Study of spin current in ~~three-dimensional~~ Dirac systems

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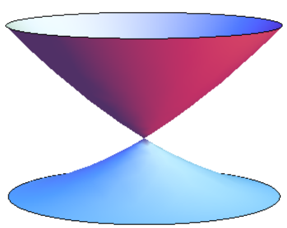
Spin Hall effect in Dirac systems

Motivation:

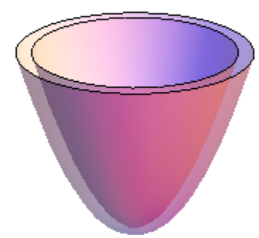
- Investigate the spin transport in Dirac systems with spin-mixing γ matrices
- Define the proper spin current in strong spin-orbit system.

Why Dirac ?



$k^2 <$ spin-orbit

well investigated model



$k^2 >$ spin-orbit

Model: Surface state of a topological insulator

Simplest example of these systems!

$$\mathcal{H} = \alpha(-k_x\sigma_y + k_y\sigma_x) + \frac{(k^2 - k_F^2)}{2m}\mathbf{1} \qquad \Sigma = -\frac{i}{2\tau}\mathbf{1}$$

How should we define the spin current?

Conventional Definition : $j_i^{S_z} = \frac{\{S^z, v_i\}}{2}$

Our Definition : Derive the diffusion equation from quantum kinetic equation and define spin current.

Method

Dyson equation in Keldysh formalism

- Electric field in x dierection
- Wigner transformation
- Derivative expansion
- Born approximation

Quantum kinetic equation

- Quasi-particle approximation $1/\tau \ll$ (other energy scales)
- $\omega\tau \ll 1$
- Solved by iteration (2nd order)

Diffusion equation

- Define spin current
- Calculate spin Hall conductivity

Result (1)

Diffusion equations

$$\frac{\partial N}{\partial t} = D\nabla^2 N + \partial_x(-De\nu E_x) + \mathcal{V}\partial_x S^y - \mathcal{V}\partial_y S^x$$

$$\frac{\partial S_x}{\partial t} = \frac{D}{4}\frac{\partial^2 S_x}{\partial x^2} + \frac{3D}{4}\frac{\partial^2 S_x}{\partial y^2} + \frac{D}{2}\frac{\partial^2 S_y}{\partial x \partial y} - \frac{\mathcal{V}}{4}\partial_y N - \frac{1}{2\tau^3 m\alpha}\partial_x S_z - \frac{S^x}{2\tau}$$

$$\frac{\partial S_y}{\partial t} = \frac{3D}{4}\frac{\partial^2 S_y}{\partial x^2} + \frac{D}{4}\frac{\partial^2 S_y}{\partial y^2} + \frac{D}{2}\frac{\partial^2 S_x}{\partial x \partial y} + \frac{\mathcal{V}}{4}(\partial_x N - e\nu E_x) - \frac{1}{2\tau^3 m\alpha}\partial_x S_z - \frac{S^y}{2\tau}$$

$$\frac{\partial S_z}{\partial t} = -\frac{(-m\alpha + \sqrt{m^2\alpha^2 + p_F^2})^2}{8}\nabla^2 S_z + \frac{1}{2\tau(-m\alpha + \sqrt{m^2\alpha^2 + p_F^2})}(\partial_x S^x + \partial_y S^y) - \frac{S^z}{\tau}$$

previous study: PRL. **105**, 066802 (2010)

Electric current

$$J_x = -D\partial_x N + De\nu E_x - \mathcal{V}S^y = \frac{I}{e}$$

$$J_y = -D\partial_y N + \mathcal{V}S^x = 0$$

Spin current

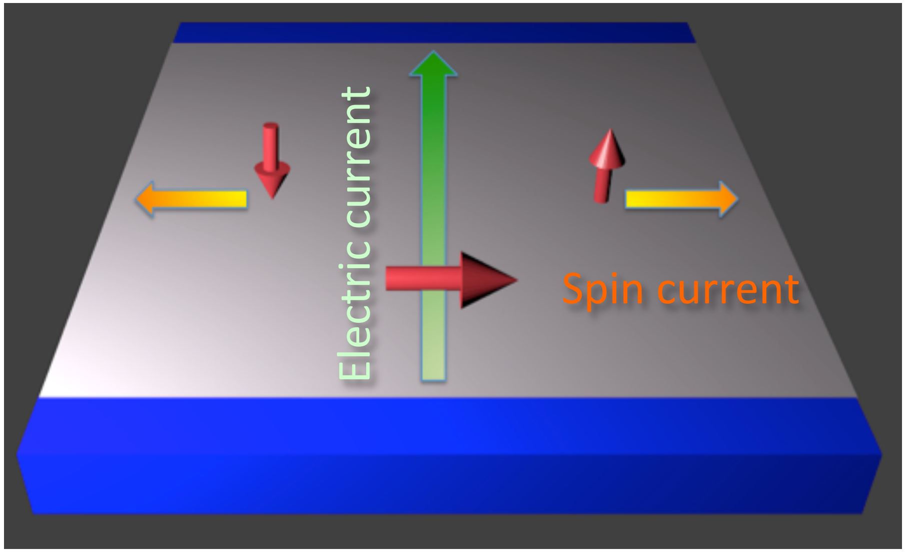
$$J_y^{S_z} = \frac{(-m\alpha + \sqrt{m^2\alpha^2 + p_F^2})^2}{8}\partial_y S^z - \frac{1}{2\tau(-m\alpha + \sqrt{m^2\alpha^2 + p_F^2})}S^y$$

Result (2)

- Steady state@Bulk

$$S_y = -\frac{I}{2\mathcal{V}e}, \qquad J_y^{S_z} = \frac{\mathcal{V}I}{8eD\left(-m\alpha + \sqrt{m^2\alpha^2 + p_F^2}\right)}, \qquad I = 2De^2\nu E_x$$

$$\boxed{\sigma_{xy}^{S_z} = \frac{e}{8\pi}}$$



- Universal value
- The existence of k^2 term is not important.

Spin current flows vertical to spin-polarized electric current.