

# **Odd-frequency superconductivity in $\text{Sr}_2\text{RuO}_4$ measured by Kerr effect**

Annica Black-Schaffer



UPPSALA  
UNIVERSITET

NQS2017 workshop  
Kyoto, November 6<sup>th</sup> 2017

# Outline



UPPSALA  
UNIVERSITET

- Introduction to odd-frequency ( $\omega$ ) superconductivity
  - SF, SN junctions
- Odd- $\omega$  bulk superconductivity in multiband systems
  - Simple two-band superconductors
- Odd- $\omega$  superconductivity and Kerr effect in  $\text{Sr}_2\text{RuO}_4$

# Different Phases of Matter



UPPSALA  
UNIVERSITET

Ordered states → symmetry of order parameter,  $\Delta$

Superconductivity:  $\Delta \sim F = \langle \psi_\alpha \psi_\beta \rangle$

Fermi-Dirac  
statistics

- spin-singlet, *s*-wave  
 $\uparrow\downarrow - \downarrow\uparrow$  
- spin-triplet, *p*-wave  
 $\uparrow\downarrow + \downarrow\uparrow$   
 $\uparrow\uparrow, \downarrow\downarrow$  



UPPSALA  
UNIVERSITET

# Different Phases of Matter

Ordered states → symmetry of order parameter,  $\Delta$

Superconductivity:  $\Delta \sim F = \langle \psi_\alpha(t) \psi_\beta(0) \rangle$  ( $t \leftrightarrow \omega$ , frequency)

Fermi-Dirac  
statistics

- even- $\omega$ , spin-singlet, s-wave  
 $\uparrow\downarrow - \downarrow\uparrow$  
- odd- $\omega$ , spin-triplet, s-wave  
 $\uparrow\downarrow + \downarrow\uparrow$   
 $\uparrow\uparrow, \downarrow\downarrow$  

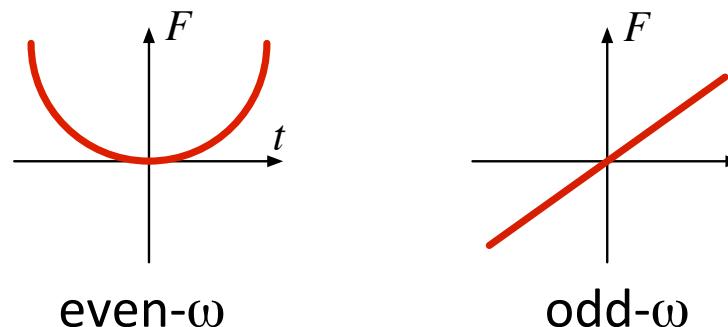


UPPSALA  
UNIVERSITET

# Odd- $\omega$ Pairing

BCS order parameter:  $F(\mathbf{r}, t; \mathbf{r}', t' \rightarrow t) = \langle \psi(\mathbf{r}, t) \psi(\mathbf{r}', t' \rightarrow t) \rangle$   
vanishes for odd- $\omega$  pairing

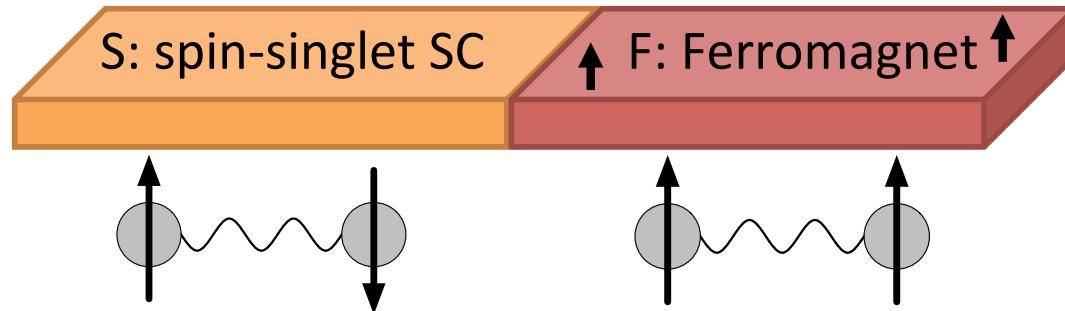
Equal-time odd- $\omega$  order parameter:  $\left. \frac{dF(\mathbf{r}, t; \mathbf{r}', t')}{dt} \right|_{t \rightarrow t'}$



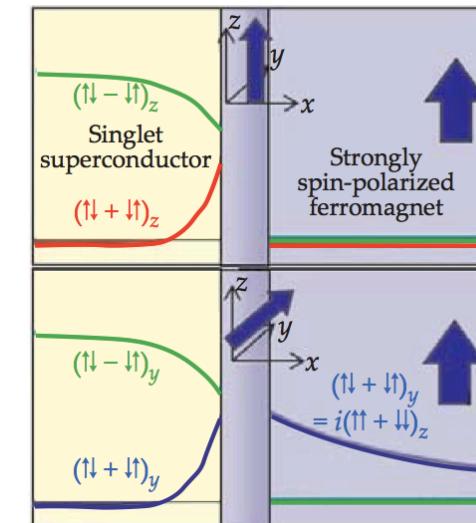
# SF Interface



UPPSALA  
UNIVERSITET



Spin-rotation symmetry breaking



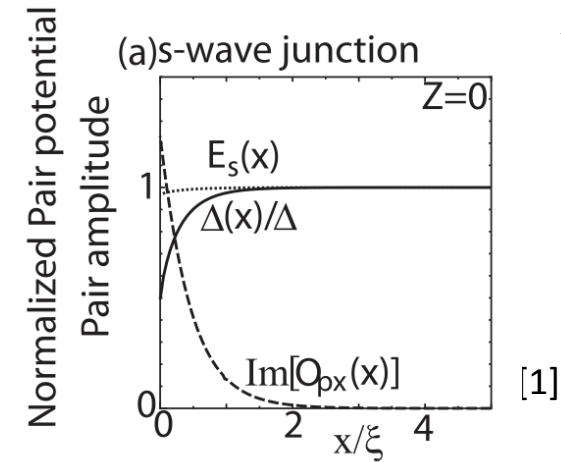
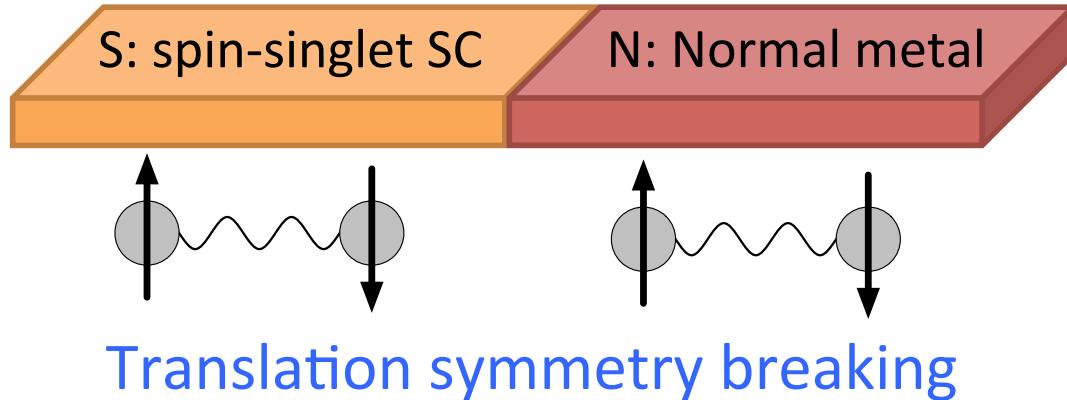
[1]

Spin-singlet  $s$ -wave SC  $\rightarrow$  odd- $\omega$  spin-triplet  $s$ -wave pairing

- Long-range superconducting proximity effect in F
- $s$ -wave = disorder robust

[1]: Eschrig, Phys. Today 64, 43 (2011)

# SN Interface



Spin-singlet *s*-wave SC  $\rightarrow$  odd- $\omega$  spin-singlet *p*-wave pairing

- Only high-transparency junctions
- *p*-wave = only ballistic systems

[1]: Tanaka et al, PRL 99, 037005 (2007)



UPPSALA  
UNIVERSITET

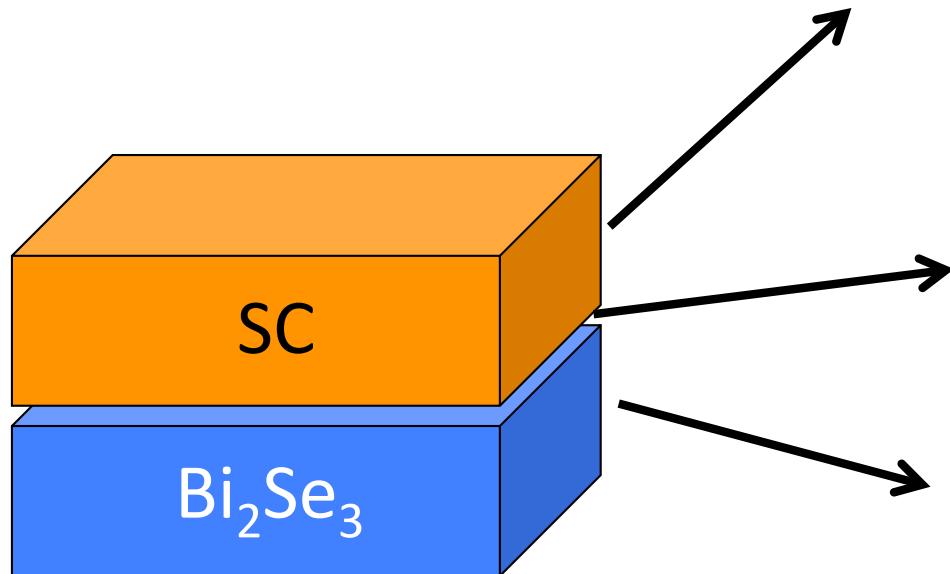
# Odd- $\omega$ Bulk Superconductivity in Multiband Systems

- A clue from  $\text{Bi}_2\text{Se}_3$
- Simple two-band superconductors



UPPSALA  
UNIVERSITET

# $\text{Bi}_2\text{Se}_3$ – SC Hybrid Structure



$$H_{\text{SC}} = \sum_{\mathbf{k},\sigma} (-2 \cos(k_x a) - 2 \cos(k_y a) + \mu_{\text{SC}}) c_{\mathbf{k}\sigma}^\dagger c_{\mathbf{k}\sigma} + \frac{1}{2} \sum_{\mathbf{k},\sigma,\sigma'} \Delta_{\sigma\sigma'}(\mathbf{k}) c_{\mathbf{k}\sigma}^\dagger c_{-\mathbf{k}\sigma'}^\dagger - \Delta_{\sigma\sigma'}^*(-\mathbf{k}) c_{-\mathbf{k}\sigma} c_{\mathbf{k}\sigma'}$$

2D superconductor

$$H_T = - \sum_{\mathbf{k},\sigma} T_1 c_{\mathbf{k}\sigma}^\dagger b_{1\mathbf{k}\sigma} + T_2 c_{\mathbf{k}\sigma}^\dagger b_{2\mathbf{k}\sigma} + \text{H.c.}$$

Local tunneling

$$H_{\text{TI}} = \gamma_0 - 2 \sum_{\mathbf{k},i} \gamma_i \cos(k_i a) + \sum_{\mathbf{k},\mu} d_\mu \Gamma_\mu$$

$\text{Bi}_2\text{Se}_3$  ( $\tau$  orbital Pauli matrix) [1]

$$\left. \begin{array}{l} d_0 = \epsilon - 2 \sum_i t_i \cos(k_i a), d_i = -2\lambda_i \sin(k_i a) \\ \Gamma_0 = \tau_x \otimes \sigma_0, \Gamma_x = -\tau_z \otimes \sigma_y, \Gamma_y = \tau_z \otimes \sigma_x, \Gamma_z = \tau_y \otimes \sigma_0 \end{array} \right\}$$

ABS and Balatsky, PRB 87, 220506(R) (2013), [1]: Rosenberg and Franz, PRB 85, 195119 (2012)

# Superconducting Symmetries in $\text{Bi}_2\text{Se}_3$



UPPSALA  
UNIVERSITET

Singlet/triplet, spatial ( $s/p/d$ ), even/odd- $\omega$ , even/odd orbital

Superconductor			Even-frequency		Odd-frequency	
$\Gamma$	Basis function	$J_z$	Even-orbital	Odd-orbital	Even-orbital	Odd-orbital
A <sub>1g</sub>	$\psi = 1$	0	A <sub>1g</sub> singlet, A <sub>2u</sub> triplet ( $m_s = \pm 1$ )	-	-	A <sub>1g</sub> singlet, A <sub>2u</sub> triplet ( $m_s = \pm 1$ )
B <sub>1g</sub>	$\psi = k_x^2 - k_y^2$	$\pm 2$	B <sub>1g</sub> singlet, B <sub>2u</sub> triplet ( $m_s = \pm 1$ )	-	-	B <sub>1g</sub> singlet, B <sub>2u</sub> triplet ( $m_s = \pm 1$ )
B <sub>2g</sub>	$\psi = 2k_x k_y$	$\pm 2$	B <sub>2g</sub> singlet, B <sub>1u</sub> triplet ( $m_s = \pm 1$ )	-	-	B <sub>2g</sub> singlet, B <sub>1u</sub> triplet ( $m_s = \pm 1$ )
A <sub>1u</sub>	$\mathbf{d} = (k_x, k_y, 0)$	0	A <sub>1u</sub> triplet ( $m_s = \pm 1$ )	A <sub>1g</sub> triplet ( $m_s = 0$ )	A <sub>1g</sub> triplet ( $m_s = 0$ )	A <sub>1u</sub> triplet ( $m_s = \pm 1$ )
A <sub>2u</sub>	$\mathbf{d} = (k_y, -k_x, 0)$	0	A <sub>2u</sub> triplet ( $m_s = \pm 1$ ), A <sub>1g</sub> singlet	-	-	A <sub>2u</sub> triplet ( $m_s = \pm 1$ ), A <sub>1g</sub> singlet
B <sub>1u</sub>	$\mathbf{d} = (k_x, -k_y, 0)$	$\pm 2$	B <sub>1u</sub> triplet ( $m_s = \pm 1$ ), B <sub>2g</sub> singlet	B <sub>1g</sub> triplet ( $m_s = 0$ )	B <sub>1g</sub> triplet ( $m_s = 0$ )	B <sub>1u</sub> triplet ( $m_s = \pm 1$ ), B <sub>2g</sub> singlet
B <sub>2u</sub>	$\mathbf{d} = (k_y, k_x, 0)$	$\pm 2$	B <sub>2u</sub> triplet ( $m_s = \pm 1$ ), B <sub>1g</sub> singlet	B <sub>2g</sub> triplet ( $m_s = 0$ )	B <sub>2g</sub> triplet ( $m_s = 0$ )	B <sub>2u</sub> triplet ( $m_s = \pm 1$ ), B <sub>1g</sub> singlet
E <sub>2u</sub> <sup>+</sup>	$\mathbf{d} = (0, 0, k_x + ik_y)$	1	E <sub>2u</sub> <sup>+</sup> triplet ( $m_s = 0$ )	A <sub>1g</sub> triplet ( $m_s = 1$ ), B <sub>1g</sub> +iB <sub>2g</sub> triplet ( $m_s = -1$ )	A <sub>1g</sub> triplet ( $m_s = 1$ ), B <sub>1g</sub> +iB <sub>2g</sub> triplet ( $m_s = -1$ )	E <sub>2u</sub> <sup>+</sup> triplet ( $m_s = 0$ )
E <sub>2u</sub> <sup>-</sup>	$\mathbf{d} = (0, 0, k_x - ik_y)$	-1	E <sub>2u</sub> <sup>-</sup> triplet ( $m_s = 0$ )	A <sub>1g</sub> triplet ( $m_s = -1$ ), B <sub>1g</sub> -iB <sub>2g</sub> triplet ( $m_s = 1$ )	A <sub>1g</sub> triplet ( $m_s = -1$ ), B <sub>1g</sub> -iB <sub>2g</sub> triplet ( $m_s = 1$ )	E <sub>2u</sub> <sup>-</sup> triplet ( $m_s = 0$ )

ABS and Balatsky, PRB 87, 220506(R) (2013)



UPPSALA  
UNIVERSITET

# Frequency and Orbital

Complete reciprocity in oddness in frequency and orbital index

Superconductor			Even-frequency	Odd-frequency	Odd-orbital	
$\Gamma$	Basis function	$J_z$	Even-orbital	Odd-orbital	Even-orbital	Odd-orbital
A <sub>1g</sub>	$\psi = 1$	0	A <sub>1g</sub> singlet, A <sub>2u</sub> triplet ( $m_s = \pm 1$ )	-	-	A <sub>1g</sub> singlet, A <sub>2u</sub> triplet ( $m_s = \pm 1$ )
B <sub>1g</sub>	$\psi = k_x^2 - k_y^2$	$\pm 2$	B <sub>1g</sub> singlet, B <sub>2u</sub> triplet ( $m_s = \pm 1$ )	-	-	B <sub>1g</sub> singlet, B <sub>2u</sub> triplet ( $m_s = \pm 1$ )
B <sub>2g</sub>	$\psi = 2k_x k_y$	$\pm 2$	B <sub>2g</sub> singlet, B <sub>1u</sub> triplet ( $m_s = \pm 1$ )	-	-	B <sub>2g</sub> singlet, B <sub>1u</sub> triplet ( $m_s = \pm 1$ )
A <sub>1u</sub>	$\mathbf{d} = (k_x, k_y, 0)$	0	A <sub>1u</sub> triplet ( $m_s = \pm 1$ )	A <sub>1g</sub> triplet ( $m_s = 0$ )	A <sub>1g</sub> triplet ( $m_s = 0$ )	A <sub>1u</sub> triplet ( $m_s = \pm 1$ )
A <sub>2u</sub>	$\mathbf{d} = (k_y, -k_x, 0)$	0	A <sub>2u</sub> triplet ( $m_s = \pm 1$ ) A <sub>1g</sub> singlet	-	-	A <sub>2u</sub> triplet ( $m_s = \pm 1$ ), A <sub>1g</sub> singlet
B <sub>1u</sub>	$\mathbf{d} = (k_x, -k_y, 0)$	$\pm 2$	B <sub>1u</sub> triplet ( $m_s = \pm 1$ ) B <sub>2g</sub> singlet	B <sub>1g</sub> triplet ( $m_s = 0$ )	B <sub>1g</sub> triplet ( $m_s = 0$ )	B <sub>1u</sub> triplet ( $m_s = \pm 1$ ), B <sub>2g</sub> singlet
B <sub>2u</sub>	$\mathbf{d} = (k_y, k_x, 0)$	$\pm 2$	B <sub>2u</sub> triplet ( $m_s = \pm 1$ ) B <sub>1g</sub> singlet	B <sub>2g</sub> triplet ( $m_s = 0$ )	B <sub>2g</sub> triplet ( $m_s = 0$ )	B <sub>2u</sub> triplet ( $m_s = \pm 1$ ), B <sub>1g</sub> singlet
E <sub>2u</sub> <sup>+</sup>	$\mathbf{d} = (0, 0, k_x + ik_y)$	1	E <sub>2u</sub> <sup>+</sup> triplet ( $m_s = 0$ )	A <sub>1g</sub> triplet ( $m_s = 1$ ), B <sub>1g</sub> +iB <sub>2g</sub> triplet ( $m_s = -1$ )	A <sub>1g</sub> triplet ( $m_s = 1$ ), B <sub>1g</sub> +iB <sub>2g</sub> triplet ( $m_s = -1$ )	E <sub>2u</sub> <sup>+</sup> triplet ( $m_s = 0$ )
E <sub>2u</sub> <sup>-</sup>	$\mathbf{d} = (0, 0, k_x - ik_y)$	-1	E <sub>2u</sub> <sup>-</sup> triplet ( $m_s = 0$ )	A <sub>1g</sub> triplet ( $m_s = -1$ ), B <sub>1g</sub> -iB <sub>2g</sub> triplet ( $m_s = 1$ )	A <sub>1g</sub> triplet ( $m_s = -1$ ), B <sub>1g</sub> -iB <sub>2g</sub> triplet ( $m_s = 1$ )	E <sub>2u</sub> <sup>-</sup> triplet ( $m_s = 0$ )

ABS and Balatsky, PRB 87, 220506(R) (2013)



UPPSALA  
UNIVERSITET

# Multiband Superconductors

- S: Spin (even: spin-triplet; odd: spin-singlet)
  - P: Spatial parity (even:  $s,d$ -wave; odd:  $p,f$ -wave)
  - O: Orbital or band parity (even; odd orbital)
  - T: Time (even; odd-frequency)
- } SPOT = -1

S = 0	P	T	O	S = 1	P	T	O
even- $\omega$	+	+	+	even- $\omega$	-	+	+
even- $\omega$	-	+	-	even- $\omega$	+	+	-
odd- $\omega$	+	-	-	odd- $\omega$	+	-	+
odd- $\omega$	-	-	+	odd- $\omega$	-	-	-



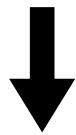
UPPSALA  
UNIVERSITET

# Simple Two-Band Superconductor

$$H_{ab} = \sum_{k\sigma} \varepsilon_a(k) a_{k\sigma}^\dagger a_{k\sigma} + \varepsilon_b(k) b_{k\sigma}^\dagger b_{k\sigma}$$

$$+ \sum_k \Delta_a(k) a_{k\uparrow}^\dagger a_{-k\downarrow}^\dagger + \Delta_b(k) b_{k\uparrow}^\dagger b_{-k\downarrow}^\dagger + \text{H.c.}$$

Interband  
hybridization



$$H_{cd} = \sum_{k\sigma} \varepsilon_c(k) c_{k\sigma}^\dagger c_{k\sigma} + \varepsilon_d(k) d_{k\sigma}^\dagger d_{k\sigma} \quad \text{Diagonal bands}$$

$$+ \sum_k \Delta_c(k) c_{k\uparrow}^\dagger c_{-k\downarrow}^\dagger + \Delta_d(k) d_{k\uparrow}^\dagger d_{-k\downarrow}^\dagger + \text{H.c.} \quad \text{Intraband pairing}$$

$$+ \sum_k \Delta_{cd}(k) [c_{k\uparrow}^\dagger d_{-k\downarrow}^\dagger + d_{k\uparrow}^\dagger c_{-k\downarrow}^\dagger] + \text{H.c.}$$

Interband pairing

$$\Delta_{cd} = \frac{(\Delta_b - \Delta_a)|\Gamma|}{\sqrt{(\varepsilon_a - \varepsilon_b)^2 + 4|\Gamma|^2}}$$



UPPSALA  
UNIVERSITET

# Time-Dependent Pairing

Time-ordered s-wave interband pairing:

$$F^\pm(\tau) = \frac{1}{2N_k} \sum_k T_\tau \langle c_{-k\downarrow}(\tau) d_{k\uparrow}(0) \pm d_{-k\downarrow}(\tau) c_{k\uparrow}(0) \rangle$$

$F^e = F^+(\tau \rightarrow 0^+)$  Even- $\omega$ , even-interband pairing

$$F_\omega^o = \left. \frac{\partial F^-}{\partial \tau} \right|_{\tau \rightarrow 0^+}$$

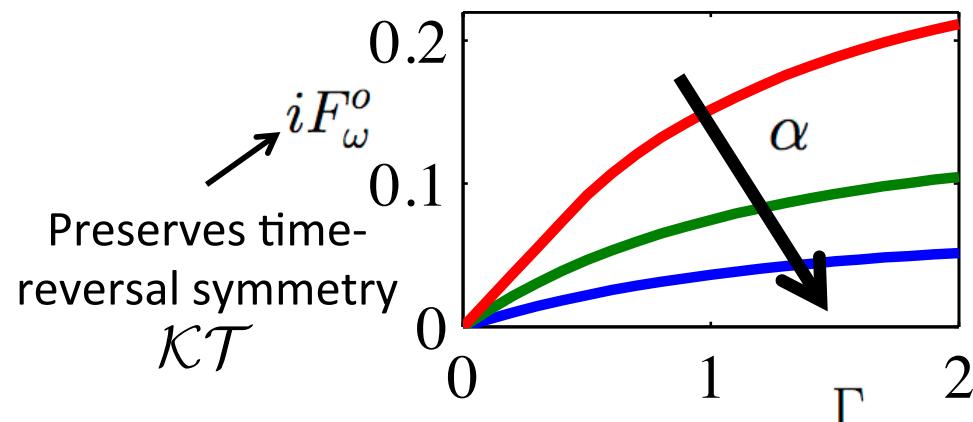
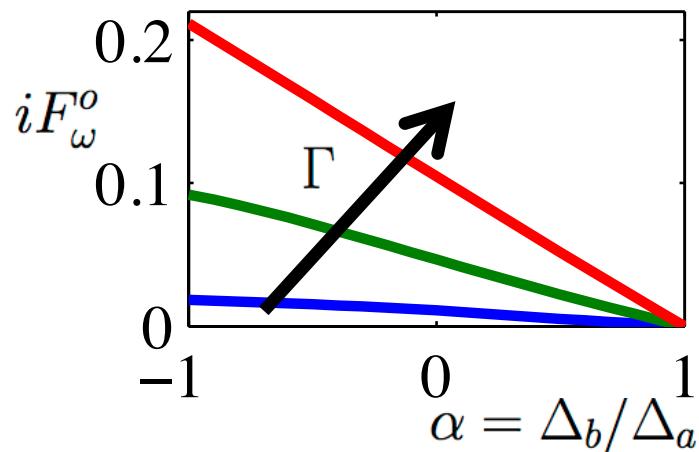
Odd- $\omega$ , odd-interband pairing



UPPSALA  
UNIVERSITET

# Odd- $\omega$ , Odd-Interband Pairing

For  $\begin{cases} \varepsilon_a = \varepsilon_b \\ \Delta_a = -\Delta_b \end{cases}$   $\rightarrow$   $\begin{cases} \varepsilon_{c,d} = \varepsilon_a \mp \Gamma \\ \Delta_c = \Delta_d = 0 \\ \Delta_{cd} = \Delta_a \text{ Interband pairing} \end{cases}$   $\Gamma < \Delta_a$   $\rightarrow$   $\begin{cases} F^e = -\frac{1}{2N_k} \sum_k \frac{\Delta_a}{\sqrt{\varepsilon_a^2 + |\Delta_a|^2}} \\ \text{BCS equation} \\ F_\omega^o = i\Gamma F^e \\ \text{Odd-}\omega \end{cases}$



ABS and Balatsky, PRB 88, 104514 (2013)

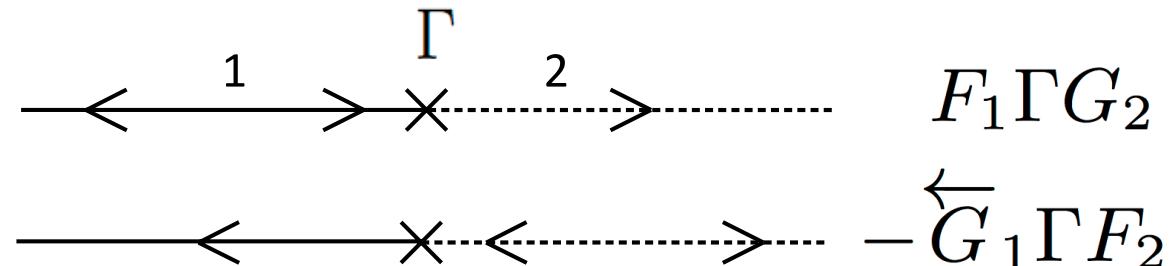


UPPSALA  
UNIVERSITET

# Using Perturbation Theory

$$H = \sum_{k\sigma} \varepsilon_1(k) a_{k\sigma}^\dagger a_{k\sigma} + \varepsilon_2(k) b_{k\sigma}^\dagger b_{k\sigma} + \boxed{\sum_{k\sigma} \Gamma(k) a_{k\sigma}^\dagger b_{k\sigma} + \text{H.c.}}$$
$$+ \sum_k \Delta_1(k) a_{k\uparrow}^\dagger a_{-k\downarrow}^\dagger + \Delta_2(k) b_{k\uparrow}^\dagger b_{-k\downarrow}^\dagger + \text{H.c.}$$

Interband pairing  $F_{12}$ :





UPPSALA  
UNIVERSITET

# Interband Pairing

Perturbation theory to infinite order in  $\Gamma$

Odd-interband:  $F_{12}^{\text{odd}}(k, i\omega) = \frac{F_{12} - F_{21}}{2} = i\cancel{\omega}\Gamma(\Delta_1 - \Delta_2)/D$

Even-interband:  $F_{12}^{\text{even}}(k, i\omega) = \frac{F_{12} + F_{21}}{2} = \Gamma(\Delta_1\varepsilon_2 - \Delta_2\varepsilon_1)/D$

$$\begin{cases} D = (\omega^2 + E_1^2)(\omega^2 + E_2^2) - \Gamma^2[2(\varepsilon_1\varepsilon_2 - \omega^2) - \Delta_2^*\Delta_1 - \Delta_1^*\Delta_2] + \Gamma^4 \\ E_j^2 = \varepsilon_j^2 + |\Delta_j|^2 \end{cases}$$

**Odd-frequency pairing:  $\Gamma \neq 0, \Delta_1 \neq \Delta_2$**

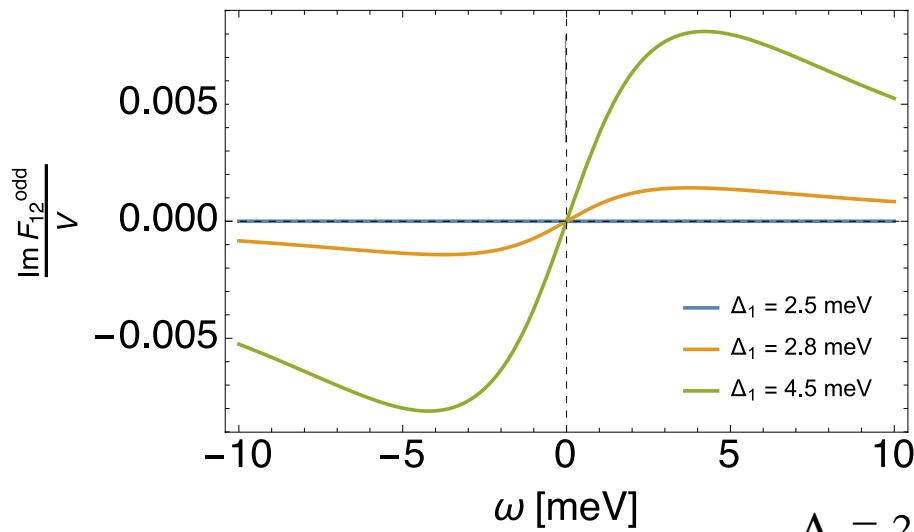
Komendova, Balatsky, and ABS, PRB 92, 04517 (2015)

# Interband Frequency Dependence

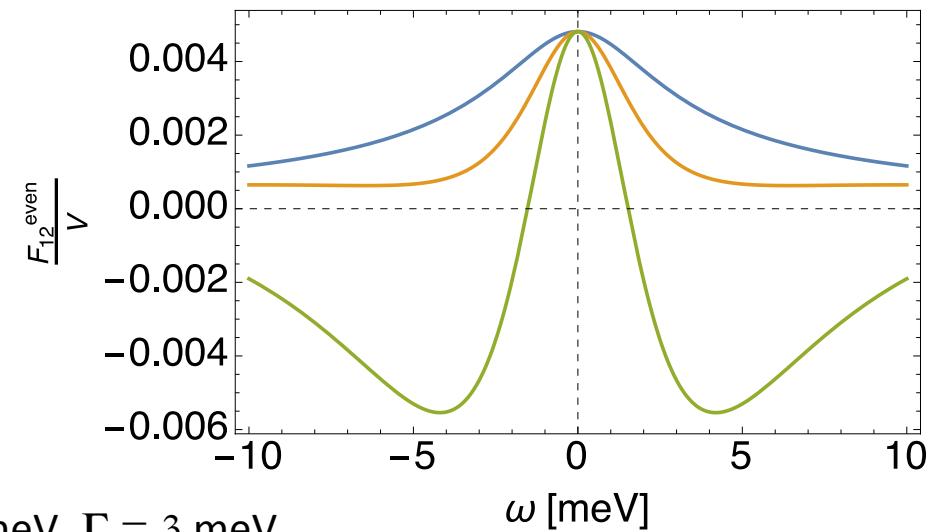


UPPSALA  
UNIVERSITET

## Odd-frequency



## Even-frequency



Odd-frequency pairing:  $\Gamma \neq 0, \Delta_1 \neq \Delta_2$

Komendova, Balatsky, and ABS, PRB 92, 04517 (2015)



UPPSALA  
UNIVERSITET

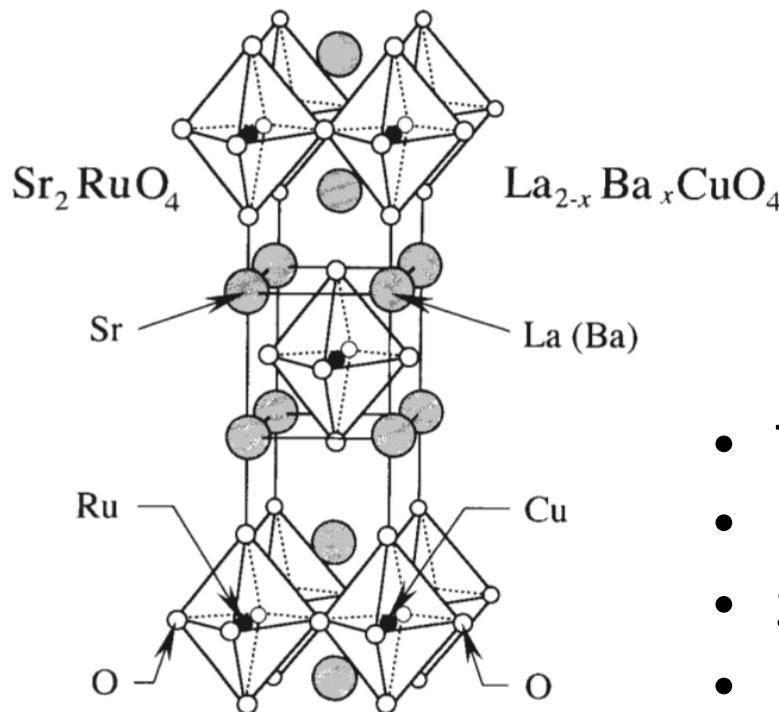
# Odd- $\omega$ Superconductivity in $\text{Sr}_2\text{RuO}_4$

- Superconductivity in  $\text{Sr}_2\text{RuO}_4$
- Two- and three-orbital models
- Odd- $\omega$  pairing measured by Kerr effect

# Strontium Ruthenate, $\text{Sr}_2\text{RuO}_4$



UPPSALA  
UNIVERSITET



## Superconductivity in a layered perovskite without copper

Y. Maeno\*, H. Hashimoto\*, K. Yoshida\*,  
S. Nishizaki\*, T. Fujita\*, J. G. Bednorz†,  
& F. Lichtenberg†‡

Nature 372, 532 (1994)

- $T_c = 1.5\text{K}$
- Non-*s*-wave (disorder sensitive)
- Spin-triplet (neutron scattering, Knight shift)
- Breaks time-reversal symmetry (Kerr effect)

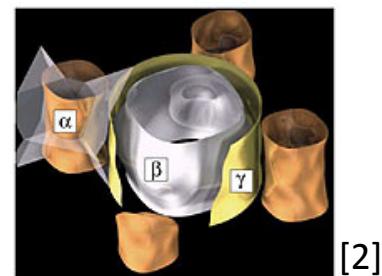
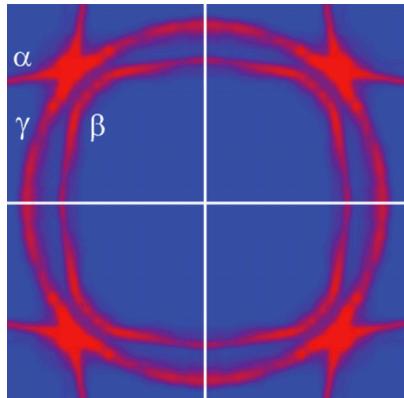
→ Spin-triplet chiral  $p_x + ip_y$ -wave symmetry



UPPSALA  
UNIVERSITET

# Properties of $\text{Sr}_2\text{RuO}_4$

Three Fermi surfaces (FSs)



Three Ru 4d orbitals:

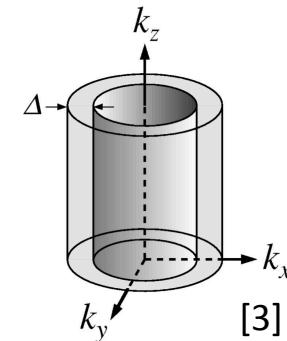
- $xy \rightarrow \gamma$  (electron-like)
- $xz, yz \rightarrow \beta$  (electron-like) and  $\alpha$  (hole-like)

Superconducting state

$$\begin{pmatrix} \Delta_{\uparrow\uparrow}(\mathbf{k}) & \Delta_{\uparrow\downarrow}(\mathbf{k}) \\ \Delta_{\downarrow\uparrow}(\mathbf{k}) & \Delta_{\downarrow\downarrow}(\mathbf{k}) \end{pmatrix} = \begin{pmatrix} -d_x(\mathbf{k}) + id_y(\mathbf{k}) & \psi(\mathbf{k}) + d_z(\mathbf{k}) \\ -\psi(\mathbf{k}) + d_z(\mathbf{k}) & d_x(\mathbf{k}) + id_y(\mathbf{k}) \end{pmatrix}$$

Triplet **d**-vector for chiral state

$$\mathbf{d}(\mathbf{k}) = (0, 0, k_x \pm ik_y)$$



Fully gapped on  
cylindrical FS

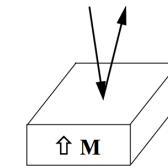
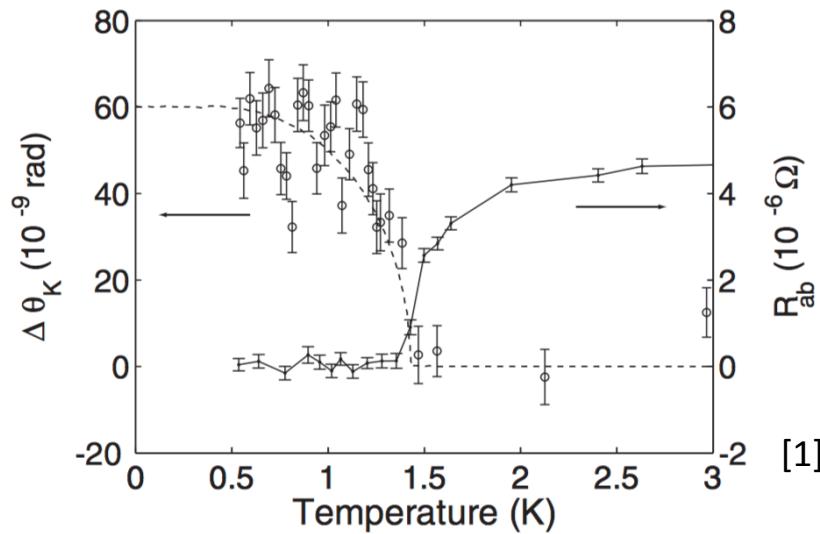
[1]: Damascelli et al, PRL 85, 5194 (2000), [2]: Phys. Today 54 (2001), [3]: Mackenzie and Maeno, RMP 75, 657 (2003)



UPPSALA  
UNIVERSITET

# Kerr Effect in $\text{Sr}_2\text{RuO}_4$

Reflected light has a slightly rotated plane of polarization if material breaks time-reversal symmetry (TRS)



SC state in  $\text{Sr}_2\text{RuO}_4$  breaks TRS

But ...

Clean **single-band** chiral SC has zero  
Kerr effect

→ Interband pairing with relative  
superconducting phases [2]

Electric-field driven interband transitions with relative SC phases  
→ finite transverse Hall current response → finite Kerr effect

[1]: Xia et al, PRL 97, 167002 (2006), [2]: Taylor and Kallin, PRL 108, 157001 (2012); Wysokinski et al, PRL 108, 077004 (2012)



UPPSALA  
UNIVERSITET

# Two-Orbital Model for $\text{Sr}_2\text{RuO}_4$

Hamiltonian  $\sum_{\mathbf{k}} \Psi_{\mathbf{k}}^{\dagger} \hat{H}_{\mathbf{k}} \Psi_{\mathbf{k}}$  with  $\hat{H}_{\mathbf{k}} = \begin{pmatrix} \hat{H}_0(\mathbf{k}) & \check{\Delta}(\mathbf{k}) \\ \check{\Delta}^{\dagger}(\mathbf{k}) & -\hat{H}_0(-\mathbf{k}) \end{pmatrix}$

- $xz$  (1),  $yz$  (2) orbitals  
 $\rightarrow \alpha, \beta$  bands
- Spin-triplet  $d \parallel \hat{z}$

$$\hat{H}_0(\mathbf{k}) = \begin{pmatrix} \xi_1 & \epsilon_{12} \\ \epsilon_{12} & \xi_2 \end{pmatrix}, \quad \check{\Delta}(\mathbf{k}) = \begin{pmatrix} \Delta_1 & \Delta_{12} \\ \Delta_{12} & \Delta_2 \end{pmatrix}$$

Intraorbital energy

Interorbital hybridization

Interorbital pairing

Intraorbital pairing

# Odd- $\omega$ Pairing and Kerr Effect



UPPSALA  
UNIVERSITET

Odd- $\omega$ , odd-interorbital pairing:

$$F_{12} - F_{21} = i\omega[(\Delta_2 - \Delta_1)\epsilon_{12} + \Delta_{12}(\xi_1 - \xi_2)]/D \quad (D \sim \omega^2)$$

Interorbital hybridization      Interorbital pairing  
+ gap asymmetry      + dispersion asymmetry

Kerr effect: [1]

$$\sigma_H \propto \epsilon_{12}\text{Im}(\Delta_1^*\Delta_2) + \xi_1\text{Im}(\Delta_2^*\Delta_{12}) - \xi_2\text{Im}(\Delta_1^*\Delta_{12})$$

Interorbital hybridization      Interorbital pairing  
+ gap asymmetry      + dispersion asymmetry

→ Intrinsic Kerr effect evidence of odd- $\omega$  superconductivity

Komendova and ABS, PRL 119, 087001 (2017), [1]: Taylor and Kallin, PRL 108, 157001 (2012)

# Generic Three-Orbital Model



$$\hat{H}_0(\mathbf{k}) = \begin{pmatrix} \xi_1 & \epsilon_{12} & \epsilon_{13} \\ \epsilon_{12} & \xi_2 & \epsilon_{23} \\ \epsilon_{13} & \epsilon_{23} & \xi_3 \end{pmatrix}, \quad \check{\Delta}(\mathbf{k}) = \begin{pmatrix} \Delta_1 & \Delta_{12} & \Delta_{13} \\ \Delta_{12} & \Delta_2 & \Delta_{23} \\ \Delta_{13} & \Delta_{23} & \Delta_3 \end{pmatrix}$$

$xy$  (1),  $xz$  (2),  $yz$  (3) orbitals  $\rightarrow \gamma, \alpha, \beta$  bands

General interorbital pairing:

$$F_{AS} = \sum_{i,j,k=1,\dots,N} \epsilon_{ijk} F_{ij}$$

$$F_{AS}(-\omega) = -F_{AS}(\omega)$$

odd- $\omega$ , odd-interorbital

$$F_S = \sum_{i \neq j=1,\dots,N} F_{ij}$$

$$F_S(-\omega) = F_S(\omega)$$

even- $\omega$ , even-interorbital



UPPSALA  
UNIVERSITET

# Examples Odd- $\omega$ Pairing

- No interorbital pairing and  $\epsilon_{ij} = \Gamma$ :

$$F_{\text{odd}} = 2\Gamma i\omega [\Delta_1(\epsilon_2 - \epsilon_3)(\epsilon_2 + \epsilon_3 + \Gamma) + |\Delta_1|^2(\Delta_3 - \Delta_2) \\ + \text{two cyclic permutations}] / D_3$$

- Only  $\epsilon_{23} \neq 0, \Delta_{23} \neq 0$  ( $xz, yz \rightarrow \alpha, \beta$  bands with hybridization,  $xy \rightarrow \gamma$  band)

$$F_{\text{odd}} = 2i\omega [\Delta_{23}(\epsilon_3 - \epsilon_2) + \epsilon_{23}(\Delta_3 - \Delta_2)] / D'_3$$

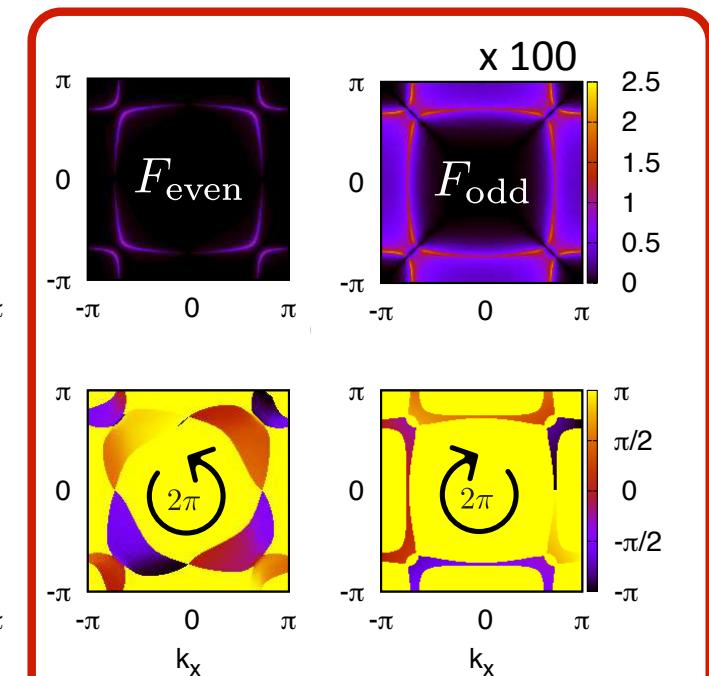
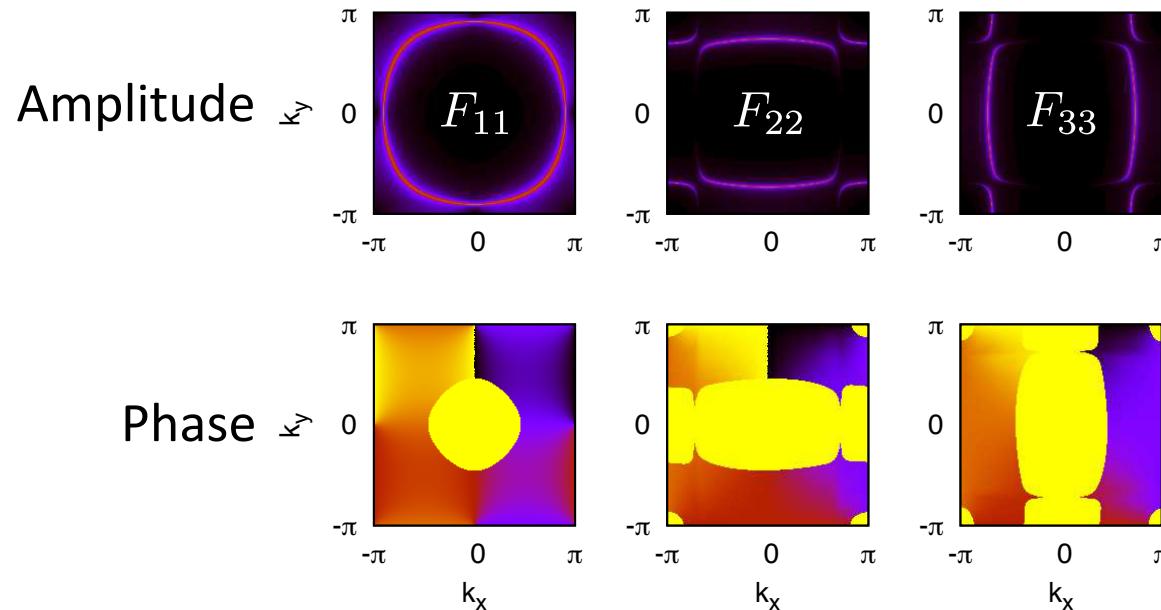
→ Odd- $\omega$  pairing requires only limited interorbital processes, exactly same as finite Kerr rotation [1]



UPPSALA  
UNIVERSITET

# Pair Amplitudes in $\text{Sr}_2\text{RuO}_4$

Only  $\epsilon_{23} \neq 0, \Delta_{23} \neq 0$

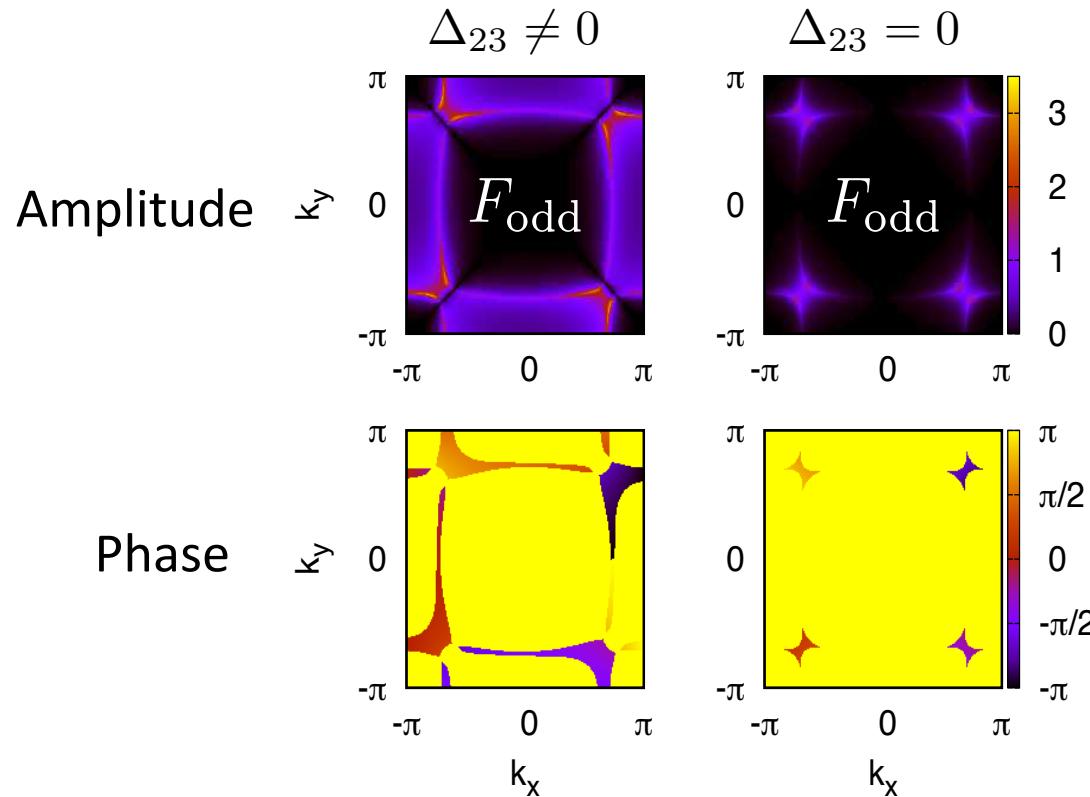


- $\alpha, \beta$  bands only
- Chiral  $p$ -wave symmetry

# Odd- $\omega$ Pairing in k-Space



UPPSALA  
UNIVERSITET



- Chiral  $p$ -wave symmetry
- Peaked at  $\alpha, \beta$  hybridization
- Nodal structure dependent on exact parameters

# Summary



UPPSALA  
UNIVERSITET

- Bulk odd- $\omega$  superconductivity in multiband systems
  - Finite interband/orbital pairing or hybridization
- Odd- $\omega$  pairing in  $\text{Sr}_2\text{RuO}_4$  measured by Kerr effect
- Future directions:
  - Other odd- $\omega$  bulk superconductors ( $\text{UPt}_3$ , ...)
  - Consequences of odd- $\omega$  pairing?
  - Band/orbital  $\rightarrow$  layer, wires, ...

# Acknowledgements

In Uppsala:



Lucia Komendova  
(UU → Belgium)



Christopher Triola



Dushko Kuzmanovski



Fariborz Parhizgar

## Collaborators:

Alexander Balatsky (Nordita/UConn)  
Jacob Linder (NTNU)



UPPSALA  
UNIVERSITET

## Financial support:



Vetenskapsrådet



# Summary



UPPSALA  
UNIVERSITET

- Bulk odd- $\omega$  superconductivity in multiband systems
  - Finite interband/orbital pairing or hybridization
- Odd- $\omega$  pairing in  $\text{Sr}_2\text{RuO}_4$  measured by Kerr effect
- Future directions:
  - Other odd- $\omega$  bulk superconductors ( $\text{UPt}_3$ , ...)
  - Consequences of odd- $\omega$  pairing?
  - Band/orbital  $\rightarrow$  layer, wires, ...