

Electron transport and magnetization dynamics in metallic ferromagnets

Gen TATARA

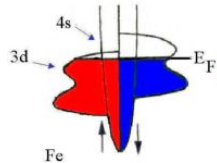
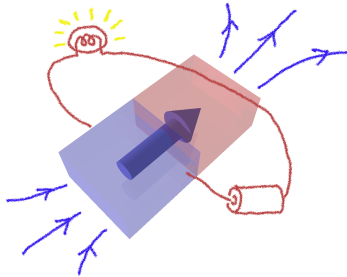


RIKEN Center for Emergent Matter Science (CEMS)



YKIS Kyoto 2014/12/04

Ferromagnetic metal



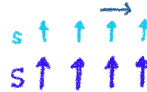
- Magnetization $\mathbf{M} (= \frac{g\mu_B}{a^3} \mathbf{S})$ (Localized spin \mathbf{S}) d electron
- Finite electric conductivity Itinerant s electron
- sd interaction
Between conduction electron spin \mathbf{s} and localized spin

$$H_{sd} = -J_{sd} \mathbf{S} \cdot \mathbf{s}$$

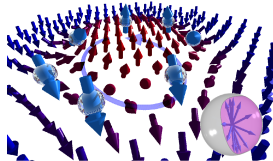
Strong \Rightarrow Electron spin follows \mathbf{S} perfectly Adiabatic limit

Ferromagnetic metal under electric current

- Uniform magnetization \Rightarrow No new feature



- Non uniform magnetization \Rightarrow Non-trivial transport



- Today's subjects

- Rotation of conduction electron spin

Spin Berry's phase

\Rightarrow Spin electromagnetic field

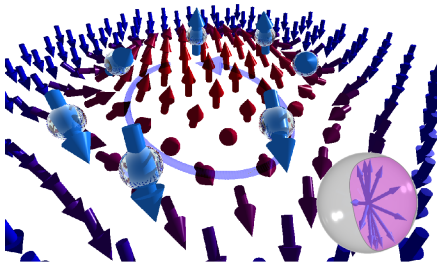
- Rotation of magnetization

Spin-transfer torque

\Rightarrow Magnetization dynamics

Spin electromagnetic field

Volovik'87, Stern'92, Barnes&Maekawa'07



Adiabatic limit

- Electron spin rotation
- ⇒ Phase $e^{i\varphi}$

$$\varphi = \int_C dr \cdot \mathbf{A}_s$$

- Spin magnetic field

$$\varphi = \int_S d\mathbf{S} \cdot \mathbf{B}_s$$

- Faraday's law is satisfied

- Spin electric field (dynamics)

$$\dot{\varphi} = - \int_C dr \cdot \mathbf{E}_s$$

$$\nabla \times \mathbf{E}_s = - \frac{\partial \mathbf{B}_s}{\partial t}$$

Electromagnetic field coupled to spin

Phase induced by localized spin

- Strong sd exchange interaction

- Electron spin \parallel localized spin
- Electron wave function

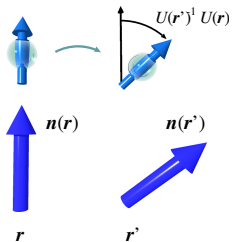
$$|\theta\phi\rangle = \cos\frac{\theta}{2}|\uparrow\rangle + e^{i\phi}\sin\frac{\theta}{2}|\downarrow\rangle$$

- Overlap

$$\langle\theta'\phi'|\theta\phi\rangle = 1 + \frac{i}{2}(1 - \cos\theta)\delta\phi \equiv e^{i\varphi}$$

- Effective vector potential $\varphi = d\mathbf{r} \cdot \mathbf{A}_s$

$$\mathbf{A}_s = \frac{1}{2}(1 - \cos\theta)\nabla\phi$$



Phase induced by localized spin

- Effective vector potential induced by sd interaction

$$\mathbf{A}_s = \frac{1}{2}(1 - \cos\theta)\partial\phi$$

- Gauge interaction

$$H_A = \int d^3r \mathbf{A}_s \cdot \mathbf{j}_s$$

$\mathbf{j}_s (= P\mathbf{j})$: Spin current (P : Spin polarization)

- Two effects

- Current-induced torque on magnetization Spin-transfer torque

$$\mathbf{B}_{\text{eff}} = \frac{\delta H_A}{\delta \mathbf{S}} = \mathbf{S} \times (\mathbf{j}_s \cdot \nabla) \mathbf{S} \Rightarrow \dot{\mathbf{S}} = (\mathbf{j}_s \cdot \nabla) \mathbf{S}$$

- Spin motive force on electron Effective electromagnetic fields

$$\begin{aligned} \mathbf{E}_s &= -\nabla A_{s,0} + \partial_t \mathbf{A}_s = -\frac{1}{2} \mathbf{n} \cdot (\dot{\mathbf{n}} \times \nabla_i \mathbf{n}) \\ \mathbf{B}_s &= \nabla \times \mathbf{A}_s = \frac{1}{4} \sum_{jk} \epsilon_{ijk} \mathbf{n} \cdot (\nabla_j \mathbf{n} \times \nabla_k \mathbf{n}) \end{aligned}$$

Magnetization dynamics under current-induced torque

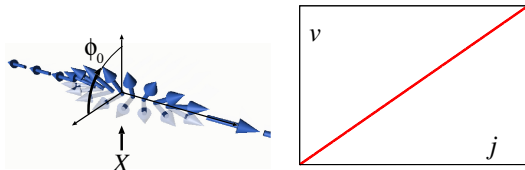
$$\dot{\mathbf{S}} = (\mathbf{j}_s \cdot \nabla) \mathbf{S}$$

- Spin-transfer torque \Rightarrow Sliding of magnetization structure

$$(\partial_t - \mathbf{j}_s \cdot \nabla) \mathbf{S} = 0$$

Magnetization dynamics under current-induced torque

$$\dot{\mathbf{S}} = (\mathbf{j}_s \cdot \nabla) \mathbf{S}$$

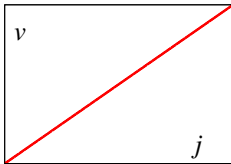
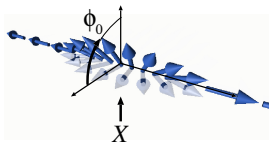


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Magnetization dynamics under current-induced torque

$$\dot{\mathbf{S}} = (\mathbf{j}_s \cdot \nabla) \mathbf{S} + \alpha (\mathbf{S} \times \dot{\mathbf{S}})$$



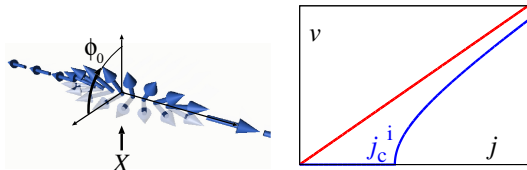
- Spin-transfer torque \Rightarrow Sliding of magnetization structure

$$(\partial_t - \mathbf{j}_s \cdot \nabla) \mathbf{S} = 0$$

- Damping (friction) Transverse torque \Rightarrow Screw motion

Magnetization dynamics under current-induced torque

$$\dot{\mathbf{S}} = (\mathbf{j}_s \cdot \nabla) \mathbf{S} + \alpha (\mathbf{S} \times \dot{\mathbf{S}}) + K_{\perp} (\mathbf{S} \times \mathbf{e}_y)$$



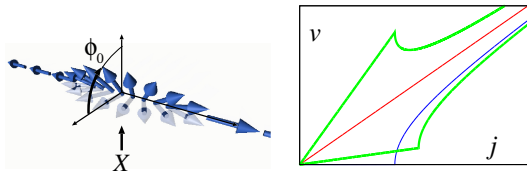
- Spin-transfer torque \Rightarrow Sliding of magnetization structure

$$(\partial_t - \mathbf{j}_s \cdot \nabla) \mathbf{S} = 0$$

- **Damping (friction)** Transverse torque \Rightarrow Screw motion
- **Anisotropy energy** $K_{\perp} (S_y)^2 \Rightarrow$ Intrinsic pinning

Magnetization dynamics under current-induced torque

$$\dot{\mathbf{S}} = (\mathbf{j}_s \cdot \nabla) \mathbf{S} + \alpha (\mathbf{S} \times \dot{\mathbf{S}}) + K_{\perp} (\mathbf{S} \times \mathbf{e}_y) + \beta [\mathbf{S} \times (\mathbf{j}_s \cdot \nabla) \mathbf{S}]$$



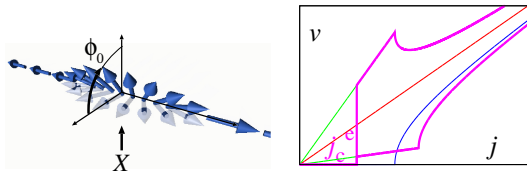
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- **Spin-orbit, spin relaxation of electron** \Rightarrow Transverse torque β

Magnetization dynamics under current-induced torque

$$\dot{\mathbf{S}} = (\mathbf{j}_s \cdot \nabla) \mathbf{S} + \alpha (\mathbf{S} \times \dot{\mathbf{S}}) + K_{\perp} (\mathbf{S} \times \mathbf{e}_y) + \beta [\mathbf{S} \times (\mathbf{j}_s \cdot \nabla) \mathbf{S}] + \tau_{\text{pin}}$$



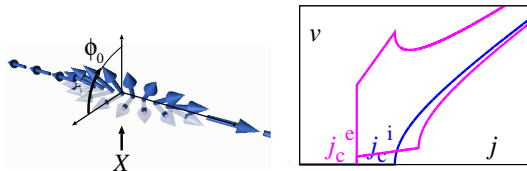
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- **(Extrinsic) Pinning**

Magnetization dynamics under current-induced torque

$$\dot{\mathbf{S}} = (\mathbf{j}_s \cdot \nabla) \mathbf{S} + \alpha (\mathbf{S} \times \dot{\mathbf{S}}) + K_{\perp} (\mathbf{S} \times \mathbf{e}_y) + \beta [\mathbf{S} \times (\mathbf{j}_s \cdot \nabla) \mathbf{S}] + \tau_{\text{pin}}$$

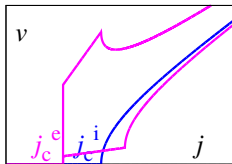


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Domain wall dynamics under current



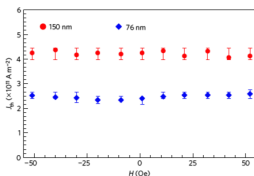
- $\beta \lesssim \alpha$ Intrinsic pinning

- Threshold current

$$j_c = \frac{eS^2}{\hbar a^3 P} K_{\perp} \lambda$$

- Stable operation

Inensitive to defects, external field



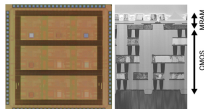
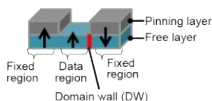
Koyama '11

- $\beta \gtrsim \alpha$ Extrinsic pinning

- Threshold current

$$j_c \propto \frac{V_{\text{pin}}}{\beta}$$

- Strong spin-orbit interaction for low j_c large β

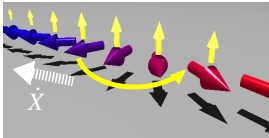


Domain wall MRAM (NEC)

Current-induced torque

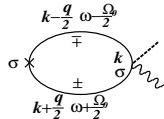
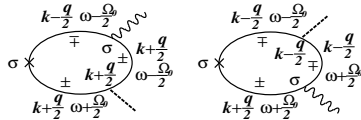
- Non-equilibrium conduction electron spin polarization $\delta \mathbf{s}$

$$\tau = J_{sd} \delta \mathbf{s} \times \mathbf{S}$$



$$\mathbf{E}, \mathbf{j} \Rightarrow \delta \mathbf{s} \Rightarrow \tau$$

- Calculation of non-equilibrium spin density



Thermally-induced torque

$$\nabla T \Rightarrow \delta \mathbf{s} \Rightarrow \boldsymbol{\tau}$$

- Luttinger's 'gravitaional potential' Ψ
- Equilibrium torque needs to be carefully subtracted

Kohno, Hatami, Bauer '14

$$H_T = \int d^3r \mathcal{E} \Psi$$

\mathcal{E} : Energy density

$$\nabla \Psi \sim \frac{\nabla T}{T}$$

- Vector potential formulation of thermal effect (?)

$$H_T = \int d^3r \mathbf{A}_T \cdot \mathbf{j}_\mathcal{E}$$

$\mathbf{j}_\mathcal{E}$: Energy current

$$\dot{\mathbf{A}}_T \sim \frac{\nabla T}{T}$$

$$\dot{S} = - \int d^3r \frac{1}{T} \nabla \cdot \mathbf{j}_\mathcal{E}$$

Entropy change

Recent topics : Interface effects

- Rashba spin-orbit interaction Inversion symmetry broken

$$H_R = i\mathbf{E}_R \cdot (\nabla \times \boldsymbol{\sigma})$$

\mathbf{E}_R : Rashba field

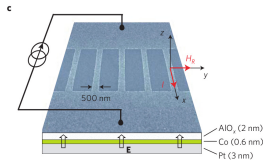
- Large force on domain wall \Rightarrow Efficient motion

Obata>'08, Manchon&Zhang'09

- Experiment

Miron'10,'11

- Pt/Co/ALO layer no inversion symmetry
- $v = 400$ m/s 100 times larger
- $j_c = 10^{12}$ A/m² same order
- Rashba turned out not to be dominant



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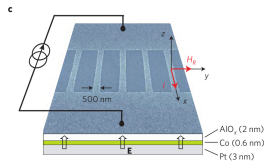
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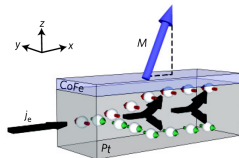


- Spin Hall torque

Emori'13

Spin Hall effect in heavy metal (Pt) layer

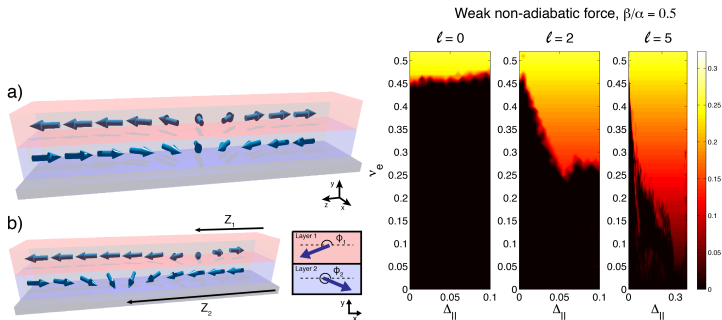
\Rightarrow Large torque $v > 100$ m/s



Recent topics : Interface effects

- Synthetic antiferromagnetic system Coupled two domain walls

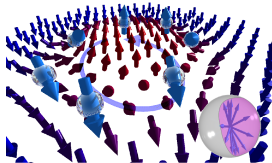
Saarikoski, Kohno, Marrows, GT'14



- Interlayer coupling removes random extrinsic pinning
Two walls help each other to depin

Fast wall motion at low current by artificial structures

Electron transport in ferromagnetic metal



- Rotation of magnetization

Spin-transfer torque

⇒ Current-induced magnetization dynamics



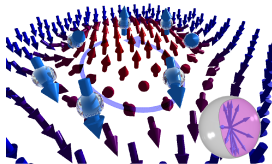
Magnetic memory (MRAM)

- Rotation of conduction electron spin

Spin Berry's phase

⇒ Spin electromagnetic field

Electron transport in ferromagnetic metal



- Rotation of magnetization

Spin-transfer torque

⇒ Current-induced magnetization dynamics



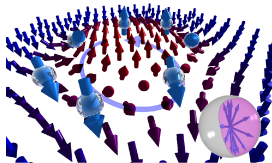
Magnetic memory (MRAM)

- Rotation of conduction electron spin

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⇒ Spin electromagnetic field

Spin electromagnetic field



- Effective vector potential

$$\mathbf{A}_s = \frac{1}{2}(1 - \cos \theta)\partial\phi$$

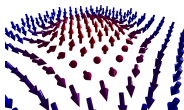
- Effective electromagnetic fields

$$\mathbf{E}_s = -\nabla A_{s,0}^z + \partial_t \mathbf{A}_s^z = -\frac{1}{2}\mathbf{n} \cdot (\dot{\mathbf{n}} \times \nabla_i \mathbf{n})$$
$$\mathbf{B}_s = \nabla \times \mathbf{A}_s^z = \frac{1}{4} \sum_{ijk} \epsilon_{ijk} \mathbf{n} \cdot (\nabla_j \mathbf{n} \times \nabla_k \mathbf{n})$$

Spin electromagnetic field

- Spin magnetic field

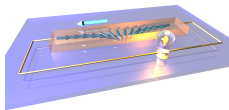
$$\mathbf{B}_{s,i} = \frac{\hbar}{4e} \sum_{jk} \epsilon_{ijk} \mathbf{n} \cdot (\nabla_j \mathbf{n} \times \nabla_k \mathbf{n})$$



- Chirality (non-coplanarity)
- Frustrated magnets, Magnetic skyrmion
 ~ 0.8 T for 30 nm size

- Spin electric field

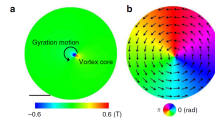
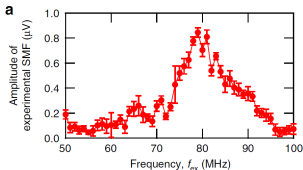
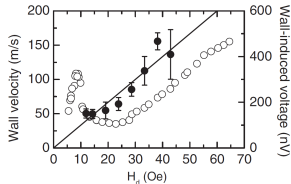
$$\mathbf{E}_{s,i} = -\frac{\hbar}{2e} \mathbf{n} \cdot (\dot{\mathbf{n}} \times \nabla_i \mathbf{n})$$



- Non-coplanarity in space-time
- Moving structures $E_s \propto v$
Domain wall, vortex, skyrmion
 ~ 0.1 V/m for 10 nm DW @ $v = 4$ m/s

Current generation from magnetization dynamics

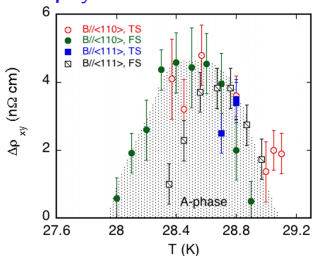
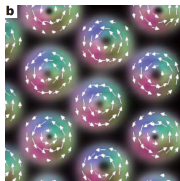
- E_s from motion of domain wall, vortex $V \sim \mu V$, $E_s \propto v$



Domain wall Yang'09

Vortex Tanabe'12

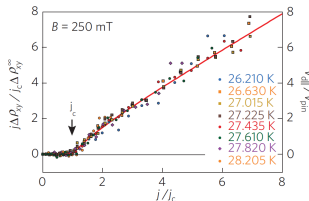
- Skyrmion lattice $\rho_{xy} \sim 4n\Omega cm \propto B_s$



Topological Hall effect B_s

Yu'10

Lee'09, Neubauer'09



Voltage $E_s \propto v$

Schulz'12

Monopole in adiabatic spin Berry's phase

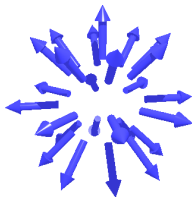
$$\mathbf{E}_s = -\frac{\hbar}{2e} [\mathbf{n} \cdot (\dot{\mathbf{n}} \times \nabla_i \mathbf{n})]$$
$$\mathbf{B}_s = \frac{\hbar}{4e} \epsilon_{ijk} \mathbf{n} \cdot (\nabla_j \mathbf{n} \times \nabla_k \mathbf{n})$$

- Satisfy Maxwell's equations with monopole

$$\nabla \times \mathbf{E}_s + \partial_t \mathbf{B}_s = \mathbf{j}_m$$
$$\nabla \cdot \mathbf{B}_s = \rho_m$$

$$\mathbf{j}_m = \frac{\hbar}{4e} \sum_{jk} \dot{\mathbf{n}} \cdot (\nabla_j \mathbf{n} \times \nabla_k \mathbf{n})$$

$$\rho_m = \frac{\hbar}{4e} \sum_{ijk} \nabla_i \mathbf{n} \cdot (\nabla_j \mathbf{n} \times \nabla_k \mathbf{n})$$



Topological monopole
(Hedgehog)

Coupling to electromagnetic fields

Effective interaction Hamiltonian

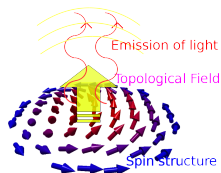
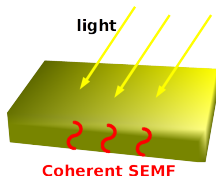
Kawaguchi, GT'14

$$H_{\text{int}} = \frac{e^2}{m} \int d^3r (2s_e \tau \mathbf{E} \cdot \mathbf{A}_s + 2s_e \tau^2 \mathbf{E} \cdot \mathbf{E}_s + b \mathbf{B} \cdot \mathbf{B}_s)$$

τ : Electron elastic lifetime

- \mathbf{A}_s is physical field Large sd splitting
- First term : Spin-transfer effect

- $B \Rightarrow B_s$: Frustration

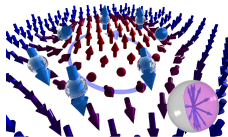


Electromagnetic excitation of spin electromagnetic fields (?)

Current generation from magnetization dynamics

- Spin Berry's phase

Gradient of spin $\nabla \mathbf{S} \Rightarrow$ Effective gauge field, $\mathbf{E}_s, \mathbf{B}_s$



- Spin-orbit interaction

$$H_{\text{SO}} = \lambda \cdot (\mathbf{p} \times \boldsymbol{\sigma})$$

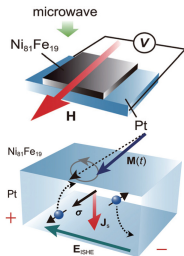
- Directly couples spin and electron motion
- Current generation ?
- Modification of spin Berry's phase ?

Kim'12, Takeuchi'12, Nakabayashi'14, Takashima, Fujimoto'14

Charge current pumped by magnetization

- Spin pumping + Inverse spin Hall

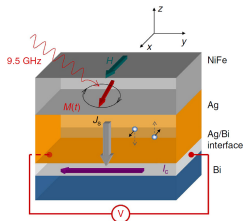
Saitoh'06



- Ferro/Pt
- Spin-orbit in heavy metal
- $\dot{M} \Rightarrow j_s \Rightarrow j$
- $4\mu\text{V}$ @ mW microwave

- Spin pumping + Inverse Edelstein

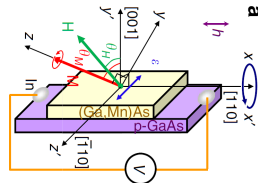
Sanchez'13



- Ferro/Ag/Bi
- Rashba spin-orbit at Ag/Bi interface
- $\dot{M} \Rightarrow j_s \Rightarrow s \Rightarrow j$

- (Ga,Mn)As

Chen'13



- Uniform ferromagnet
- Strong spin-orbit
- $\dot{M} \Rightarrow j$

Spin pumping and inverse spin Hall effects

- Spin pumping

$$j_s = g_{\uparrow} \tanh \frac{d}{2\lambda_s} \langle \mathbf{S} \times \dot{\mathbf{S}} \rangle$$

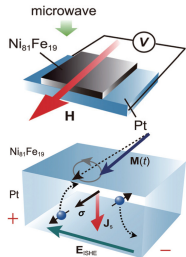
g_{\uparrow} : Mixing conductance Spin flip at interface

- Inverse spin Hall

$$V = \rho_n \theta_{\text{SHE}} j_s$$

θ_{SHE} : Spin Hall angle Spin-orbit interaction

- $V = \theta_{\text{SHE}} \rho_n g_{\uparrow} \tanh \frac{d}{2\lambda_s} \langle \mathbf{S} \times \dot{\mathbf{S}} \rangle$

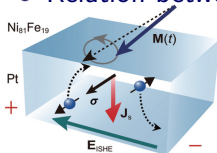


- Useful but incomplete description

- Two phenomenological parameters $\theta_{\text{SHE}}, g_{\uparrow}$
- Spin current is not defined uniquely Not conserved current
- $j \neq \theta_{\text{SHE}} j_s$ in a simple theoretical model *Takeuchi, GT'10*

Spin pumping and inverse spin Hall effects

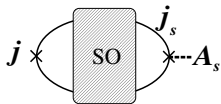
- Relation between observables



- $\dot{\mathbf{M}} \Rightarrow \mathbf{j}$

- Calculate current and motive force

- Feynman diagram

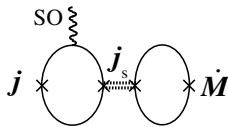


$$H_A = \int dr \mathbf{j}_s \cdot \mathbf{A}_s$$

\mathbf{j}_s : Spin current

$\mathbf{A}_s \sim \dot{\mathbf{M}}, \nabla \mathbf{M}$: Spin gauge field

- Conventional picture

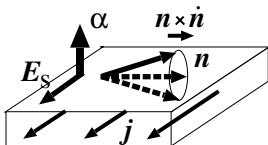


Different effect

Simple view of charge pumping

Takeuchi'12, GT PRB'13, Nakabayashi New J Phys'14

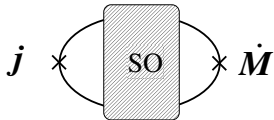
- Model



- sd interaction n : Magnetization
- Rashba & Random Spin-orbit interaction

$$H = \left(-\frac{\hbar^2}{2m} \nabla^2 - \epsilon_F \right) + \Delta_{sd} (\mathbf{n} \cdot \boldsymbol{\sigma}) + \alpha_R \cdot (\mathbf{p} \times \boldsymbol{\sigma}) + \text{spin relaxation}$$

- Diagram calculation



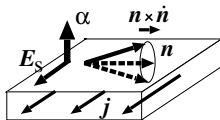
- Force $\mathbf{F} \equiv \frac{m}{en} \left\langle \frac{dj}{dt} \right\rangle$
- Pumped current \mathbf{j} with Hall contribution

Rashba-induced spin electromagnetic field

GT'13, Nakabayashi>, New J Phys.'14

- Result

$$\mathbf{j} = \frac{1}{\mu_s} \nabla \times \mathbf{B}_R + \sigma_s \mathbf{E}_R$$
$$\mathbf{F} = q_s \mathbf{E}_R + q_s (\mathbf{v} \times \mathbf{B}_R)$$



Linear order in α_R

- Rashba-induced spin electromagnetic field

$$\mathbf{E}_R = -\frac{m}{e\hbar} [\alpha_R \times (\dot{\mathbf{n}} + \beta_R (\mathbf{n} \times \dot{\mathbf{n}}))]$$
$$\mathbf{B}_R = \frac{m}{e\hbar} [\nabla \times (\alpha_R \times \mathbf{n})]$$

β_R : spin relaxation rate

- \mathbf{E}_R & \mathbf{B}_R : Effective spin electromagnetic fields
 $\mathbf{E}_R \sim 2\text{kV/m}$, $\mathbf{B}_R \sim 0.2\text{kT}$ Not charge electromagnetic fields
- Arise from Rashba interaction and spin dynamics
Generalized spin Berry's phase

Effective spin electromagnetic field

- 'Maxwell's equations'

$$\begin{aligned}\nabla \times \mathbf{E}_R + \dot{\mathbf{B}}_R &= \mathbf{j}_m \\ \nabla \cdot \mathbf{B}_R &= 0 \\ \nabla \cdot \mathbf{E}_R &= -\frac{\rho_s}{\epsilon_s} \\ \nabla \times \mathbf{B}_R - \epsilon_s \mu_s \dot{\mathbf{E}}_R &= \mu_s \mathbf{j}\end{aligned}$$

- $\mathbf{j}_m = \beta_R \nabla \times (\boldsymbol{\alpha}_R \times (\mathbf{n} \times \dot{\mathbf{n}}))$ Monopole current

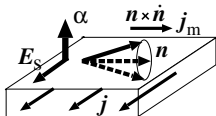
Monopole induced by magnetization dynamics and spin relaxation

\mathbf{j}_m is not spin current

Effective spin electromagnetic field

- Current pumping by magnetization dynamics in Rashba system

$$\nabla \times \mathbf{E}_R = \mathbf{j}_m$$
$$\mathbf{j}_m = \beta_R \nabla \times (\boldsymbol{\alpha}_R \times (\mathbf{n} \times \dot{\mathbf{n}}))$$



- $\dot{\mathbf{M}} \rightarrow$ Monopole current $\mathbf{j}_m \Rightarrow \mathbf{j}$
- Effective electromagnetic fields satisfying Maxwell's eq. + monopole
- Spin current is not necessary

Electromagnetic description of spintronics

Spintronics without spin current (!?)

Acknowledgements

Tokyo Metropolitan Univ. Akihito Takeuchi (→ Aoyama-gakuin),
Noriyuki Nakabayashi, Hideo Kawaguchi

RIKEN Henri Saarikoski

Nagoya Univ. Hiroshi Kohno

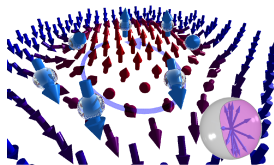
Leeds Univ. Christopher H. Marrows

Grants Kakenhi 

Japan Society for the Promotion of Science and The
Ministry of Education, Culture, Sports, Science and
Technology (MEXT), Japan

Summary

- sd exchange interaction in ferromagnetic metals
- Effective vector potential for spin
 - Current-induced torque, Spin dynamics
 - Spin Berry's phase
- Spin-orbit effects
 - Novel driving mechanism of magnetization structures
 - Spin relaxation torque (β)
 - Rashba spin-orbit interaction Interface, Multilayers
 - Modification of spin Berry's phase Spin electromagnetic field
 - Spin-charge conversion
 - current generation by spin relaxation monopole $\nabla \times \mathbf{E}_R = \mathbf{j}_m$



References

- Nakabayashi, GT, *New J Phys*, **16**,015016(2014).
- Tatara, Nakabayashi, K.-J. Lee, *Phys. Rev. B* **87**,054403(2013).
- Takeuchi, Tatara, *J. Phys. Soc. Jpn.* **81**,033705(2012).
- Kawaguchi, GT, *J. Phys. Soc. Jpn.* **83**, 074710 (2014).
- Tatara, Takeuchi, Nakabayashi, Taguchi, *J. Korean Phys. Soc.* **61**, 1331 (2012).
- GT, Kohno, Shibata, *Phys. Rep.* **468**, 213 (2008)
- 数理科学7月号(2014)