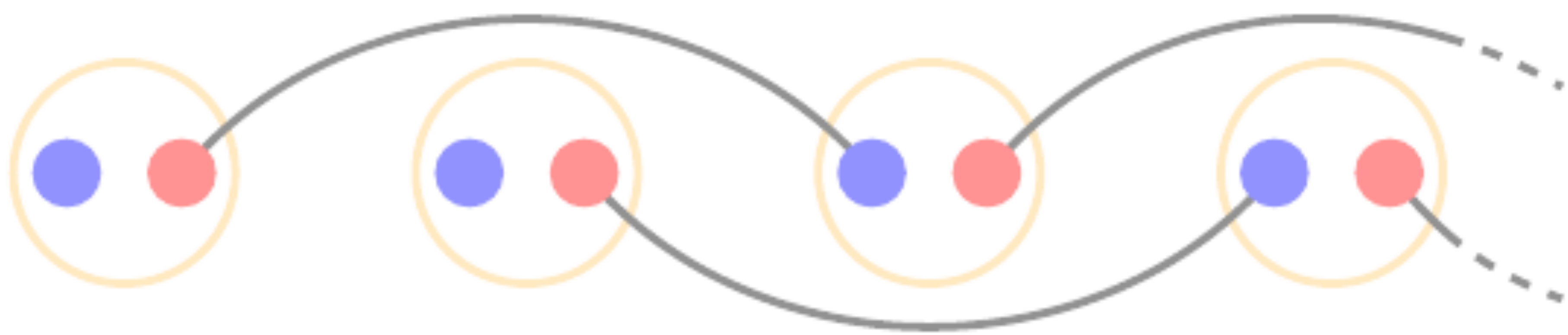
Topology and edge modes in quantum critical chains

Frank Pollmann

Technische Universität München

Verresen, Moessner, FP, arXiv:1707.05787
Verresen, Jones, FP, arXiv:1709.03508



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Novel Quantum States in Condensed Matter
Kyoto, Oct. 31st 2017

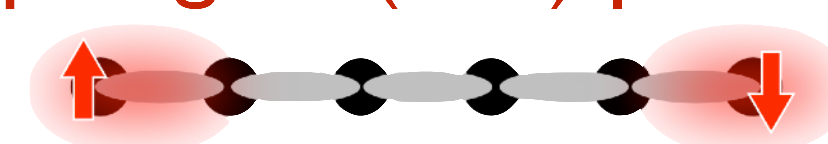
Topology and edge modes in quantum critical chains

Novel quantum phases of matter: No symmetry breaking and no local order parameters

- Integer quantum Hall  [Klitzing '80]
- Topological insulators [Kane & Mele '05]

- Fractional quantum Hall  [Tsui '82, Laughlin '83]
- Quantum spin-liquids [Anderson '73]
- **Haldane gap: Symmetry protected topological (SPT) phases**

   [Haldane '83]



Quantum critical properties of SPT phases

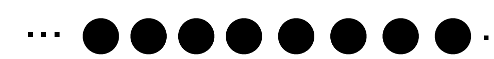
(1) Quantum critical points between SPTs?

(2) Critical SPT phases?

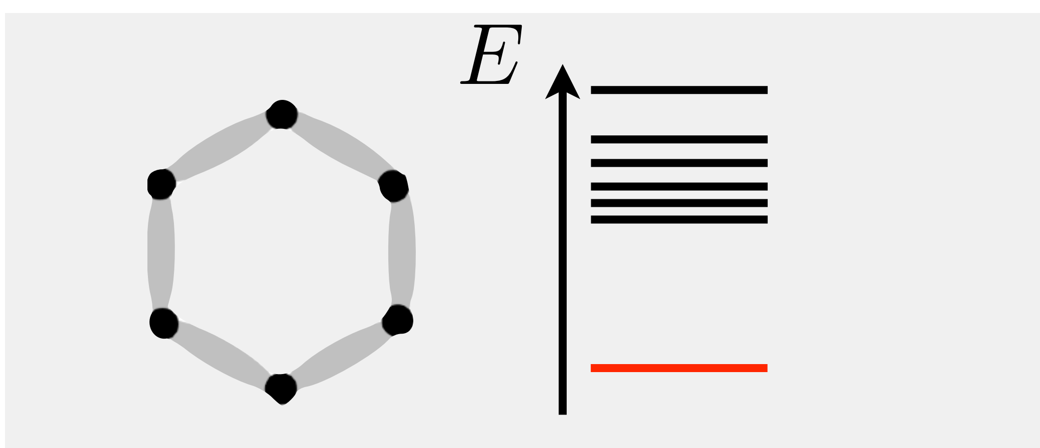
Symmetry protected topological phases

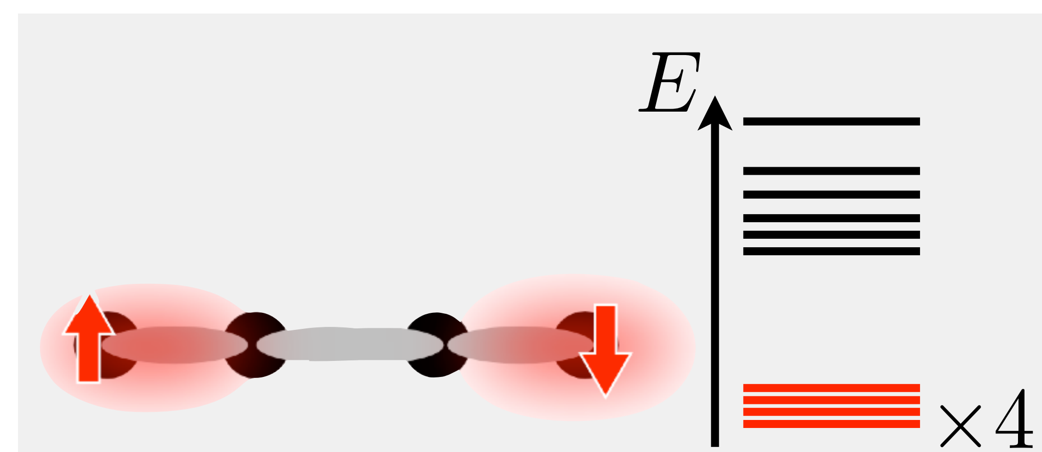
Spin-1 Heisenberg chain

$H = \sum_j \vec{S}_j \cdot \vec{S}_{j+1}$



- **Haldane phase:** Gapped and no symmetry breaking [Haldane '83]
- **Spin-1/2** excitations at the edges: Protected by symmetry [Affleck et al '87]





Complete classifications of (gapped) SPT phases in 1D: "Symmetry fractionalization"

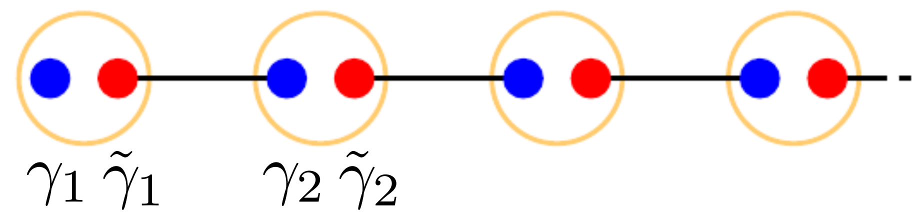
[FP, Turner, Berg, Oshikawa '10, Fidkowski, Kitaev '11, Turner, Berg, FP '11, Chen, Gu, Wen '11]

Symmetry protected topological phases

One dimensional chain of coupled Majoranas [Kitaev '01]

$$c = \frac{1}{2}(\gamma + i\tilde{\gamma}) \qquad \begin{matrix} \gamma^\dagger = \gamma & T\gamma T = \gamma \\ \tilde{\gamma}^\dagger = \tilde{\gamma} & T\tilde{\gamma}T = -\tilde{\gamma} \end{matrix}$$

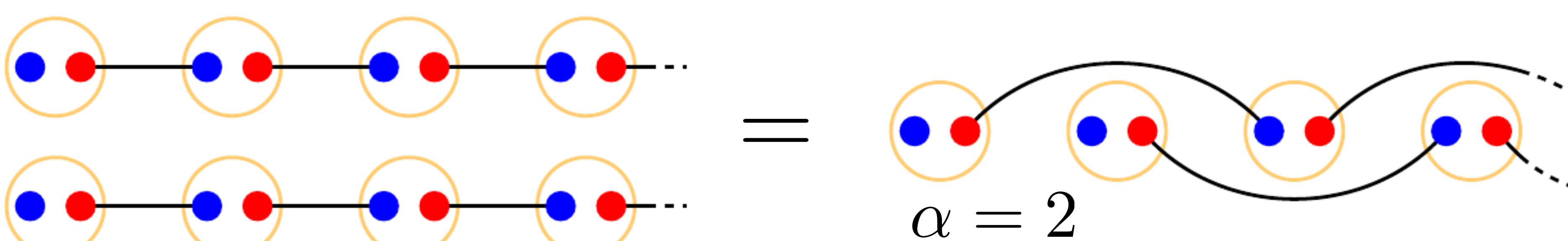
Majorana Mode →



$$H = \sum c_n^\dagger c_{n+1} + c_n^\dagger c_{n+1}^\dagger + h.c. \qquad = i \sum \tilde{\gamma}_n \gamma_{n+1}$$

Generalized "α - chain" (stacks of Kitaev chains)

$$H_\alpha = i \sum \tilde{\gamma}_n \gamma_{n+\alpha} + (T\text{-preserving interactions})$$



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Kitaev chain : BDI class

$\mathbb{Z}_8$ classification of the interacting BDI class
fermion parity (P) and time reversal (T)

[Fidkowski, Kitaev '11, Turner, Berg, FP '11, Chen, Gu, Wen '11]

	α	P	T	PT	total degeneracy
	-3	non-local fermion	left, (right)	[left], right	8
	-2		fermion on left	right	4
	-1	non-local fermion	(left)	[right]	2
Trivial →	0				1
Kitaev →	1	non-local fermion	[right]	(left)	2
SSH →	2		right	left	4
	3	non-local fermion	[left], right	left, (right)	8
Haldane→	4		left, right	left, right	4

Group structure: Adding stacks of chains

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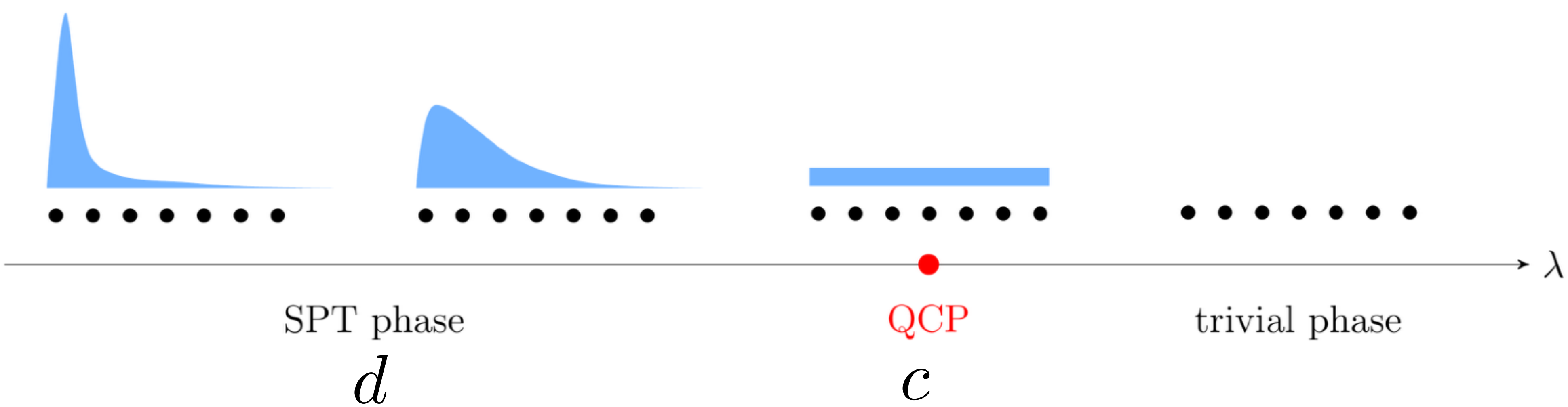
(1) Critical points between SPT phases

Transitions between SPT and trivial phases



Dimension (d) : # low-energy d.o.f. at one edge

Central charge (c) : # linearly dispersing modes

Examples: Haldane chain has $d = 2$ and $c = 1$
Kitaev chain has $d = \sqrt{2}$ and $c = 1/2$

Conjecture: $c \geq \log_2 d$

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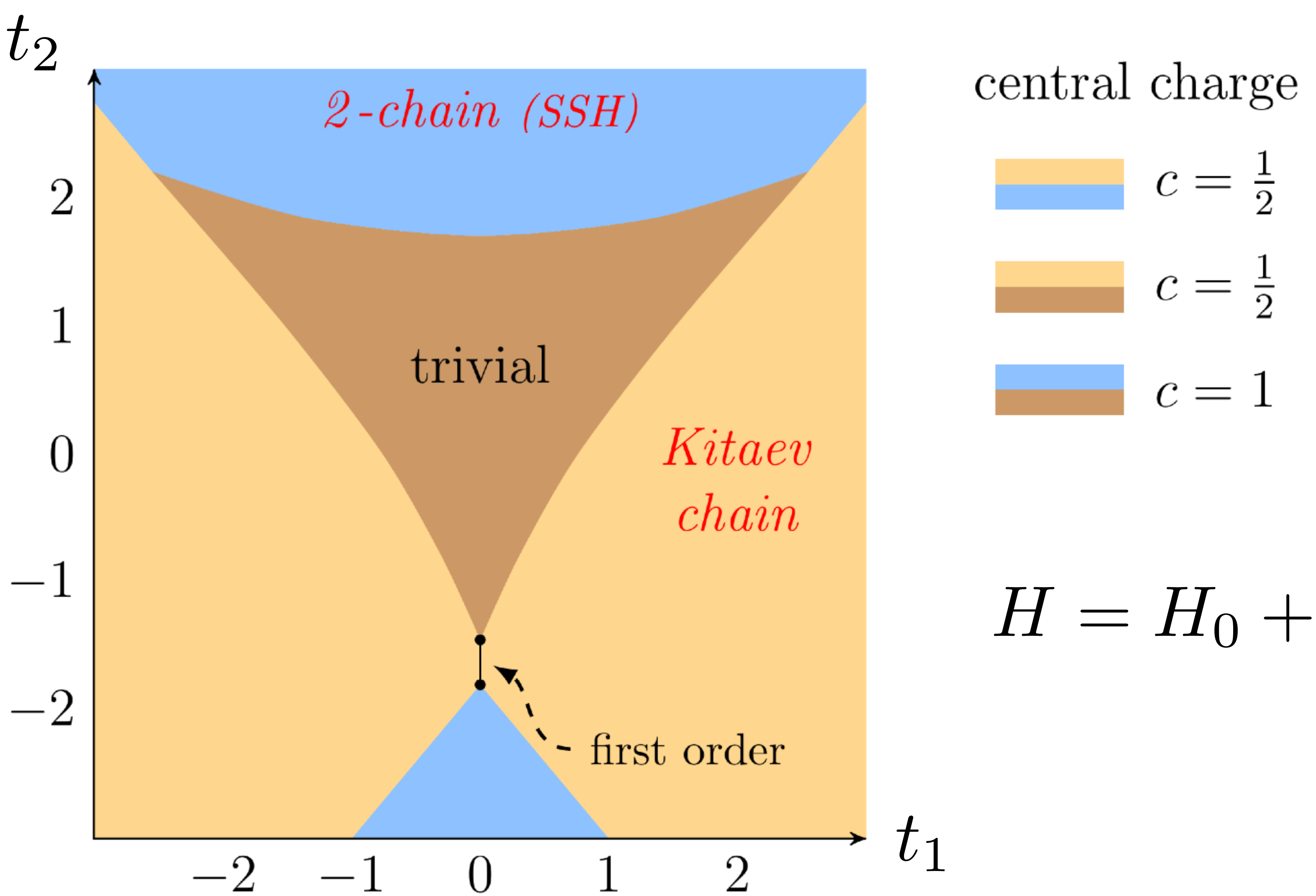
(1) Critical points between SPT phases

Testing the conjecture : α-chain



$$H = H_0 + t_1 H_1 + t_2 H_2$$

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(1) Critical points between SPT phases

Testing the conjecture: $SU(2)_k$ anyonic SPT

[Trebst, Troyer, Wang, Ludwig '08]

	k	central charge c	$\log_2 d$	$\frac{c-\log_2 d}{c}$
Kitaev chain →	2	$\frac{1}{2}$	$\frac{1}{2}$	0
Fibonacci →	3	$\frac{7}{10}$	≈ 0.6942	≈ 0.0082
	4	$\frac{4}{5}$	≈ 0.7925	≈ 0.0094
	5	$\frac{6}{7}$	≈ 0.8495	≈ 0.0089
	$\vdots$	$\vdots$	$\vdots$	$\vdots$
	k	$1-\frac{6}{(k+1)(k+2)}$	$\log_2\left(2\cos\frac{\pi}{k+2}\right)$	$0 \leq \frac{c-\log_2 d}{c} < \frac{1}{100}$
	$\vdots$	$\vdots$	$\vdots$	$\vdots$
Haldane →	∞	1	1	0

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(1) Critical points between SPT phases

Testing the conjecture: WZW $SU(2M)_1$

[Nonne, Moliner, Capponi, Lecheminant, Totsuka '13]

M	central charge c	$\log_2 d$	$\frac{c-\log_2 d}{c}$
1	1	1	0
2	3	$\log_2 6 \approx 2.59$	≈ 0.14
3	5	$\log_2 20 \approx 4.32$	≈ 0.14
4	7	$\log_2 70 \approx 6.13$	≈ 0.12
$\vdots$	$\vdots$	$\vdots$	$\vdots$
M	$c = 2M - 1$	$\log_2 \frac{(2M)!}{(M!)^2}$	$\sim \frac{\log_2 M}{4M}$ (M large)
$\vdots$	$\vdots$	$\vdots$	$\vdots$
∞	∞	∞	0

Proof? Not clear if the conjecture holds in general…

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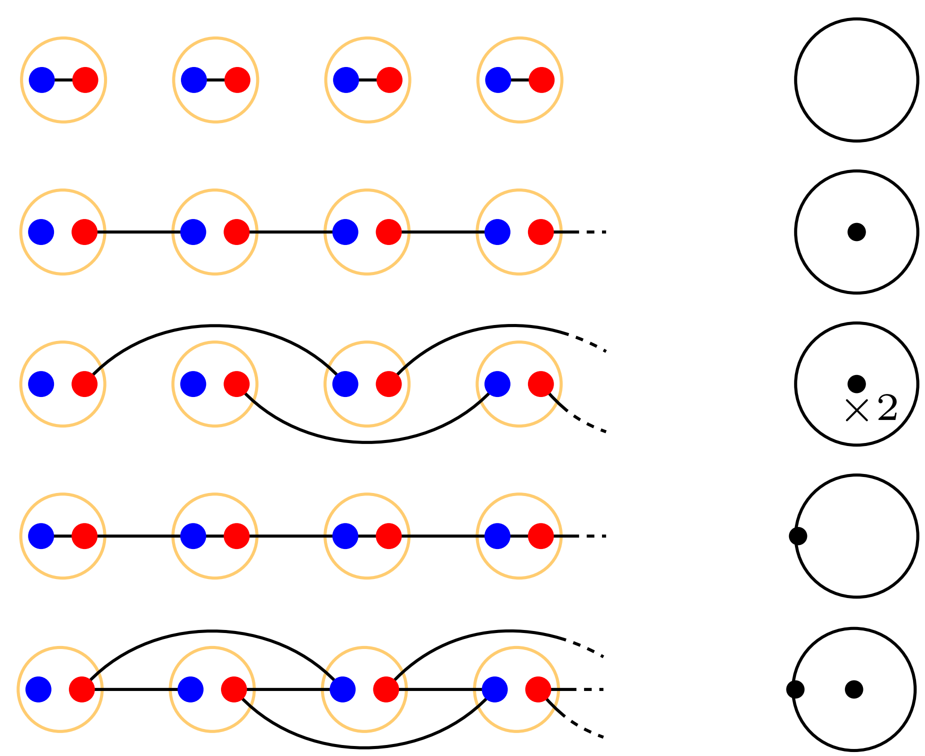
(2) Edge modes in critical chains

Fixed point example in the (non-interacting) BDI class

$$H = i \sum_{\alpha=-\infty}^{+\infty} t_\alpha \left(\sum_{n \in \text{sites}} \tilde{\gamma}_n \gamma_{n+\alpha} \right) = \sum_\alpha t_\alpha H_\alpha \qquad f(z) = \sum_{\alpha=-\infty}^{\infty} t_\alpha \, z^\alpha$$



H_0	$f(z) = 1$
H_1	$f(z) = z$
H_2	$f(z) = z^2$
$H_0 + H_1$	$f(z) = z + 1$
$H_1 + H_2$	$f(z) = z(z+1)$

Critical point $H_1 + H_2$ has localized edge modes!

No gapped sectors [Kestner et al '11, Keselman and E. Berg '15, …]

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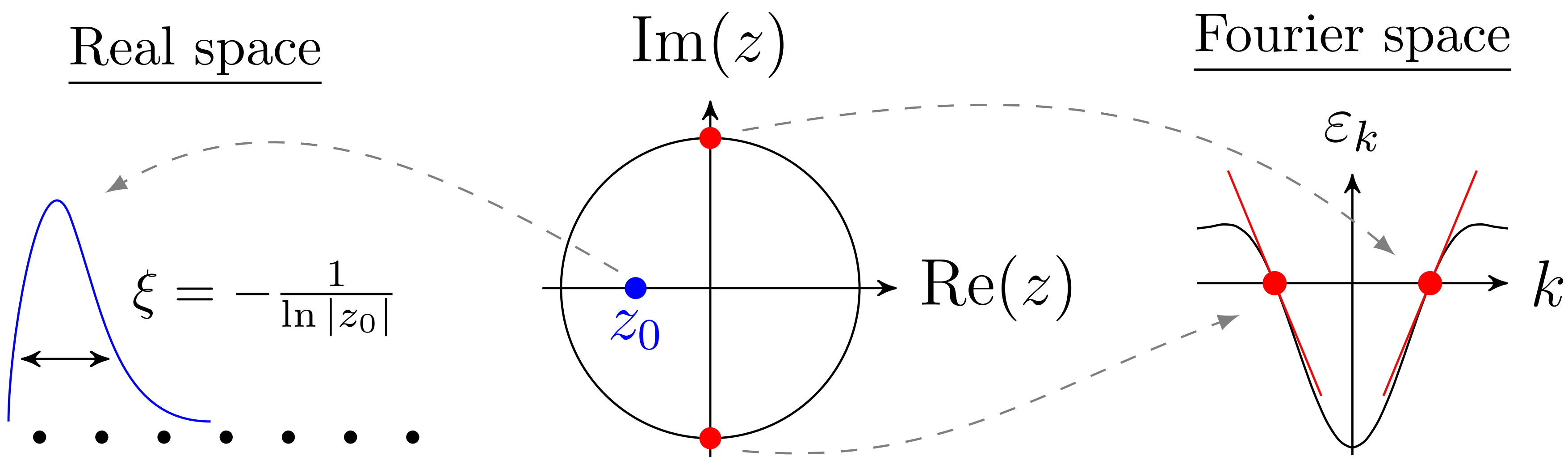
(2) Edge modes in critical chains

Physical relevance of the zeros z of $f(z)$



$f(k) = \epsilon_k e^{i\phi_k}$

$w = N_Z - N_P$ with N_Z zeros $|z| < 1$

N_P order of the pole at $z = 0$

➠Number of edge modes

$c =$ half the number of zeros with $|z| = 1$

➠Central charge if non-degenerate

see also Motrunich et al. '01:
Edge modes from transfer matrix approach

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(2) Edge modes in critical chains

Theorem 1. *If the topological invariant $\omega > 0$, then*

1. *each boundary has ω Majorana zero modes,*
2. *the modes have localization length $\xi_i = -\frac{1}{\ln(|z_i|)}$ where $\{z_i\}$ are the ω largest zeros of $f(z)$ within the unit disk,*
3. *the modes on the left (right) are real (imaginary)*

If $\omega < -2c$ (where $c =$ half the number of zeros on the unit circle), the left (right) boundary has $|\omega + 2c|$ imaginary (real) Majorana modes with localization length $\xi_i = \frac{1}{\ln(|z_i|)}$, with $\{z_i\}$ the $|\omega + 2c|$ smallest zeros outside the unit disk.

For any other value of ω, no localized edge modes exist.

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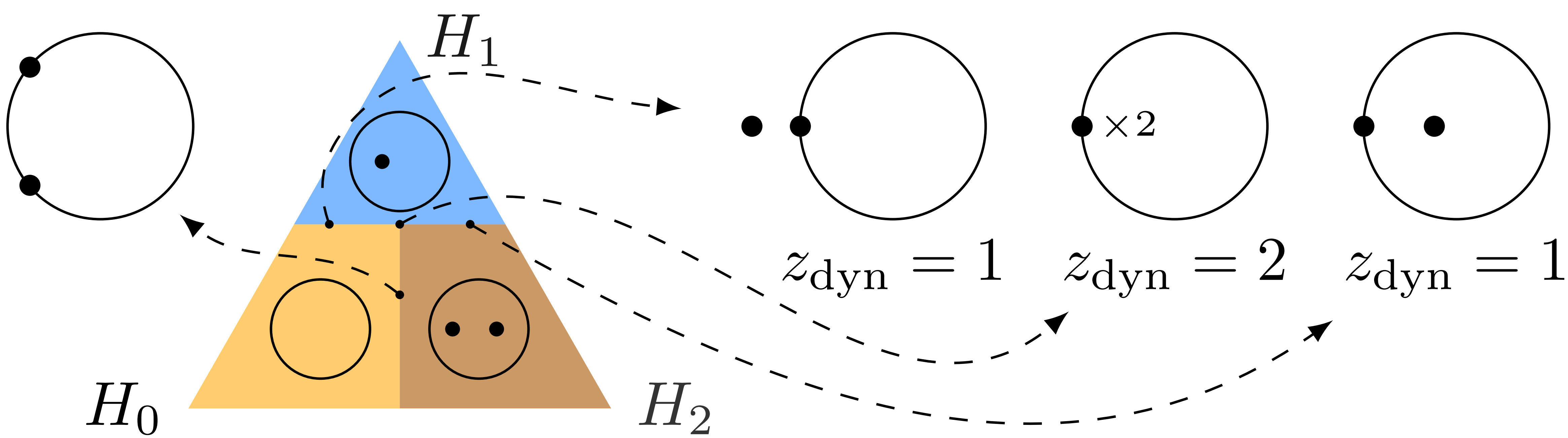
(2) Edge modes in critical chains

Classification of critical phases



Chains with different w are separated by a phase transition!

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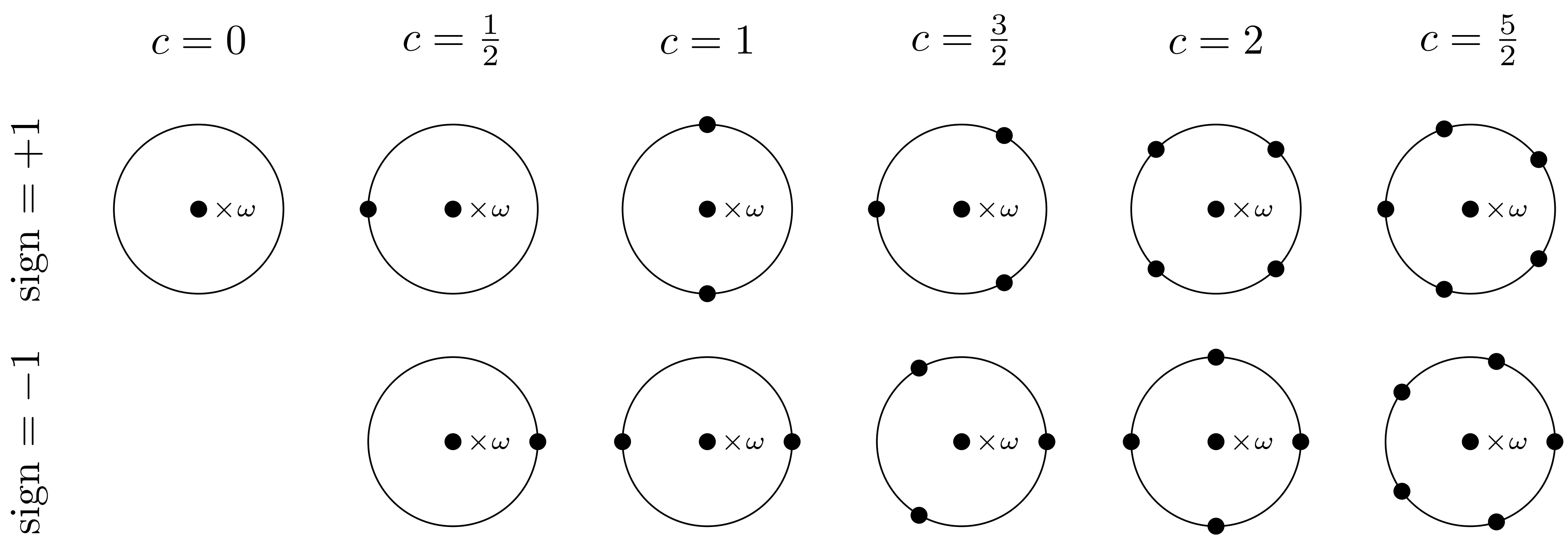
(2) Edge modes in critical chains

$t_\alpha \in \mathbb{R}:\ f(z)$ **come in complex conjugate pairs**

Canonical form $f(z) = \pm\left(z^{2c} \pm 1\right) z^{\omega}$



Different signs can be connected by allowing for unit cells

BDI Class: Critical phases are classified by c and ω

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(2) Edge modes in critical chains

Theorem 2. *The phases in the BDI class described in the bulk by a CFT and obtained by deforming a translation invariant Hamiltonian (or a stacking thereof) with an arbitrary unit cell, are classified by the semigroup $\mathbb{N} \times \mathbb{Z}$: they are labeled by the central charge $c \in \frac{1}{2}\mathbb{N}$ and the topological invariant $\omega \in \mathbb{Z}$.*

Translation invariance gives an extra $\mathbb{Z}_2$ invariant when $c = 0$ and an extra $\mathbb{Z}_2 \times \mathbb{Z}_2$ invariant when $c \neq 0$.

Theorem 3. *A phase transition between two gapped phases with winding numbers ω_1 and ω_2 obeys $c \geq \frac{|\omega_1 - \omega_2|}{2}$.*

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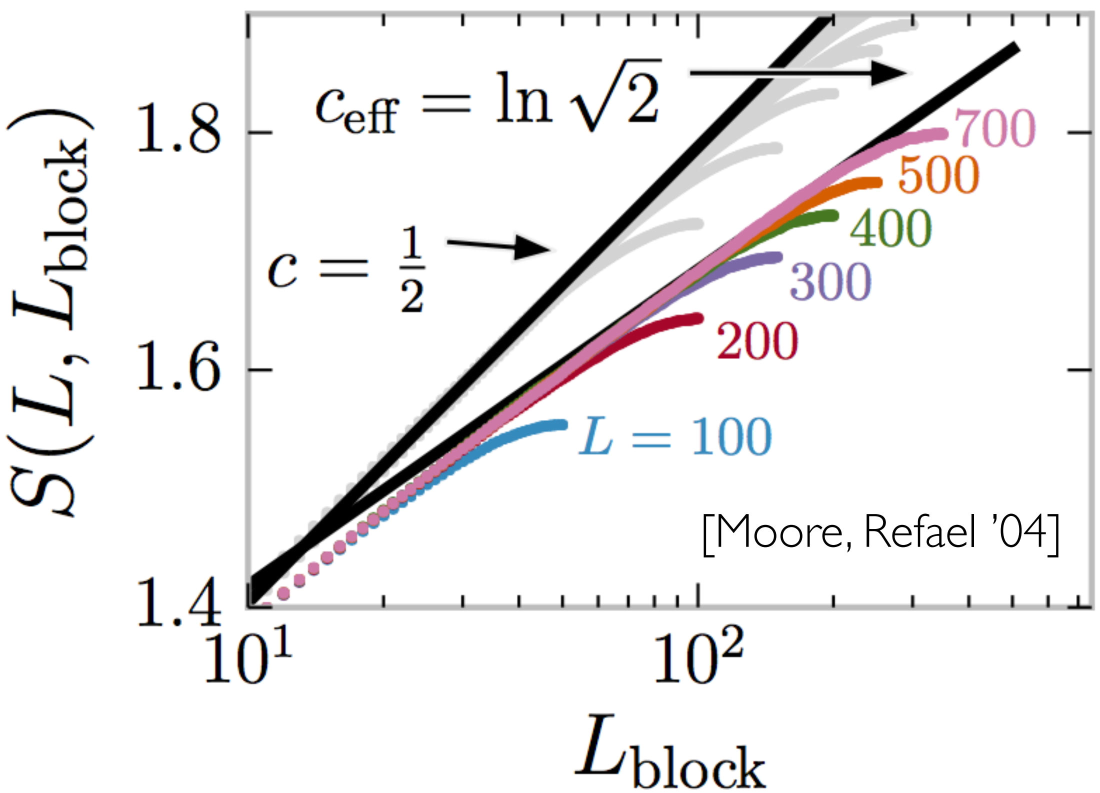
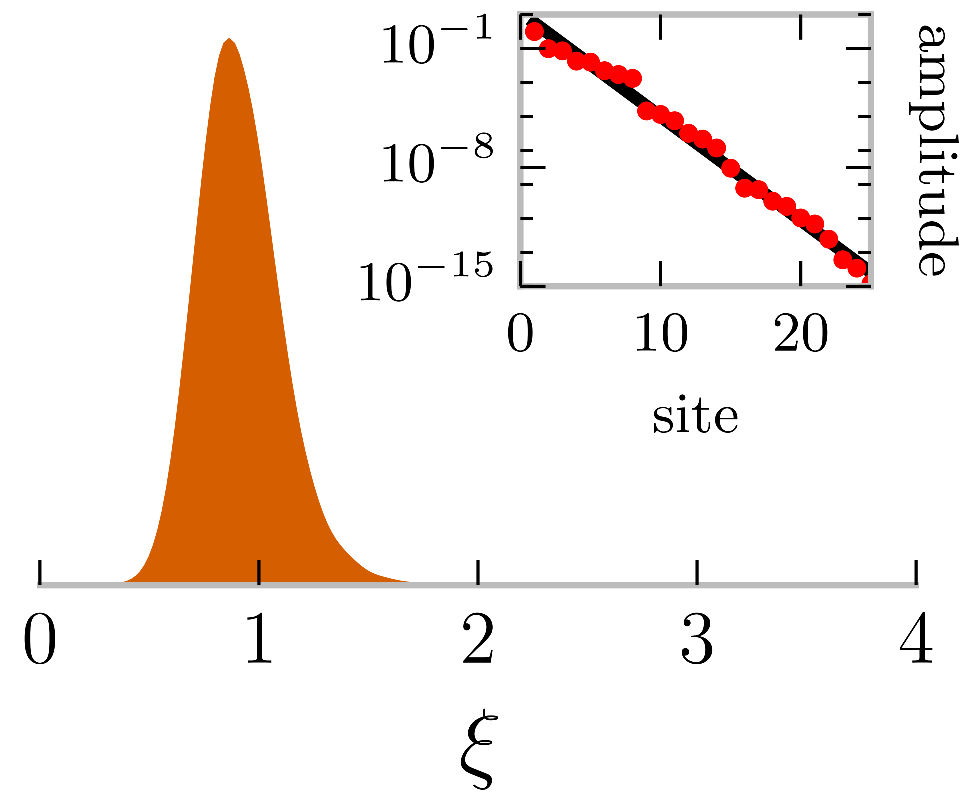
(2) Edge modes in critical chains

Stability of edge mode against disorder





$$H = i \sum_{\alpha=0}^{3} \sum_{n=1}^{L} t_\alpha^{(n)} \, \tilde{\gamma}_n \gamma_{n+\alpha}$$

with $t_1^{(n)}, t_2^{(n)} \in [0,1]$ and $t_0^{(n)}, t_3^{(n)} \in [-0.5, 0]$

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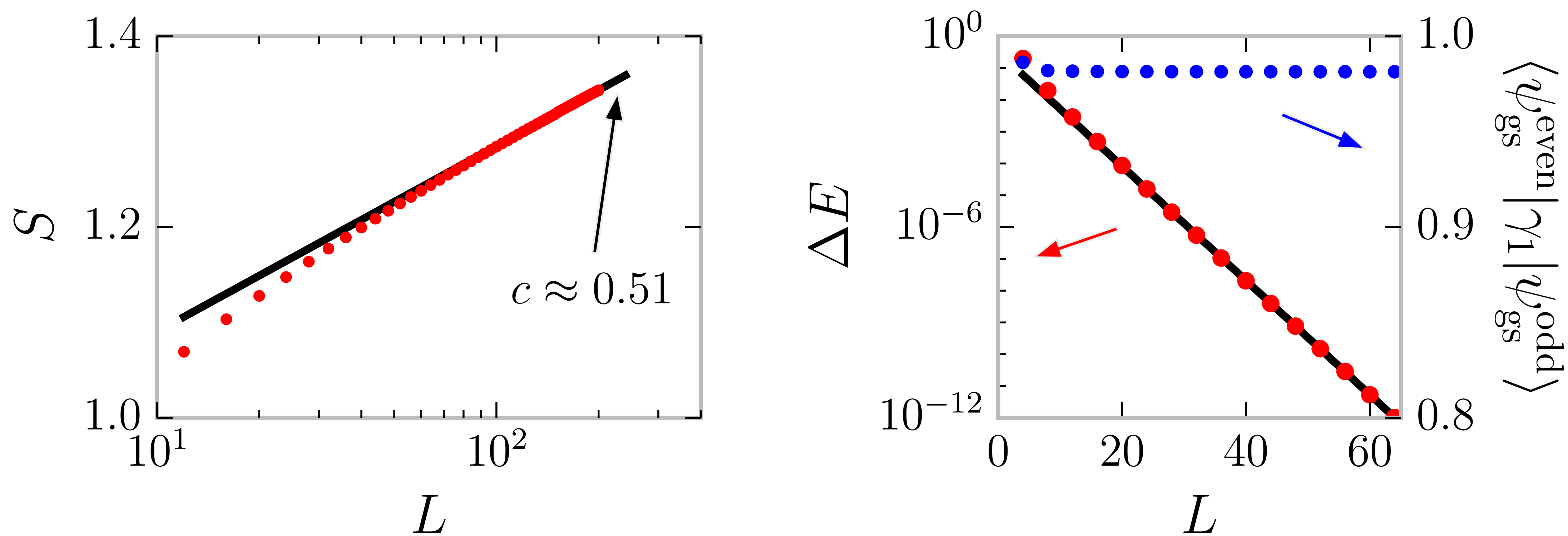
(2) Edge modes in critical chains

Stability of edge mode against interactions



$$H = H_1 + H_2 + U \sum_{n=1}^{L} \gamma_n \gamma_{n+1} \gamma_{n+2} \gamma_{n+3} + (\gamma \leftrightarrow \tilde{\gamma})$$

with $U = 0.6$

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Summary

(1) Transitions between any SPT and a trivial phase:

Conjecture: $c \geq \log_2 d$

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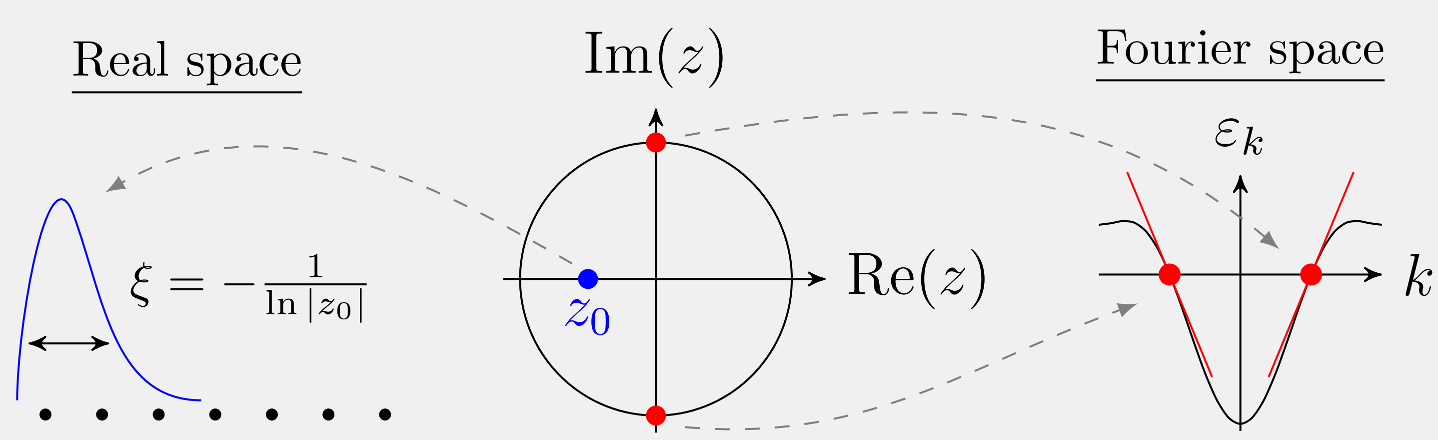
(2) Protected Edge modes in critical chains (BDI)



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Thank You!