Exotic and not-so-exotic quantum states of spinful bosons in 1D

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I: Bosons with spin on 1D lattice (Mott)

spin-ordered ground states

Po-chung CHEN (National Tsing-Hua U) + Zhi-long Xue

Ming-Chiang CHUNG (AS→ National Chung-Hsing U) + Chao-Chun Huang

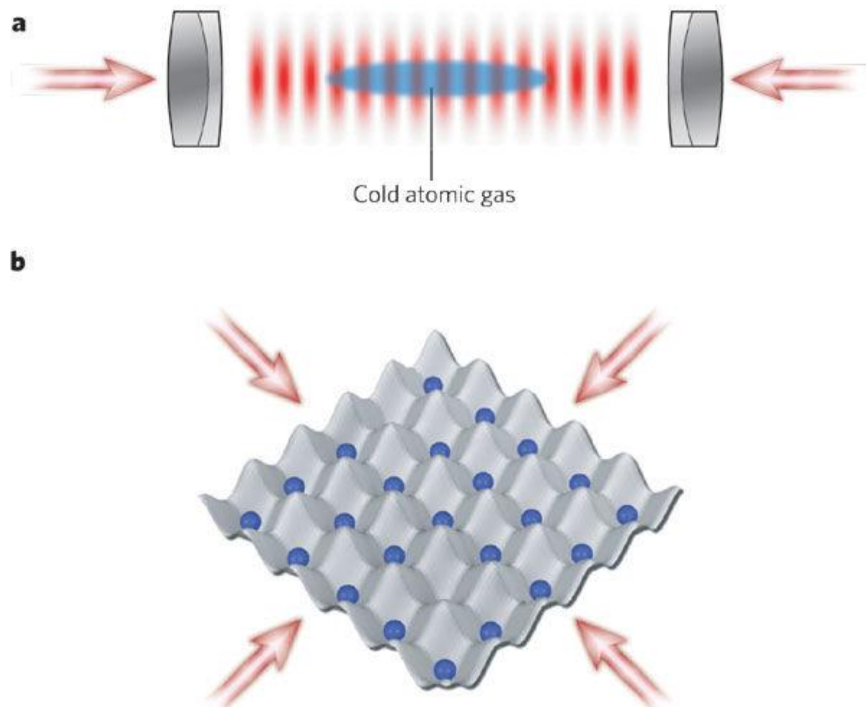
Ian McCulloch (Queensland, Australia)

PRA 85, 011601 (2012), PRL 114, 145301 (2015);

II: Spin-incoherent Luttinger liquid

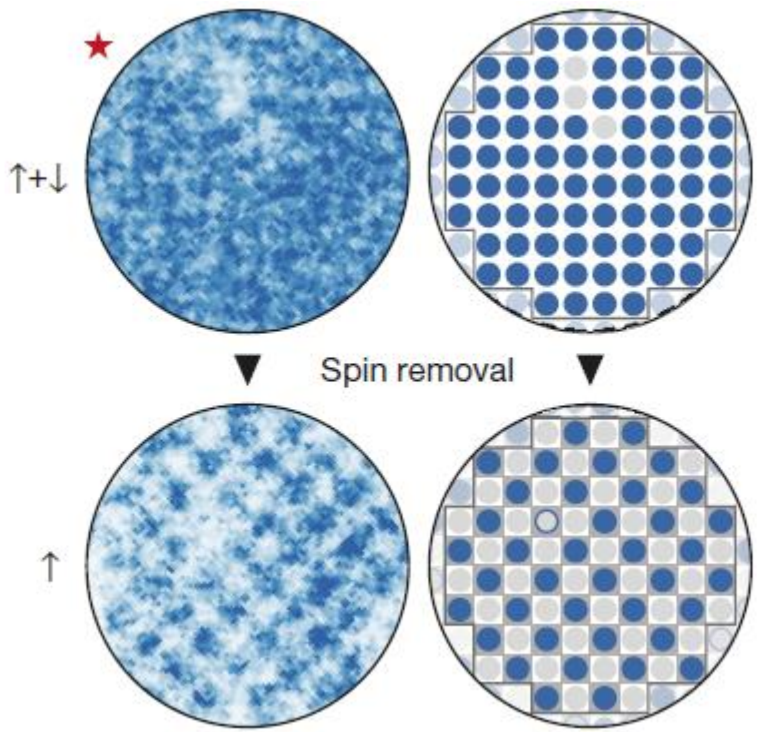
spin-disordered

Hsiang-Hua Jen (AS) **PRA 94, 033601 (2016), PRA 95, 053631 (2017)**


a
Cold atomic gas
b

^{6}Li (fermion)

↑↓= two different hyperfine states



Greiner, Nature, 2017

Bosons:

one Boson, one orbital per site (well)

Spin-1:

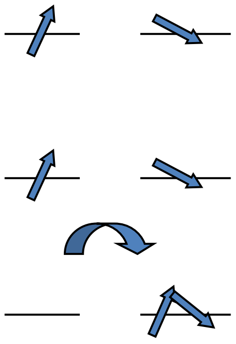
$$H = \sum H_{ij}$$

$$H_{ij} = \varepsilon_0 P_{ij}^{(0)} + \varepsilon_2 P_{ij}^{(2)}$$

$$\varepsilon_0 = -4t^2 / U_0; \varepsilon_2 = -4t^2 / U_2$$

$$U_0 \propto a_0 > 0, U_2 \propto a_2 > 0$$

$$\varepsilon_0, \varepsilon_2 < 0$$



virtual states with two particles per site

on-site energy spin dependent

Spin-2

$$H_{ij} = \varepsilon_0 P_{ij}^{(0)} + \varepsilon_2 P_{ij}^{(2)} + \varepsilon_4 P_{ij}^{(4)}$$

$$\varepsilon_0, \varepsilon_2, \varepsilon_4 < 0$$

Bosons: one Boson, one orbital per site (well)

Spin-1:

$$H = \sum H_{ij}$$

$$H_{ij} = \varepsilon_0 P_{ij}^{(0)} + \varepsilon_2 P_{ij}^{(2)}$$

$$\varepsilon_0 = -4t^2 / U_0; \varepsilon_2 = -4t^2 / U_2$$

$$U_0 \propto a_0 > 0, U_2 \propto a_2 > 0$$

$$\varepsilon_0, \varepsilon_2 < 0$$

$$H^{int}_{ij} = \varepsilon_0 + J\,(S_i \cdot S_j) + K\,(S_i \cdot S_j)^2$$

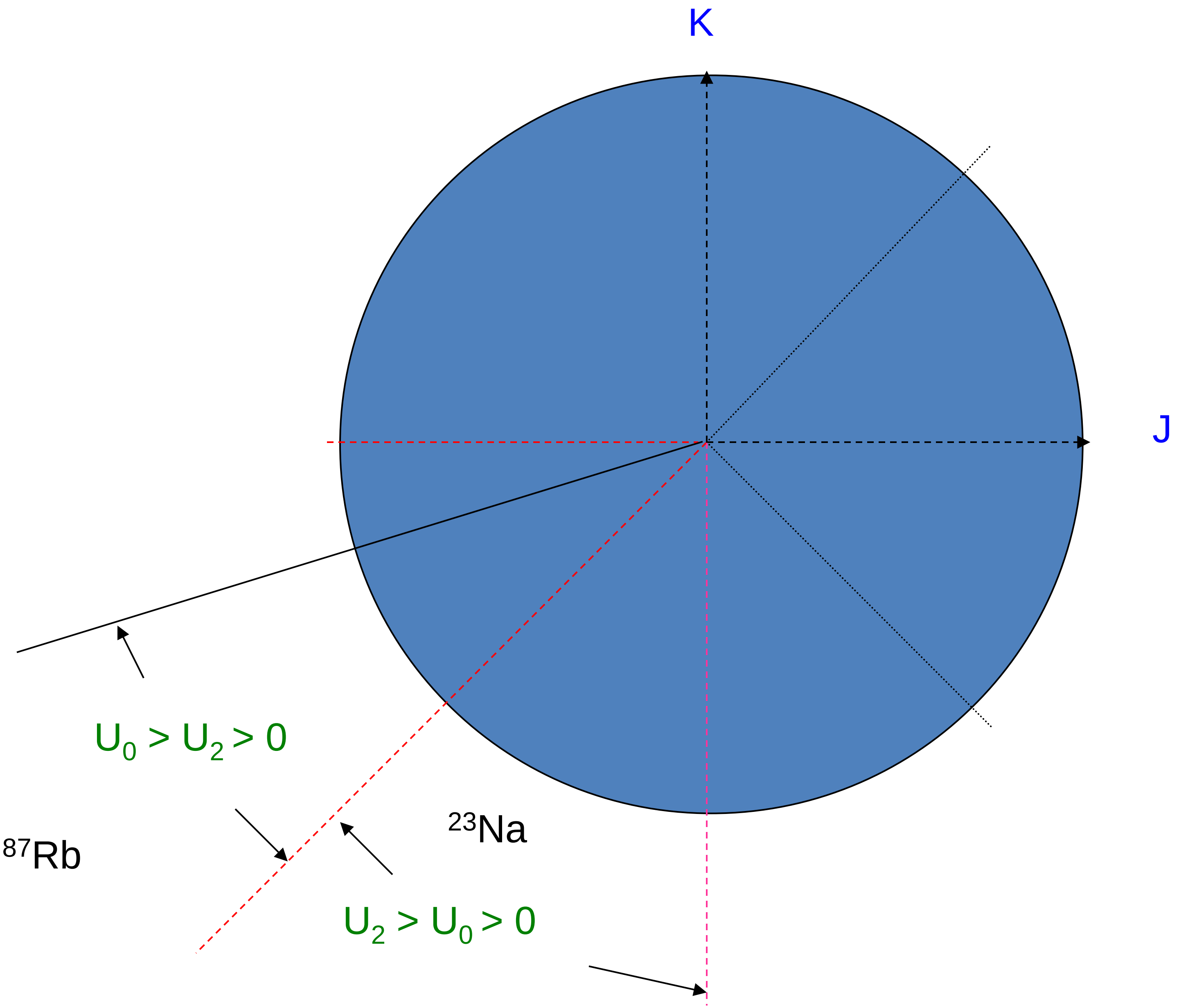
Spin-2

$$H_{ij} = \varepsilon_0 P_{ij}^{(0)} + \varepsilon_2 P_{ij}^{(2)} + \varepsilon_4 P_{ij}^{(4)}$$

$$\varepsilon_0, \varepsilon_2, \varepsilon_4 < 0$$

$$H^{int}_{ij} = \varepsilon_0 + J_1\,(S_i \cdot S_j) + \ldots + J_4\,(S_i \cdot S_j)^4$$

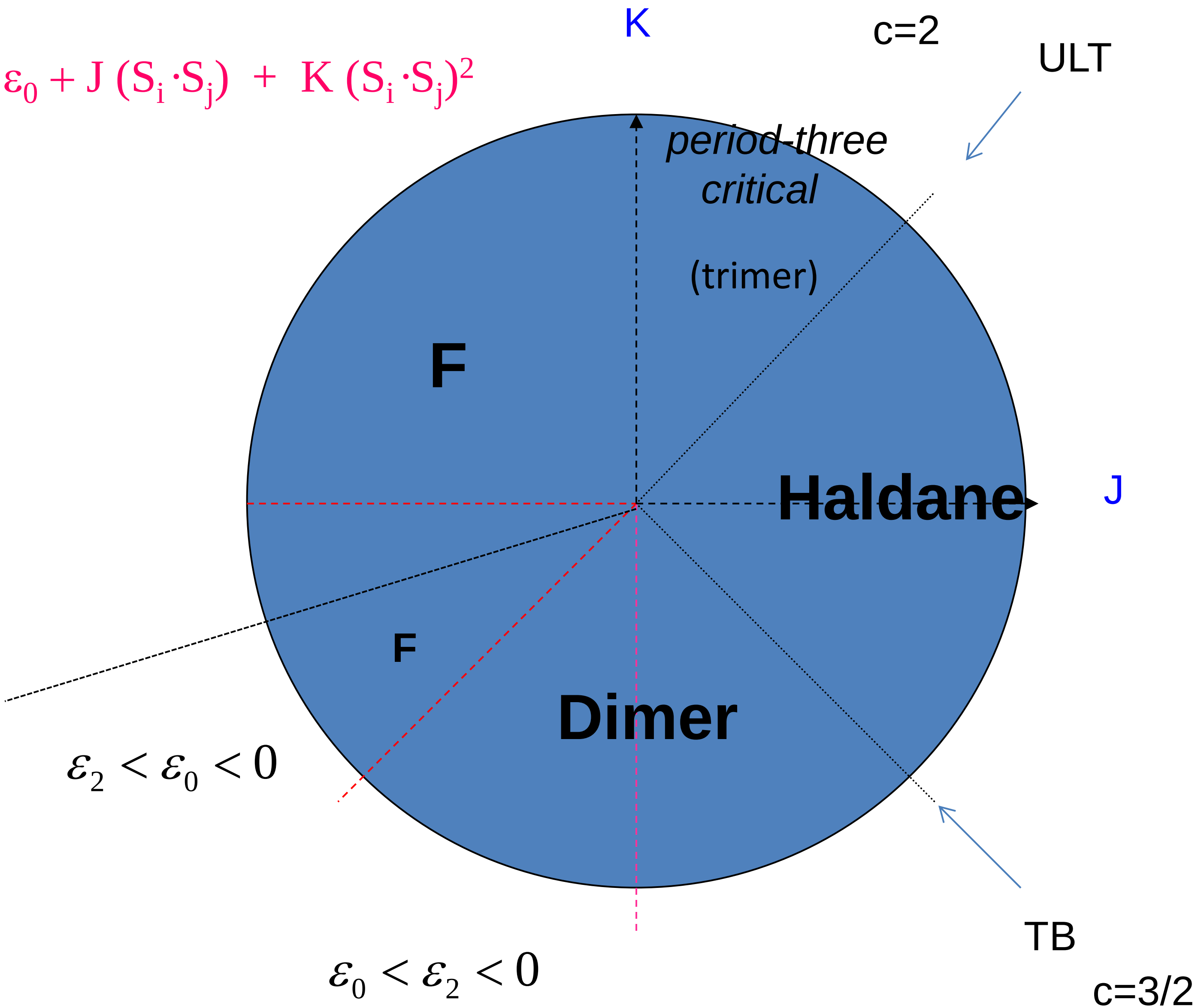
$$H^{int}_{ij} = \varepsilon_0 + J\,(S_i \cdot S_j) + K\,(S_i \cdot S_j)^2$$

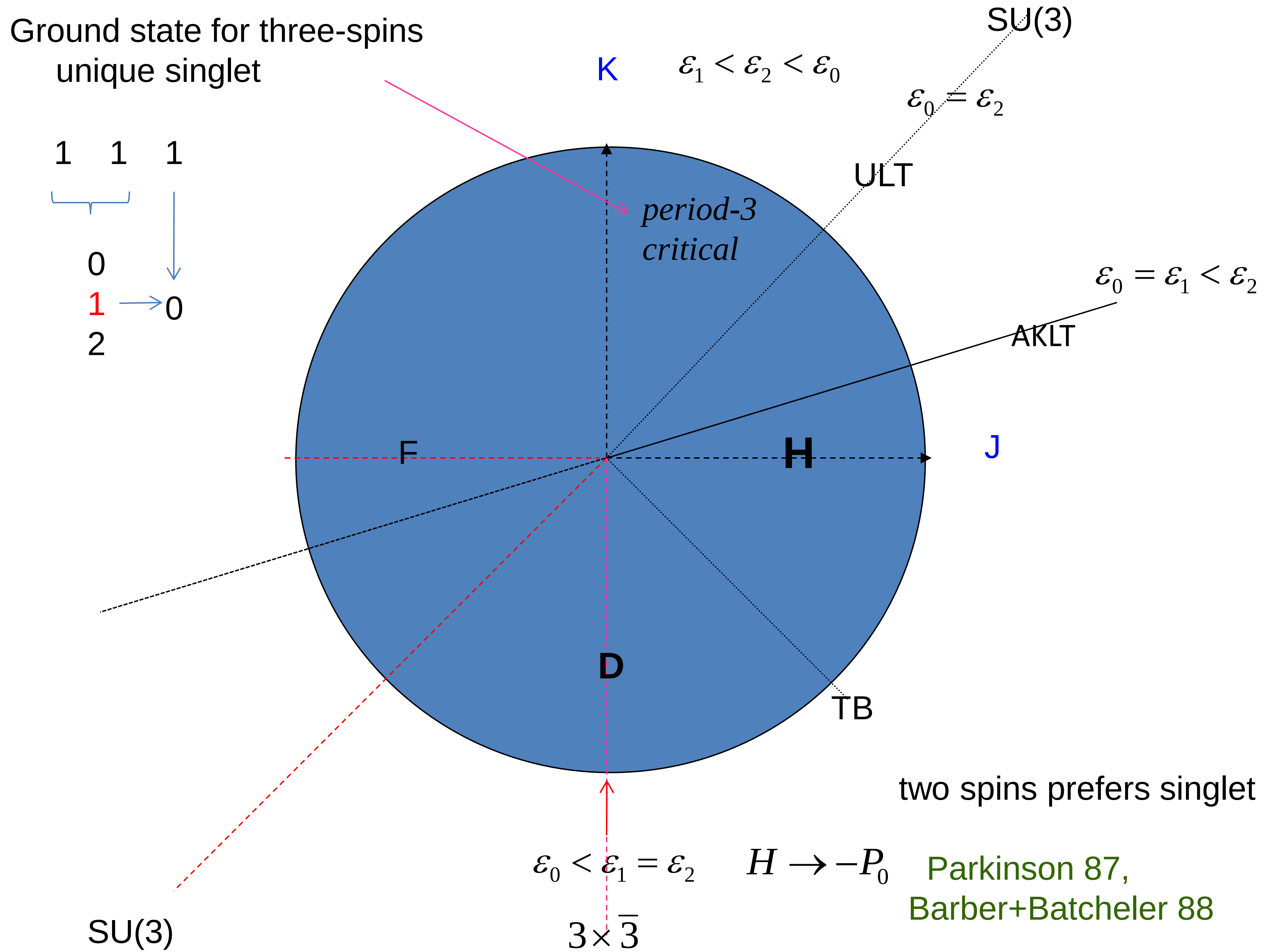


PRL 03

1D:

$$H^{int}_{ij} = \varepsilon_0 + J\,(S_i \cdot S_j) + K\,(S_i \cdot S_j)^2$$

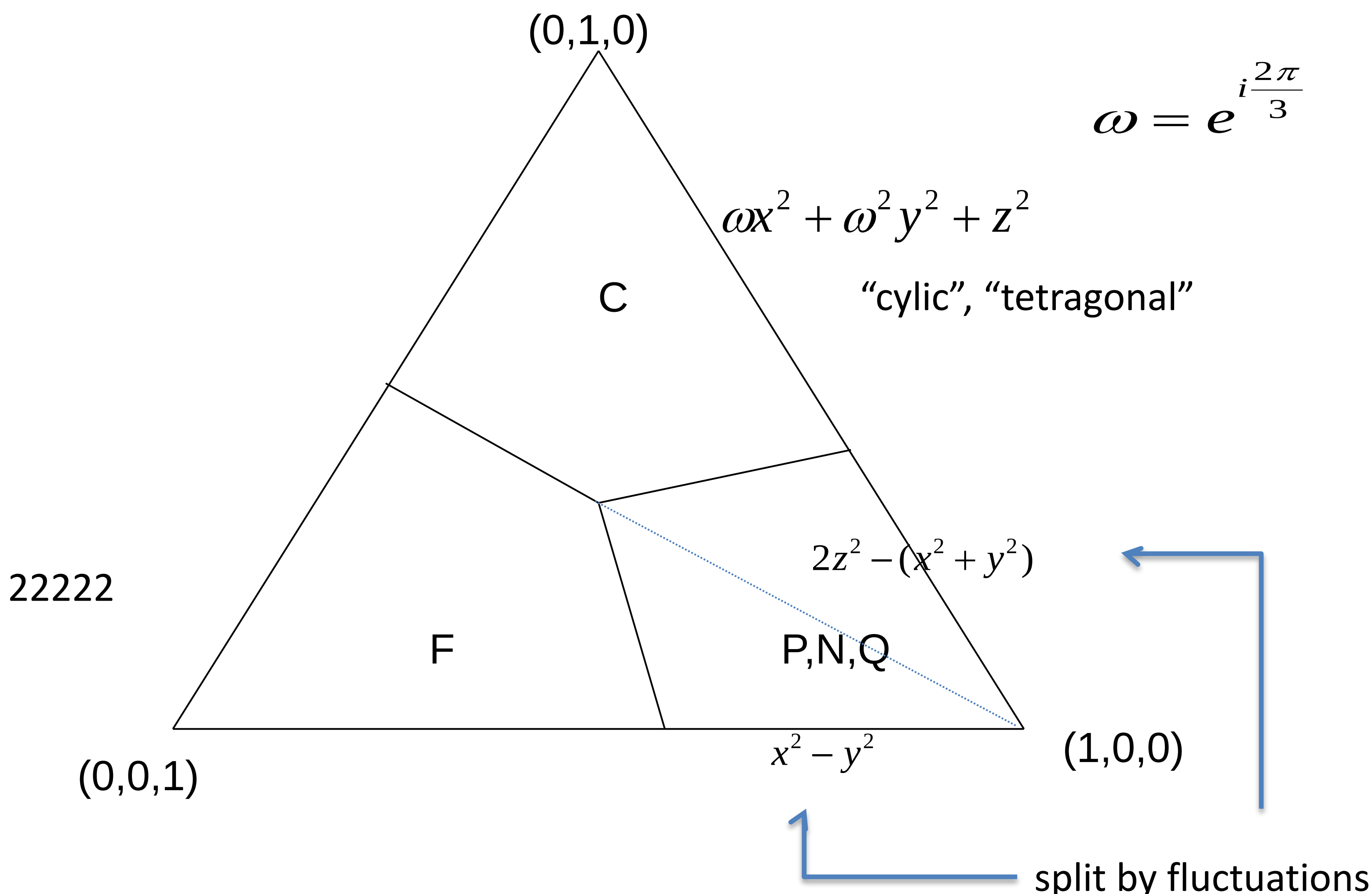



Ground state for three-spins
unique singlet
1 1 1
0
1
2
0
K
$\varepsilon_1 < \varepsilon_2 < \varepsilon_0$
SU(3)
$\varepsilon_0 = \varepsilon_2$
ULT
period-3
critical
$\varepsilon_0 = \varepsilon_1 < \varepsilon_2$
AKLT
F
H
J
D
TB
two spins prefers singlet
$\varepsilon_0 < \varepsilon_1 = \varepsilon_2$
$H \to -P_0$
Parkinson 87,
Barber+Batcheler 88
SU(3)
$3 \times \overline{3}$

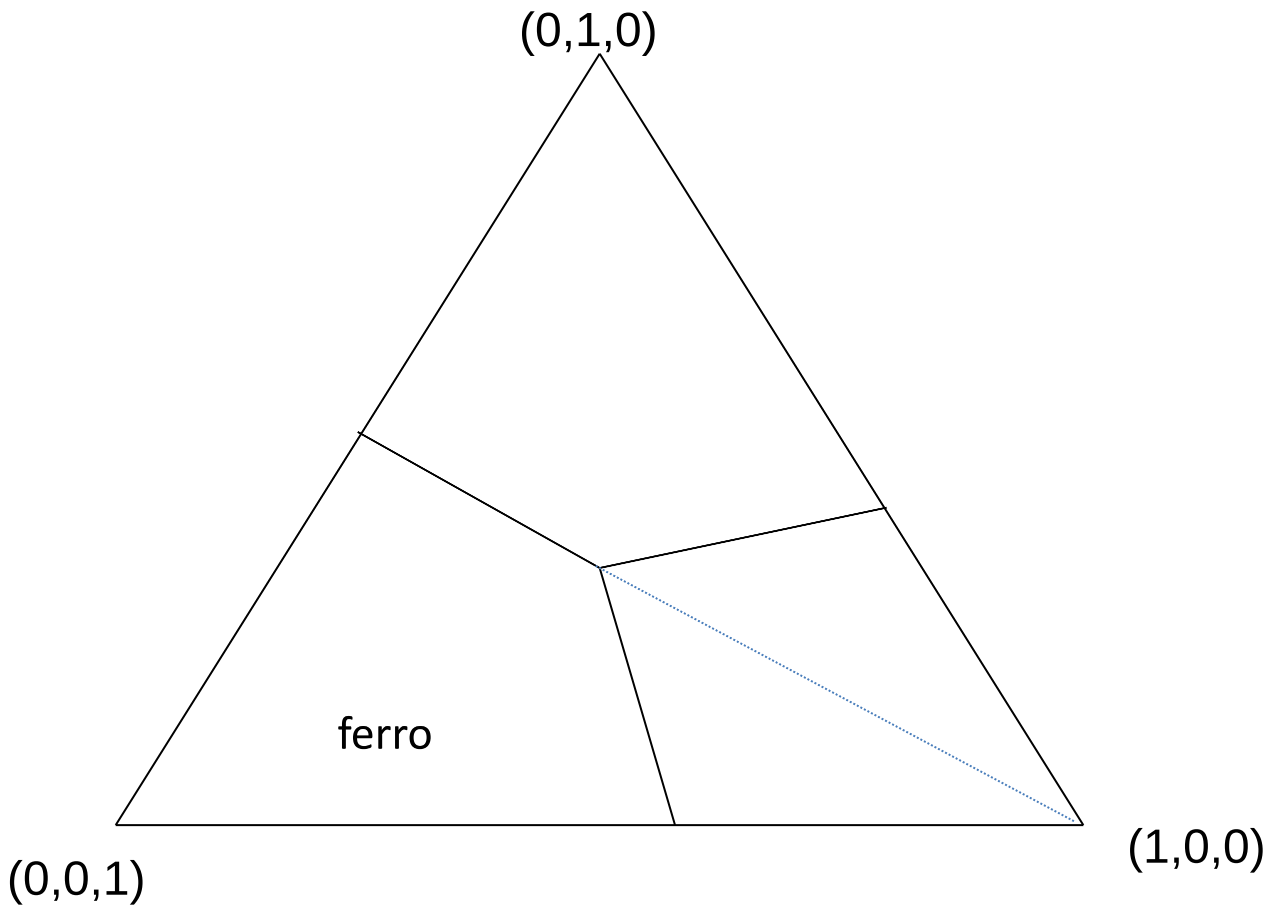
Spin-2 Mean-field:

$$(x_0, x_2, x_4) = (\varepsilon_0, \varepsilon_2, \varepsilon_4)/(\varepsilon_0 + \varepsilon_2 + \varepsilon_4)$$

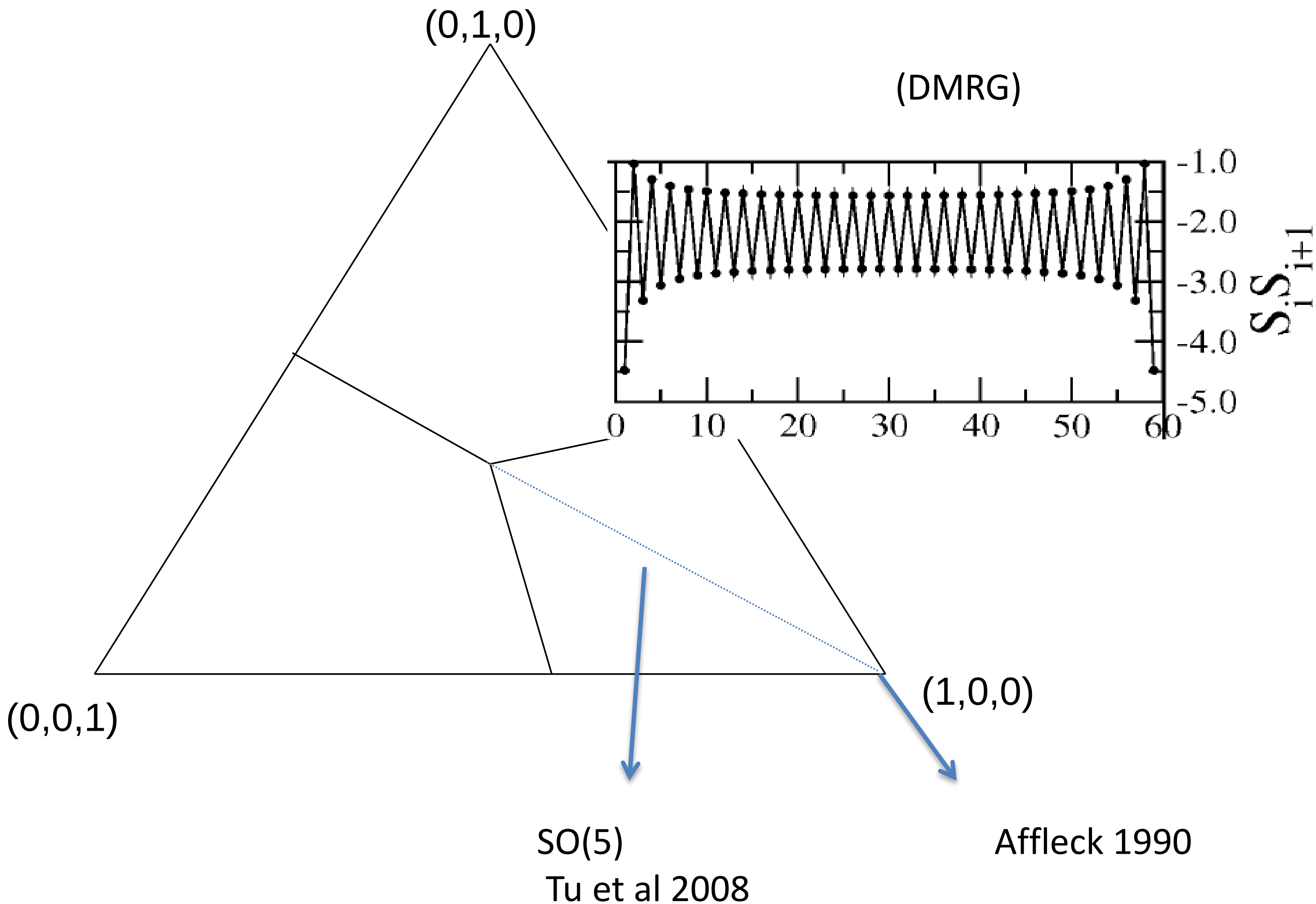
$$x_0 + x_2 + x_4 = 1 \qquad x_{0,2,4} \geq 0$$

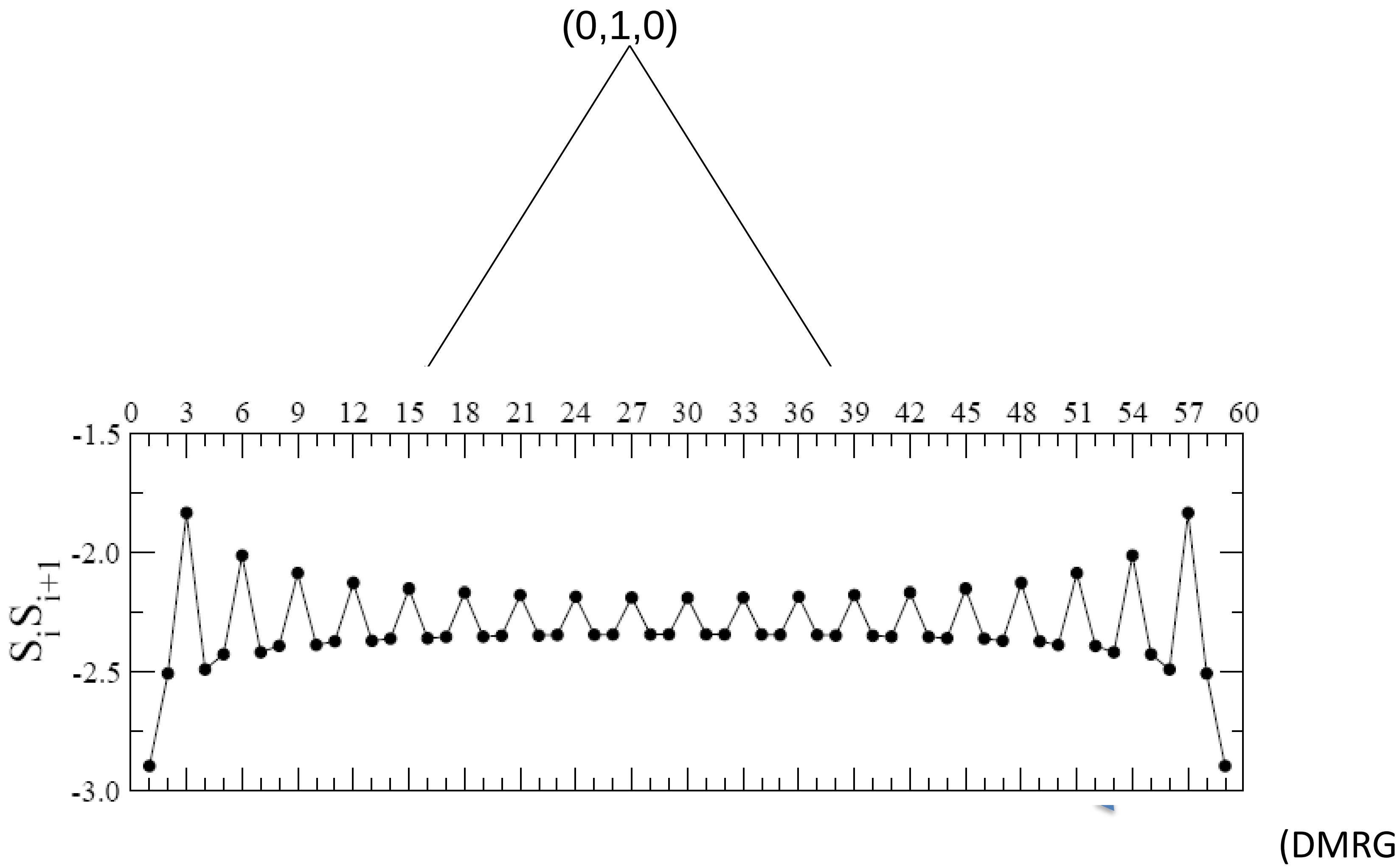


Ciobanu, Yip and Ho, 2000

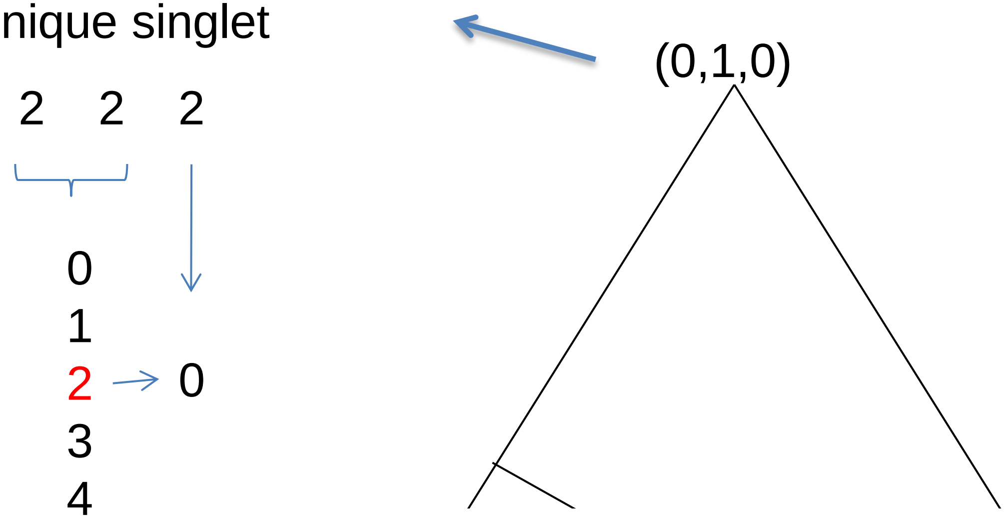


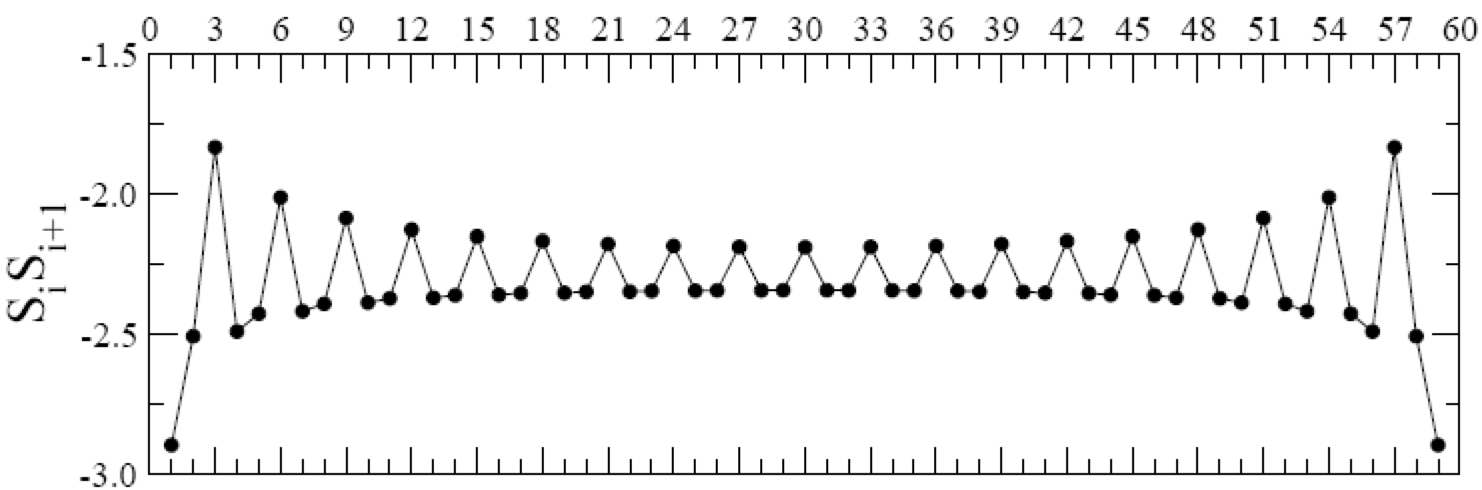
$$\varepsilon_4 < \varepsilon_{0,2}$$


(0,1,0)
(DMRG)
-1.0
-2.0
-3.0
-4.0
-5.0
$S_i S_{i+1}$
0
10
20
30
40
50
60
(0,0,1)
(1,0,0)
SO(5)
Tu et al 2008
Affleck 1990


(0,1,0)
0 3 6 9 12 15 18 21 24 27 30 33 36 39 42 45 48 51 54 57 60
-1.5
-2.0
-2.5
-3.0
$S_i S_{i+1}$
(DMRG)

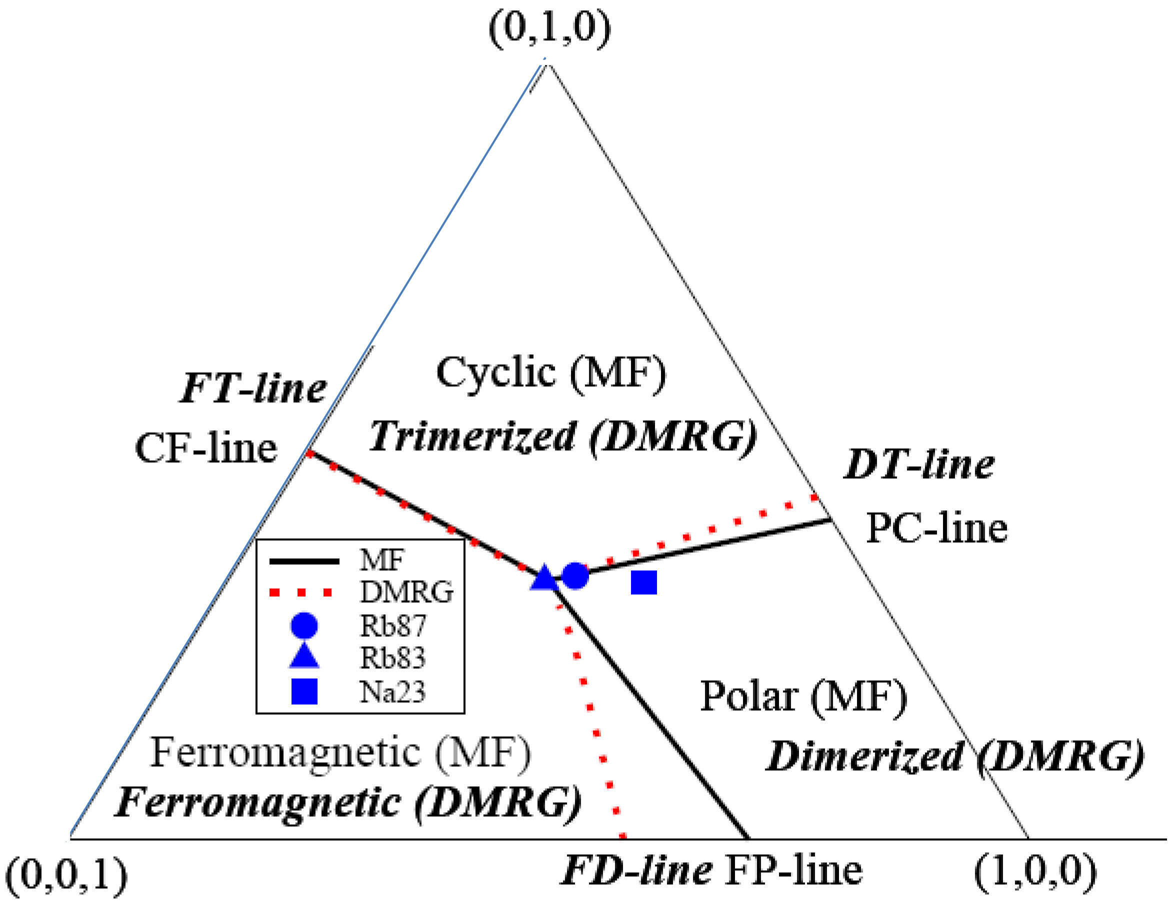
Ground state of three spins
= unique singlet

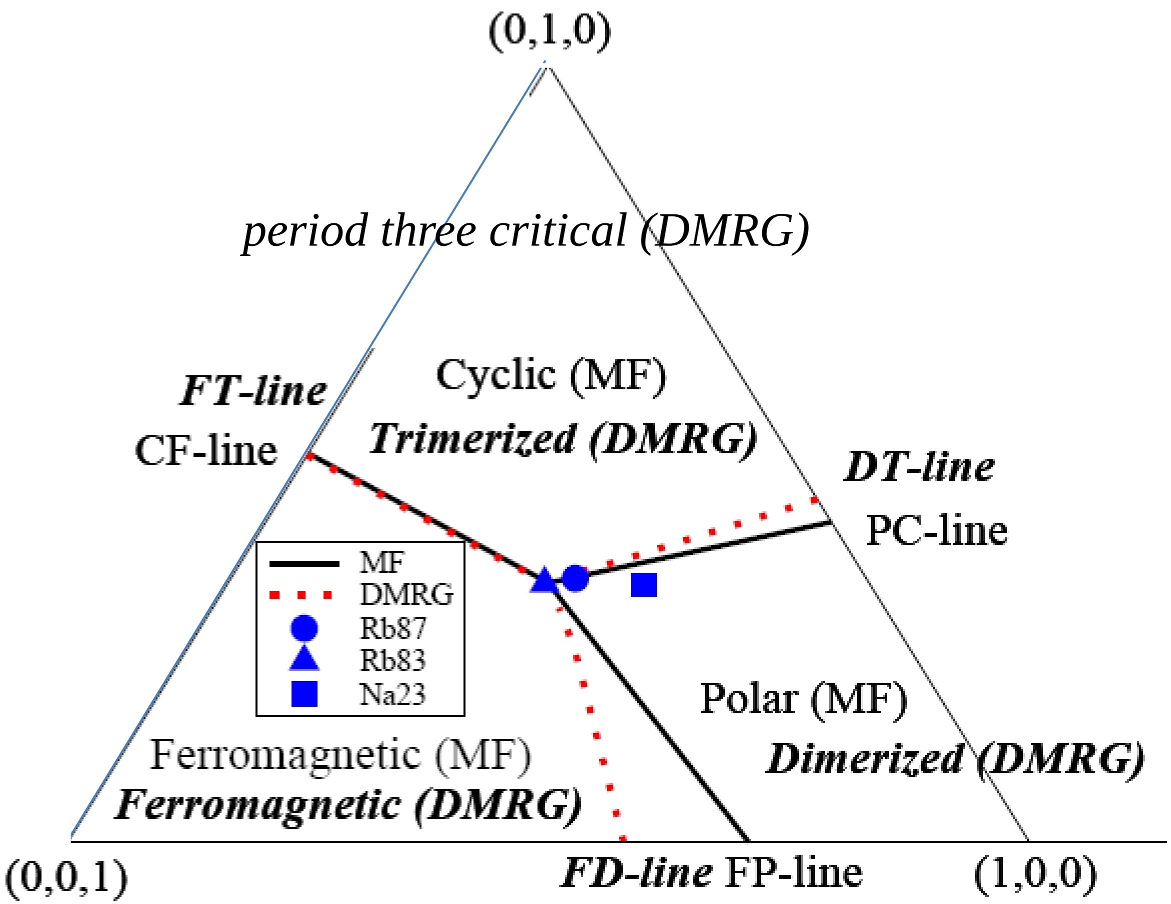




(DMRG)

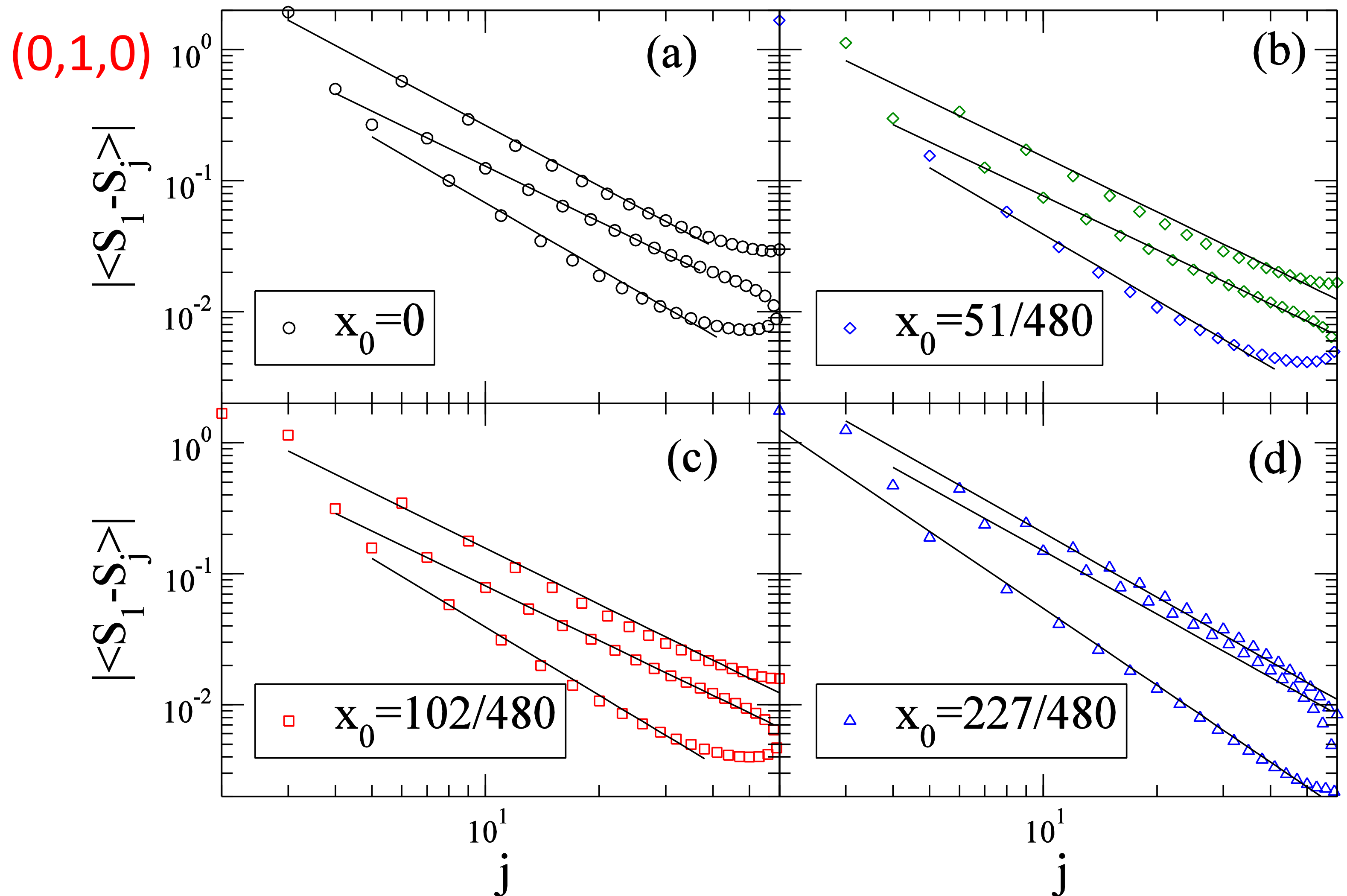
1D (DMRG, finite N)





(0,1,0)
period three critical (DMRG)
FT-line
CF-line
Cyclic (MF)
Trimerized (DMRG)
DT-line
PC-line
MF
DMRG
Rb87
Rb83
Na23
Polar (MF)
Dimerized (DMRG)
Ferromagnetic (MF)
Ferromagnetic (DMRG)
(0,0,1)
FD-line FP-line
(1,0,0)

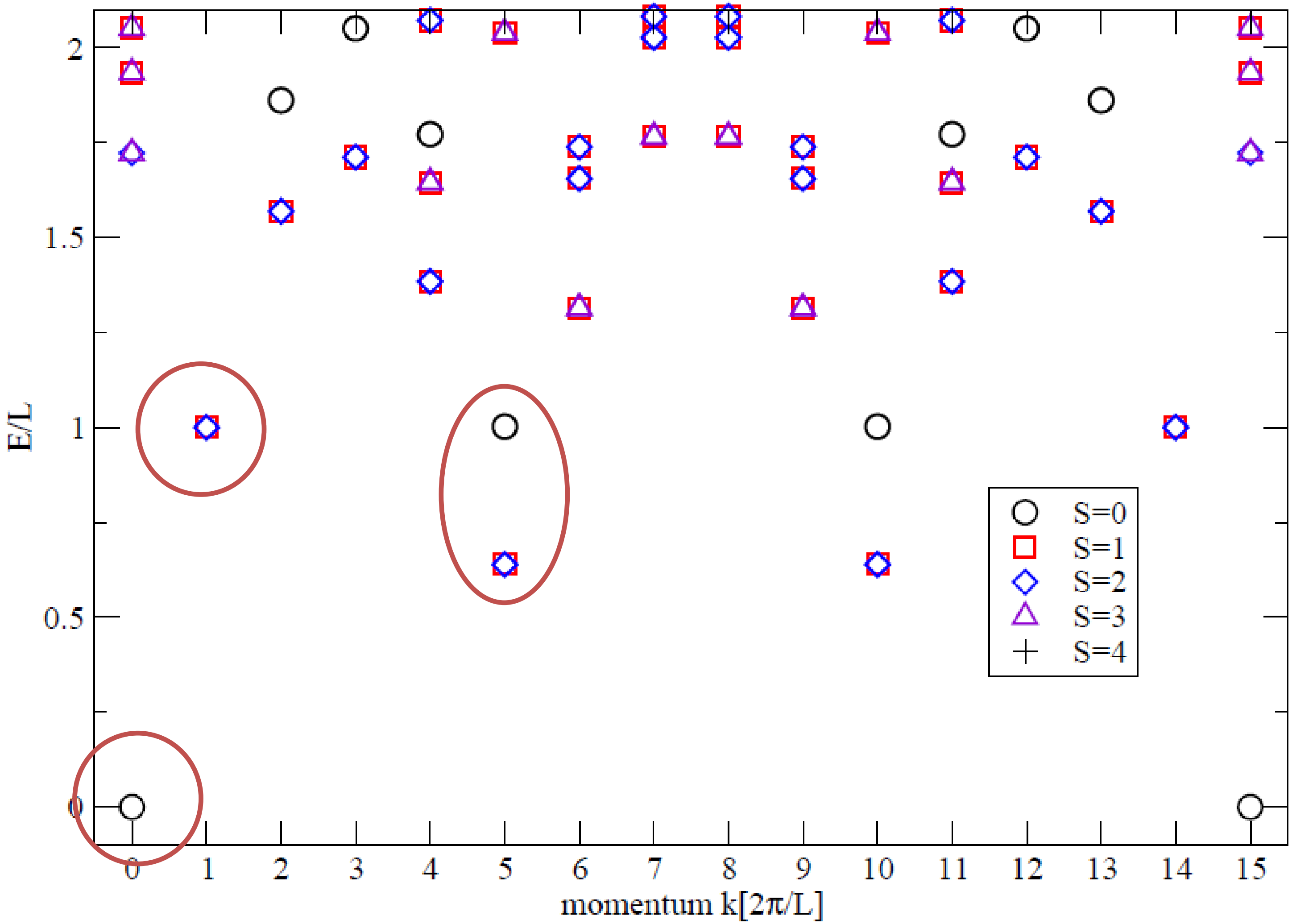
Period three-critical



Claim: CFT = $SU(3)_1$ WZW Model

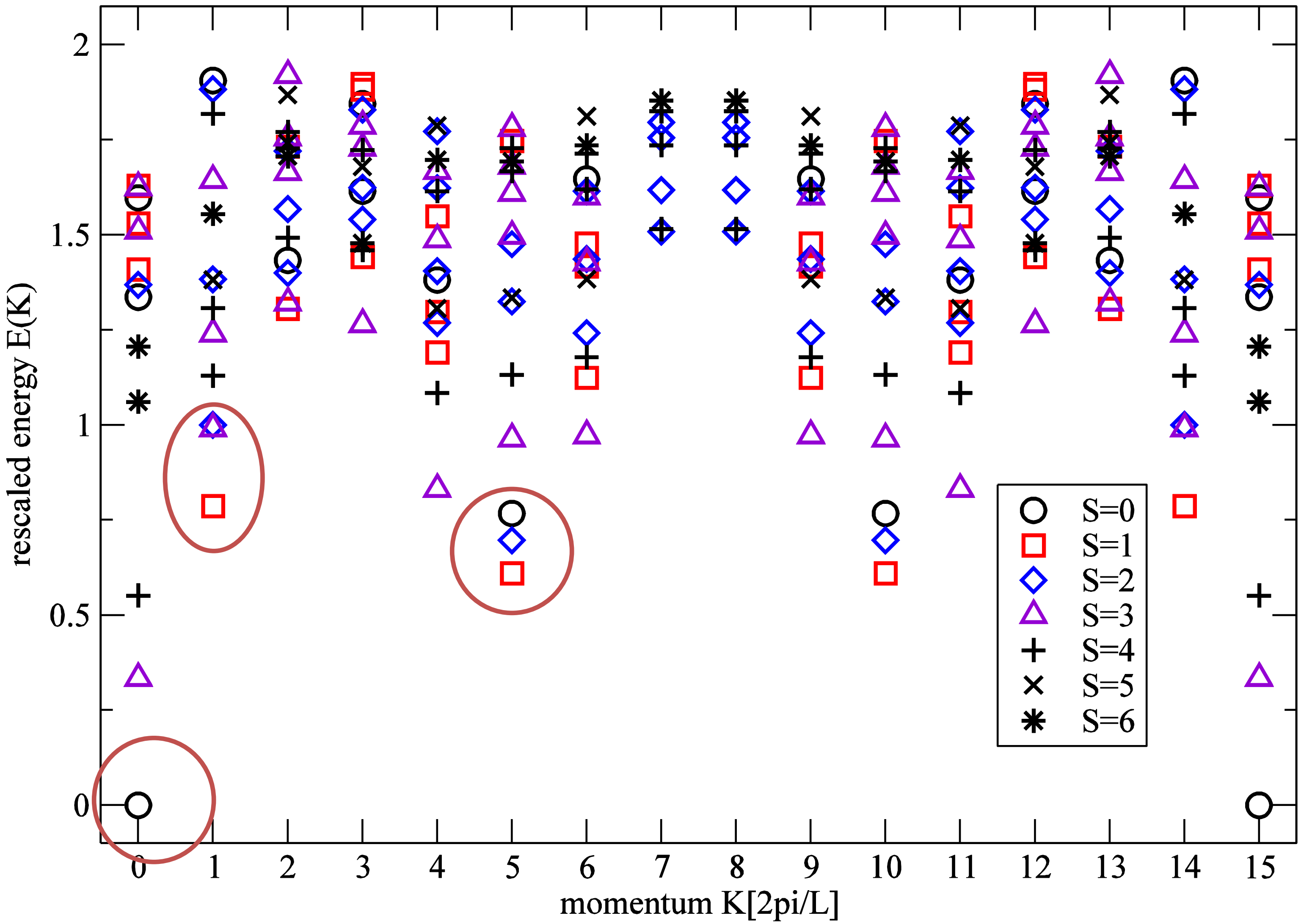
- Ground state (for L=3M)
  - S=0
- Soft mode at
  - S=0
  - S=1
  - S=2
- Spin wave excitation
  - S=1
  - S=2
- Central charge and scaling dimension:
  - $c = 2$
  - $x = 2/3$

Spin-1 $H^{int}_{ij} = J\,(S_i \cdot S_j) + K\,(S_i \cdot S_j)^2$ J=K, SU(3)



Spin-2

Excitation spectrum



Finite-Size Scaling of Ground and Excited States Energies

$$\frac{E_0(L)}{L} = \epsilon_\infty - \frac{\pi}{6L^2} c v$$

ground state

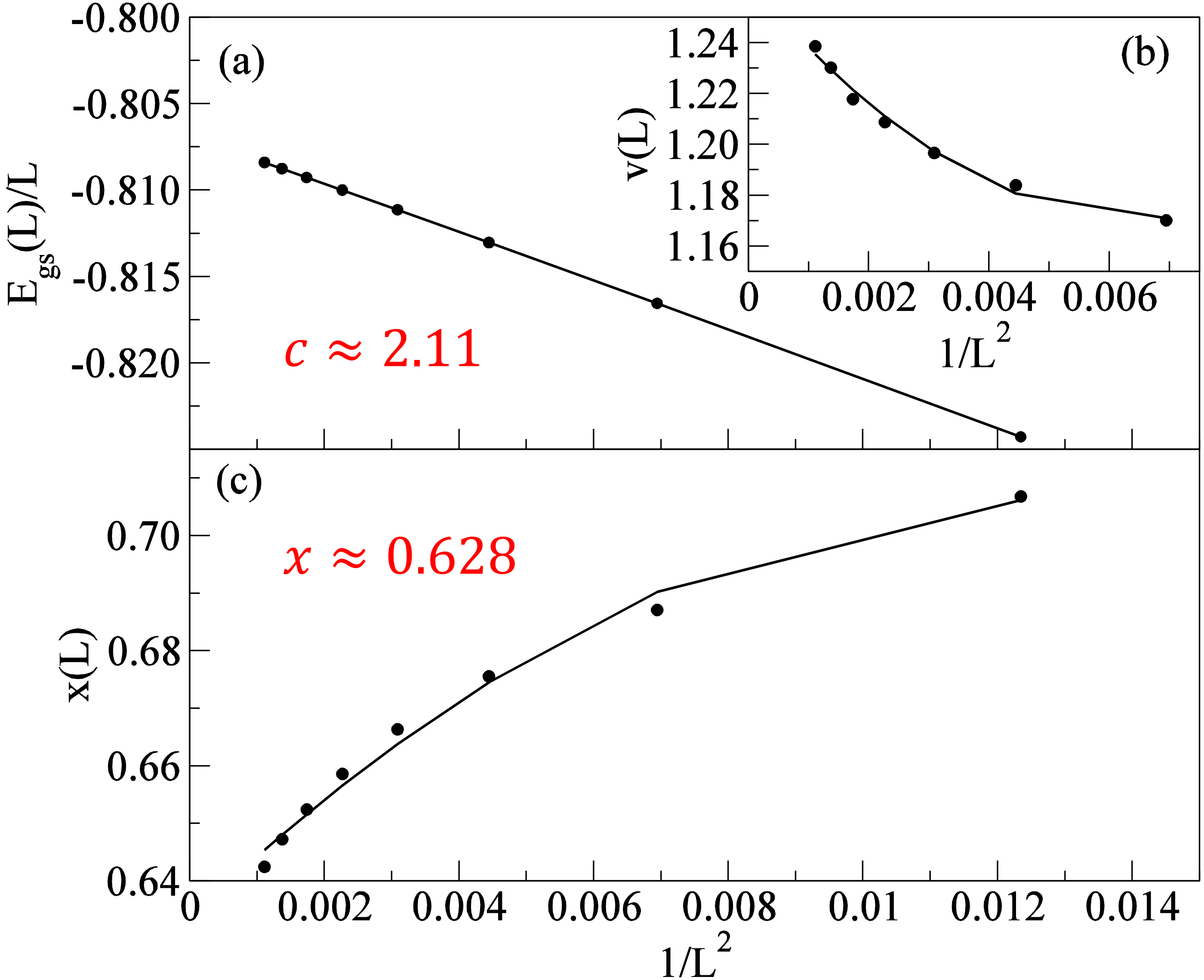
$$\frac{E_i(L) - E_0(L)}{L} = \frac{2\pi v}{L^2}\left(x_i + \frac{d_i}{\ln L}\right)$$

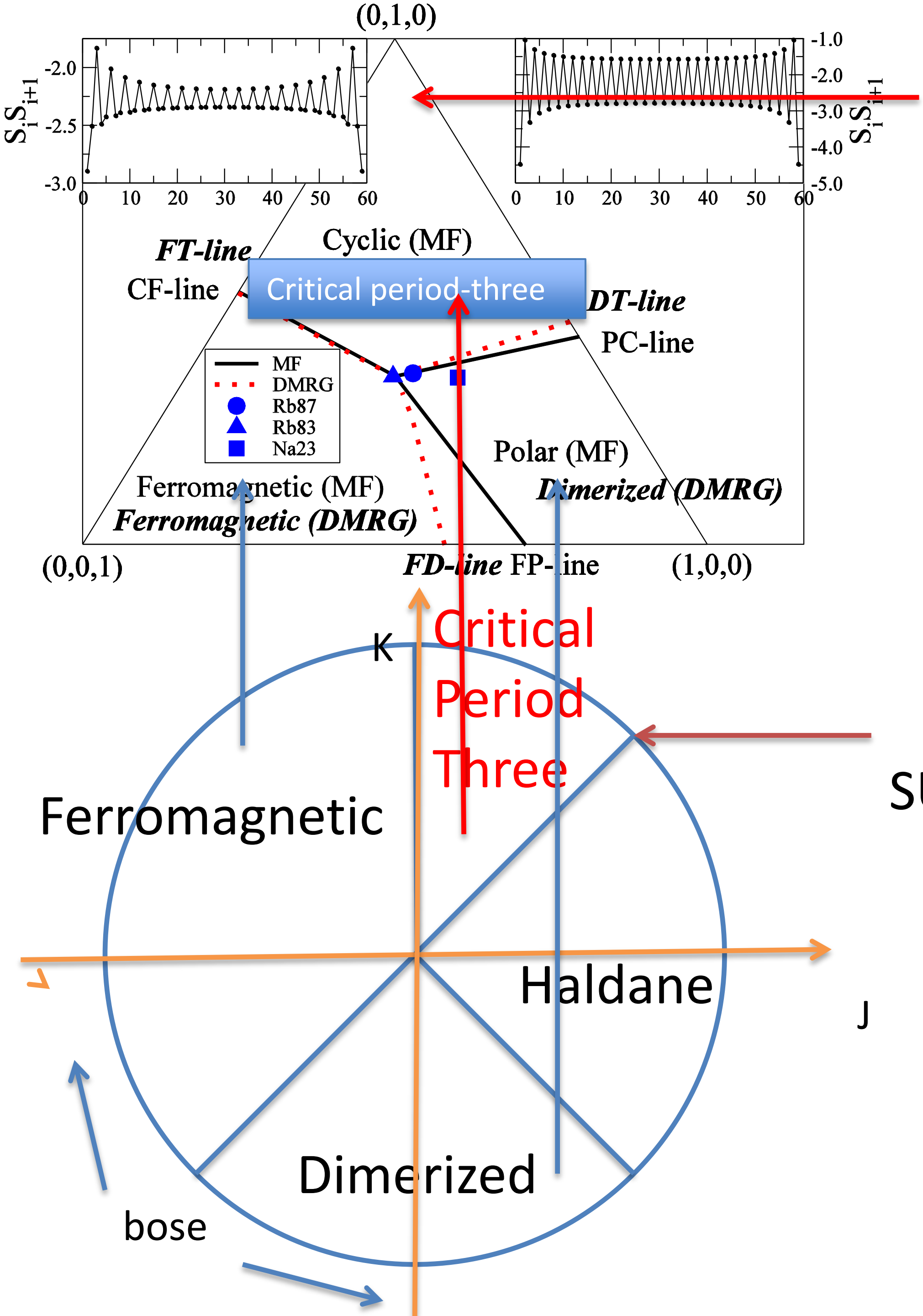
$k = 2\pi / 3$

$$\sum_S (2S+1) d_S = 0$$

(v from excitation at 2π/L)

SU(3)$_1$ WZW Model





Emergent SU(3) Symmetry

$SU(3)_1$ WZW Model

S=2, boson sector

SU(3) Symmetry

S=1

$$J(\vec{S}_1 \bullet \vec{S}_2) + K(\vec{S}_1 \bullet \vec{S}_2)^2$$

Summary:

Spinor Bosons in optical lattice

realize spin Hamiltionians usually not available in electronic systems

Spin 1

Mean-field: ferro, nematic/quadrupolar

1D: ferro, dimer

Spin 2:

Mean-field: ferro, nematic/quadrupolar, cyclic/tetragonal

1D: ferro, dimer, trimer (period-three critical)

field theory ?

(c.f. spin-1 Itoi + Kato)

Spin incoherent Luttinger liquid

1D Strongly interacting bose gas
no lattice or incommensurate

Strongly repulsive → weak exchange
(infinite repulsion, no exchange)

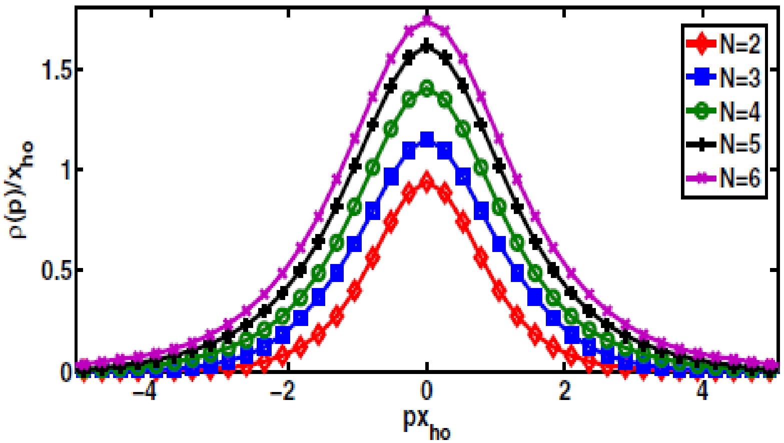
All spin configurations degenerate

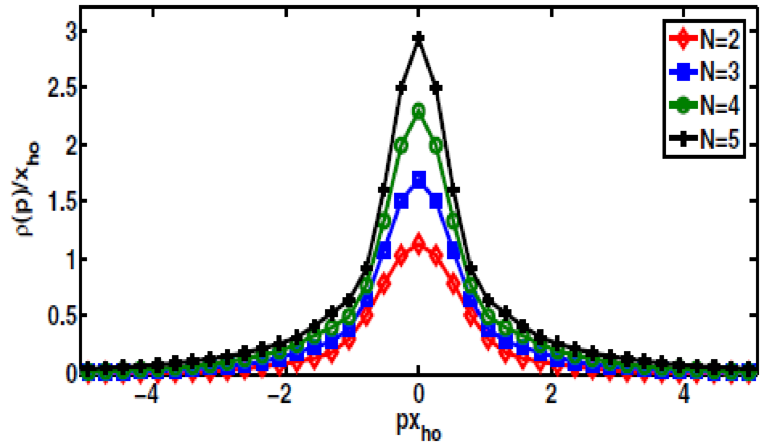
Short range order (rather than quasi-long range for spinless)

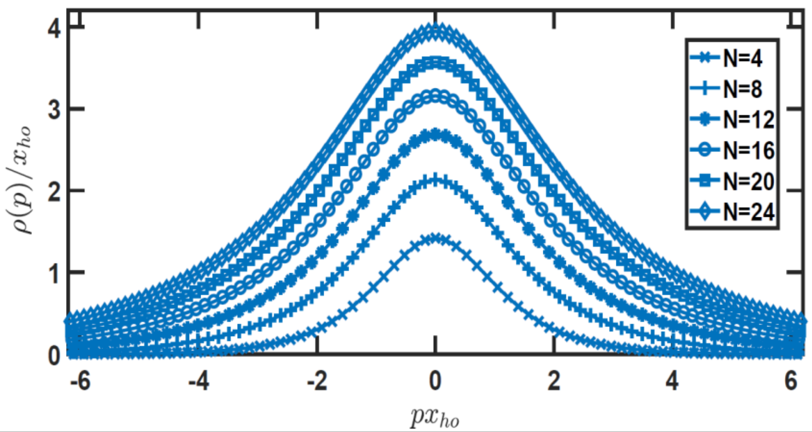
Spin 1

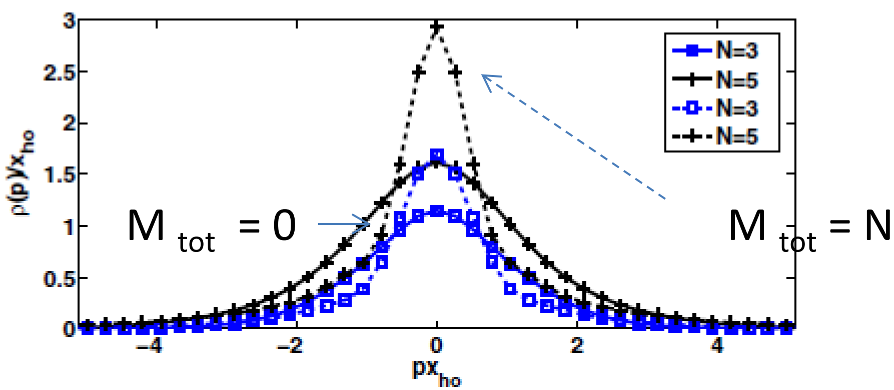
$M_{tot} = 0$ ground state manifold

Spinless, or $M_{tot} = N$









Broader momentum distribution but smaller $1/p^4$ tail

Ground state of three spins
= unique singlet

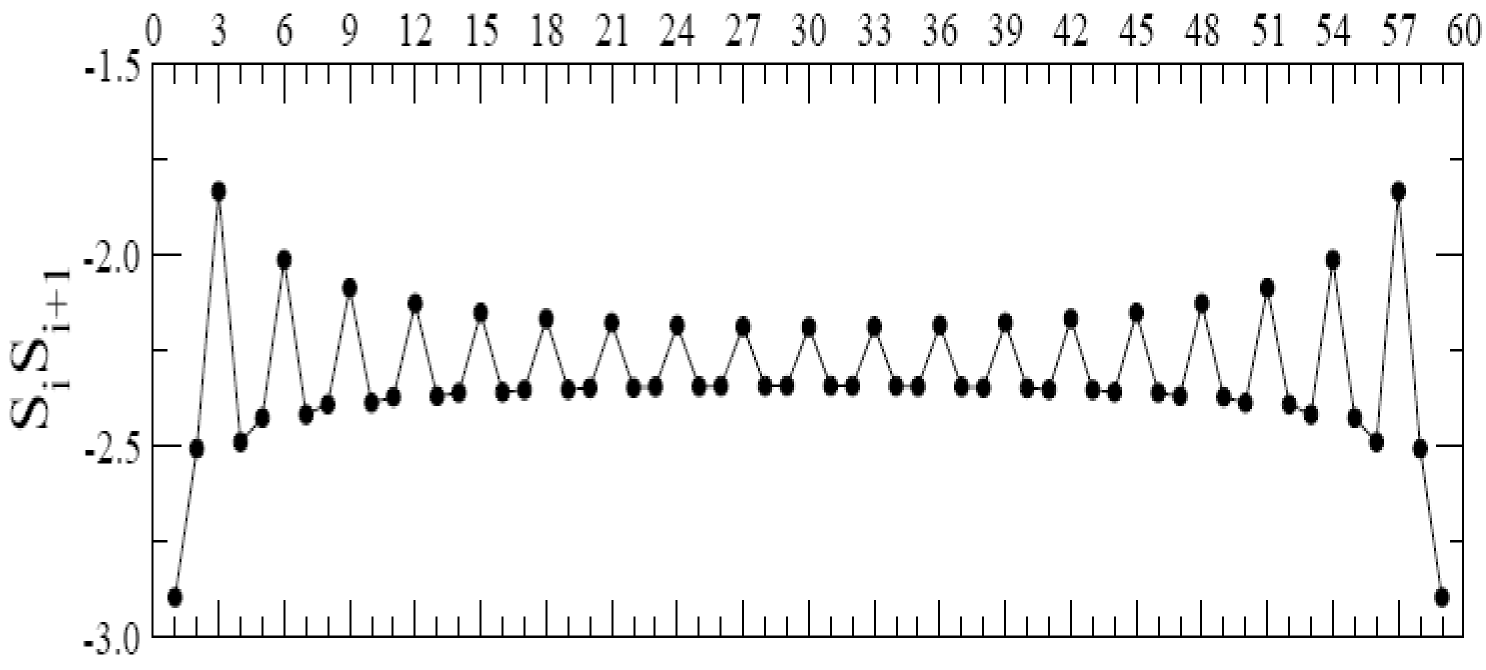
2 2 2

0
1
2 → 0
3
4

(0,1,0)

gap, order parameter
→ 0 as N→ 0

period three critical (DMRG)



(0,1,0)

Influence of a nearby symmetric phase

- strong finite-size/truncation effects
- low energy states at k=0