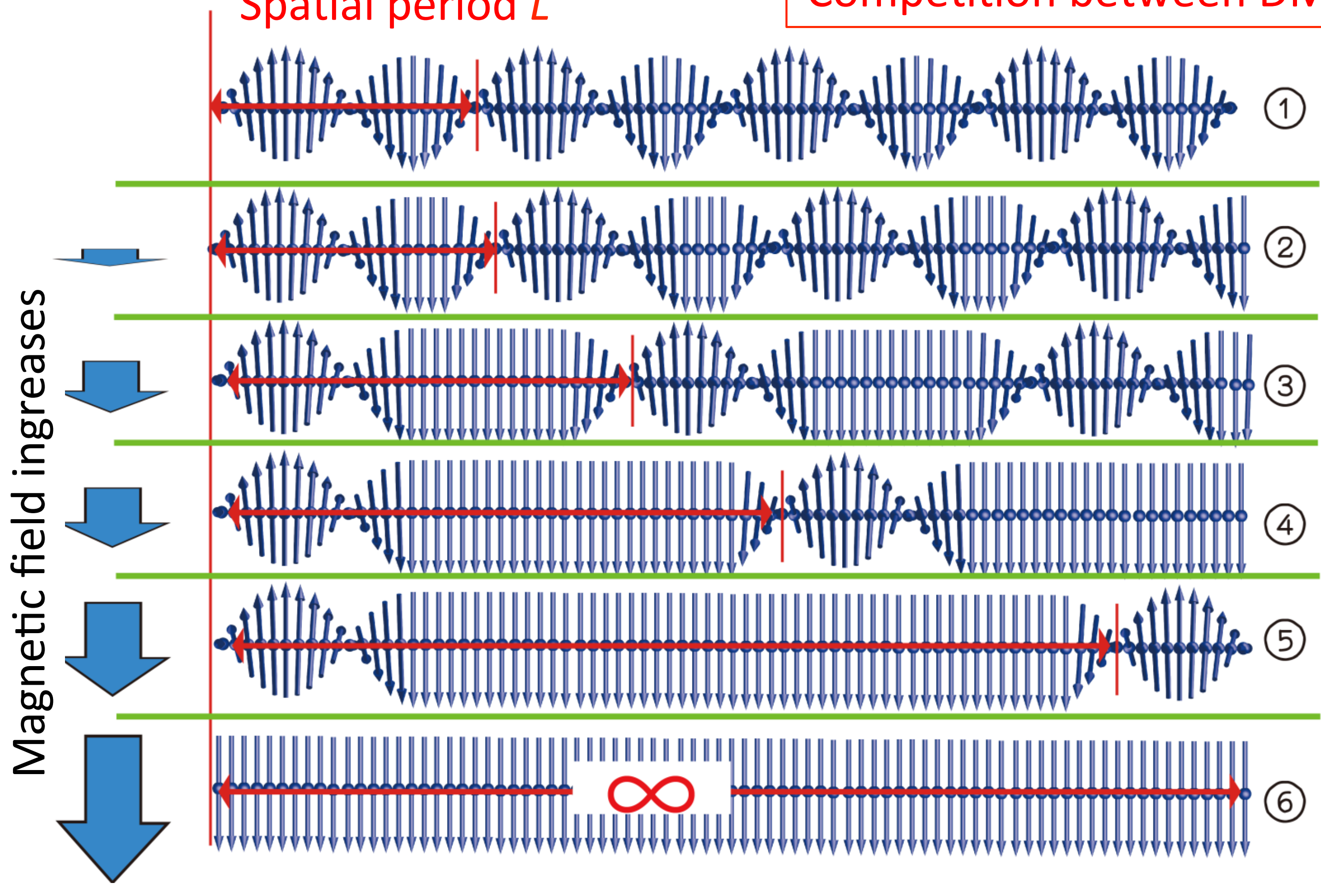
Quantitatively: reasonable?

5 x 10^{-6} rad, *t* :50 nm

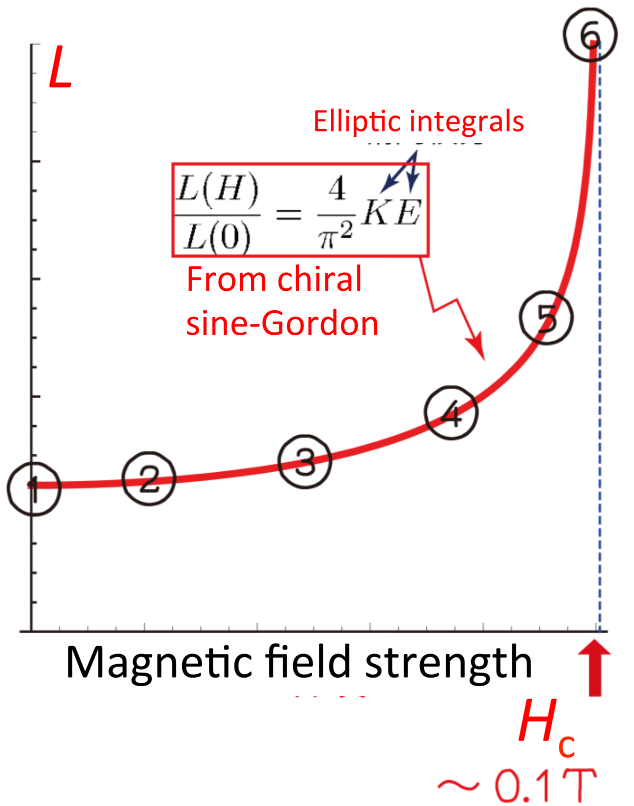
I : 0.18 T ← 0.24 T (1.5 u_B/Cr)

Ground state under perpendicular mag. field : Chiral soliton lattice（CSL） structure

Competition between DM and Zeeman



CSL =Magnetic Kink Crystal



- DM interaction = macroscopic but weaker than J
- Energy scale to control mag. texture is very weak
- Non-linear, asymmetric mag. texture

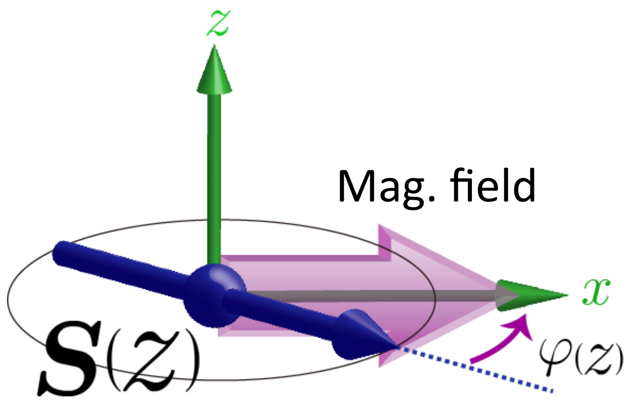
Chiral sine-Gordon model

- effective 1D model→'soliton'

$$\mathscr{H}_{\mathrm{CSL}} = \frac{JS^2}{a_0}\int_0^L dz \left[\frac{1}{2}\left(\partial_z\varphi\right)^2 - Q_0\left(\partial_z\varphi\right) - m^2\cos\varphi\right]$$

$$m^2 = \frac{\tilde{H}}{JSa_0^2}$$

I.E. Dzyaloshinskii, Sov. Phys. JETP 19, 960 (1964)
P.G.de Gennes, Solid State Commun. 6, 163 (1968)



- Helical period at zero field

$$L_0 = \frac{2\pi}{Q_0} = \frac{a_0}{\arctan\left(D/J\right)} \simeq \frac{J}{D}a_0$$

- Stationary solution

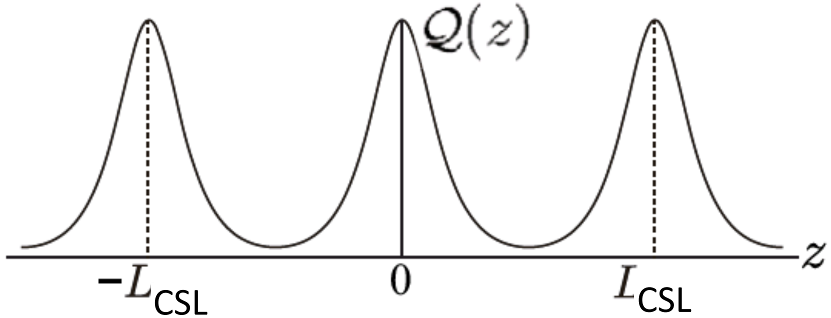
$$\cos\left(\frac{\varphi_0(z)}{2}\right) = \mathrm{sn}\left(\frac{m}{\kappa}z\right)$$

⇨

$$L_{\mathrm{CSL}} = \frac{2\kappa K(\kappa)}{m} = \frac{8K(\kappa)E(\kappa)}{\pi Q_0}$$

- Topological charge

$$\mathscr{Q}(z) = \frac{1}{2\pi}\partial_z\varphi_0(z) = \frac{1}{\pi}\frac{m}{\kappa}\mathrm{dn}\left(\frac{m}{\kappa}z\right)$$



Topology protected by geometric chirality

Energy minimization with respect to elliptic modulusκ

$$\mathscr{E}_{\text{kink}}(\kappa) = \frac{1}{L_{\text{MKC}}}\int_{-L_{\text{MKC}}/2}^{L_{\text{MKC}}/2} dz \left[4\left(\frac{m}{\kappa}\right)^2 \text{dn}^2\left(\frac{m}{\kappa}z\right) - 2\left(\frac{m}{\kappa}\right)^2 - \partial_z\varphi_0\right]$$

$$= 2m^2\left(\frac{2E(\kappa)}{\kappa^2 K(\kappa)} - \frac{1}{\kappa^2} - \frac{\pi Q_0}{2m}\frac{1}{\kappa K(\kappa)}\right),$$

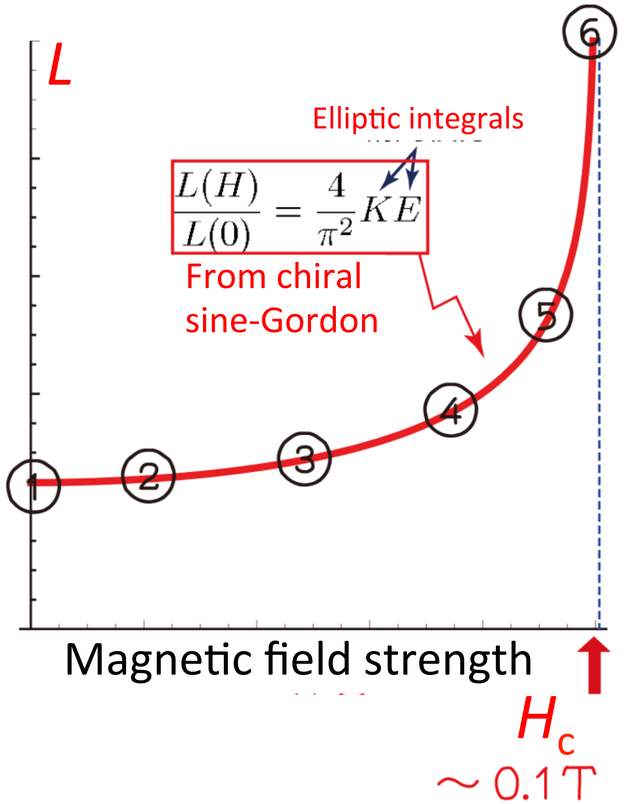
⇩

$$\sqrt{\frac{H}{H_c}} = \frac{\kappa}{E(\kappa)}$$

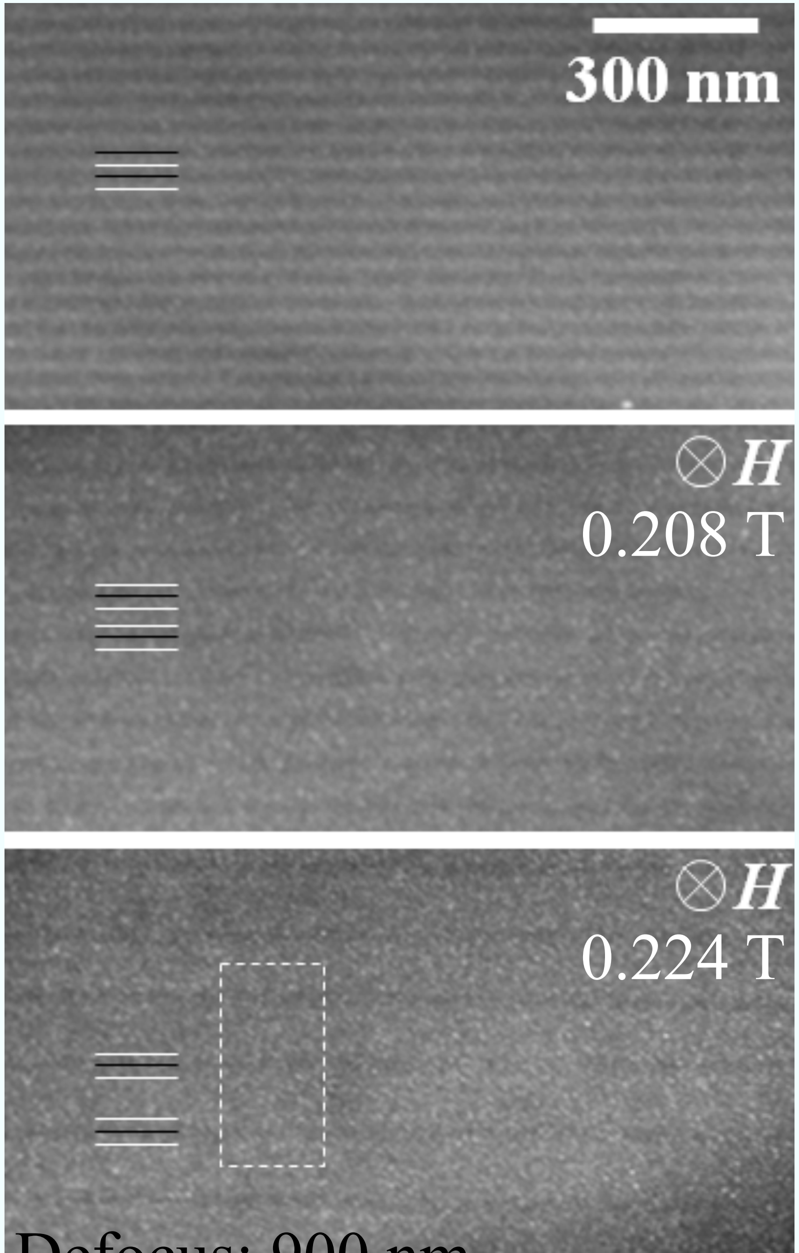
$$\frac{L_{\text{CSL}}}{L_0} = \frac{4K(\kappa)E(\kappa)}{\pi^2}$$

$$H_c = JS\left(\frac{\pi Q_0 a_0}{4}\right)^2 \sim JS\left(\frac{\pi}{4\times 40}\right)^2$$

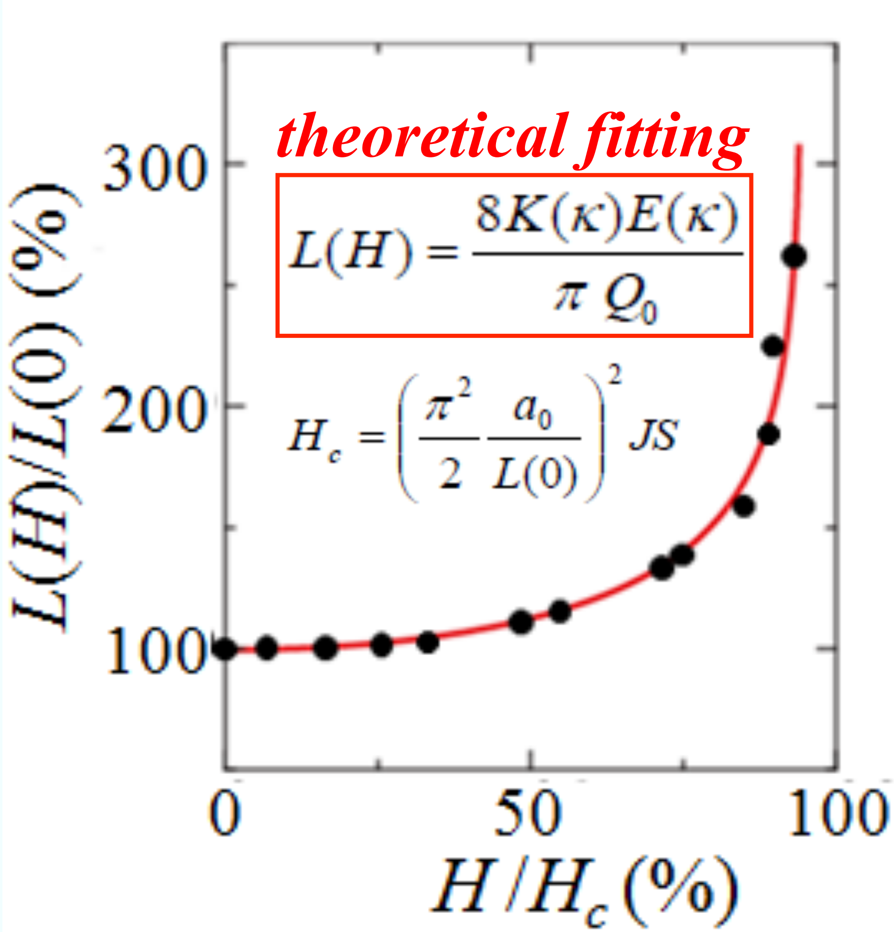
$$\sim 0.1\text{K} \sim 0.1\text{T} \longrightarrow JS \times \left(\frac{D}{J}\right)^2$$



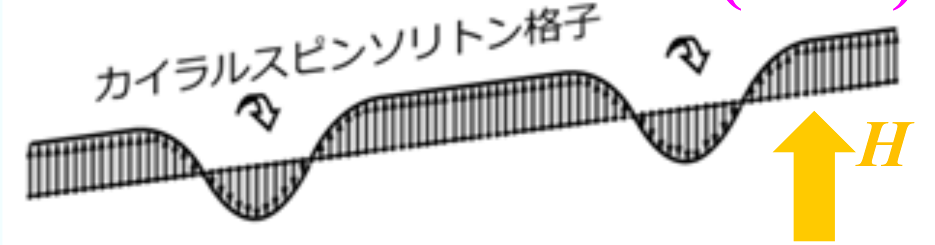
Chiral soliton lattice (CSL) in magnetic fields



Defocus: 900 nm



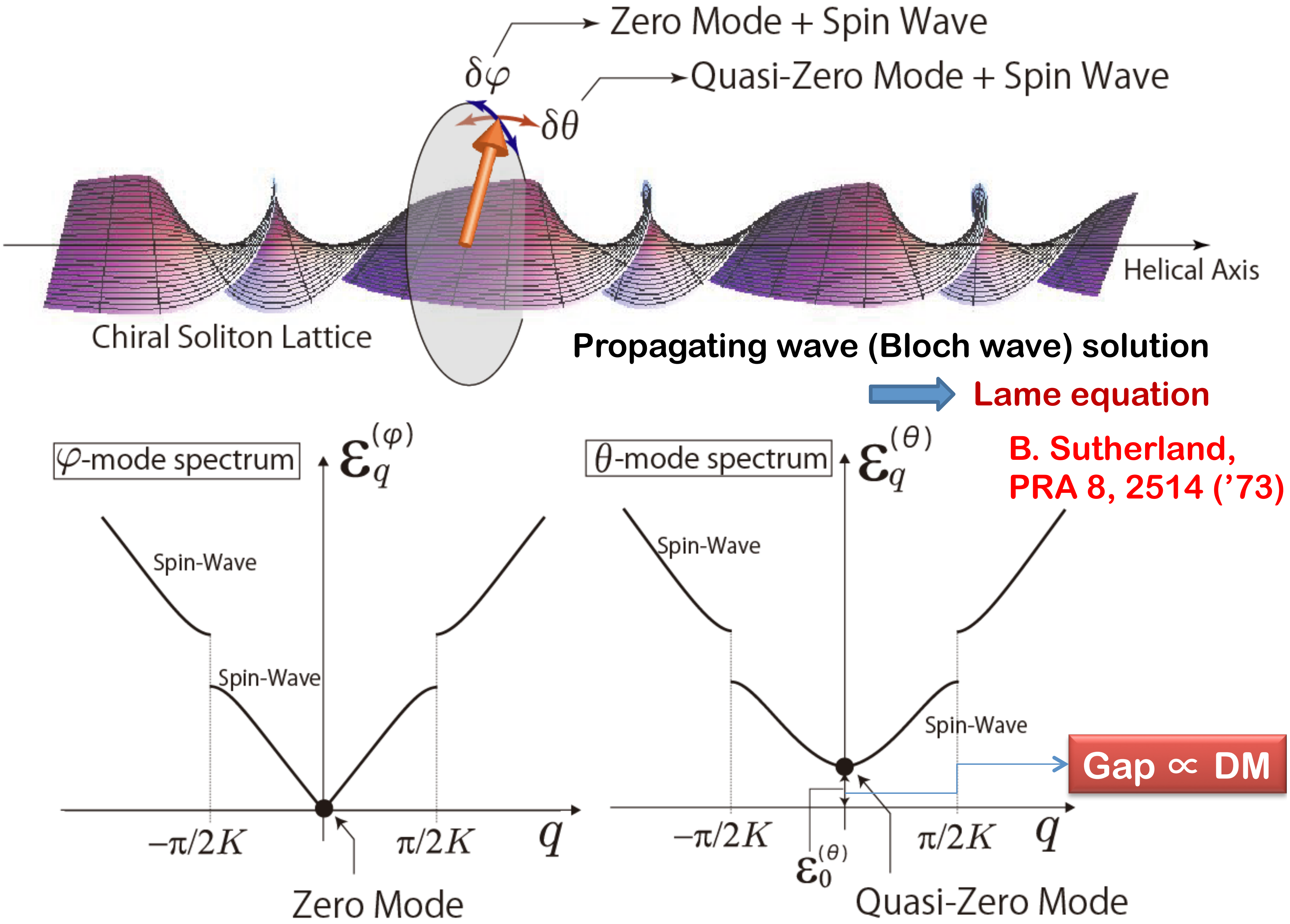
Chiral soliton lattice (CSL)



So far, consistency of theories and experiments were well confirmed.

From now on, theoretical proposals only (as yet)…

Elementary excitations over the CSL state



Propagating wave (Bloch wave) solution

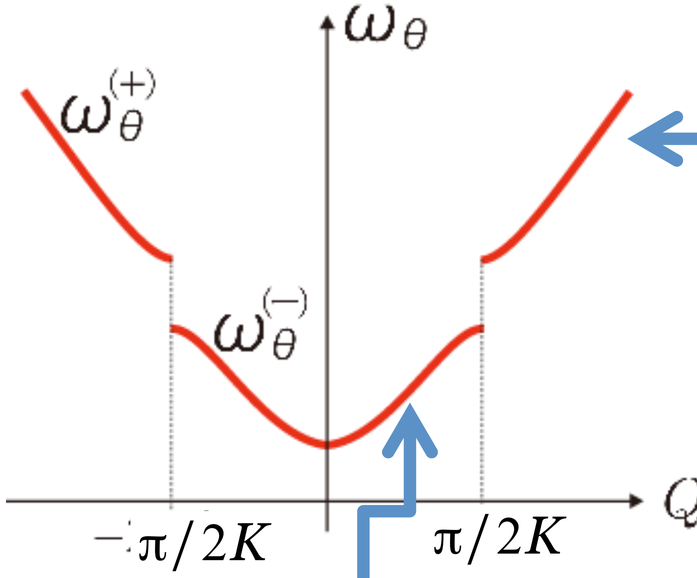
Lame equation

B. Sutherland, PRA 8, 2514 ('73)

CSL Phonon Wave Functions

Lame equation

$$\frac{d^2 u_a(\bar{z})}{d\bar{x}^2} = [2\kappa^2 \mathrm{sn}^2(\bar{x},\kappa) - (\kappa^2 - 4\bar{q}_0 + 4 + \omega_{\theta;a}^2)]u_a(\bar{z})$$



$$\Lambda_a^{(+)}(\bar{z}) = N_a \frac{\vartheta_4\left(\frac{\pi}{2K}(z - ia)\right)}{\vartheta_4\left(\frac{\pi}{2K}\bar{z}\right)} e^{-i\bar{Q}_a\bar{z}},$$

$$\bar{Q} = Z(a,\kappa') + \frac{\pi a}{2KK'} + \frac{\mathrm{dn}(a,\kappa')\mathrm{cn}(a,\kappa')}{\mathrm{sn}(a,\kappa')},$$

$$\omega_{\theta;a}^{(+)} = \sqrt{4(\bar{Q}_0 - 1) + \frac{1}{\mathrm{sn}^2(a,\kappa')}};$$

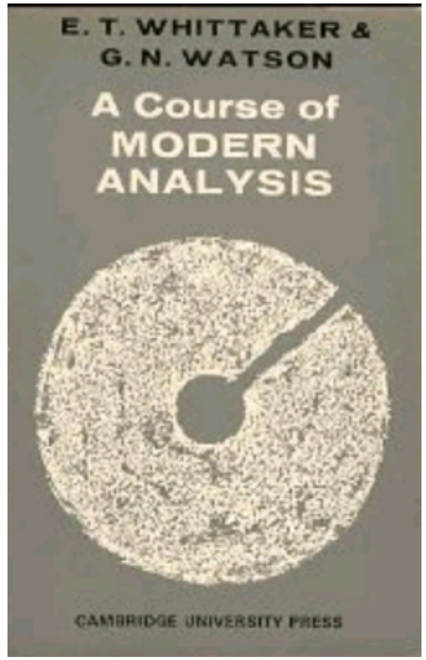
$$-K' < a \le K'$$

Hidden parameter

$$\Lambda_a^{(-)}(\bar{z}) = N_a \frac{\vartheta_4\left(\frac{\pi}{2K}(\bar{z} - ia - K)\right)}{\vartheta_4\left(\frac{\pi}{2K}\bar{z}\right)} e^{-i\bar{Q}_a\bar{z}},$$

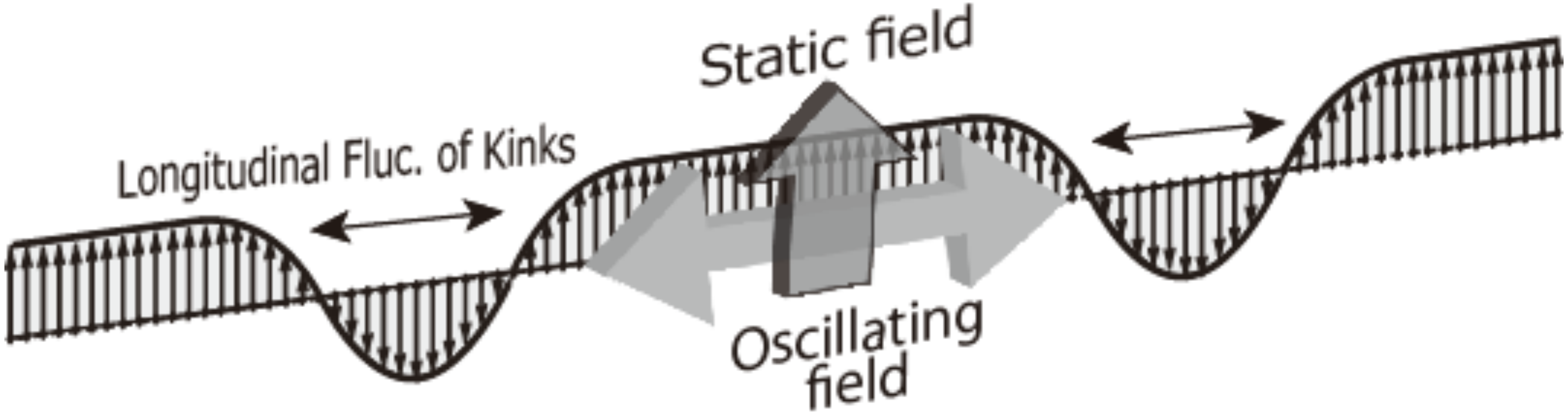
$$\bar{Q} = Z(a,\kappa') + \frac{\pi a}{2KK'},$$

$$\omega_{\theta;a}^{(-)} = \sqrt{4(\bar{Q}_0 - 1) + \kappa'^2 \mathrm{sn}^2(a,\kappa')}$$



ESR(CSL phonon resonance)

JK and Ovchinnikov, **Phys. Rev. B 79**, 220405**(R)** (2009)

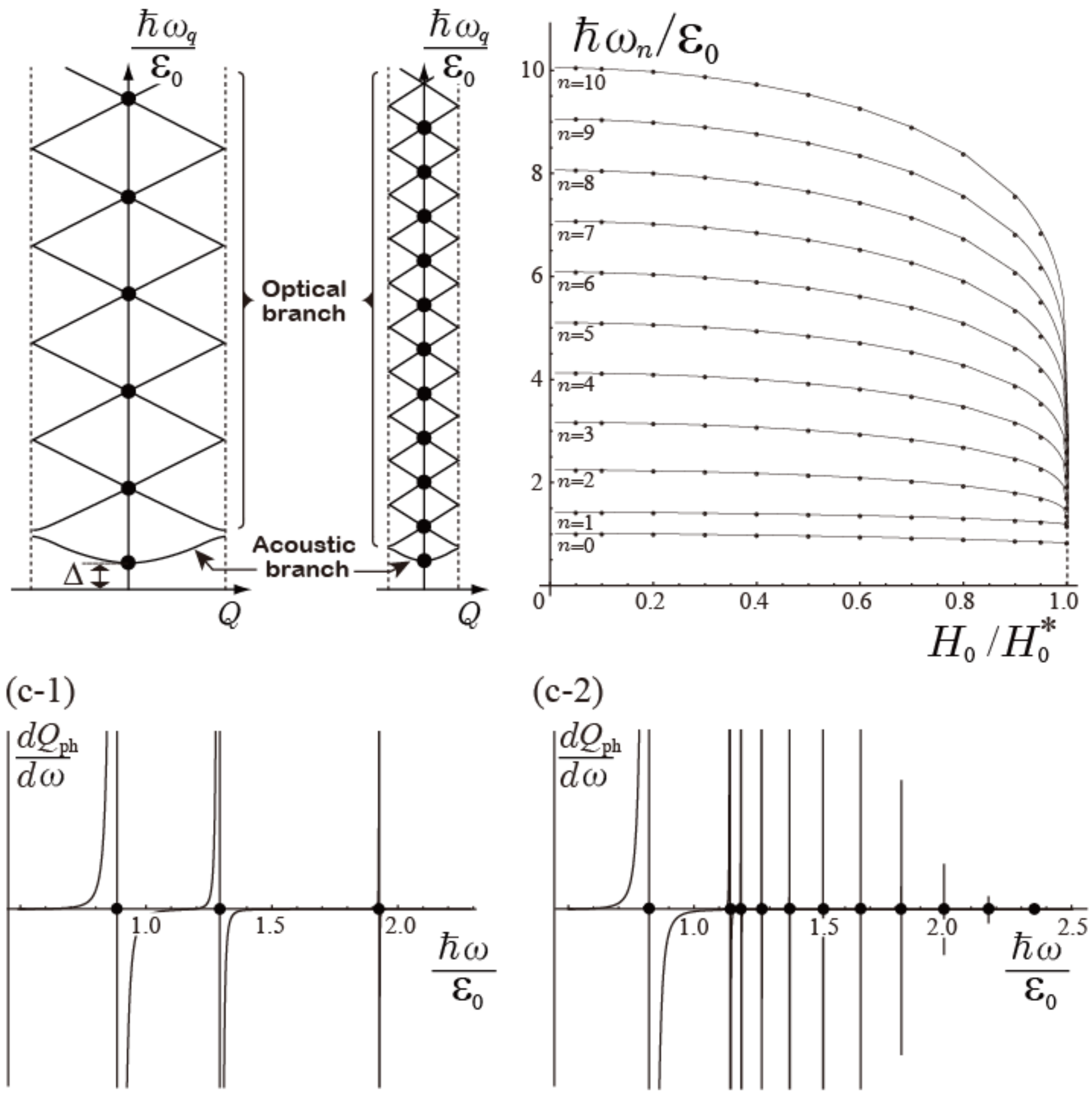


Reciprocal lattice const. of CSL

$$G_{\mathrm{CSL}} = \frac{2\pi}{L_{\mathrm{CSL}}} = \frac{\pi^2}{4KE} Q_0$$

CSL Phonon w.f.

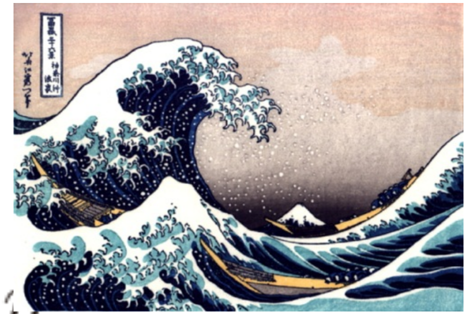
$$u(z,t) = \sum_q \sum_{n=-\infty}^{\infty} \left[\frac{U_n}{\sqrt{2\omega_q}} e^{-i(q-nG_{\mathrm{CSL}})z + i\omega_q t} b_q^\dagger + \mathrm{h.c.} \right]$$

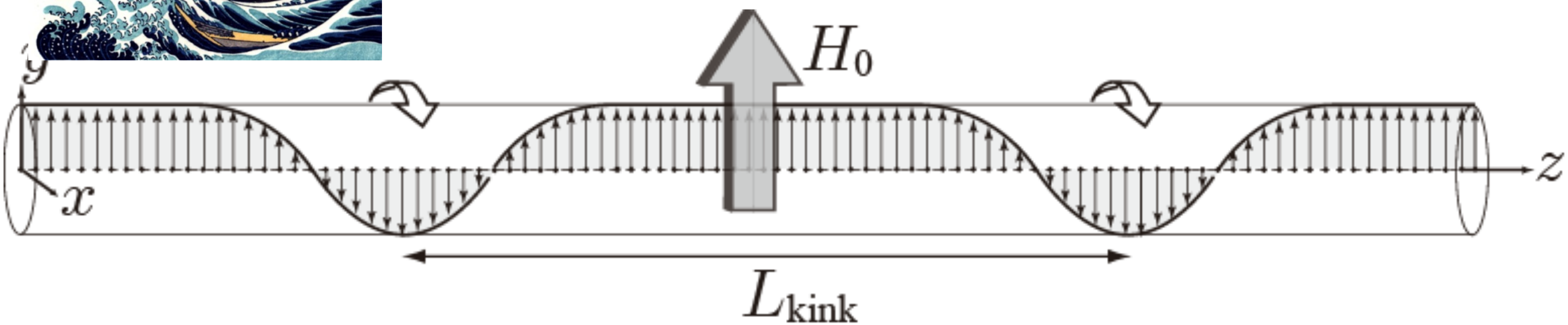


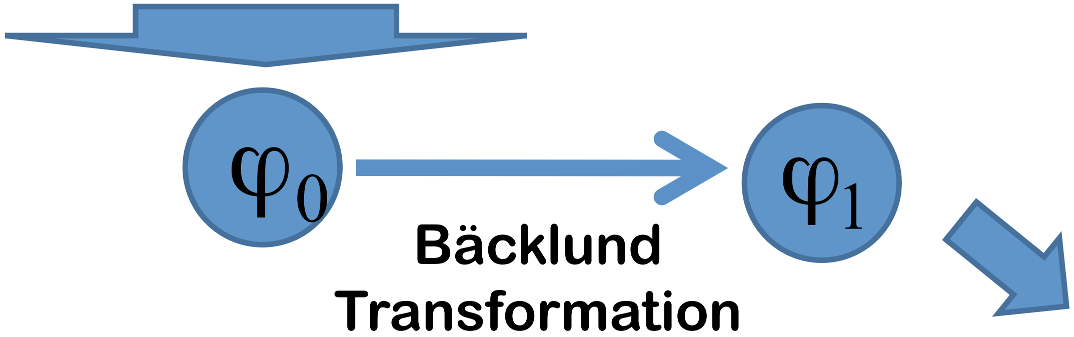
$$\frac{\hbar\omega_n}{\varepsilon_0} \simeq n\frac{\pi}{K} \simeq \frac{n\pi}{\log\left(4/\sqrt{1 - H_0/H_0^*}\right)}$$

New Soliton Solution

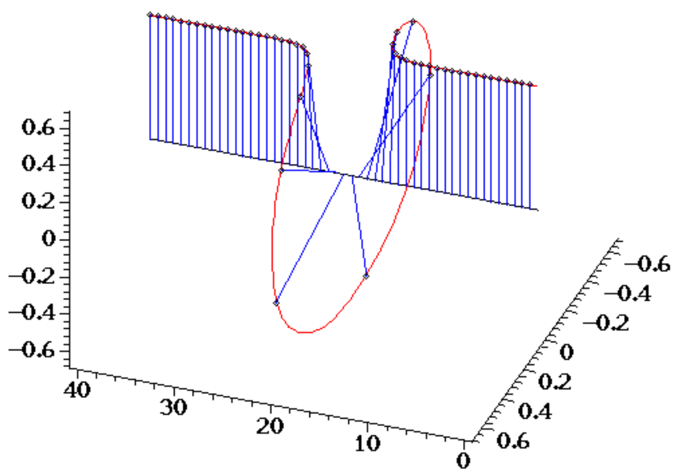
A.B.Borisov, JK, I.G.Bostrem, and A.S.Ovchinnikov, Phys. Rev. B79,134436 (2009)







Soliton conjugated to soliton lattice



Topological excitation over topological vacuum

⇒ Everything protected by geometric chirality

So far, I presented all about CSL

Next, let us move on to coupling of CSL with itinerant quantum spins

$$\hat{\mathcal{H}}_{\mathrm{el}} = -t \sum_{<i,j>,\sigma} \hat{c}^{\dagger}_{i,\sigma} \hat{c}_{j,\sigma}$$

$$-J_{\mathrm{sd}} \sum_{i} \boldsymbol{S}_i \cdot \hat{\boldsymbol{s}}_i$$

Classical Local Spin

$$\boldsymbol{S} = \chi^{\dagger} \hat{\boldsymbol{S}} \chi = S \hat{\boldsymbol{n}}$$

$$\hat{\boldsymbol{n}} = (\sin\theta \cos\varphi, \sin\theta \sin\varphi, \cos\theta)$$

$$\cos\left(\frac{\varphi_0(z)}{2}\right) = \mathrm{sn}\left(\frac{m}{\kappa} z\right)$$

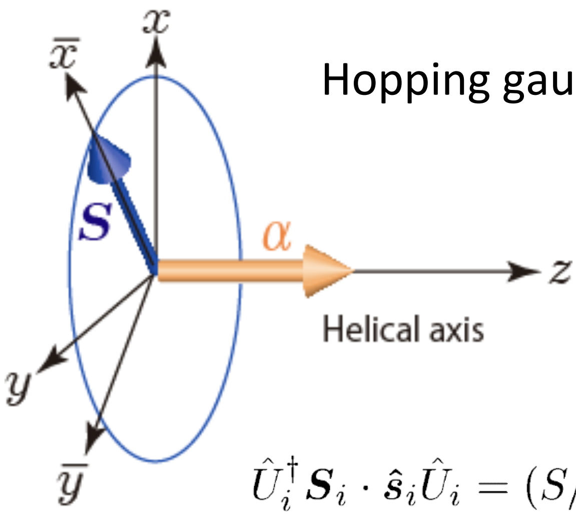
Quantum spin Carried by Itinerant Electron

$$\hat{c}(z) = \hat{U}(z) \hat{a}(z)$$ Internal SU(2) rotation

$$\hat{U}(z) = \exp\left[\frac{i}{2} \boldsymbol{\alpha}(z) \cdot \boldsymbol{\sigma}\right]$$

Two gauge choices
To make different physics “visible”

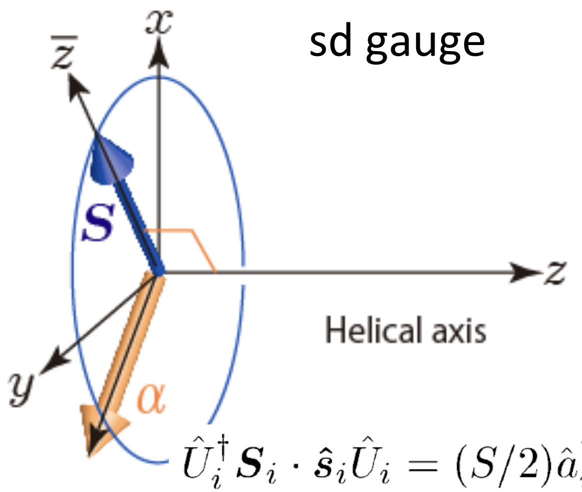
$$\boldsymbol{\alpha}(z) = \varphi_0(z)(0, 0, 1)$$



Hopping gauge

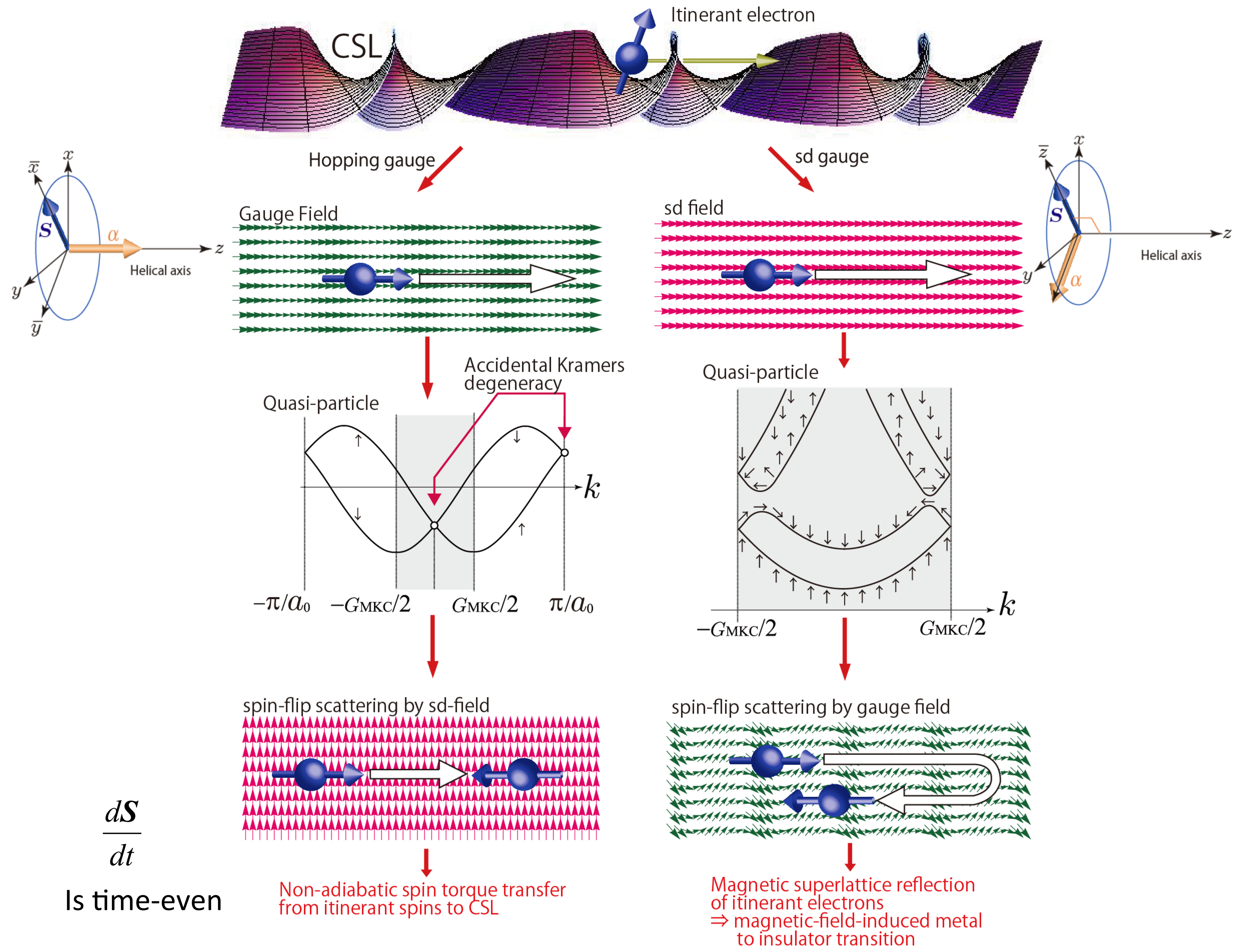
$$\hat{U}_i^{\dagger} \boldsymbol{S}_i \cdot \hat{\boldsymbol{s}}_i \hat{U}_i = (S/2) \hat{a}_i^{\dagger} \sigma^{\bar{x}} \hat{a}_i$$

$$\boldsymbol{\alpha}(z) = \frac{\pi}{2} (-\sin\varphi(z), \cos\varphi(z), 0)$$



sd gauge

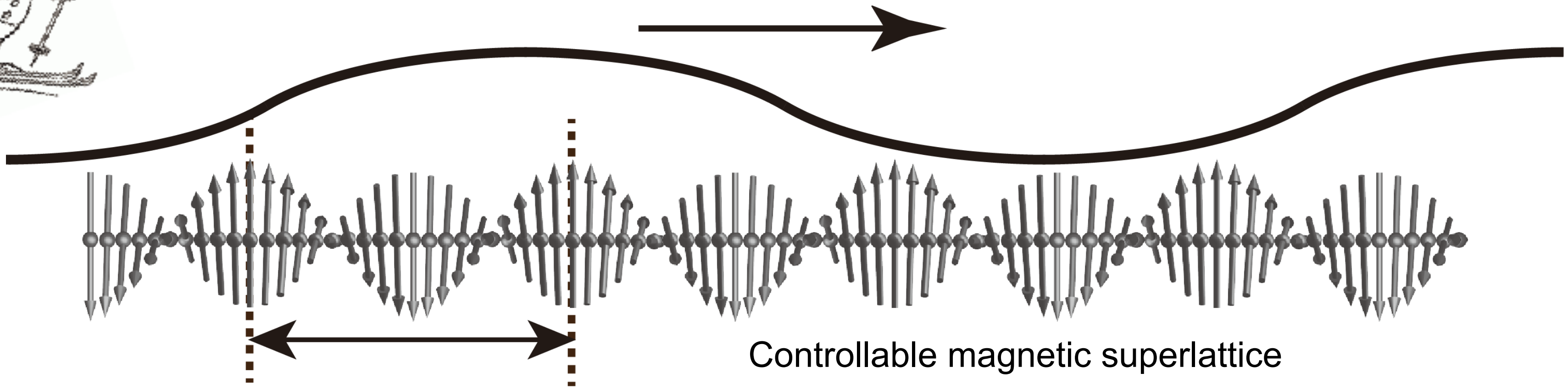
$$\hat{U}_i^{\dagger} \boldsymbol{S}_i \cdot \hat{\boldsymbol{s}}_i \hat{U}_i = (S/2) \hat{a}_i^{\dagger} \sigma^{\bar{z}} \hat{a}_i$$

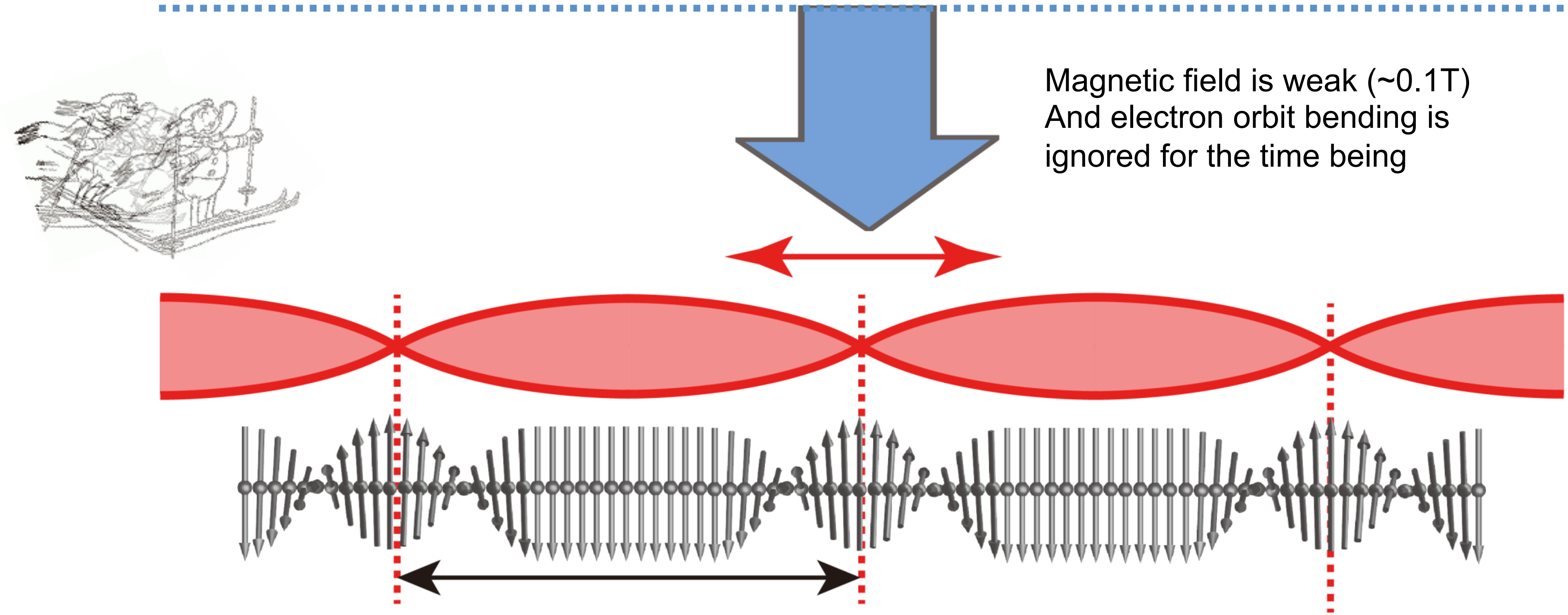


CSL
Itinerant electron
Hopping gauge
sd gauge
Gauge Field
sd field
Helical axis
Helical axis
Accidental Kramers degeneracy
Quasi-particle
Quasi-particle
$-\pi/a_0$ $-G_{MKC}/2$ $G_{MKC}/2$ π/a_0
k
$-G_{MKC}/2$ $G_{MKC}/2$
spin-flip scattering by sd-field
spin-flip scattering by gauge field
Non-adiabatic spin torque transfer from itinerant spins to CSL
Magnetic superlattice reflection of itinerant electrons
⇒ magnetic-field-induced metal to insulator transition
$\frac{dS}{dt}$
Is time-even

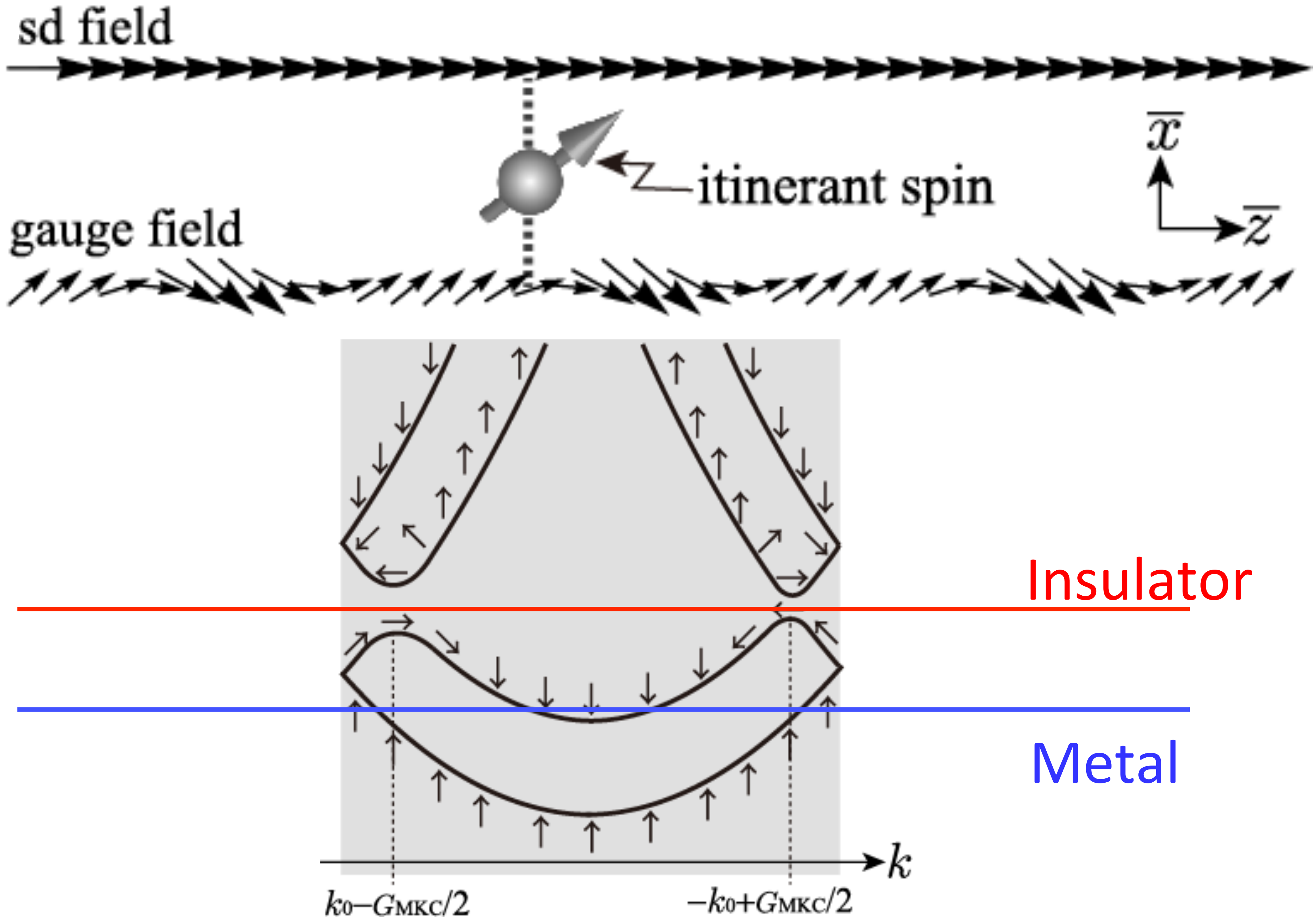
Magnetic superlattice potential acting on itinerant electrons

JK, Proskurin and Ovchinnikov, **Phys. Rev. Lett.** 107, 017205 (2011)

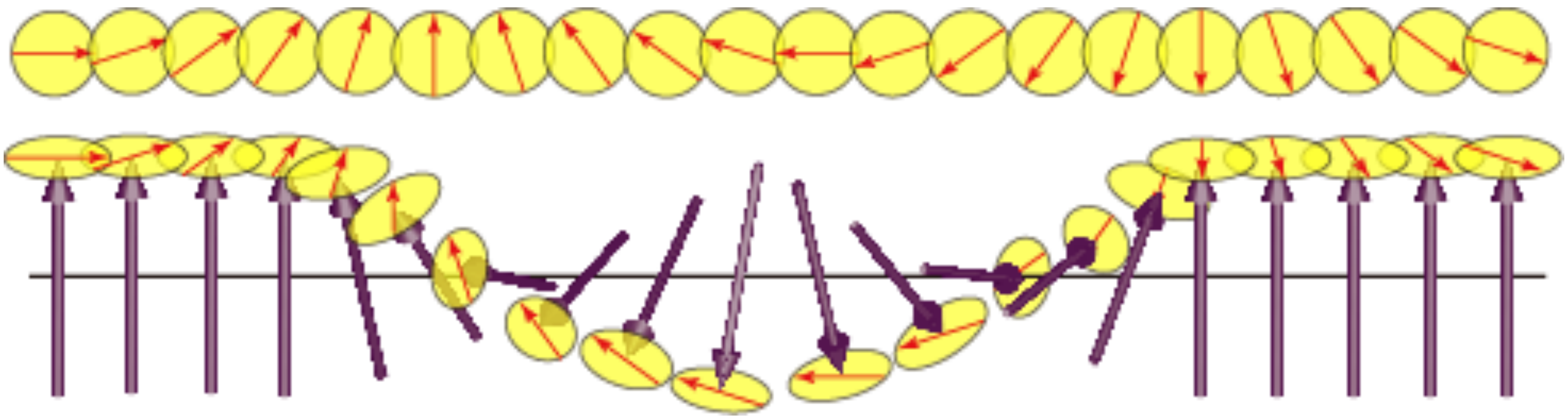




Magnetic field is weak (~0.1T)
And electron orbit bending is
ignored for the time being


sd field
itinerant spin
gauge field
x
z
Insulator
Metal
k
k0−GMKC/2
−k0+GMKC/2

Heli-cycloidal spin structure in the insulating state



$$|\varphi;\pm\rangle = \frac{1}{\sqrt{2}}(e^{-inG_{\text{CSL}}z/2}|k,\uparrow\rangle \pm e^{inG_{\text{CSL}}z/2}|k,\downarrow\rangle)$$

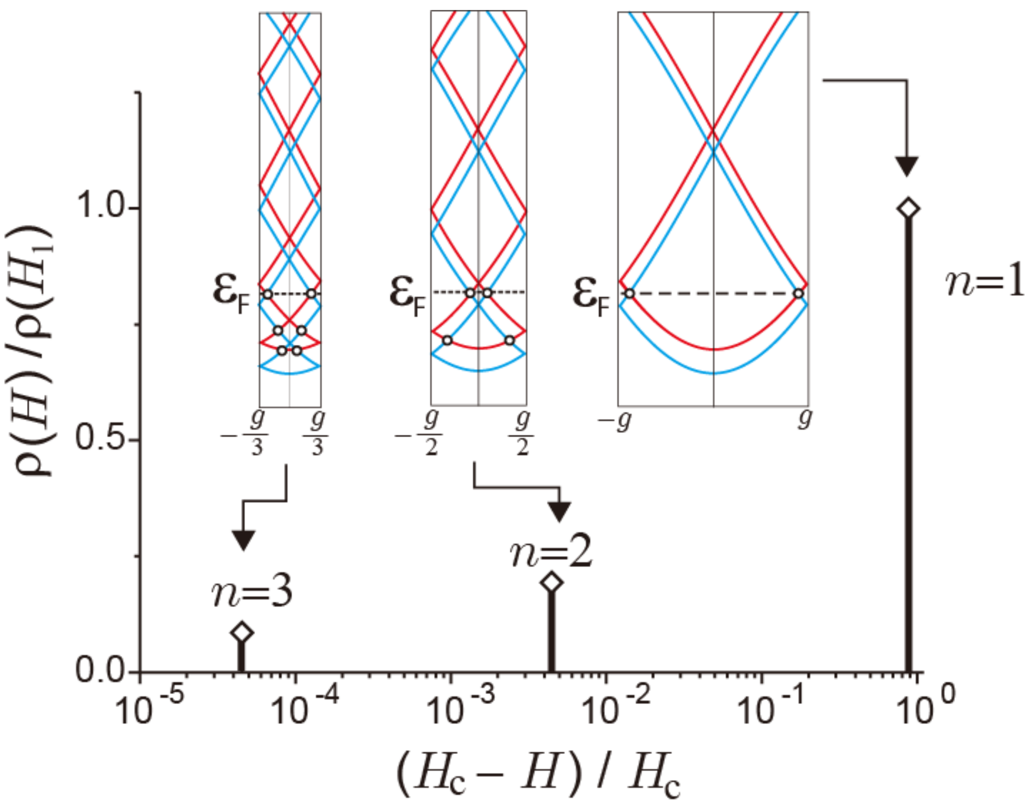
Zubarev's nonequilibrium density operator approach

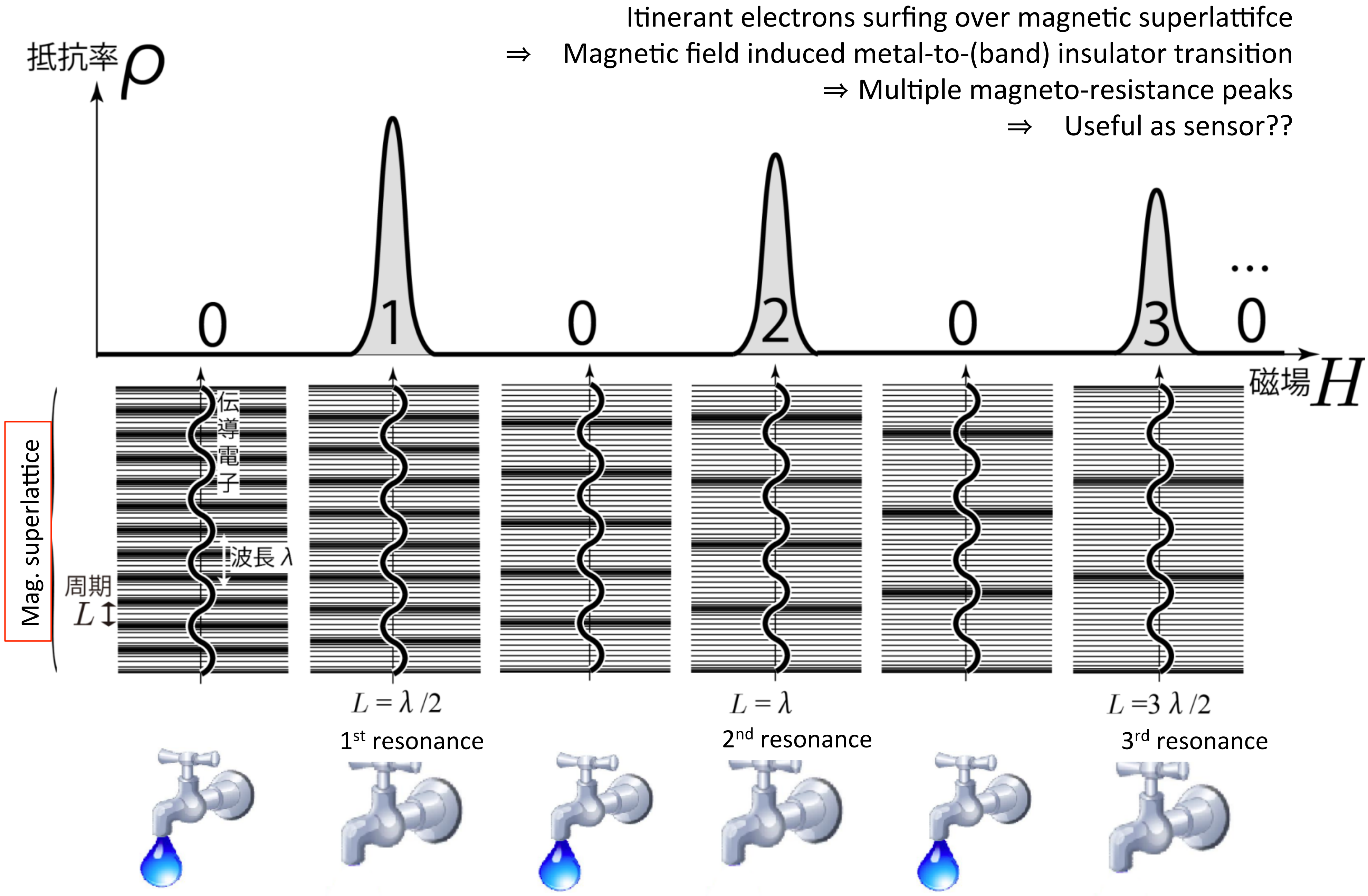
$$\rho(H)/\rho_{\max} = \mathcal{N}(H)/\mathcal{N}_{\max}$$

$$\mathcal{N}(H) = \lim_{\omega\to 0}\langle \dot{\mathcal{J}};\dot{\mathcal{J}}\rangle_{\omega+i\varepsilon}$$

$$\mathcal{J} = -e\sum_{k,\sigma} v_k b^{\dagger}_{k\sigma} b_{k\sigma}$$

$$\langle \dot{\mathcal{J}};\dot{\mathcal{J}}\rangle_{\omega+i\varepsilon} = \int_0^\infty dt e^{i\omega t-\varepsilon t}\int_0^1 dx \langle \dot{\mathcal{J}}(t);\dot{\mathcal{J}}(i\beta\hbar x)\rangle_{\mathrm{eq}}$$

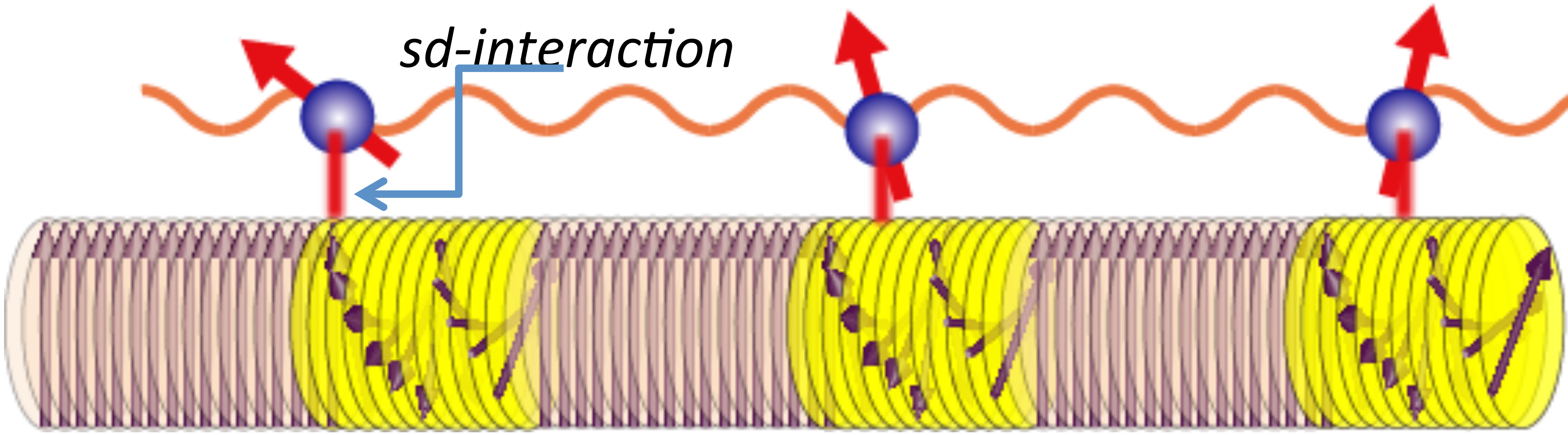




Itinerant electrons surfing over magnetic superlattifce
⇒ Magnetic field induced metal-to-(band) insulator transition
⇒ Multiple magneto-resistance peaks
⇒ Useful as sensor??
抵抗率 ρ
0
1
0
2
0
3
0
...
磁場 H
Mag. superlattice
伝導電子
波長 λ
周期 L
L = λ /2
1st resonance
L = λ
2nd resonance
L =3 λ /2
3rd resonance

Spin Torque Transfer Mechanisms

JK, Ovchinnikov, and Proskurin**, Phys. Rev. B 82**, 064407 (2010)



$$V \sim 10\,\mathrm{m/s} \quad \text{for} \quad j \sim 10^7\,\mathrm{A/m^2}$$

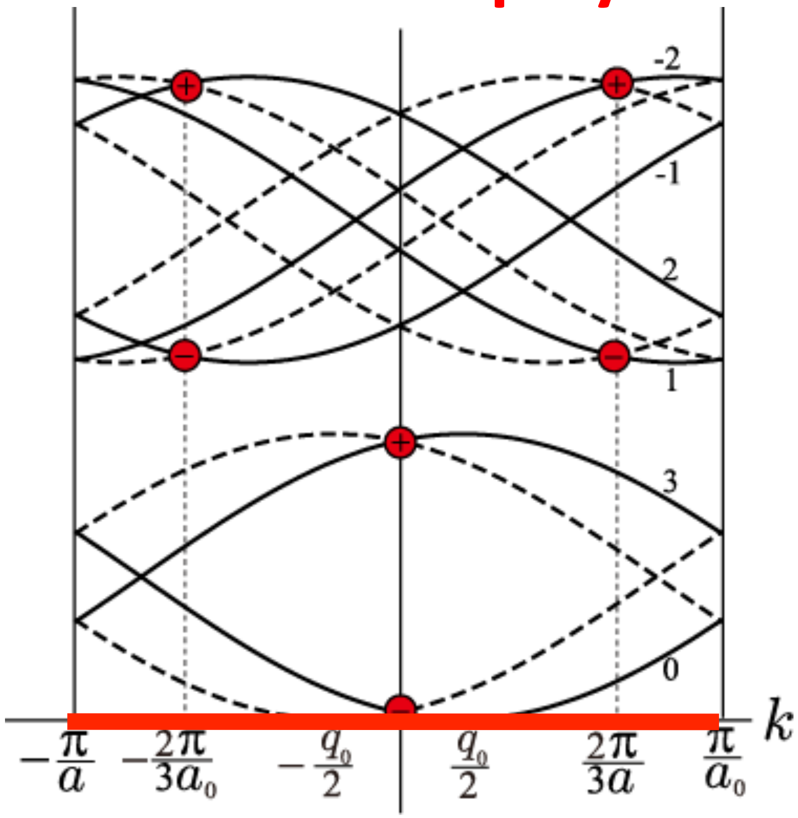
Fine interplay of quantum phase of Bloch electron and Semiclassical phase of magnetic kink crystal

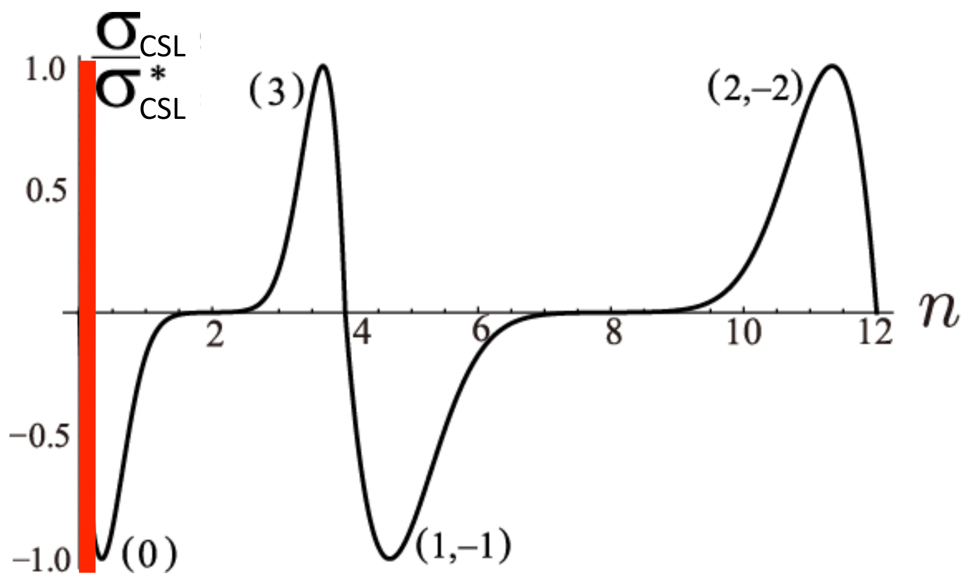
JK, Ovchinnikov, and Proskurin**, Phys. Rev. B 82**, 064407 (2010)

Spin Current Diode Effect

Chiral Band + Chiral Non-linear Texture

= Interplay of electron phase & spin phase





$$V = \sigma_{CSL} E$$

Sliding velocity Of CSL

Sliding conductivity

Electric field

Summary

- Chiral helimagnet $Cr_{1/3}NbS_2$
  - Hexagonal is good (no geometric frustration of crystallographic axes)
  - Well localized classical spin S=3/2 and itinerant quantum S=1/2
- Chiral sine-Gordon model
  - Chiral Soliton Lattice = asymmetric incommensurate spin phase object protected by geometric chirality
  - Ground state and elementary excitations fully available

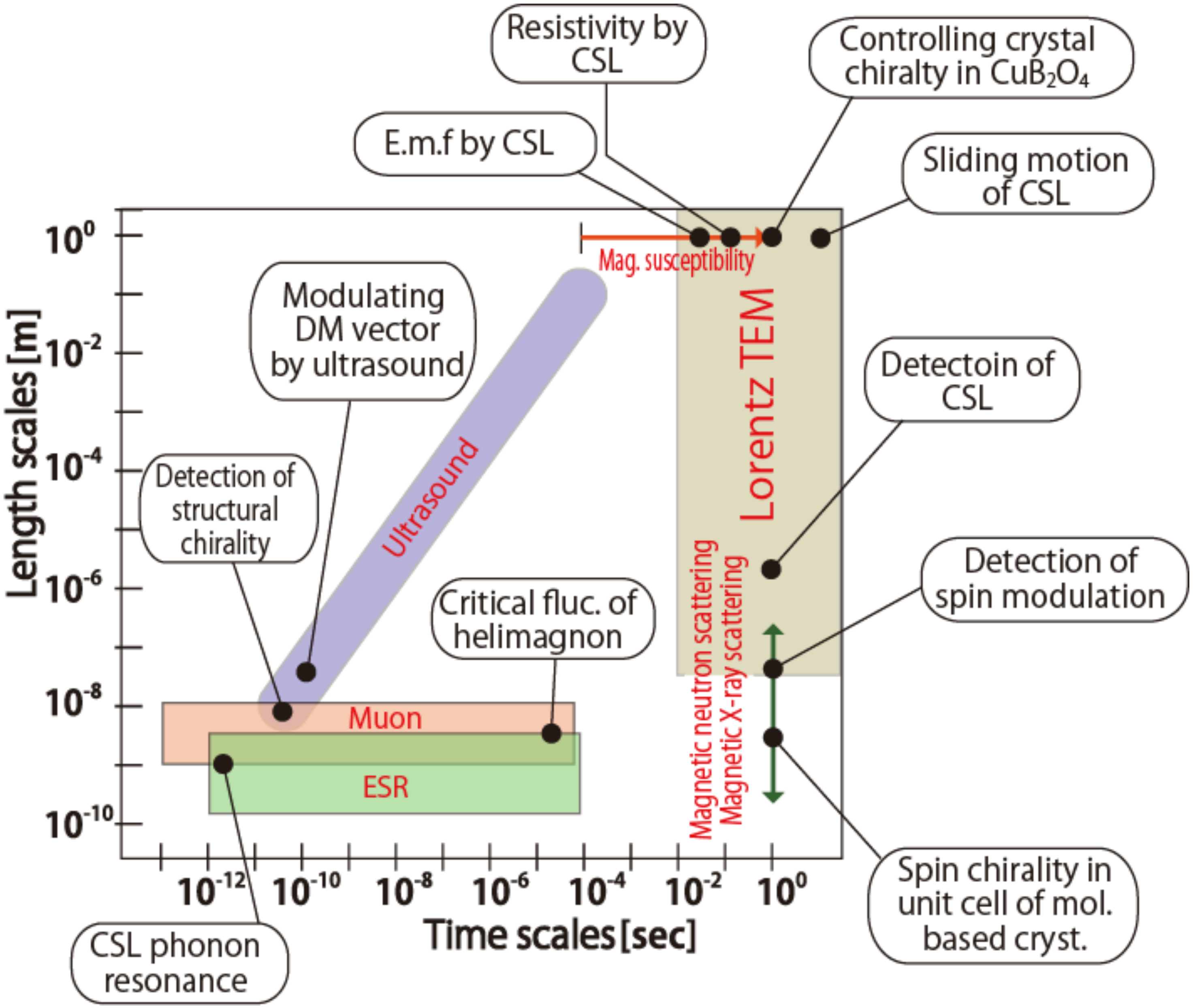
    ⇒ Fairyland of elliptic functions
  - New soliton surfing over the magnetic superlattice
- Coupling of localized and itinerant spins
  - Ground state as magnetic superlattice

    ⇒ multiple magneto-resistance peaks
  - Excitations as spin torque supplier

    ⇒ sliding motion of CSL magnetic superlattice

    ⇒Magnetic Current Diode effect

Geometric chirality of natural crystal gives us rich physics connecting classical and quantum degrees of freedom



Resistivity by CSL
Controlling crystal chiralty in CuB_2O_4
E.m.f by CSL
Sliding motion of CSL
Mag. susceptibility
Modulating DM vector by ultrasound
Lorentz TEM
Detectoin of CSL
Ultrasound
Detection of structural chirality
Detection of spin modulation
Critical fluc. of helimagnon
Magnetic neutron scattering
Magnetic X-ray scattering
Muon
ESR
Spin chirality in unit cell of mol. based cryst.
CSL phonon resonance
Length scales [m]
10^0
10^{-2}
10^{-4}
10^{-6}
10^{-8}
10^{-10}
10^{-12}
10^{-10}
10^{-8}
10^{-6}
10^{-4}
10^{-2}
10^0
Time scales [sec]