Spin Hall Effect and Quantum Spin Vorticity Principle

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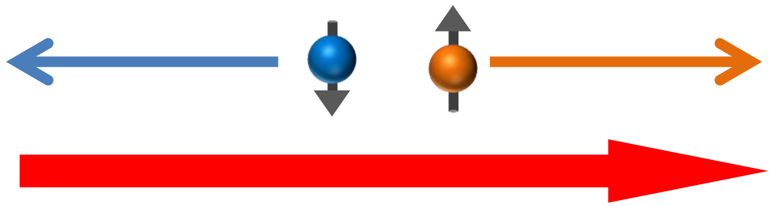
Introduction

The concept of spin current is widely used in the field of spintronics.

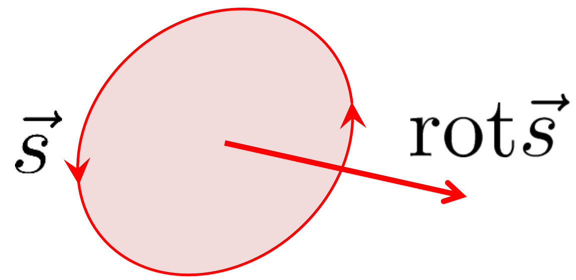
However, spin current is introduced as the flow of spin, whose definitions owe to observation methods.



spin current

The spin vorticity is

- an observable physical quantity defined as the rotation of the spin angular momentum density
- the mechanical physical quantity derived by the invariance under the general coordinate transformation
- introduced naturally on the basis of the "quantum electron spin vorticity principle"



spin vorticity

In this study, we discuss theoretical aspects of the spin Hall effect and inverse spin Hall effect on the basis of the "quantum electron spin vorticity principle" without introducing the spin current but the spin vorticity.

Quantum electron spin vorticity principle

$$\varepsilon^{A\mu\nu}(\mathrm{SUGRA}) + \tau^{A\mu\nu}(\mathrm{SUGRA}) = 0$$

Anti-symmetric component of geometrical tensor

$$\varepsilon^{A\mu\nu} = \left(\varepsilon^{\Pi\mu\nu} - \varepsilon^{\Pi\mu\nu}\right)/2$$

Anti-symmetric component of stress tensor

$$\tau^{A\mu\nu} = \left(\tau^{\Pi\mu\nu} - \tau^{\Pi\mu\nu}\right)/2$$

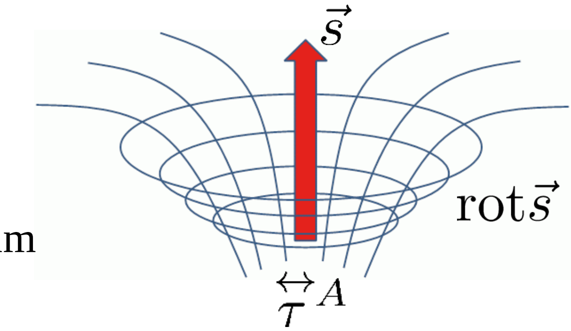
limit to Minkowski space

Spin vorticity of electron

$$\mathrm{rot}\vec{s} = \frac{1}{2}\left(\bar{\psi}\vec{\gamma}\left(i\hbar D_0\right)\psi + h.c.\right) - \vec{\Pi}$$

Spin vorticity

Kinetic momentum



Equation of motion of electronic spin

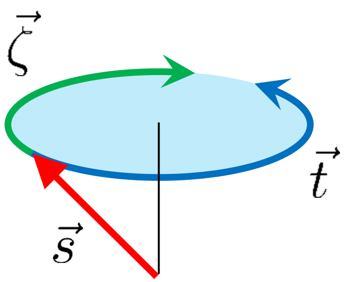
$$\frac{\partial}{\partial t}\vec{s} = \vec{t} + \vec{\zeta}$$

Spin torque

Zeta force

$$t^k = -\varepsilon_{kln}\tau^{Aln}$$

$$\zeta^k = -\partial_k \phi_5$$

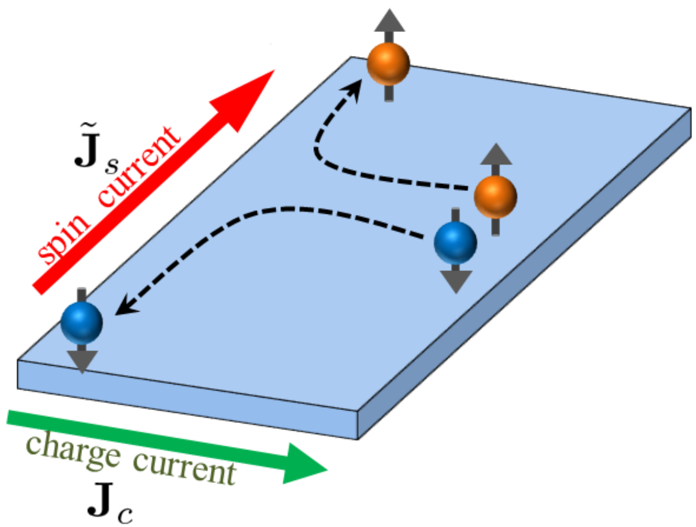


The "quantum electron spin vorticity principle", which is derived in the framework of the supergravity, leads to the equation for the vorticity of spin and the equation of motion of local spin. These equations include local contributions, while the EOMs of spin based on the quantum mechanics do not.

A. Tachibana, "Quantum electron spin vorticity principle", Annual Meeting of Japan Society for Molecular Science, 1E19 (2013);
A. Tachibana, J. Comput. Chem. Jpn., 13, 18 (2014); A. Tachibana , J. Math. Chem. 50, 669-688 (2012); A. Tachibana, Electronic Stress with Spin Vorticity. In *Concepts and Methods in Modern Theoretical Chemistry*, S. K. Ghosh and P. K. Chattaraj Eds., CRC Press, Florida (2013), pp 235-251.
A. Tachibana, Indian Journal of Chemistry A, Vol. 53A, Aug-Sept (2014), pp. 1031-1035.

★Spin Hall effect

【Conventional concept】

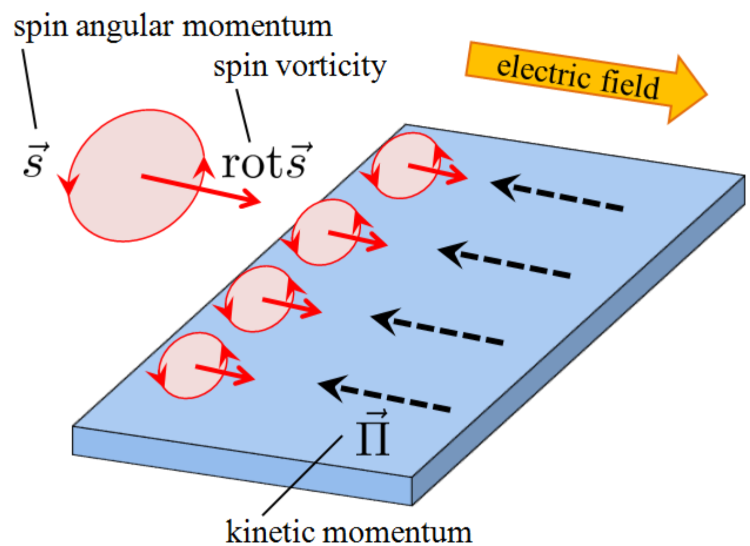


A conversion of a charge current into a transverse spin current

$$\tilde{\mathbf{J}}_s = \theta_{\mathrm{H}} \hat{\sigma} \times \mathbf{J}_c$$

spin current　　spin　　charge current

【Spin vorticity principle】



A conversion of the kinetic momentum of electron into the electronic spin vorticity

$$\mathrm{rot}\,\hat{\vec{s}} = \hat{\overleftrightarrow{\lambda}}\,\hat{\vec{j}}_e$$

spin vorticity　　charge current

★Summary

The conservation law of momentum

$$\frac{\partial}{\partial t}\left(\vec{\Pi}+\frac{1}{2}\mathrm{rot}\vec{s}\right)=\vec{L}+\vec{\tau}^{S}$$

$$\begin{cases}\dfrac{\partial}{\partial t}\vec{\Pi}=\vec{L}+\vec{\tau}^{S}+\mathrm{div}\,\overleftrightarrow{\tau}^{A}\\[2ex] \dfrac{\partial}{\partial t}\left(\mathrm{rot}\,\vec{s}(\vec{r})\right)=-2\,\mathrm{div}\,\overleftrightarrow{\tau}^{A}(\vec{r})=\mathrm{rot}\,\vec{t}(\vec{r})\end{cases}$$

The spin vorticity principle

$$\mathrm{rot}\vec{s}=\frac{1}{2}\left(\bar{\psi}\vec{\gamma}\left(i\hbar D_0\right)\psi+h.c.\right)-\vec{\Pi}$$

The rotation of the spin torque as the driving force is converted into the kinetic momentum through the generation of the spin vorticity.

The time evolution of the electron spin is driven by the antisymmetric component of the electric stress tensor through the vorticity.

Spin Hall effect：
A conversion of the kinetic momentum of electron into the electronic spin vorticity, and then spin angular momentum is generated at the both edges of the conductor.

Inverse spin Hall effect：
A conversion of the electronic spin vorticity generated by an applied spin torque into the kinetic momentum of electron in the conductor.

【Concept based on the spin vorticity principle】