Novel Quantum States in Condensed Matter
at Yukawa Institute of Theoretical Physics
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Flat-band Andreev bound states and Odd-frequency pairs

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MEXT of Japan

Core-to-core by JSPS

Outline

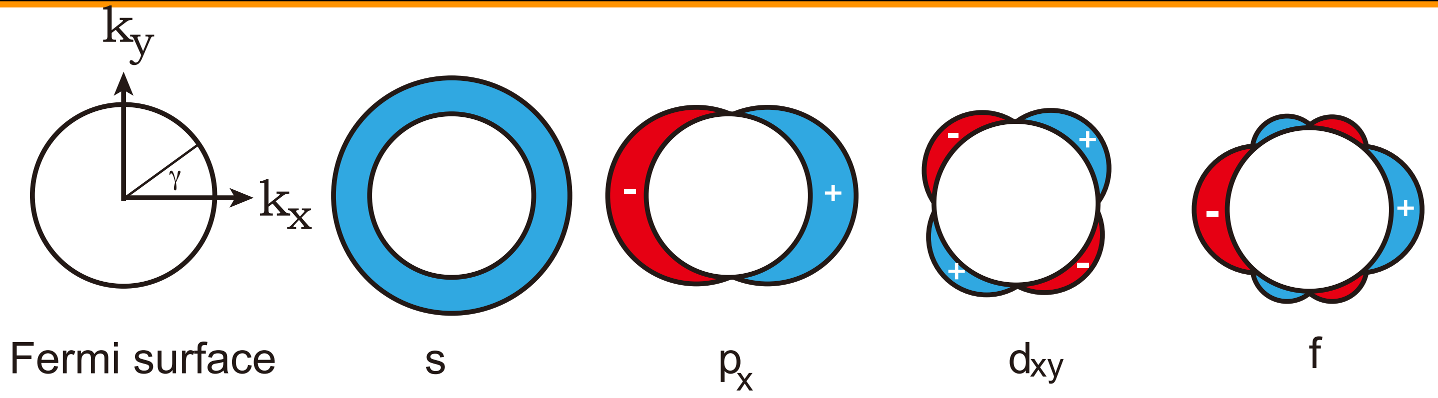
Flat-band Andreev bound states in a nodal SC

- Conductance in a NS hybrid
- Paramagnetic response of a superconducting disk
- Relation to Majorana physics

Tunable φ–junction with a QAHI

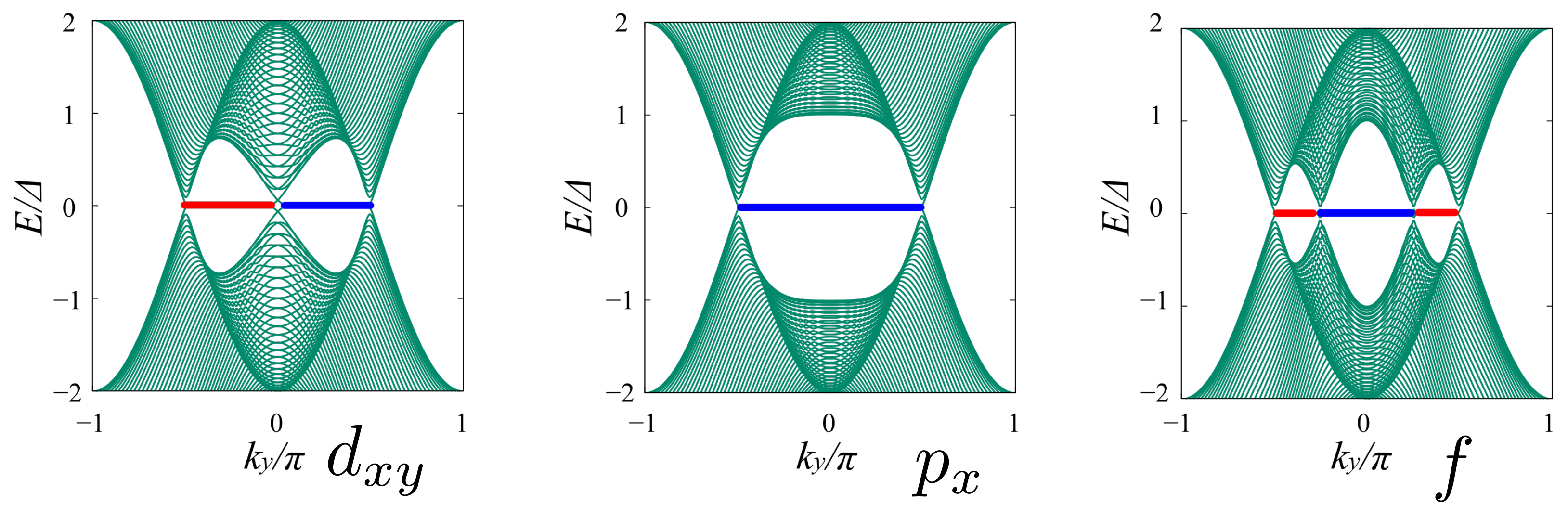
Summary

Unconventional Superconductors



Sign change is necessary to be nontrivial

Nodal! (out of the ten-fold symmetry classes)



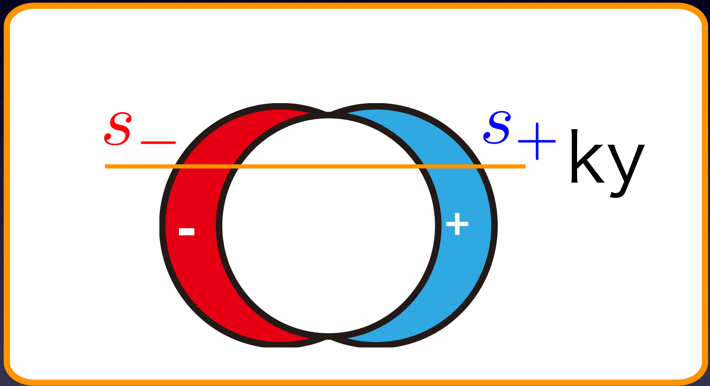
Andreev bound states with flat dispersion at a clean surface x=0

Topological characterization

Dimensional reduction

M. Sato. et. al, PRB(2011)

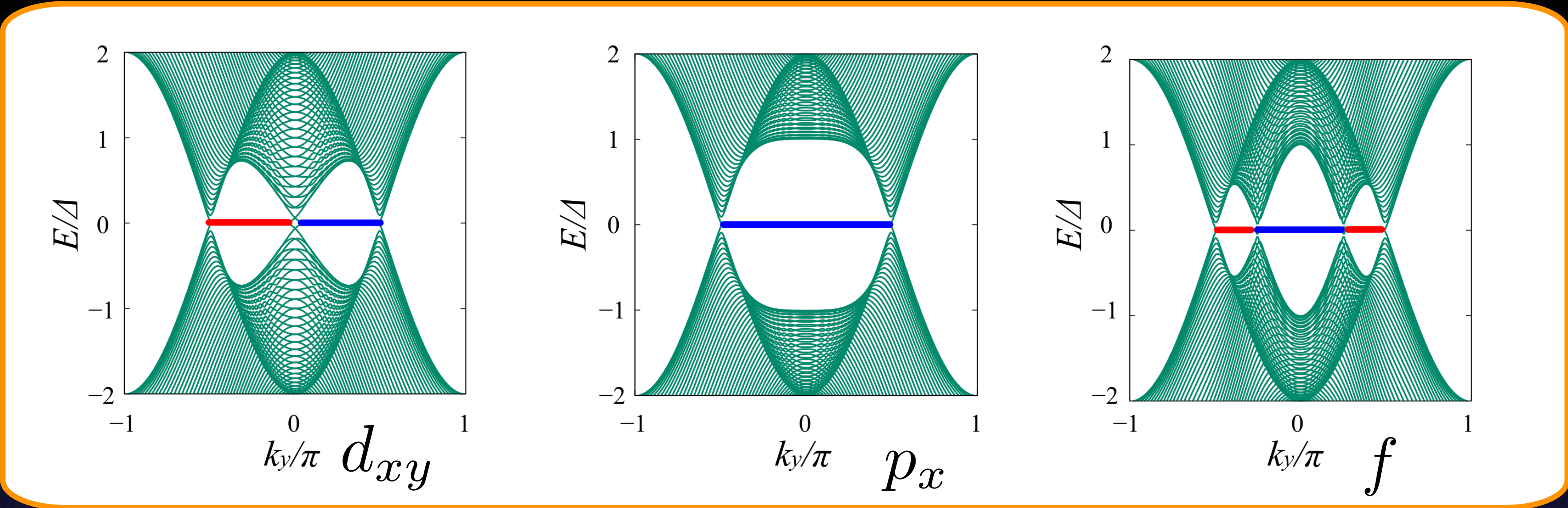
Fix ky and consider 1D BZ



$$s_\pm = \frac{\Delta(\pm k_x, k_y)}{|\Delta(\pm k_x, k_y)|}$$

$$H = \begin{bmatrix} \xi & \Delta(\boldsymbol{k}) \\ \Delta(\boldsymbol{k}) & -\xi \end{bmatrix} = \Delta(\boldsymbol{k})\tau_1 + \xi\tau_3$$

$$w_{1\mathrm{D}}(k_y) = \frac{i}{2\pi}\int_{-\pi}^{\pi} dk_x \mathrm{Tr}[\tau_2 H(k_x)^{-1}\partial_{k_x} H(k_x)] = \frac{1 - s_+ s_-}{2} s_+$$



$$w_{1\mathrm{D}} = 1 \,\mathrm{or}\, -1$$

Theory

Experiment

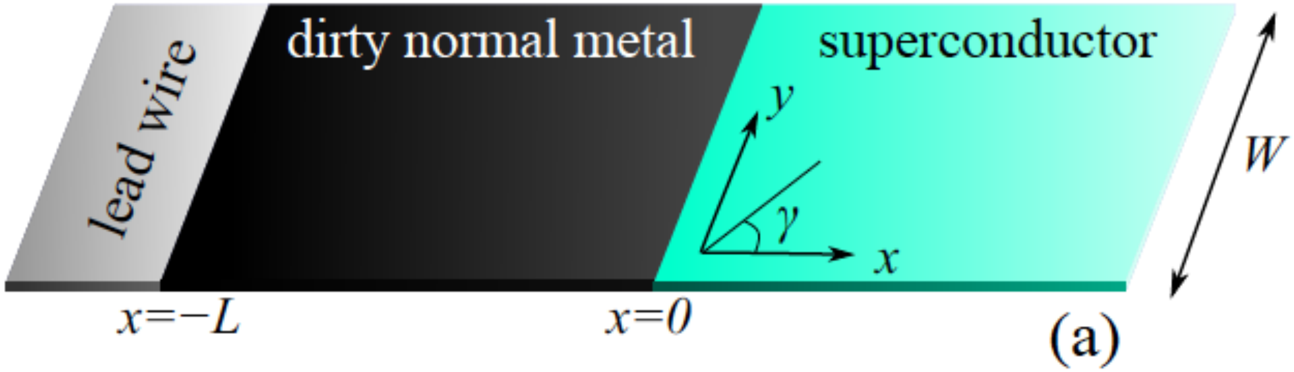
Clean
(Translational symmetry) ⟷ Potential disorder

High degeneracy ⟵ High symmetry

What happen on the flat ZESs under the potential disorder?

How the flat ZESs affect observable values?

Zero-bias conductance in a NS junction



$$\hat{H}_{\mathrm{BdG}} = \begin{bmatrix} \xi_{\boldsymbol{r}} + V(\boldsymbol{r}) & \Delta(\boldsymbol{r}) \\ \Delta(\boldsymbol{r}) & -\xi_{\boldsymbol{r}} - V(\boldsymbol{r}) \end{bmatrix},$$

$$\xi_{\boldsymbol{r}} = -\frac{\hbar^2\nabla^2}{2m} - \mu_F,$$

$$V(\boldsymbol{r}) = V_{\mathrm{imp}}(\boldsymbol{r})\Theta(-x)\Theta(x+L) + v_0\delta(x),$$

$$\Delta(\boldsymbol{r}) = \begin{cases} \Delta, & s \\ -2\Delta\,\partial_x\partial_y/k_F^2, & d_{xy} \\ -i\Delta\,\partial_x/k_F & p_x \\ -i\Delta\,\partial_x(k_F^2 + 2\partial_y^2)/k_F^3, & f, \end{cases}$$

Classical Ohm's law

$$G_{\mathrm{NS}}^{-1} = R_{\mathrm{NS}} = R_B + R_N$$

$$\lim_{R_N \to \infty} G_{\mathrm{NS}} = 0$$

would be expected

Quasiclassical Usadel equation in N

$$\hbar D \frac{\partial^2 \theta(x,\epsilon)}{\partial x^2} + 2i\,\epsilon\,\sin\theta(x,\epsilon) = 0$$

Quantum Ohm's law

Tanaka et. al. PRB (2004)

$$R_{\mathrm{NS}} = \frac{1}{G_Q I_B} + \frac{R_{\mathrm{N}}}{L}\int_{-L}^{0} \frac{dx}{\cosh^2\left(\mathrm{Im}(\theta(x,\epsilon))\right)}$$

$$G_Q = \frac{2e^2}{h}$$

with $\epsilon = 0$

In a singlet superconductor

$$\mathrm{Im}(\theta) = 0 \quad \Rightarrow \quad \lim_{R_N\to\infty} G_{\mathrm{NS}} = 0$$

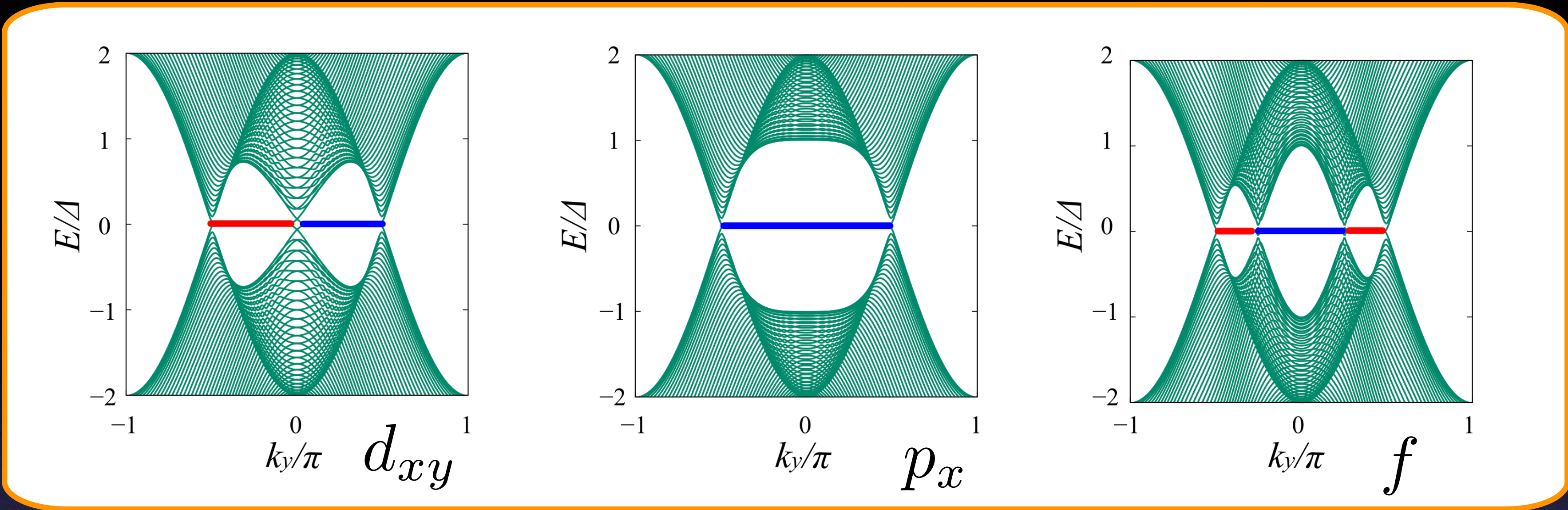
In a triplet superconductor

$$\theta = i(x/L+1)\,\beta_0$$

$$\beta_0 = 2G_Q R_N\, \underline{N_{\mathrm{ZES}}}$$

$$\Rightarrow \quad \lim_{R_N\to\infty} G_{\mathrm{NS}} = \frac{4e^2}{h}|N_{\mathrm{ZES}}|$$

Flat surface Andreev bound states



Topological classification
of nodal SC
Dimensional reduction

$$w_{1\mathrm{D}} = \underline{1} \, \mathrm{or} \, \underline{-1}$$

$$N_{\mathrm{ZES}} = \sum_{k_y} w_{1\mathrm{D}}(k_y)$$

topological invariant

Chiral symmetry of Hamiltonian

$$H_{\mathrm{BdG}} = (\xi + V)\hat{\tau}_3 + \Delta\hat{\tau}_1$$

$$\{H_{\mathrm{BdG}}, -\hat{\tau}_2\}_+ = 0$$

eigenvalue of $-\hat{\tau}_2$ $\lambda = \underline{1} \, \mathrm{or} \, \underline{-1}$

$$N_{\mathrm{ZES}} = N_+ - N_-$$

an invariant in differential equation

In mathematics,

Atiyah-Singer Index theorem

connects topology and analysis

$$N_{\mathrm{ZES}} = \sum_{k_y} w_{\mathrm{1D}} = N_{+} - N_{-}$$

topological invariant

The number of ZES

belong to $\lambda = \pm 1$

chiral symmetry

$$\{\hat{H}_{\mathrm{BdG}}, -\hat{\tau}_2\}_+ = 0$$

Sato et.al., PRB (2011)
Ikegaya, YA, PRB (2015)

eigenvalue of $-\hat{\tau}_2$ $\lambda = 1\,\mathrm{or}\,-1$

chirality

(1) ZES: eigenstate of $-\hat{\tau}_2$

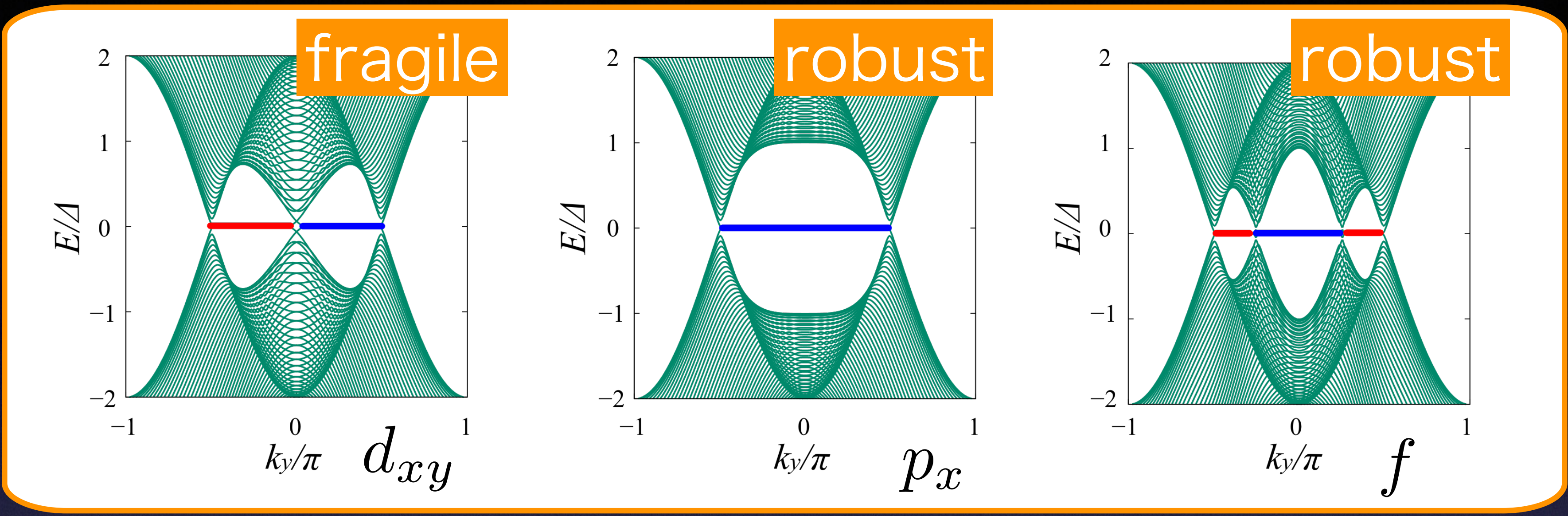
(2) non ZES: linear combination of $\lambda = 1$ and $\lambda = -1$

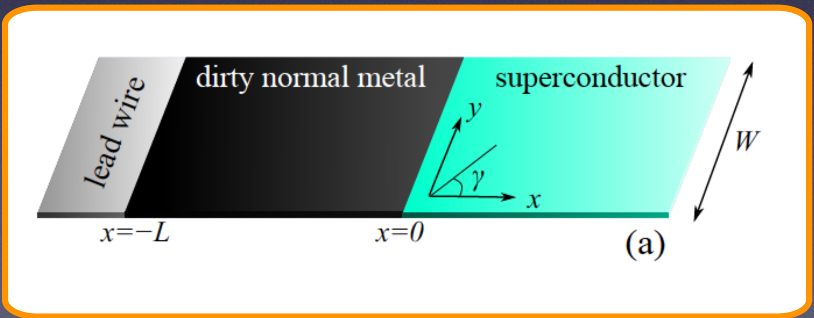
$$\chi_{E\neq 0} = a_+\chi_+ + a_-\chi_-,$$

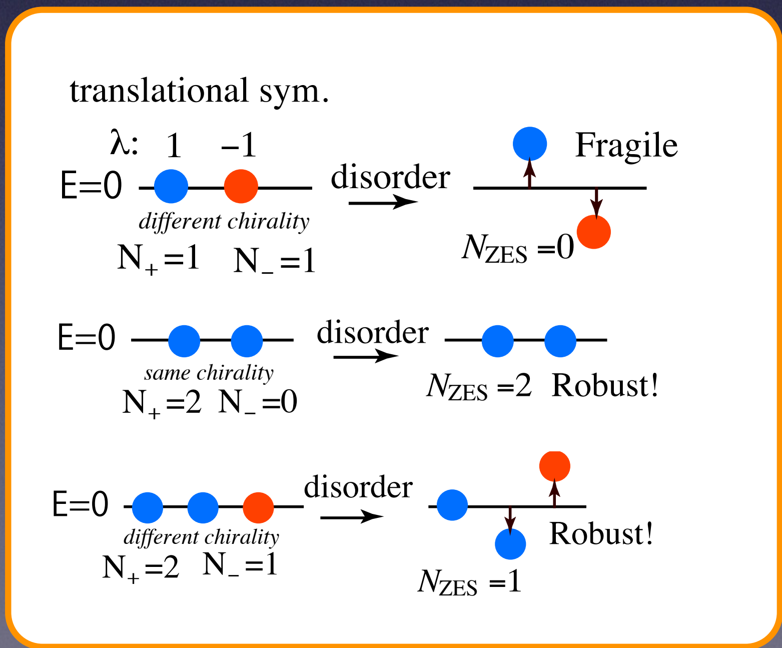
$$|a_+| = |a_-|$$

one-by-one

ZESs at a clean surface







dxy

px

f

In physics,

$$N_{\rm ZES} = \sum_{k_y} w_{\rm 1D} = N_+ - N_-$$ Atiyah-Singer index

describes the number of zero-energy states

that penetrate into dirty normal metal

and

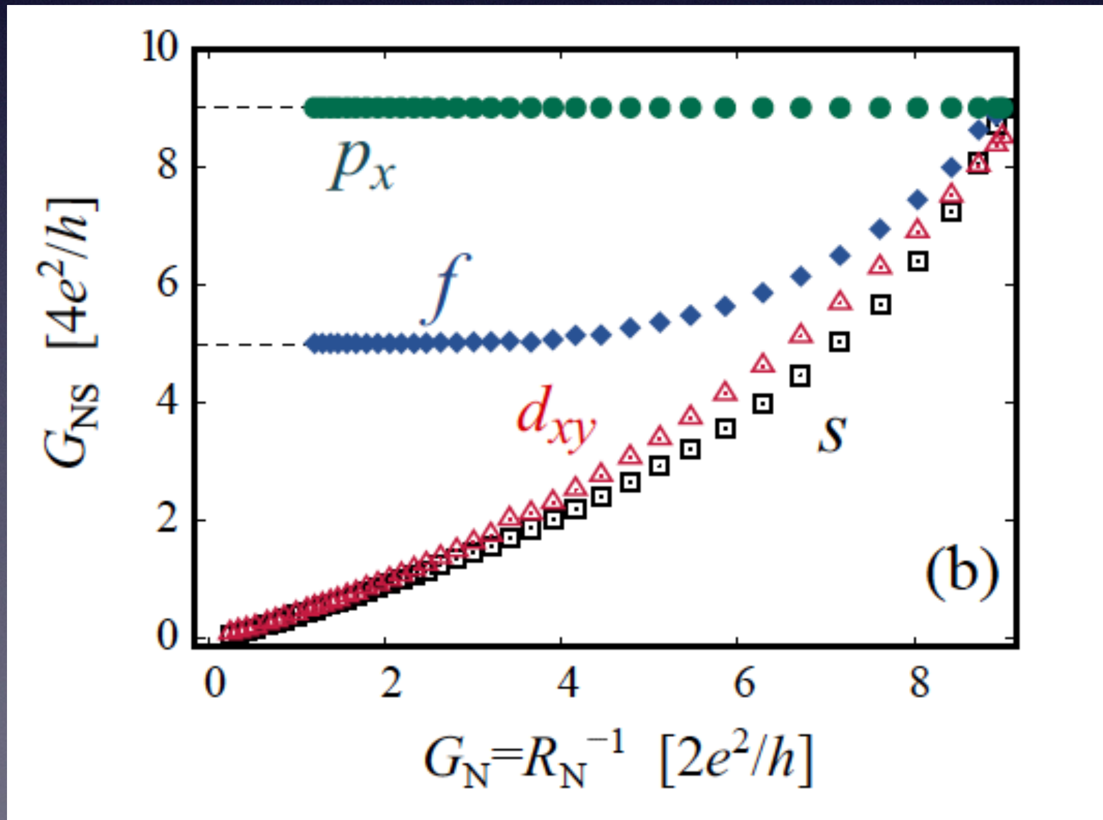
form resonant transmission channels

↓

$$\lim_{R_N \to \infty} G_{\rm NS} = \frac{4e^2}{h} |N_{\rm ZES}|$$

Quantization of

Conductance minimum



	Classification Schnyder et.al. (2008)	Real SC
Pair potential	full gap	nodal (to be nontrivial)
Translational symmetry	not necessary	necessary
Topo # in bulk	Z	W(k) (if TRS is preserved)
ZESs at a clean surface	|Z|	$\sum_k \|W(k)\|$
ZESs at a dirty surface	|Z|	$\|N_{\mathrm{ZES}}\|$

Ikegaya, Suzuki, Tanaka, YA, PRB 94, 054512 (2016) spin-triplet SC

Conductance minimum is quantized at Atiyah-Singer index

degenerate ZESs

Cooper pairs?

	Spin	X	Parity	X	Frequency
In SC	triplet		p-wave (odd)		even
In dirty N	triplet		s-wave (even)		odd

to satisfy a requirement of Fermi-Dirac statistics

Symmetry Classification

$$f_{\sigma,\sigma'}(r - r') = -\langle \psi_\sigma(\boldsymbol{r})\psi_{\sigma'}(\boldsymbol{r}')\rangle$$

Fourier trans. → $f_{\sigma,\sigma'}(p)$

Spin	Orbital
singlet	s, d (even-parity)
triplet	p, f (odd-parity)

spin X orbital = -1 Fermi-Dirac statistics

Spin-flip potential mix spin-singlet and spin-triplet

Surface & interface mix even- and odd-parity

Odd-freq. Pairs

General definition of pairing function

$$f_{\sigma,\sigma'}(r-r',\tau-\tau') = -\langle T_\tau \psi_\sigma(r,\tau)\psi_{\sigma'}(r'\tau')\rangle$$

$\rightarrow$ $f_{\sigma,\sigma'}(p,\omega_n)$

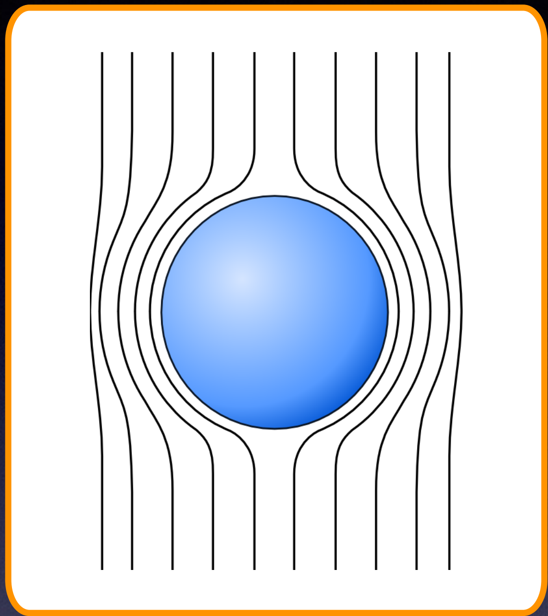
Fourier trans.

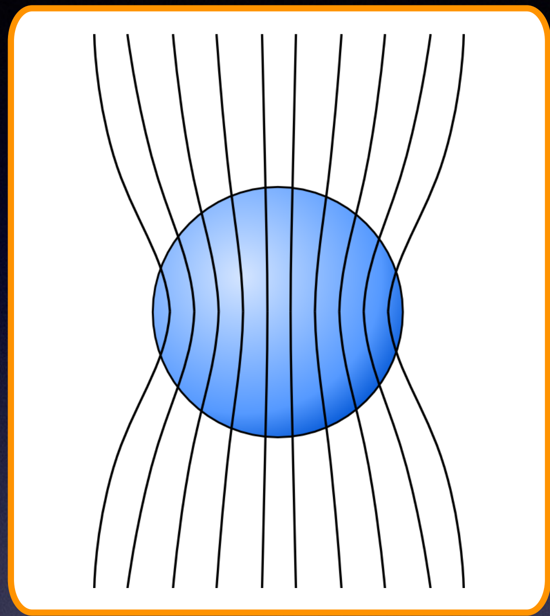
spin X orbital X frequency = -1

Topological surfaces (ZES)

generate odd-freq. pairs

Paramagnetic response of a small superconductor





Diamagnetic

Paramagnetic

Odd-frequency pairs are paramagnetic!

YA, Golubov, Fominov, Tanaka, PRL **107**, 087001 (2011)

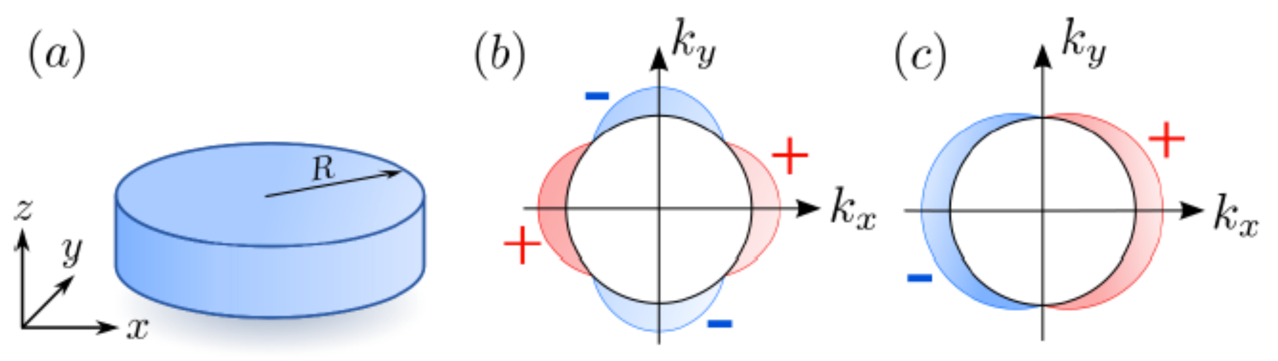
Small unconventional superconductors

are paramagnetic

due to odd-freq. pair at their surface

Suzuki and YA, PRB **89**, 184508 (2014)

We consider…



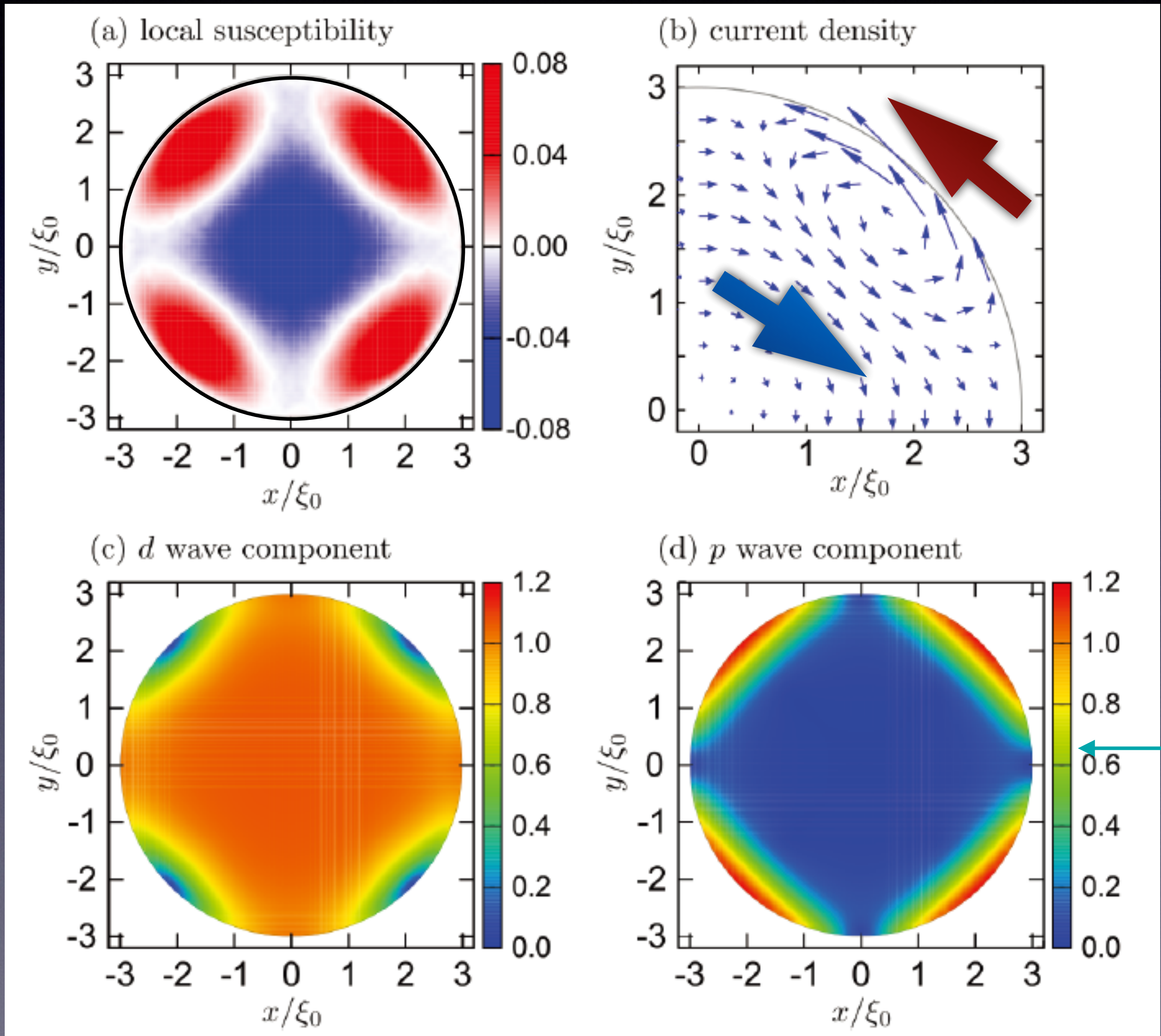
Solve Eilenberger and Maxwell Eqs. simultaneousely

Pair potential and vector potential

are determined self-consistently on 2D disks

Paramagnetic response of a singlet d-wave SC

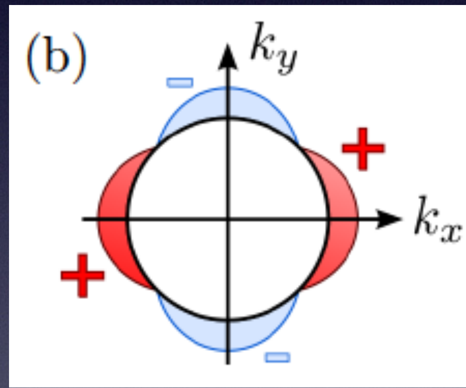
$$\chi(\boldsymbol{r}) = [H(\boldsymbol{r}) - H_{ex}]/[4\pi H_{ex}]$$



$R = 3\xi_0$

$\lambda_L = 3\xi_0$

$H_{ex} = 0.001 H_{c2}$

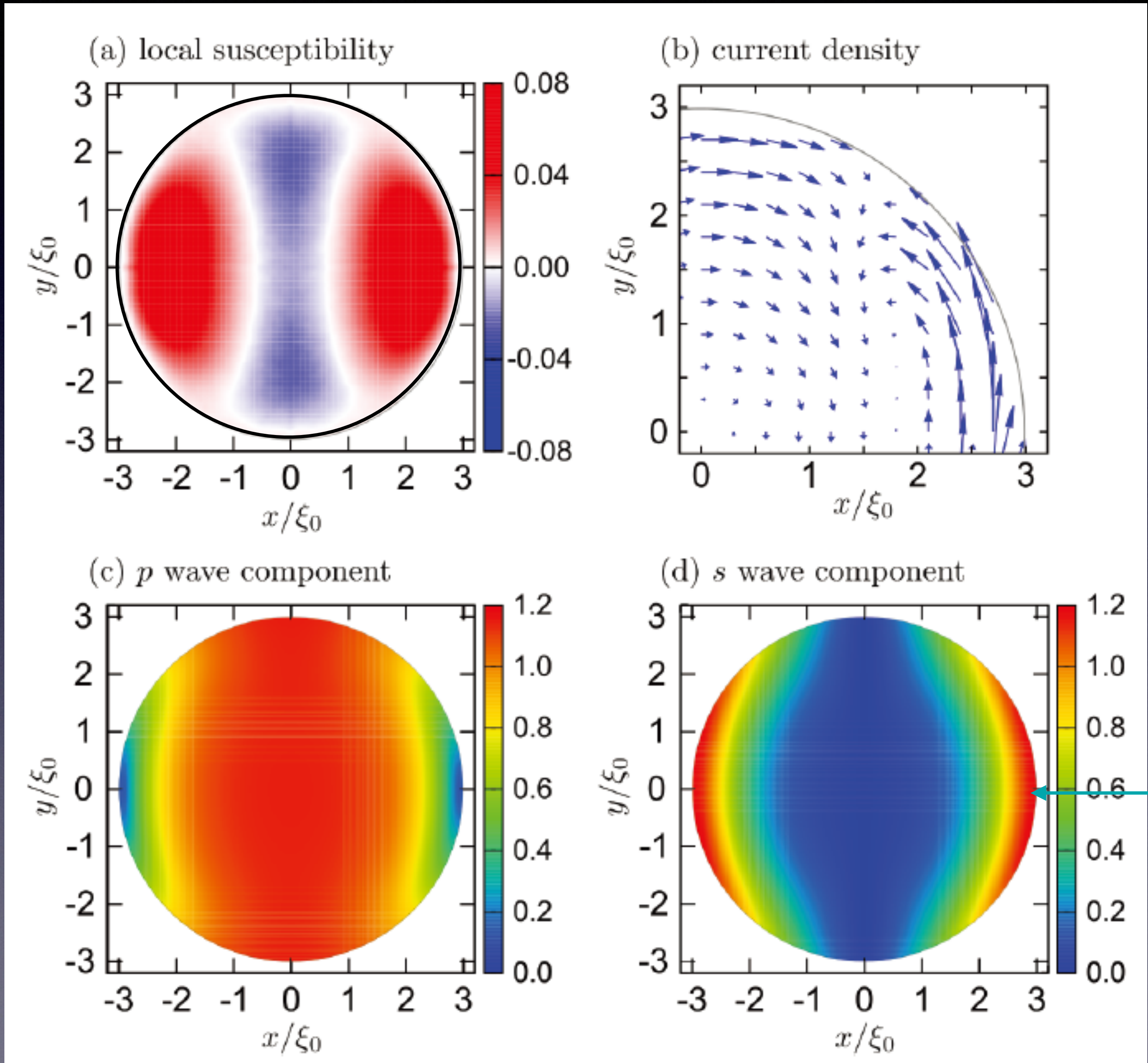


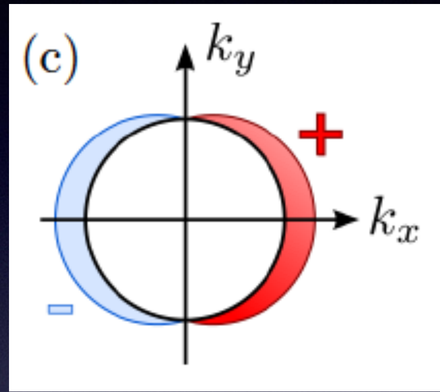
subdominant component

odd-freq.

paramagnetic

Paramagnetic response of a triplet p-wave SC



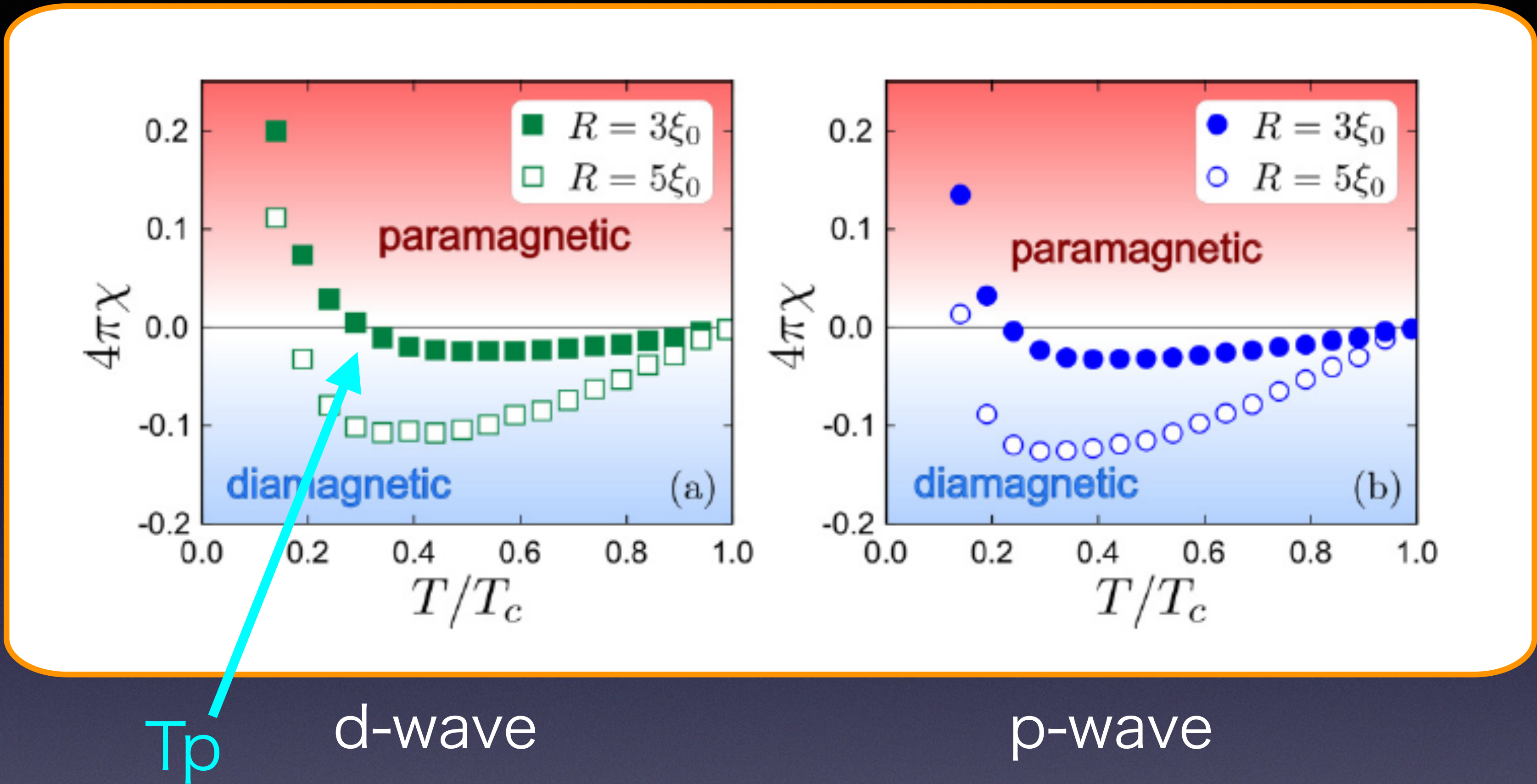


subdominant component

odd-freq.

paramagnetic

Susceptibility v.s. Temperature



d-wave

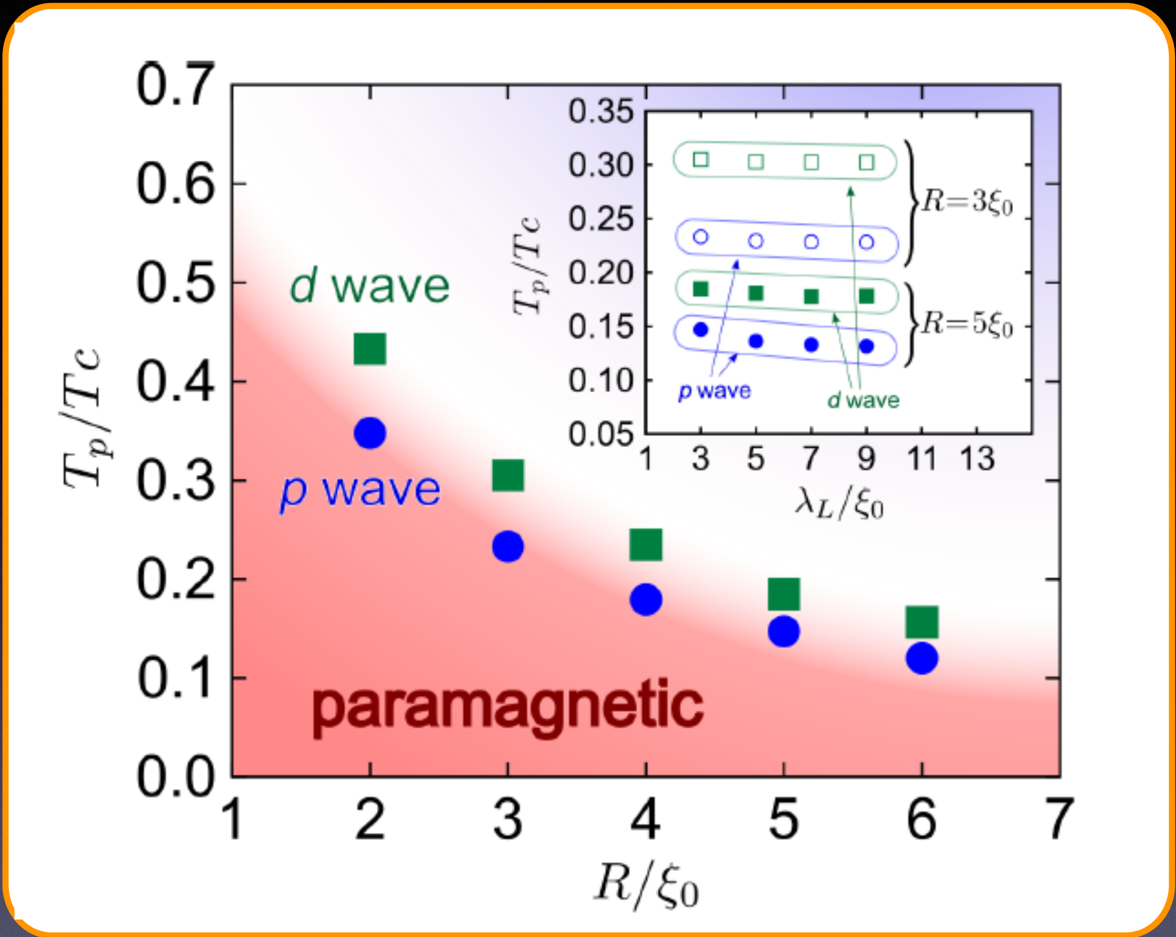
p-wave

Crossover to paramagnetic phase at low temperature

odd-freq. pairs are paramagnetic

energetically localize near E=0

Crossover temperature v.s. Size of disk



odd-freq. pairs are confined at surface within ξ_0

In larger discs, relative area of ‘surface’ becomes smaller

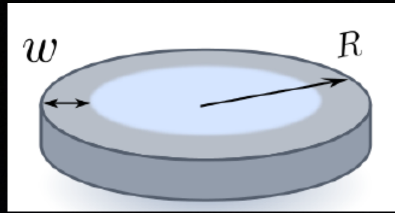
Any difference between p and d?

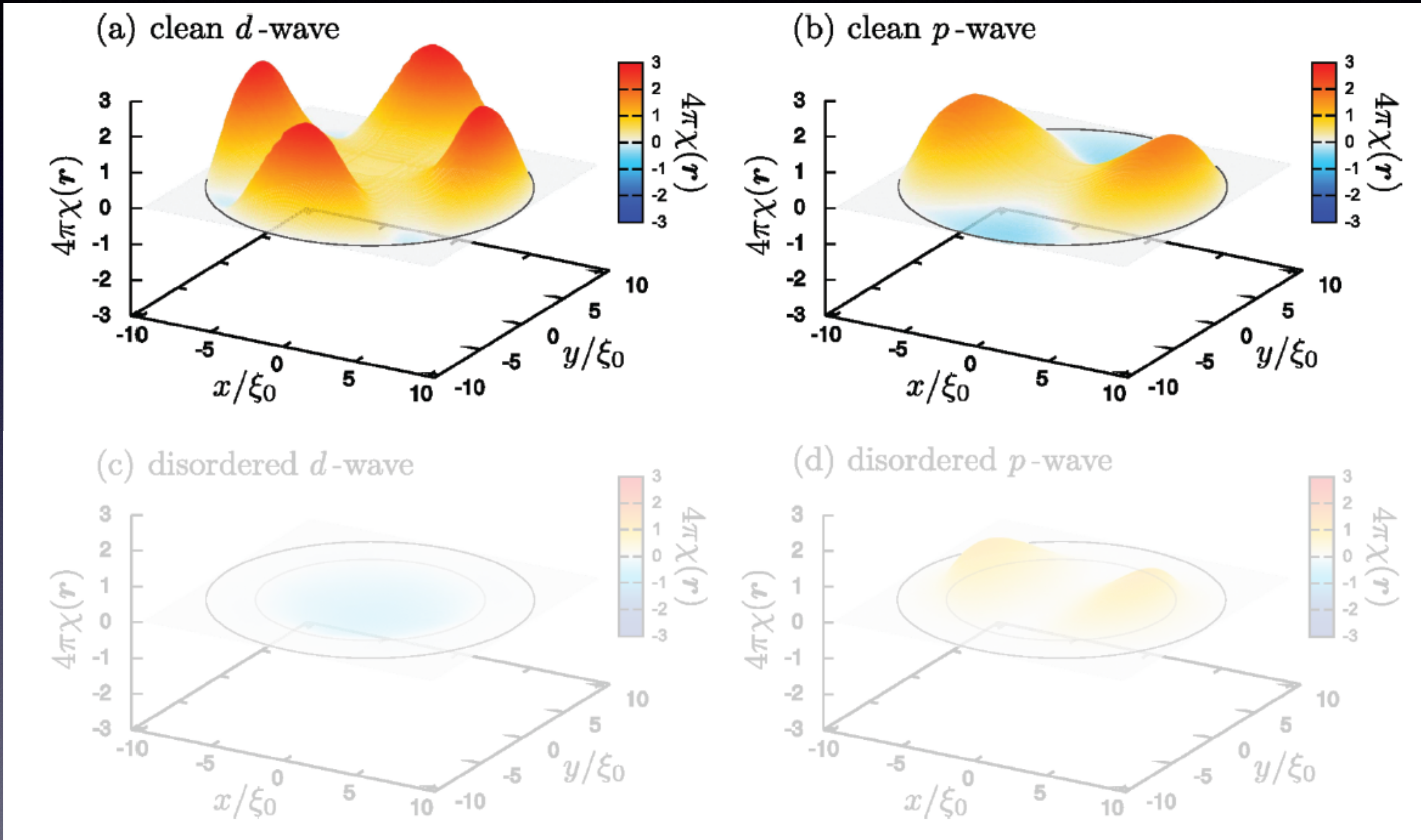
Yes!

in the presence of surface roughness

Effects of surface roughness

Suzuki and Asano, PRB 91, 214510 (2015)





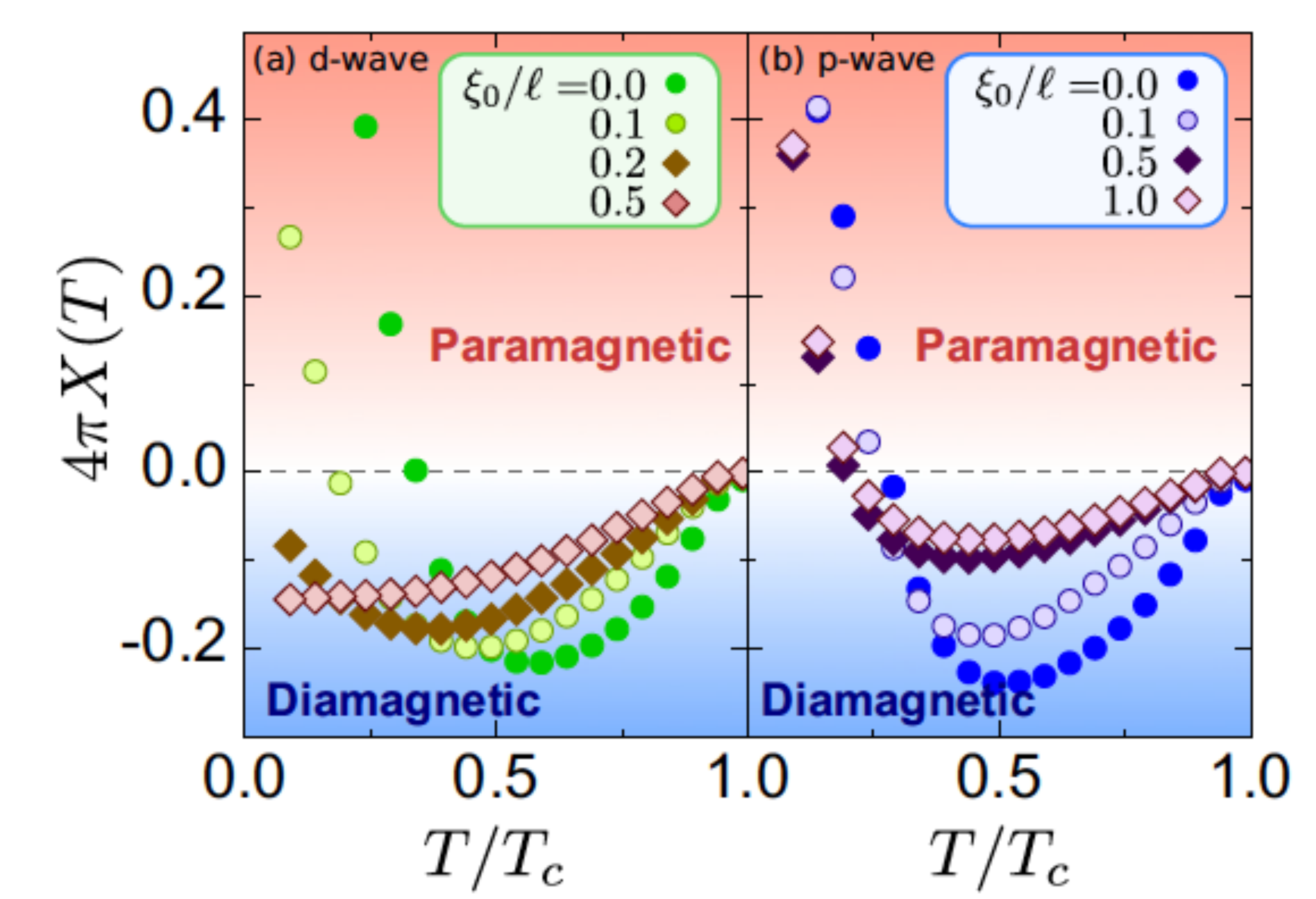
Odd-w p-wave

$N_{\mathrm{ZES}} = 0$

Odd-w s-wave

$N_{\mathrm{ZES}} \neq 0$

Susceptibility v.s. Temperature under surface roughness



Relating papers on d-wave SC

Higashitani, JPSJ **66**, 2556 (1997)

Fogelstrom, Rainer, and Sauls, PRL **79**, 281 (1997)

Barash, Kalenkov, and Kurkijarvi, PRB **62**, 6665 (2000)

Zare, Dahm, and Schophl, PRL **104**, 237001 (2010)

Vorontsov, PRL**102**, 177001 (2009).

Hakansson, Lofwander and Fogelstrom, Nat. Phys. **11**, 755 (2015).

energetics of flat-band ZESs

Our papers on d, p, chiral-d, chiral-p, chiral-f

Suzuki and YA, PRB **89**, 184508 (2014)

Suzuki and YA, PRB **91**, 214510 (2015)

Suzuki and YA, PRB **94**, 155302 (2016)

odd-frequency pairs

Trouble!

A spin-triplet p-wave superconductor
has never been discovered yet!

$$N_{\mathrm{ZES}} \neq 0$$

Why don't we make it?   Sure! Why not!

Ikegaya, Kobayashi, YA, in preparation

What we have done

spin-triplet p-wave

A sufficient condition for $N_{\mathrm{ZES}} \neq 0$

Necessary conditions?

single-band BdG Hamiltonian

↓

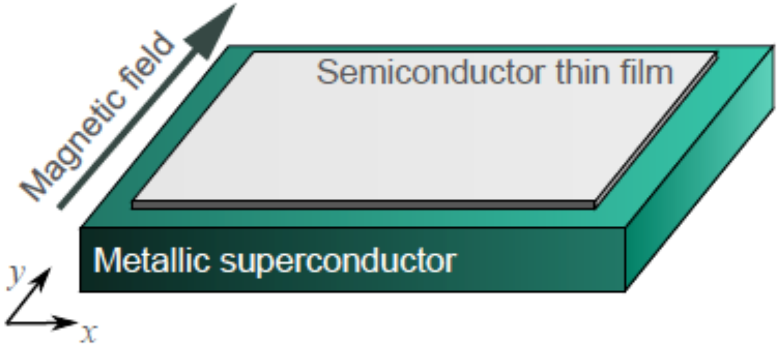
must belong to the class BDI

↓

specify realistic models

Solutions

$N_{\mathrm{ZES}} = \text{Majorana number}$



$$\check{H}_{\mathrm{D}}(\boldsymbol{k}) = \begin{bmatrix} \hat{h}_{\mathrm{D}}(\boldsymbol{k}) & \hat{\Delta}_{\mathrm{D}}(\boldsymbol{k}) \\ -\hat{\Delta}^*_{\mathrm{D}}(-\boldsymbol{k}) & -\hat{h}^*_{\mathrm{D}}(-\boldsymbol{k}) \end{bmatrix},$$

$$\hat{h}_{\mathrm{D}}(\boldsymbol{k}) = \varepsilon(\boldsymbol{k})\sigma_0 + \beta k_x \sigma_3 + \sum_{j=1,2} V_j \sigma_j,$$

$$\hat{\Delta}_{\mathrm{D}}(\boldsymbol{k}) = i\Delta_s \sigma_2,$$

$$\check{H}_{\mathrm{P}}(\boldsymbol{k}) = \begin{bmatrix} \hat{h}_{\mathrm{P}}(\boldsymbol{k}) & \hat{\Delta}_{\mathrm{P}}(\boldsymbol{k}) \\ -\hat{\Delta}^*_{\mathrm{P}}(-\boldsymbol{k}) & -\hat{h}^*_{\mathrm{P}}(-\boldsymbol{k}) \end{bmatrix},$$

$$\hat{h}_{\mathrm{P}}(\boldsymbol{k}) = \varepsilon(\boldsymbol{k})\sigma_0 + \sum_{j=1,2} V_j \sigma_j$$

$$\hat{\Delta}_{\mathrm{P}}(\boldsymbol{k}) = i\frac{\Delta_p}{k_{\mathrm{F}}}\left[k_x \hat{\sigma}_1 + k_y \hat{\sigma}_2\right]\hat{\sigma}_2,$$

Dresselhaus [110]

+

in-plane Zeeman

Alicea, PRB 81, 125381 (2010)

You, Oh, Vedral, PRB 87, 054501 (2013)

Majorana!

2D helical p-wave

+

in plane Zeeman

Mizushima, Sato, Machida, PRL 109, 165031 (2012)

Wong, Oriz, Law, Lee, PRB 88, 060504 (2014)

SCs with $N_{\mathrm{ZES}} \neq 0$

?

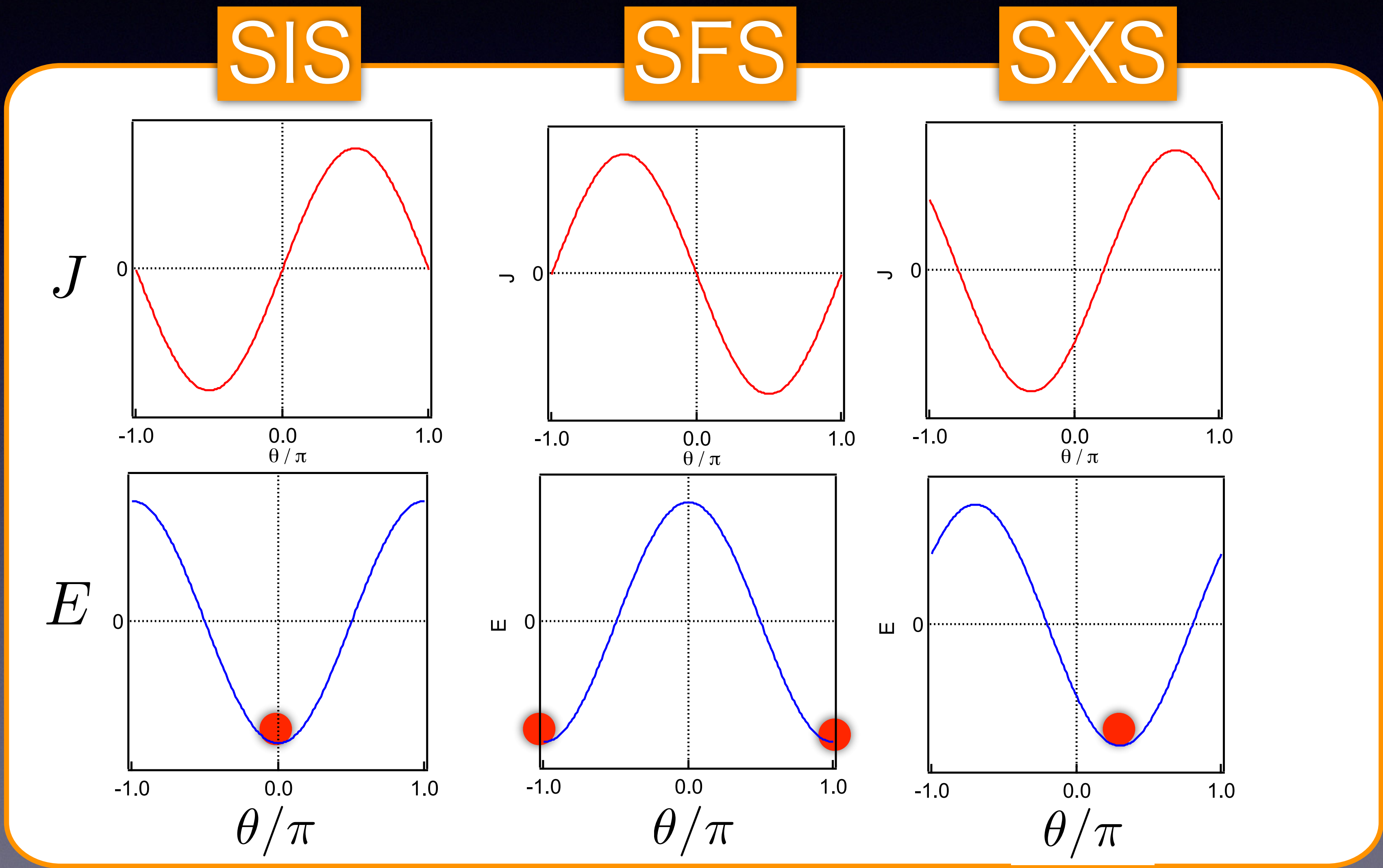
Majorana SCs

Tunable φ-junction with a QAHI

Sakurai, Ikegaya, and YA, arXiv:1709.02338.

Josephson Junction

$$J(\theta) \propto \partial_\theta E(\theta)$$

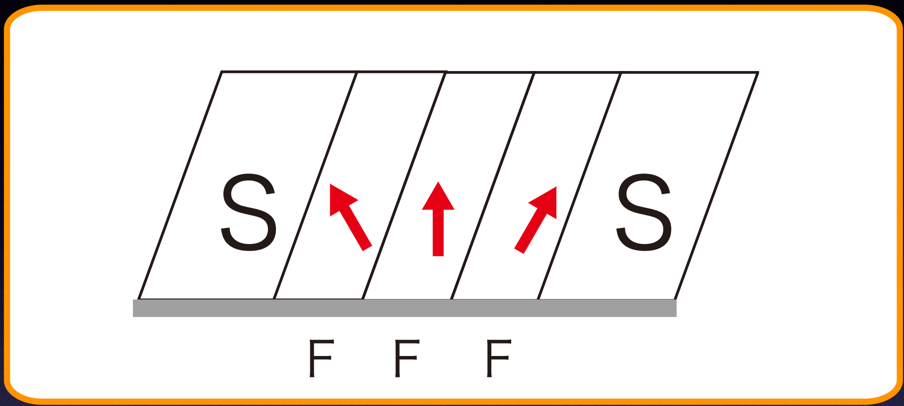


0-junction π-junction φ-junction

φ-junction

$$J = J_0 \sin(\theta - \varphi) = J \sin\theta \cos\varphi - J_0 \cos\theta \sin\varphi$$

Current at zero phase difference



Breaking

TRS + Inversion

$$J \propto (\boldsymbol{M}_1 \times \boldsymbol{M}_2 \cdot \boldsymbol{M}_3) \cos\theta + J_0 \sin(\theta)$$

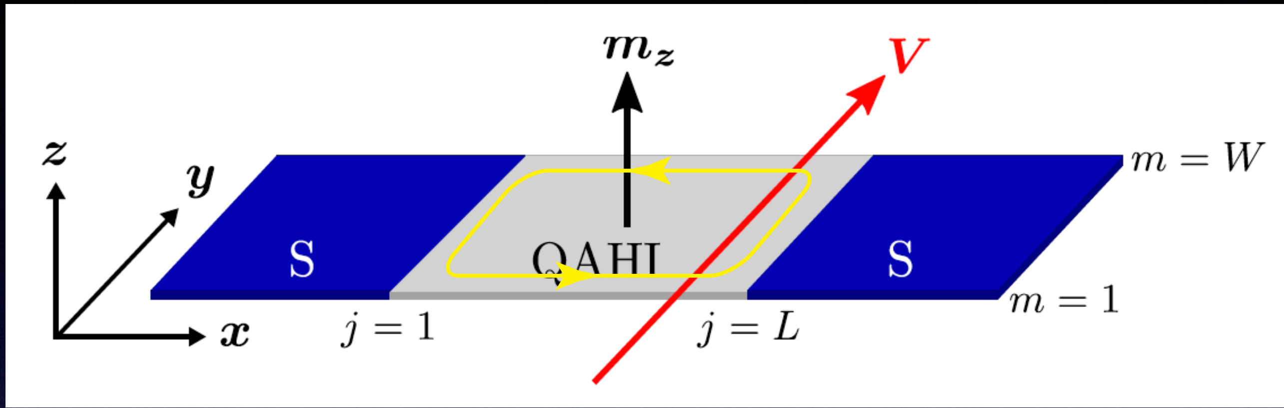
YA et. al, PRB 2007

built-in φ value

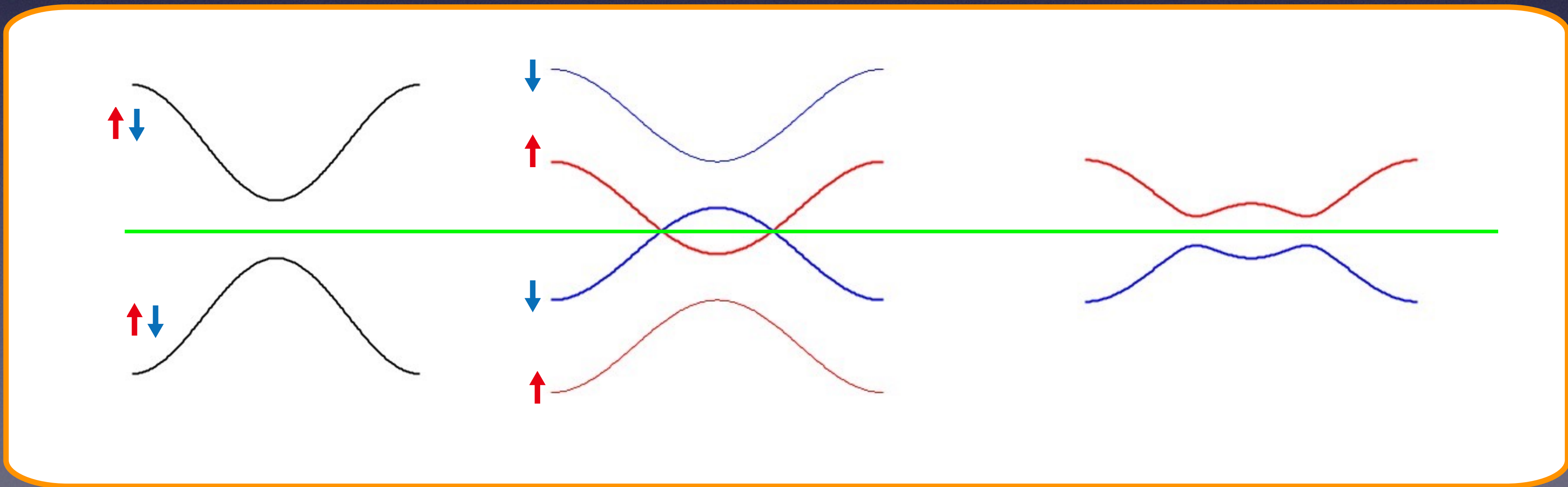
Heim, et. al., J. Phys. 25, 215701 (2013).
Reynoso,et. al., PRL 101, 107001 (2008).
Dell'Anna, et. al, PRB 75, 085305 (2007).
Zazunov, et. al., PRL 103, 147004 (2009).
Campagnano, et. al., J. Phys.27, 2053012015).
Tanaka, et. al., PRL 103, 107002 (2009).
Dolcini, et. al., PRB 92,035428 (2015)
Buzdin, PRL 101, 107005 (2008)
….

Yokoyama, Eto, Nazarov, PRB 89, 195407 (2014).

Quantum Anomalous Hall Insulator



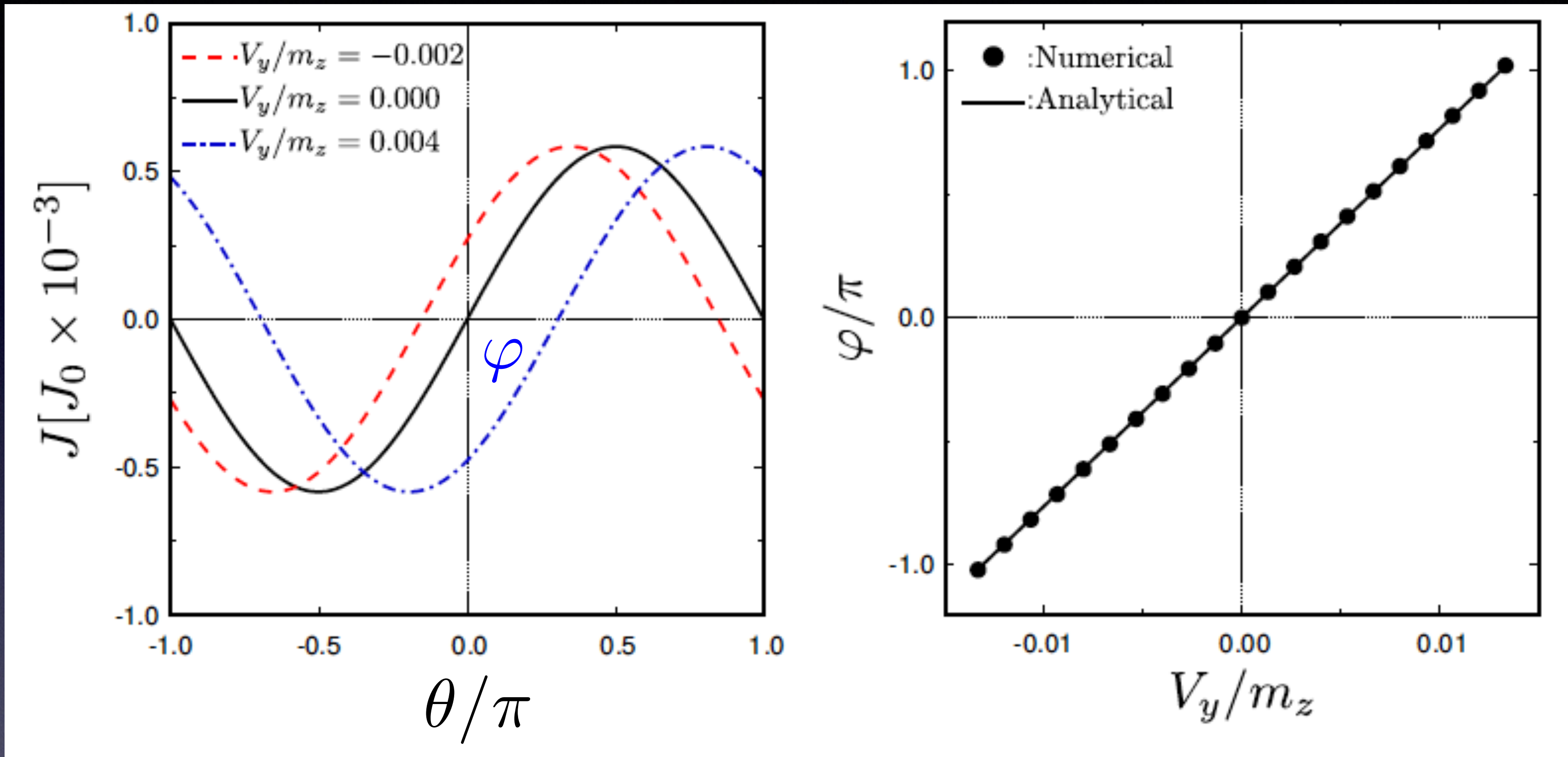
$$\hat{H}_Q(\boldsymbol{r}) = (\varepsilon_{\boldsymbol{r}} - m_z)\hat{\sigma}_3 + i\lambda\partial_x\hat{\sigma}_2 - i\lambda\partial_y\hat{\sigma}_1$$

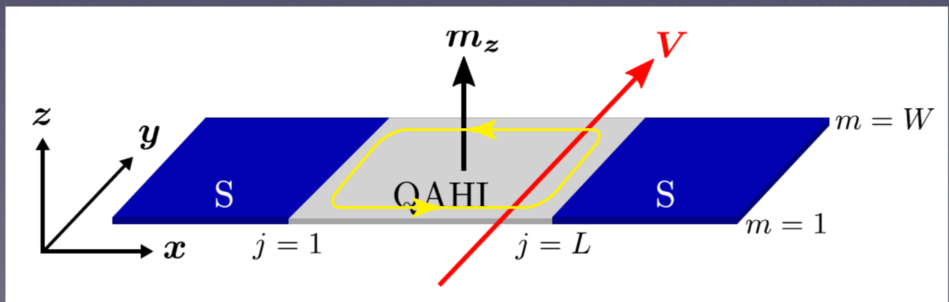


Zeeman

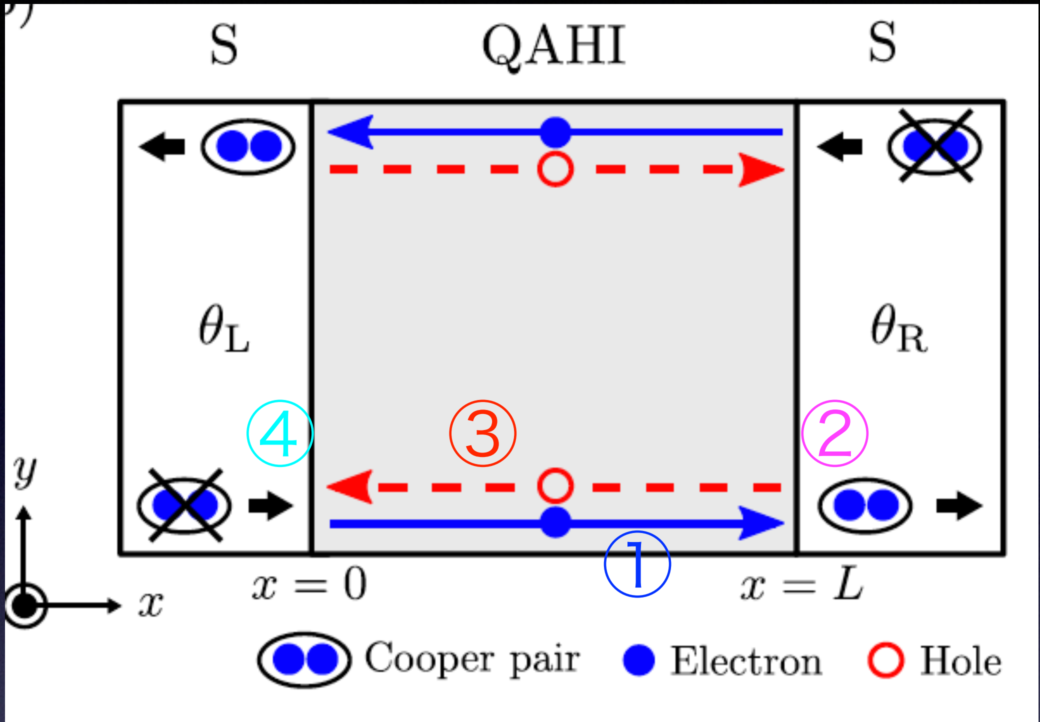
Spin-orbit

Current-phase relationship (CPR)





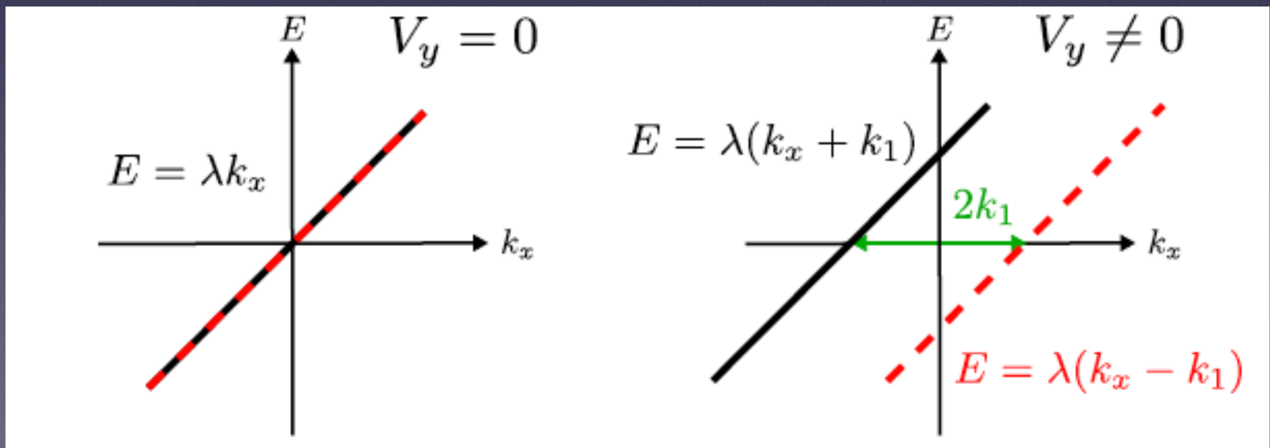
Andreev reflections



$$J = \frac{e\Delta}{\hbar} t_0^2 t_I \sin(\theta - \varphi),$$
$$\varphi = 2k_1 L = \frac{2V_y L}{\lambda}.$$

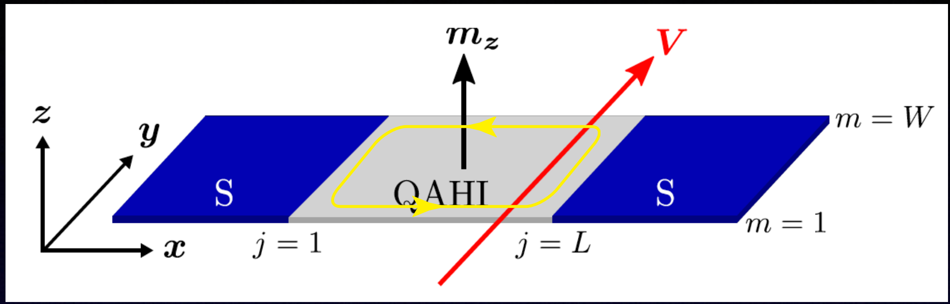
$$\underset{①}{e^{ik_e L}}\ \underset{②}{e^{-i\theta_R}}\ \underset{③}{e^{-ik_h L}}\ \underset{④}{e^{i\theta_L}}$$

$$= e^{i\theta}\, e^{i\underbrace{(k_e - k_h)L}_{-\varphi}}$$



$$k_e = k_h = 0 \qquad\qquad k_e = -k_1, k_h = k_1$$

Magnetic mirror reflection symmetry



$$\check{H} = \check{H}_L + \check{H}_R + \check{H}_Q$$

$$\check{H}_{L,R}(-\theta_{L,R}) = \check{H}^*_{L,R}(\theta_{L,R})$$

$\check{H}^*_Q = \check{H}_Q$ $\quad E(\theta) = E(-\theta)$ $\quad$ 0 or π

$\check{H}^*_Q \neq \check{H}_Q$ $\quad E(\theta) \neq E(-\theta)$ $\quad$ φ-junction

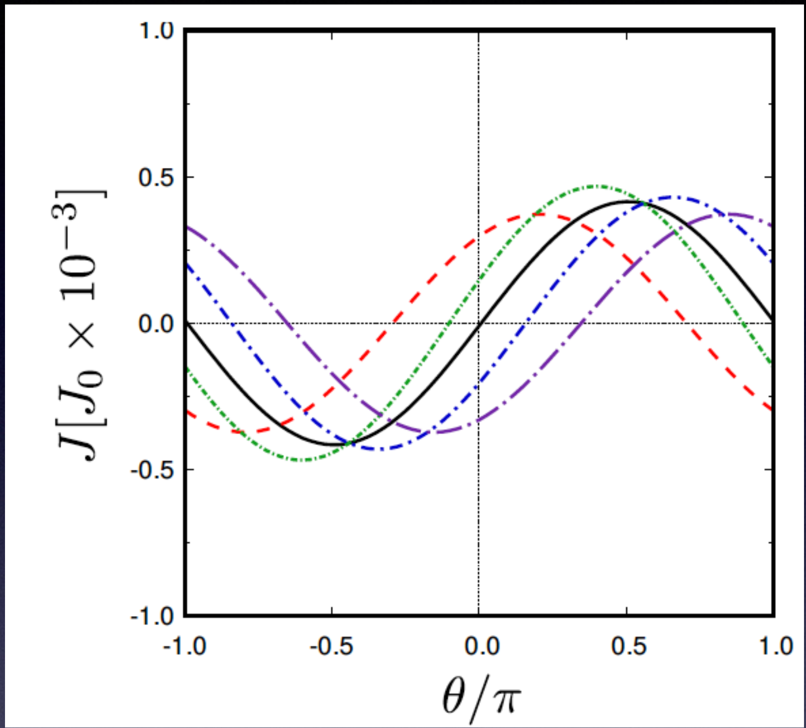
Zeeman random

$$\hat{H}_{\mathrm{Q}}(\boldsymbol{r}) = (\varepsilon_{\boldsymbol{r}} - m_z)\hat{\sigma}_3 + i\lambda\partial_x\hat{\sigma}_2 - i\lambda\partial_y\hat{\sigma}_1 - V_y\hat{\sigma}_2 \quad + V(x,y)$$

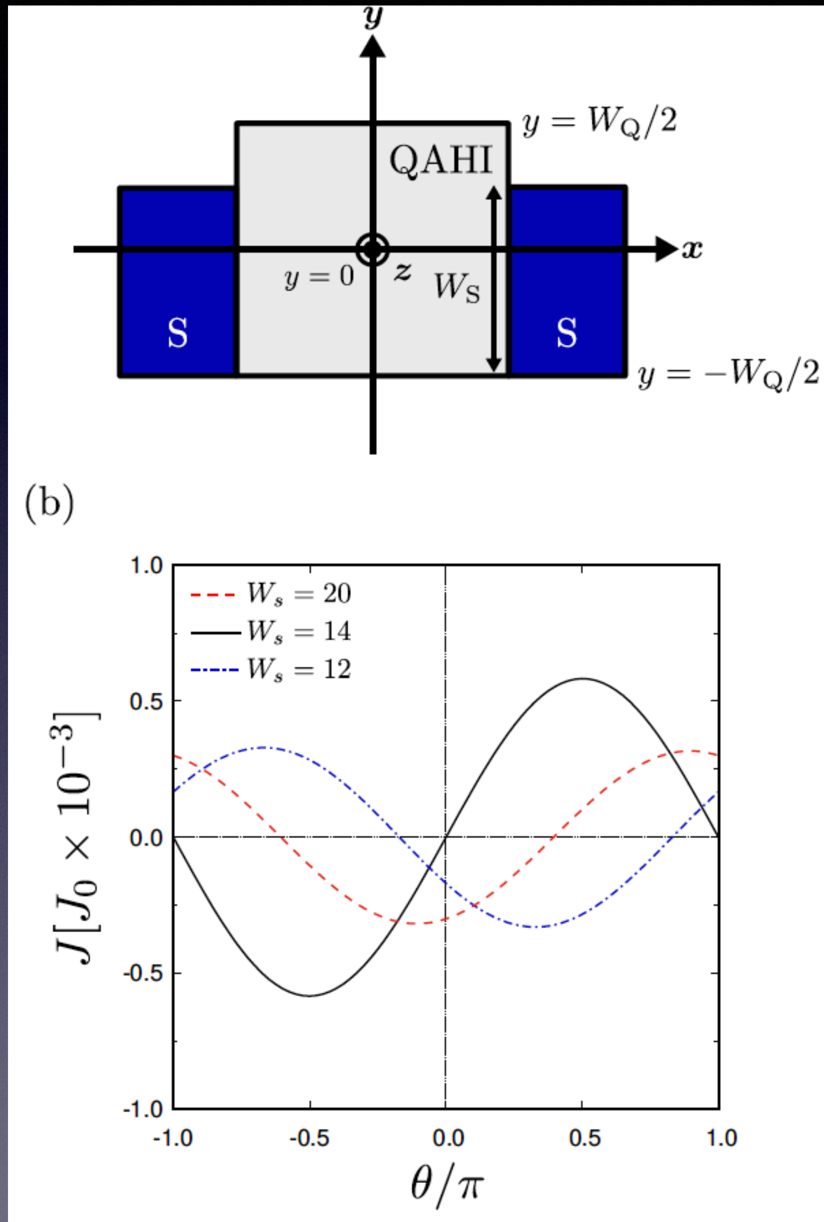
$$\hat{H}^*_{\mathrm{Q}}(\boldsymbol{r}) = (\varepsilon_{\boldsymbol{r}} - m_z)\hat{\sigma}_3 + i\lambda\partial_x\hat{\sigma}_2 + i\lambda\partial_y\hat{\sigma}_1 + V_y\hat{\sigma}_2 \quad + V(x,-y)$$

This sign can be changed by $y \to -y$

Impurity potential



Changing width



Zeeman field

Impurities

Junction shape

→ φ-junction

Summary

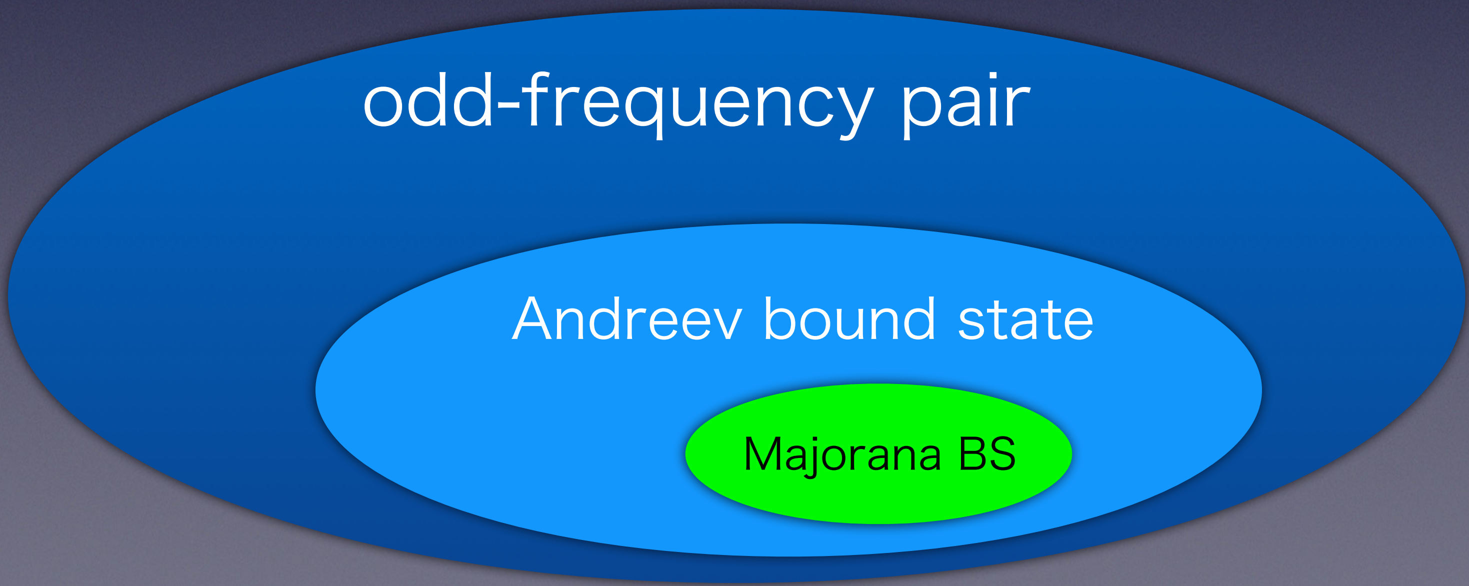
Flat-band Andreev bound states in a nodal SC

Conductance minimum and index theorem

Flat-band ZESs = Majorana

Paramagnetic response of a small superconductor

Flat-band ZESs = odd-frequency Cooper pairs



Collaborators

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Core-to-core by JSPS

Discussion

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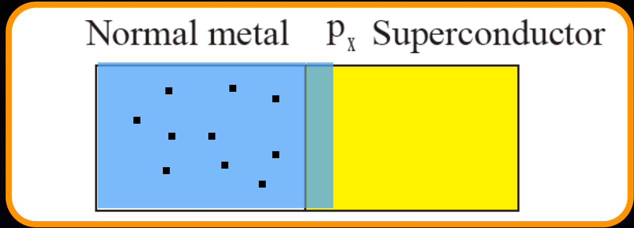
QP in normal metal

At E=0



In the ballistic limit

$$\psi_N(\boldsymbol{r}) = \sum_{n=1}^{N_c} \left[\begin{pmatrix} 1 \\ r_n^{he} \end{pmatrix} e^{ik_n x} + \begin{pmatrix} r_n^{ee} \\ 0 \end{pmatrix} e^{-ik_n x} \right] Y_n(y)$$

$$r_n^{ee} = 0, \quad r_n^{he} = -i$$ Perfect Andreev reflection

$$\psi_N(\boldsymbol{r}) = \sum_{n=1}^{N_c} \begin{pmatrix} 1 \\ -i \end{pmatrix} e^{ik_n x} Y_n(y)$$

$$\lambda = 1$$ Purely chiral

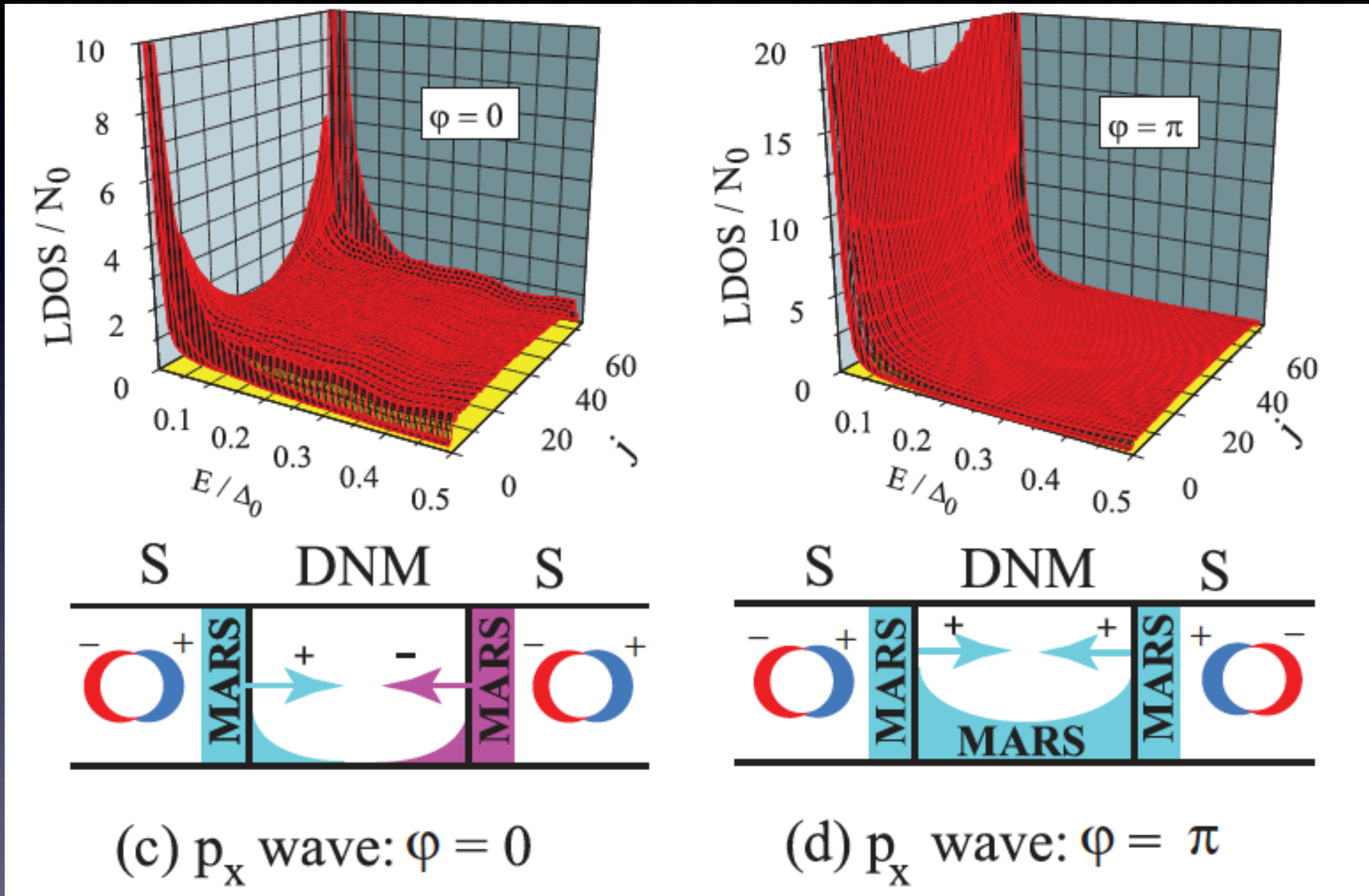
dirty case $\psi_N(\boldsymbol{r}) = \begin{pmatrix} 1 \\ -i \end{pmatrix} Z(\boldsymbol{r})$

eigen state of $-\hat{\tau}_2$

Chiral Symmetry protects the degeneracy of ZESs

Ikegaya, YA, Tanaka, PRB 91, 174511 (2015)

Penetration of Majorana fermions

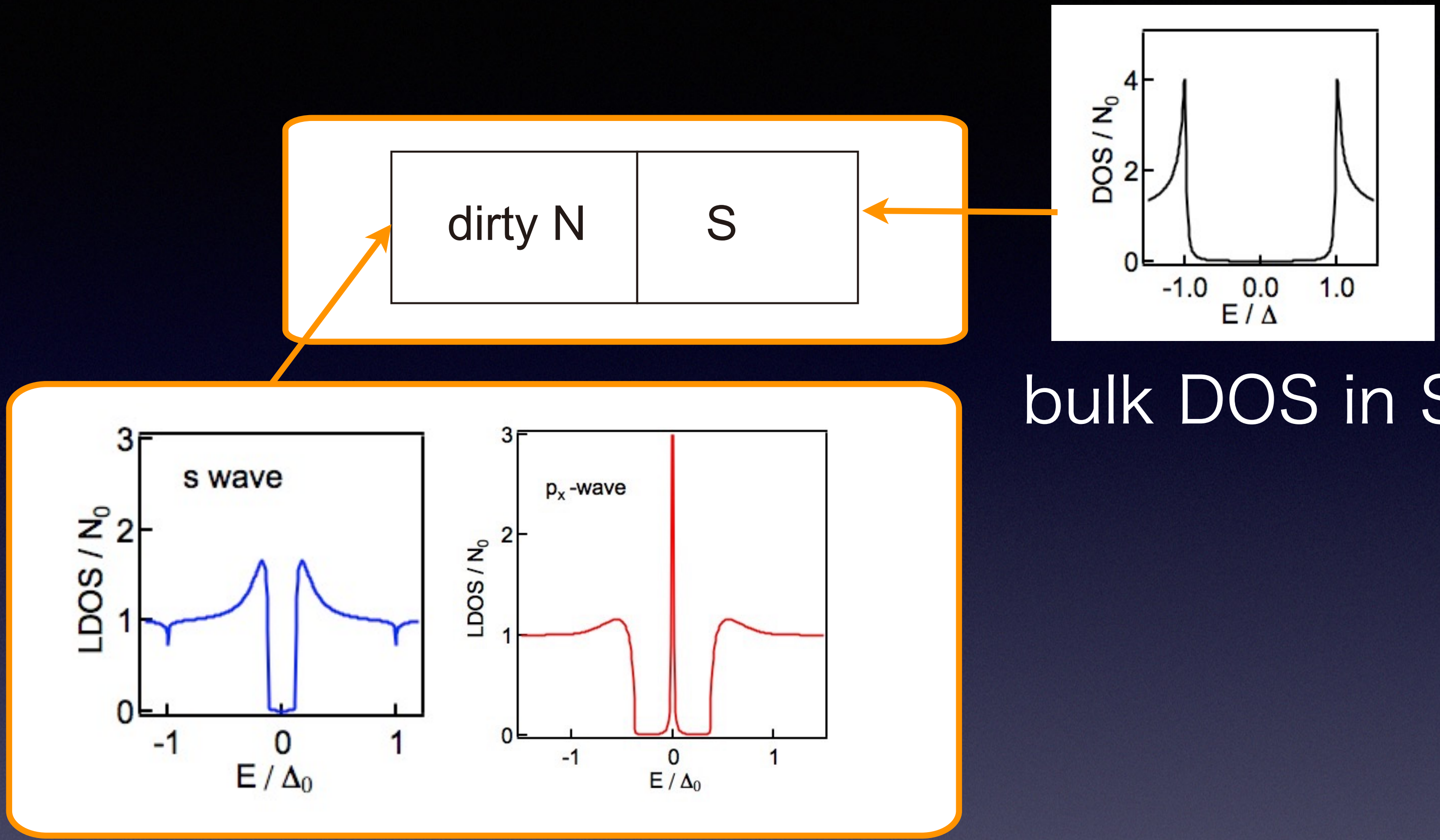


YA, Tanaka, Kashiwaya, PRL 96, 097007 (2006)

Ikegaya, YA, J. Phys Condens. Matter 28, 375702 (2016)

Origin of fractional Josephson effect

DOS in a dirty normal metal



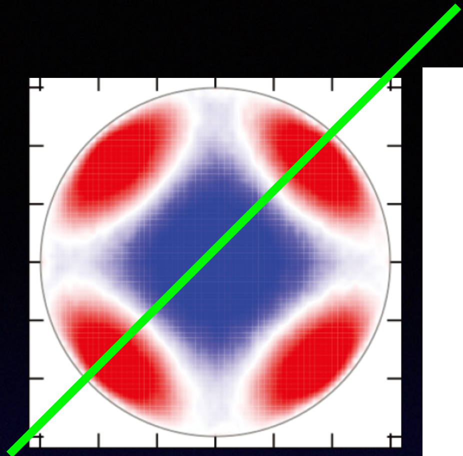
bulk DOS in S

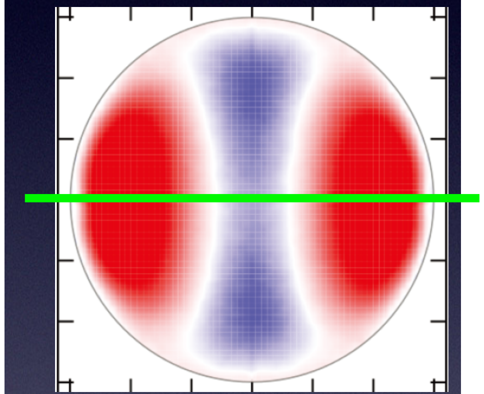
S: singlet s-wave
even-freq. → gap

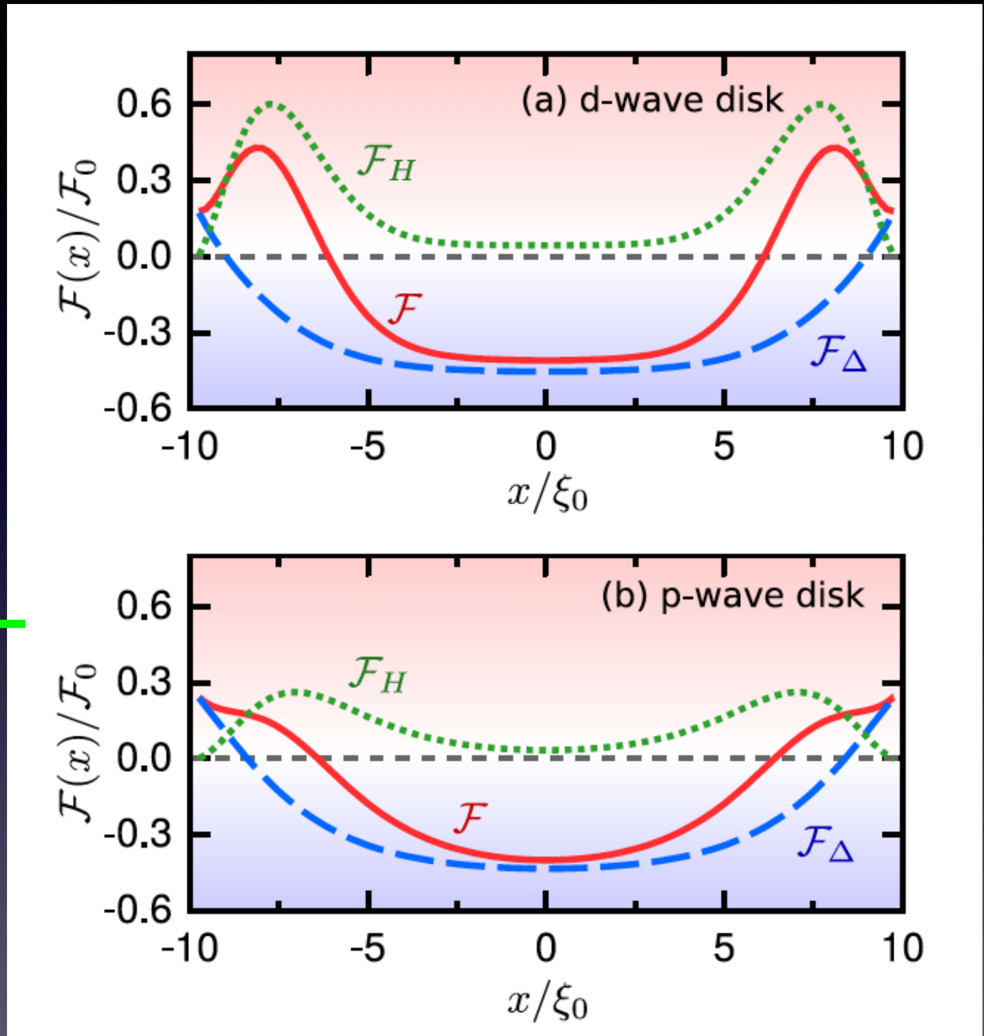
S: triplet px-wave
odd-freq. → peak

$$N(E=0)/N_0 \approx \cosh[2G_Q R_{\rm N} N_{\rm ZES}] \gg 1$$

Free-energy







$$F_S - F_N = \int d\boldsymbol{r}\mathcal{F}(\boldsymbol{r}),$$

$$\mathcal{F}(\boldsymbol{r}) = \mathcal{F}_\Delta(\boldsymbol{r}) + \mathcal{F}_H(\boldsymbol{r}),$$

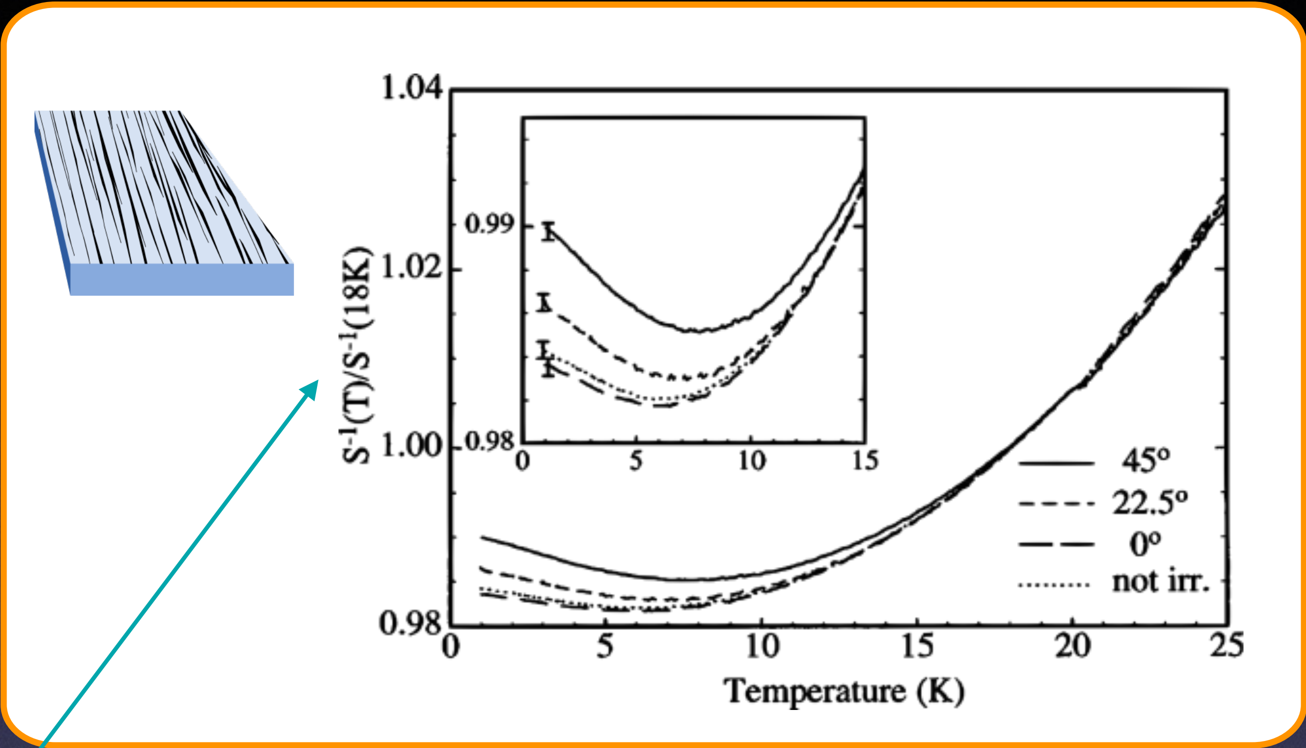
$$\mathcal{F}_H(\boldsymbol{r}) = \frac{\{H(\boldsymbol{r}) - H^{\text{ext}}\}^2}{8\pi},$$

$$\mathcal{F}_\Delta(\boldsymbol{r}) = \mathcal{F}_f(\boldsymbol{r}) + \mathcal{F}_g(\boldsymbol{r}),$$

Inhomogeneous superconducting state

Paramagnetic but $F_S - F_N < 0$

A relating experiment on HTSC films



$$\lambda \propto \frac{1}{\sqrt{n_s}}$$

'pair density'

positive : diamagnetic

negative: paramagnetic

H. Walter et al. ,

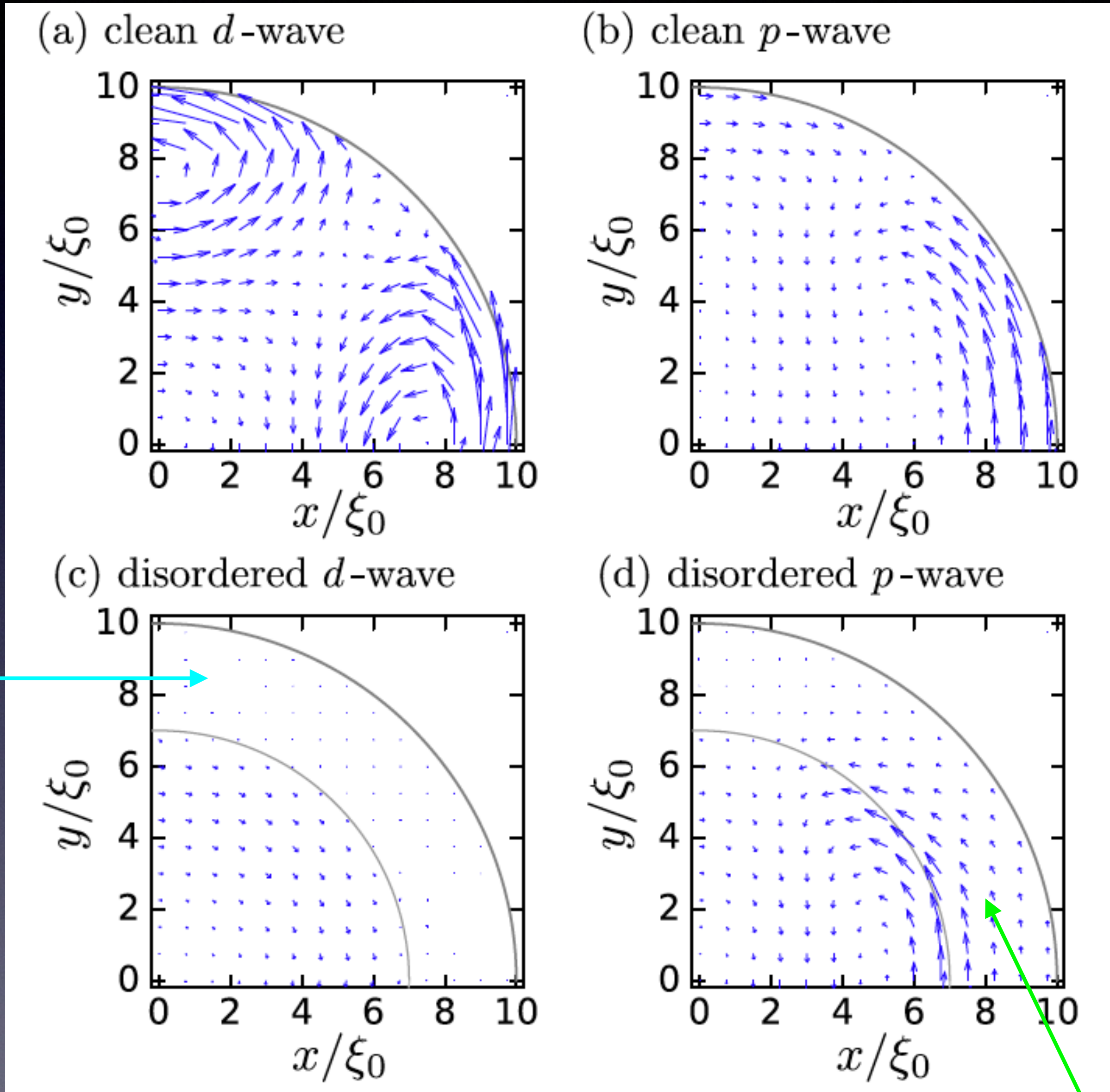
Phys. Rev. Lett. **80**, 3598 (1998)

Paramagnetic effect in the experiment is very weak !

Why?

Current profile

No pairs!



s-wave pairs exist (para)

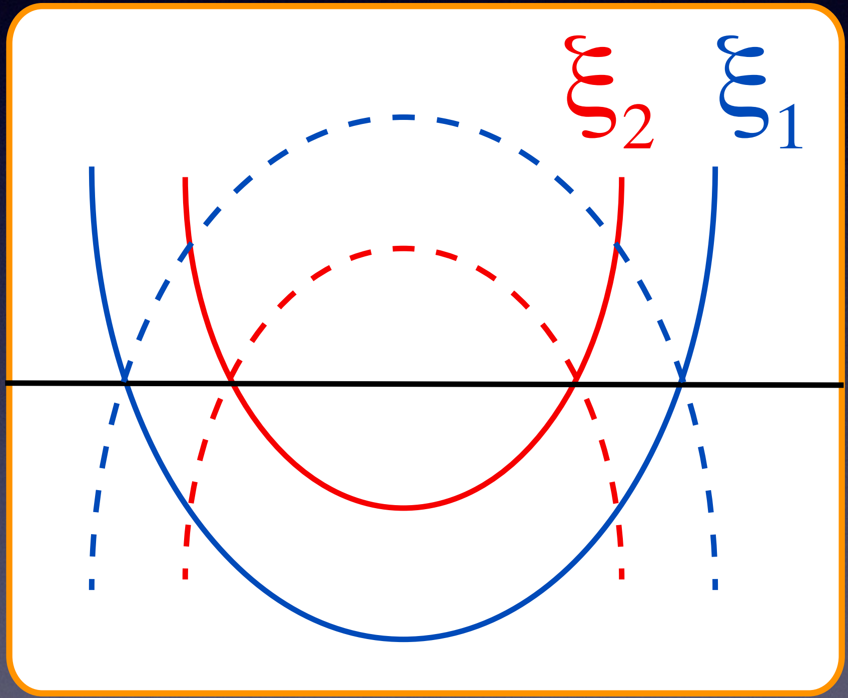
Odd-frequency pairs
in
two-band superconductors

Multi-band Superconductors

MgB2, Iron pnivtides

A. M. Black-Schaffer and A. V. Balatsky, PRB 88, 104514 (2013).

s-wave, equal-time order parameter



$$H = \begin{bmatrix} \xi_1 & V & \Delta_1 & \\ V & \xi_2 & & \Delta_2 \\ \Delta_1^* & & -\xi_1 & -V \\ & \Delta_2^* & -V & -\xi_2 \end{bmatrix}$$

V: hybridization (real)

Hybridization generates
odd-frequency odd-interband pairs! $\Delta_1 \neq \Delta_2$

$$H = \begin{bmatrix} \xi_1 & V & \Delta_1 & 0 \\ V^* & \xi_2 & 0 & \Delta_2 \\ \Delta_1^* & 0 & -\xi_1 & -V^* \\ 0 & \Delta_2^* & -V & -\xi_2 \end{bmatrix} \qquad V = v_1 + i v_2$$

$$[i\omega - H]\check{G} = 1 \qquad \check{G}(k, i\omega) = \begin{bmatrix} \mathcal{G} & \mathcal{F} \\ \underline{\mathcal{F}} & \underline{\mathcal{G}} \end{bmatrix}$$

Analyze the anomalous Green function $\mathcal{F}(k, i\omega)$

Odd-frequency pairs: Diamagnetic or Paramagnetic?

Magnetic response

$$\boldsymbol{j} = -\frac{ne^2}{mc} Q \boldsymbol{A}$$

$$Q = \frac{n_s}{n} = T \sum_{\omega_n} \frac{1}{V_{vol}} \sum_k \mathrm{Tr}\left[\mathcal{G}\underline{\mathcal{G}} + \mathcal{F}\underline{\mathcal{F}} - \mathcal{G}_N \underline{\mathcal{G}}_N\right]$$

$$\mathcal{F} = [f_0 + \boldsymbol{f} \cdot \hat{\boldsymbol{\rho}}] i \hat{\rho}_2 \qquad \sum_{k, \omega_n} f_\nu \underline{f}_\nu \quad \begin{matrix} > 0 & \text{Dia} \\ < 0 & \text{Para} \end{matrix}$$

Change basis

$$\Delta_{\pm} = \frac{\Delta_1 \pm \Delta_2}{2}, \quad \xi_{\pm} = \frac{\xi_1 \pm \xi_2}{2}$$

$$H = \begin{bmatrix} \xi_+ + \xi_- & V & \Delta_+ + \Delta_- & 0 \\ V^* & \xi_+ - \xi_- & 0 & \Delta_+ - \Delta_- \\ \Delta_+^* + \Delta_-^* & 0 & -\xi_+ - \xi_- & -V^* \\ 0 & \Delta_+^* - \Delta_-^* & -V & -\xi_+ + \xi_- \end{bmatrix}$$

ξ_-:Band asymmetry	Orbital Zeeman
Δ_-: Difference in pair potential	Equal-orbital pair
Δ_+: Average of pair potential	Equal-orbital pair
V : Hybridization	Orbital-flipping

Eilenberger Eq.

$$i\hbar v_F \hat{k}\cdot\nabla_r \check{g} + [\check{H},\check{g}] = 0,$$

$$\check{H}(r,k,i\omega_n) = \begin{bmatrix} \hat{\xi}(r,k,i\omega_n) & \hat{\Delta}(r,k) \\ \hat{\underset{\sim}{\Delta}}(r,k) & \hat{\underset{\sim}{\xi}}(r,k,i\omega_n) \end{bmatrix},$$

$$\check{g}(r,k,i\omega_n) = \begin{bmatrix} \hat{g}(r,k,i\omega_n) & \hat{f}(r,k,i\omega_n) \\ -\hat{\underset{\sim}{f}}(r,k,i\omega_n) & -\hat{\underset{\sim}{g}}(r,k,i\omega_n) \end{bmatrix},$$

$$\hat{\xi}(r,k,i\omega_n) = i\omega_n + (ev_F/c)k\cdot A(r),$$

Maxwell Eq.

$$\nabla\times \boldsymbol{H} = \frac{4\pi}{c} j$$

Current

$$j(r) = \frac{\pi e v_F N_0}{2i} T \sum_{\omega_n} \int \frac{dk}{2\pi} \mathrm{Tr}[\check{T}_3\, k\, \check{g}(r,k,\omega_n)],$$

Meissner effect by bulk condensate

Pair potential