

# Estimation of Pseudo Magnetic Field for Isotropic/Anisotropic Dirac Cones

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2 Nov 2017

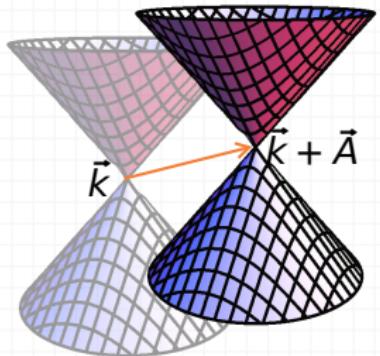
arXiv:1707.08601

# Motivation & Background

Landau levels without an external magnetic field.

## Essence

Dirac cones shift  
as gauge field

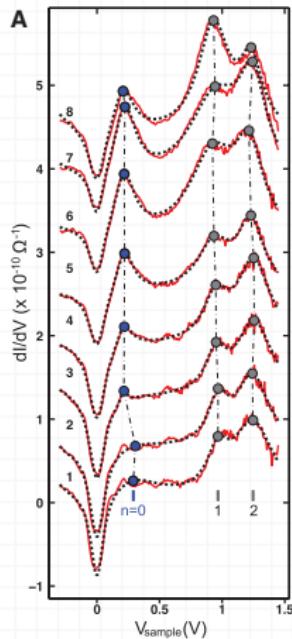
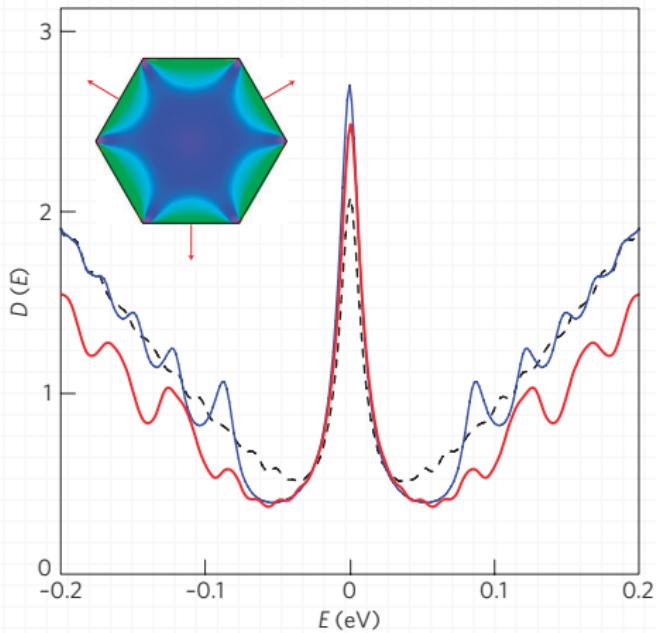


Any system with Dirac cones!

*Even for a system inert to magnetic field: charge neutral particles, photons, phonons...*

# Example: Graphene under Strain

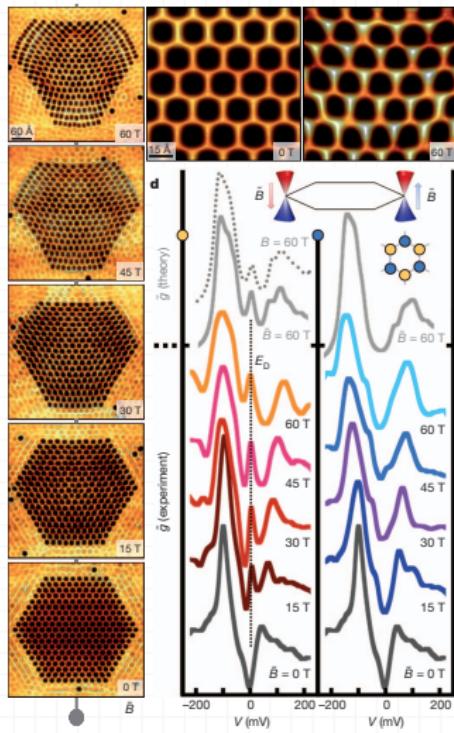
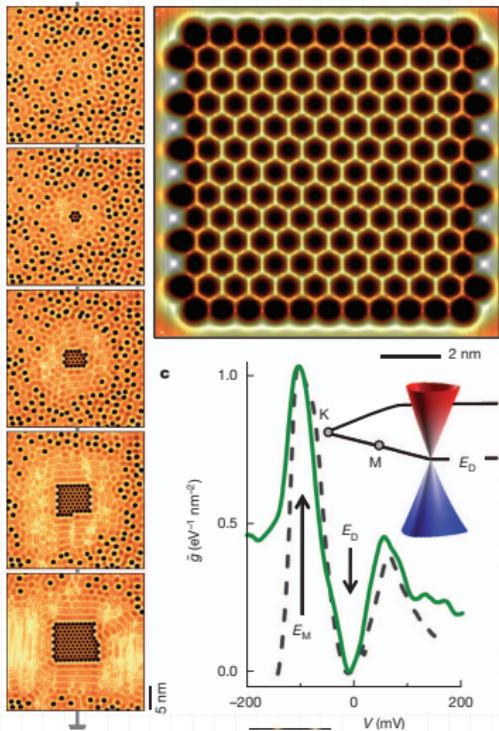
<sup>†</sup>Theory: F. Guinea *et al.*, Nat. Phys. **6**, 30 (2010). Exp.: N. Levy *et al.*, Science **329**, 544 (2010).



# Example: Artificial System

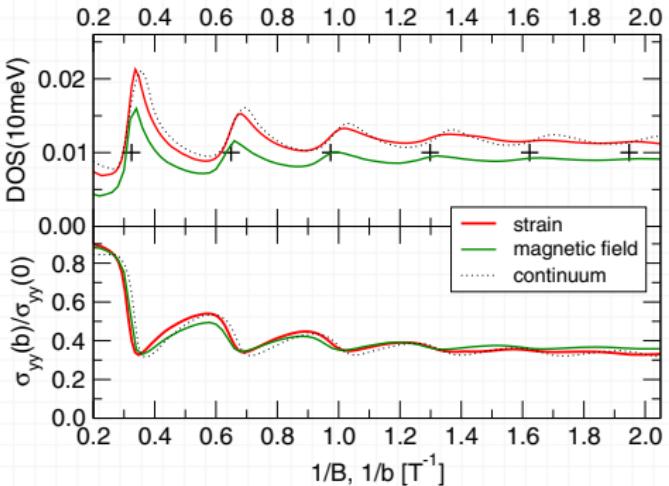
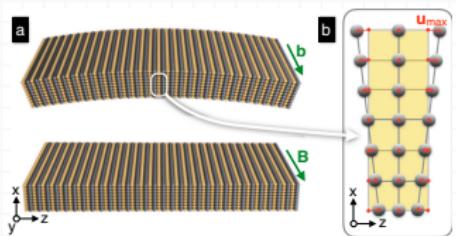
K. K. Gomes *et al.*, Nature 483, 306 (2012).

2D electrons on Cu surface with arranged molecule deposition



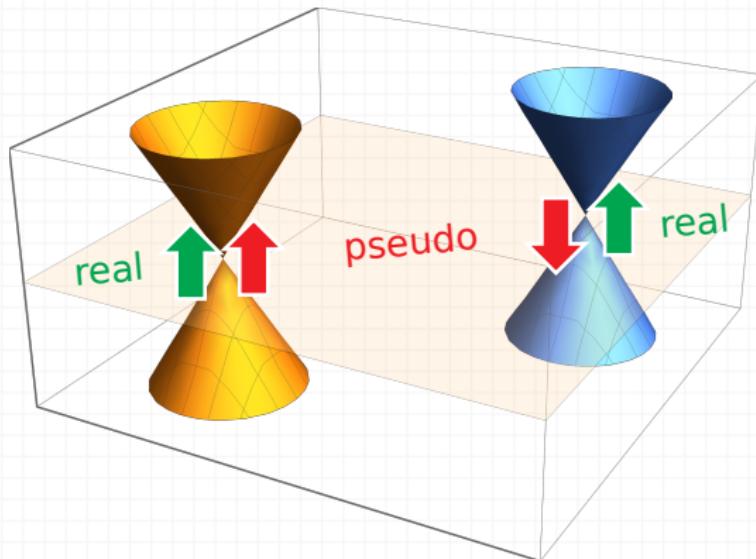
# Quantum Oscillation

## Strained 3D Weyl semimetal



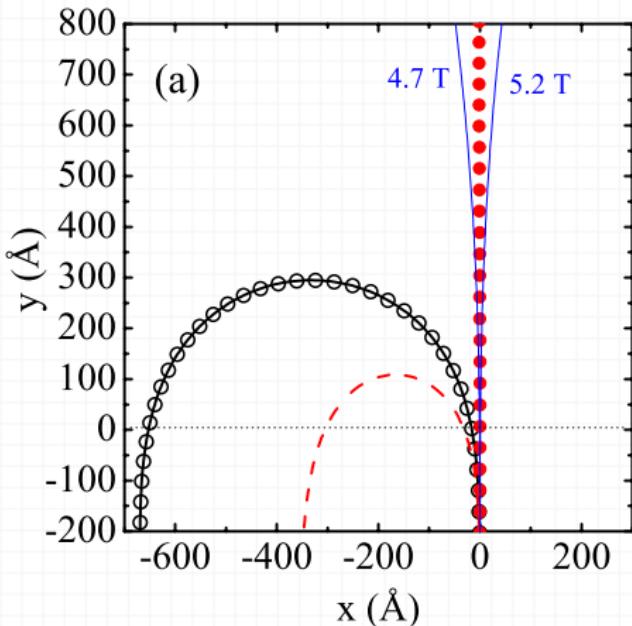
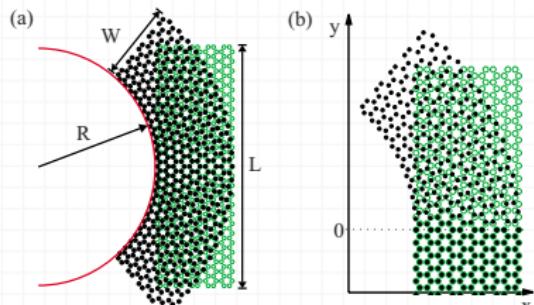
T. Liu, D. I. Pikulin, and M. Franz, Phys. Rev. B 95, 041201 (2017).

# Valley Imbalance



# Valley Imbalance

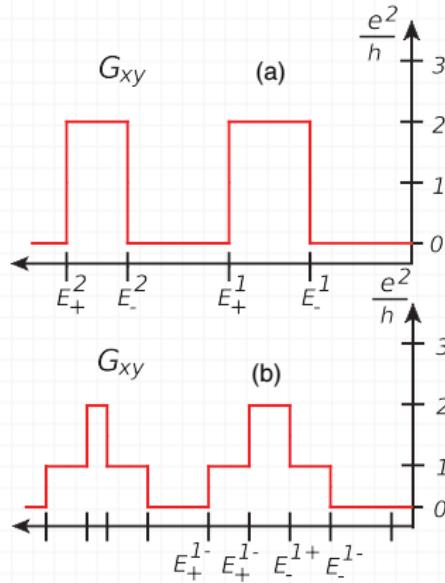
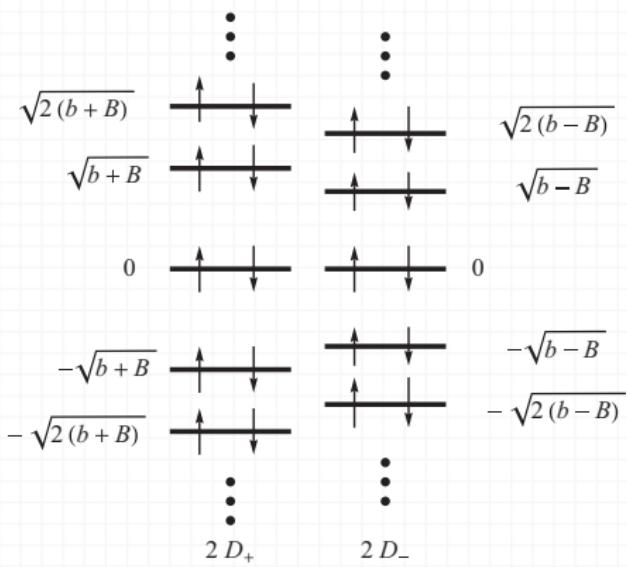
Valley dependent Lorentz force in strained graphene



A. Chaves *et al.*, Phys. Rev. B **82**, 205430 (2010).

# Valley Imbalance

## Landau level splitting in strained graphene



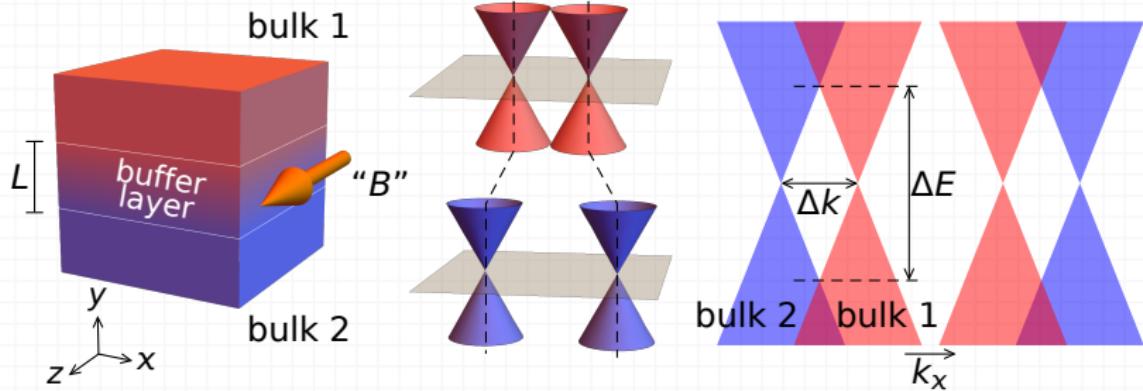
B. Roy, Z.-X. Hu, and K. Yang, Phys. Rev. B 87, 121408 (2013).

# Topic

1. Simple setup for pseudo magnetic field generation
  - ▶ not necessary strain
2. Concise formula to estimate pseudo magnetic field
  - ▶ Counting number of “observable” Landau levels
  - ▶ Effects of anisotropy of Dirac cones
3. Application to an existing material
  - ▶ 3D Dirac cones in an antiperovskite family

# Setup

## “Simplest” configuration



## Important Parameters

- ▶  $L$ : thickness of the buffer layer
- ▶  $\Delta k$ : size of the Dirac cone shift

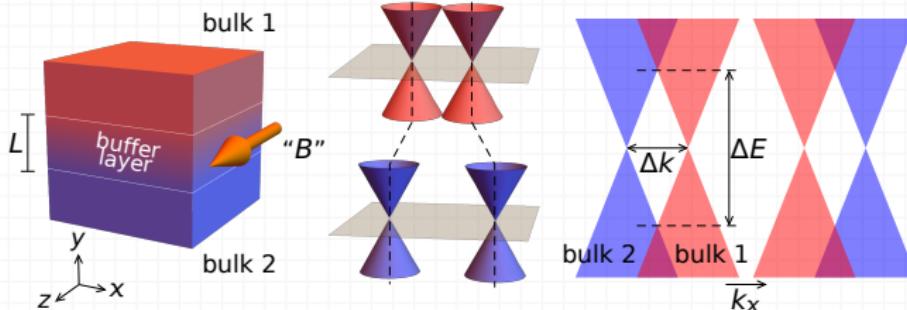
See also, A. G. Grushin *et al.*, Phys. Rev. X **6**, 041046 (2016). C. Brendel *et al.*, Proc. Natl. Acad. Sci. USA **114**, 3390 (2017). H. Abbaszadeh *et al.*, arXiv:1610.06406.

# Formulation

$$H_{\vec{k}}^{(\pm)} = \hbar v (\vec{k} \pm \vec{k}_0) \cdot \vec{\sigma} \longleftrightarrow H^{(\pm)} = \hbar v (-i\vec{\nabla} \pm \vec{k}_0(y)) \cdot \vec{\sigma}$$

$$\vec{A}^{(\pm)} = \mp \frac{\hbar}{e} \vec{k}_0(y), \quad |\vec{B}| = |\vec{\nabla} \times \vec{A}| \sim \frac{\hbar \Delta k}{e L} = \frac{h}{ea^2} \frac{R}{N}$$

$$\Delta k = \frac{2\pi R}{a}, \quad L = Na$$



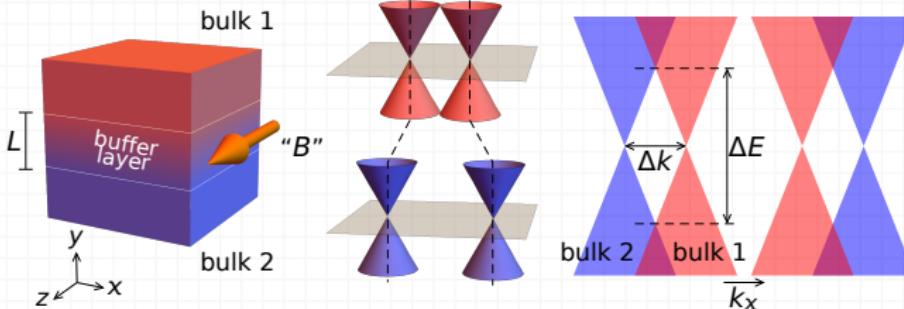
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$R$ : Dirac cone shift,  $N$ : buffer thickness



# Formulation

$R$ : Dirac cone shift,  $N$ : buffer thickness

- ▶ typical case

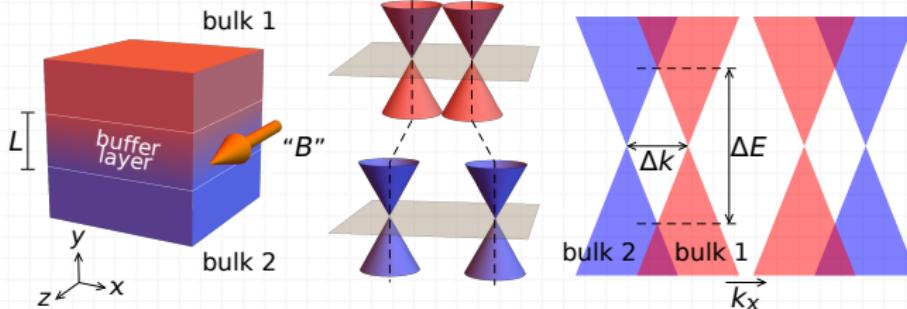
$$a \sim 5\text{\AA}$$

$\rightarrow$

$$|\vec{B}| \sim 1.6 \times 10^4 \times \frac{R}{N} [\text{T}]$$

- ▶ observable Landau levels

$$E_n = \sqrt{\frac{4\pi\nu^2\hbar^2R|n|}{Na^2}} < \frac{\hbar\nu\Delta k}{2} \quad \rightarrow \quad |n| < \frac{\pi}{4}NR$$



# Toy Model

$$H_{\vec{k}} = [1 + \delta + 2(\cos k_x + \cos k_y)]\sigma_z + 2\alpha \sin k_y \sigma_y$$



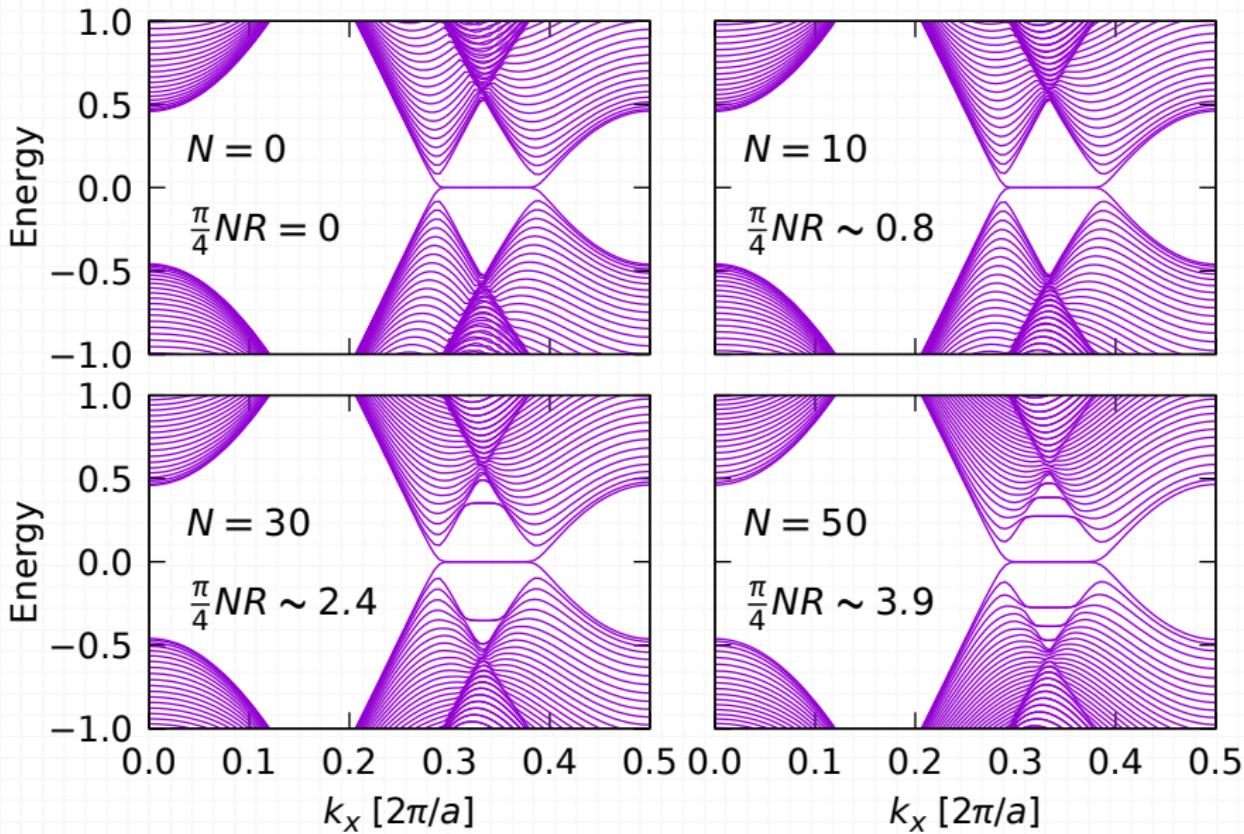
$$H_{\vec{k}} \sim -\sqrt{3}[(\tilde{k}_x - \tilde{\delta})\sigma_z + \tilde{\alpha}k_y\sigma_y]$$

$$\tilde{k}_x = k_x - \frac{2\pi}{3}, \quad \tilde{\delta} = \frac{\delta}{\sqrt{3}}, \quad \tilde{\alpha} = \frac{2\alpha}{\sqrt{3}}$$

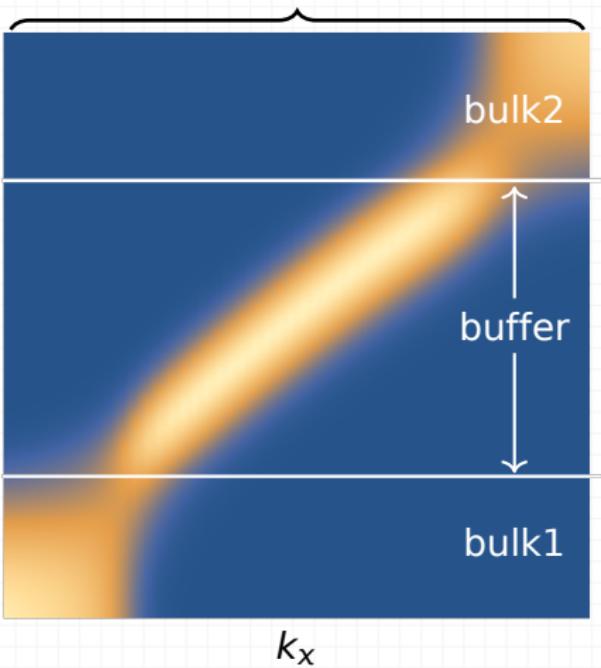
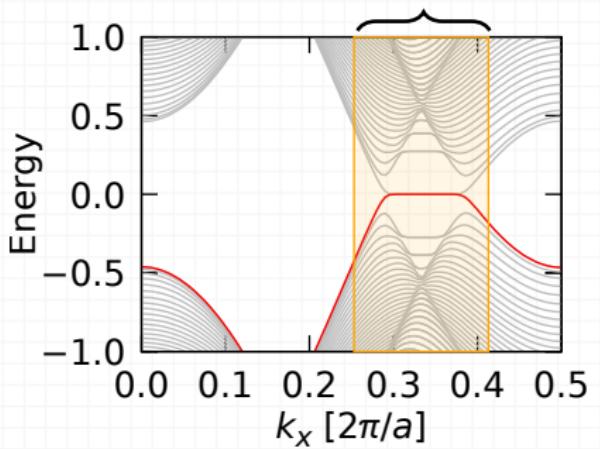
$$\tilde{\delta} \leftrightarrow A_x \quad \& \quad \tilde{\alpha} \leftrightarrow v_y/v_x$$

# Results

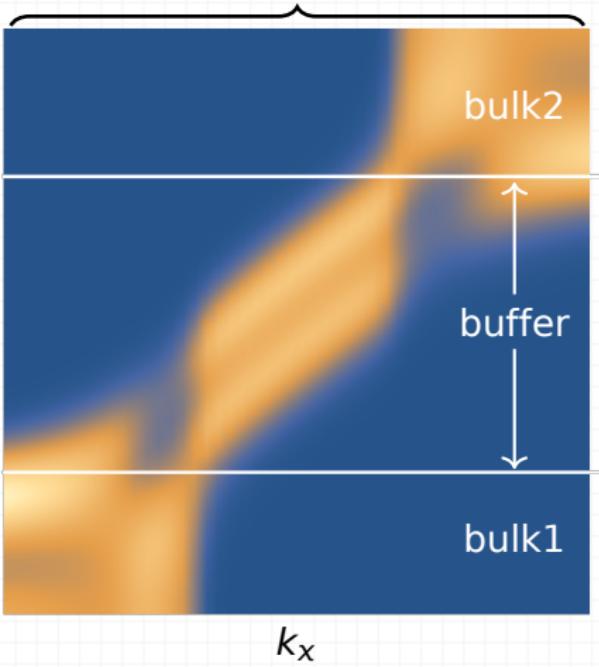
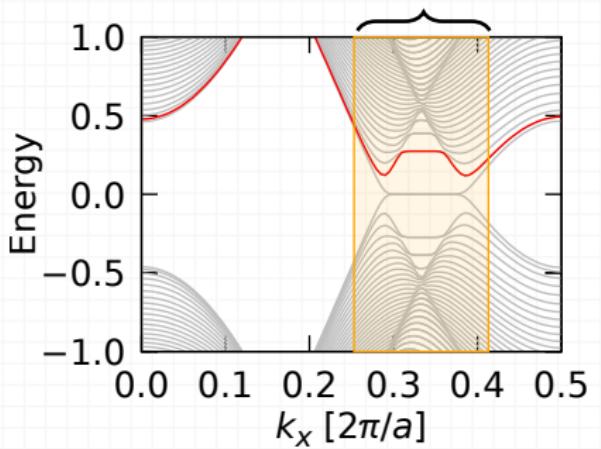
*R*: Dirac cone shift, *N*: buffer thickness



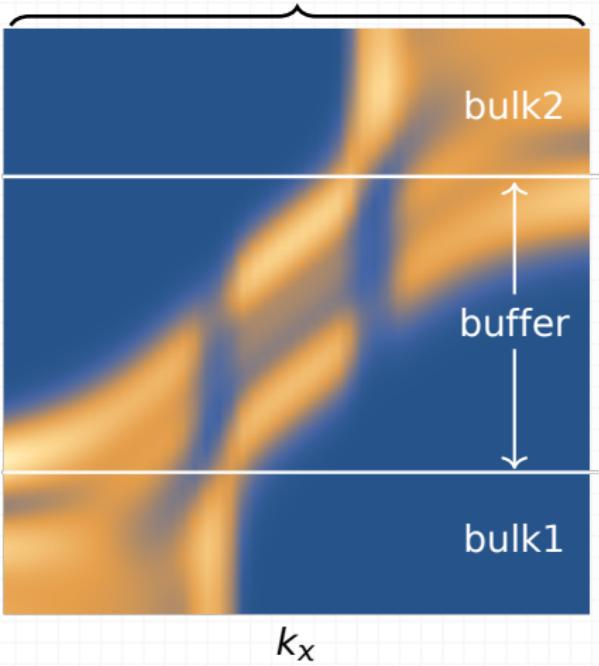
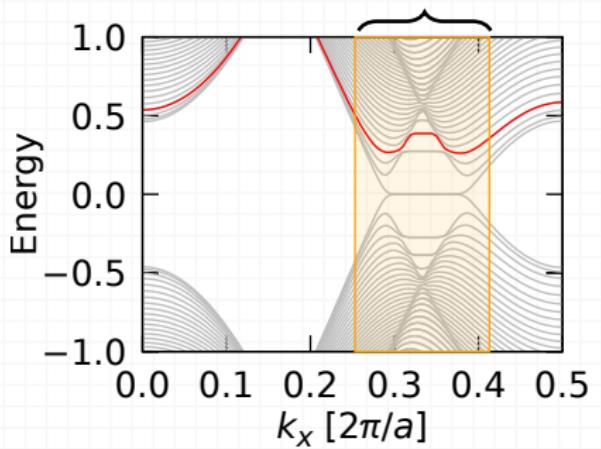
## Discussion



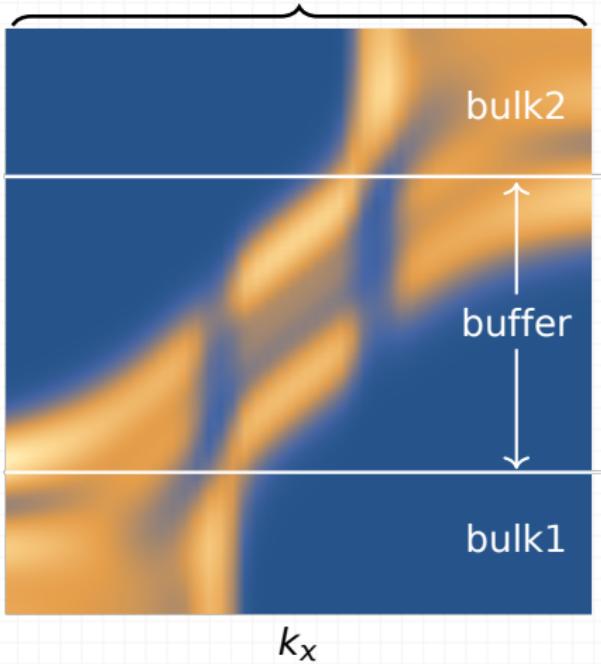
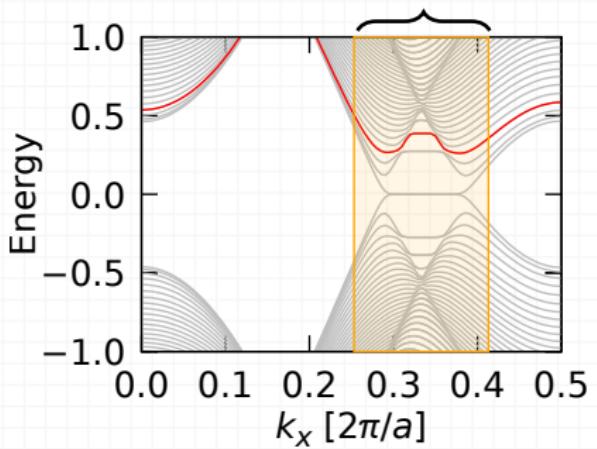
## Discussion



## Discussion



## Discussion



wave function tail hits the boundary → no longer Landau level

- extension of the wave function  $\sim \sqrt{n}l_B \propto \sqrt{nN/R}$
- extension < thickness  $\rightarrow n < NR$

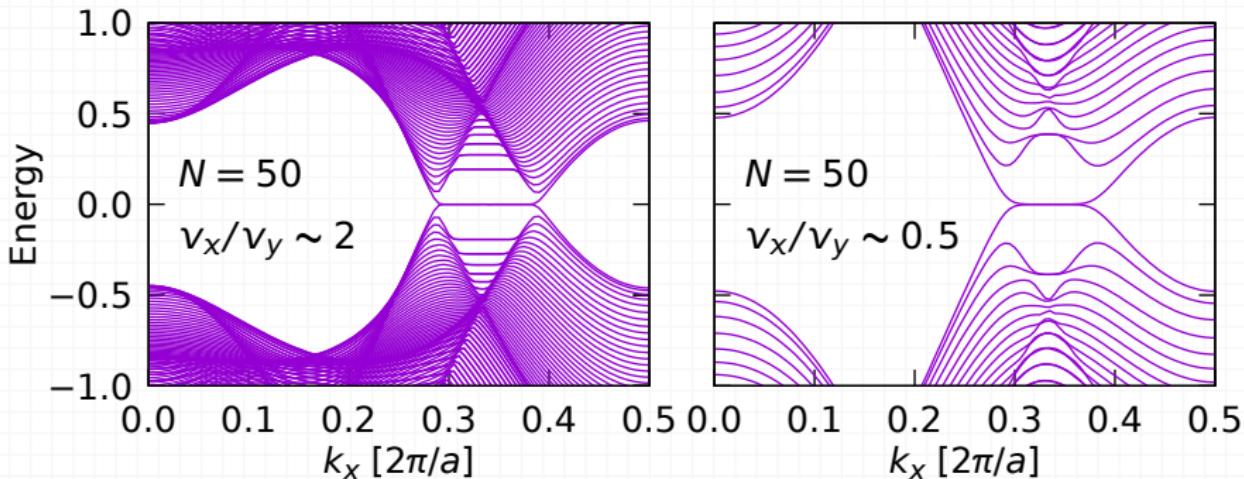
# Anisotropy

$R$ : Dirac cone shift,  $N$ : buffer thickness

$$\Delta E = \hbar v_x \Delta k$$

$$E_n = \sqrt{\frac{4\pi v_x v_y \hbar^2 R |n|}{Na^2}}$$

$$|n| < \frac{\pi v_x}{4 v_y} NR$$

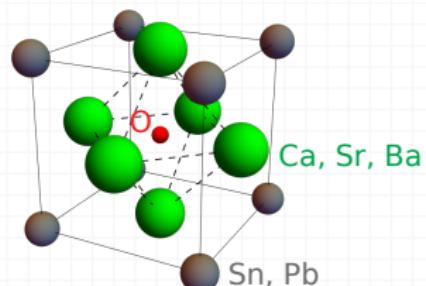
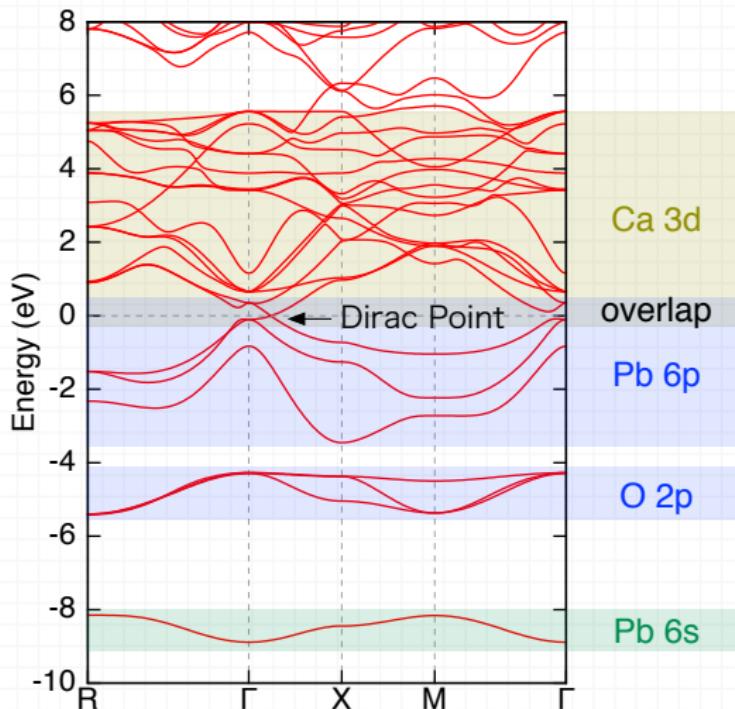


Anisotropy is advantageous for observing the LL structure!

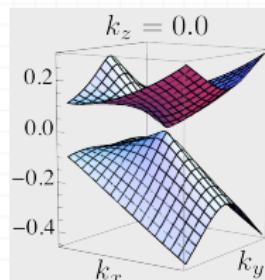
# Materials

TK and M. Ogata, J. Phys. Soc. Jpn. **80**, 083704 (2011).

- Antiperovskite  $A_3EO$  ( $A=Ca,Sr,Ba$  and  $E=Sn,Pb$ ) family



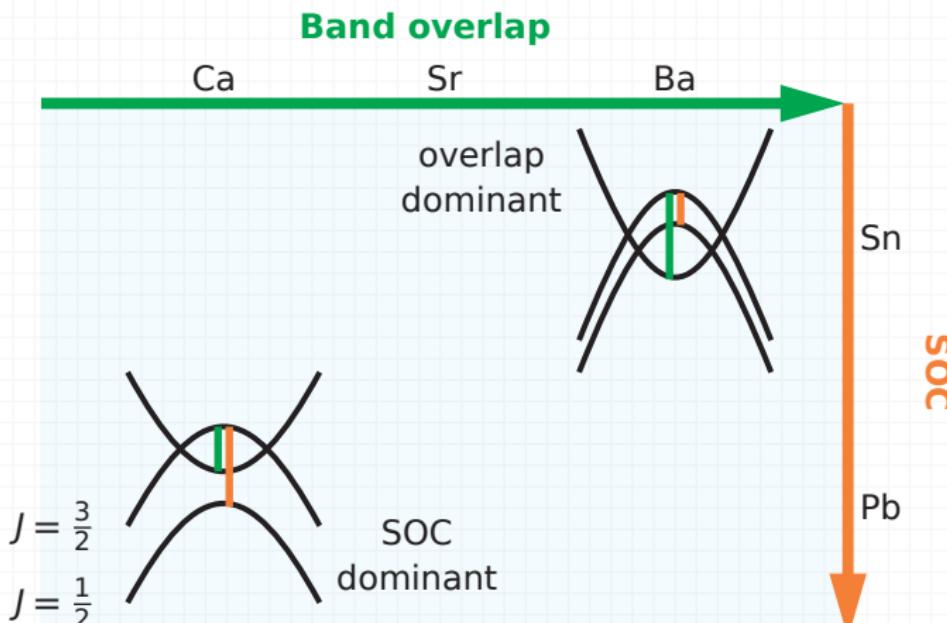
3D Dirac cones (with tiny mass) by d-p overlap!



3D linear dispersion

# Materials

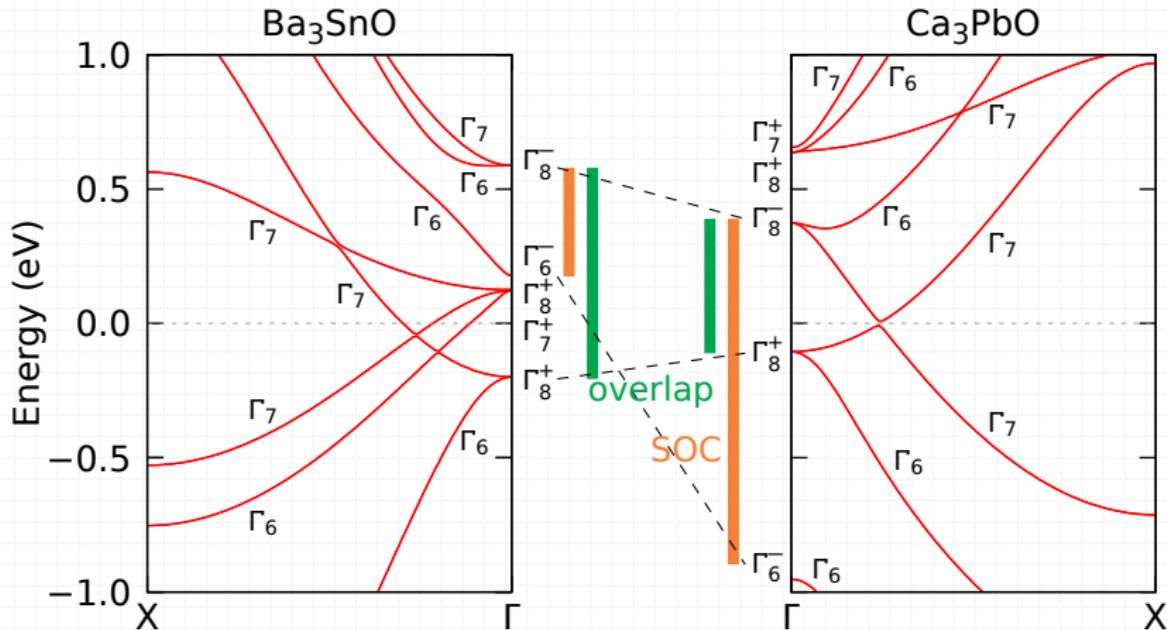
TK and M. Ogata, arXiv:1705.08934, to appear in PRMaterials.



(Potentially) tunable!

## Materials

- $\text{Ba}_3\text{SnO}$  (band inversion dominant) vs  $\text{Ca}_3\text{PbO}$  (SOC dominant)

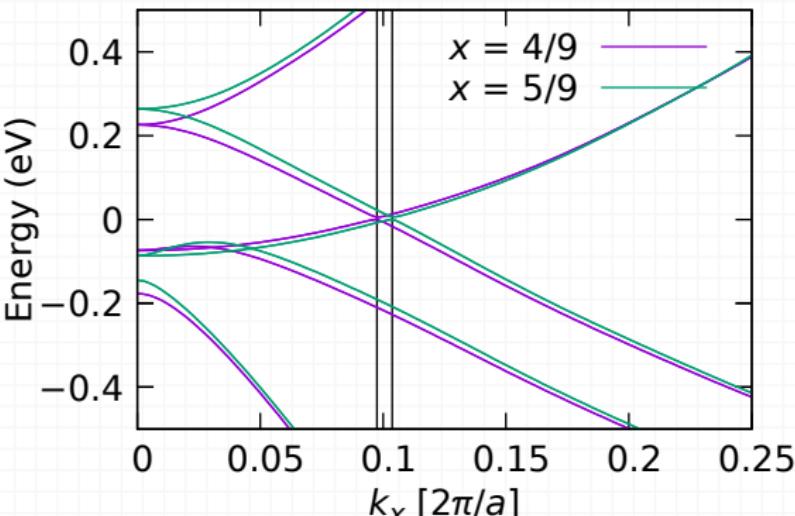


## Strategy

- ▶ Inducing Dirac cone shift by modulating chemical composition
  - ▶  $\text{Ca}_3\text{SnO} \leftrightarrow \text{Sr}_3\text{SnO}$
- ▶ Estimating  $R$  instead of  $|B_{\text{pseudo}}|$ , to avoid computational burden

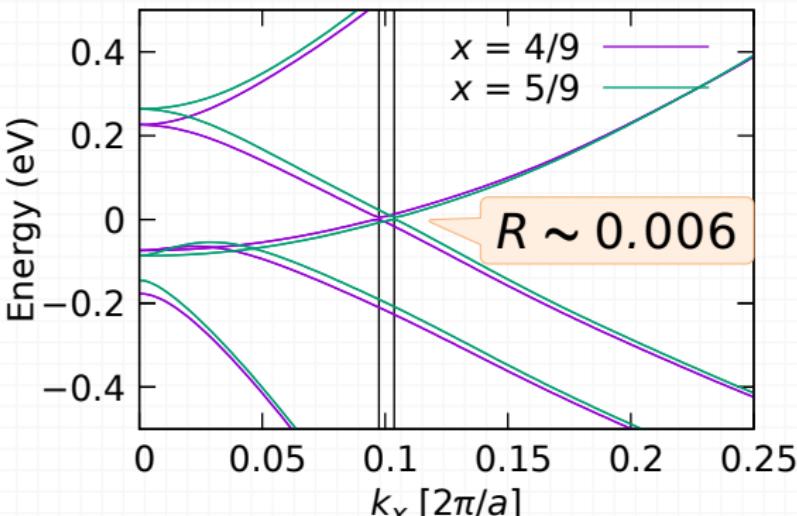
## (Quasi) Ab-Initio Estimation: Wannier Interpolation

1. Derive effective models for the two end materials  $\text{Ca}_3\text{SnO}$  and  $\text{Sr}_3\text{SnO}$
2. Interpolate the parameters to obtain a model for  $\text{Ca}_3(1-x)\text{Sr}_{3x}\text{SnO}$



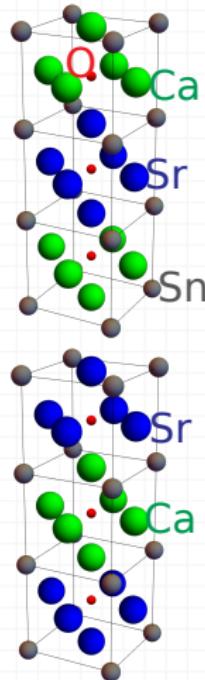
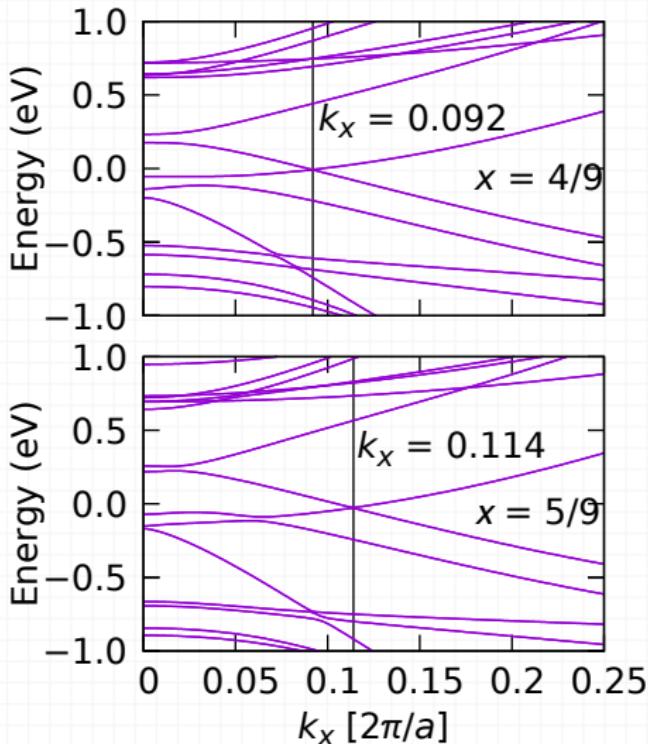
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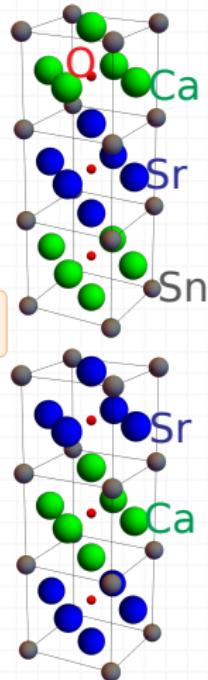
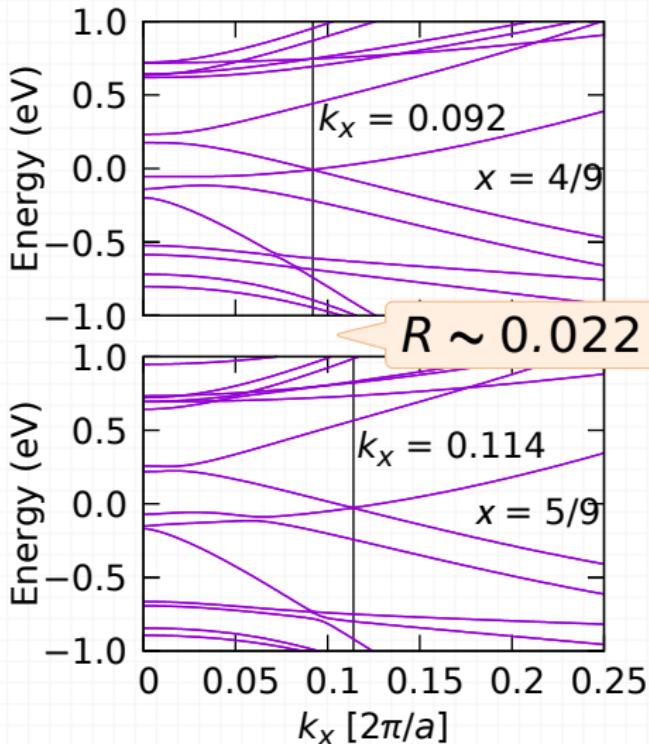
# (Quasi) Ab-Initio Estimation

- heterostructure  $\text{Ca}_3(1-x)\text{Sr}_{3x}\text{SnO}$ ,  $a = (a_{x=0} + a_{x=1})/2$



# (Quasi) Ab-Initio Estimation

- heterostructure  $\text{Ca}_3(1-x)\text{Sr}_{3x}\text{SnO}$ ,  $a = (a_{x=0} + a_{x=1})/2$



## Fabrication of Films

- ▶  $\text{Sr}_3\text{PbO}$ , molecular beam epitaxy, thickness 200nm-300nm  
*D. Samal, H. Nakamura, and H. Takagi, APL Mater. 4, 076101 (2016).*
  
- ▶  $\text{Ca}_3\text{SnO}$ , pulsed laser deposition  
*M. Minohara et al., arXiv:1710.03406.*

## Summary

TK, arXiv:1707.08601

- ▶ Concise formulae for the pseudo magnetic field & pseudo Landau levels

$$B \sim \frac{\hbar}{ea^2} \frac{R}{N}, \quad |n| < \frac{\pi v_x}{4 v_y} NR$$

- ▶ Anisotropic Dirac cones are better to observe LL structures.
- ▶ Estimation of  $R$  for an existing material

## Perspective

- ▶ Interesting physical consequences!
  - ▶ eg. coexistence with a *real* magnetic field