Bulk-edge correspondence in topological transport and pumping

Ken Imura

Hiroshima University

BEC: bulk-edge correspondence

bulk properties	edge/surface properties
- energy bands, band gap	mid-gap edge states
gapped, insulating ⟷	gapless, metallic

- band structure & wave function

"topologically"

nontrivial vs. trivial

- number of
- presence/absence of

edge states

bulk-edge correspondence

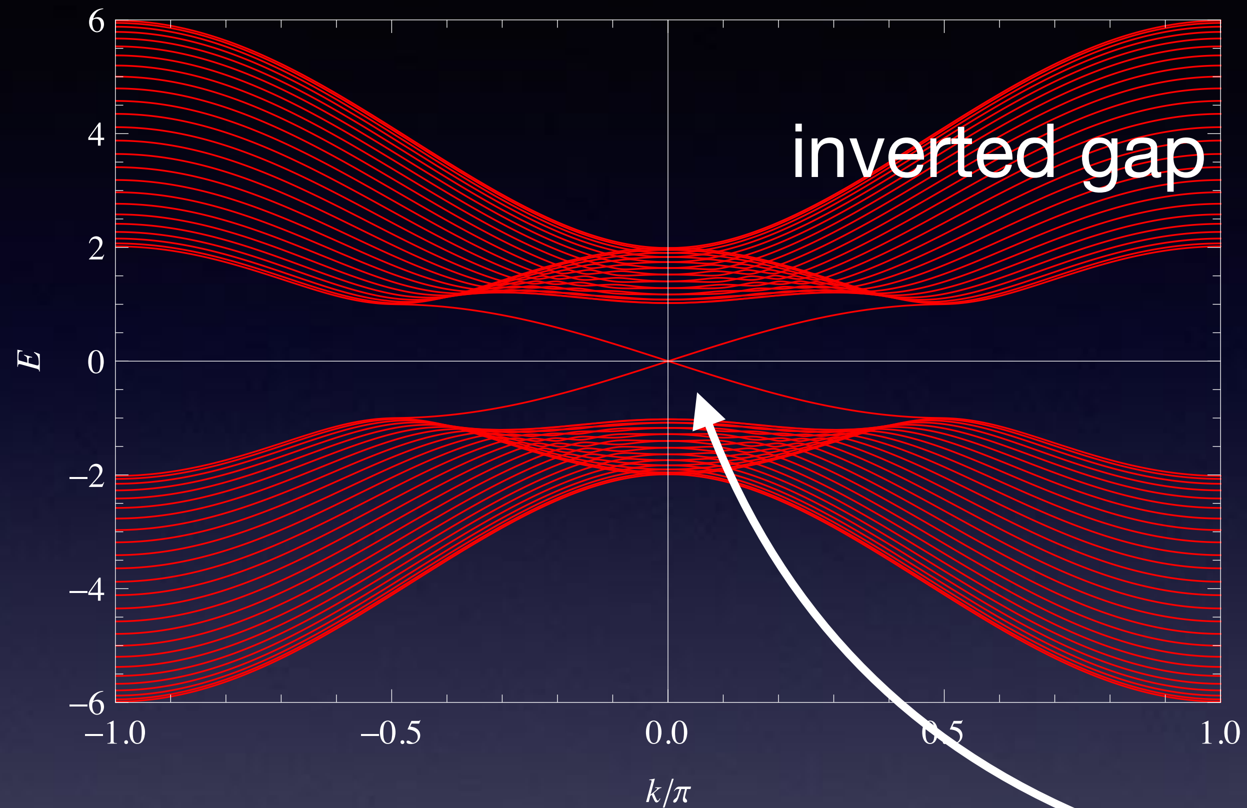
- topological invariant:

$$C = \frac{1}{8\pi}\int d^2k \; \epsilon_{\mu\nu} \boldsymbol{n} \cdot [\partial_{k_\mu} \boldsymbol{n} \times \partial_{k_\nu} \boldsymbol{n}]$$

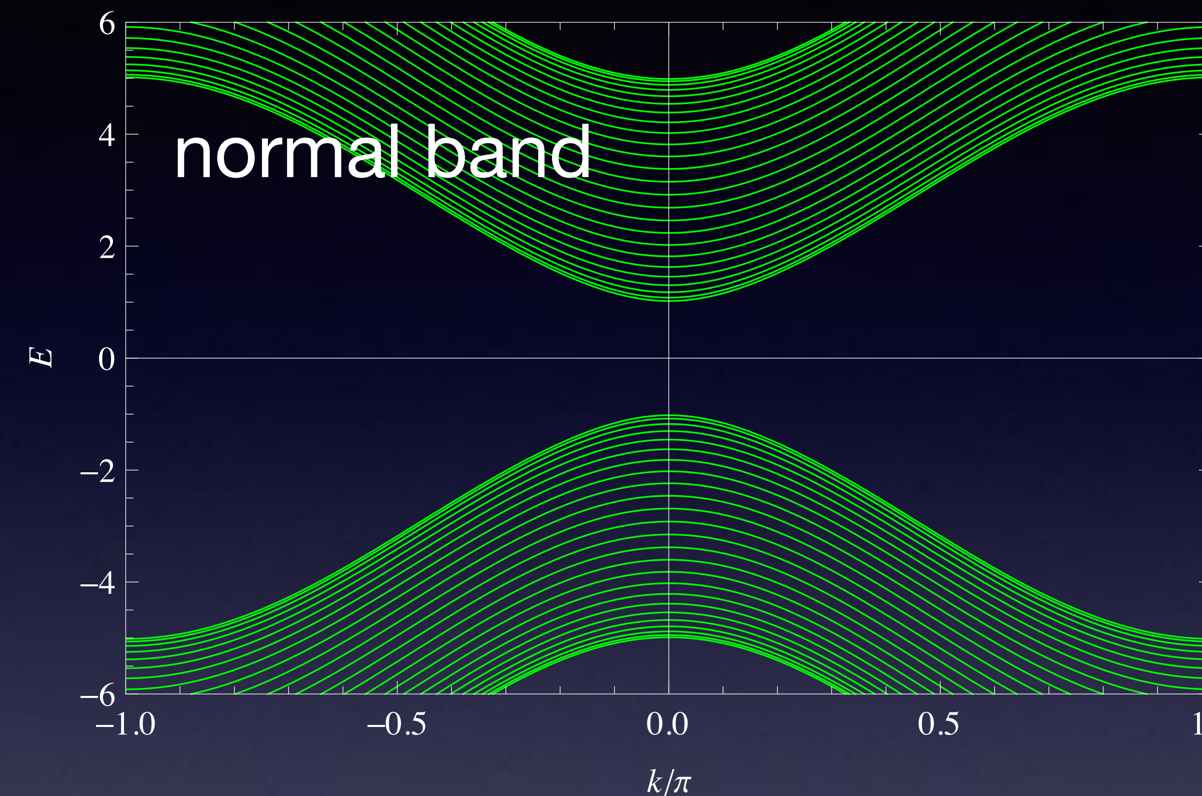
Z vs. Z2 types

Topological vs. non-topological band structures

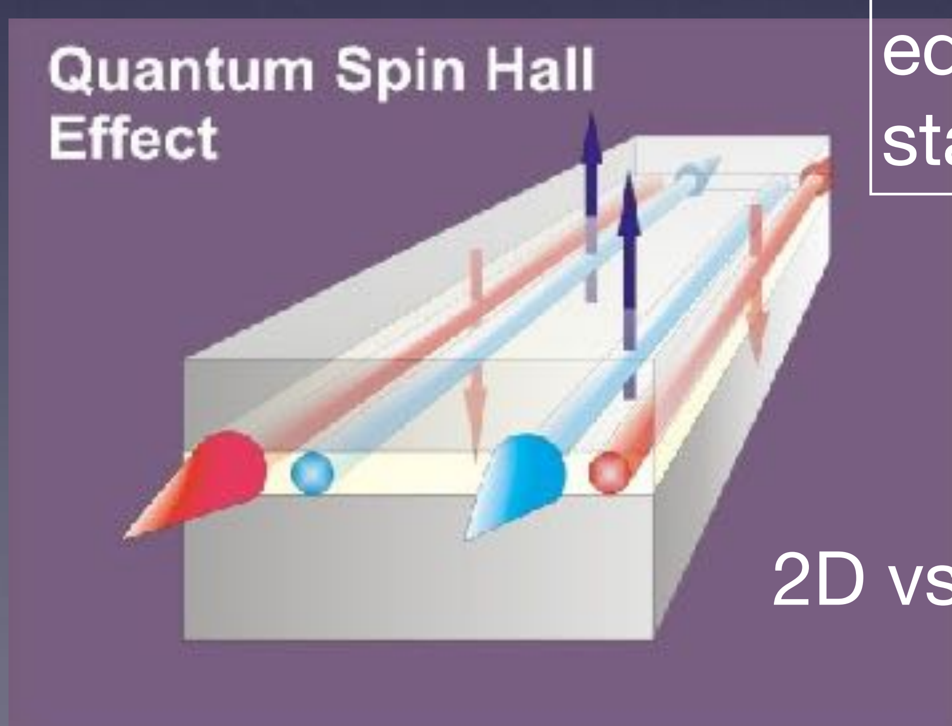
TI: topological insulator

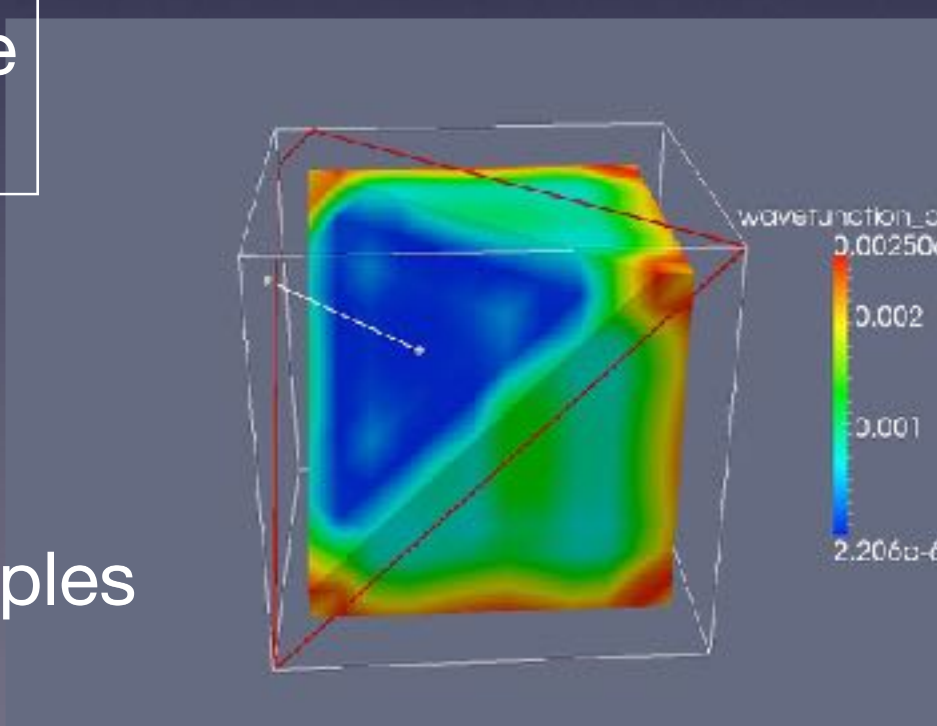


OI: ordinary insulator



edge/surface states





2D vs. 3D examples

2D example: how to characterize the bulk

$$H = p_x\sigma_x + p_y\sigma_y + m(\boldsymbol{p})\sigma_z$$
$$= P_\mu(\boldsymbol{p})\sigma_\mu$$

$$m(\boldsymbol{p}) = m_0 + m_2\boldsymbol{p}^2$$

- The winding number

$$N_2 = -\frac{1}{8\pi}\int d^2p\ \epsilon_{\mu\nu}\ \boldsymbol{n}\cdot[\partial_{p_\mu}\boldsymbol{n}\times\partial_{p_\nu}n],\qquad n_\mu(\boldsymbol{p}) = \frac{P_\mu(\boldsymbol{p})}{\sqrt{P_\mu P_\mu}}$$

mapping: $\boldsymbol{p}\to n_\mu(\boldsymbol{p})$ $\quad\mathbb{R}^2\to\mathbb{S}^2$

$$p = |\boldsymbol{p}|\to\infty \qquad\qquad p = 0$$
$$\boldsymbol{n}(\boldsymbol{p})\to(0,0,\mathrm{sgn}(m_2)) \qquad \boldsymbol{n}(\boldsymbol{p})\to(0,0,\mathrm{sgn}(m_0))$$

stereographic projection ➔ $\mathbb{S}^2\to\mathbb{S}^2$ $\qquad\pi_2(\mathbb{S}^2) = 0,\pm1,\pm2,\cdots$

$$N_2 = \frac{\mathrm{sgn}(m_2)-\mathrm{sgn}(m_0)}{2}$$

BEC in different formats

here, two specific examples:

Case 1: topological insulator thin films

Phys. Rev. B 92, 235407 (2015)

- correspondence in physical properties

Phys. Rev. B 94, 235414 (2016)

bulk *penetration of top/bottom "surface" wave function into the <<bulk>> of auxiliary 3D system*

edge *1D helical modes circulating around a thin-film*

Case 2: topological quantum pump

arXiv:1706.04493

- Laughlin's argument, a version of BEC
- pump version: more intuitive interpretation

Case 1: topological insulator thin films

Model: standard Wilson-Dirac type

$$H^{\mathrm{WD}}_{\mathrm{bulk}}(\boldsymbol{k}) = m_{3\mathrm{D}}(\boldsymbol{k})\tau_z \otimes 1_2 + \sum_{\mu=x,y,z} t_\mu \sin k_\mu \tau_x \otimes \sigma_\mu \qquad 4 \times 4$$

spin & orbital

- gap/mass and Wilson terms

$$m_{3\mathrm{D}}(\boldsymbol{k}) = m_0 - \sum_{\mu=x,y,z} b_\mu \cos k_\mu$$

- Topological classification

⟶ periodic table (ten-fold way)

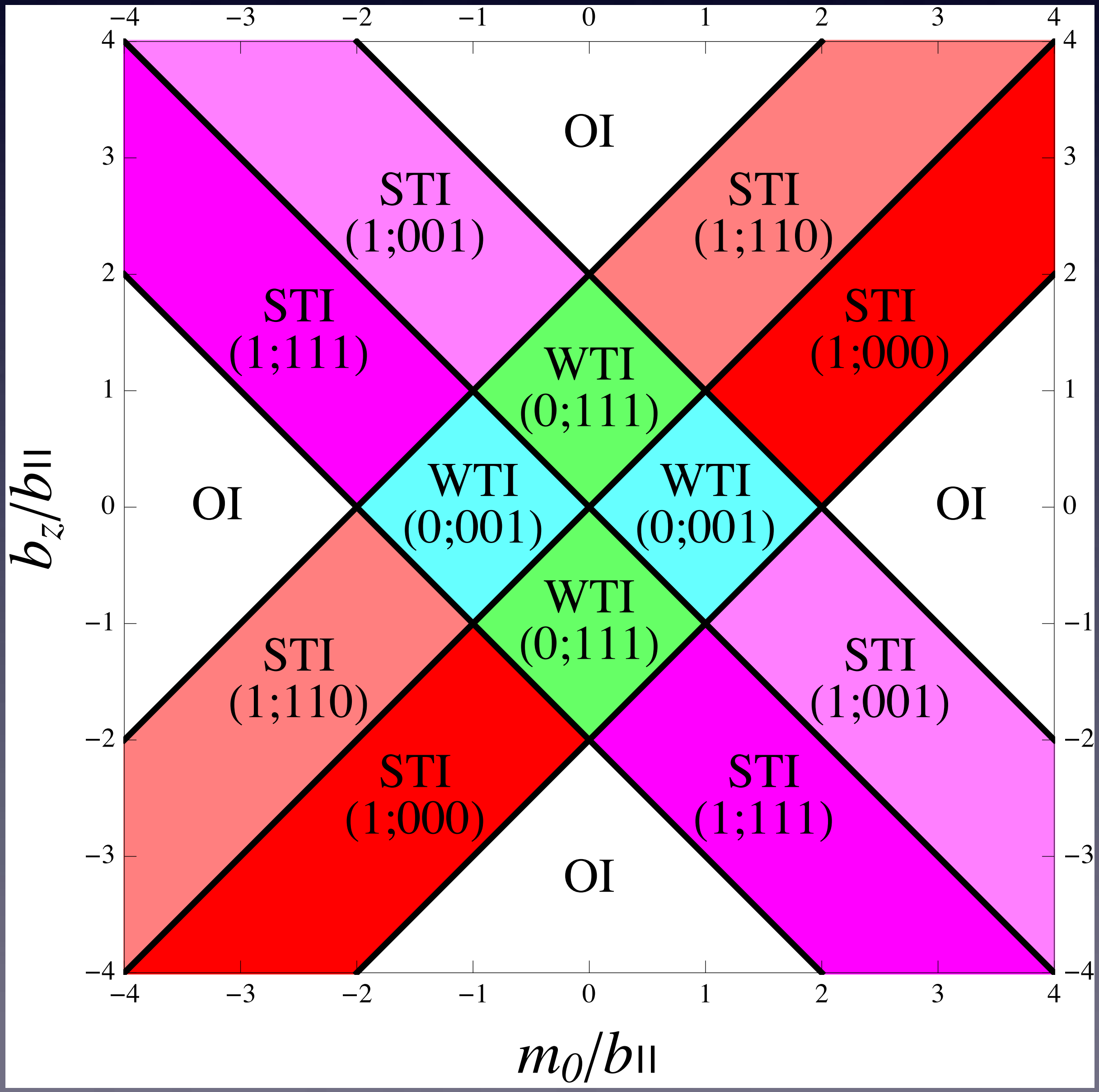
Ryu & Schnyder, PRB 2010

Present model: 3D, class AII

Diagnosis: Z2 type

16 different types of topological phases: 8 STI, 7 WTI, 1 OI

Z2 indices: $\nu_0, (\nu_1, \nu_2, \nu_3)$



The "periodic table" of topological insulators (ten-fold way)

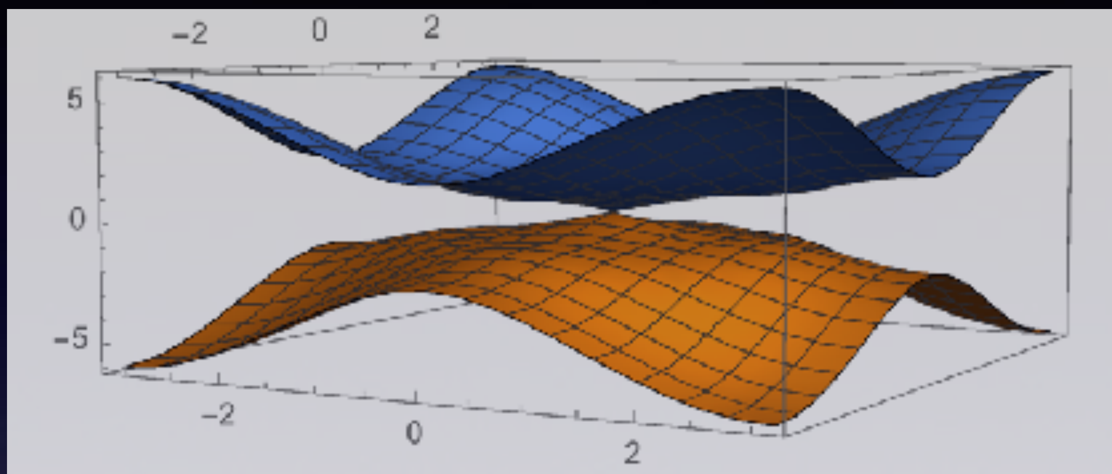
		Symmetry			$\delta=d-D$							
s	AZ	Θ^2	Ξ^2	Π^2	0	1	2	3	4	5	6	7
0	A	0	0	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0
1	AIII	0	0	1	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$
0	AI	1	0	0	$\mathbb{Z}$	0	0	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$
1	BDI	1	1	1	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$
2	D	0	1	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$2\mathbb{Z}$	0
3	DIII	−1	1	1	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$2\mathbb{Z}$
4	AII	−1	0	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0
5	CII	−1	−1	1	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0
6	C	0	−1	0	0	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0
7	CI	1	−1	1	0	0	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$

Ryu & Schnyder, PRB 2010; Teo & Kane, PRB 2010

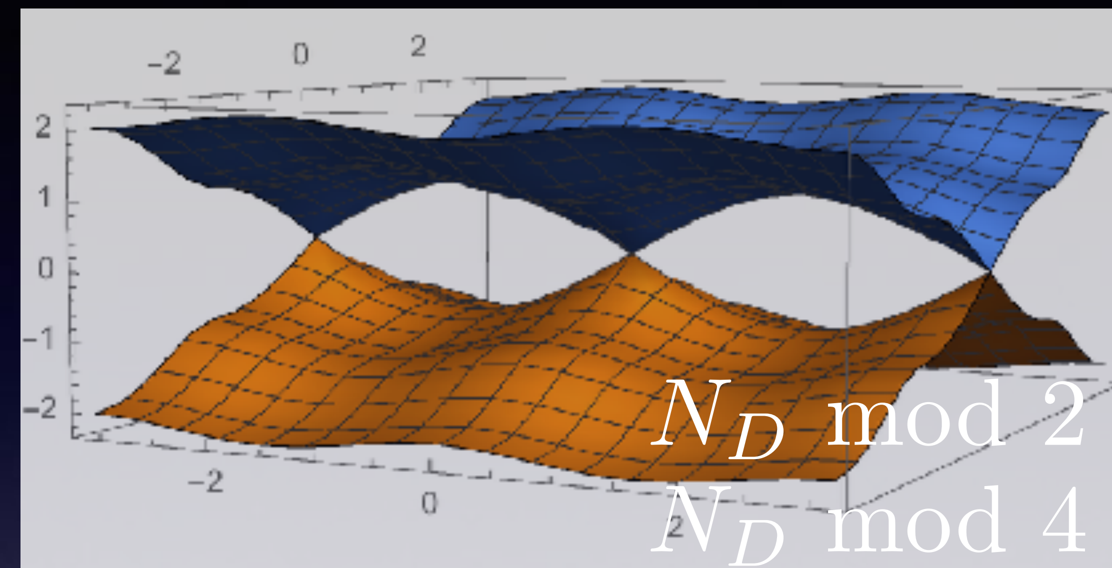
STI/WTI: *two types of topological insulators*

strong vs. weak

In reciprocal space



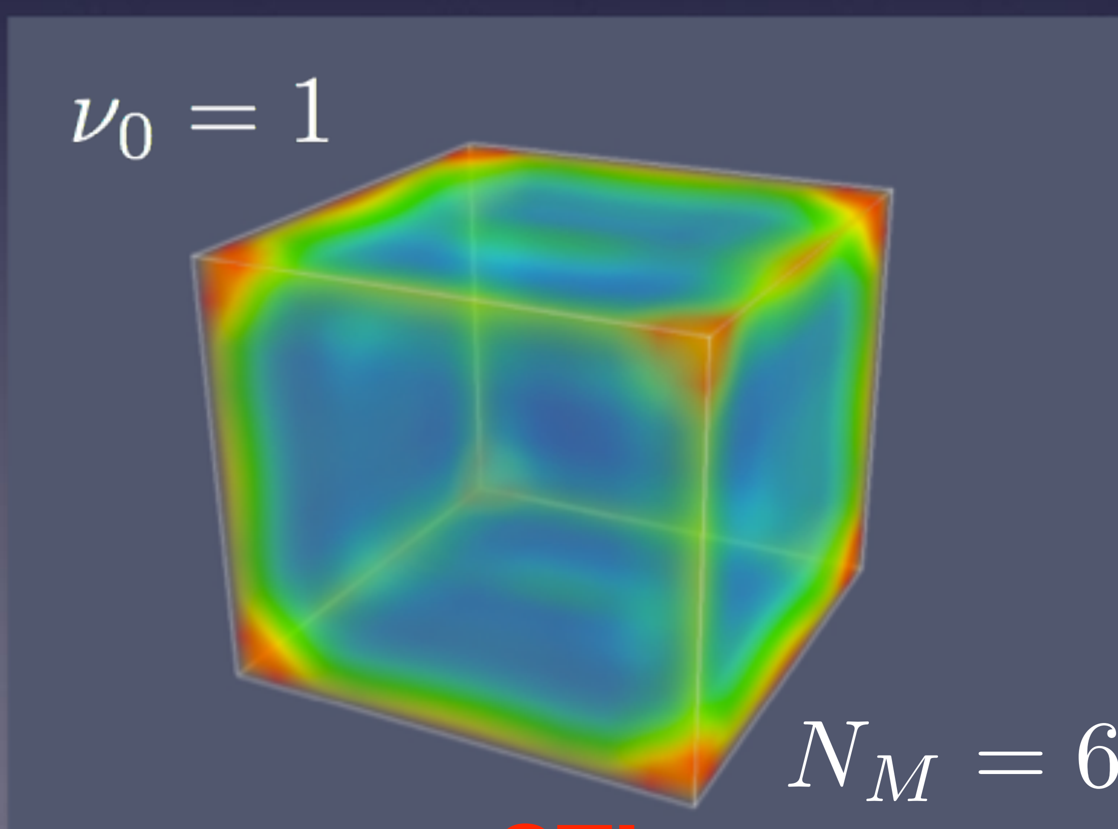
$N_D \bmod 2 = 1$



$N_D \bmod 2 = 0$

$N_D \bmod 4 = 2$

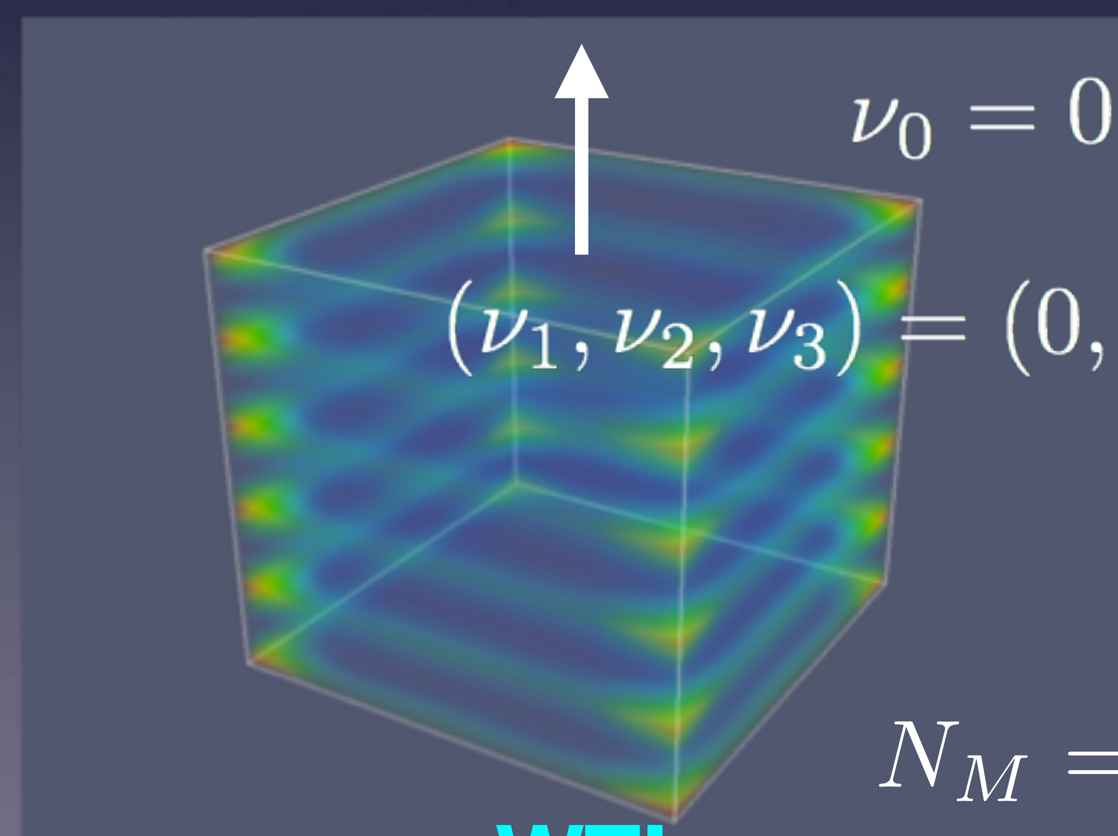
In real space



$\nu_0 = 1$

$N_M = 6$

STI



$\nu_0 = 0$

$(\nu_1, \nu_2, \nu_3) = (0, 0, 1)$

$N_M = 4$

WTI

Reduction to a thin film

- tight-binding construction:

$$H^{\mathrm{WD}}_{\mathrm{film}}(\boldsymbol{k}_{2\mathrm{D}}) = 1_{N_z} \otimes \left(m_{2\mathrm{D}}(\boldsymbol{k}_{2\mathrm{D}})\gamma_0 + \sum_{\mu=x,y} t_\mu \sin k_\mu \gamma_\mu \right)$$

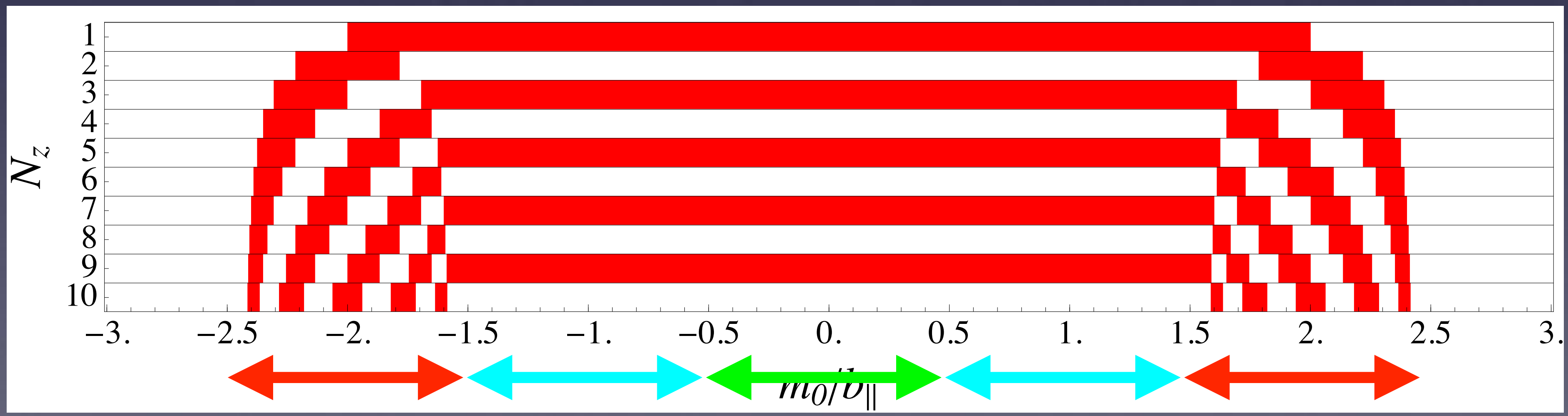
$$-\frac{b_z}{2}\begin{pmatrix} 0 & 1 & & \\ 1 & \ddots & \ddots & \\ & \ddots & \ddots & 1 \\ & & 1 & 0 \end{pmatrix} \otimes \gamma_0 + \frac{t_z}{2}\begin{pmatrix} 0 & -i & & \\ i & \ddots & \ddots & \\ & \ddots & \ddots & -i \\ & & i & 0 \end{pmatrix} \otimes \gamma_3$$

2D gap/mass and Wilson terms:

$$m_{2\mathrm{D}}(\boldsymbol{k}_{2\mathrm{D}}) = m_0 - \sum_{\mu=x,y} b_\mu \cos k_\mu$$

TI thin film = stacked 2D QSH layers

- Topological property as a quasi 2D system → still Z2 type



$\nu = 1$

$\nu = 0$

STI | WTI 001 | WTI 111 | WTI 001 | STI

$b_z/b_{||} = 0.5$

Phys. Rev. B 92, 235407 (2015)

Two characteristic patterns? ⟶ brick vs. stripe

1) stripe pattern: *even-odd feature w.r.t. Nz*

WTI situation

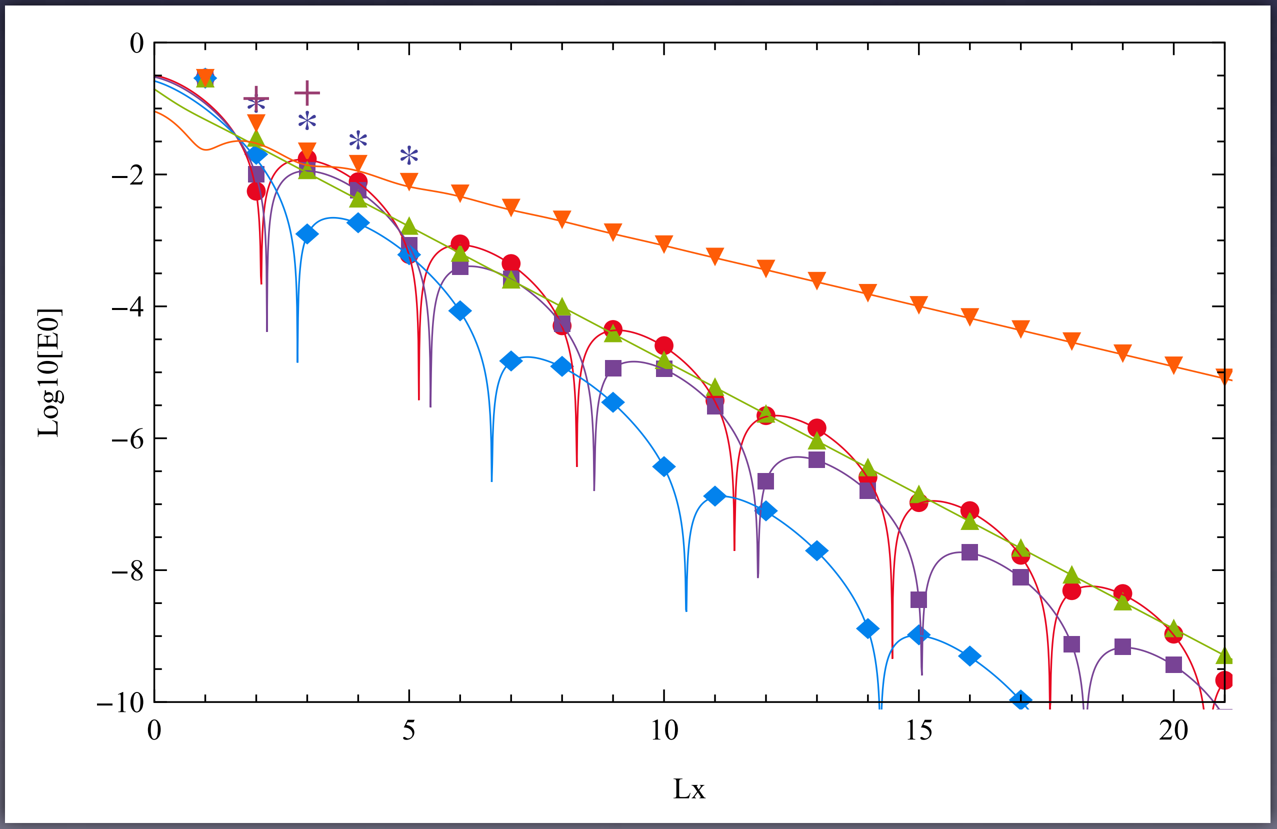
Nz: even — hybridization of gapless helical edge modes

formation of the hybridization gap $\nu = 0$

Nz: odd — a single gapless combination remains $\nu = 1$

edge point of view

2) brick pattern: *oscillation of the surface wave function*



in an auxiliary 3D semi-infinite system

STI situation

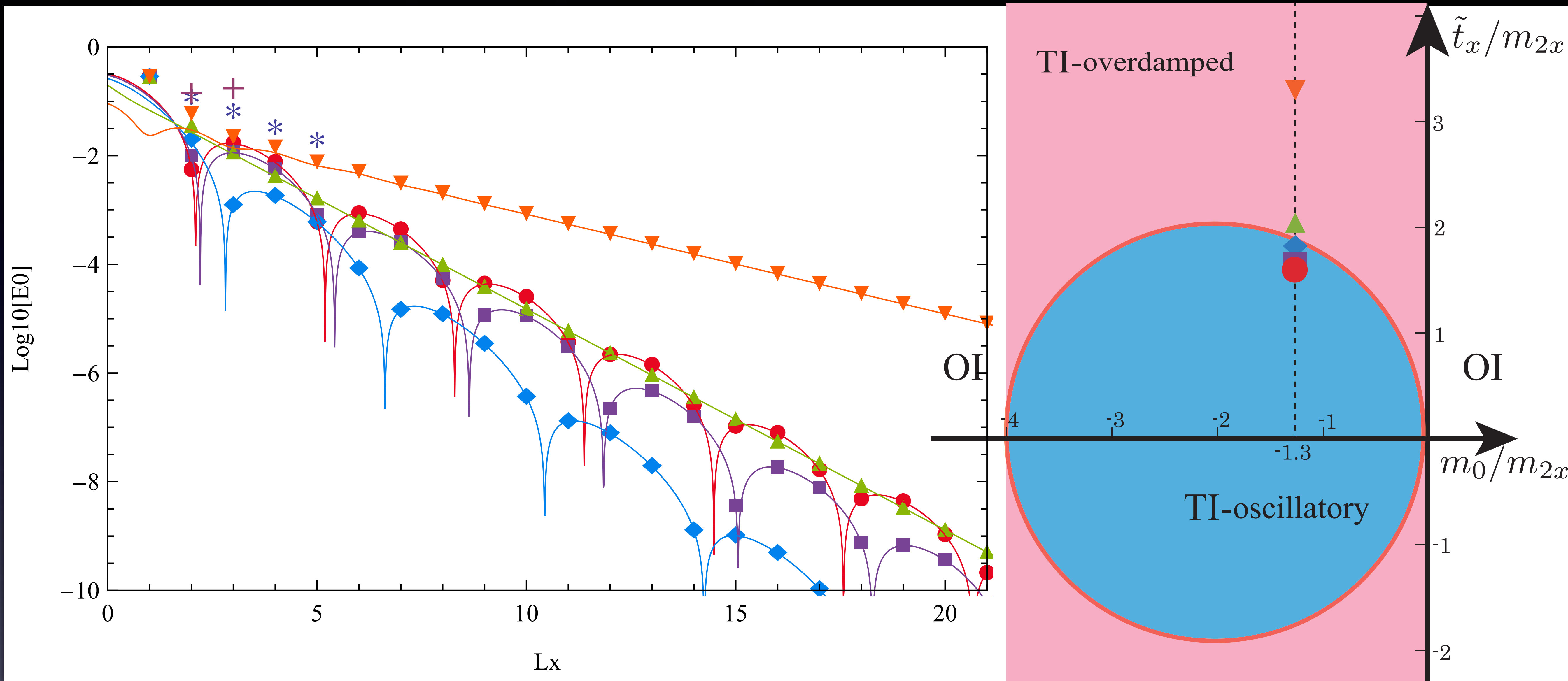
size gap ⟷ oscillation of the surface wane function

Phys. Rev. B 89, 125425 (2014)

surface/bulk point of view

Oscillatory vs. over-damped regimes

Phys. Rev. B 89, 125425 (2014)



$$2E_0(L_x) \simeq \frac{4}{\mathcal{N}} \frac{1-\left(\frac{\epsilon_{2x}}{m_{2x}}\right)^2}{\left|\frac{1}{\bar{m}_{1+}} - \frac{1}{\bar{m}_{1-}}\right|} |\psi_{\text{semi}}(x = L_x + 1)|.$$

Energy gap ⟷ oscillation of the surface wave function

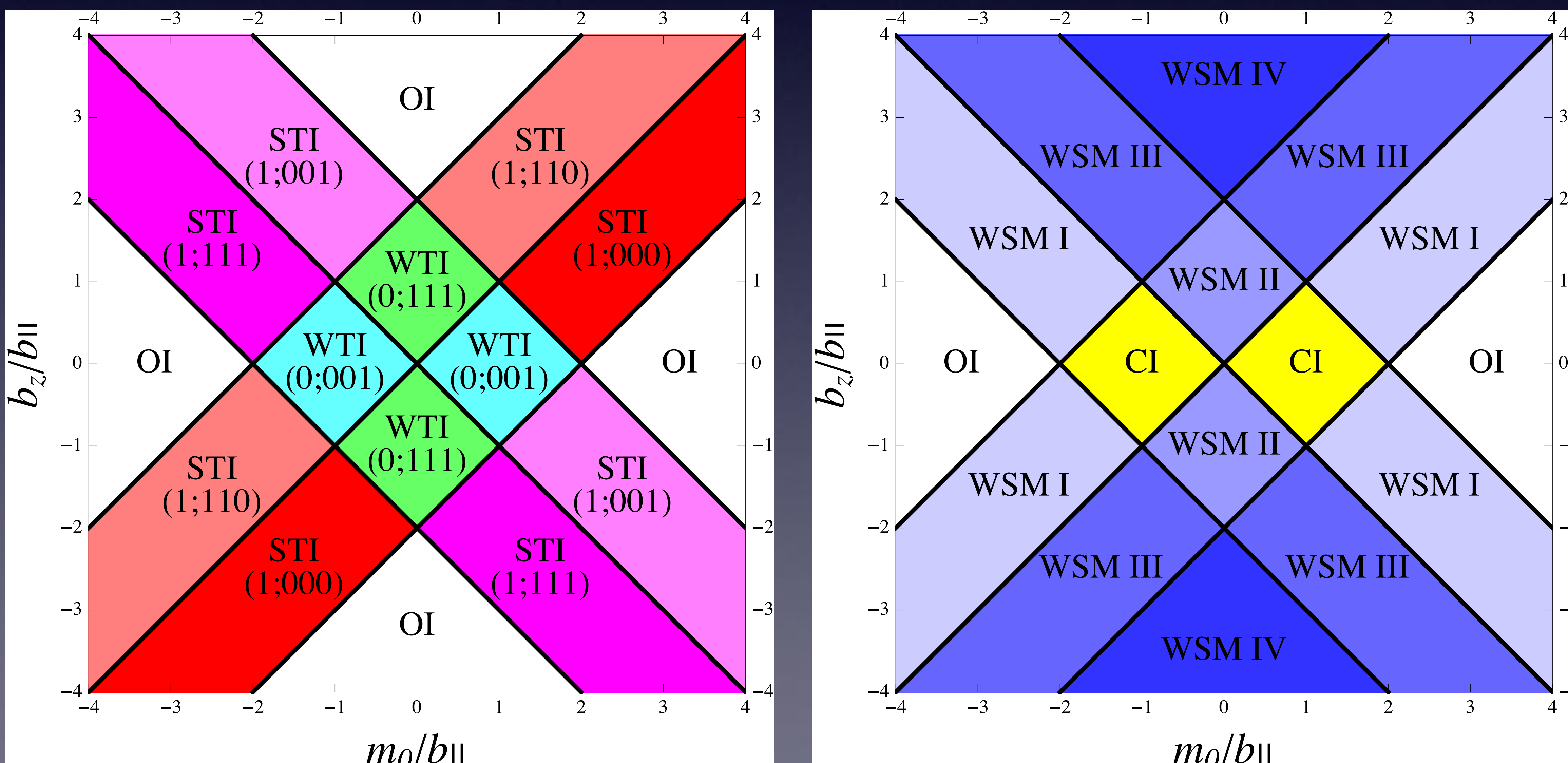
Case of Weyl semimetal thin films

- Model Hamiltonian: $$H^{\mathrm{CI}}_{\mathrm{bulk}}(\boldsymbol{k}) = m_{3\mathrm{D}}(\boldsymbol{k})\tau_z + \sum_{\mu=x,y} t_\mu \sin k_\mu \tau_\mu$$

gap/mass and Wilson terms are the same as the TI case:

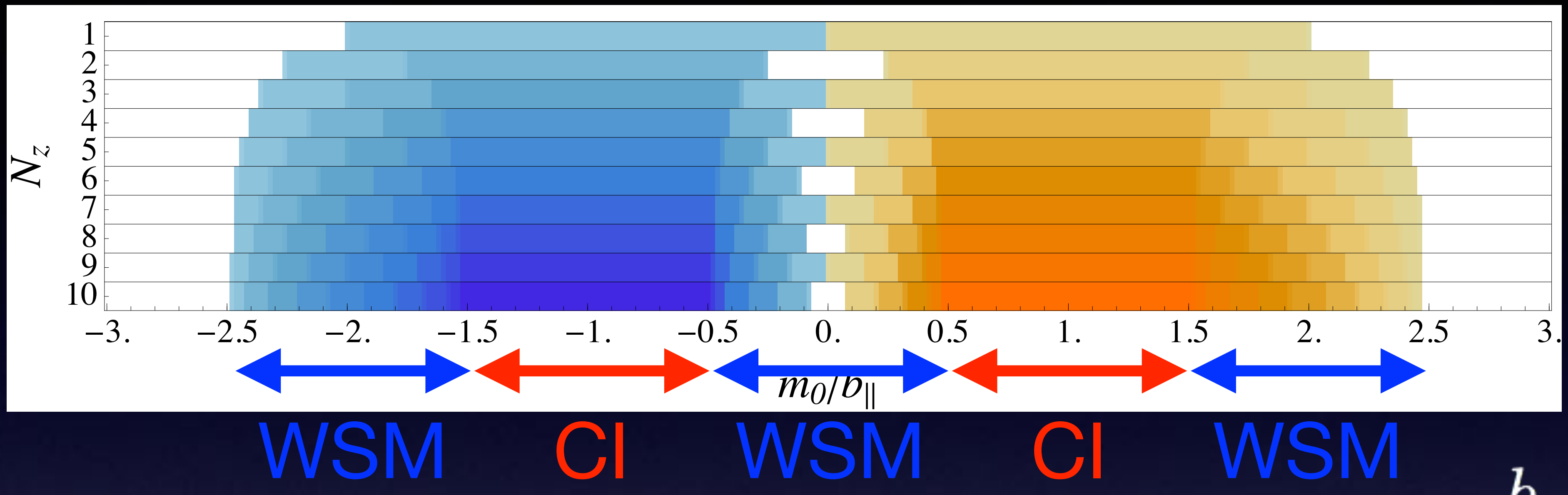
$$m_{3\mathrm{D}}(\boldsymbol{k}) = m_0 - \sum_{\mu=x,y,z} b_\mu \cos k_\mu$$

⟶ similar phase diagram



Phys. Rev. B 94, 235414 (2016)

Thin film case: brick and stripe patterns



$$b_z/b_{\|} = 0.5$$

straight regular pattern   cf. stripe pattern

⟶ CI (Chern insulator)

= stacked QAH layers

$$H_{2D} = m_{2D}(\boldsymbol{k}_{2D})\tau_z + \sum_{\mu=x,y} t_\mu \sin k_\mu \tau_\mu$$

$$m_{2D}(\boldsymbol{k}_{2D}) = m_0 - \sum_{\mu=x,y} b_\mu \cos k_\mu$$

contributions from each layer:

$$\sigma_{xy} = \frac{e^2}{h}$$

they all add up in the CI phase:

$$\sigma_{xy} = \mathcal{N}\frac{e^2}{h} \qquad |\mathcal{N}| = N_z$$

case of WSM thin films

brick regions ⟷ WSM (Weyl semimetal)

cross sections at kz = fixed in the reciprocal space

QAH, i.e., $\mathcal{C}(k_z) = \pm 1$ if $-k_0 < k_z < k_0$

OI, i.e., $\mathcal{C}(k_z) = 0$ otherwise

WSM = partially broken CI $|\mathcal{N}| < N_z$

2D topological character of the constituent QAH layers are only partially maintained

---

Similarly, in TI thin films

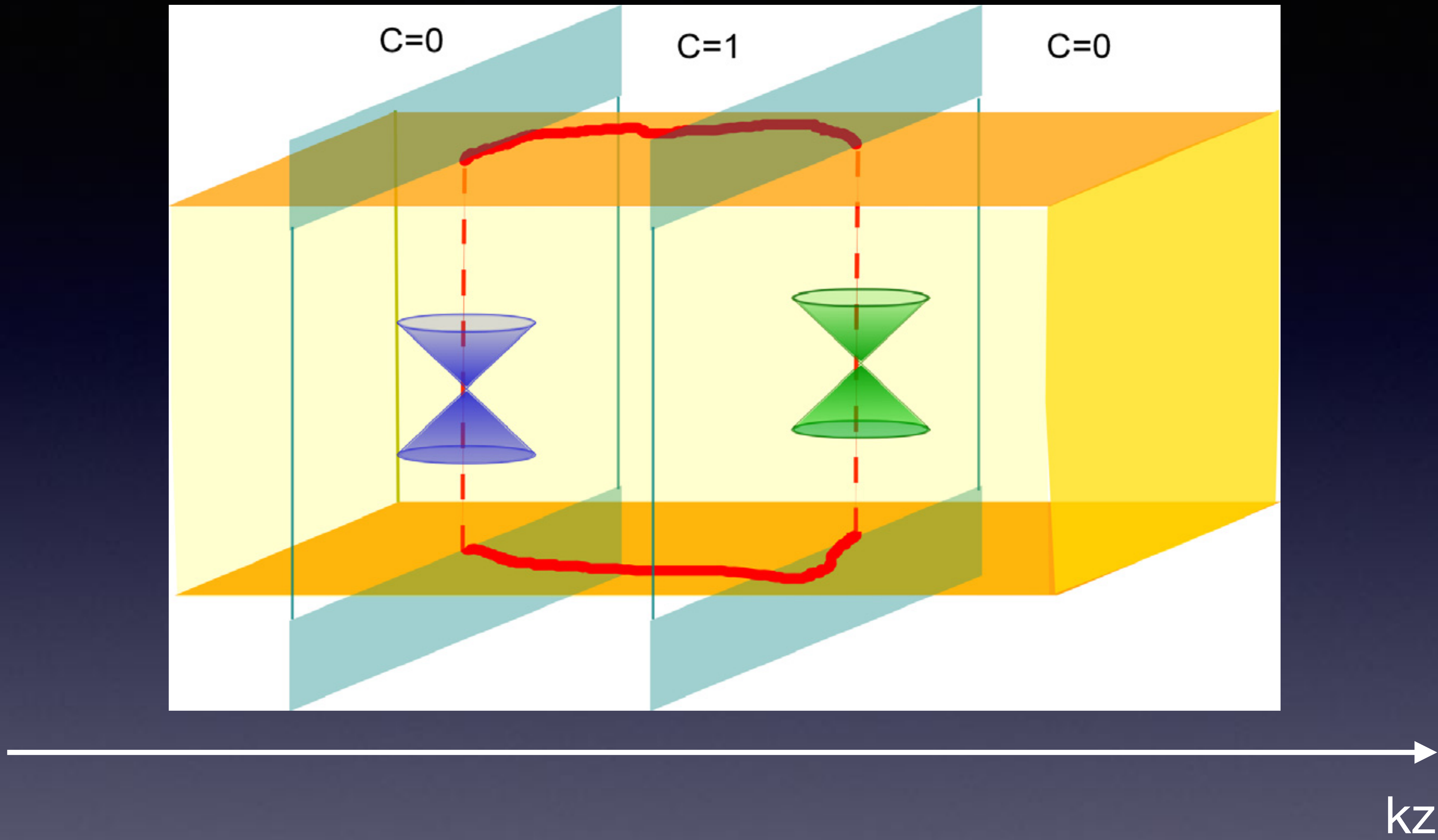
topological nature of constituent layers is

stripe pattern ⟷ WTI fully respected in the stacked system

brick pattern ⟷ STI partially broken

STI as partially broken WTI

Hosur & Qi, CRP '13



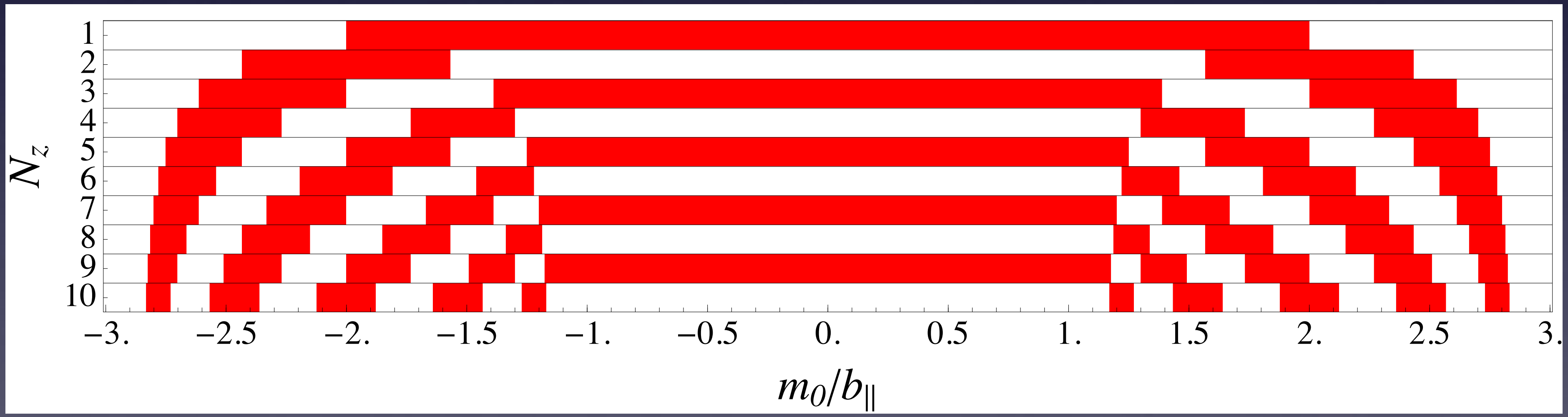
What is the precise relation between the two systems?

TI vs. WSM cases

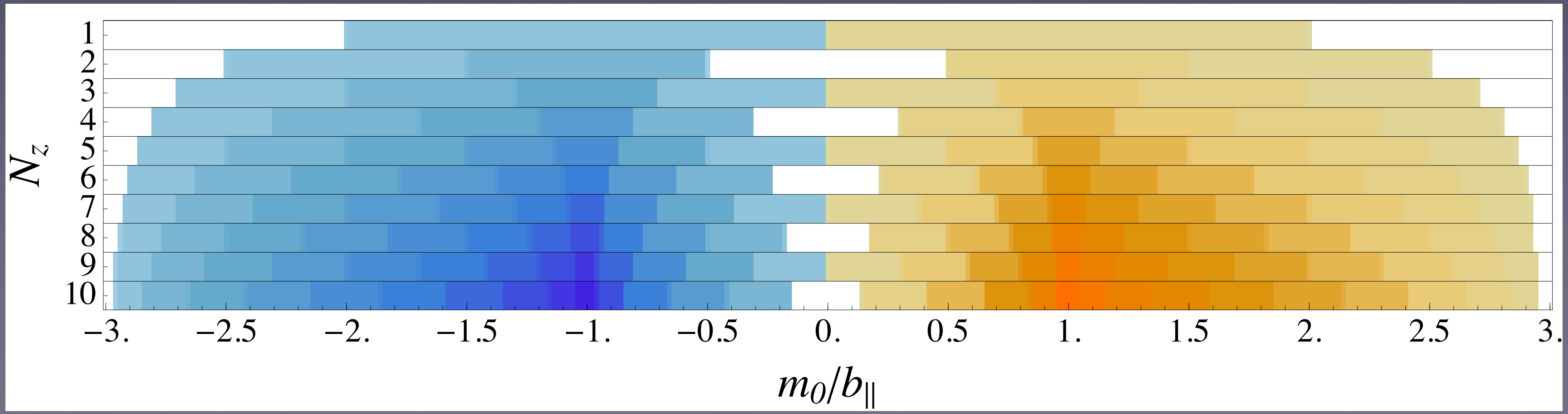
- both 3D bulk & 2D thin-film phase diagrams look very similar

Remark: In the limit of $t_z \to 0$ All the phase boundaries *forming the brick and stripe patterns* <u>coincide</u> in TI and WSM thin-film cases

TI:



WSM:



$b_z/b_{||} = 1$

The reason is simple…

1) TI thin films:

$$H^{\rm WD}_{\rm film}(\boldsymbol{k}_{\rm 2D}) = 1_{N_z} \otimes \left(m_{\rm 2D}(\boldsymbol{k}_{\rm 2D})\gamma_0 + \sum_{\mu=x,y} t_\mu \sin k_\mu \gamma_\mu \right)$$

$$-\frac{b_z}{2}\begin{pmatrix} 0 & 1 & & \\ 1 & \ddots & \ddots & \\ & \ddots & \ddots & 1 \\ & & 1 & 0 \end{pmatrix} \otimes \gamma_0 + \frac{t_z}{2}\begin{pmatrix} 0 & -i & & \\ i & \ddots & \ddots & \\ & \ddots & \ddots & -i \\ & & i & 0 \end{pmatrix} \otimes \gamma_3$$

2) WSM case:

$$H^{\rm CI}_{\rm film}(\boldsymbol{k}_{\rm 2D}) = 1_{N_z} \otimes \left(m_{\rm 2D}(\boldsymbol{k}_{\rm 2D})\tau_z + \sum_{\mu=x,y} t_\mu \sin k_\mu \tau_\mu \right)$$

$$-\frac{b_z}{2}\begin{pmatrix} 0 & 1 & & \\ 1 & \ddots & \ddots & \\ & \ddots & \ddots & 1 \\ & & 1 & 0 \end{pmatrix} \otimes \tau_z$$

$$m_{\rm 2D}(\boldsymbol{k}_{\rm 2D}) = m_0 - \sum_{\mu=x,y} b_\mu \cos k_\mu$$

common in the two cases

So, the two systems are indeed very similar…

Then, how is *the nature of brick patterns* in the WSM case?

Reminder: nature of brick patterns

oscillation of the surface wave function in an auxiliary 3D semi-infinite system

However, in the WSM model, *there is no surface state on top and at bottom*

Answer:

As approaching the limit $t_z \to 0$

The surface Dirac cones in the WTI/STI model *sink into the bulk*, *transforming into a pair of Dirac/Weyl cones*

$$@ \quad k_z = \pm \arccos\left(\frac{m_0}{b_z} - 2\frac{b_\parallel}{b_z}\right) \equiv \pm k_0$$

$$\longrightarrow \quad N_D = N_W$$

$N_D =$ # of surface Dirac cones in the WTI/STI model

$N_W =$ # of Weyl pairs in the CI/WSM model

A short summary

- Relation between the STI/WTI vs. WSM type models

"A thin-film point of view"

WTI ⟷ CI -------------- stripe region

STI ⟷ WSM ---------- brick region

- How about the role of <<bulk-edge>> correspondence?

bulk		edge
penetration of the surface wave function into the bulk	⟷	number of chiral edge modes in thin-film systems

(auxiliary 3D system)

One-to-one correspondence in <<physical properties>> at the edge and in the bulk

A short detour on the role of disorder

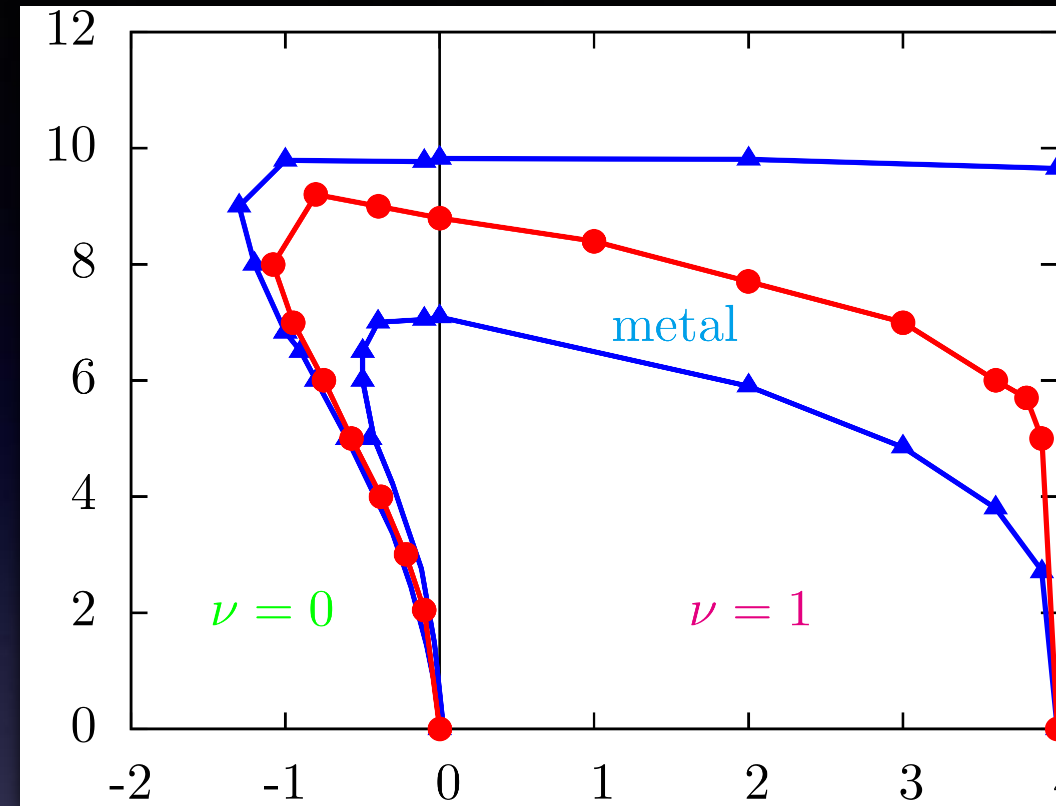
Phase diagram of 2D disordered TI

BHZ + Rashba vs. TI thin film

- No direct transition between different topological phases in 2D

potential disorder marginal in 2D

- metal in between (symplectic symmetry class)

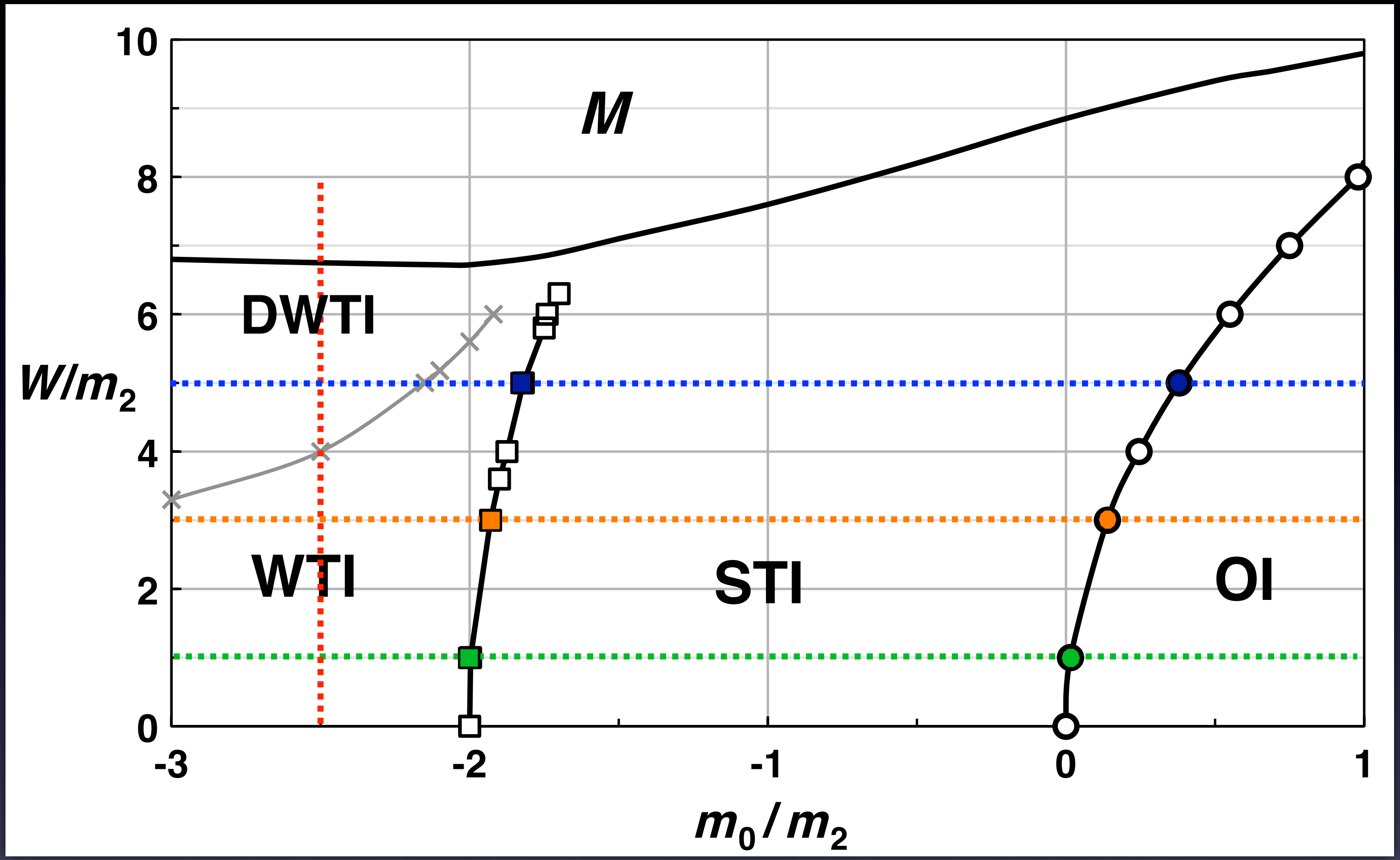


BHZ + Rashba (localization length)

JPSJ 80, 053703 (2011)

$W=$ 9.5	1.8	1.8	1.9	2	2.1	2.2	2.3	2.4	2.5	2.5	2.6	2.6	2.7	2.7	2.7	2.7	2.7	2.7	2.7	2.6	2.6	2.6	2.5	2.5	2.4	2.4	2.3	2.2	2.1	2.1	2	1.9	1.8	1.8	1.6	1.5
$W=$ 9.0	1.9	2	2.1	2.2	2.3	2.3	2.4	2.4	2.5	2.6	2.6	2.6	2.6	2.6	2.6	2.6	2.5	2.5	2.5	2.4	2.4	2.3	2.3	2.2	2.1	2.1	2	2	1.9	1.8	1.8	1.7	1.7	1.6	1.5	1.3
$W=$ 8.5	1.9	2	2.1	2.2	2.3	2.3	2.4	2.4	2.4	2.5	2.5	2.5	2.4	2.4	2.4	2.3	2.3	2.2	2.2	2.1	2	2	1.9	1.9	1.8	1.8	1.8	1.7	1.7	1.7	1.6	1.6	1.6	1.4	1.2	1
$W=$ 8.0	1.9	2	2.1	2.1	2.2	2.2	2.3	2.3	2.3	2.3	2.3	2.3	2.2	2.2	2.1	2	2	1.9	1.8	1.8	1.7	1.6	1.6	1.5	1.5	1.5	1.5	1.6	1.6	1.6	1.6	1.6	1.4	1.1	0.8	0.4
$W=$ 7.5	1.9	1.9	2	2.1	2.1	2.2	2.2	2.2	2.1	2.1	2	1.9	1.9	1.8	1.7	1.6	1.5	1.4	1.3	1.2	1.2	1.2	1.2	1.3	1.3	1.4	1.5	1.6	1.7	1.7	1.6	1.2	0.8	0.5	0.2	0.1
$W=$ 7.0	1.8	1.9	1.9	2	2	2	2	1.9	1.9	1.8	1.7	1.6	1.4	1.3	1.1	1	0.9	0.8	0.8	0.8	0.8	0.9	1	1.1	1.3	1.5	1.7	1.8	1.8	1.5	1	0.6	0.3	0.1	0	0
$W=$ 6.5	1.8	1.8	1.8	1.9	1.9	1.8	1.7	1.6	1.5	1.3	1.1	0.9	0.7	0.5	0.5	0.4	0.4	0.4	0.4	0.4	0.5	0.6	0.8	1	1.3	1.7	1.9	1.8	1.4	0.8	0.4	0.1	0	0	0	0
$W=$ 6.0	1.8	1.8	1.8	1.8	1.7	1.5	1.3	1.1	0.8	0.6	0.4	0.3	0.2	0.1	0.1	0.1	0.1	0.1	0.1	0.2	0.3	0.4	0.7	1.1	1.5	1.9	1.8	1.3	0.7	0.3	0.1	0	0	0	0	0
$W=$ 5.5	2	2	1.9	1.7	1.4	1.1	0.7	0.5	0.3	0.1	0.1	0	0	0	0	0	0	0	0	0.1	0.2	0.4	0.7	1.3	1.9	1.9	1.3	0.7	0.3	0.1	0	0	0	0	0	0
$W=$ 5.0	2.5	2.4	2	1.5	0.9	0.4	0.2	0.1	0	0	0	0	0	0	0	0	0	0	0	0.1	0.2	0.5	1.1	1.7	2	1.4	0.7	0.3	0.1	0	0	0	0	0	0	0
$W=$ 4.5	3	2.7	1.9	1.1	0.5	0.2	0.1	0	0	0	0	0	0	0	0	0	0	0	0	0.1	0.3	0.8	1.5	2	1.6	0.8	0.3	0.1	0	0	0	0	0	0	0	0
$W=$ 4.0	3.2	2.8	1.7	0.8	0.3	0.1	0	0	0	0	0	0	0	0	0	0	0	0	0.1	0.2	0.5	1.2	1.9	1.8	1.1	0.4	0.2	0	0	0	0	0	0	0	0	0
$W=$ 3.5	3.4	2.8	1.6	0.7	0.3	0.1	0	0	0	0	0	0	0	0	0	0	0	0	0.1	0.3	0.8	1.6	2	1.4	0.6	0.2	0.1	0	0	0	0	0	0	0	0	0
$W=$ 3.0	3.5	2.8	1.5	0.6	0.2	0.1	0	0	0	0	0	0	0	0	0	0	0	0	0.2	0.5	1.1	1.9	1.8	1	0.4	0.1	0	0	0	0	0	0	0	0	0	0
$W=$ 2.5	3.7	2.8	1.4	0.5	0.2	0.1	0	0	0	0	0	0	0	0	0	0	0	0.1	0.2	0.7	1.5	2	1.4	0.6	0.2	0	0	0	0	0	0	0	0	0	0	0
$W=$ 2.0	3.8	2.8	1.3	0.5	0.2	0	0	0	0	0	0	0	0	0	0	0	0	0.1	0.3	0.9	1.8	1.9	1.1	0.4	0.1	0	0	0	0	0	0	0	0	0	0	0
$W=$ 1.5	3.9	2.8	1.3	0.4	0.1	0	0	0	0	0	0	0	0	0	0	0	0	0.1	0.4	1.1	1.9	1.7	0.9	0.3	0.1	0	0	0	0	0	0	0	0	0	0	0
$W=$ 1.0	3.9	2.8	1.2	0.4	0.1	0	0	0	0	0	0	0	0	0	0	0	0	0.2	0.5	1.3	2	1.5	0.7	0.2	0.1	0	0	0	0	0	0	0	0	0	0	0
$W=$ 0.5	4	2.8	1.2	0.4	0.1	0	0	0	0	0	0	0	0	0	0	0	0.1	0.2	0.6	1.4	2	1.4	0.6	0.2	0.1	0	0	0	0	0	0	0	0	0	0	0
$W=$ 0.0	4	2.8	1.2	0.4	0.1	0	0	0	0	0	0	0	0	0	0	0	0.1	0.2	0.6	1.4	2	1.4	0.6	0.2	0.1	0	0	0	0	0	0	0	0	0	0	0
m_0/m_2:	-3	-2.9	-2.8	-2.7	-2.6	-2.5	-2.4	-2.3	-2.2	-2.1	-2	-1.9	-1.8	-1.7	-1.6	-1.5	-1.4	-1.3	-1.2	-1.1	-1	-0.9	-0.8	-0.7	-0.6	-0.5	-0.4	-0.3	-0.2	-0.1	0	0.1	0.2	0.3	0.4	0.5

$N_z = 3$ thin-film calculation (conductance) *Phys. Rev. B 92, 235407 (2015)* cf. Phase diagram of 3D disordered TI



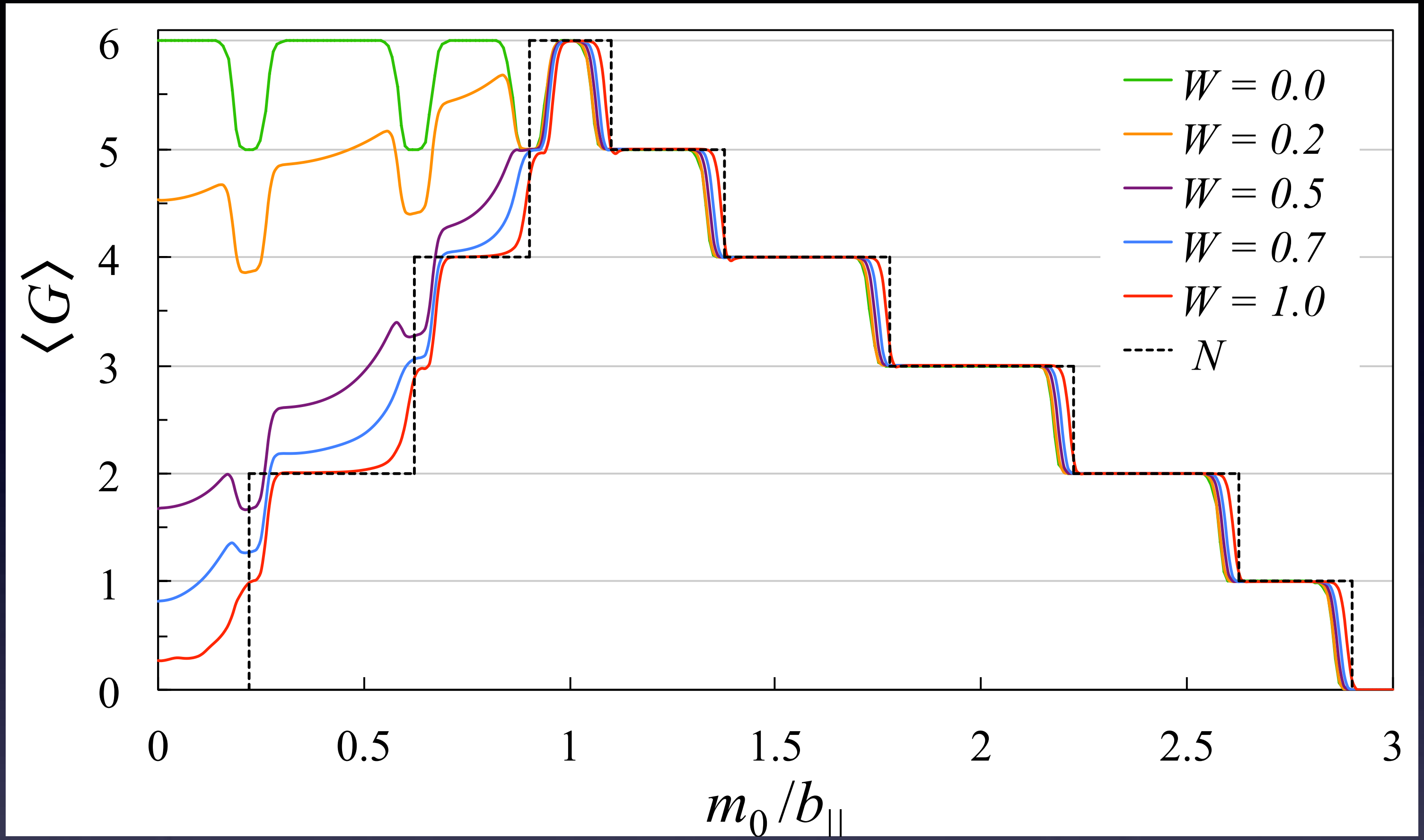
Direct transitions!

between different topological phases in 3D

potential disorder irrelevant in 3D

Phys. Rev. Lett. 110, 236803 (2013)

Case of disordered WSM thin films

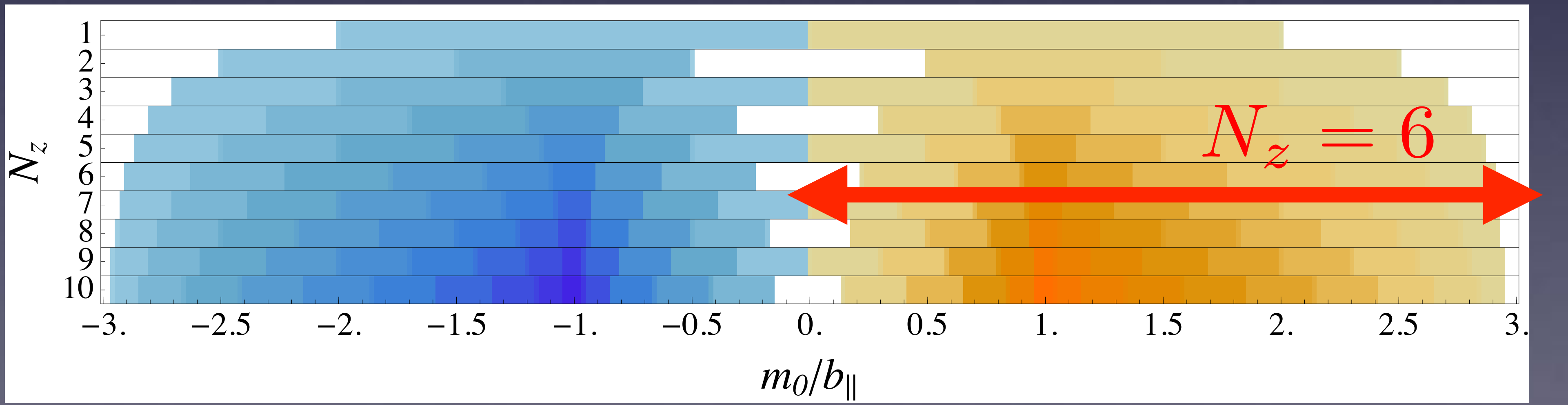


- two-terminal conductance

Phys. Rev. B 94, 235414 (2016)

co-propagating regime

Co- vs. Counter-propagating regimes



$b_z/b_{||} = 1$

counter-propagating regime

Two-terminal vs. Hall conductances

Chern number = Hall conductance

$$G_H = (\mathcal{N}_+ - \mathcal{N}_-)\frac{e^2}{h} = \mathcal{N}\frac{e^2}{h}$$

($\mathcal{N}_\pm$ *# of left- and right-going chiral modes*)

while the two-terminal conductance

$$G = (\mathcal{N}_+ + \mathcal{N}_-)\frac{e^2}{h}$$

measures the number of transmitting channels

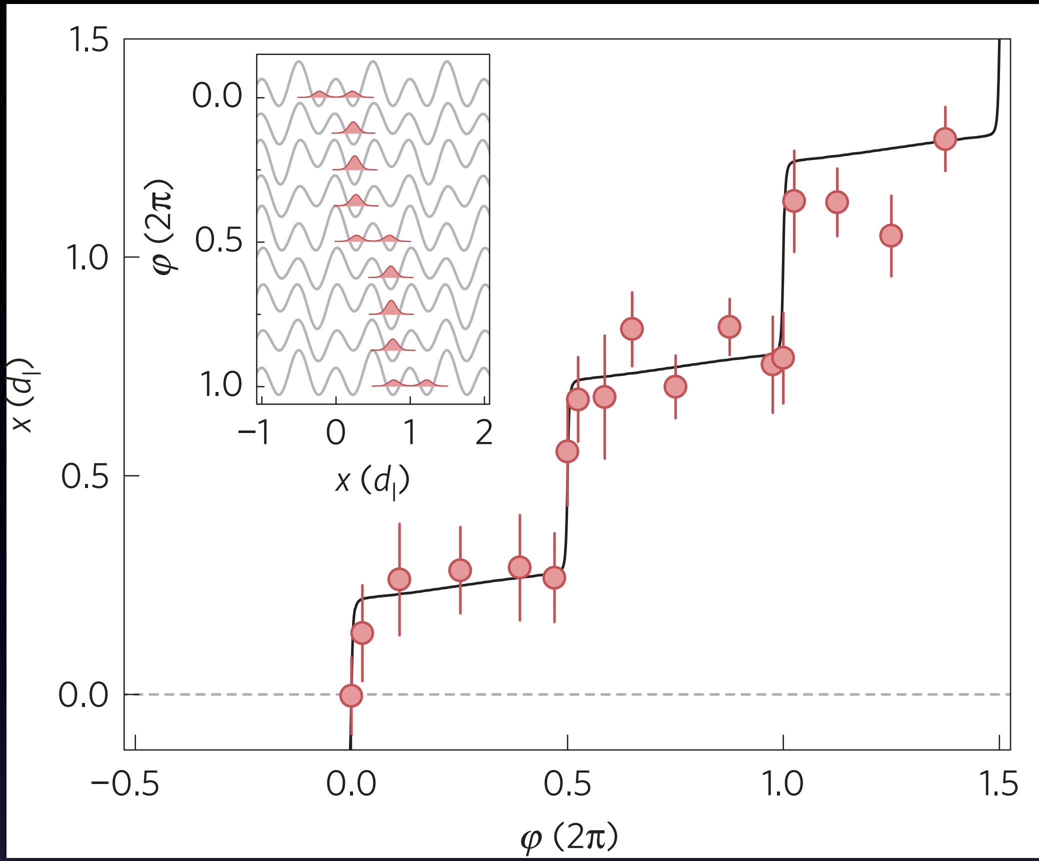
They differ
- in the presence of counter-propagating modes &
- in the clean limit

Relaxation of counter-propagating modes at the edge recovers

$$G = (\mathcal{N}_+ - \mathcal{N}_-)\frac{e^2}{h}$$

Case 2: topological quantum pump

Why pumping?



1) Experiments in cold atoms

Nakajima et al., Nature Phys., 2015;
Lohse et al., ibid.

2) Nobel prize in physics 2016

TKNN vs. Thouless pump

QHE	⟷	topological pumping
2D		1+1D
k_x, k_y		k_x, t

Thouless, PRB 1983

correspondence:

$$k_y \leftrightarrow 2\pi \frac{t}{T}$$

- mathematically equivalent (classification, etc.)
- physics different: different physical quantities, different physical pictures

cf. Laughlin's argument: pump version?
original: for QHE ⟶ *Hatsugai & Fukui, PRB 2016*

Topological pumping *in the snapshot picture* (adiabatic limit)

Hatsugai & Fukui, PRB 2016

Harper/AA model (pump version)

AA = Aubry-Andre

$$H(t) = \sum_{x=-L/2}^{L/2} \left[t_x |x+1\rangle\langle x| + (h.c.) + V(x,t)|x\rangle\langle x| \right]$$

"Laughlin's geometry"

- periodic in t
- finite (w/ edges) in the x-direction

V(x,t): periodic in time

$$V(x,t) = 2t_y \cos\left[2\pi\left(\frac{t}{T} - \phi x\right)\right]$$

What is related to the Chern number?

Ans.: pumped charge $\longleftrightarrow \sigma_{xy}$

= change of polarization over the pumping cycle

= center-of-mass position

$$\bar{x}(t) = \frac{1}{L}{\sum_{\alpha}}' x|\psi_\alpha(x,t)|^2$$

But, because of p.b.c. $\bar{x}(t_0+T) - \bar{x}(t_0) = 0$ So, no pumping???

Polarization/center of mass

$$\bar{x}(t) = \frac{1}{L}\sum_{\alpha}{}' x|\psi_\alpha(x,t)|^2$$

filled states

half-integral jumps!

$$\Delta\bar{x} = \frac{1}{2}\mathrm{sgn}(x_{\mathrm{edge}})[-\mathrm{sgn}(\mathrm{slope})]$$
$$= \pm\frac{1}{2}$$

jumps: edge effects

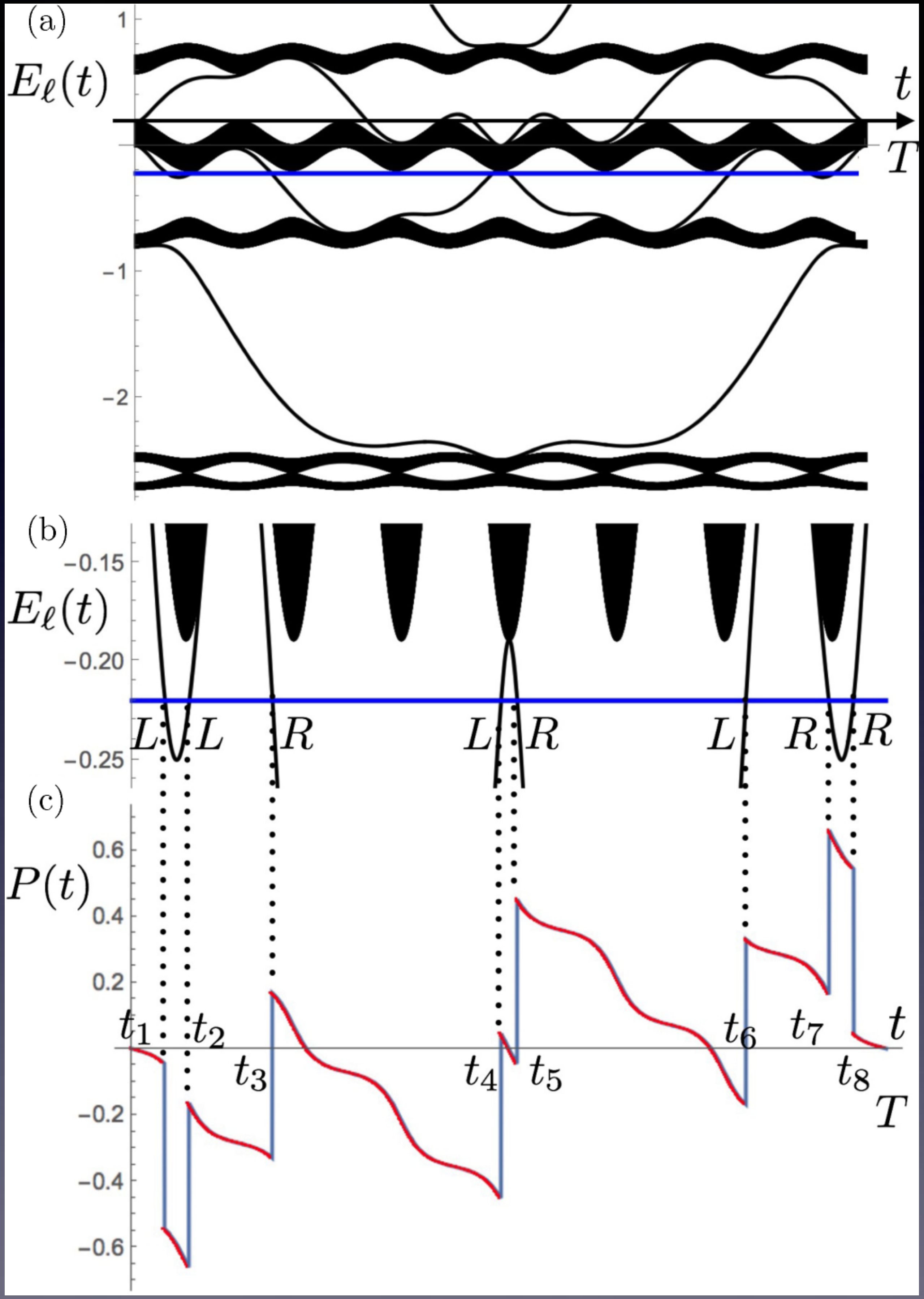
continuous part: bulk contribution

“bulk contribution” is relevant

→ skip jumps & reconstruct the continuous part:

$$\Delta\bar{x}_{\mathrm{net}} = -\sum_{\{j_n\}}\Delta\bar{x}_{\mathrm{jump}}(t_{j_n})$$

Recall: $\bar{x}(t_0+T) - \bar{x}(t_0) = 0$



Hatsugai & Fukui, PRB 2016

Consideration on **Jumps vs. continuous part** *in the polarization curve*

or edge vs. bulk contributions

or consideration on the adiabatic conditions:

The adiabatic condition: $T \gg \hbar/\Delta\epsilon$

bulk: $\Delta\epsilon_{\rm bulk} = \epsilon_g$

edge: $\Delta\epsilon_{\rm edge} \to 0$

$$t_{\rm edge} = \hbar/\Delta\epsilon_{\rm edge} \to \infty$$

Realistic situation: $t_{\rm bulk} \ll T_{\rm exp} \ll t_{\rm edge}$

i.e., bulk: adiabatic
edge: sudden

Jumps due to the edge modes are not seen in experiments

Rather, half-integral jumps *emergent in the adiabatic limit* are <<origin>> of the quantization of pumped (topological) charge

A short summary: Origin of quantization = half-integral jumps

$$\Delta\bar{x}_{\rm net} = -\sum_{\{j_n\}} \Delta\bar{x}_{\rm jump}(t_{j_n})$$

edge quantity

$$= {\sum_n}' C_n$$

filled bands

bulk topological invariant

BEC

- pump version of Laughlin's argument

- pump version of BEC

BEC: bulk-edge correspondence

A remaining issue:

QHE vs. pump

- check & quantify the robustness against disorder

$$H(t) = \sum_{x=-L/2}^{L/2} \left[t_x |x+1\rangle\langle x| + (h.c.) + [V(x,t) + W(x)]|x\rangle\langle x| \right]$$

$W(x) \in [-W/2, W/2]$ W: strength of impurity arXiv:1706.04493

In the presence of disorder

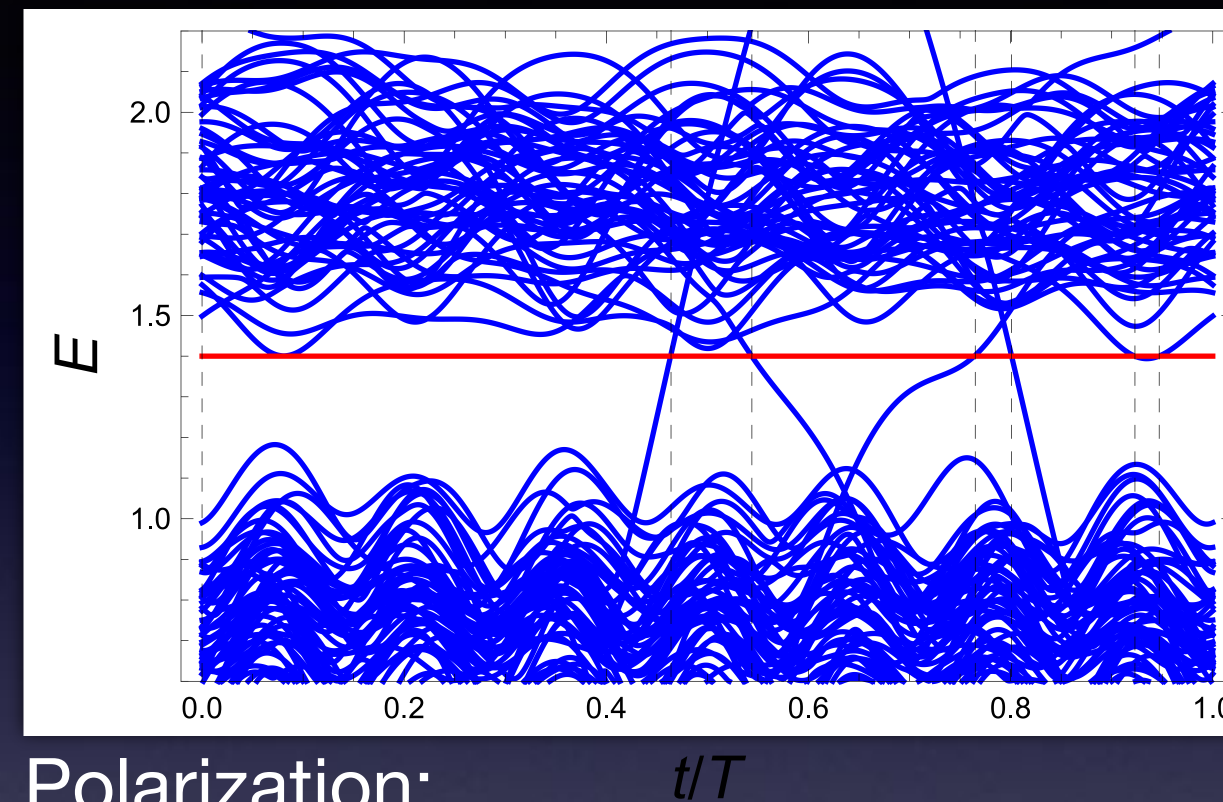
- two types of jumps appear
  1) quantized jumps
     - *edge state origin*
  2) non-quantized jumps
     - *impurity origin*

- At weak disorder they appear separately in time

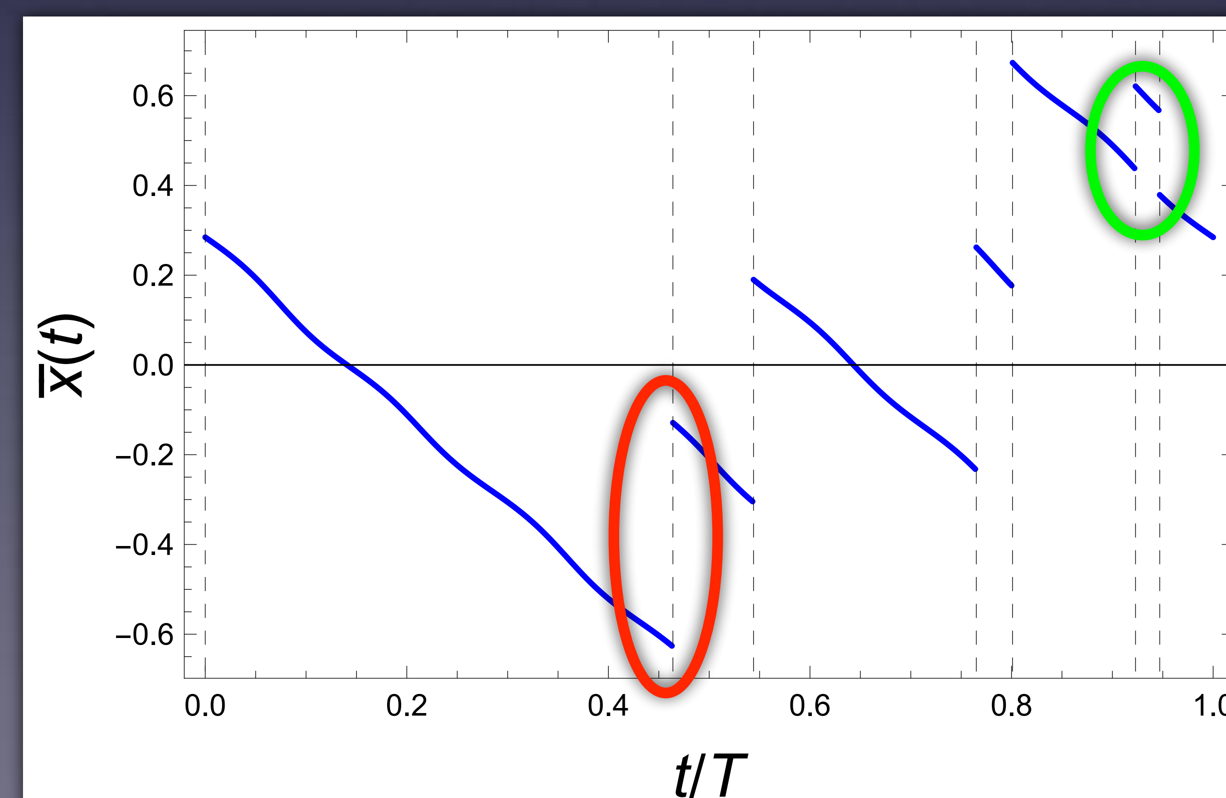
- As far as they are separable, the pumped charge is still quantized

$$\bar{x}(t) = \frac{1}{L}{\sum_{\alpha}}' x|\psi_\alpha(x,t)|^2$$

Snapshot spectrum:



Polarization:



Quantized vs. non-quantized jumps

- Non-quantized jumps: jumps due to impurities

$$\Delta \bar{x} = \langle x \rangle_{\mathrm{imp}}[-\mathrm{sgn}(\mathrm{slope})]$$

occupy/empty

- appear in pairs: appear/disappear
- irrelevant to the pumped charge

- Quantized jumps:

jumps due to edge states

$$\Delta \bar{x} = \frac{1}{2}\mathrm{sgn}(x_{\mathrm{edge}})[-\mathrm{sgn}(\mathrm{slope})] = \pm\frac{1}{2}$$

R or L

$$x_{\mathrm{edge}} = \pm L/2$$

- also appear in pairs, but …

→ quantized pumped charge

$$\Delta \bar{x}_{\mathrm{net}} = -\sum_{\{j_n\}} \Delta \bar{x}_{\mathrm{jump}}(t_{j_n}) = 0, \pm 1, \pm 2, \cdots$$

$$= \sum_n{}' C_n$$

Edge quantity

Bulk topological invariant

Conclusions

- two examples, in which

  BEC manifests as a one-to-one relation between <<visible>> physical quantities in the bulk and at the edge

- highlighted *a rather specific role of*

  bulk in case 1: topological insulator thin films

  cf. *penetration of the“surface” wave function into the <<bulk>> in the auxiliary 3D system*

  edge in case 2: topological quantum pumping

  cf. *half-integral jumps in polarization as the <<origin>> of topological quantization*