Learning to Love Disorder:

Spin-charge Conversion and Other Interesting Effects in 2D Spin-Orbit Coupled Systems

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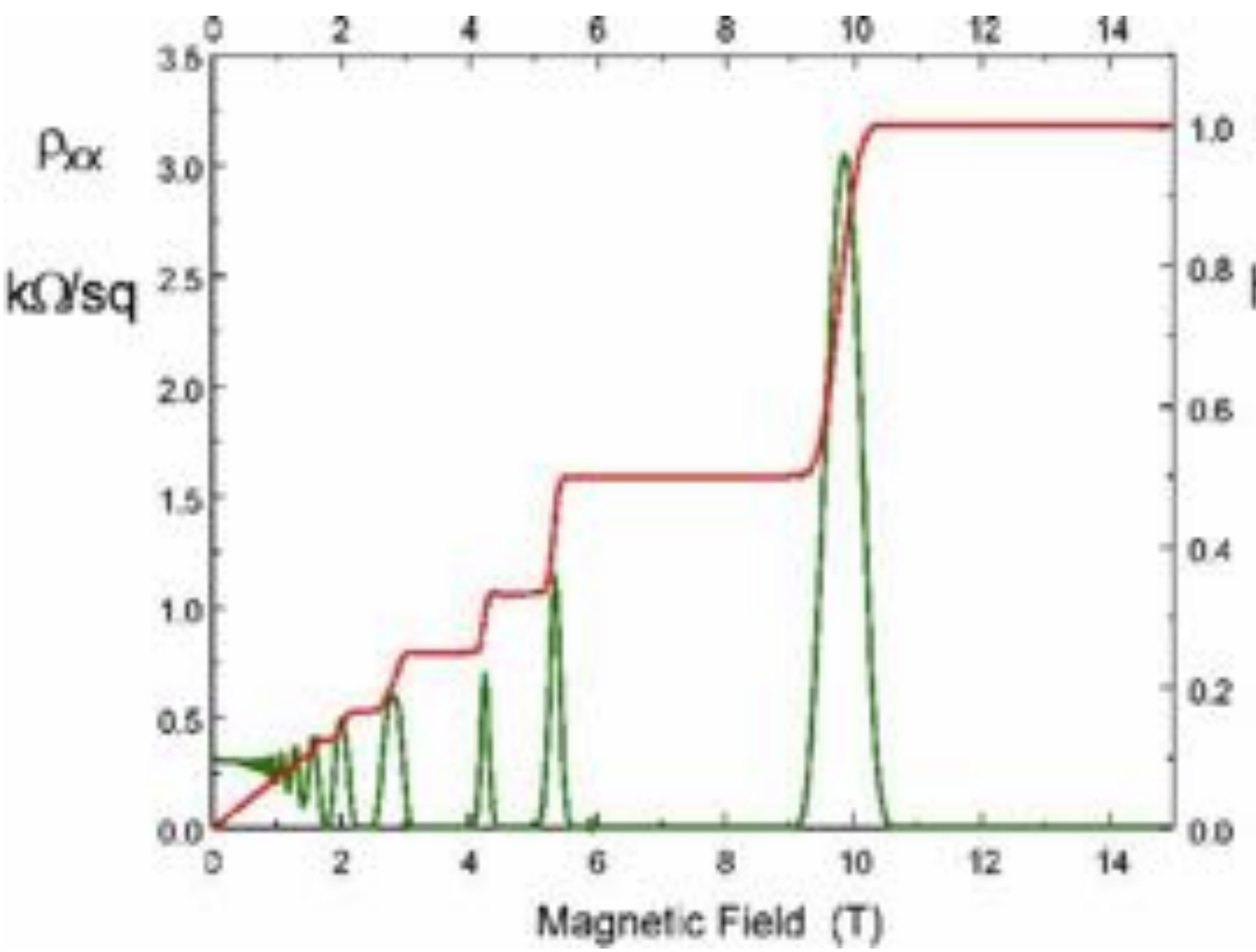
On the importance of disorder

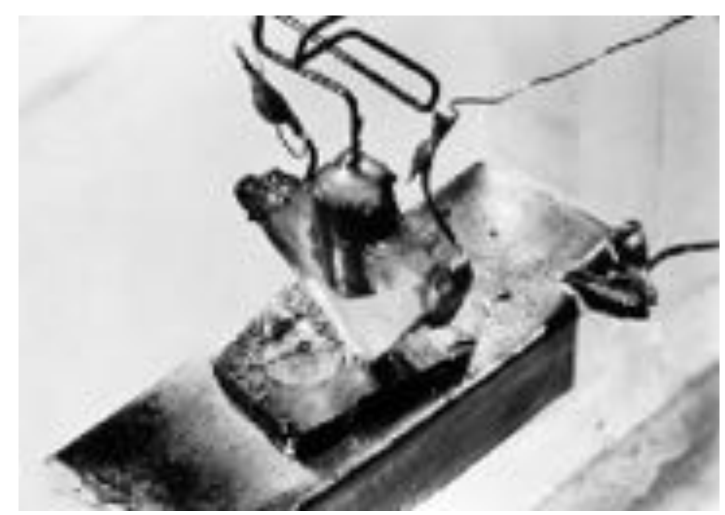
近藤さん

Disorder can be useful…

Quantum Hall Plateaux



John Bardeen



Outline: Part I

- *Extrinsic Spin Hall Effect and Mott Scattering*
- *Indirect Magneto-electric Coupling: Edelstein Effect*
- *Direct Magneto-electric Coupling: Anisotropic Spin Precession*
- *What about experiments? Non local probes, Hanle, and all that*

Outline: Part II

- *Topology = Dimensional reduction?*
- *Impurity in a 1D Channel w and w/o interactions: Kane & Fisher*
- *Magnetic Impurity near the non-interacting edge of a 2D QSHI*
- *Magnetic Impurity near the interacting edge of a 2D QSHI*

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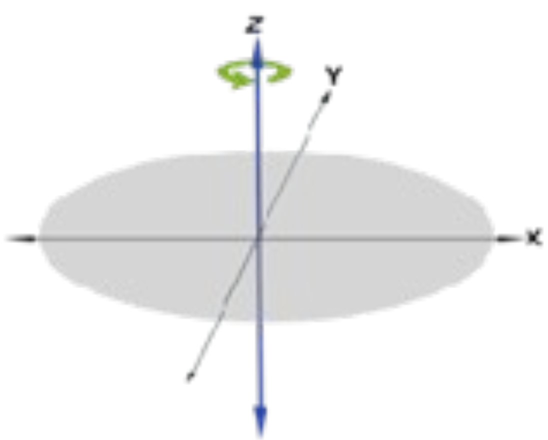
SHE and Symmetry (2D)

2D rotations $J_z = L_z + S_z \quad [H, J_z] = 0 \quad [\mathrm{U}(1)\,\text{group}]$



$$\boldsymbol{J} = (J_x, J_y)$$

Charge current

(2D vector)

Rank-2 Tensor?

$$\mathcal{J}^\alpha = (\mathcal{J}_x^\alpha, \mathcal{J}_y^\alpha)$$

$$\mathcal{J}^\alpha \to \mathcal{J}^z \oplus (\mathcal{J}^x, \mathcal{J}^y)$$

(2D vector)

Under reflection x → -x and y → y and S_z → -S_z

$$J_x \to -J_x \quad J_y \to J_y \quad \text{vs.} \quad \mathcal{J}_y^z \to J_x^z \quad \mathcal{J}_y \to -\mathcal{J}_y^z$$

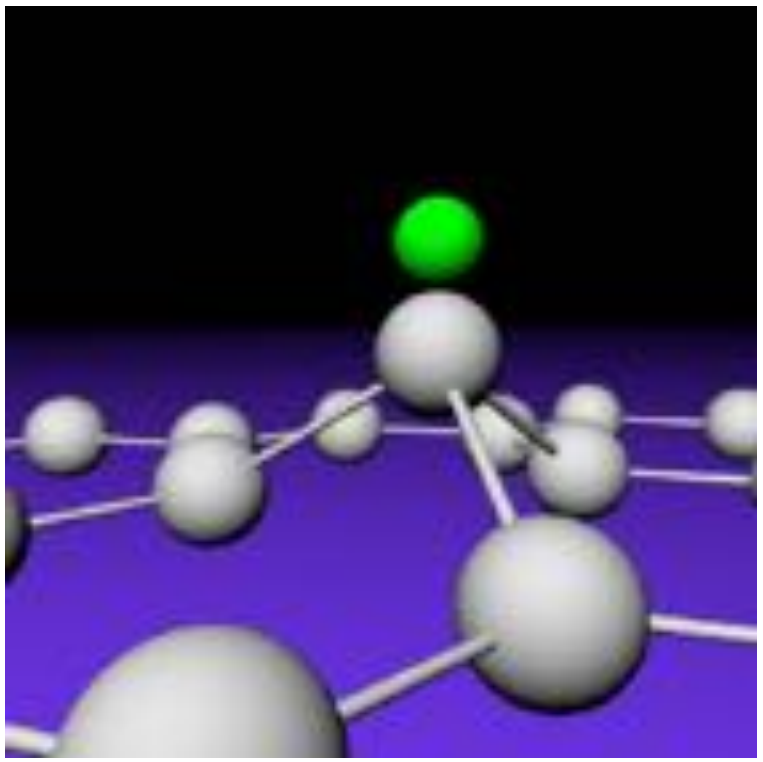
Different sign determined by TRS (by Onsager's reciprocity)

$$\mathcal{J}_y^z = \theta_{SHE}\, J_x \quad J_x = -\theta_{SHE}\, \mathcal{J}_y^z$$

Symmetry arguments are fine, but what are the mechanisms?

Motivation: Functionalized Graphene

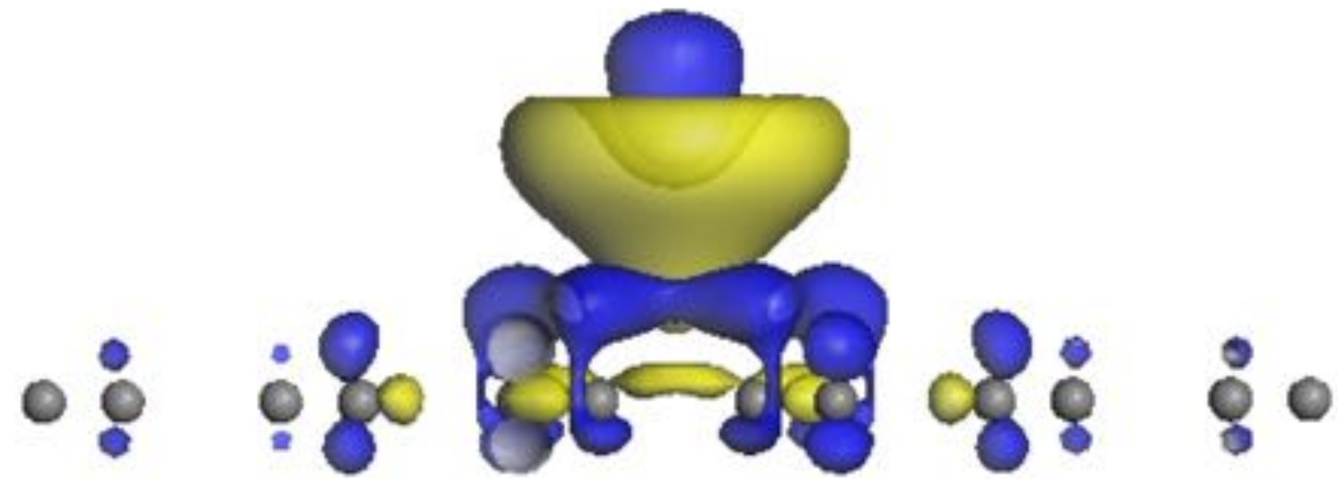
Chemisorption: H, F, …



AH Castro Neto & F Guinea Phys Rev Lett (2009)

Physisorption:

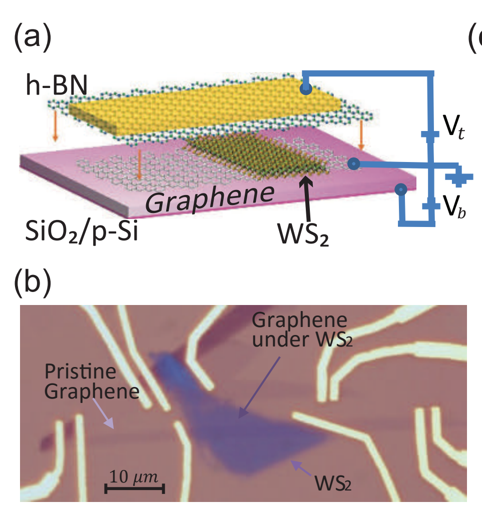
Cu, Ag, Au, Th, In,…

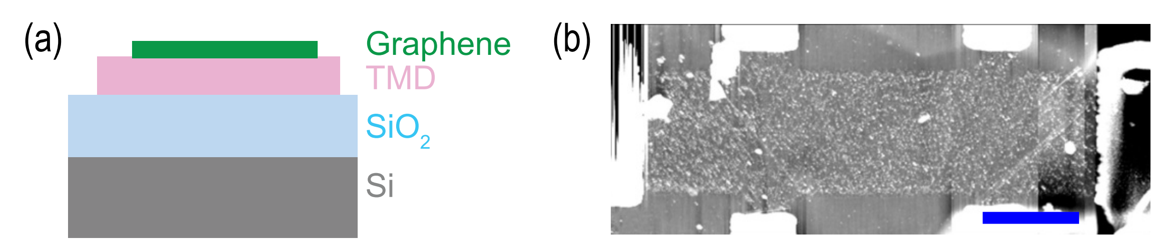


C Weeks et al Phys Rev X (2012)

Substrates:

Z Wang et al Phys. Rev. (2016)
B Wang et al 2D Mater (2016)

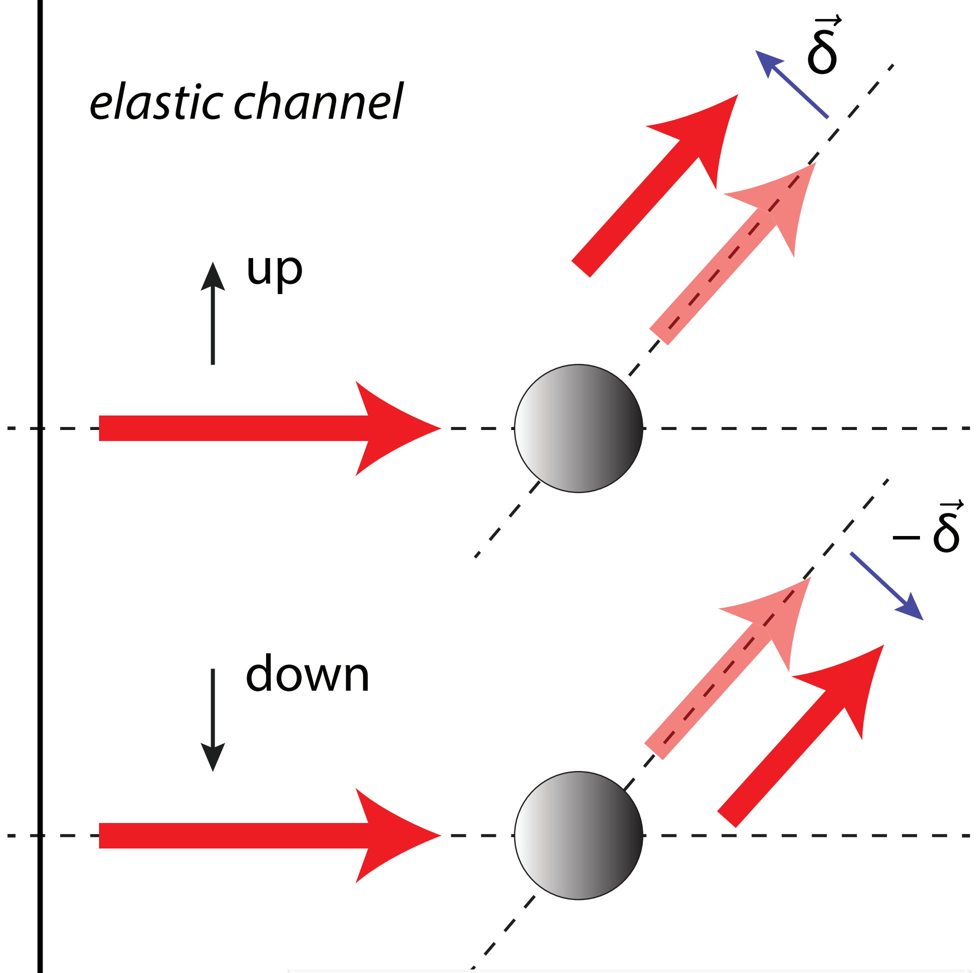




TMD = WSe_2, MoS_2

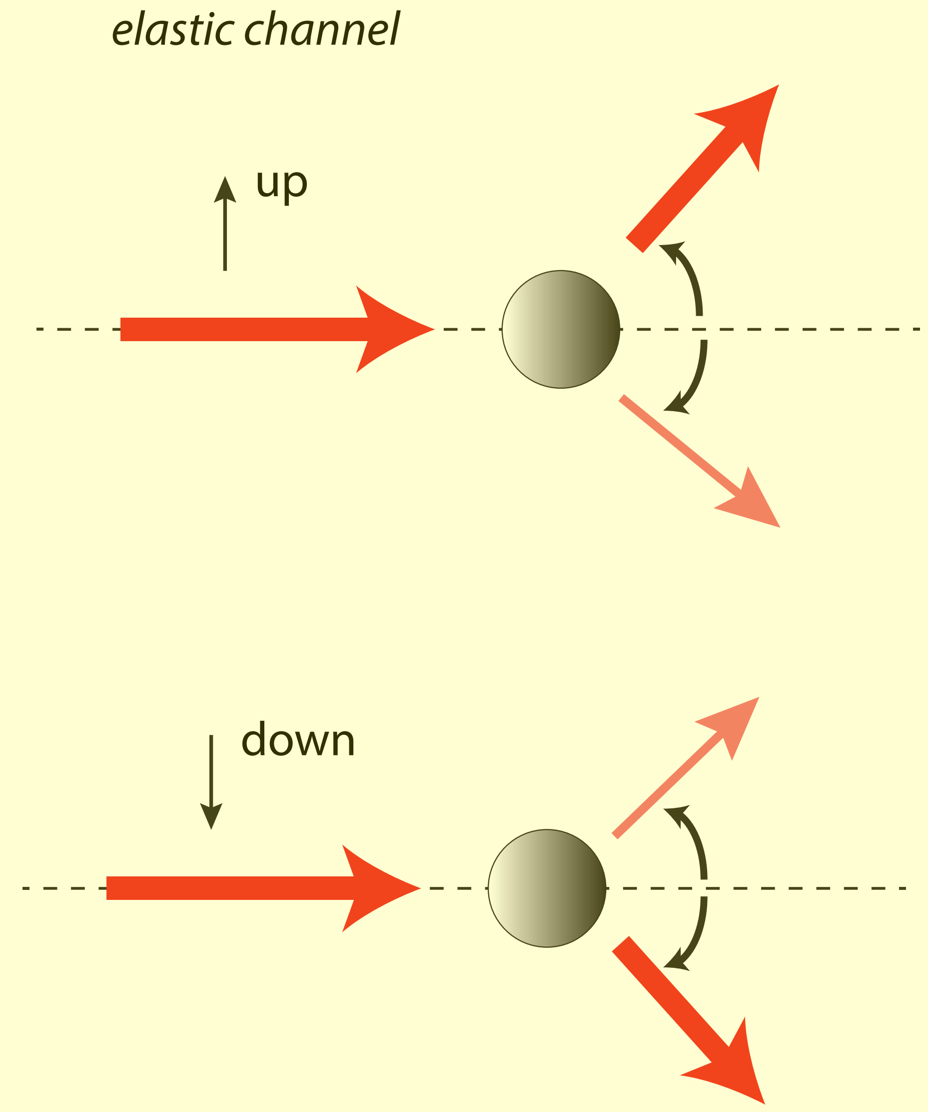
Extrinsic Mechanisms for SHE

Quantum Side Jump



$$n_{imp} < 1\%$$

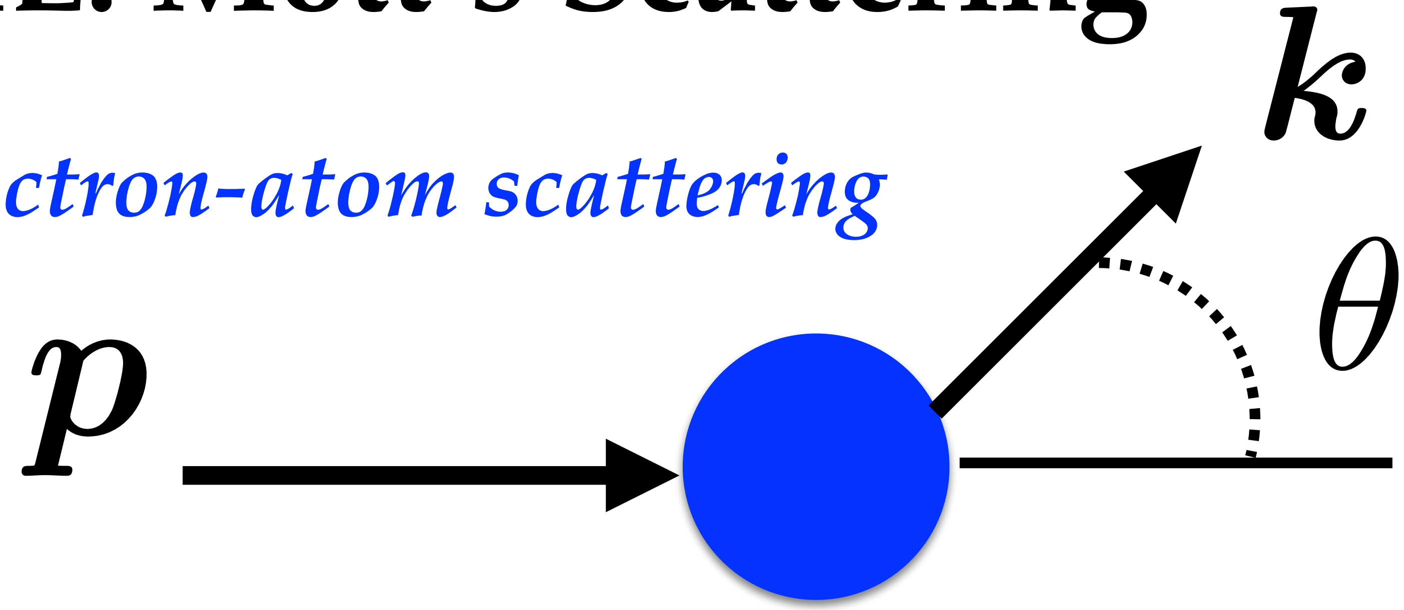
Skew Scattering ($n_{imp} \ll 1$)



Nagaosa, Sinova, Onoda, MacDonald, Ong, Rev. Mod. Phys. (2010)

Extrinsic SHE: Mott's Scattering

Electron-atom scattering



2 x 2 Scattering matrix (to all orders…)

$$T_{\boldsymbol{kp}} = A_{\boldsymbol{kp}}\,1 + \boldsymbol{B}_{\boldsymbol{kp}} \cdot \boldsymbol{s}$$

SOC

$$\boldsymbol{B}_{\boldsymbol{kp}} = S(\theta)\,(\boldsymbol{k} \times \boldsymbol{p}) \quad \left[\cos\theta = \hat{\boldsymbol{k}} \cdot \hat{\boldsymbol{p}}\right]$$

(3D Rotational Sym.)

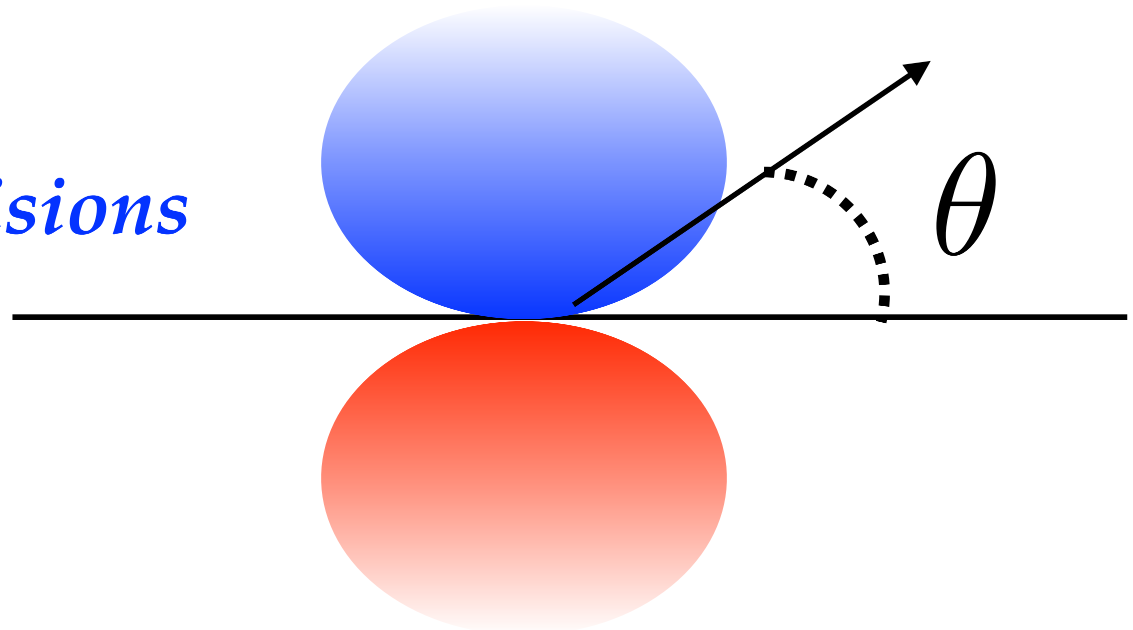
$$\rho_{\mathrm{in}}(\boldsymbol{p}) = \frac{1}{2}\mathbf{1} \qquad \rho_{\mathrm{out}}(\boldsymbol{k}) = T_{\boldsymbol{kp}}\rho_{\mathrm{in}}(\boldsymbol{p})T^{\dagger}_{\boldsymbol{kp}} = \frac{1}{2}T_{\boldsymbol{kp}}T^{\dagger}_{\boldsymbol{kp}}$$

Unpolarized!

$$\langle S^z \rangle_{\mathrm{out}} = \mathrm{Tr}\left[s^z \rho_{\mathrm{out}}(\boldsymbol{k})\right] = \mathrm{Re}\left[A^{*}_{\boldsymbol{kp}} B^{z}_{\boldsymbol{kp}}\right] \propto \sin\theta$$

Polarization!

Forces from collisions

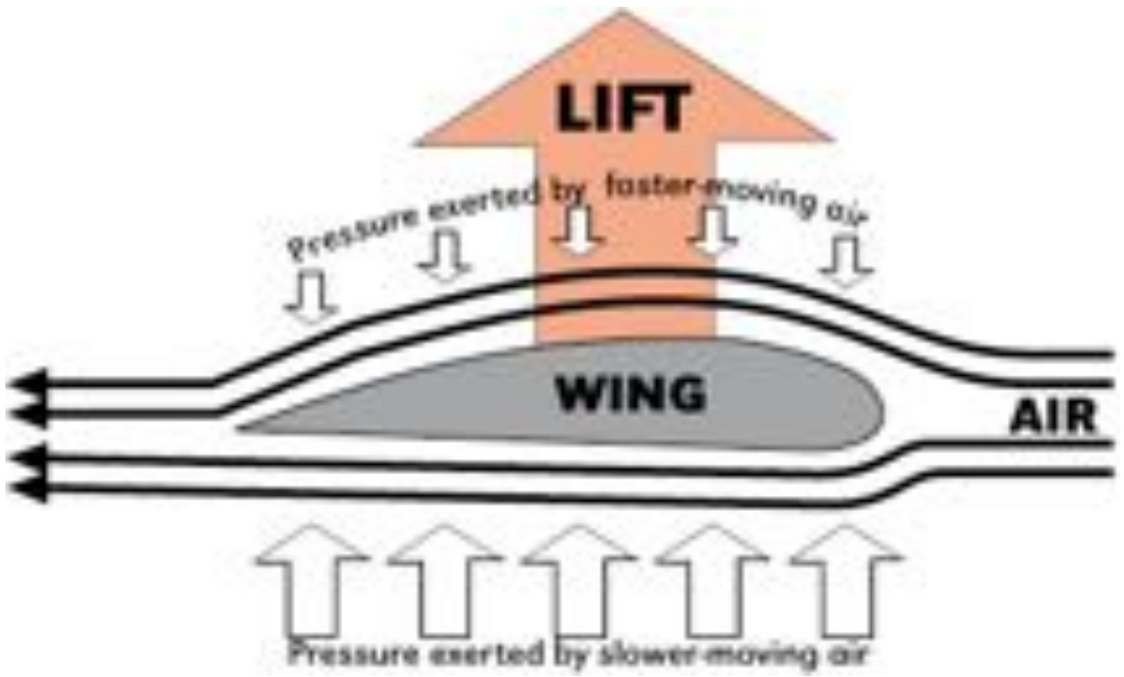


"Orbital" pseudo magnetic field from collisions

$$B^z_{\boldsymbol{kp}} = S(\theta)\,(\boldsymbol{k}\times\boldsymbol{p})\cdot\hat{\boldsymbol{z}} \propto \sin\theta$$

Lorentz-like force

$$\boldsymbol{F}_s \propto n_{\text{imp}}\,\hat{\boldsymbol{y}}s^z \int d\theta\,\sin\theta\,\text{Re}\left[A_{\boldsymbol{kp}}\left(B^z_{\boldsymbol{kp}}\right)^*\right] \qquad (\boldsymbol{J} = J_x\hat{\boldsymbol{x}})$$



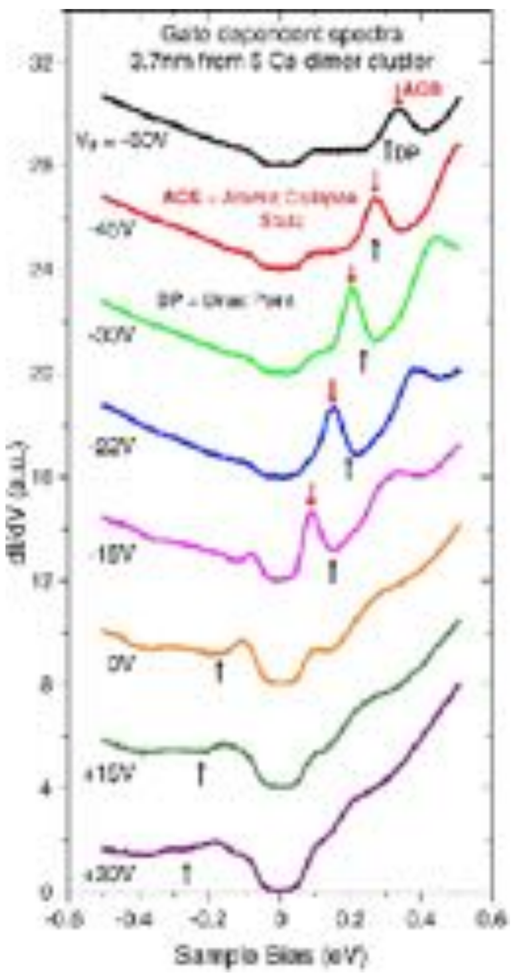
Forces "emerge" from collisions (akin to the Bernoulli principle)

Graphene: Resonant Enhancement

Graphene is very prone to resonant scattering

Linear Density of States

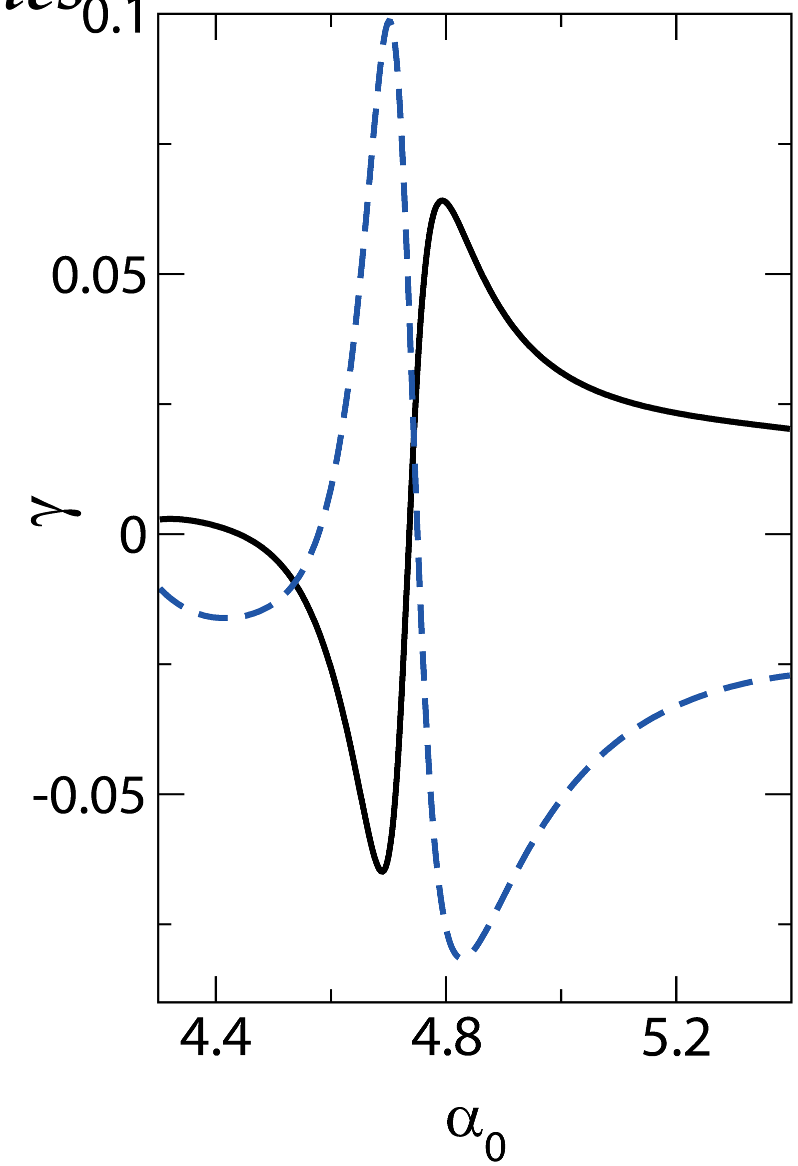
$$\rho(\epsilon) \sim |\epsilon|$$

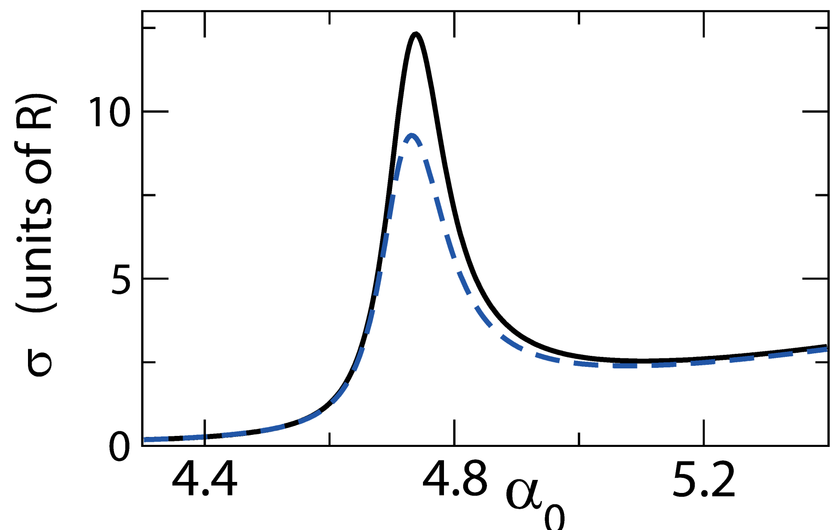


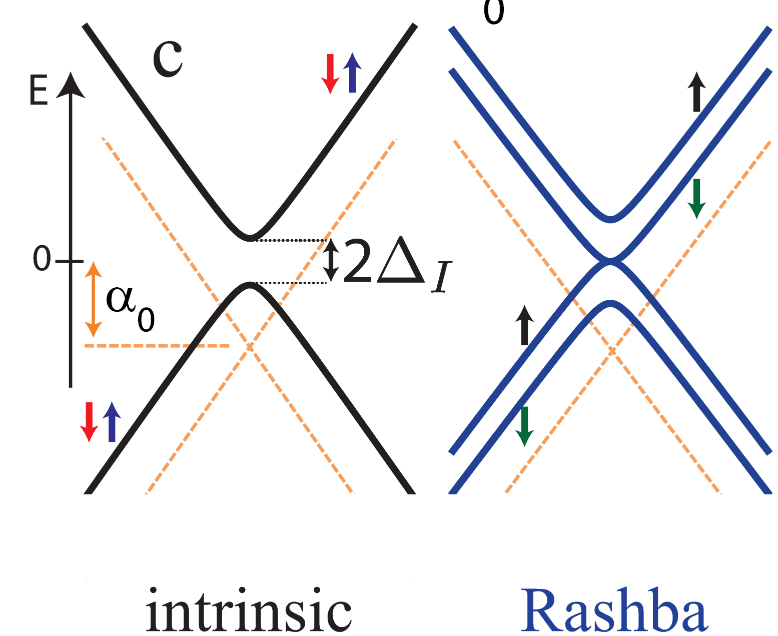
Yang et al. Science (2013)

Graphene

γ = spin Hall angle (T = 0)



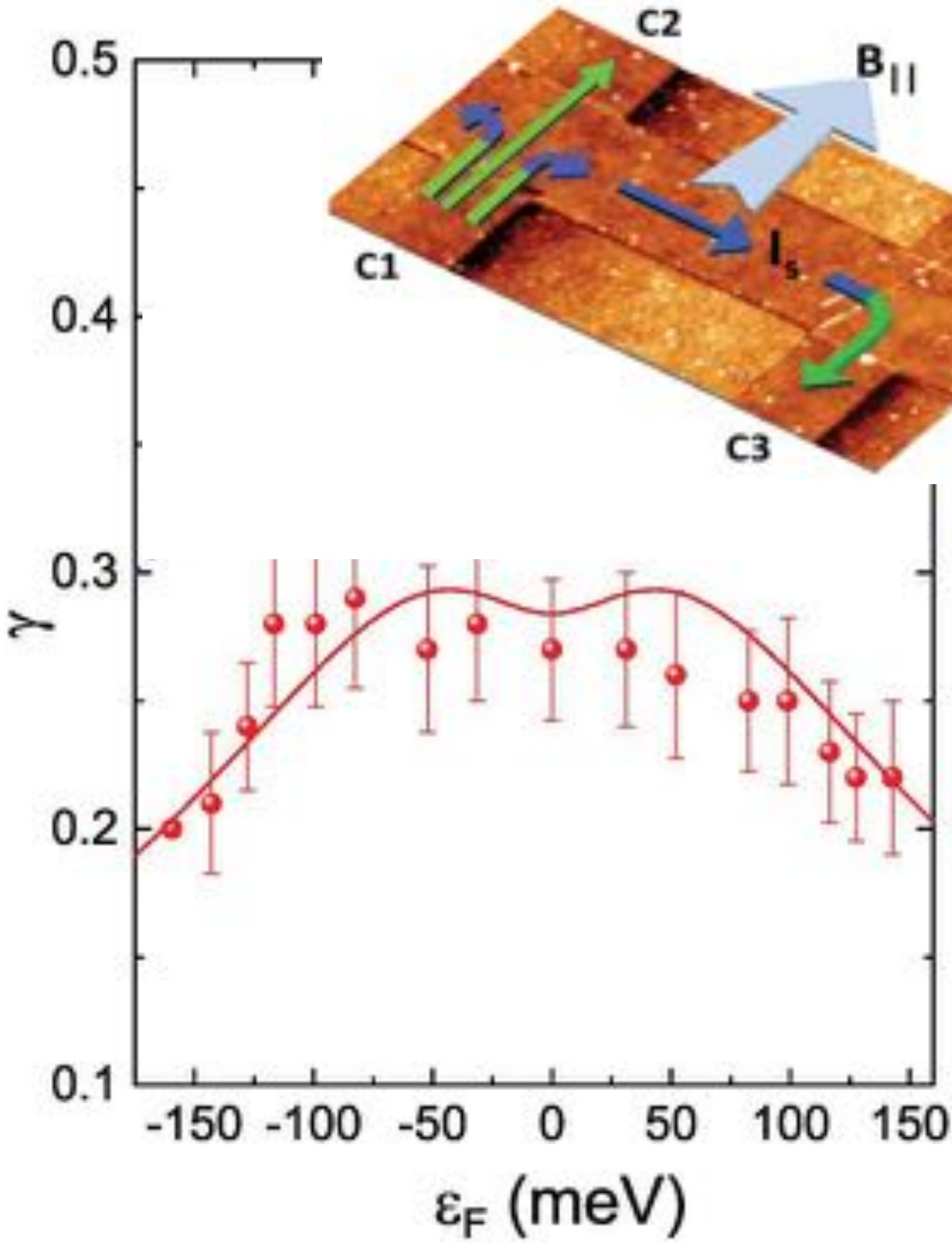




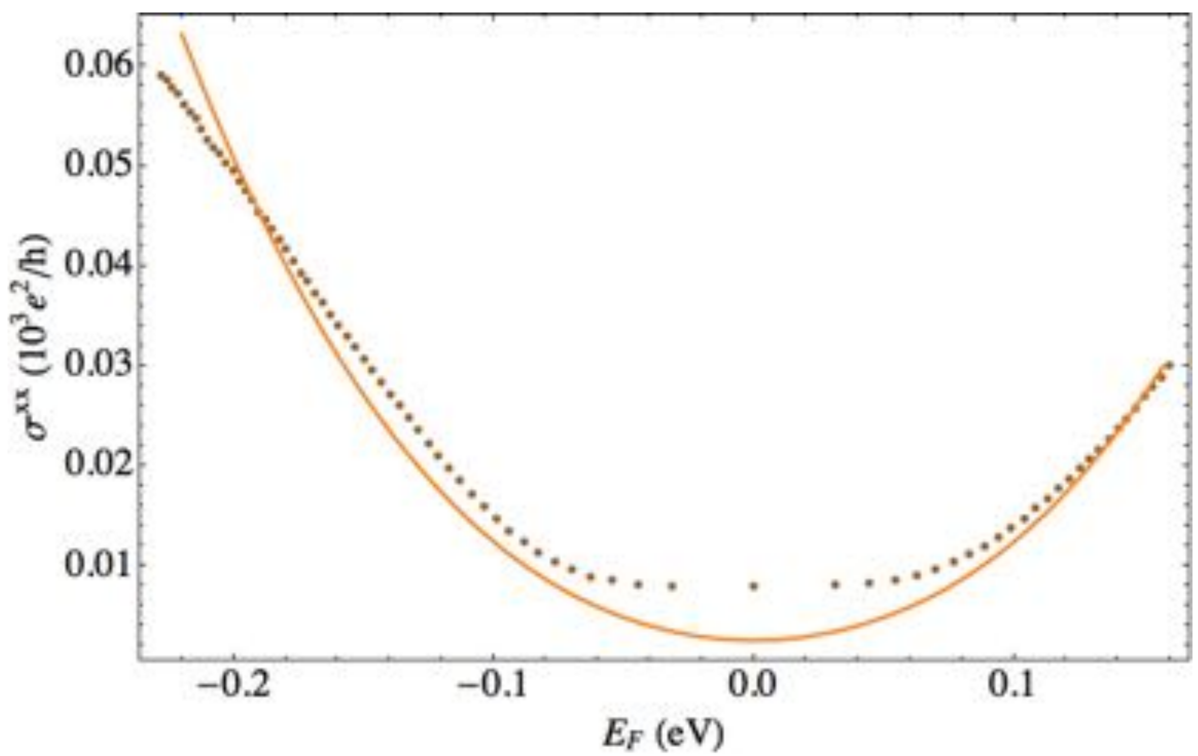
A Ferreira, T Rappoport, MAC, AH Castro Neto Phys Rev Lett (2014)

SHE in CVD Graphene

Spin Hall Angle



Electric Conductivity



J Balakrishnan et al Nat. Comm. (2014)

Model: Kane-Mele SOC impurities plus scalar resonant scatterers

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2D Magnetoelectric Effect & Symmetry

provided INVERSION SYMMETRY IS BROKEN

$\boldsymbol{\mathcal{M}}$ *must be related to* $\boldsymbol{J}$

$$\boldsymbol{\mathcal{M}} = (\mathcal{M}^x, \mathcal{M}^y, \mathcal{M}^z) = \mathcal{M}^z \oplus \boldsymbol{\mathcal{M}}_{\parallel} = (\mathcal{M}^x, \mathcal{M}^y)$$

(pseudo-vector under 3D rotation)

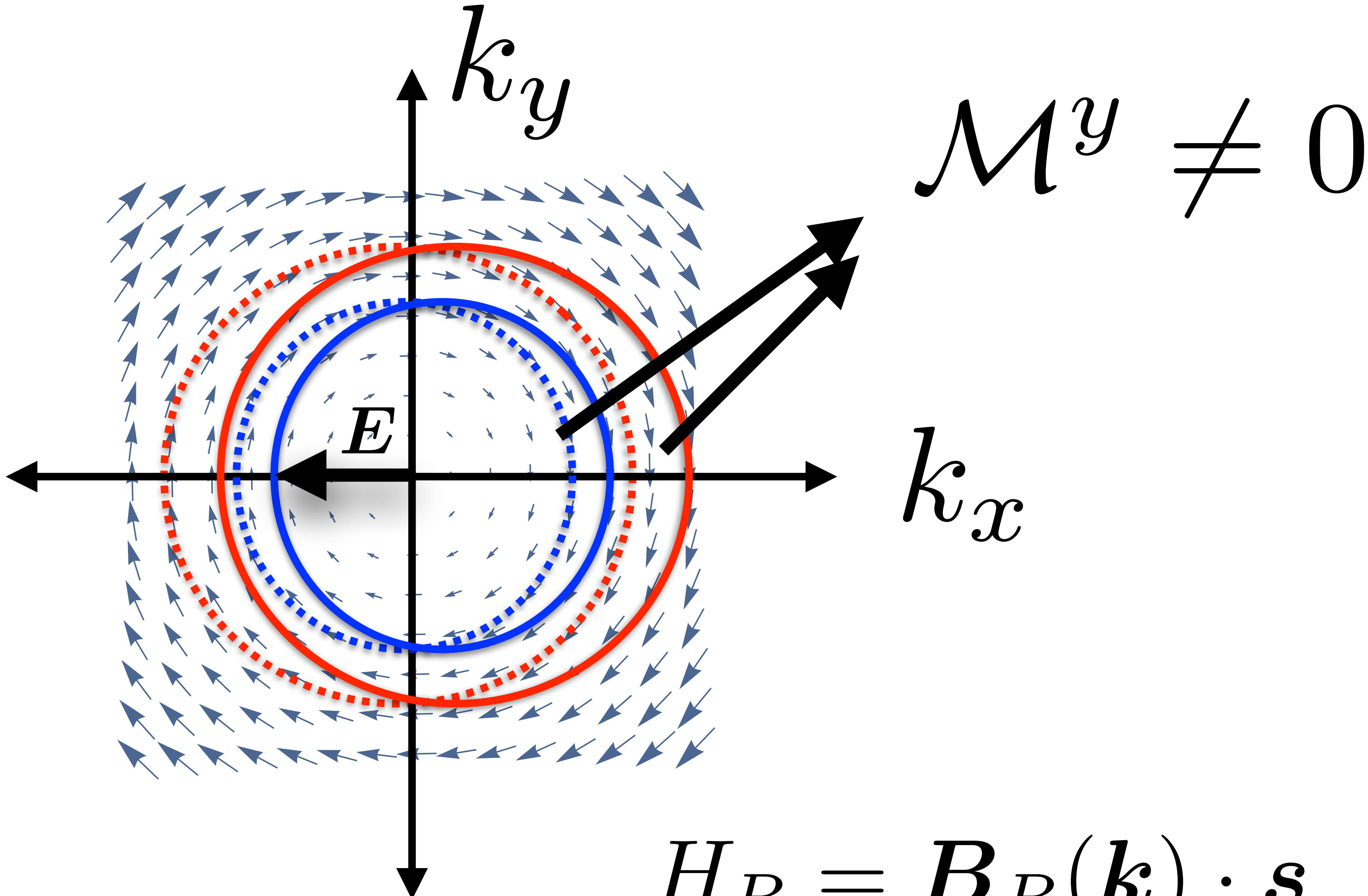
$$\boldsymbol{J} = (J_x, J_y)$$

Charge current (2D vector)

Under reflection $x \to -x$ *and* $y \to y$ *and* $S_z \to -S_z$

$$\mathcal{M}^y = \alpha_{CISP}\, J_x \qquad J_x = \tilde{\alpha}_{CISP}\, \mathcal{M}^y$$

CISP from Edelstein Effect



$$H_R = \boldsymbol{B}_R(\boldsymbol{k}) \cdot \boldsymbol{s}$$

$$\boldsymbol{B}_R(\boldsymbol{k}) = \alpha_R \left(\hat{\boldsymbol{z}} \times \boldsymbol{k}\right)$$

Indirect DMC: Eldestein Effect



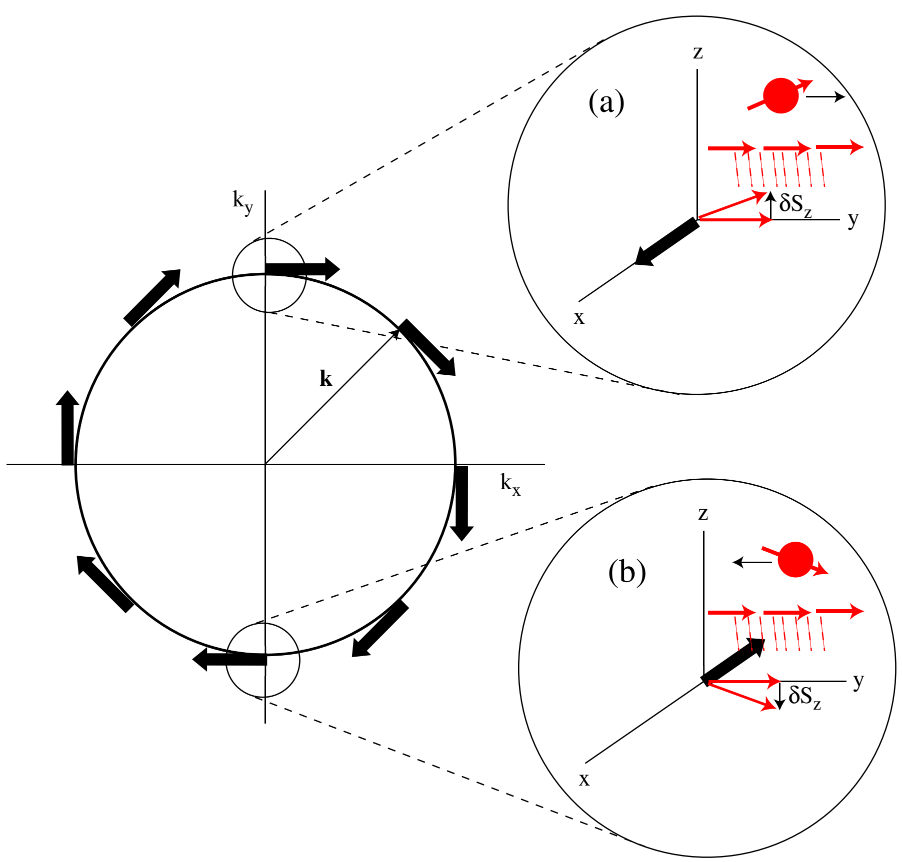
Two-step process

$\boldsymbol{J}$

SHE

$$\delta \dot{S}^y = -2m\alpha J_y^z - \frac{\delta S^y}{\tau_{\mathrm{EY}}}$$

$\mathcal{M}^y \leftarrow \mathcal{J}^z$



Rashba SOC

$$H_R = \boldsymbol{B}_R(\boldsymbol{k}) \cdot \boldsymbol{s}$$

$$\boldsymbol{B}_R(\boldsymbol{k}) = \alpha_R \left(\hat{\boldsymbol{z}} \times \boldsymbol{k}\right)$$

K Shen, G Vignale & R Raimondi PRL (2014)
R Raimondi, P Schwab, C Gorinni, and G Vignale Ann Phys (Berlin) 2012

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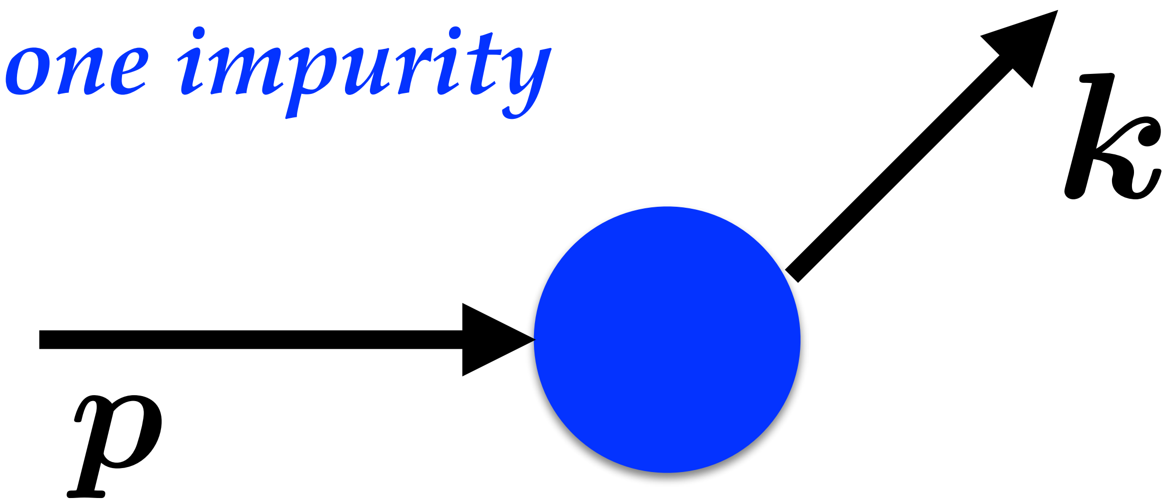
Revisiting Mott: Anisotropic Spin Precession

Understanding disorder from one impurity

$$T_{\boldsymbol{kp}} = A_{\boldsymbol{kp}} \, 1 + \boldsymbol{B}_{\boldsymbol{kp}} \cdot \boldsymbol{s}$$



Unpolarized $\rho_{\text{in}}(\boldsymbol{p}) = \frac{1}{2}\mathbf{1}$ $\qquad \rho_{\text{out}}(\boldsymbol{k}) = T_{\boldsymbol{kp}} \rho_{\text{in}}(\boldsymbol{p}) T^{\dagger}_{\boldsymbol{kp}} = \frac{1}{2} T_{\boldsymbol{kp}} T^{\dagger}_{\boldsymbol{kp}}$

Mott's scattering

$$\langle \boldsymbol{S} \rangle = \mathrm{Tr}\left[\boldsymbol{s} \rho_{\text{out}}(\boldsymbol{p})\right] = \mathbf{Re}\left[A^{*}_{\boldsymbol{kp}} \boldsymbol{B}_{\boldsymbol{kp}}\right] + i\left(\boldsymbol{B}^{*}_{\boldsymbol{kp}} \times \boldsymbol{B}_{\boldsymbol{kp}}\right)$$ *ASP*

Isotropic TR-invariant

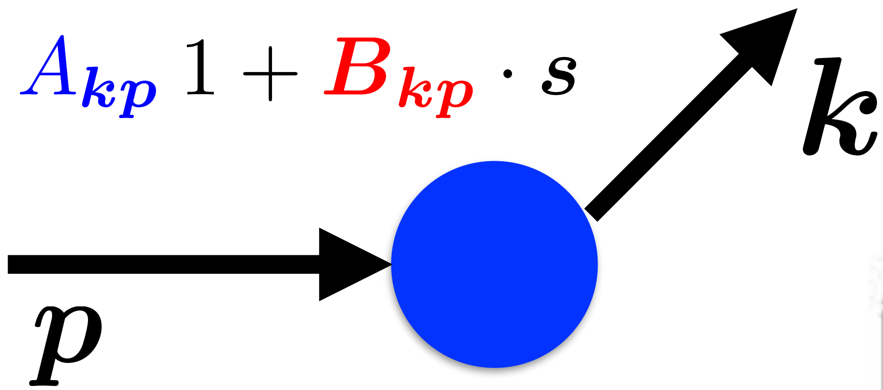
$$\boldsymbol{B}_{\boldsymbol{kp}} = S(\theta)\left(\boldsymbol{k} \times \boldsymbol{p}\right) \propto \boldsymbol{B}^{*}_{\boldsymbol{kp}} \Rightarrow \mathrm{ASP} = 0$$

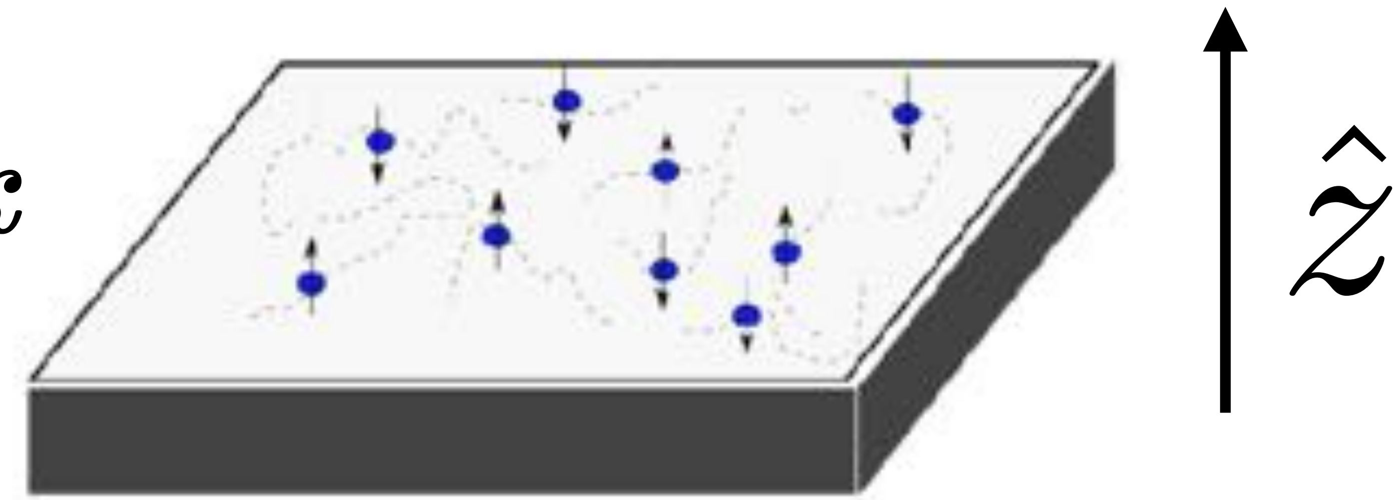
No ASP scattering!

ASP in 2D Materials

C Huang, Y Chong, and MAC, Phys Rev B (2016)

$$T_{\boldsymbol{kp}} = A_{\boldsymbol{kp}}\, 1 + \boldsymbol{B}_{\boldsymbol{kp}} \cdot \boldsymbol{s}$$





3D Rotational Invariance broken to 2D rotation

$$\boldsymbol{B}_{\boldsymbol{kp}} = \boldsymbol{B}^{\parallel}_{\boldsymbol{kp}} + \hat{\boldsymbol{z}}\, B^{z}_{\boldsymbol{kp}}$$

"Zeeman-like"

Orbital-like

(Rashba disorder fluctuations)

MM Glazov, E Ya Sherman, VK Dugaev Physica E (2010)
JP Binder et al Nature Physics (2016)

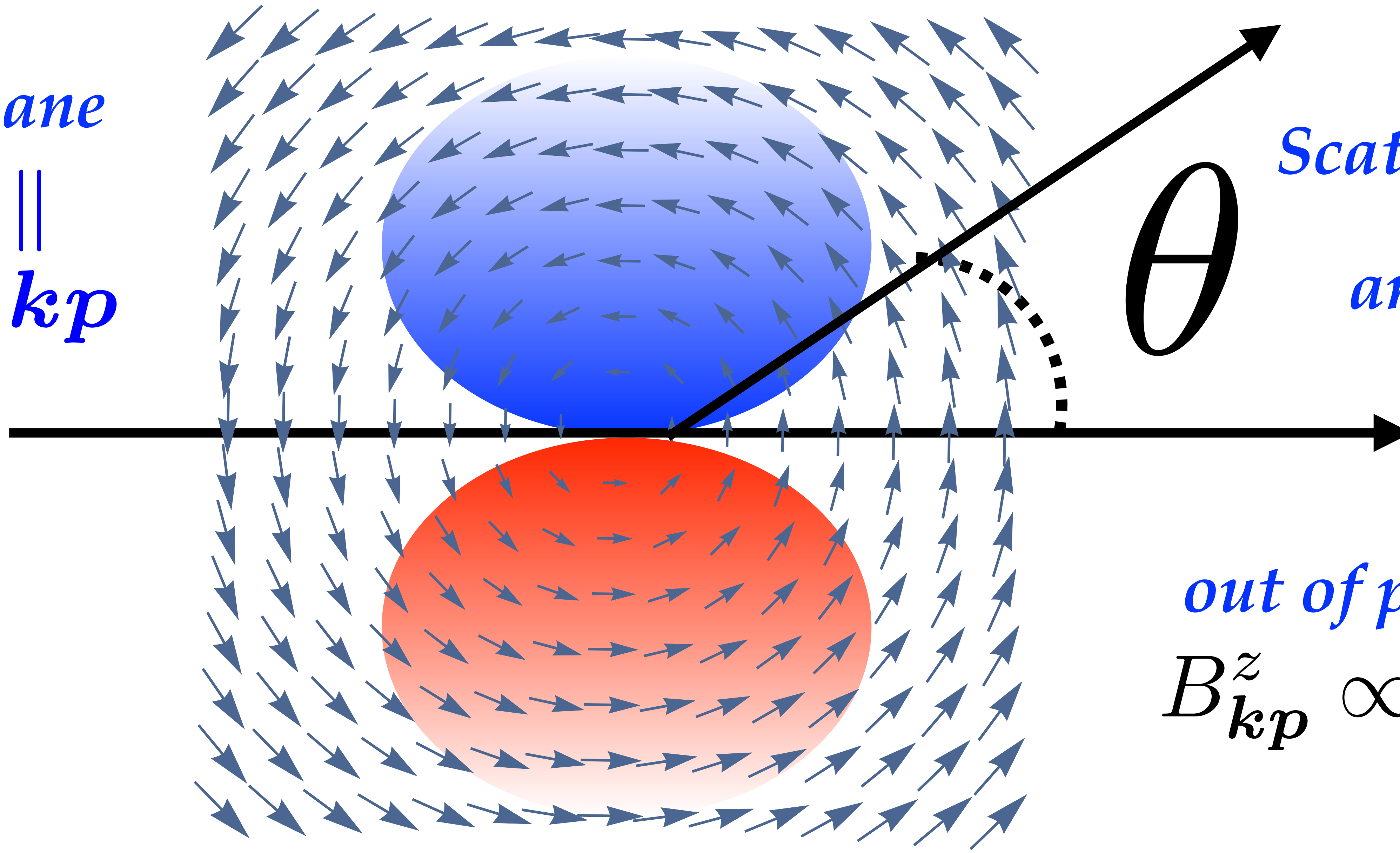
2D Mott's scattering

$$B^{z}_{\boldsymbol{kp}} = S(\theta)\,(\boldsymbol{k} \times \boldsymbol{p}) \cdot \hat{\boldsymbol{z}}$$

Anisotropic Precession Scattering (ASP)

C Huang, Y Chong, and MAC, Phys Rev B (2016)

in-plane $B^{\|}_{\boldsymbol{kp}}$



Scattering θ *angle*

out of plane $B^{z}_{\boldsymbol{kp}} \propto \sin\theta$

$$\text{ASP rate} = i\left(\boldsymbol{B}_{\boldsymbol{kp}} \times \boldsymbol{B}^{*}_{\boldsymbol{kp}}\right) \propto iB^{\|}_{\boldsymbol{kp}}\left(B^{z}_{\boldsymbol{kp}}\right)^{*} \times \hat{\boldsymbol{z}} \propto \langle S^{y}\rangle_{\text{out}}$$

Electron current $\rightarrow$ *spin polarization perp. to* $\boldsymbol{E}$ (i.e. $\mathcal{M}^{y}$)

Kohn & Luttinger: Strong disorder

C Huang, Y Chong, and MAC, Phys Rev B (2016)

Yidong Chong

C-L Huang

$$\delta n_k = n_k - n_k^0$$

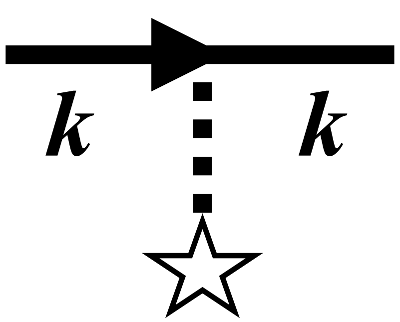
Spin

$$\delta n_k = \boxed{\rho_k} \mathbf{1} + \boxed{\boldsymbol{m}_k} \cdot \boldsymbol{s}$$

Charge

$$\partial_t \rho_k + e\mathbf{E} \cdot \frac{\nabla_k n_k^0}{\hbar} = \mathcal{I}_1[\rho_k, \mathbf{m}_k]$$

$$\partial_t \mathbf{m}_k + \left(\frac{\gamma}{\hbar}\boldsymbol{\mathcal{H}} - \boxed{\frac{n_{\text{imp}}}{\hbar}\text{Re}\,\mathbf{B}_{kk}}\right) \times \mathbf{m}_k = \boldsymbol{\mathcal{I}}_2[\rho_k, \mathbf{m}_\mathbf{k}]$$



Impurity (Rashba)
self-energy

Collision integral

$$\mathcal{I}_1, \boldsymbol{\mathcal{I}}_2 \propto n_{\text{imp}}$$

Collision Integral

Conductivity | SHE / ISHE | CISP

Charge $$\mathcal{I}_1 = \frac{n_{\rm imp}}{2\pi\hbar}\int d^2p \left[c_1(\rho_p - \rho_k) + \boldsymbol{c}_2 \cdot (\boldsymbol{m}_p - \boldsymbol{m}_k) - \boldsymbol{c}_3 \cdot (\boldsymbol{m}_p + \boldsymbol{m}_k) \right] \delta(\epsilon_p - \epsilon_k)$$

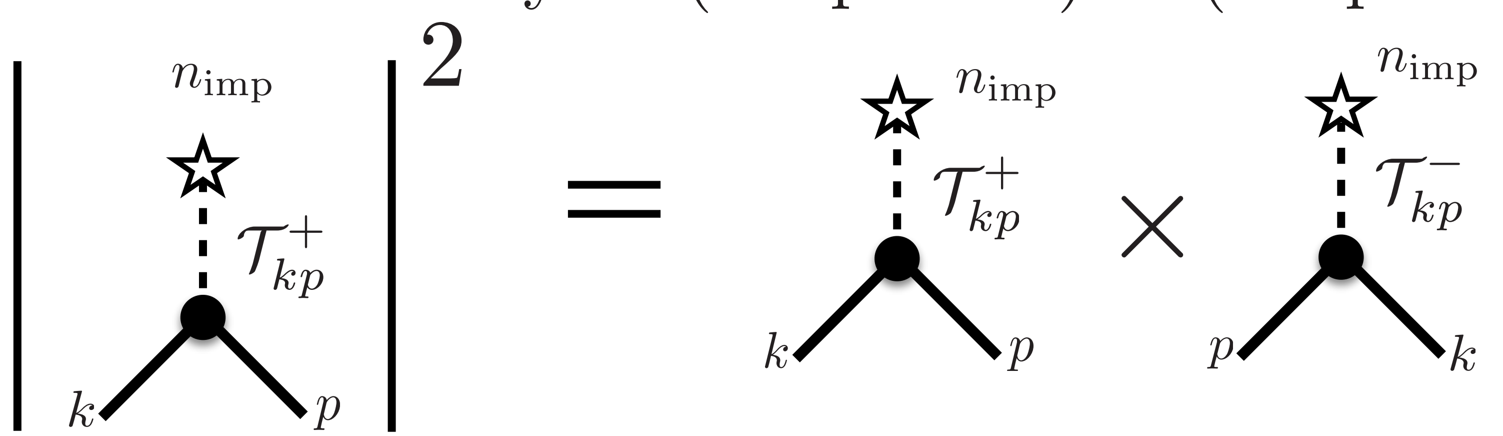
Spin $$\boldsymbol{\mathcal{I}}_2 = \frac{n_{\rm imp}}{2\pi\hbar}\int d^2p \left[c_1(\boldsymbol{m}_p - \boldsymbol{m}_k) + \boldsymbol{c}_2(\rho_p - \rho_k) + \boldsymbol{c}_3(\rho_p - \rho_k) + \boldsymbol{\mathcal{K}} \right] \delta(\epsilon_p - \epsilon_k)$$

Drude relaxation rate c_1

Skew scattering rate $\boldsymbol{c}_2$

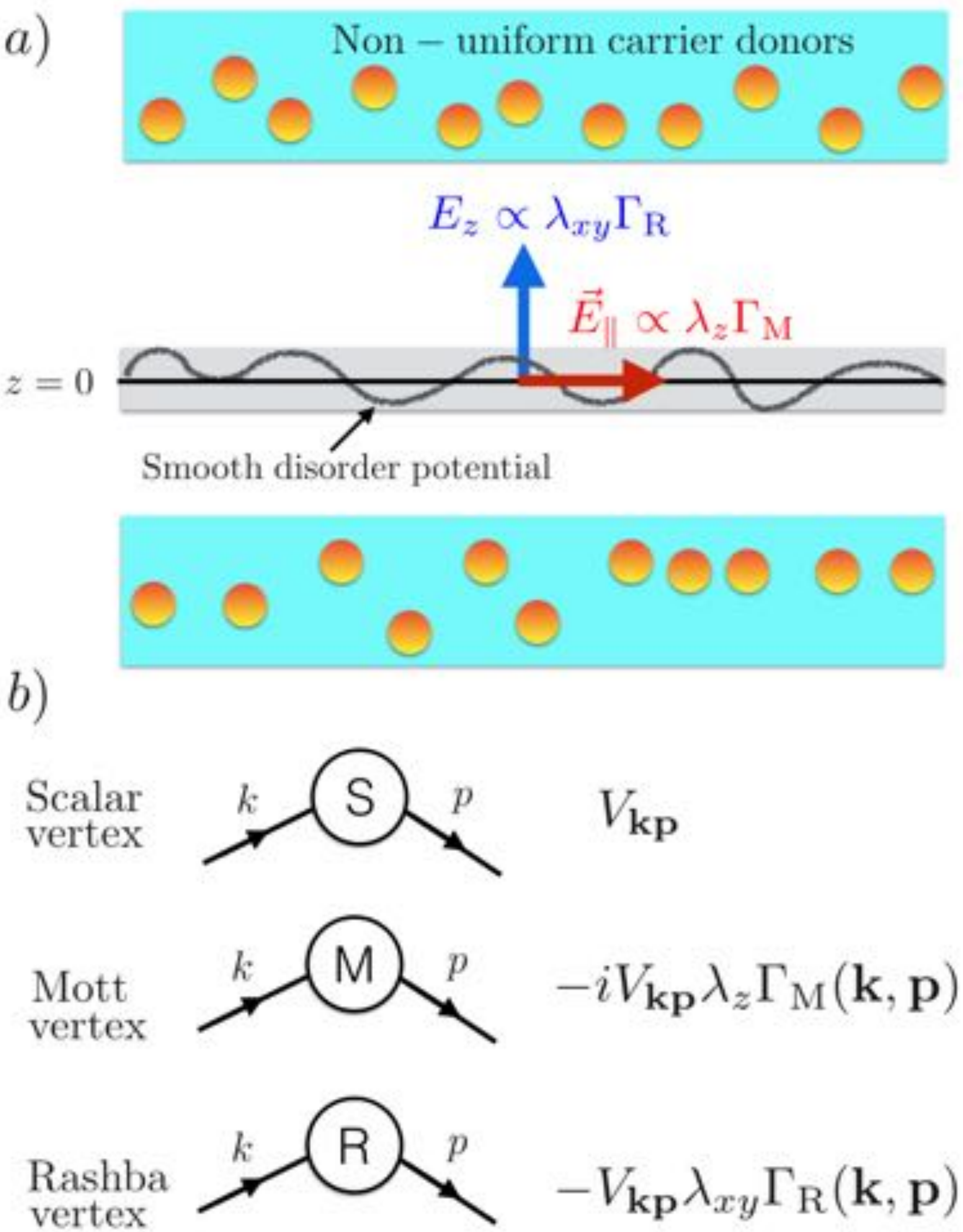
Anisotropic spin precession (ASP) scattering rate $\boldsymbol{c}_3$

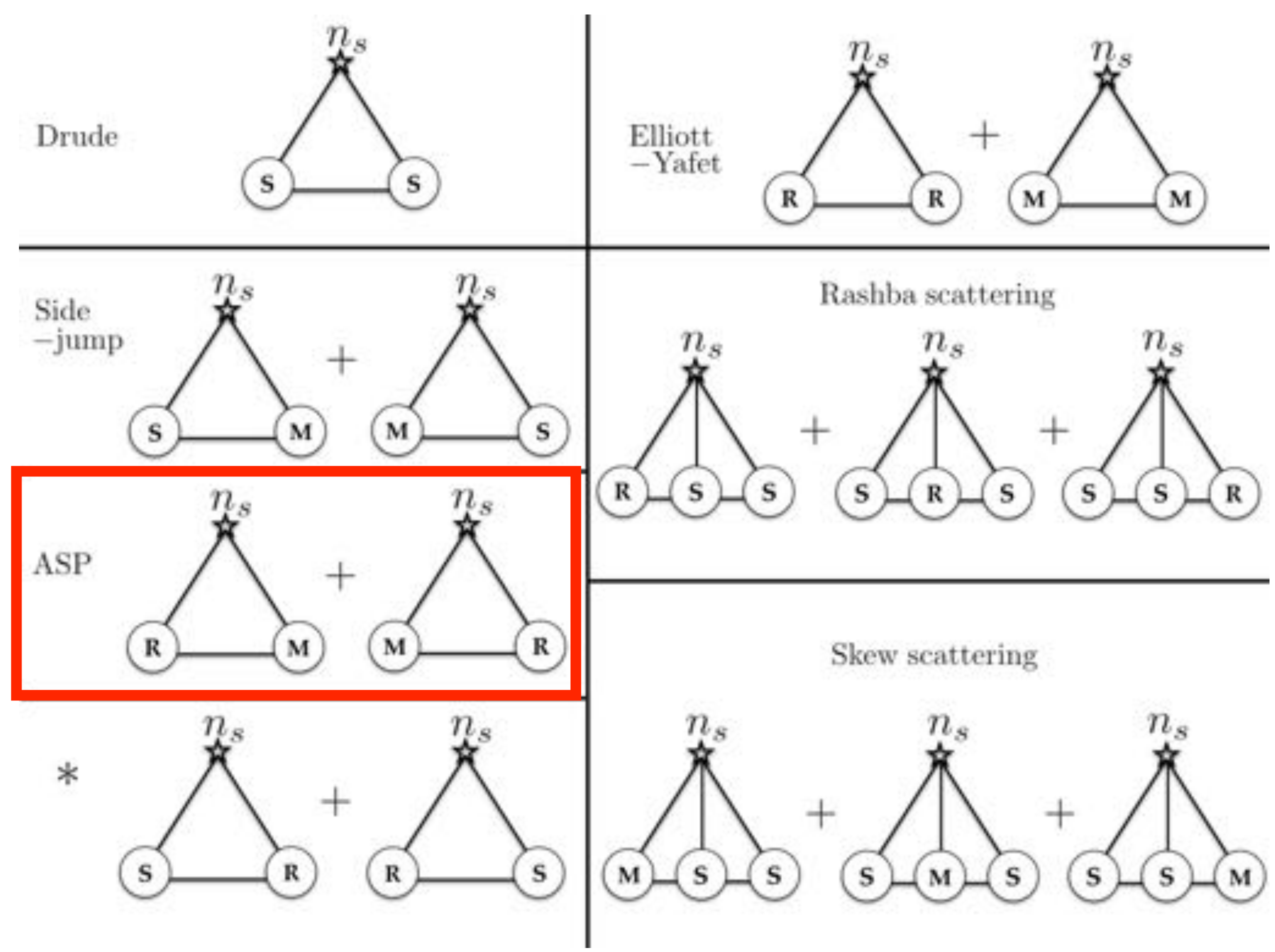
Scattering rate $\propto$ Probability $=$ (Amplitude) $\times$ (Amplitude)*



C Huang, Y Chong, and MAC, Phys Rev B (2016)

Keldysh: Smooth weak disorder





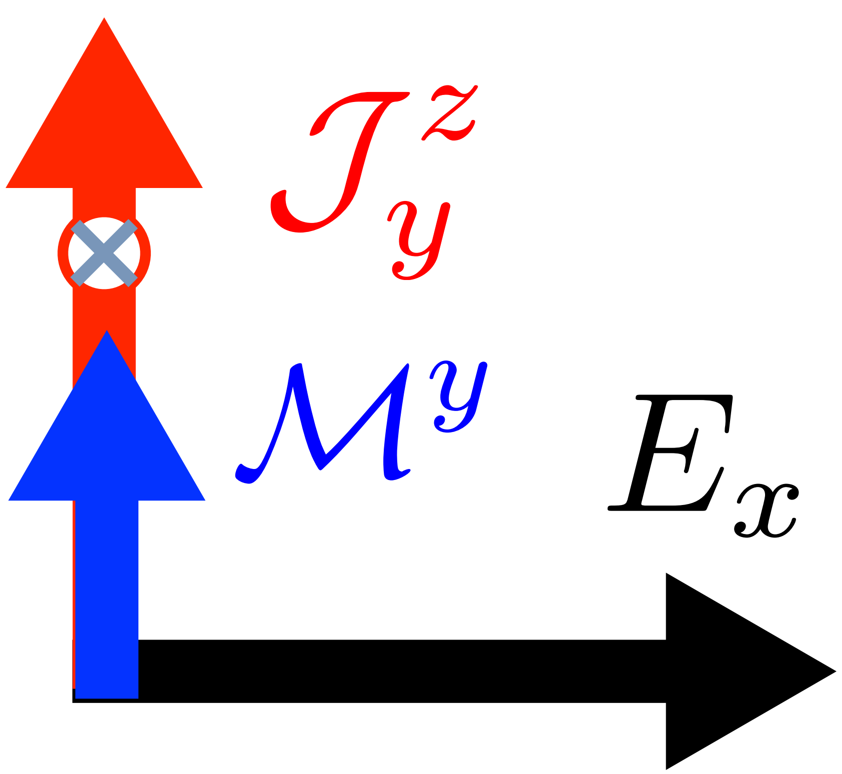
C Huang, M Milletari, and MAC, arxiv:17061316 (2017) (accepted in PRB)

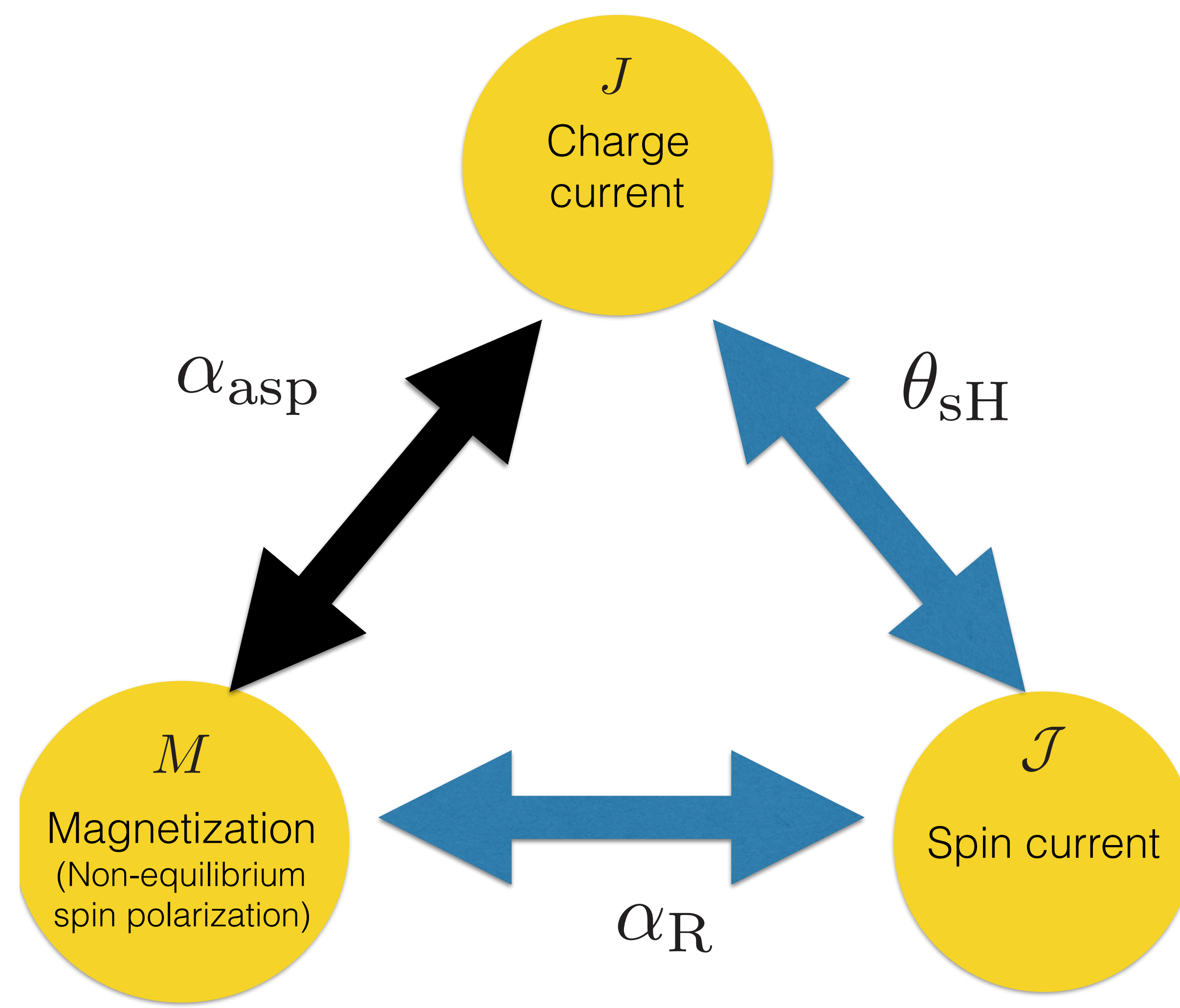
Linear Response & Direct Magnetoelectric Coupling (DMC)

C Huang, Y Chong, and MAC, Phys Rev B (2016)

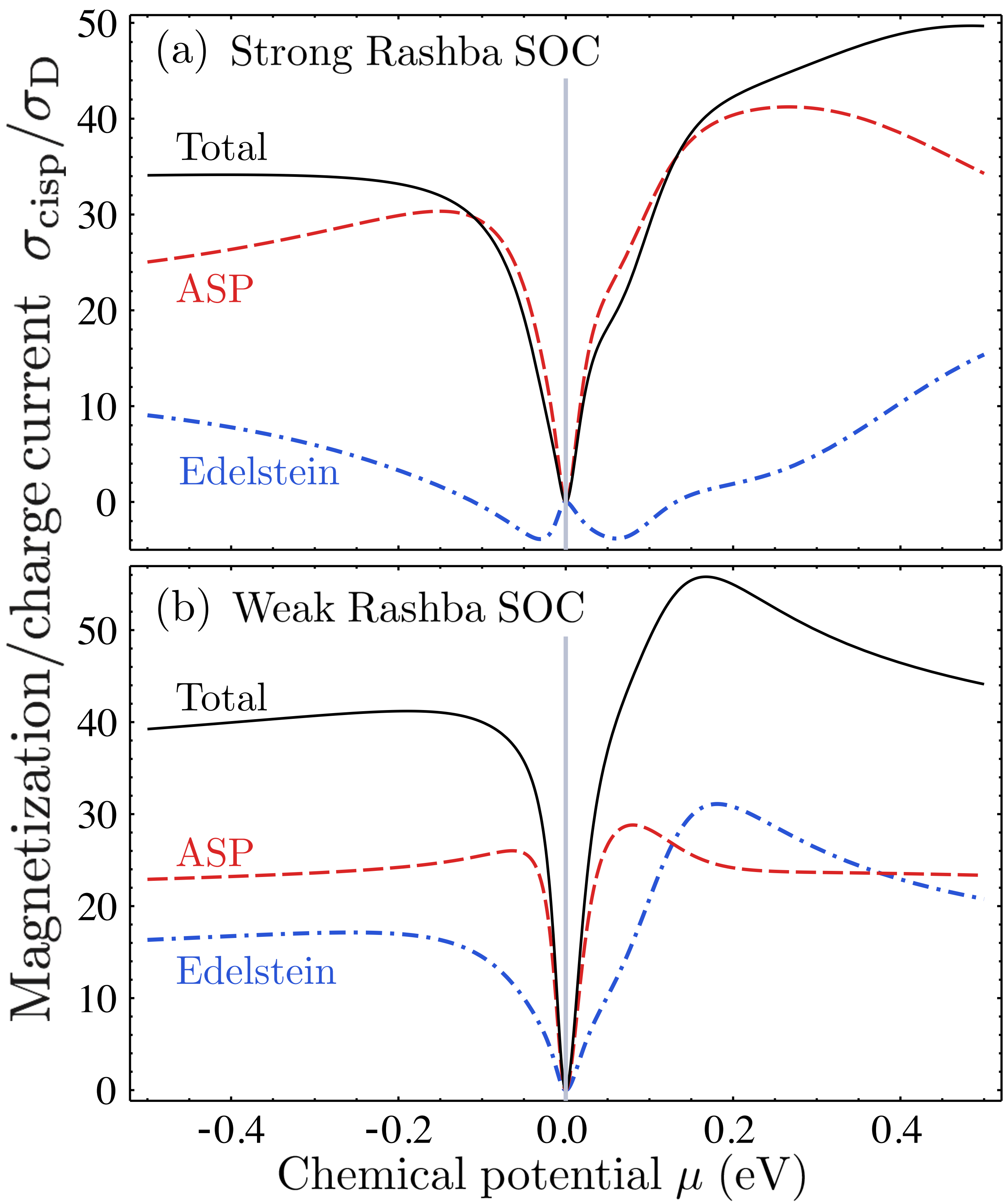
$$\begin{pmatrix} \color{blue}{O}\; J_x \\ \color{red}{E}\; \mathcal{J}^z_y \\ \color{blue}{O}\; \mathcal{M}^y \end{pmatrix} = \begin{pmatrix} 0 & \theta_{\mathrm{sH}} & \tau_{\mathrm{D}}\alpha_{\mathrm{asp}} \\ -\theta_{\mathrm{sH}} & 0 & \tau_{\mathrm{D}}\alpha_{\mathrm{R}} \\ \tau_{\mathrm{EY}}\alpha_{\mathrm{asp}} & -\tau_{\mathrm{EY}}\alpha_{\mathrm{R}} & 0 \end{pmatrix} \begin{pmatrix} J_x \\ \mathcal{J}^z_y \\ \mathcal{M}^y \end{pmatrix} + \sigma_{\mathrm{D}} \begin{pmatrix} E_x \\ 0 \\ 0 \end{pmatrix}$$

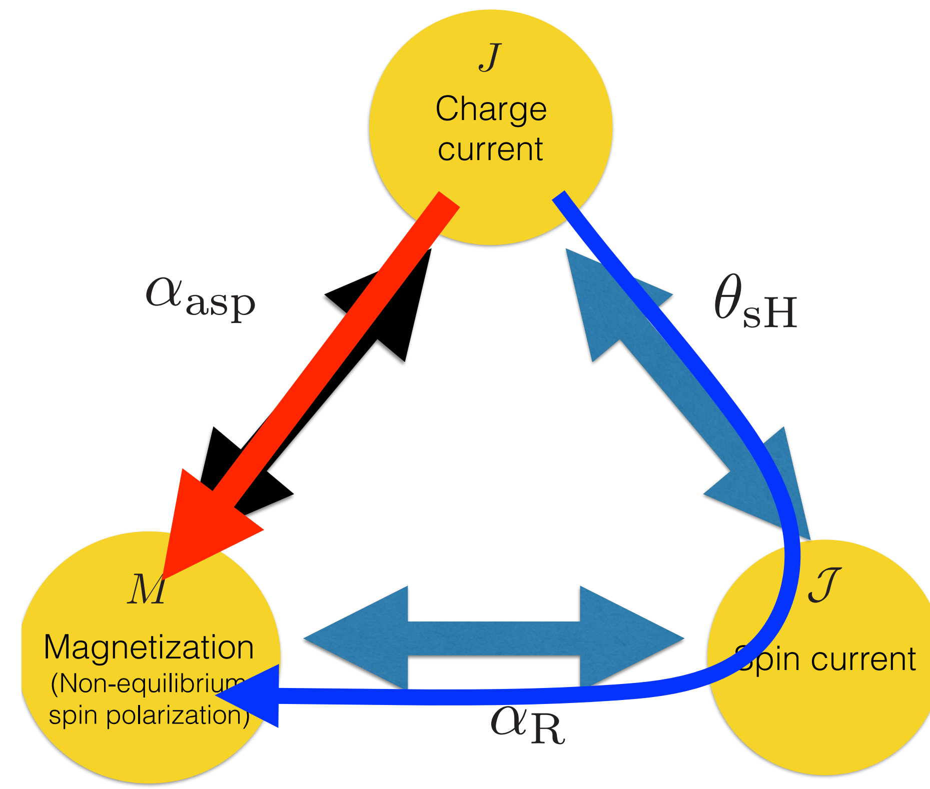
Onsager $R_{ij} = \epsilon_i \epsilon_j R_{ji} \quad (\text{TR parity } \epsilon_i = \pm 1)$





Extrinsic CISP? Yes! Two mechanisms





$$\mathcal{M}^y = \sigma_{\rm cisp} E_x$$

$$\sigma_{\rm cisp} = \sigma_{\rm D}\left(\theta_{\rm sH}\alpha_{\rm R}\tau_s + \alpha_{\rm asp}\tau_s\right)$$

Edelstein effect:
SHE ×Rashba

ASP
Scattering

Controlled by gating

C Huang, Y Chong, and MAC, Phys Rev B (2016)

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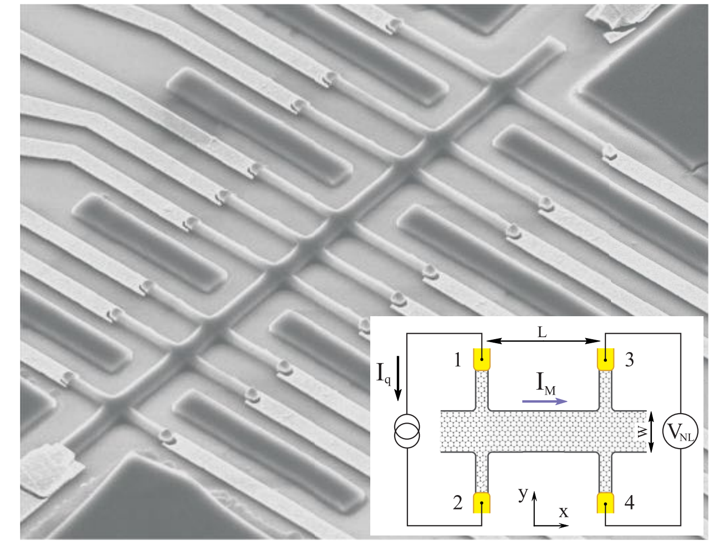
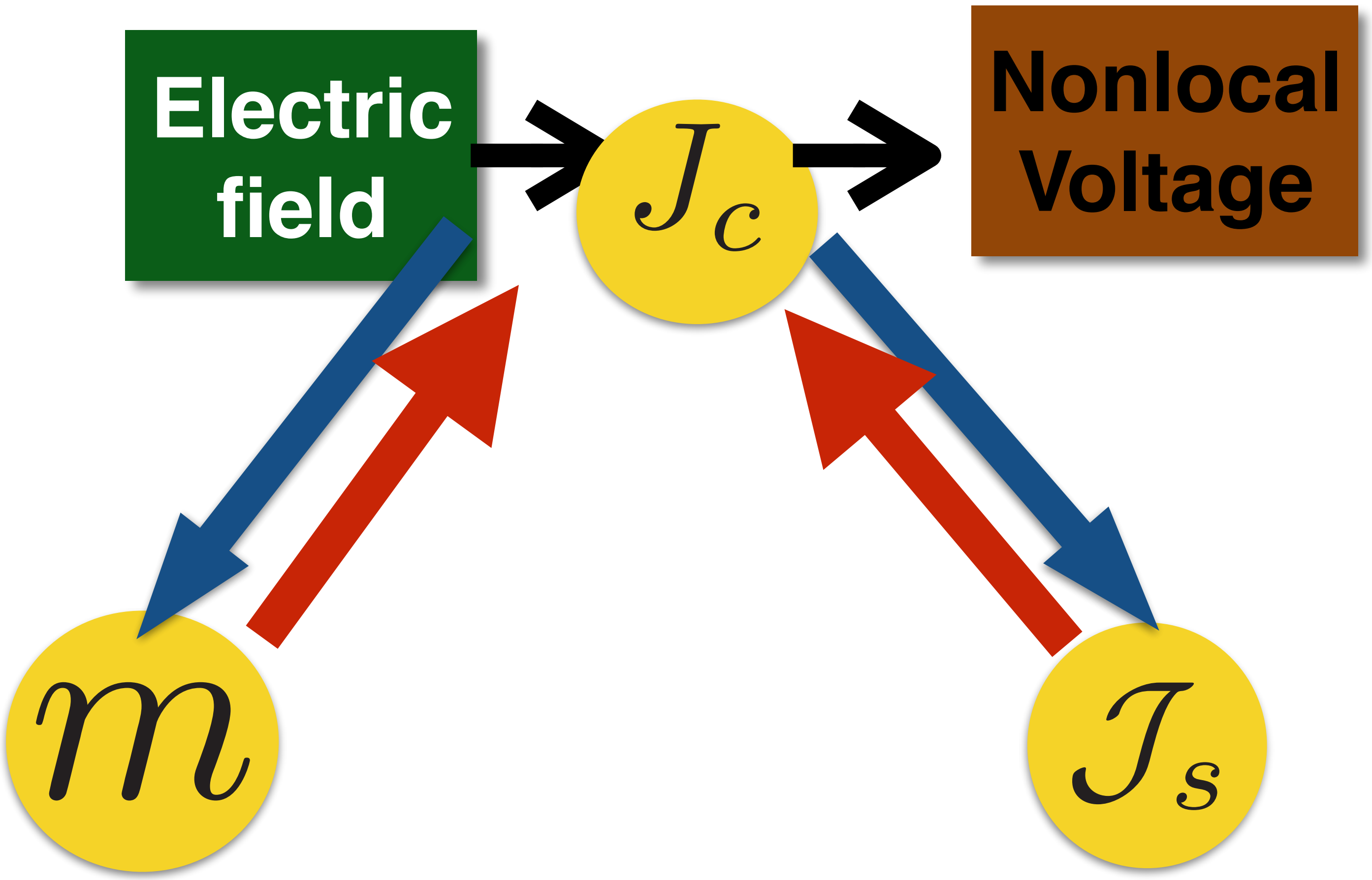
Nonlocal Resistance

(or Mott's double scattering)



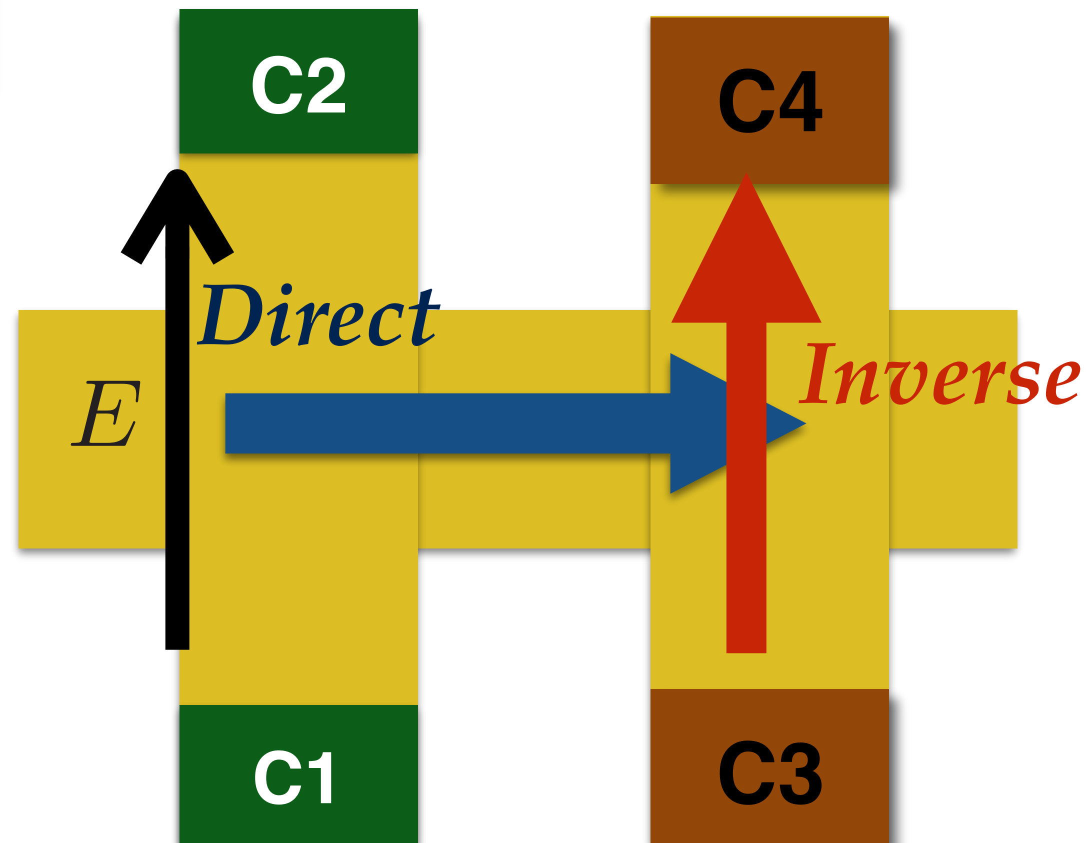


Direct Magnetoelectric Coupling (ASP)

C Huang, Y Chong, and MAC, arXiv:1702.04955 (2017)

SHE

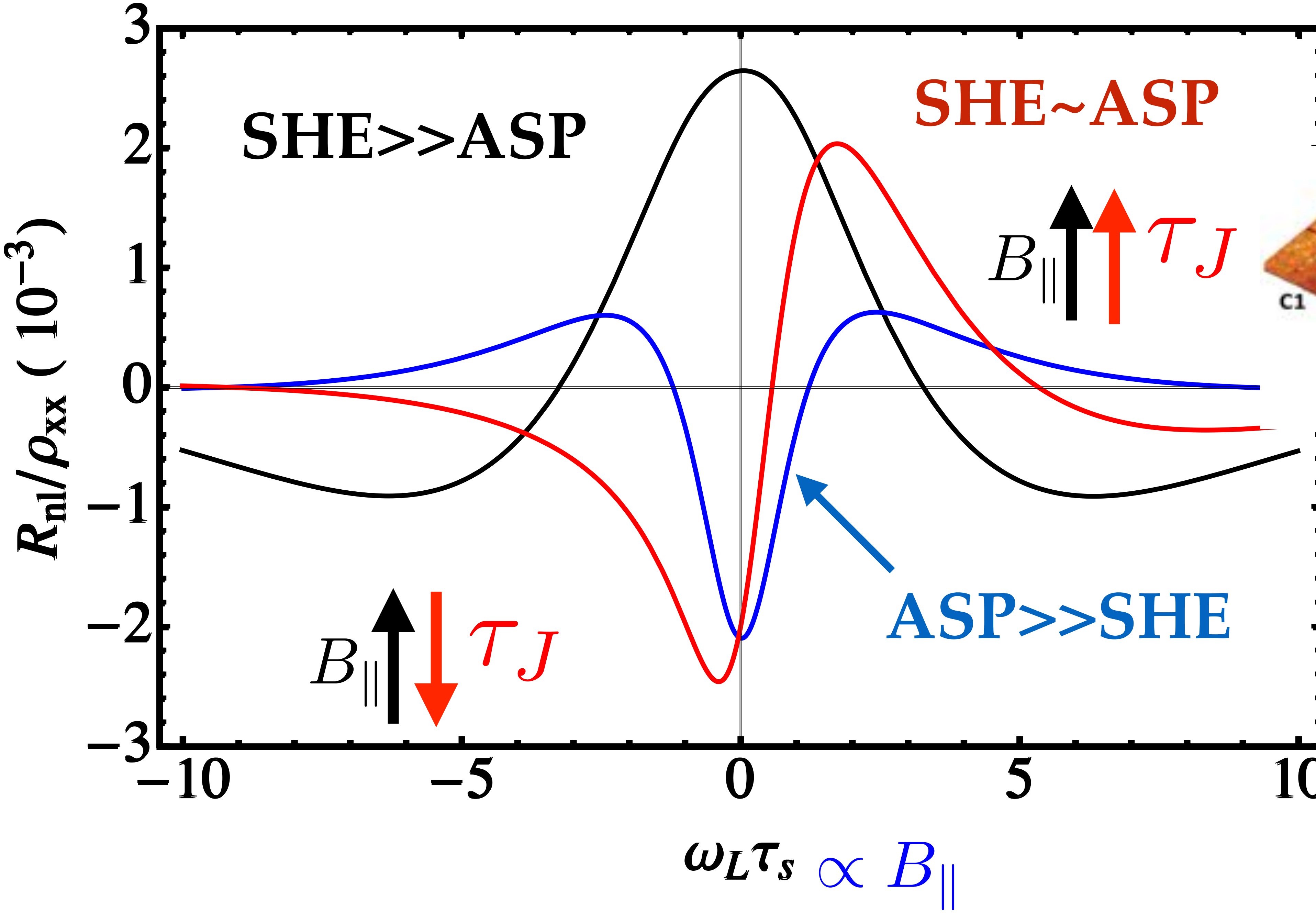
J Hirsch PRL (1999)
E M Hankiewiz et al PRB (2004)
D Abanin et al PRB (2007)

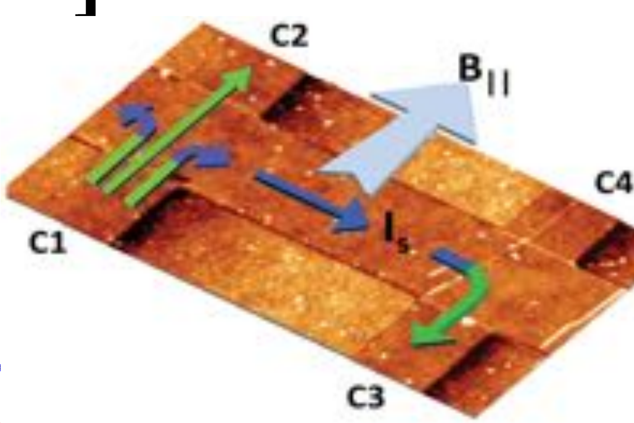


Anomalous Hanle spin precession

C Huang, Y Chong, and MAC, Phys. Rev. Lett. (2017)

Hanle effect is qualitatively different for different spin-charge conversion mechanism. It may become asymmetric.



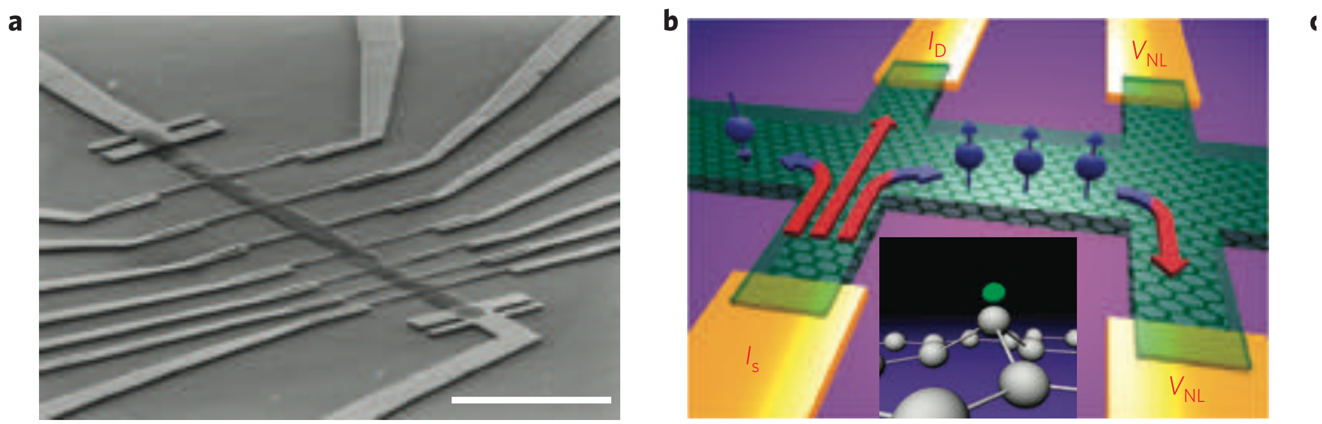


LETTERS
PUBLISHED ONLINE: 17 MARCH 2013 | DOI: 10.1038/NPHYS2576

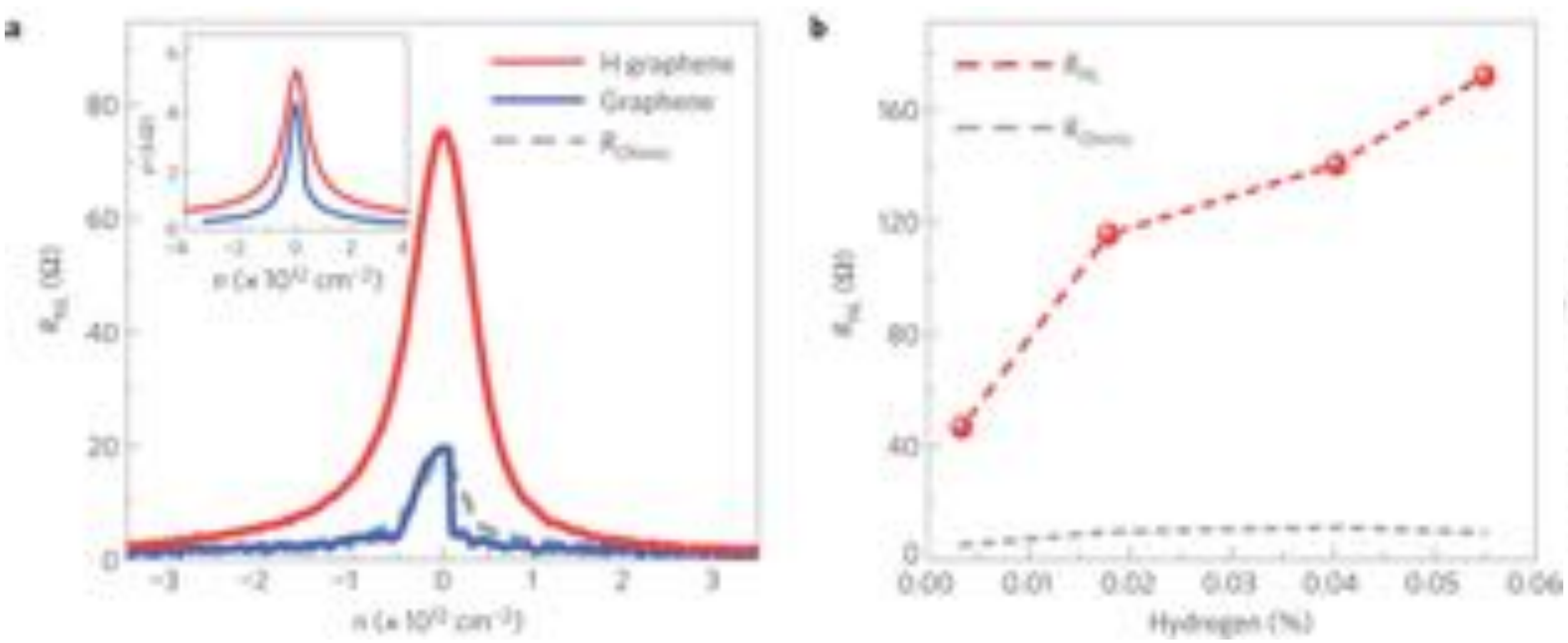
nature physics

Colossal enhancement of spin–orbit coupling in weakly hydrogenated graphene

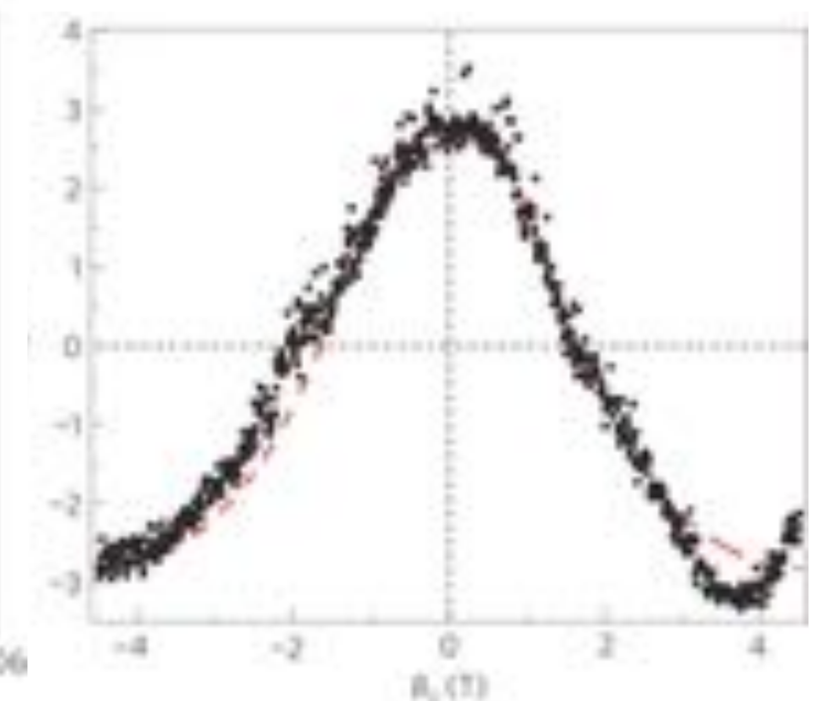
Jayakumar Balakrishnan[1,2†], Gavin Kok Wai Koon[1,2,3†], Manu Jaiswal[1,2‡], A. H. Castro Neto[1,2,4] and Barbaros Özyilmaz[1,2,3,4★]



Non-local resistance

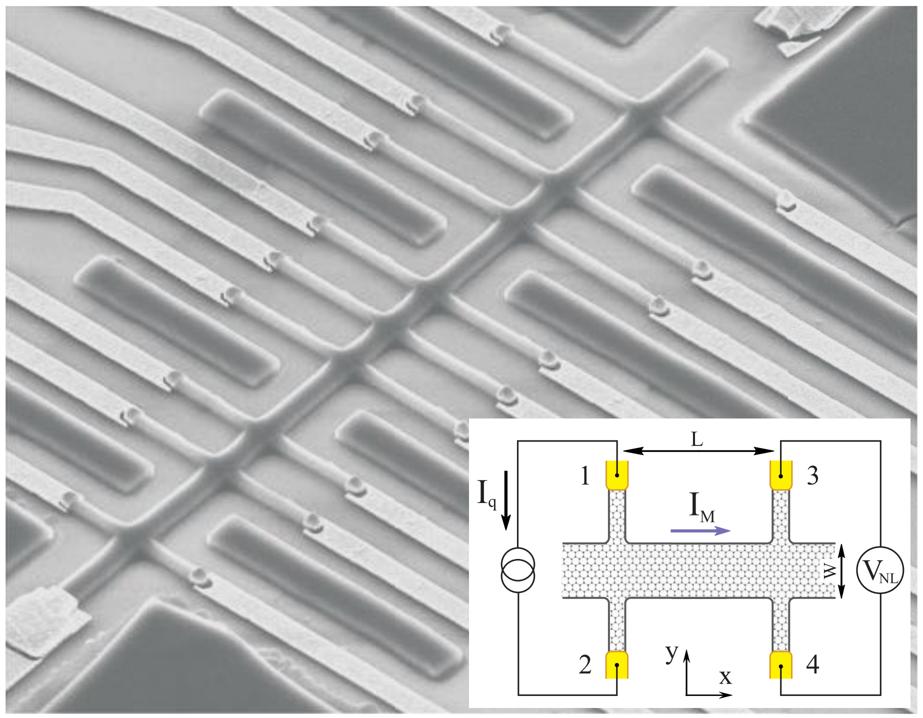


Hanle precession



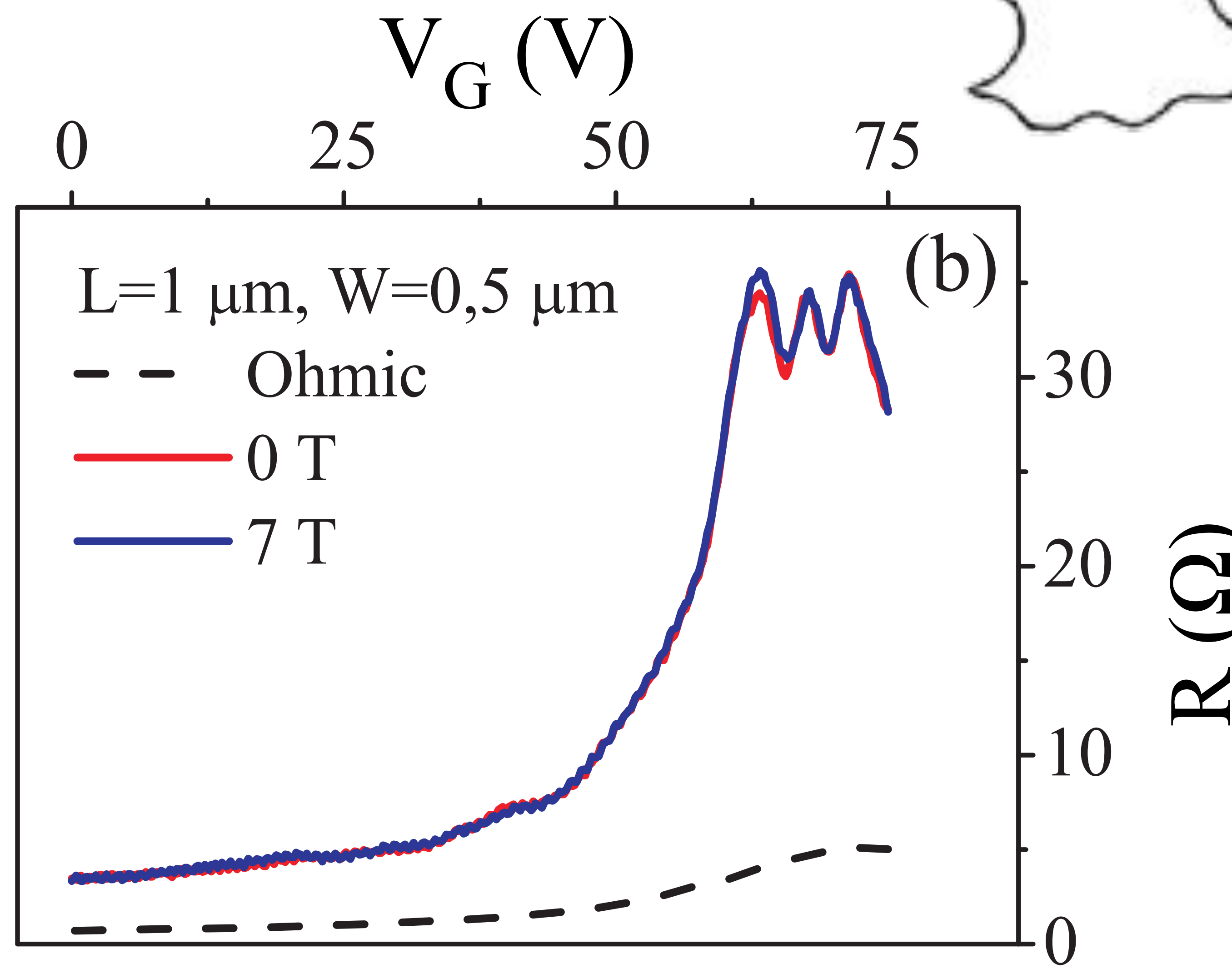
"Spooky" Non-local signals…

Weakly Hydrogenated Graphene



AA Kaverzin and BJ van Wees PRB (2015)

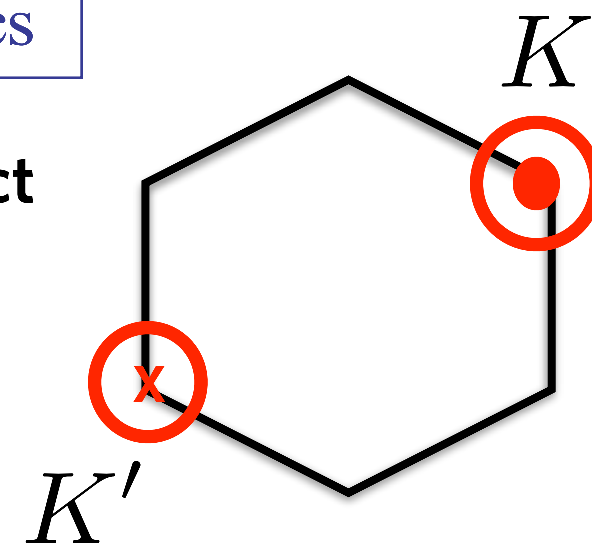
No Hanle precession!?

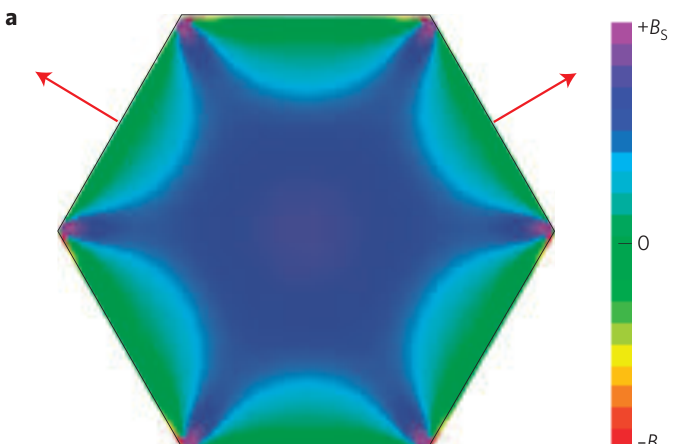


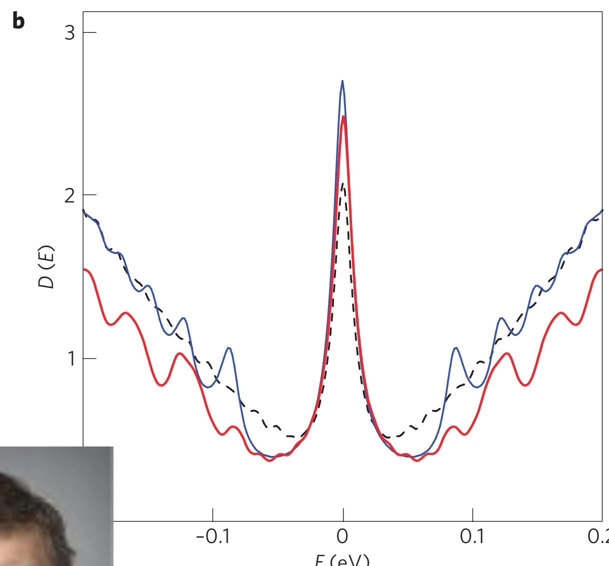
LETTERS
PUBLISHED ONLINE: 27 SEPTEMBER 2009 | DOI: 10.1038/NPHYS1420
nature physics

Energy gaps and a zero-field quantum Hall effect in graphene by strain engineering

F. Guinea[1]*, M. I. Katsnelson[2] and A. K. Geim[3]*



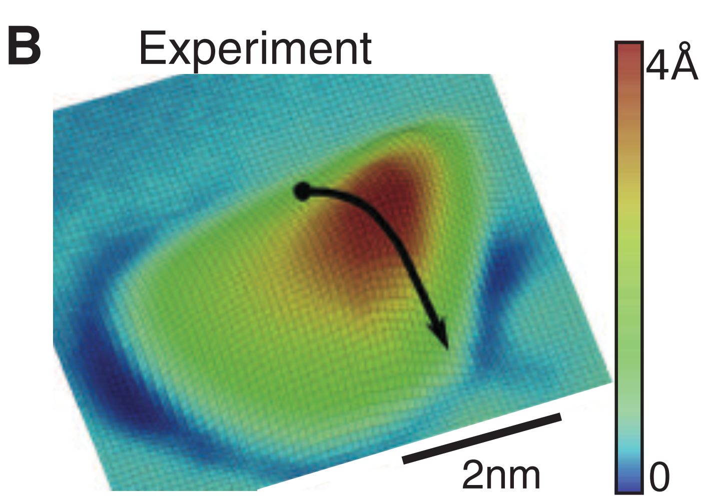


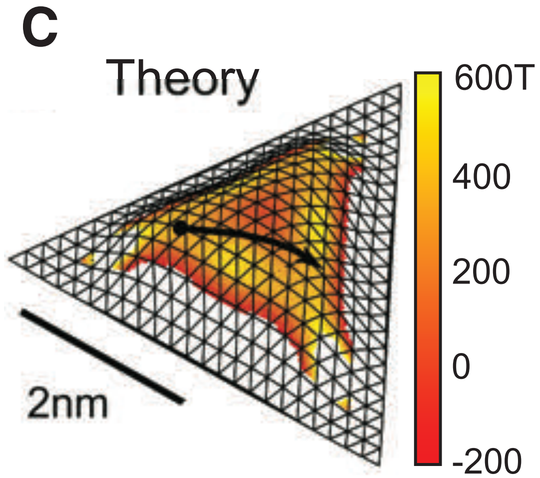


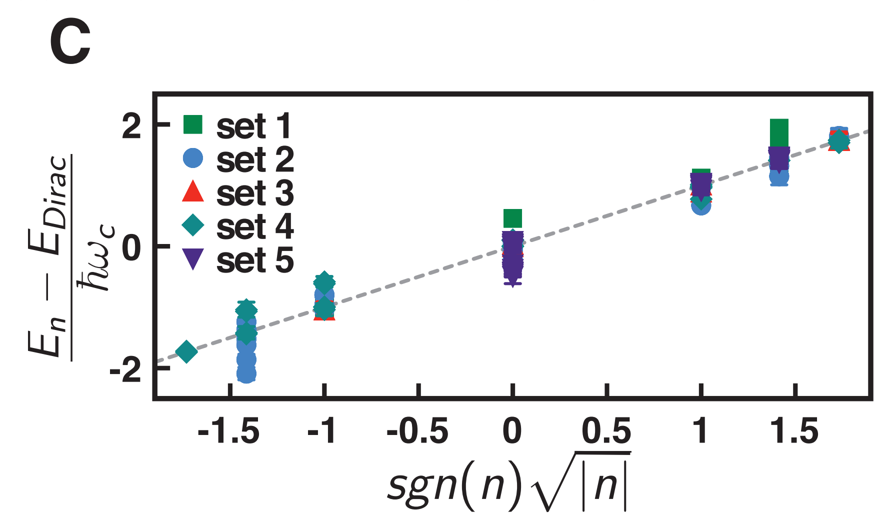
Strain-Induced Pseudo–Magnetic Fields Greater Than 300 Tesla in Graphene Nanobubbles

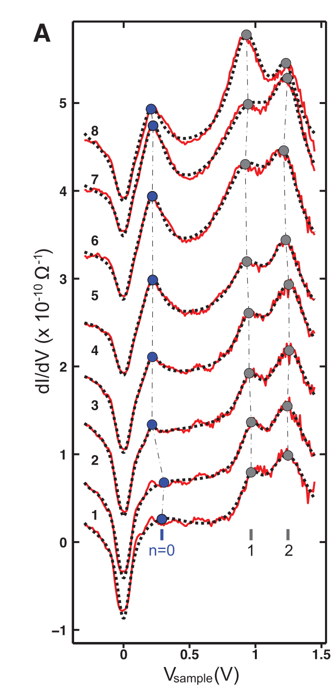
N. Levy,[1,2]*† S. A. Burke,[1]*‡ K. L. Meaker,[1] M. Panlasigui,[1] A. Zettl,[1,2] F. Guinea,[3] A. H. Castro Neto,[4] M. F. Crommie[1,2]§

30 JULY 2010 VOL 329 SCIENCE









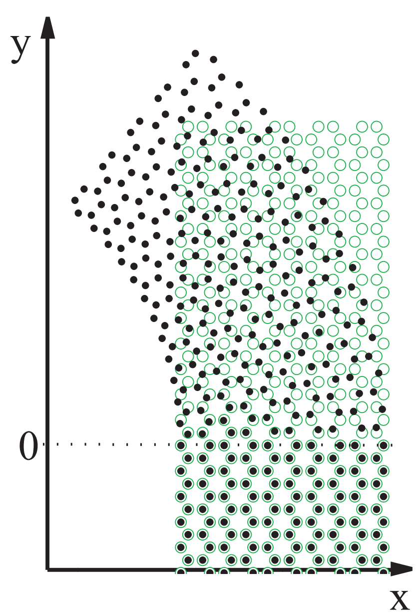
Strained, semi-classically

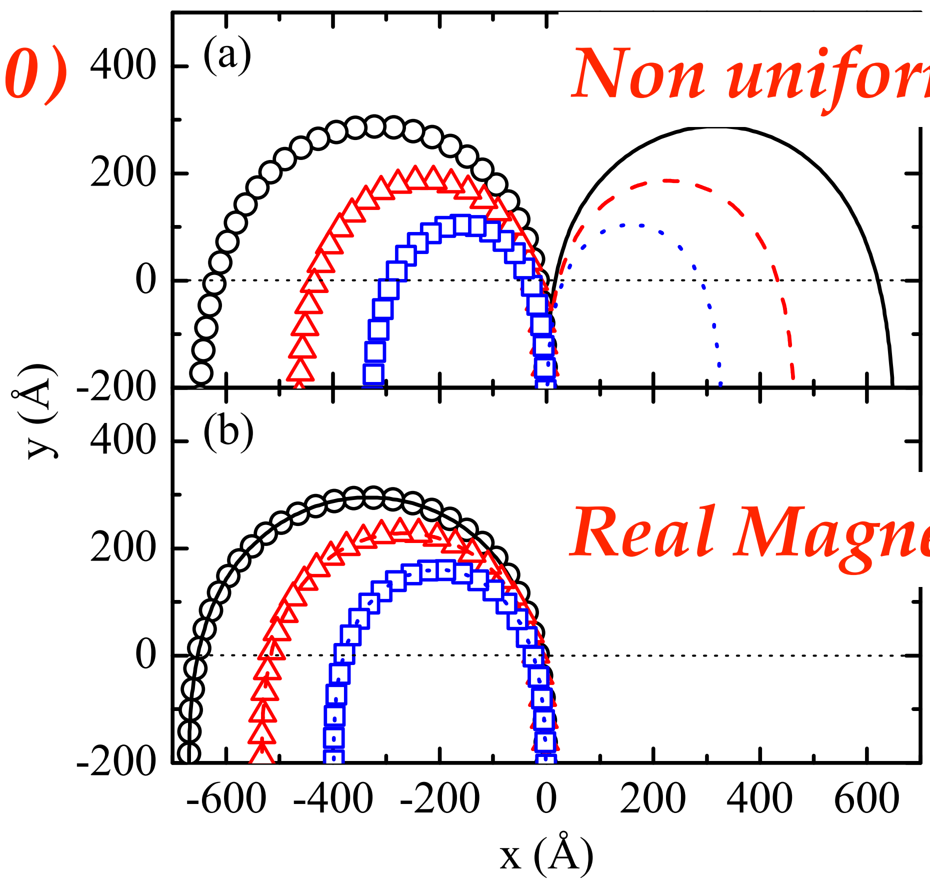
Semiclassical equations of motion

$$\boldsymbol{u}_k = \boldsymbol{\nabla}_k \epsilon_k \quad \epsilon_k = \pm v_F |\boldsymbol{k}|$$

$$\dot{\boldsymbol{r}} = \boldsymbol{u}_k, \quad \dot{\boldsymbol{k}} = (e\boldsymbol{E} + \tau_z \dot{\boldsymbol{r}} \times \boldsymbol{\mathcal{B}}_s) \quad \boldsymbol{\mathcal{B}}_s = \boldsymbol{\nabla} \times \boldsymbol{\mathcal{A}}_s(\boldsymbol{r})$$

A Chavez et al Phys Rev B (2010)





Semiclassical Transport

$$\partial_t \delta n_k + \dot{\boldsymbol{r}} \cdot \nabla_r \delta n_k + \dot{\boldsymbol{k}} \cdot \nabla_k \left[n_k^0 + \delta n_k \right] = \mathcal{I}[\delta n_k]$$

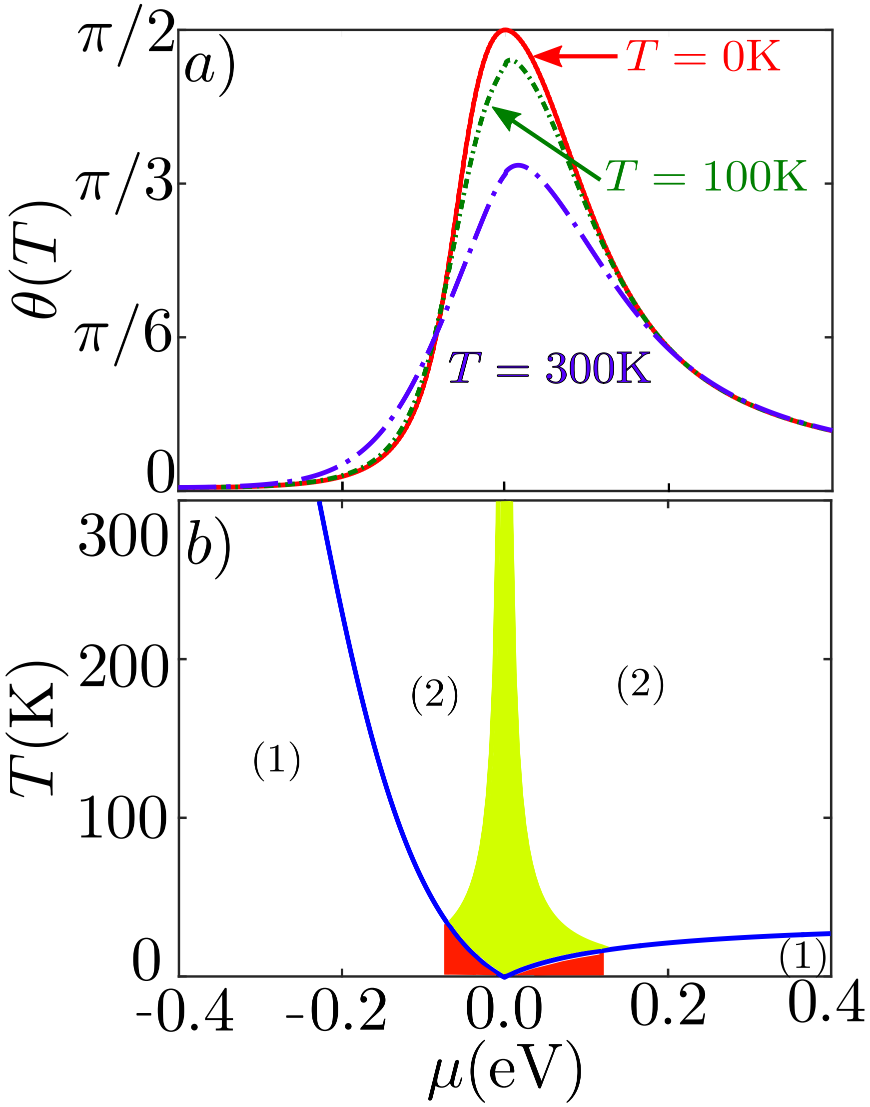
$$\delta n_k = \rho_k \mathbf{1} + \boldsymbol{\mathcal{P}}_k \cdot \boldsymbol{\tau}$$

Describes quantum coherence between valleys

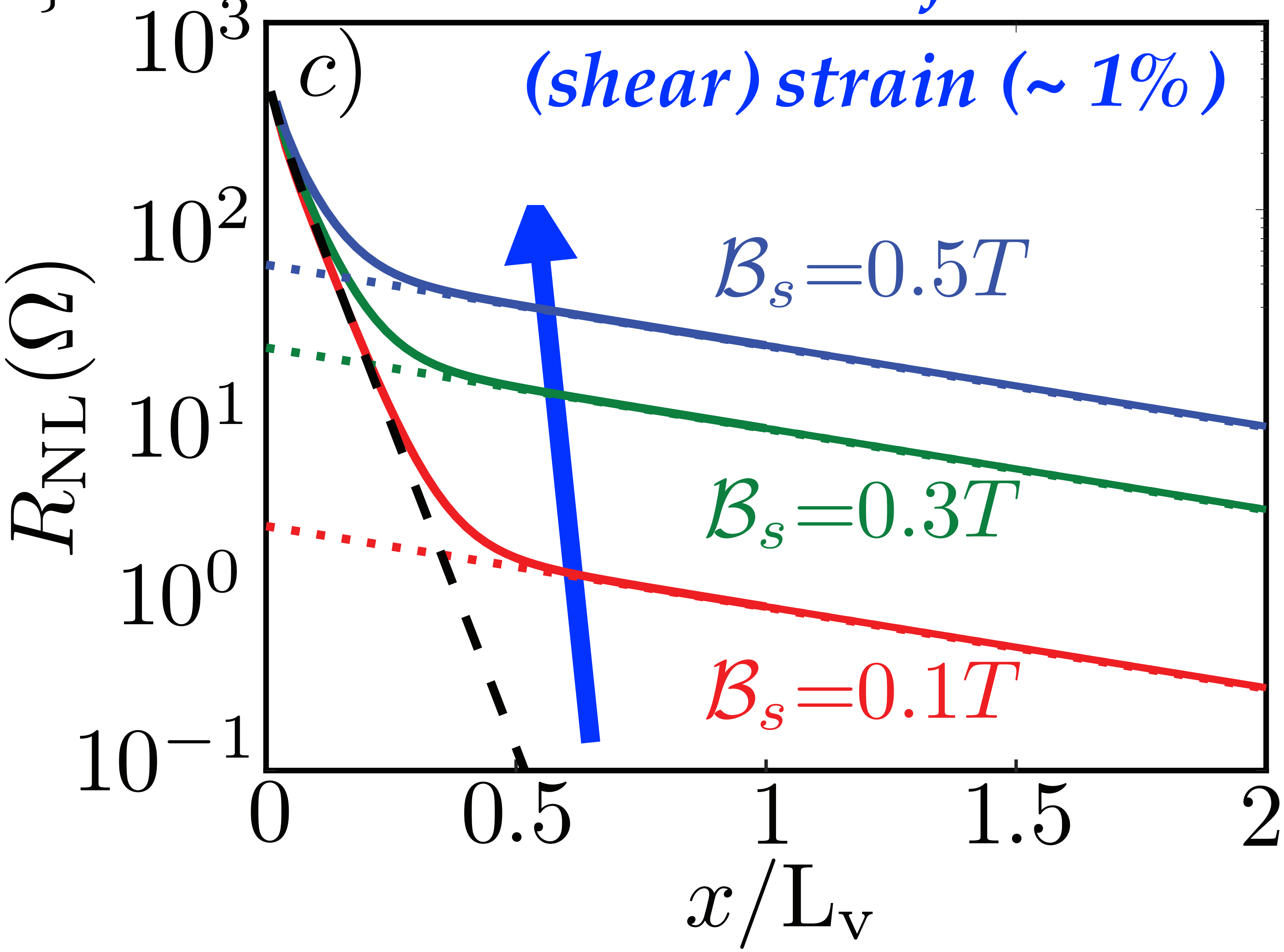
Strain induced Valley Hall Effect

X-P Zhang, CL Huang, and MAC 2D Materials (2017)

$\omega_c \ll \max\{\tau_D^{-1}, k_B T\}$



Non uniform

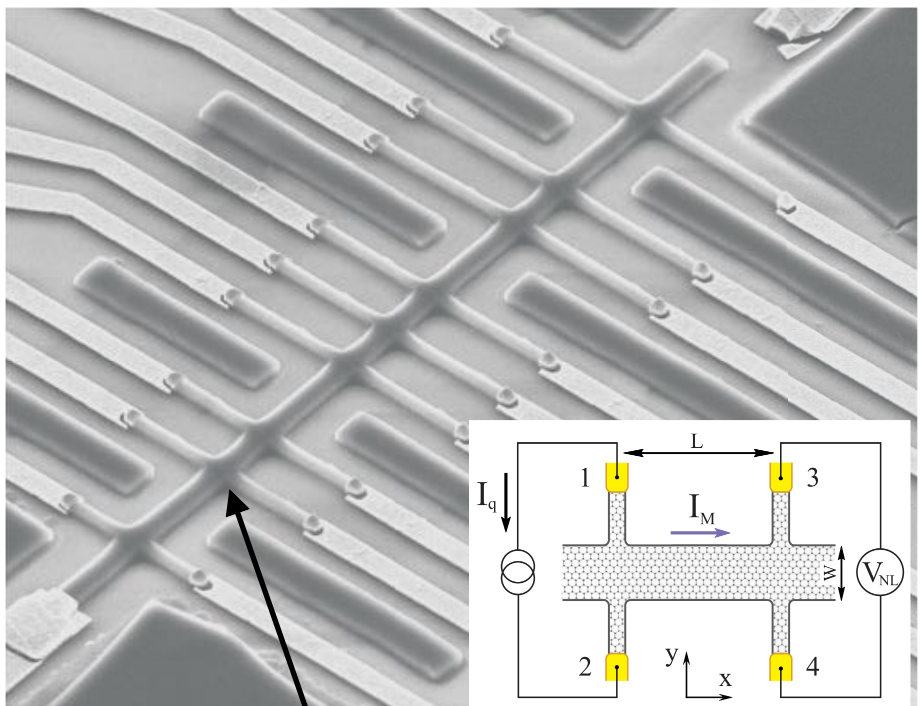


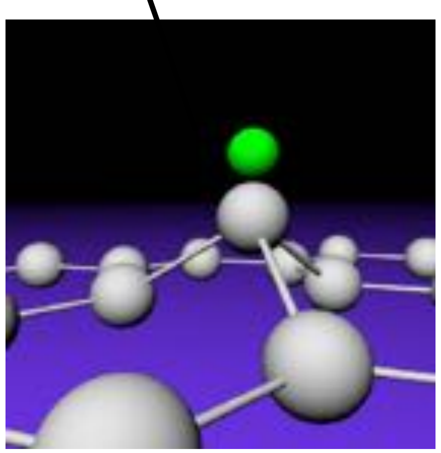
Valley currents do not exhibit the Hanle effect!

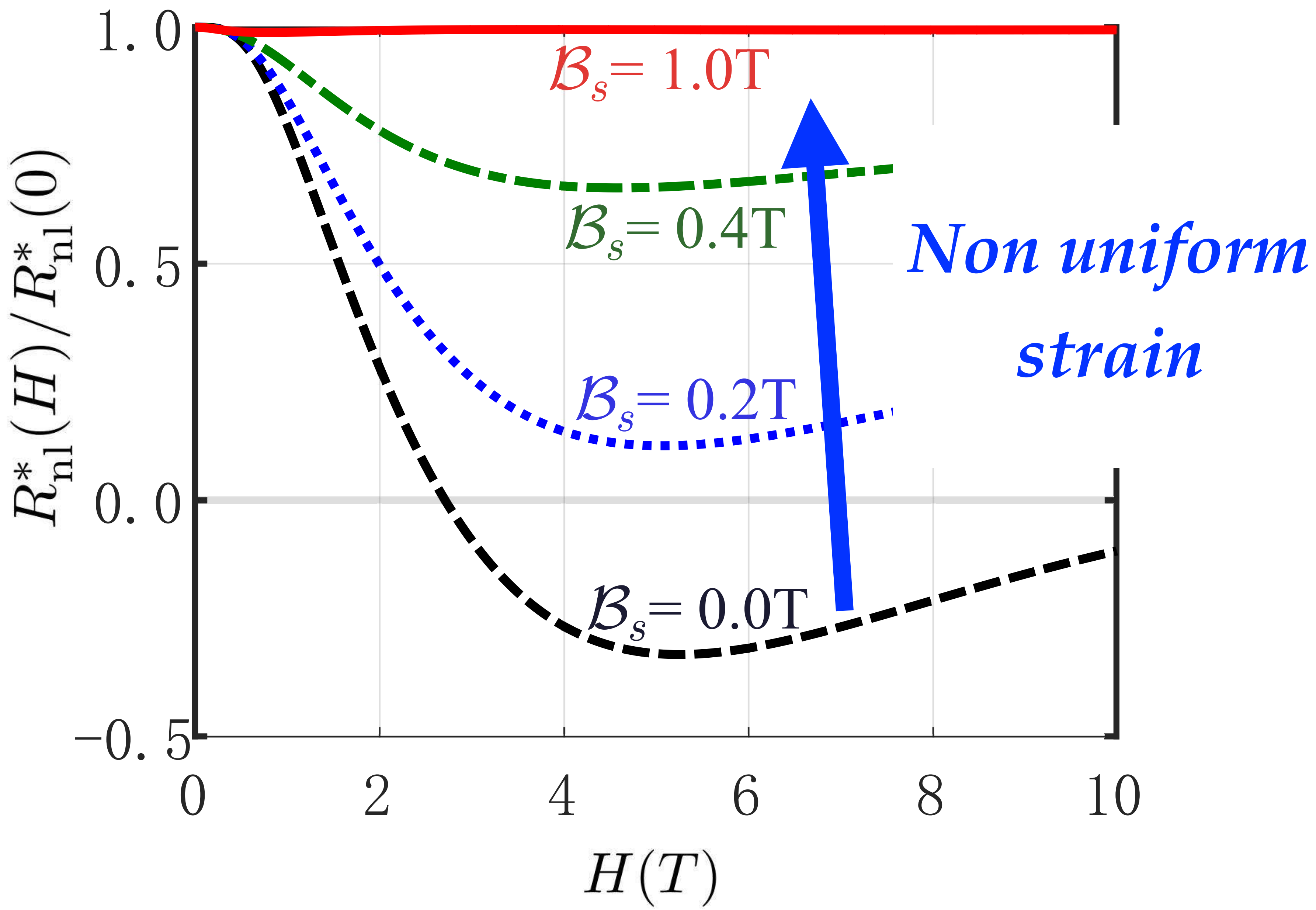
Interplay of VHE and SHE

X-P Zhang, CL Huang, and MAC in preparation

Hanle oscillations are suppressed!







Hydrogen+ nonuniform strain

Outline: Part II

- *Topology = Dimensional reduction?*
- *Impurity in a 1D Channel w and w/o interactions: Kane & Fisher*
- *Magnetic Impurity near the non-interacting edge of a 2D QSHI*
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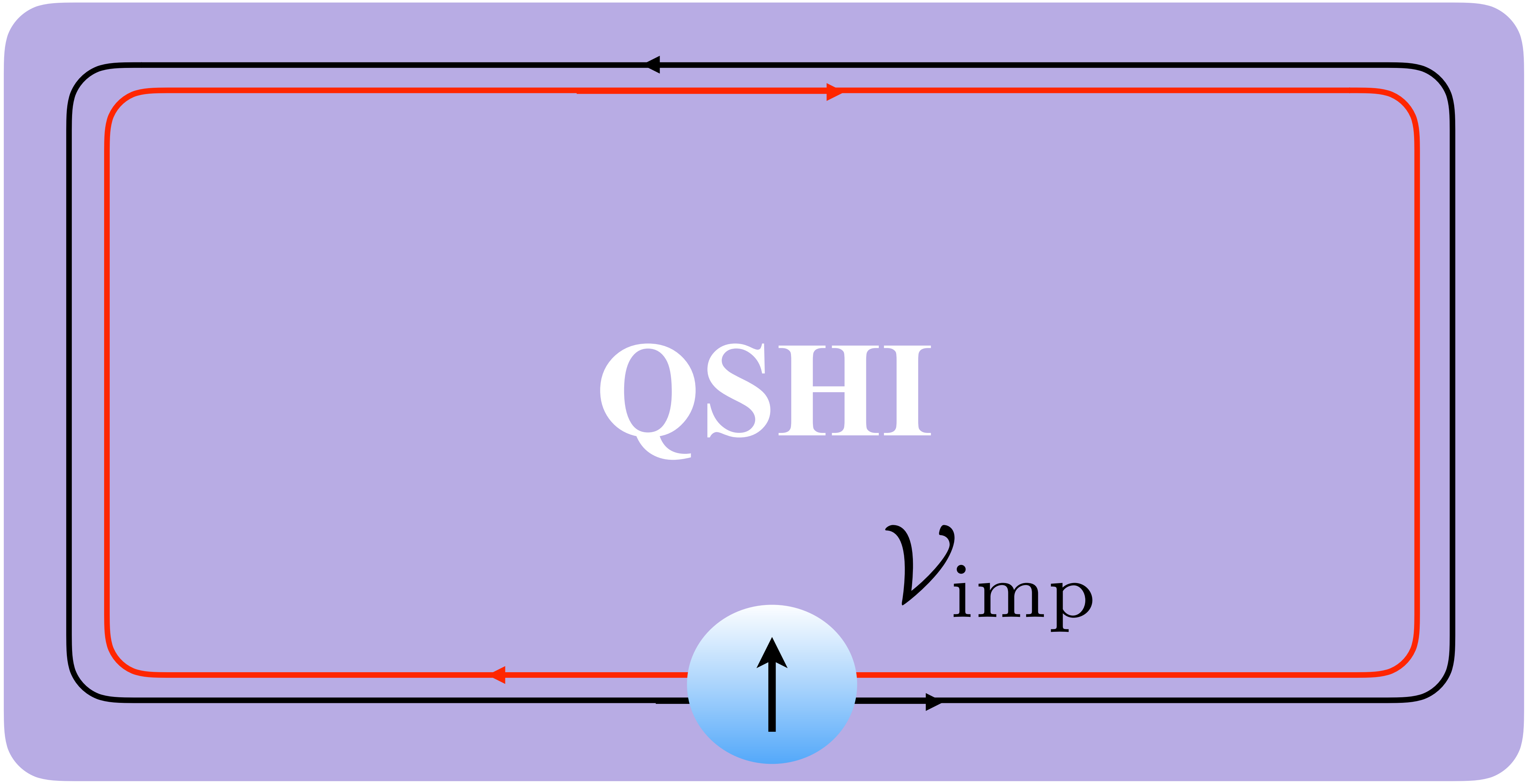
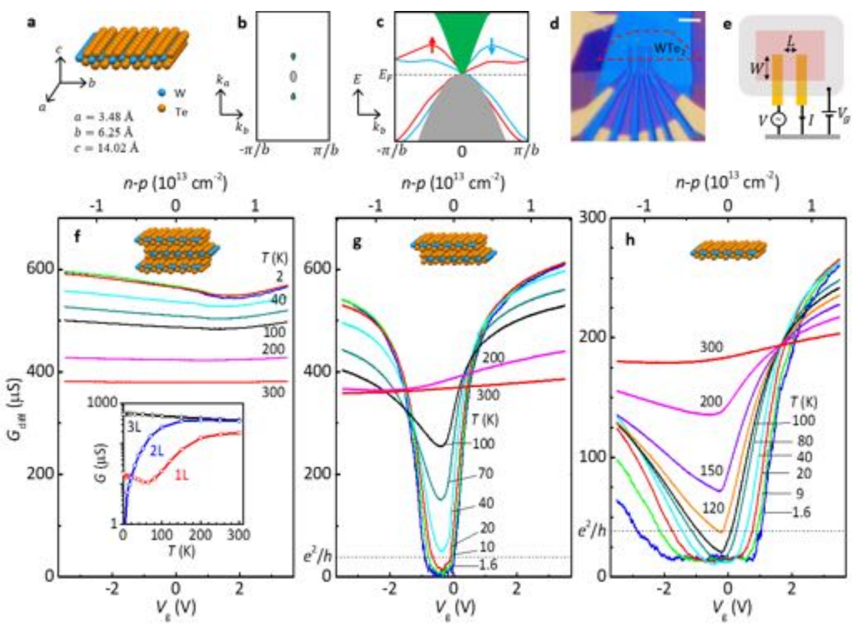
Low energy description:
Topology = Dimensional Reduction?



Quantum Spin Hall Insulators

Z Fei et al Nat. Phys (2017)

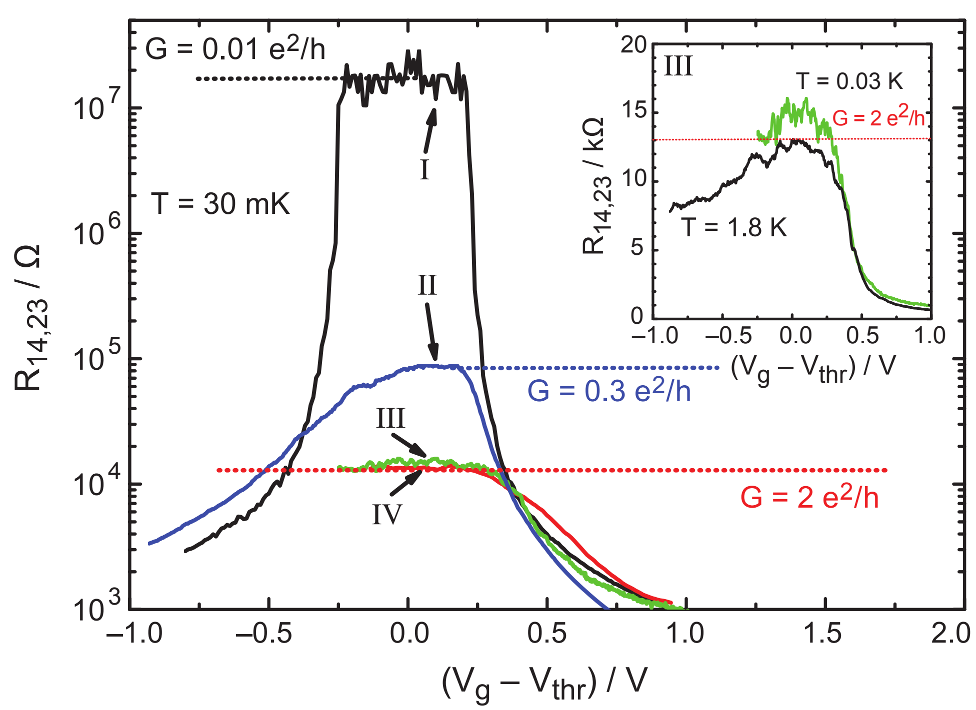


Quantum Spin Hall Effect and Topological Phase Transition in HgTe Quantum Wells

B. Andrei Bernevig,[1,2] Taylor L. Hughes,[1] Shou-Cheng Zhang[1]*

We show that the quantum spin Hall (QSH) effect, a state of matter with topological properties distinct from those of conventional insulators, can be realized in mercury telluride–cadmium telluride semiconductor quantum wells. When the thickness of the quantum well is varied, the electronic state changes from a normal to an “inverted” type at a critical thickness d_c. We show that this transition is a topological quantum phase transition between a conventional insulating phase and a phase exhibiting the QSH effect with a single pair of helical edge states. We also discuss methods for experimental detection of the QSH effect.

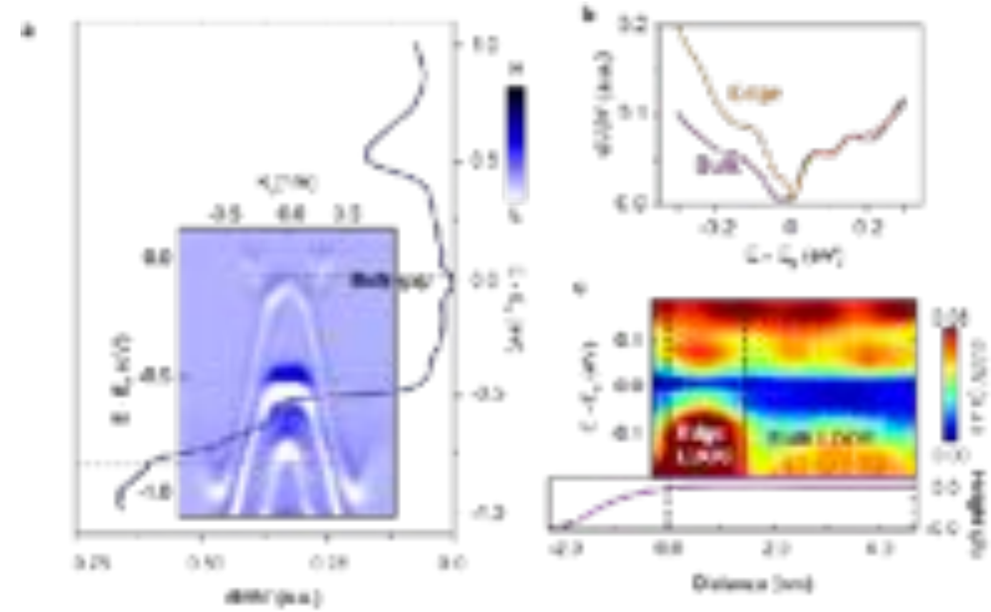
SCIENCE VOL 314 15 DECEMBER 2006



M König et al Science (2007)

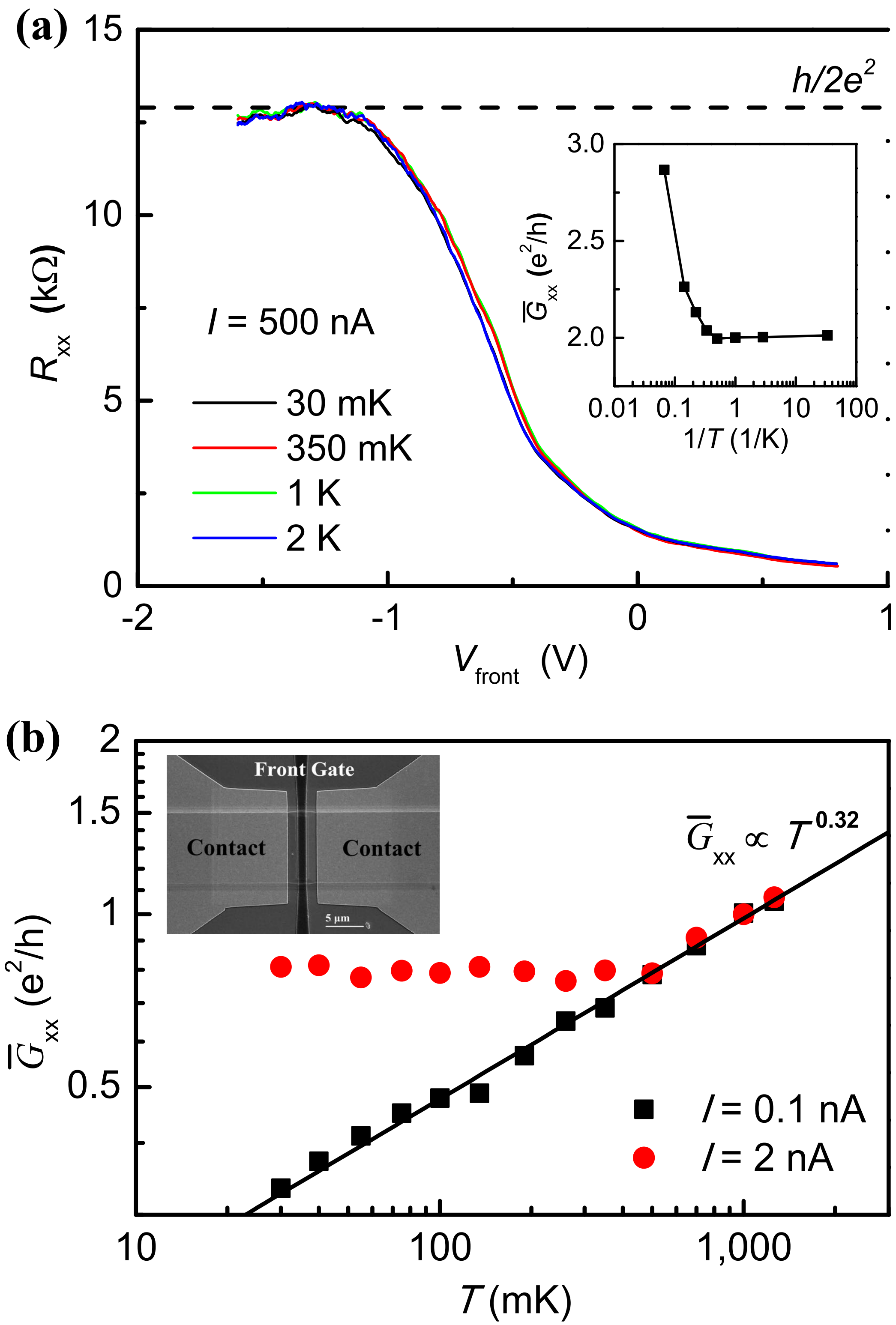
nature physics
LETTERS
PUBLISHED ONLINE: 26 JUNE 2017 | DOI: 10.1038/NPHYS4174

Quantum spin Hall state in monolayer 1T′-WTe_2



S Tang et al Nat. Phys (2017)

Absence of conductance quantization



In samples with long edge channels two-terminal conductance is NOT quantized!
DEVIATIONS FROM $2e^2/h$

The mechanism for backscattering is not clear, but conductance exhibits quantum critical SCALING with T and V

Are 1D models of the edge channels always complete?

Review: G Dolcetto, M Sassetti, T L Schmidt arxiv:1511.0614 (2015)

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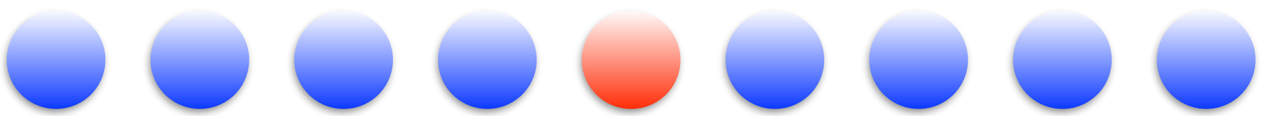
Structureless Impurity in 1D

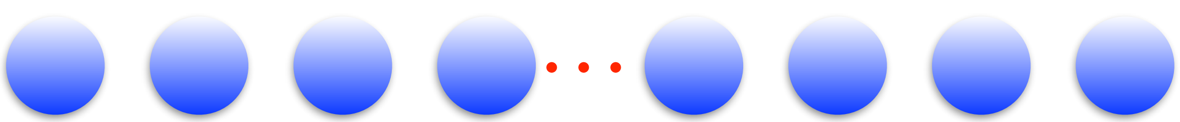
Non-interacting electrons

Weak impurity: Potential scattering



$$H = -t\sum_n \left[c_n^\dagger c_{n+1} + c_{n+1}^\dagger c_n\right] + \epsilon_0 c_0^\dagger c_0$$

Strong impurity: Weak tunneling link



$$H = -t\sum_n \left[c_n^\dagger c_{n+1} + c_{n+1}^\dagger c_n\right] - t_0 \left[c_1^\dagger c_{-1} + c_{+1}^\dagger c_{-1}\right]$$

Strong scatterer limit $|\epsilon_0| \gg t$ $\quad t_0 = -\frac{t^2}{\epsilon_0}$

Lippmann-Schwinger Equation

$$T(\epsilon) = G_0(\epsilon) + G_0(\epsilon)VT(\epsilon) \qquad G_0(\epsilon) = (\epsilon^+ - H_0)^{-1}$$

Conductance from Landauer-Buttiker

$$\mathcal{T}(\epsilon) \sim T(\epsilon) \qquad G(\epsilon) = |\mathcal{T}(\epsilon)|^2 = |\mathcal{T}_0|^2 = \text{const.}$$

(For ε near the center of the band)

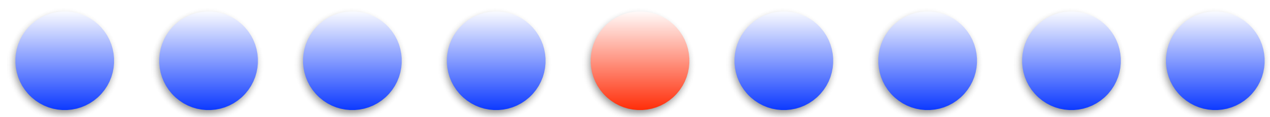
The Kane-Fisher Problem: Impurity in 1D

C Kane and MPA Fisher
Phys Rev Lett 1992

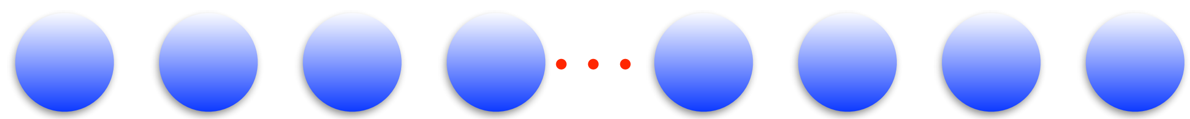
Add interactions

$$H = \sum_n \left[-t \left(c_n^\dagger c_{n+1} + c_{n+1}^\dagger c_n \right) + V c_n^\dagger c_{n+1}^\dagger c_{n+1} c_n \right] + \epsilon_0 c_0^\dagger c_0$$

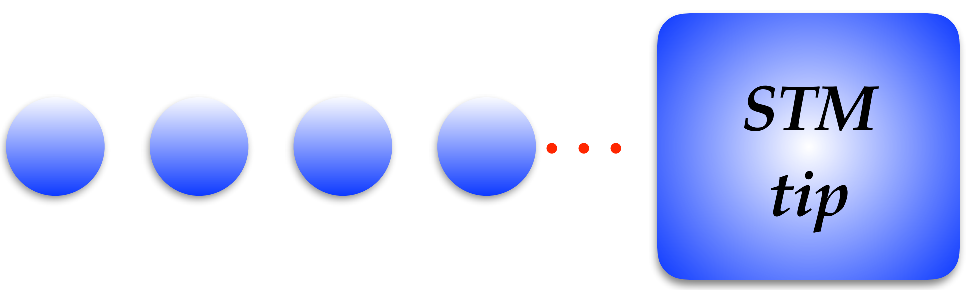
Weak impurity: Potential scattering



Strong impurity: Weak tunneling link



Tunneling density of states



$$\frac{dt_0}{d\ell} = \left(1 - \frac{1}{K}\right) t_0$$

$$\rho(\omega) \sim |\omega|^{\frac{1}{K}-1}$$

$$\omega \sim k_B T \qquad \rho(T) \sim T^{\frac{1}{K}-1}$$

$$G(T) \sim |t_0(T)|^2 \begin{cases} \to 0 & K < 1 \quad (V > 0) \\ \to t & K > 0 \quad (V < 0) \end{cases}$$

F Guinea and M Ueda Z Phys B (1991)

Another way to understand it …

D Yue, L Glazman, K A Mateev Phys Rev B (1994)

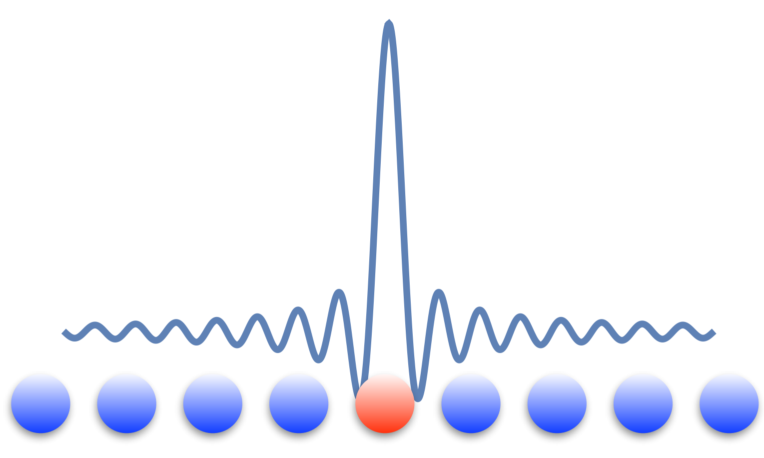
Hartree approximation to interactions

$$H_{\text{int}} = V \sum_n c_n^\dagger c_{n+1}^\dagger c_{n+1} c_n \rightarrow H_{\text{int}}^{MFA} = V \sum_n [\langle n_n \rangle n_{n+1} + \langle n_n \rangle n_{n+1}]$$

$$\langle n_n \rangle \sim \frac{\sin(2k_F x_n)}{x_n}$$

Impurity induced Fiedel Oscillation

Solve external + Hartree potential



Transmission coefficient develops log singularity (becomes a power-law when resumed)

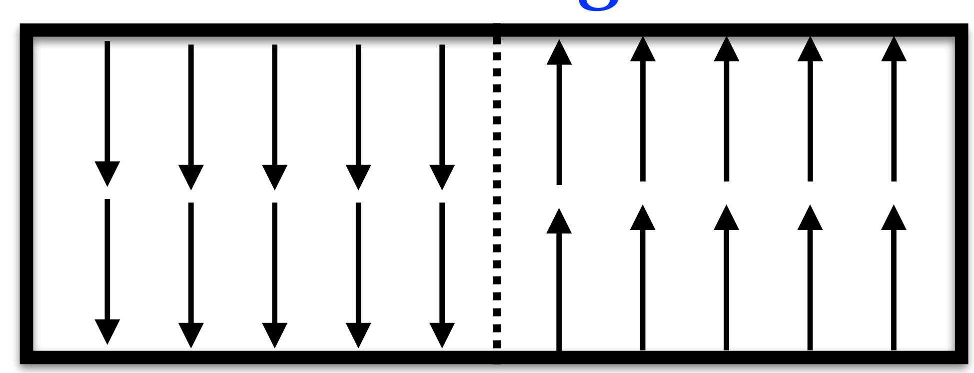
$$\mathcal{T}(\epsilon) = \mathcal{T}_0 \left[1 + c|\mathcal{R}_0|^2 V \log\left|\frac{\epsilon_F - \epsilon}{D}\right| + \cdots \right] \sim |\epsilon - \epsilon_F|^{c|\mathcal{R}_0|^2 V}$$

Phase diagram

$t_0 = 0 \quad (\epsilon_0 = +\infty)$

Impurity strength

$\epsilon_0 = 0 \quad (t_0 = t)$



Interaction

$V < 0 \quad V = 0 \quad V > 0$

C Kane and MPA Fisher Phys Rev Lett 1992

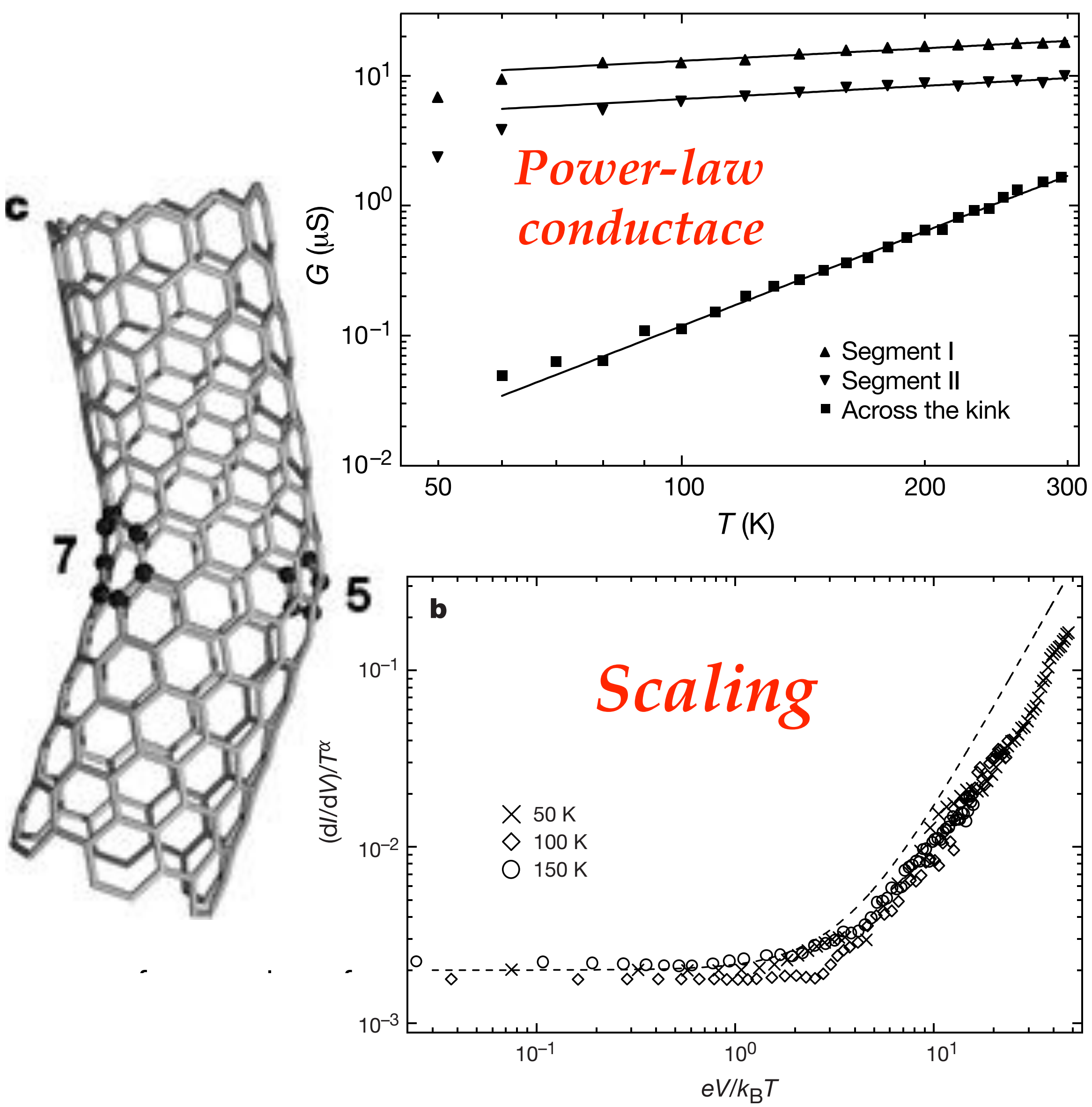
Experiment and Numerics

Experiment *Z Yao et al Nature (1999)*

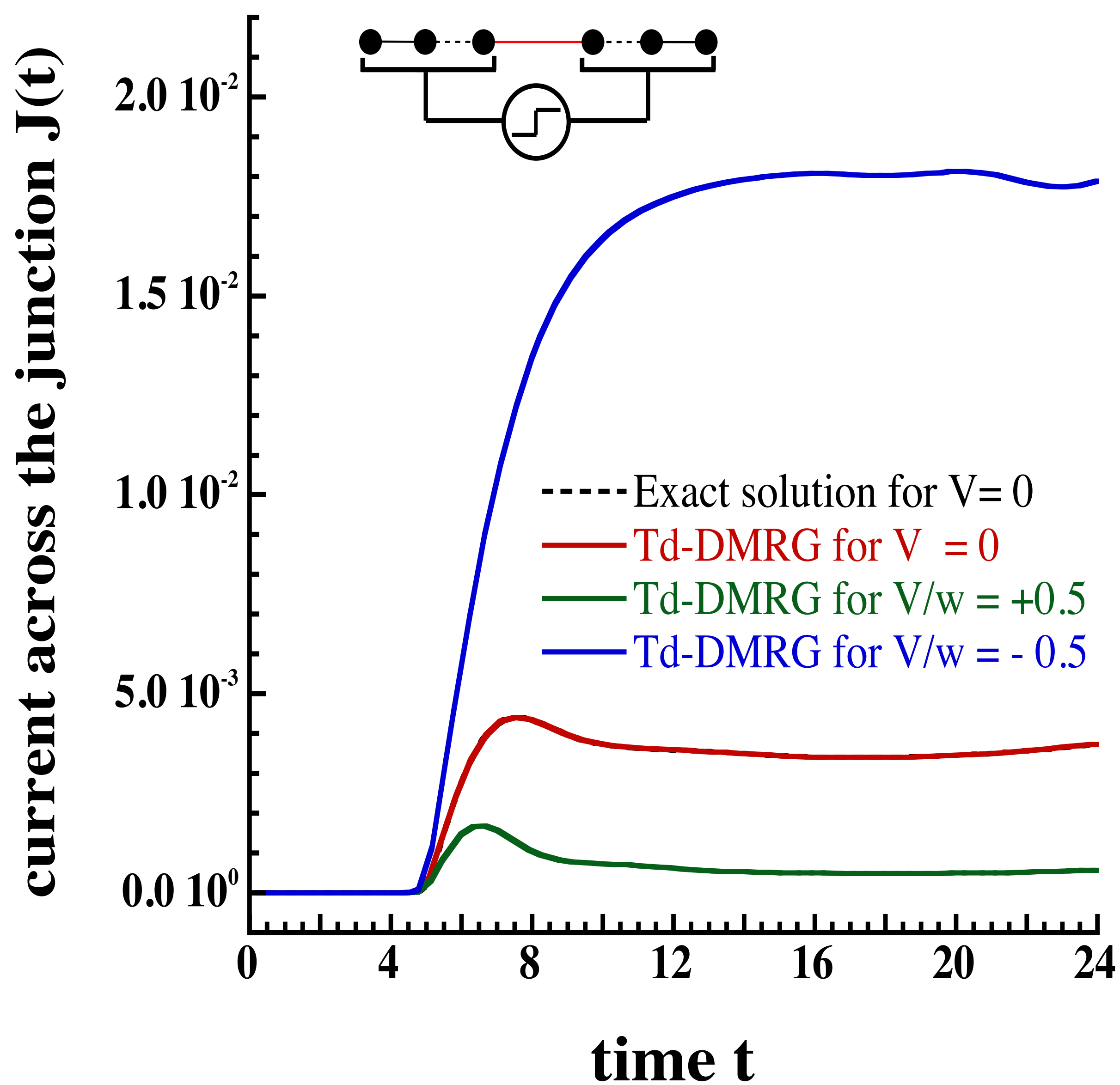
Carbon nanotube intramolecular junctions

Zhen Yao*, Henk W. Ch. Postma*, Leon Balents† & Cees Dekker*

* *Department of Applied Sciences and DIMES, Delft University of Technology, Lorentzweg 1, 2628 CJ Delft, The Netherlands*
† *Bell Laboratories, Lucent Technologies, Murray Hill, New Jersey 07974, USA*



Numerics: Time-dependent DMRG



MAC and JB Marston Phys Rev Lett (2001)

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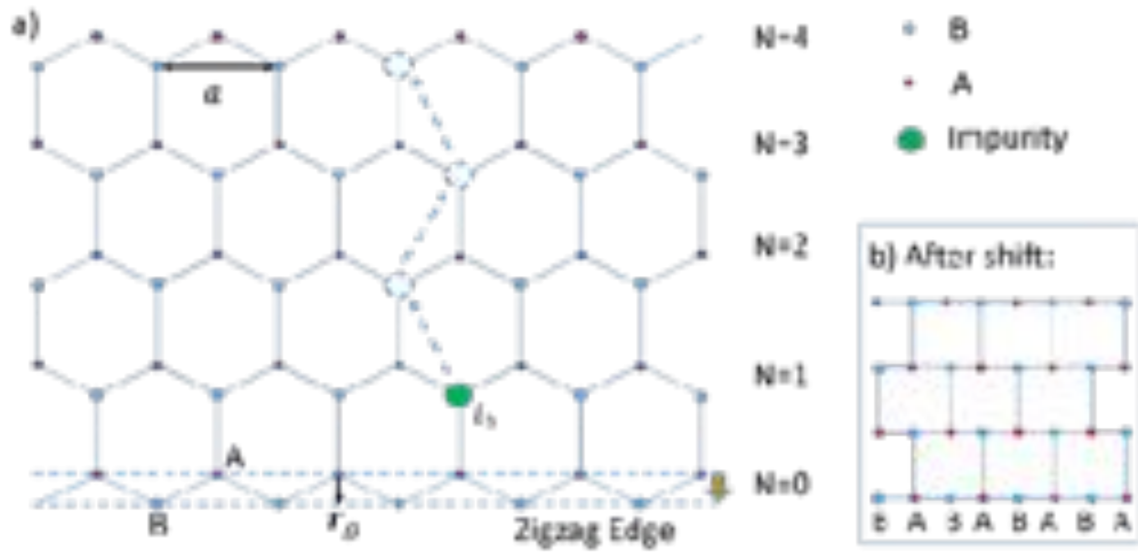
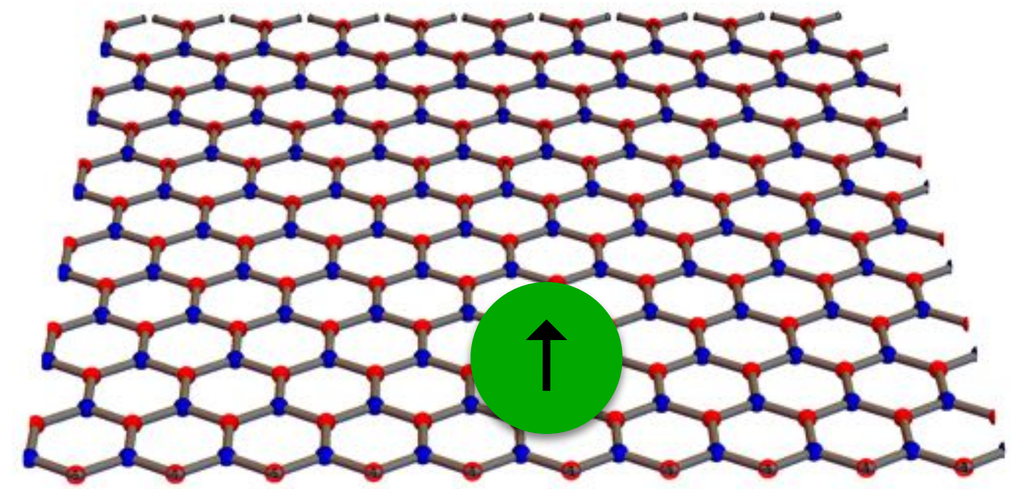
Backsatterer in a non interacting TI

Semi-infinite Kane-Mele model (with zigzag edge) + TRS breaking impurity (backscatter)





$$H_0 = H_{KM} = -t \sum_{\langle i,j \rangle} c_i^\dagger c_j + i\lambda_{\text{SO}} \sum_{\langle\langle i,j \rangle\rangle} \nu_{ij} c_i^\dagger s^z c_j$$

No interactions: Solve Scattering problem (Lippmann-Schwinger equation)

$$|\Psi_s(k_x)\rangle = |\Phi_s(k_x)\rangle + \mathcal{V}_{\text{int}} G_0(\epsilon) |\Psi_s(k_x)\rangle \qquad \mathcal{V}_{\text{imp}} = \lambda_{\text{imp}}\, s^x$$

$$\mathcal{H}_0^s(k_x, \hat{\beta})\Phi_s(k_x, y) = \epsilon \Phi_s(k_x, y),$$

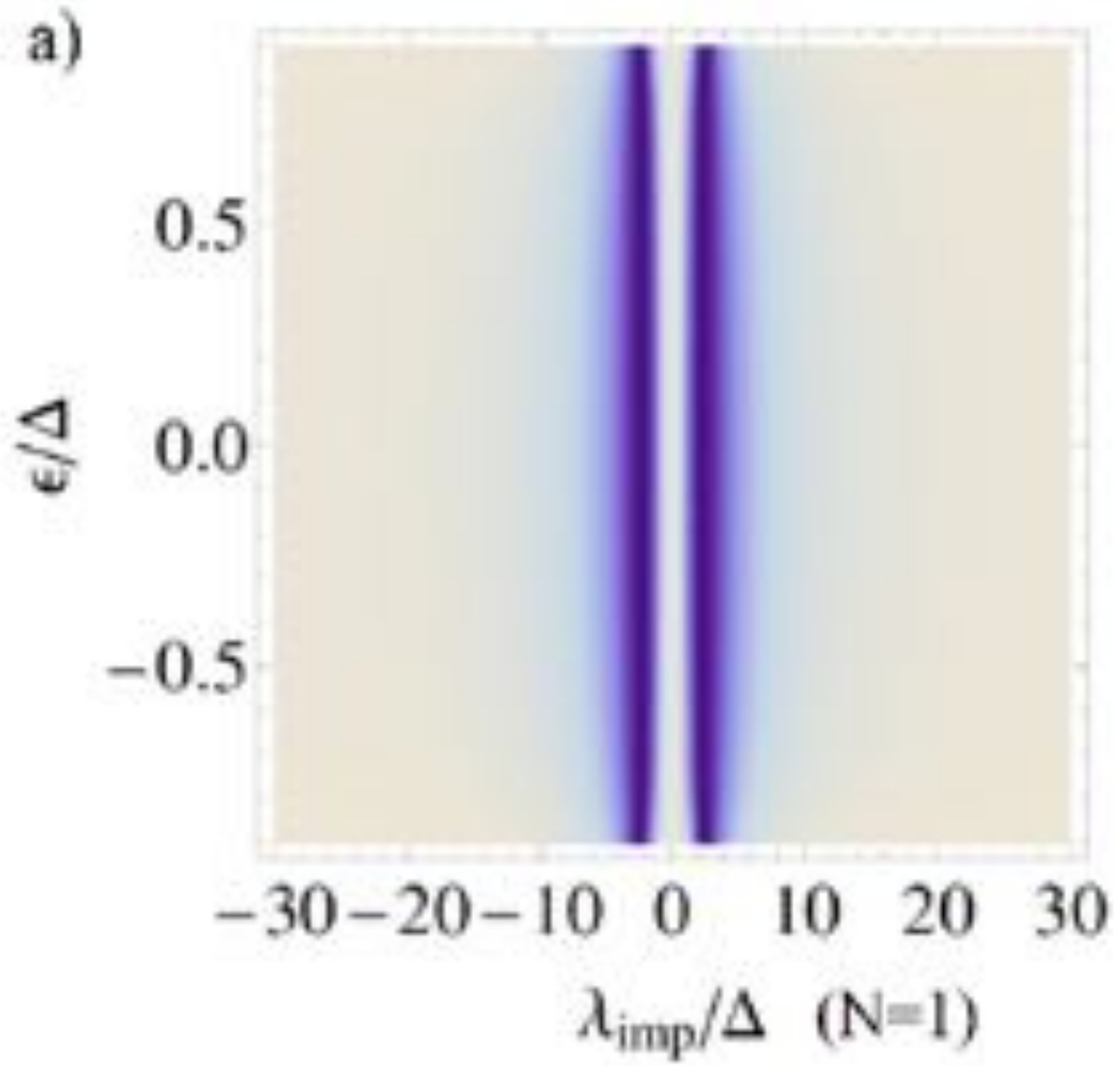
Spectrum of the pristine system

Look for solutions of the form

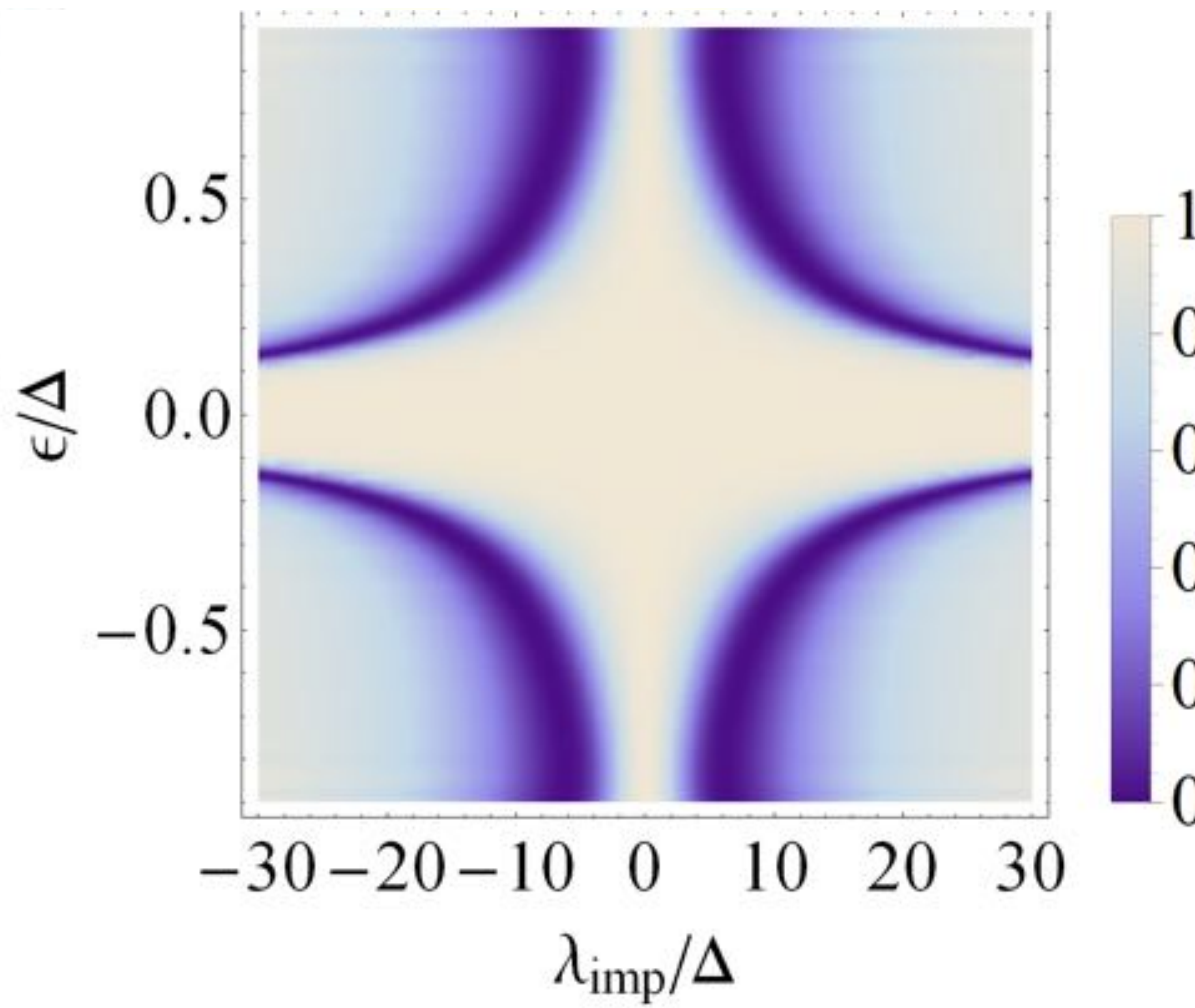
$$\Phi_s(k_x, y) \sim e^{-\kappa y}$$

Transmission coefficient

Impurity on 1st atomic row



Beyond 1st row

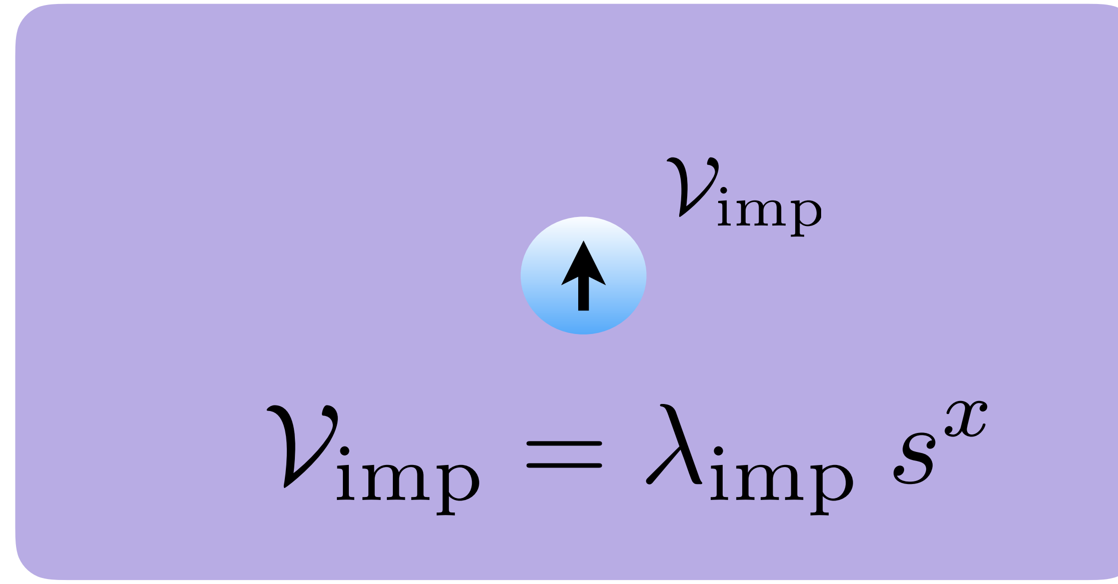


No coupling to bulk states
(zero weight on the 1st atomic row)
Indendent of the energy (like 1D system)

Energy dependent scattering

What is going on?

Impurity in the bulk of the crystal

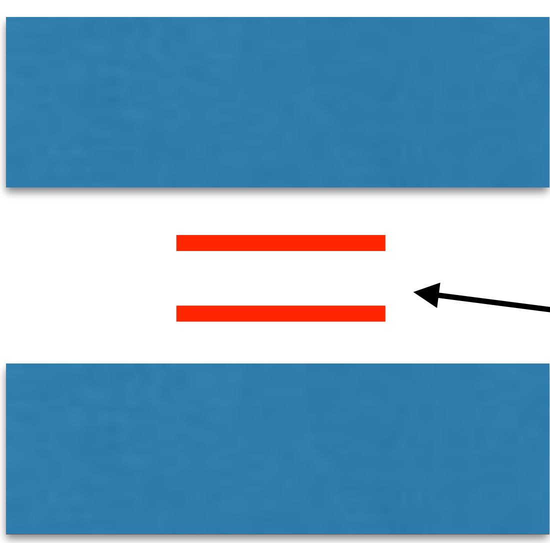


Lippmann-Schwinger Equation

$$T(\epsilon) = \mathcal{V}_{\text{imp}} + \mathcal{V}_{\text{imp}} G_0(\epsilon) T(\epsilon)$$

$$T(\epsilon) = \frac{\mathcal{V}_{\text{imp}}}{1 - G_0(\epsilon)\mathcal{V}_{\text{imp}}}$$

Bound state? $\quad 1 - G_0(\epsilon)\mathcal{V}_{\text{imp}} = 0 \qquad \epsilon \in \text{Reals}$



Bulk Green's function (particle-hole symmetric)

$$G_0(\epsilon) \sim \epsilon \quad |\epsilon| < \frac{\Delta}{2} \quad (\text{Im}\, G_0(\epsilon) = 0)$$

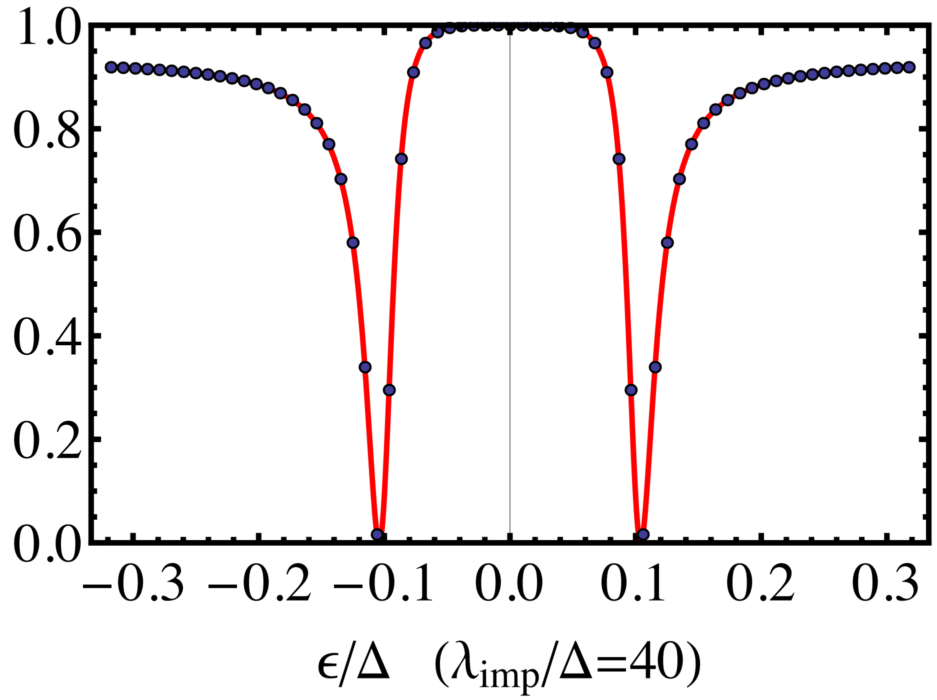
Bound state?

$$1 - G_0(\epsilon)\mathcal{V}_{\text{imp}} = 0 \Rightarrow 1 - g_0 \lambda_{\text{imp}}\, \epsilon\, s^x = 0$$

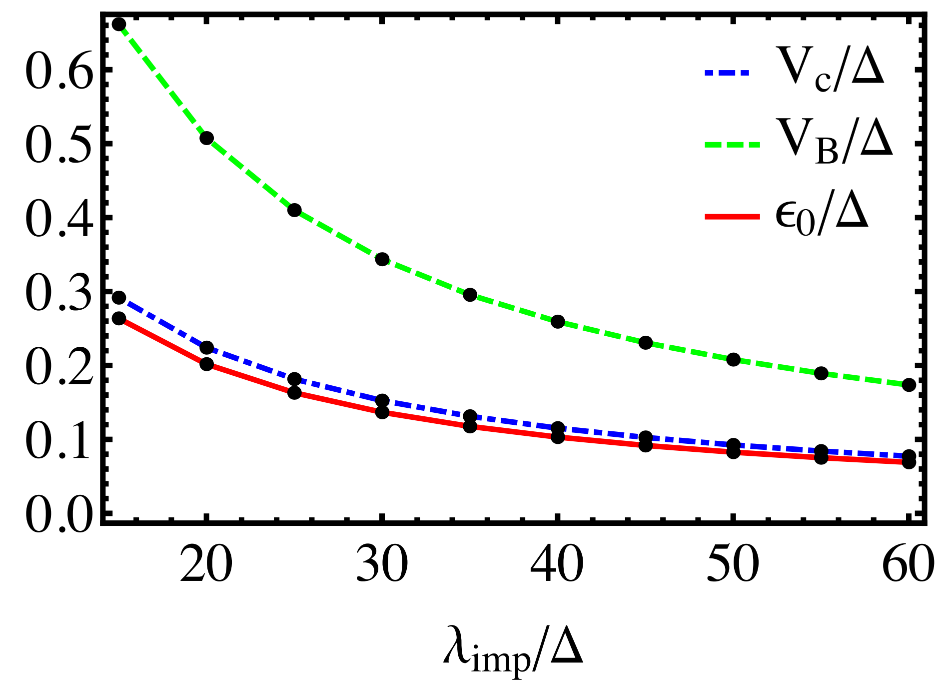
$$\epsilon_{\text{Bound}} = \epsilon_0 \sim \pm \frac{1}{\lambda_{\text{imp}}}$$

Fitting an effective low energy model

Transmission coefficient



Model parameters (N = 2)



Use discrete symmetries of the microscopic model
(TRS + π-rotation, TRS+p-h transformation)

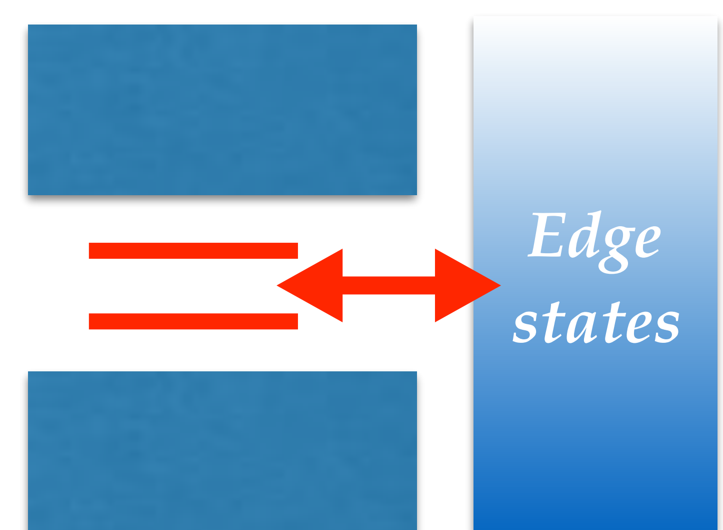
$$H_{\rm eff} = H_B + H_+\left[u, t_+\psi(0)\right] + H_-\left[d, t_-\psi(0)\right],$$

$$H_B = iv_F \int dx\, \psi^\dagger s^z \partial_x \psi + V_B a_0 \psi^\dagger(0) s^x \psi(0),$$

$$H_\pm[f,\chi] = \pm\epsilon_0\ \left(f^\dagger f - \tfrac{1}{2}\right) + V_c a_0^{1/2}\left[f^\dagger \chi + {\rm h.c.}\right]$$

Two-level Fano model

Bound states resonate with edge



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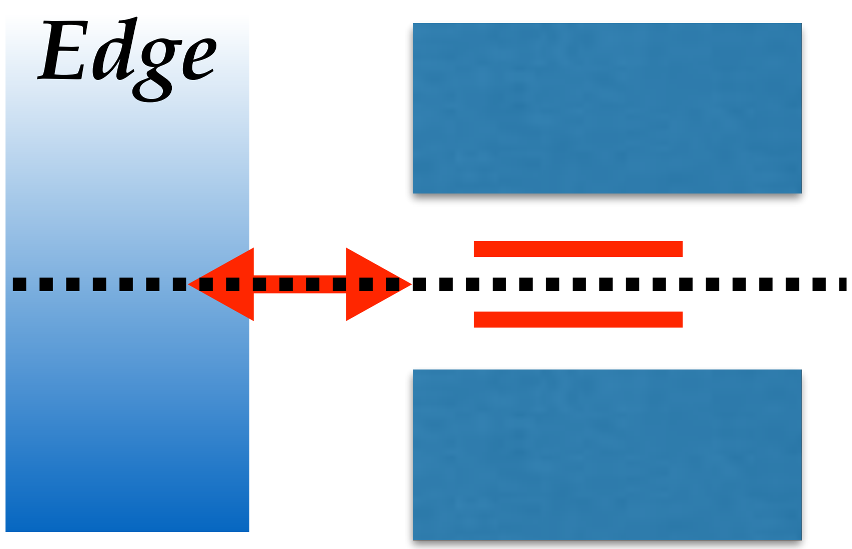
Adding interactions



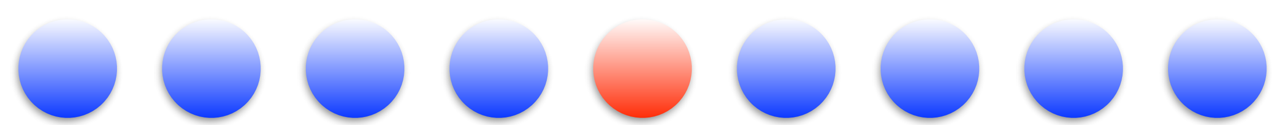
Non-resonant case = Kane-Fisher

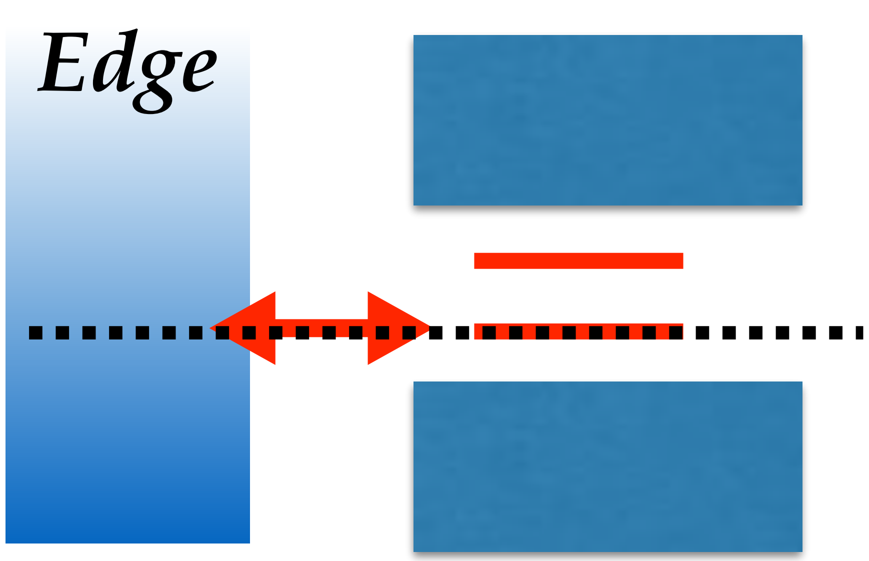
$\epsilon_F \neq \pm\epsilon_0$ *Integrate out both bound states*

$$V_B' \simeq V_B - \left[\frac{V_c^2}{\epsilon_0 - \epsilon_F} + \frac{V_c^2}{\epsilon_0 + \epsilon_F}\right]$$

$$\epsilon_0 \propto V_B'$$

Back to Kane and Fisher

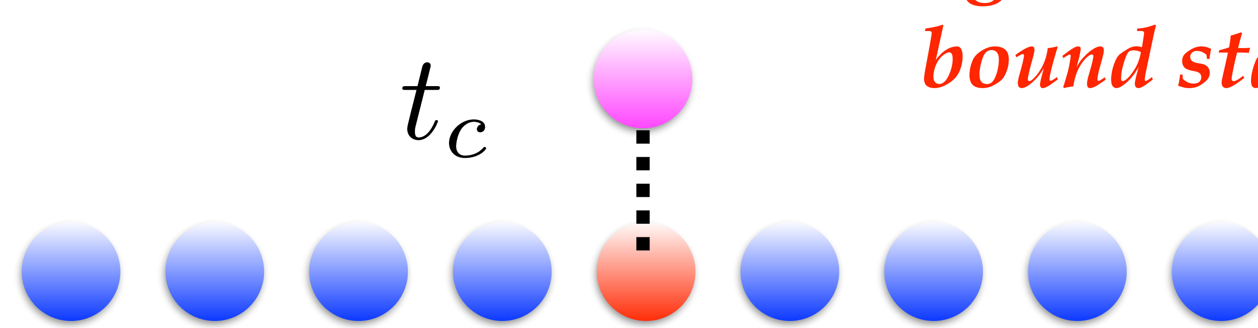




$\epsilon_F = \pm\epsilon_0$

Resonant case

Integrate out one bound state



$$\epsilon_0 \propto V_B'$$

Adding interactions: RG Flow

$$\frac{dy_B}{d\ln\xi} = (1-K)\,y_B + y_t^2.$$

$$\frac{dy_t}{d\ln\xi} = \left[1 - K/4 - (1-\delta_F)^2 K^{-1}/4\right] y_t + y_t(y_B + v_B),$$

$$\frac{d\delta_F}{d\ln\xi} = 4(1-\delta_F)y_t^2,$$

$$\frac{dv_B}{d\ln\xi} = (1-K)v_B.$$

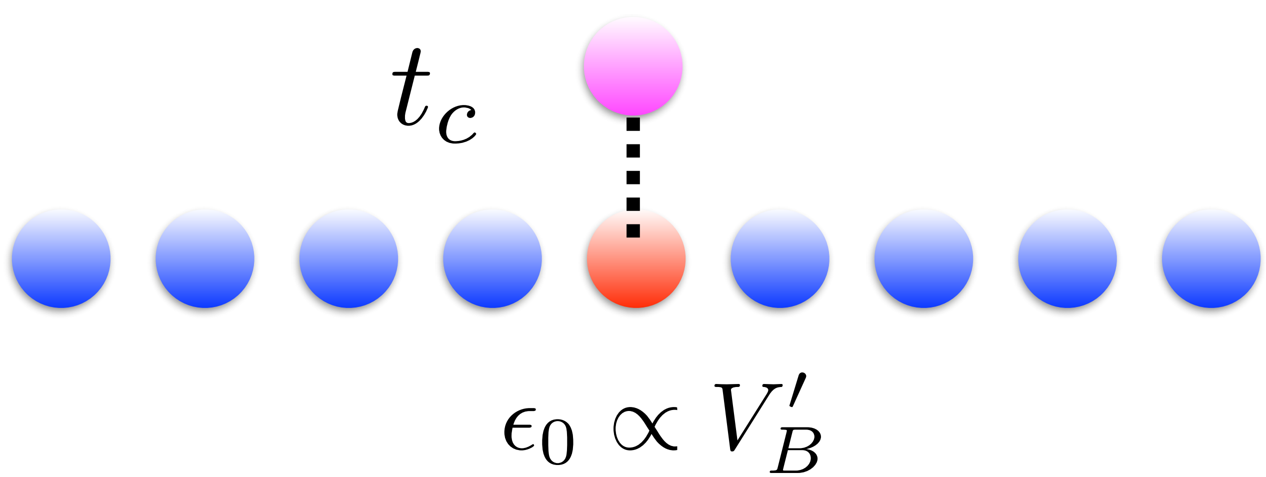
Compare to Kane-Fisher

$$\frac{dy_B}{d\log\xi} = (1-K)y_B$$

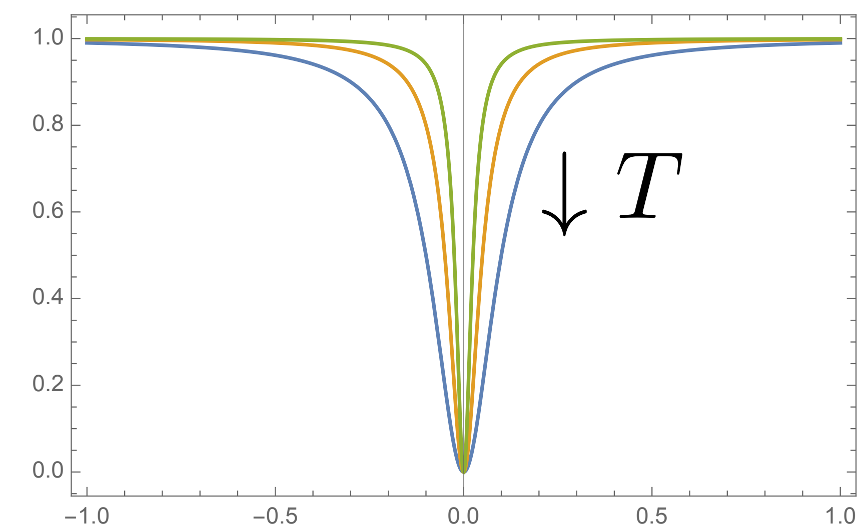
Tunneling flows to strong coupling also for moderately attractive interactions!!

M Goldstein and R Berkovits Phys Rev Lett (2011)

Side-coupled resonant level



Broadening of transmission resonance at low T



Summary & Conclusions

- Resonant enhancement of Skew Scattering in Graphene
- Direct coupling between non equilibrium spin polarization and charge current induced by impurities: Anisotropic spin precession (ASP)
- ASP can lead to negative non local resistance and asymetry in the Hanle precession
- Strong coupling limit of a magnetic impurity near the edge of a QSHI induces resonant states and non-trivial behavior of the transmission at resonance

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