Superlattice systems as a testbed of correlated topological classification

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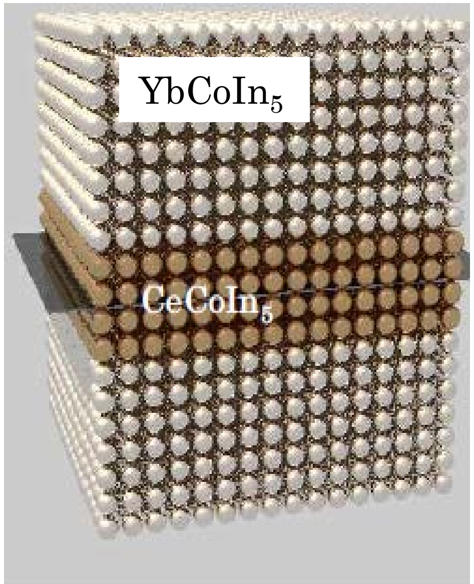
Plan of this talk

Main topic

Experimental platform of
reduction of topological classification

Part 1

Superlattice of $CeCoIn_5/YbCoIn_5$



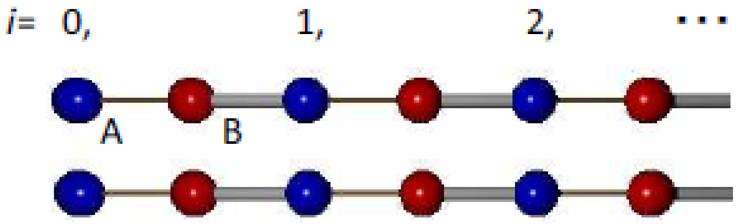
$$\mathbb{Z}^2 \to \mathbb{Z} \times \mathbb{Z}_8$$

TY-Daido-Yanase-Kawakami
PRL 118, 147001 (2016)

Part 2

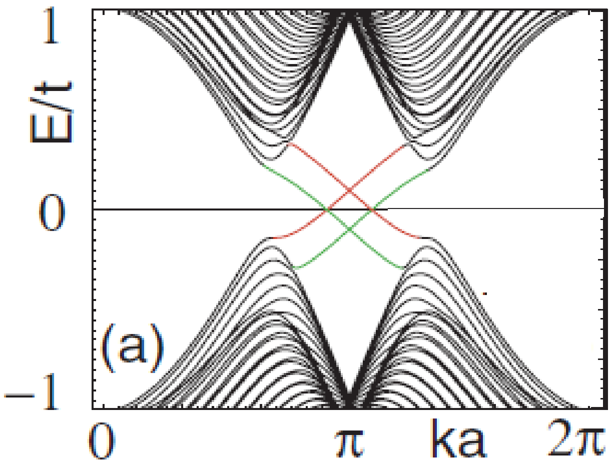
Ultracold dipolar fermions

$$\mathbb{Z} \to \mathbb{Z}_4$$



TY-Danshita-Peters-Kawakami
arXiv. 1711.xxxx

Introduction



C. L. Kane *et al.* (2005)

Topological insulators

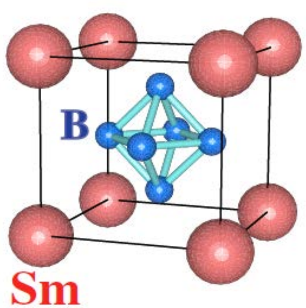
Gapless edge states
(robust against
non-magnetic perturbations)

⬍

Nontrivial band structure (Bulk)

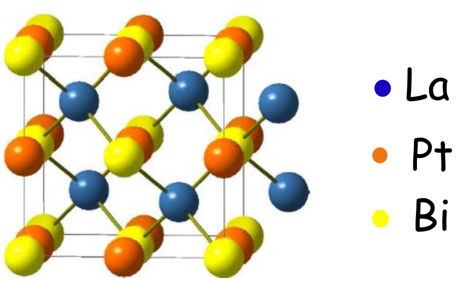
~Topological insulators in correlated systems~

SmB_6 (Kondo insulator)



Dzero *et al.* (2010)

LaPtBi etc.
(Heusler compounds)



S. Chadov *et al.* 2010

Topological phase in *d*,*f* electron systems

Topological and strong correlation

Coulomb interaction + Topology

new phenomena

- Fractional topological ins.
- Topological Mott ins.
- Reduction of topological classification

e.g., 1D class BDI, $\mathbb{Z} \to \mathbb{Z}_8$

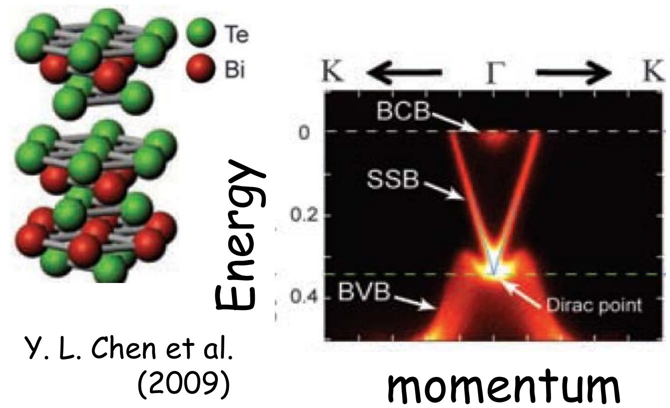
Classification of TIs/TSCs in free fermions

time-reversal   particle-hole

	TRS	PHS	SLS	$d=1$	$d=2$	$d=3$
A (unitary)	0	0	0	-	Z	-
AI (orthogonal)	+1	0	0	-	-	-
AII (symplectic)	−1	0	0	-	Z_2	Z_2
AIII (chiral unitary)	0	0	1	Z	-	Z
BDI (chiral orthogonal)	+1	+1	1	Z	-	-
CII (chiral symplectic)	−1	−1	1	Z	-	Z_2
D	0	+1	0	Z_2	Z	-
C	0	−1	0	-	Z	-
DIII	−1	+1	1	Z_2	Z_2	Z
CI	+1	−1	1	-	-	Z

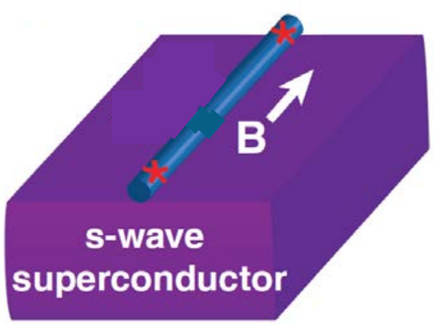
A.P. Schyder *et al.* ('08), A. Kitaev ('09), S. Ryu *et al.* ('10)

Z_2-insulator in 3D (Bi_2Te_3, Bi_2Se_3)



Y. L. Chen et al. (2009)

nanowire



V. Mourik et al. (2012)

Searching topological material

Classifying TI/TSC : useful

Reduction of topological classification

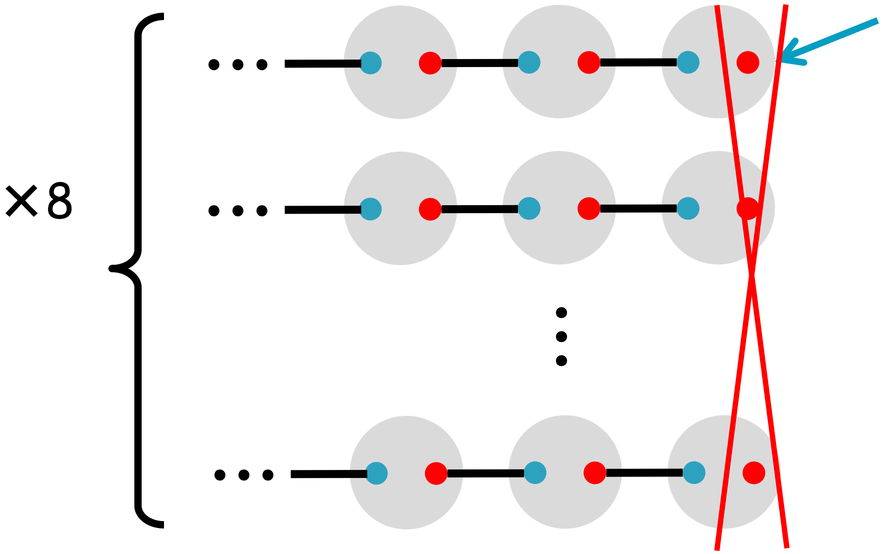
- Correlation can reduce Z classification

e.g., 1D class BDI, $\mathbb{Z} \to \mathbb{Z}_8$

Fidkowski and Kitaev (2010)

Kitaev chain (TRS, PHS)

classification Z
=[# of gapless edges]



$H_{\mathrm{int}} = \cancel{i\gamma_1\gamma_2}$

Time-reversal: $\to -i\gamma_1\gamma_2$

$H_{\mathrm{int}} = \gamma_1\gamma_2\gamma_3\gamma_4 + \gamma_5\gamma_6\gamma_7\gamma_8 + \cdots$

Gap out edge modes

	# of gapless edges							Classification result
Free-fermions	1	2	⋯	8	9	10	⋯	Z
correlated fermions	1	2	⋯	0	1	2	⋯	Z_8

[no gapless edge]=[trivial phase]

Kitaev chain ×8 :
topologically trivial!

The reduction of topological classification is addressed by many groups.

Y.-M Lu and A. V. Vishwanath (2012);
M. Levin and A. Stern (2012);
H. Yao and S. Ryu (2013);
S. Ryu and S.-C. Zhang (2012);
C. Wang, A. C. Potter, and T. Senthil (2014);
C.-T. Hsieh, T. Morimoto, and S. Ryu (2014);
Y.-Z. You and C. Xu (2014);
H. Isobe and L. Fu (2015);
T. Y and A. Furusaki (2015);
T. Morimoto, A. Furusaki, and C. Mudry (2015)

The periodic table in correlated systems is obtained in 1, 2, and 3D

Class	T	C	Γ_5	$d=1$	$d=2$	$d=3$
A	0	0	0	0	$\mathbb{Z}$	0
AIII	0	0	1	$\mathbb{Z}_4$	0	$\mathbb{Z}_8$
AI	$+1$	0	0	0	0	0
BDI	$+1$	$+1$	1	$\mathbb{Z}_8, \mathbb{Z}_4$	0	0
D	0	$+1$	0	$\mathbb{Z}_2$	$\mathbb{Z}$	0
DIII	-1	$+1$	1	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}_{16}$
AII	-1	0	0	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$
CII	-1	-1	1	$\mathbb{Z}_2, \mathbb{Z}_2$	0	$\mathbb{Z}_2$
C	0	-1	0	0	$\mathbb{Z}$	0
CI	$+1$	-1	1	0	0	$\mathbb{Z}_4$

T. Morimoto, A. Furusaki, and C. Mudry (2015)

Motivation

The reduction is a recent progress of the theoretical sides.

But...

No candidate materials for
the reduction of the classification

We propose

The CeCoIn5/YbCoIn5 superlattice as a candidate material

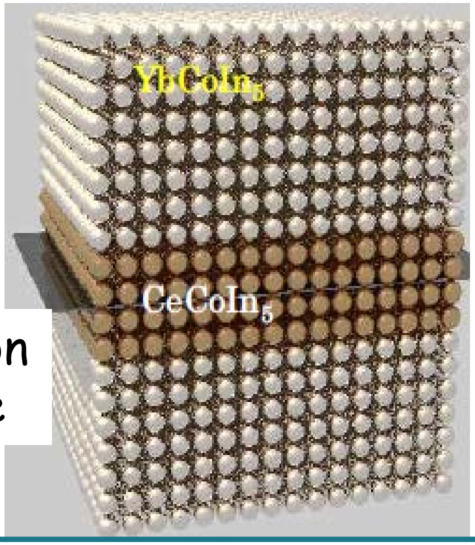
Experimental observations

Correlated lectrons are confined in $\mathrm{CeCoIn_5}$ -layers

superconducting phase for *T*~1K

Y. Mizukami, *et al.*, (2011)
S.K. Goh *et al.*, (2012)
M. Shimozawa *et al.*, (2014)

reflection plane



We find that

the superlattice: topological crystalline superconductor

mean-field level

Correlation

# of CeCoIn5 layers	$(\nu_{\mathrm{M}}, \nu_{\mathrm{tot}})$	# of Majorana	protection
2	(4,0)	4	yes
3	(1,0)	1	yes
4	(8,0)	8	NO

The superlattice: a candidate material for the reduction $\mathbb{Z}\times\mathbb{Z} \longrightarrow \mathbb{Z}\times\mathbb{Z}_8$

Results

- at a mean-field level

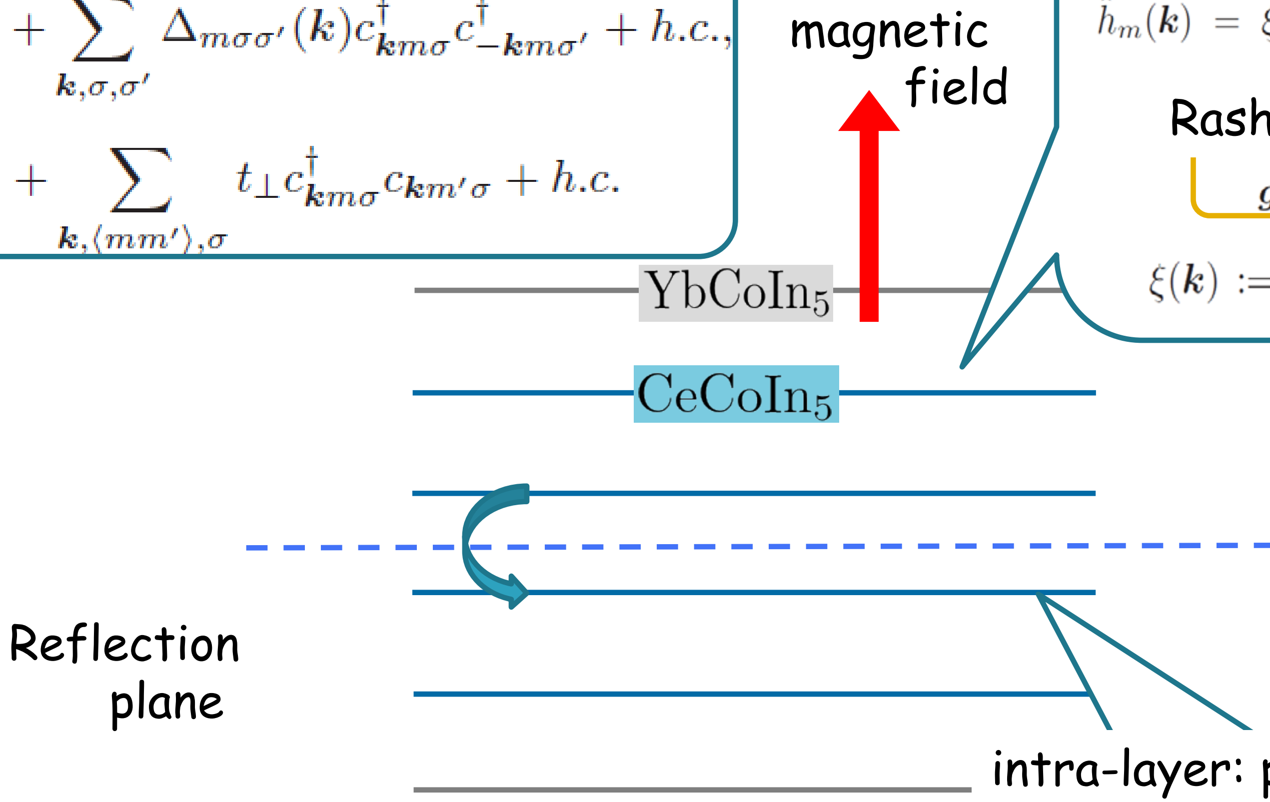
Topological crystalline superconductor

# of $CeCoIn_5$ layers	$(\nu_{\mathrm{M}}, \nu_{\mathrm{tot}})$	# of Majorana
2	(4,0)	4
3	(1,0)	1
4	(8,0)	8

Non-interacting case: BdG-Hamiltonian with magnetic field

BdG-Hamiltonian for CeCoIn5 layers

$$H = \sum_{\boldsymbol{k},m,\sigma,\sigma'} c^\dagger_{\boldsymbol{k}m\sigma}[\hat{h}_m(\boldsymbol{k})]_{\sigma\sigma'} c_{\boldsymbol{k}m\sigma'} + \sum_{\boldsymbol{k},\sigma,\sigma'} \Delta_{m\sigma\sigma'}(\boldsymbol{k}) c^\dagger_{\boldsymbol{k}m\sigma} c^\dagger_{-\boldsymbol{k}m\sigma'} + h.c., + \sum_{\boldsymbol{k},\langle mm'\rangle,\sigma} t_\perp c^\dagger_{\boldsymbol{k}m\sigma} c_{\boldsymbol{k}m'\sigma} + h.c.$$



intra-layer: normal part

Zeeman term

$$\hat{h}_m(\boldsymbol{k}) = \xi(\boldsymbol{k})\sigma^0 + \underline{\alpha_m \boldsymbol{g}(\boldsymbol{k})\cdot\boldsymbol{\sigma}} - \underline{\mu_B H \sigma^z}$$

Rashba term

$$\boxed{\boldsymbol{g}(\boldsymbol{k}) := (-\sin(k_y), \sin(k_x), 0)^T}$$

$$\xi(\boldsymbol{k}) := -2t\left(\cos(k_x) + \cos(k_y)\right) - \mu$$

intra-layer: pairing potential

$$\Delta_m(\boldsymbol{k}) = i\left(\psi_m(\boldsymbol{k}) - \boldsymbol{d}_m(\boldsymbol{k})\cdot\boldsymbol{\sigma}\right)\sigma^y$$

$\boxed{d_{x^2-y^2}}$

p-wave

Non-interacting case: symmetry of BdG-Hamiltonian

$$H = \frac{1}{2}\sum_{\boldsymbol{k}} \Psi^\dagger_{\boldsymbol{k}} \mathcal{H}(\boldsymbol{k}) \Psi_{\boldsymbol{k}}$$

Nambu operator

$$\Psi_{\boldsymbol{k}} := \oplus_\sigma (c_{\boldsymbol{k}1\sigma}, \cdots, c_{\boldsymbol{k}4\sigma}, c^\dagger_{-\boldsymbol{k}1\sigma}, \cdots, c^\dagger_{-\boldsymbol{k}4\sigma})^T$$

reflection symmetry R

$$\mathcal{H} = \begin{pmatrix} \mathcal{H}_+ & \\ & \mathcal{H}_- \end{pmatrix}$$

$$\mathrm{R}^2 = -1$$
$$R|\pm\rangle = \pm i|\pm\rangle$$

Symmetry class of $\mathcal{H}_+$ and $\mathcal{H}_-$

time-reversal	×	
particle-hole	✔	$C^2 = 1$

→ Class D

→ $\mathbb{Z}^2$-classification

	TRS	PHS	SLS	d=1	d=2	d=3
A (unitary)	0	0	0	-	Z	-
⋮						
D	0	+1	0	Z_2	Z	-
C	0	−1	0	-	Z	-
DIII	−1	+1	1	Z_2	Z_2	Z
CI	+1	−1	1	-	-	Z

Chern numbers in the superconducting phase

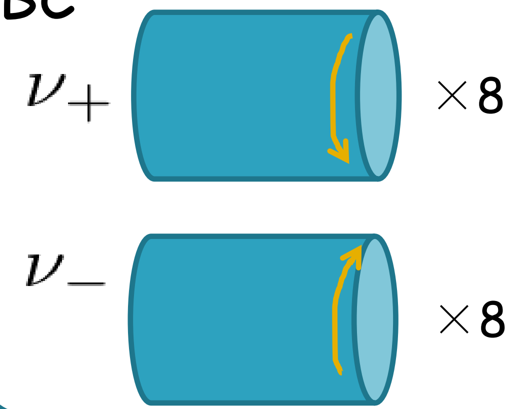
Block-diagonalize with reflection

$$\mathcal{H} = \begin{pmatrix} \mathcal{H}_+ & \\ & \mathcal{H}_- \end{pmatrix}$$

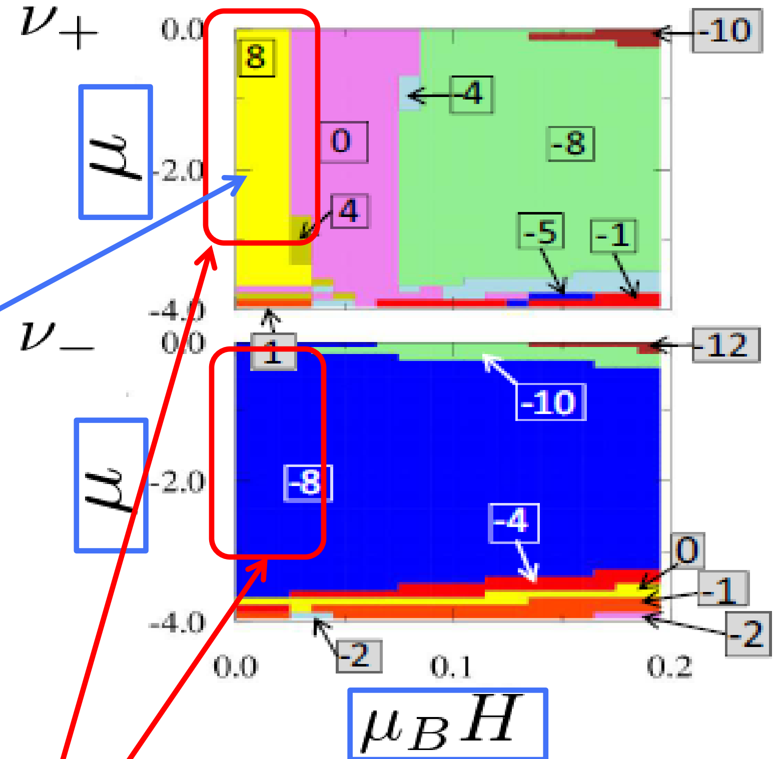
$\mathcal{H}_\pm$ is characterized by Chern#

→ $\mathbb{Z}^2$-classification

OBC



PBC:Chern number $\nu_\pm$



[mirror Chern #] $\nu_{\mathrm{M}} = \frac{\nu_+ - \nu_-}{2}$

[total Chern #] $\nu_{\mathrm{tot}} = \nu_+ + \nu_-$

Topological crystalline superconductor with $\nu_{\mathrm{M}} = 8$ and $\nu_{\mathrm{tot}} = 0$

Results

- At the mean-field level

Topological crystalline superconductor

Correlation

mean-field level

# of $CeCoIn_5$ layers	$(\nu_{\mathrm{M}}, \nu_{\mathrm{tot}})$	# of Majorana	protection
2	(4,0)	4	yes
3	(1,0)	1	yes
4	(8,0)	8	NO

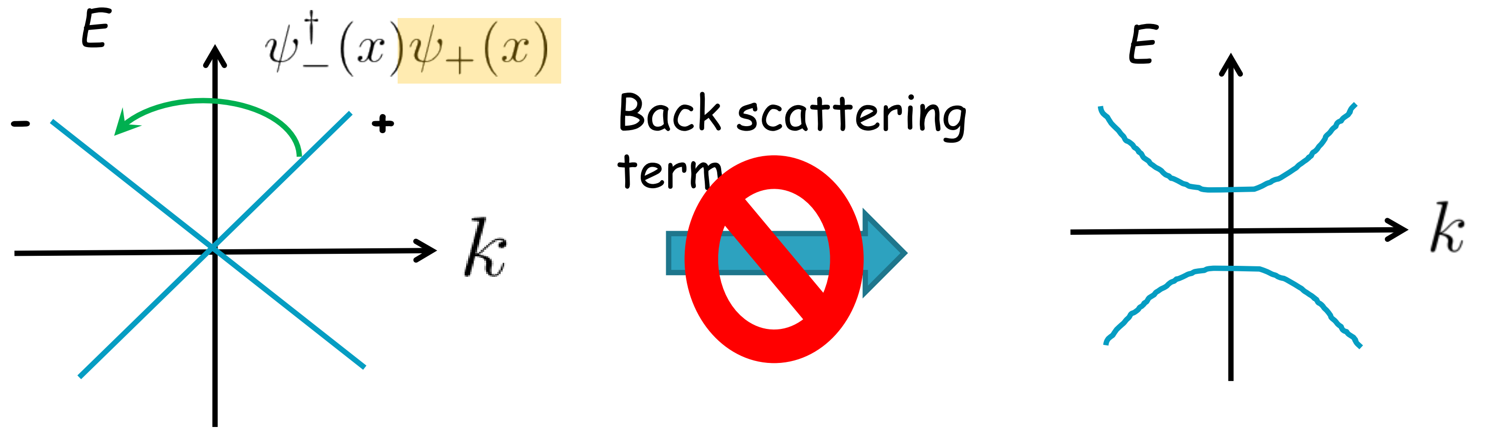
Gapping out respecting R-symmetry

complex fermion ← Two pairs of Majorana

$$\psi_\pm(x) := \eta_{1\pm}(x) + i\eta_{2\pm}(x) \qquad R\begin{pmatrix}\psi_+(x)\\ \psi_-(x)\end{pmatrix}R^{-1} = \begin{pmatrix}-\psi_+(x)\\ \psi_-(x)\end{pmatrix}$$

Two helical Majorana modes

$$H_0 = \int dx[v_F\psi_+^\dagger(x)\partial_x\psi_+(x) - v_F\psi_-^\dagger(x)\partial_x\psi_-(x)]$$



$\psi_-^\dagger(x)\psi_+(x)$ breaks R-symmetry

Symmetry protected gapless modes

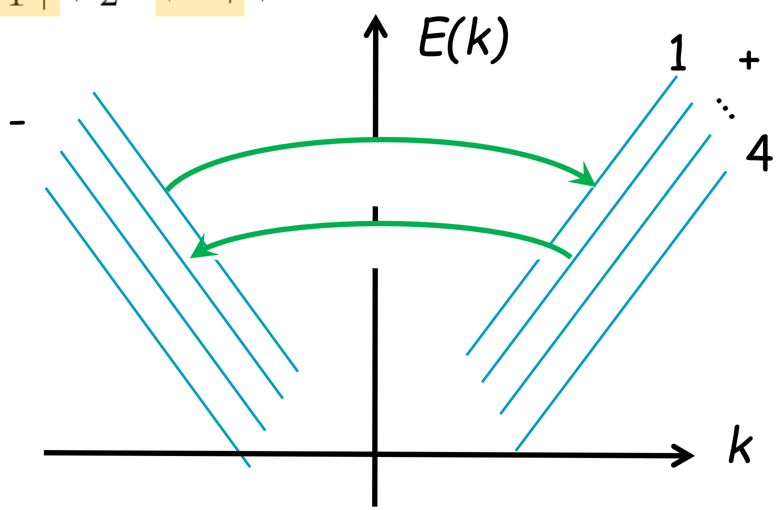
$$R\begin{pmatrix}\psi_+(x)\\ \psi_-(x)\end{pmatrix}R^{-1}=\begin{pmatrix}-\psi_+(x)\\ \psi_-(x)\end{pmatrix}$$

# of helical complex fermion	Symmetry protection
1	Yes
2	Yes
3	Yes
4	NO

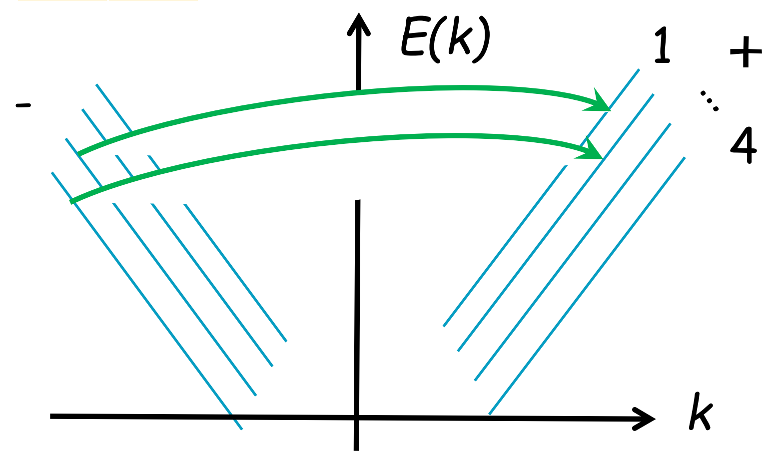
8 pairs of helical Majorana

$$\begin{aligned}H_{\rm int}=&\int dx[\psi^\dagger_{1+}(x)\psi^\dagger_{2-}(x)\psi_{2+}(x)\psi_{1-}(x)+h.c.,]\\&+\int dx[\psi^\dagger_{3+}(x)\psi^\dagger_{4-}(x)\psi_{4+}(x)\psi_{3-}(x)+h.c.,]\\&+\int dx[\psi^\dagger_{1+}(x)\psi^\dagger_{2+}(x)\psi_{3-}(x)\psi_{4-}(x)+h.c.,]\\&+\int dx[\psi^\dagger_{3+}(x)\psi^\dagger_{4+}(x)\psi_{1-}(x)\psi_{2-}(x)+h.c.,]\end{aligned}$$

$\psi^\dagger_{1+}\psi^\dagger_{2-}\psi_{2+}\psi_{1-}+h.c.$



$\psi^\dagger_{1+}\psi^\dagger_{2+}\psi_{3-}\psi_{4-}+h.c.$



Conclusion

We propose the $CeCoIn_5/YbCoIn_5$ superlattice system as a plat form of reduction of topological classification

$$\mathbb{Z} \times \mathbb{Z} \longrightarrow \mathbb{Z} \times \mathbb{Z}_8$$

# of $CeCoIn_5$ layers	$(\nu_{\mathrm{M}}, \nu_{\mathrm{tot}})$	# of Majorana	Protection (correlated)
2	(4,0)	4	yes
3	(1,0)	1	yes
4	(8,0)	8	NO

This might be observed with systematic STM measurement for 2,3,4,5,6,...layers

Part 2: Testbed of $\mathbb{Z} \to \mathbb{Z}_4$ in cold atoms

TY-Danshita-Peters-Kawakami arXiv:1711.xxxx

Motivation

For more direct observation,
it is better if the interaction can be tuned...

difficult in real materials...

Interactions can be tuned in cold atoms

The testbed of $\mathbb{Z} \to \mathbb{Z}_4$ can be build up by loading ^{161}Dy atoms to a one-dimensional lattice

Simple model of $\mathbb{Z} \to \mathbb{Z}_4$

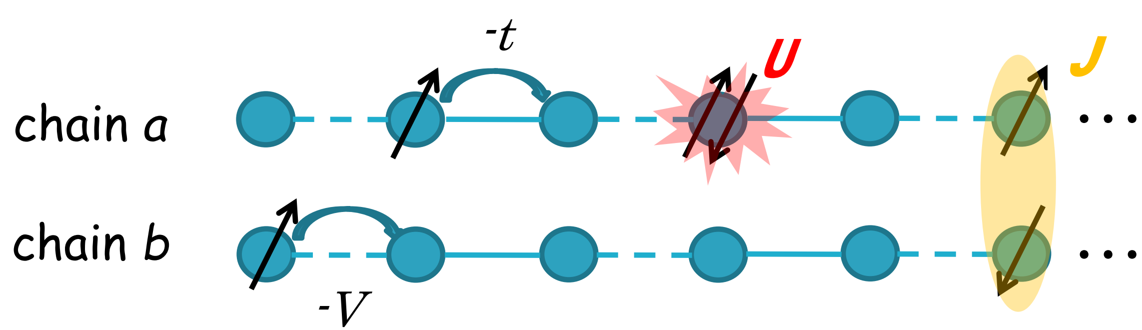
Toy model:

2-leg Su-Schrieffer-Heeger model with interactions

$$H = H_{tSSH} + \boxed{U \sum_{i\alpha} (n_{i\alpha\uparrow} - \frac{1}{2})(n_{i\alpha\downarrow} - \frac{1}{2})} + \boxed{J \sum_i \boldsymbol{S}_{ai} \cdot \boldsymbol{S}_{bi}}$$

Non-interacting part

$$H_{tSSH} = -t \sum\nolimits_{i \in odd} c^{\dagger}_{i+1\alpha\sigma} c_{i\alpha\sigma} - V \sum\nolimits_{i \in even} c^{\dagger}_{i+1\alpha\sigma} c_{i\alpha\sigma} + h.c.$$

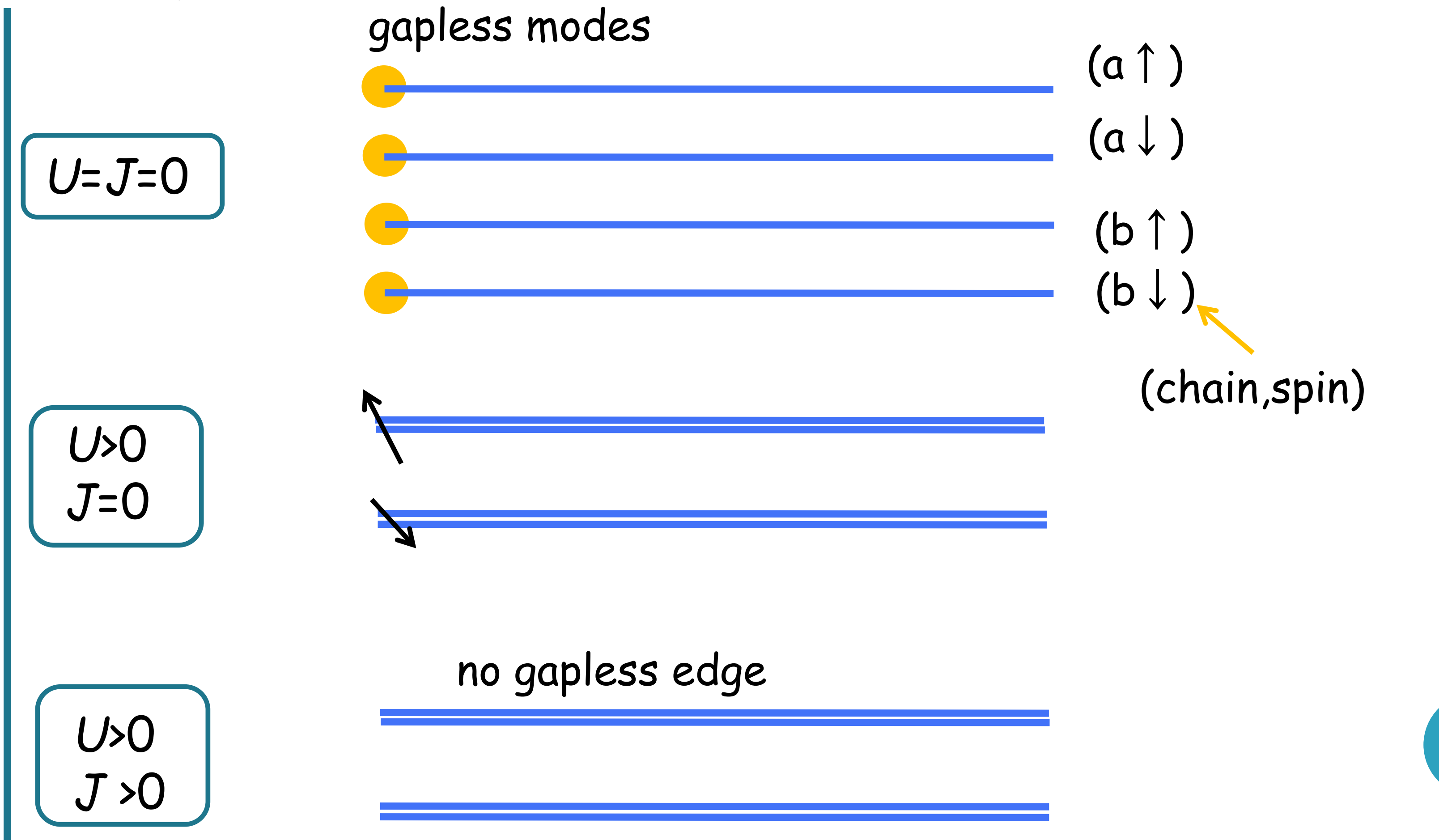


1D class AIII: Z (for free fermions)

Simple model of $\mathbb{Z} \to \mathbb{Z}_4$

$$H = H_{tSSH} + U\sum_{i\alpha}(n_{i\alpha\uparrow} - \frac{1}{2})(n_{i\alpha\downarrow} - \frac{1}{2}) + J\sum_{i} \boldsymbol{S}_{ai}\cdot\boldsymbol{S}_{bi}$$

Intuitive picture



(1) How to prepare
the above toy model or other similar?

(2) How to observe
the destruction of gapless edges?

(1) How to prepare

the above toy model or other similar?

(2) How to observe

the destruction of gapless edges?

Similar model can be build up by loading

^{161}Dy : strong magnetic dipole-dipole interaction

$$U = \frac{\mu_0 (2\mu_B)^2}{4\pi r^3}\left[\boldsymbol{S}_1 \cdot \boldsymbol{S}_2 - \frac{3}{r^2}(\boldsymbol{S}_1 \cdot \boldsymbol{r})(\boldsymbol{S}_2 \cdot \boldsymbol{r})\right],$$

$$S^z = 21/2, 19/2, \cdots, -21/2$$

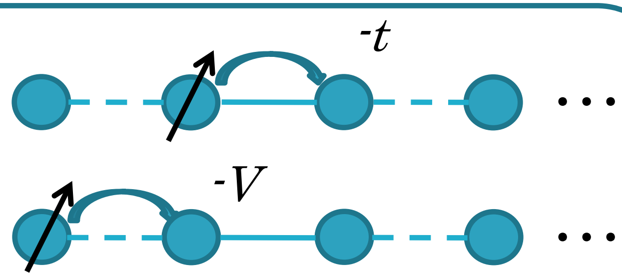
$$\Downarrow$$

$$S^z = 21/2, 19/2$$

[optical pumping] + [Zeno effect]

Effective two-leg ladder of spin-1/2

$$H = H_{tSSH} + U\sum_{i\alpha}(n_{i\alpha\uparrow} - \frac{1}{2})(n_{i\alpha\downarrow} - \frac{1}{2}) + J\sum_i h_i$$

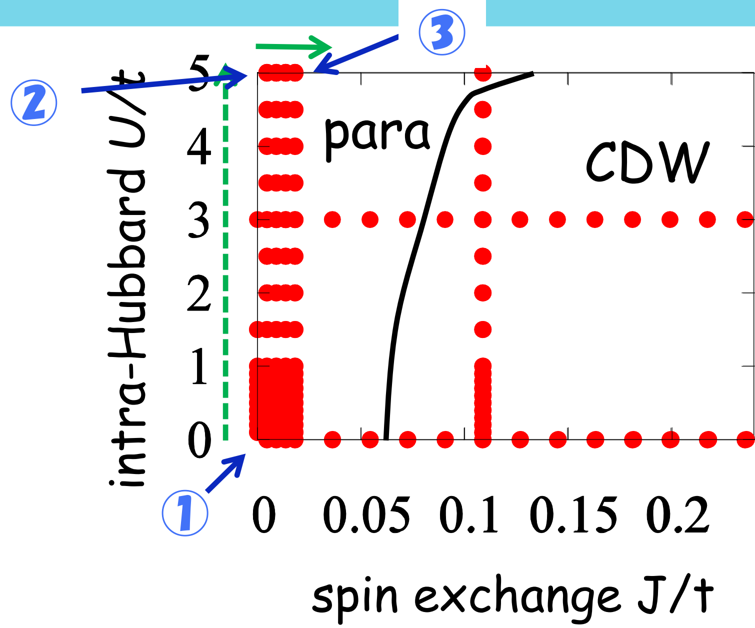


$$h_i = \underbrace{A_1(\tilde{S}^x_{ia}\tilde{S}^x_{ib} + \tilde{S}^y_{ia}\tilde{S}^y_{ib})}_{\text{spin exchange interaction}} - A_2\tilde{S}^z_{ia}\tilde{S}^z_{ib} - A_3(n_{ia}-1)(n_{ib}-1) - A_4[(n_{ia}-1)\tilde{S}^z_{ib} + (n_{ib}-1)\tilde{S}^z_{ia}],$$

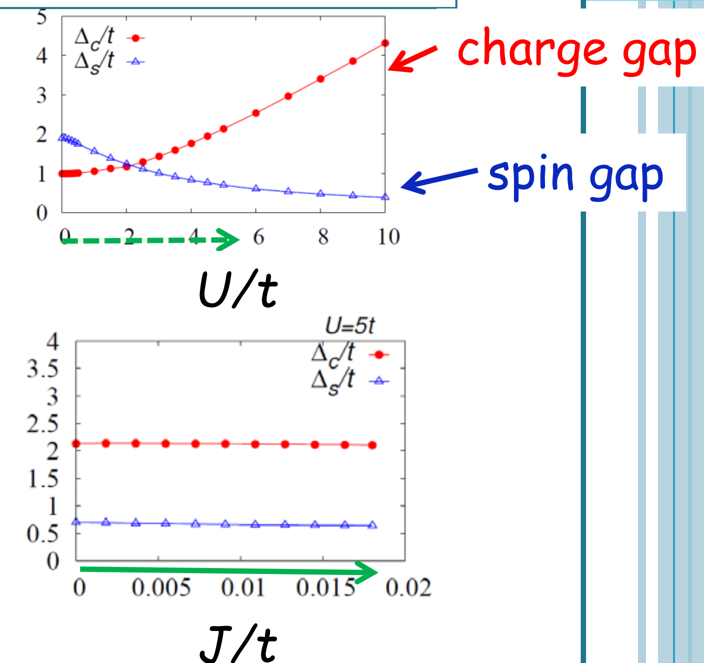
spin exchange interaction

$$\begin{pmatrix} A_1 \\ A_2 \\ A_3 \\ A_4 \end{pmatrix} = \begin{pmatrix} 16^2/21 \\ 16/21 \\ 2 * 160^2/21 \\ 20 * (16/21)^2 \end{pmatrix}$$

Numerical results: bulk properties

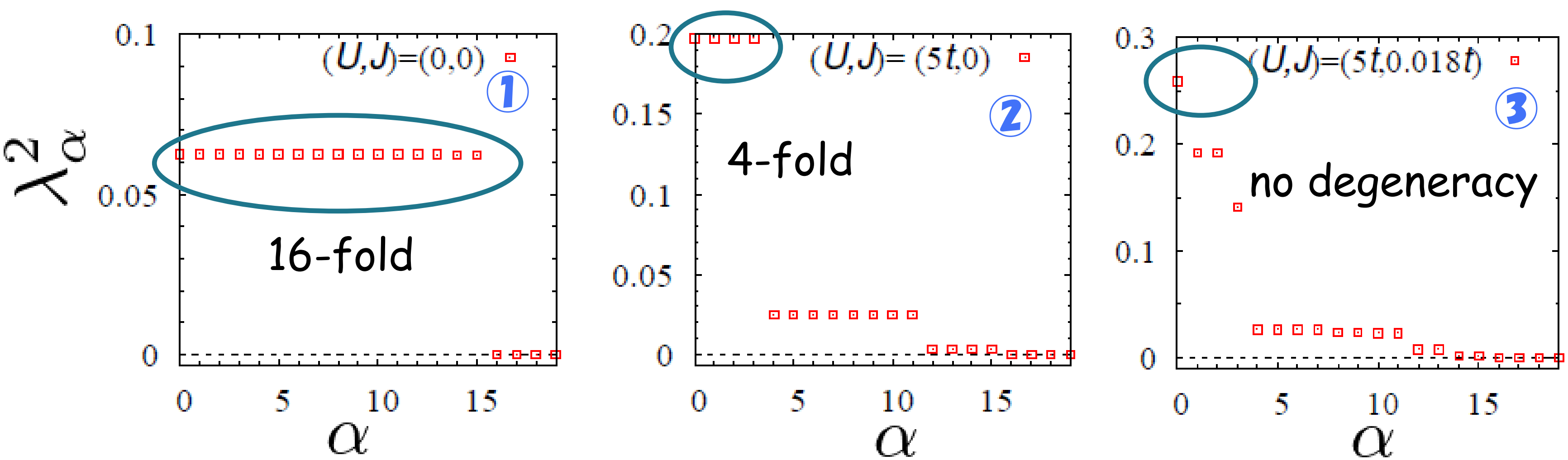


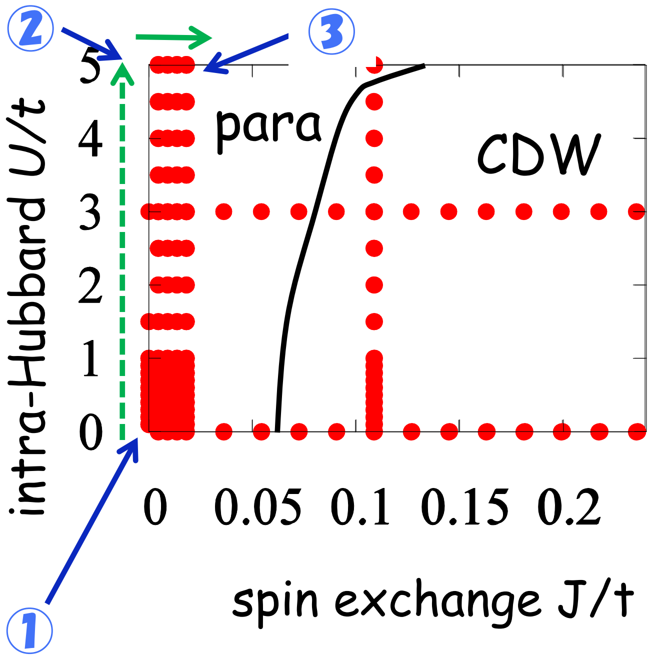
(PBC) bulk gap: finite



Entanglement spectrum E_α

$$E_\alpha = -2\log\lambda_\alpha$$

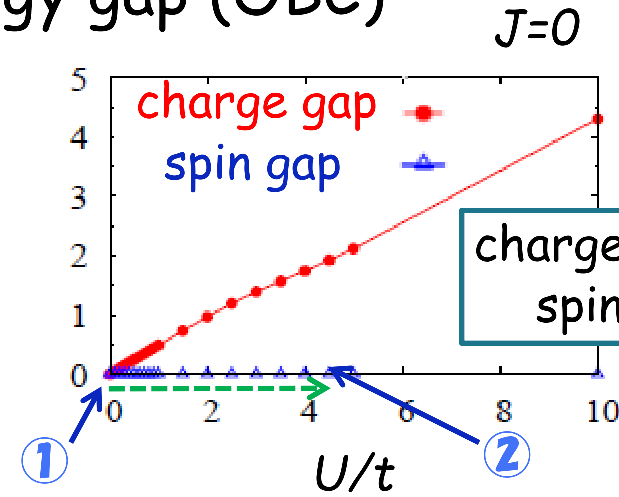




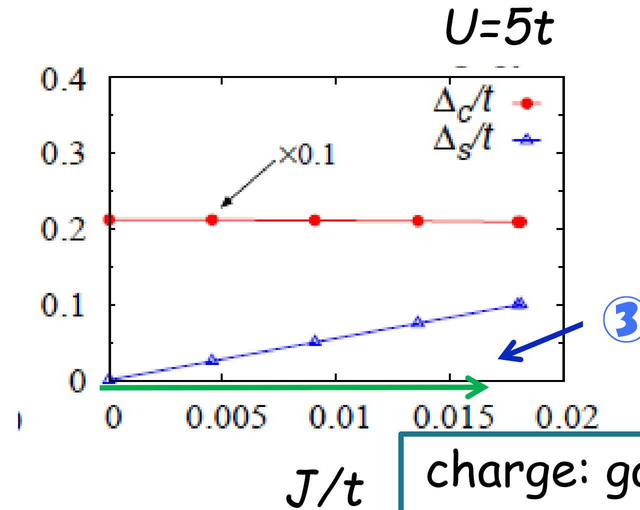
parameter set	Degeneracy of ES
①	16-fold
②	4-fold
③	no degeneracy

※ Bulk is gapped

Energy gap (OBC)



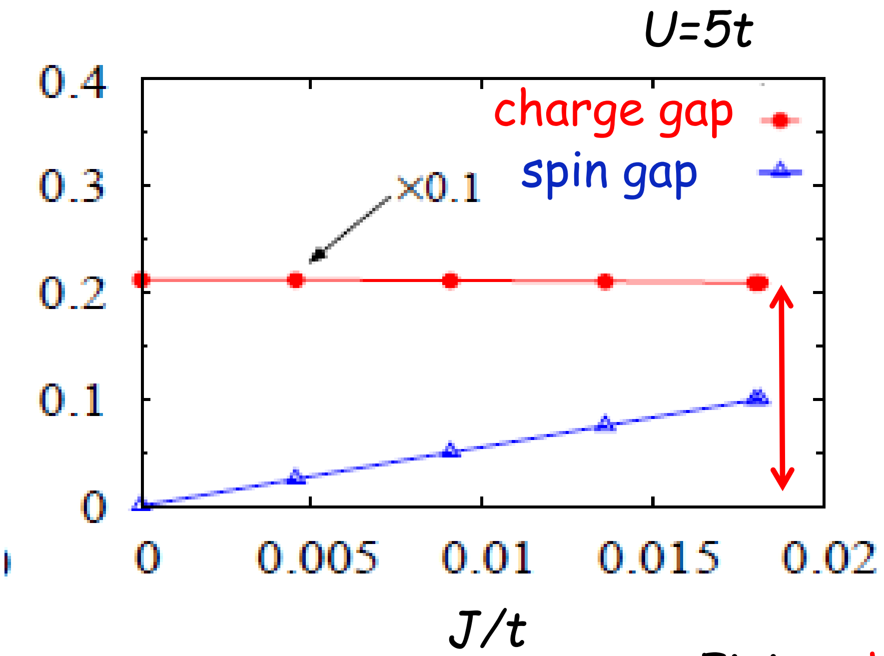
charge: gapped
spin: gapless



charge: gapped
spin: gapped

All of edge modes are destroyed by U and J

(2) How to observe the destruction of gapless edges?

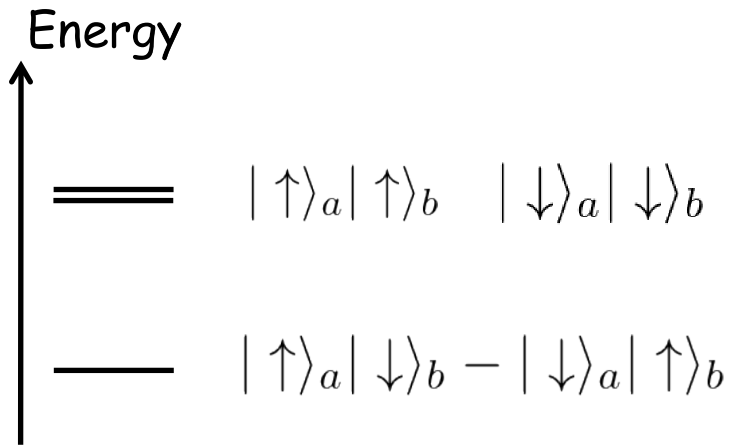


Finite charge gap@ edges

Radio frequency spectroscopy (~[ARPES measurement])

How to observe spin gap?

How to observe the spin gap?



Spin gap:

Observing time evolution

$|1\rangle,\ |2\rangle$: Eigenstates of $\mathcal{H}$

$$|\psi(0)\rangle = c_1|1\rangle + c_2|2\rangle$$

$$\langle A(t)\rangle = \sum_i |c_i|^2\langle i|A|i\rangle + 2a_{12}\cos(\omega_{21}t + \delta_{12})$$

$$\omega_{21} = E_2 - E_1$$
$$a_{12}e^{i\delta_{12}} := c_1^* c_2\langle 1|A|2\rangle$$

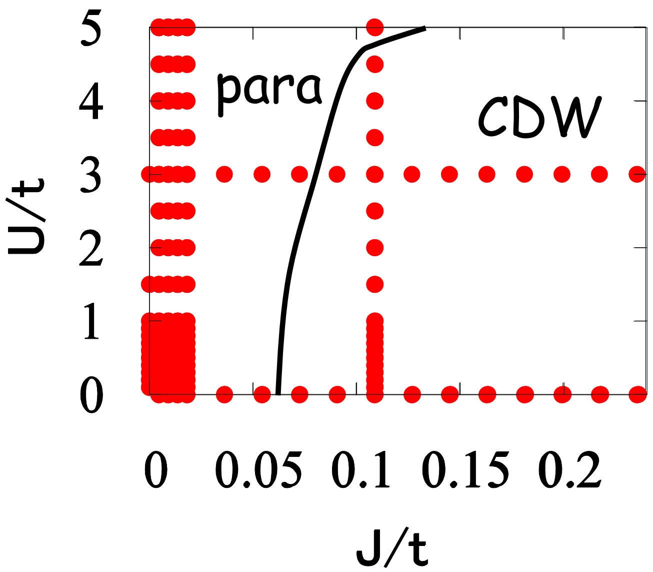
- Superposed state can be prepared by shining a half-π pulse
- Oscillation of $\langle S_a^x\rangle$, tells us the gap size → [gap size]~1nK

Summary of part 2

loading ^{161}Dy atoms,
one can prepare a testbed of

$$\mathbb{Z} \to \mathbb{Z}_4$$

* Interactions can be tuned
in experiments!!



$\mathbb{Z} \to \mathbb{Z}_4$ can be observed by

- Radio frequency spectroscopy: [charge gap]$\sim$80nK
- Time-evolution of the expectation $\langle S_a^x \rangle$ [spin gap]$\sim$1nK

TY-Danshita-Peters-Kawakami arXiv 1711.xxxx

Thank you!