Novel Quantum States in Condensed Matter 2017
21 November 2017, Yukawa Institute for Theoretical Physics, Kyoto University, Japan

M. Bamba, K. Inomata, and Y. Nakamura,
Phys. Rev. Lett. **117**, 173601 (2016)

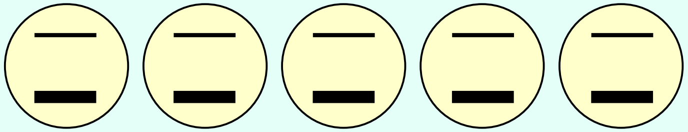
Quantum and thermal phase transitions in circuit QED system

Motoaki BAMBA

Department of Materials Engineering Science, Osaka University, Japan
& PRESTO, JST

In collaboration with Kunihiro Inomata & Yasunobu Nakamura

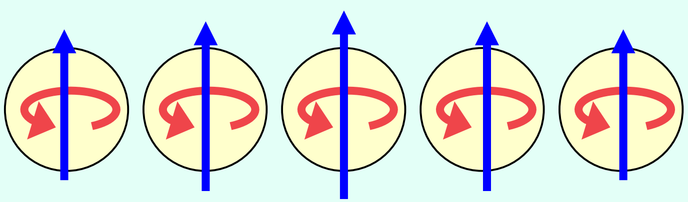
Basic concept



Many atoms in electromagnetic vacuum
(atoms in the ground state and no photon)

Super-radiant phase transition (SRPT)

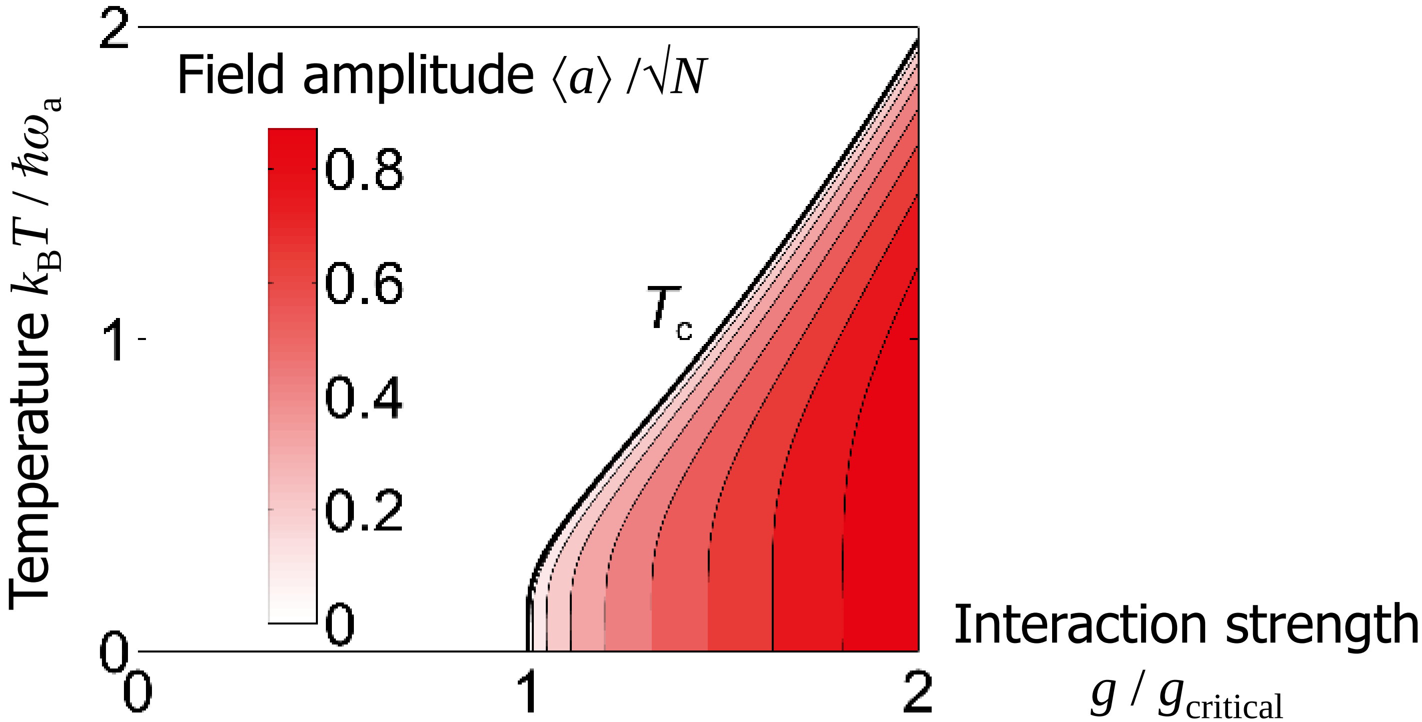
Ultra-strong interaction (correlation) between atoms and electromagnetic fields



Static magnetic field & Stationary current appear spontaneously in thermal equilibrium

Energy decrease (gain) by ultra-strong interaction > Energy increase (loss) by spontaneous field & current

Typical phase diagram & Brief summary

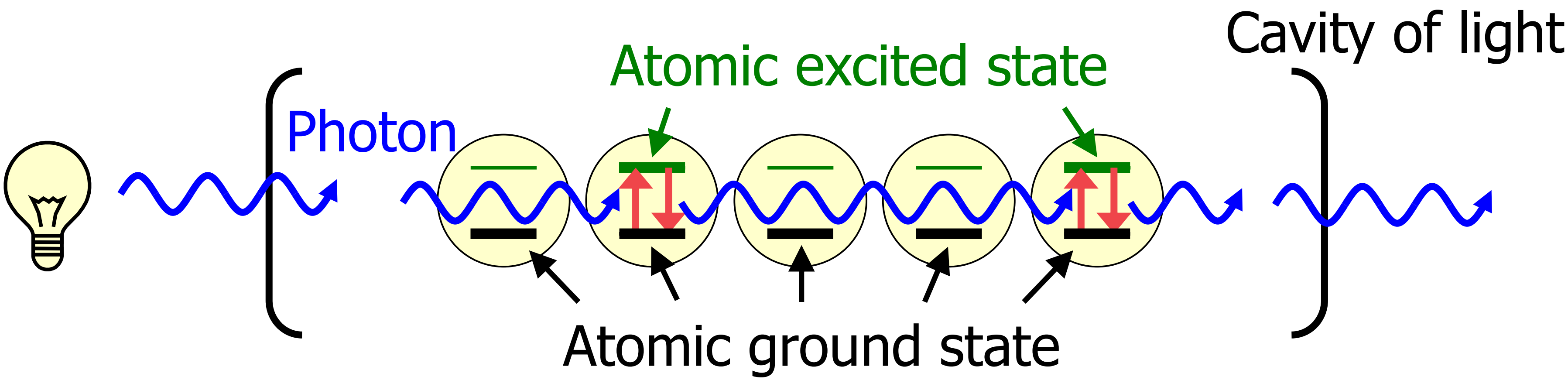


- The SRPT was proposed in 1973, but it has never been observed in experiments in thermal equilibrium.
  K. Hepp and E. H. Lieb, Ann. Phys. **76**, 360 (1973)

- We found a superconducting circuit showing the SRPT, consisting of artificial atoms and microwave resonator.
  M. Bamba, K. Inomata, and Y. Nakamura, Phys. Rev. Lett. **117**, 173601 (2016)

Typical physics and systems in quantum optics

Typical physics in quantum optics

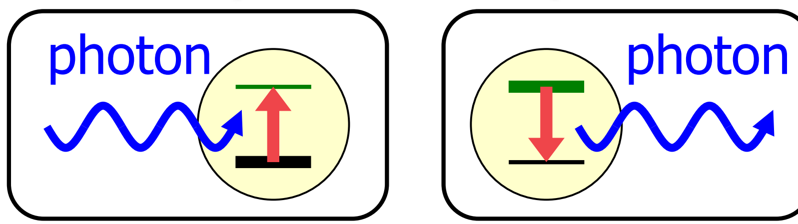


Dicke Hamiltonian

$$\hat{H}_{\mathrm{Dicke}} = \underbrace{\hbar\omega_{\mathrm{c}}\hat{a}^{\dagger}\hat{a}}_{\text{Energy of photons}} + \underbrace{\sum_{j=1}^{N}\frac{\hbar\omega_{\mathrm{a}}}{2}\hat{\sigma}_j^z}_{\text{Energy of atomic excitation}} + \frac{\hbar g}{\sqrt{N}}\sum_{j=1}^{N}(\hat{\sigma}_j^{\dagger}\hat{a} + \hat{a}^{\dagger}\hat{\sigma}_j)$$

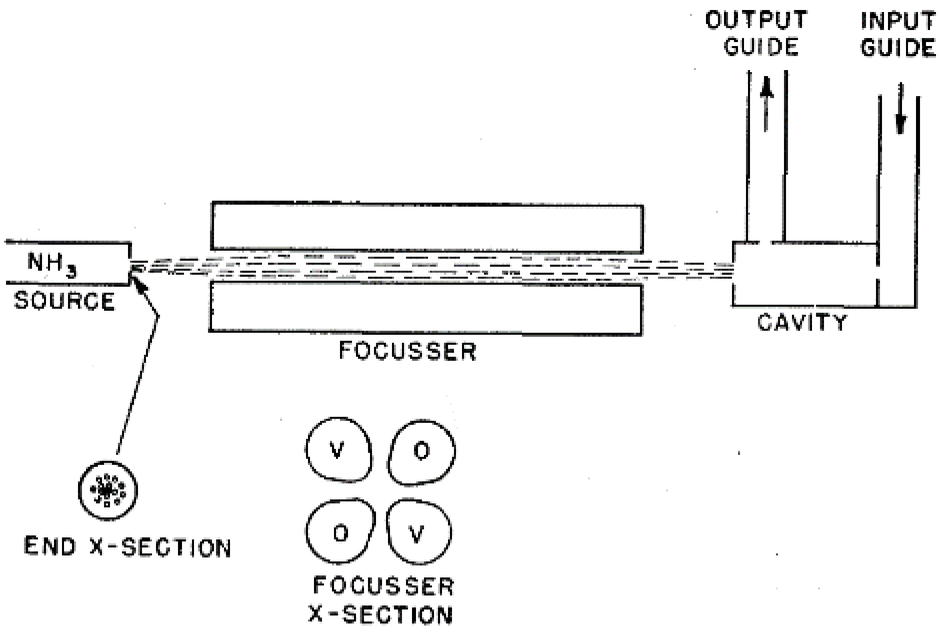
Interaction strength: g

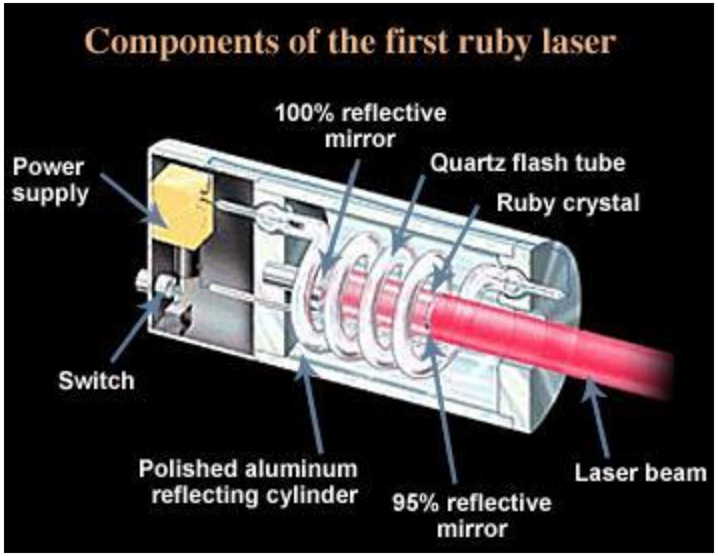
$\hat{a}$	: Annihilating a photon
$\hat{\sigma}_j$	: Lowering of atom j



Non-equilibrium dynamics of photons & atoms is typically discussed

Maser and Laser are typical systems



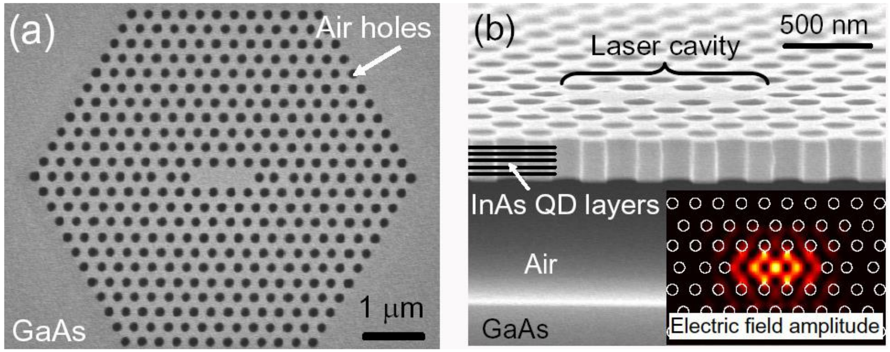


Maser (NH_3 in cavity)
J. P. Gordon, et al., Phys. Rev. **95**, 282 (1954)

Laser (Cr^{3+} in Al_2O_3 in cavity)
LaserFest
http://www.laserfest.org/lasers/how/ruby.cfm

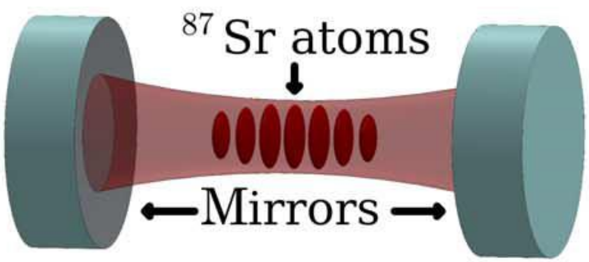
Three or four level atoms are needed for population inversion (amplification)

Other typical systems



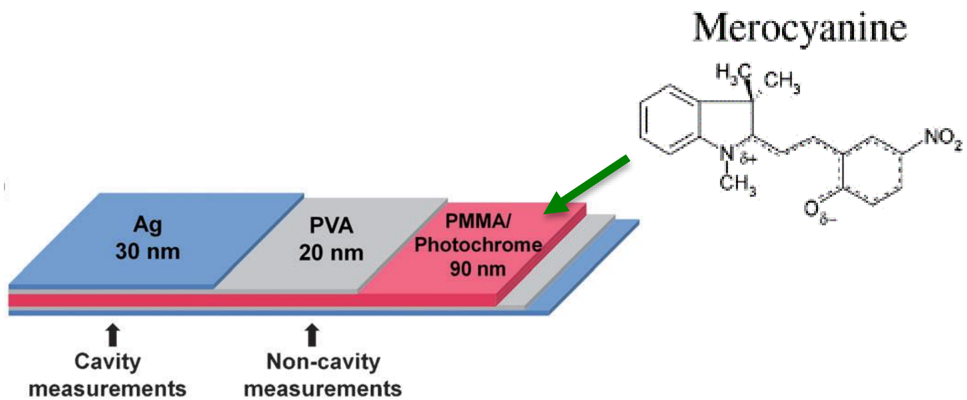
Semiconductor quantum dots
in photonic crystal cavity

M. Nomura, et al., 11 December 2007, SPIE Newsroom



Cold atoms in optical cavity

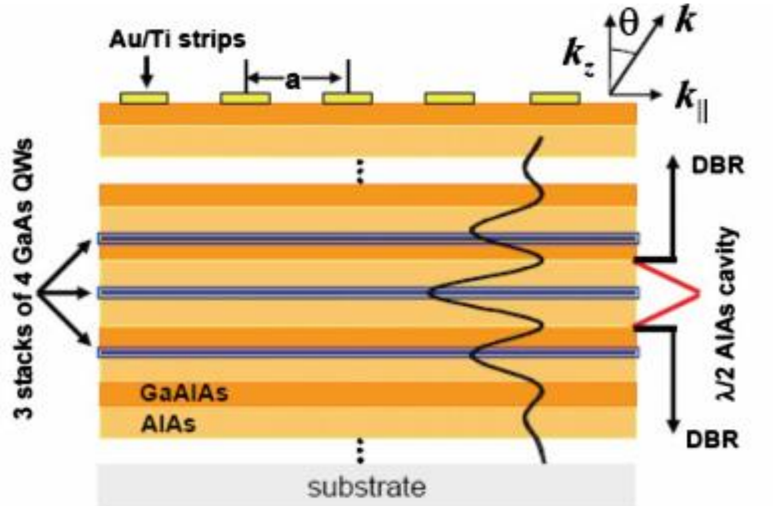
M. A. Norcia, et al., Science Advances **2**, e1601231 (2016)



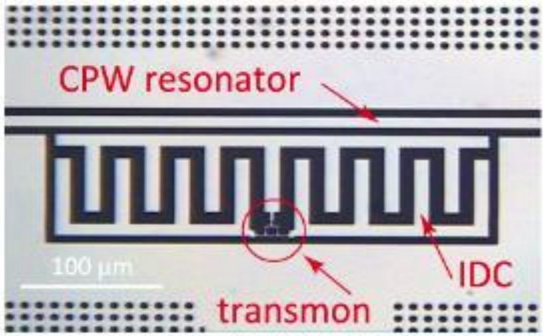
Organic molecules in micro-cavity

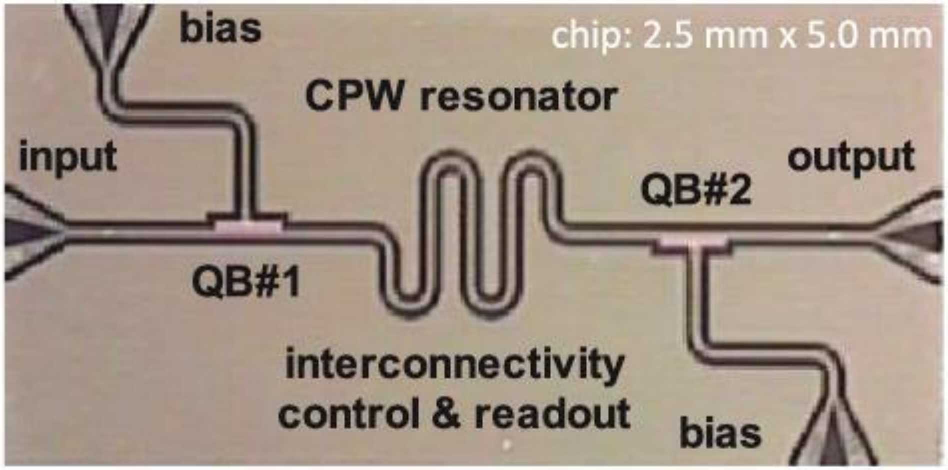
T. Schwartz, et al., PRL **106**, 196405 (2011)

Other typical systems



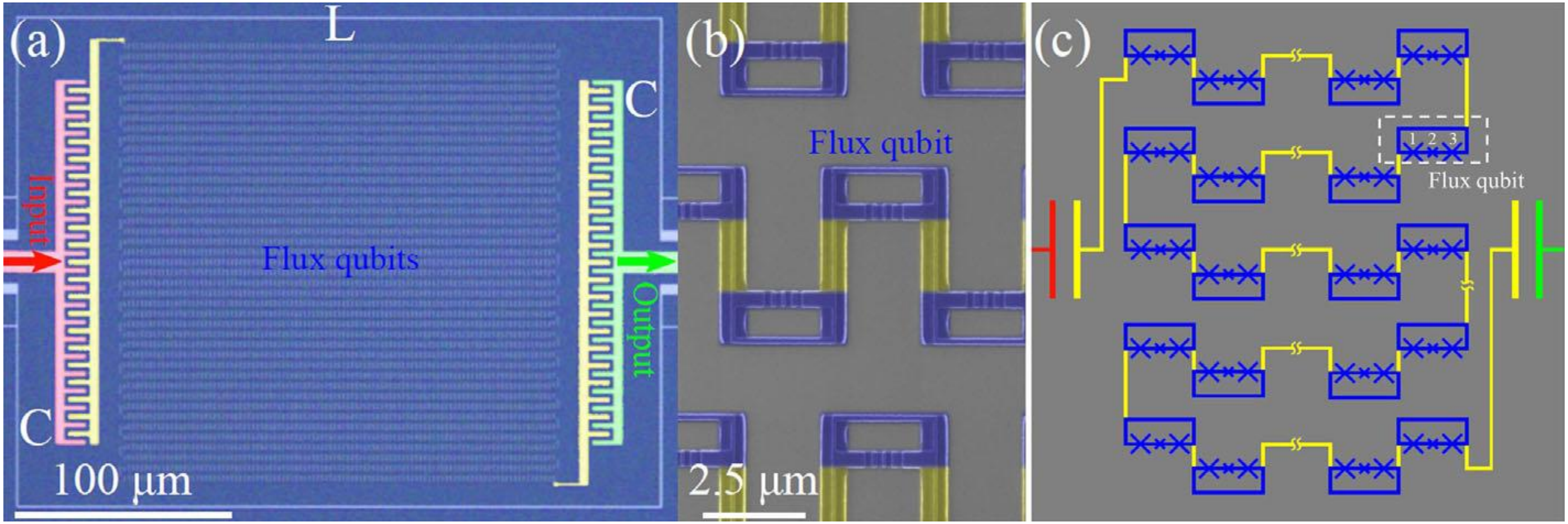
Semiconductor quantum-wells in micro-cavity
H. Deng, *et al.*, Rev. Mod. Phys. **82**, 1489 (2010)





Superconducting circuit (circuit QED system)
W. D. Oliver & P. B. Welander, MRS Bulletin **38**, 816 (2013)

Superconducting circuit with many atoms

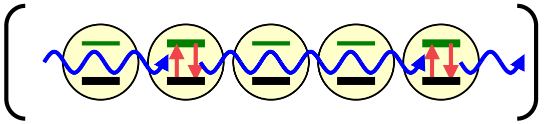


Microwave resonator (LC circuit) + 4300 artificial atoms (flux qubits)

K. Kakuyanagi, et al., Phys. Rev. Lett. **117**, 210503 (2016)

Targets of quantum optics

Non-equilibrium dynamics of photons and atoms



- Quantum information technology
  - Quantum computation (D-wave, Google, IBM, Intel, etc.)
  - Quantum communication (secured communication)
- High-sensitive sensors
  - For magnetic field (spin)
  - For temperature
  - etc.
- Fundamentals of quantum physics
  - Bell’s inequality (hidden variables)
  - etc.

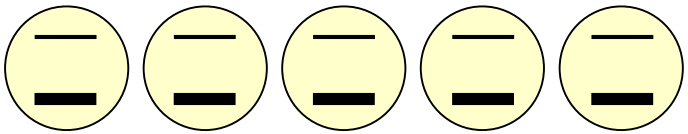
Today’s topic is a phase transition
in the thermal equilibrium,

NOT a typical phenomenon in quantum optics

K. Hepp and E. H. Lieb, Ann. Phys. **76**, 360 (1973)

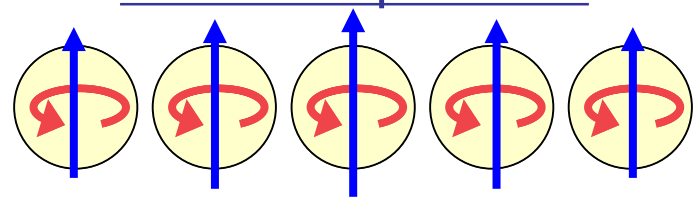
Super-radiant phase transition (SRPT)

Thermal equilibrium



Atoms in the ground state and no photon (T = 0K)

Thermal equilibrium



Static magnetic field B & Stationary current J
(Static electric field E & Static polarization P)
appear spontaneously

$$\hat{H}_{\mathrm{Dicke}} = \hbar\omega_{\mathrm{c}}\hat{a}^\dagger\hat{a} + \sum_{j=1}^{N}\frac{\hbar\omega_{\mathrm{a}}}{2}\hat{\sigma}_j^z + \frac{\hbar g}{\sqrt{N}}\sum_{j=1}^{N}(\hat{\sigma}_j^\dagger\hat{a} + \hat{a}^\dagger\hat{\sigma}_j)$$

Energy increase by field & current < Energy decrease by interaction

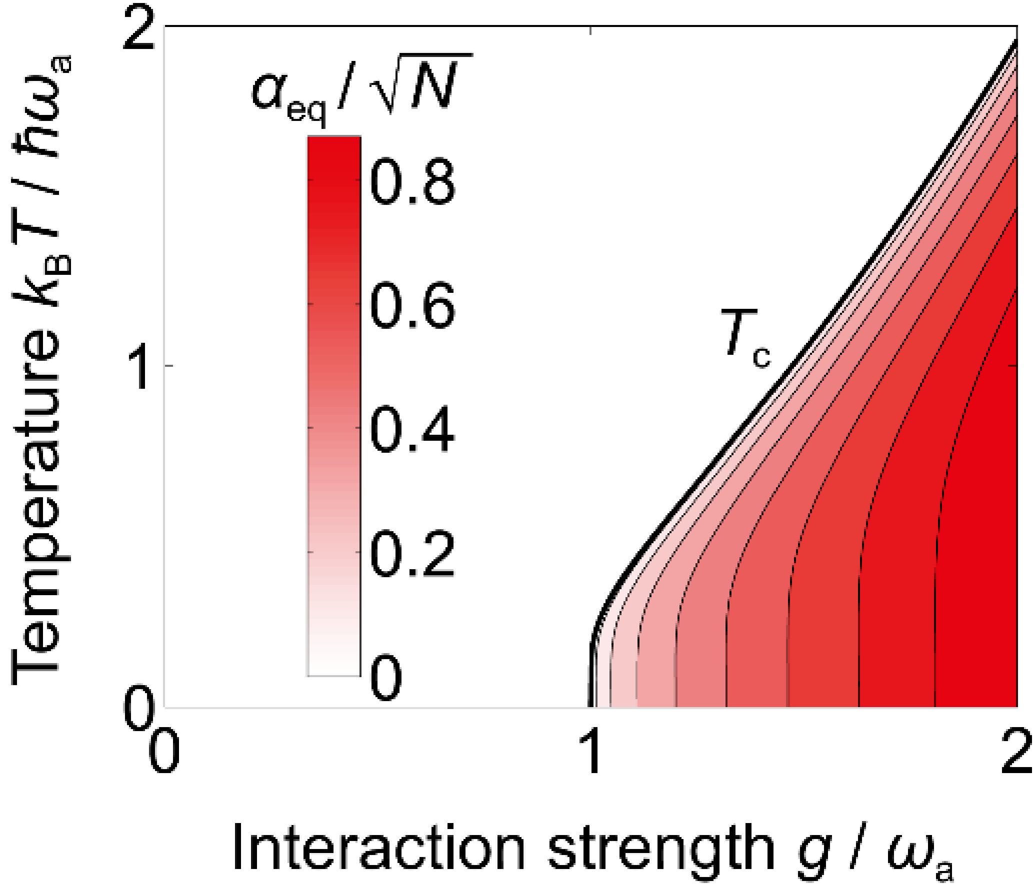
Requirements for SRPT

1. Ultra-strong interaction: $g > g_{\mathrm{critical}} = (\omega_{\mathrm{a}}\omega_{\mathrm{c}})^{1/2}$
2. Thermodynamic limit: $N \to \infty$
3. Critical temperature: $T < T_{\mathrm{c}}$ (thermal equilibrium; no light irradiation)

Photonic field gets a static amplitude spontaneously $\langle a \rangle \neq 0$

Phase diagram of Dicke Hamiltonian

In the case of $\omega_c = \omega_a$



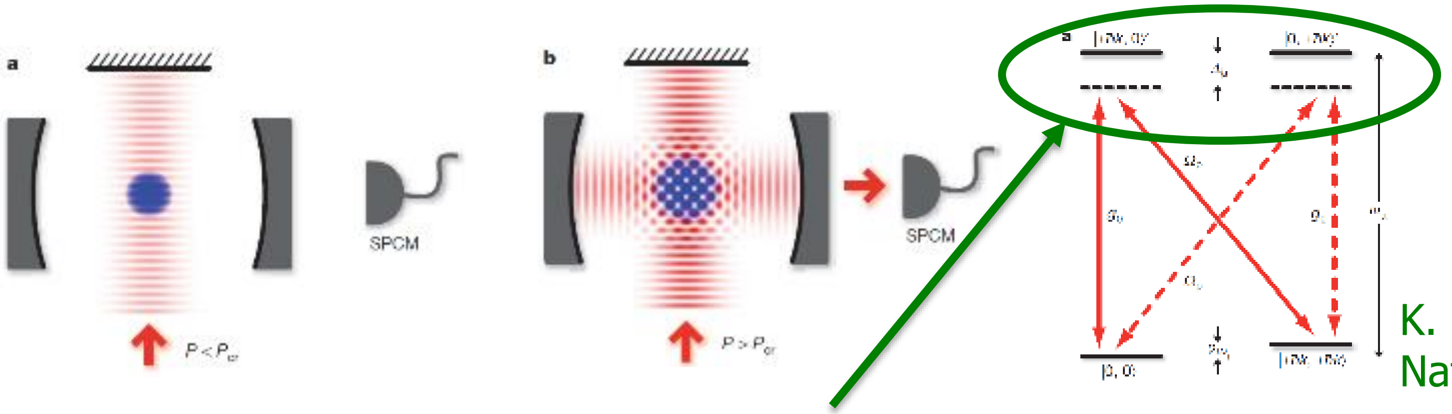
Perspectives

- Thanks to the SRPT, we can introduce the heat and phase transitions into the systems of quantum optics, where non-equilibrium dynamics of atoms and photons have long been discussed.
  - We might find phenomena involving the heat, light, current, spins, etc., and also energy conversion technologies between them.
  - The non-equilibrium statistical physics can also be developed by comparing the SRPT and the laser (non-equilibrium transition).
- Quantum information technologies are developed, since the entanglement between atoms and photons is obtained even in the thermal equilibrium.

SRPT in non-equilibrium

In non-equilibrium situation (driven by laser light)

SRPT analogue was observed in system of cold atoms



K. Baumann, *et al.*, Nature **464**, 1301 (2010)

Eliminating higher atomic levels (almost virtual excitation)

Dicke Hamiltonian is effectively implemented

$$\hat{H}_{\mathrm{Dicke}} = \hbar\omega_{\mathrm{c}}\hat{a}^{\dagger}\hat{a} + \sum_{j=1}^{N}\frac{\hbar\omega_{\mathrm{a}}}{2}\hat{\sigma}_j^z + \frac{\hbar g}{\sqrt{N}}\sum_{j=1}^{N}(\hat{\sigma}_j^{\dagger}\hat{a} + \hat{a}^{\dagger}\hat{\sigma}_j)$$

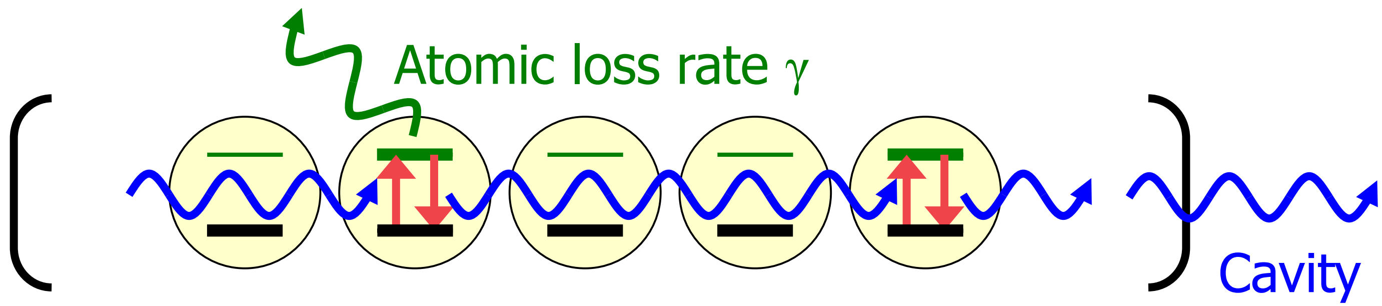
Interaction strength g is tuned by the pump power

(called “quantum” phase transition, NOT a thermal transition, temperature cannot be defined in non-equilibrium)

How about the thermal SRPT?

Requirement 1: Ultra-strong interaction

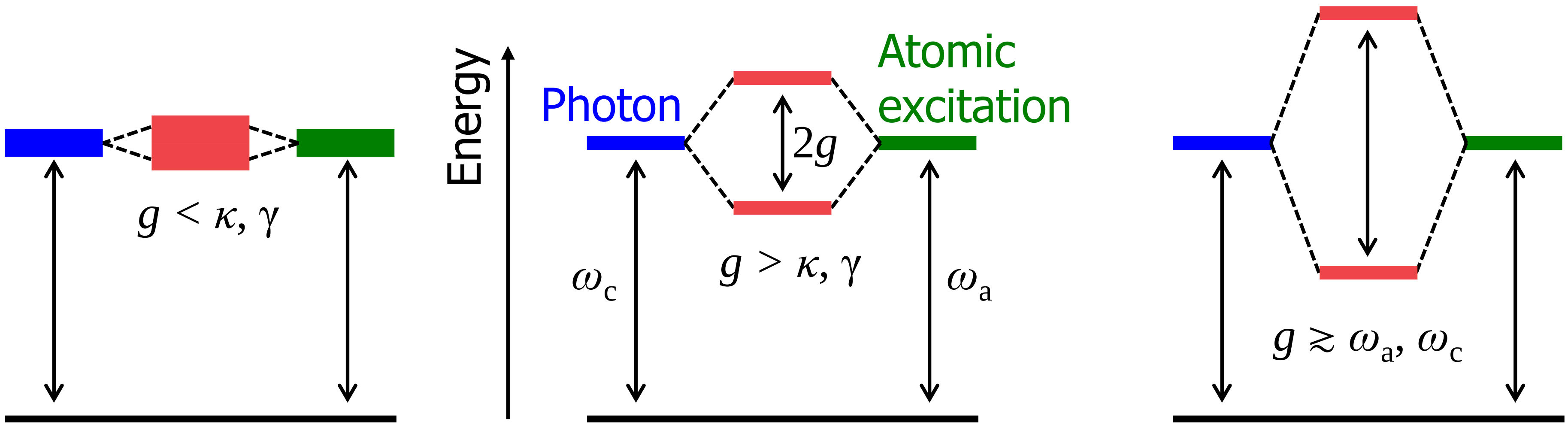
$$\hat{H}_{\mathrm{Dicke}} = \hbar\omega_{\mathrm{c}}\hat{a}^{\dagger}\hat{a} + \sum_{j=1}^{N}\frac{\hbar\omega_{\mathrm{a}}}{2}\hat{\sigma}_j^z + \frac{\hbar g}{\sqrt{N}}\sum_{j=1}^{N}(\hat{\sigma}_j^{\dagger}\hat{a} + \hat{a}^{\dagger}\hat{\sigma}_j)$$



Weak coupling

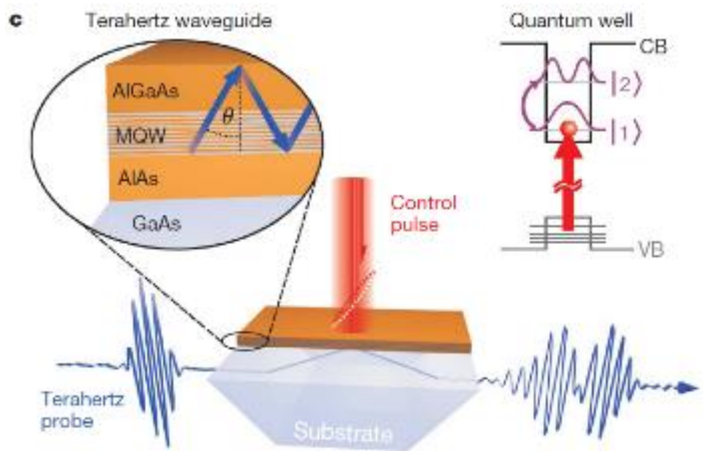
Strong coupling

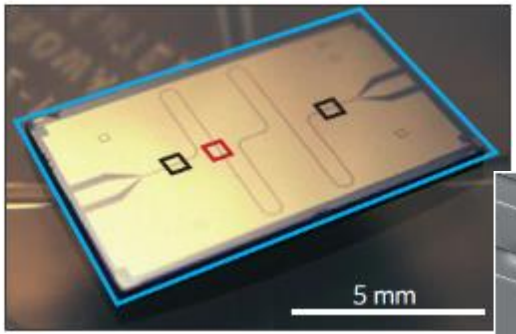
Ultra-strong coupling



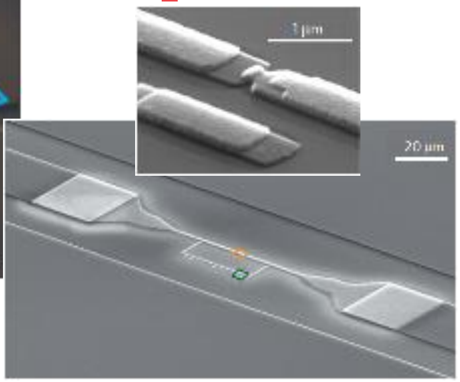
C. Ciuti, G. Bastard, & I. Carusotto, PRB **72**, 115303 (2005)

Materials showing ultra-strong interaction

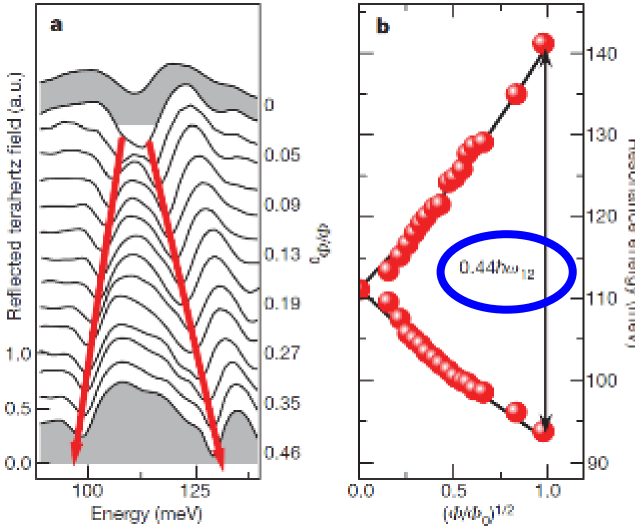




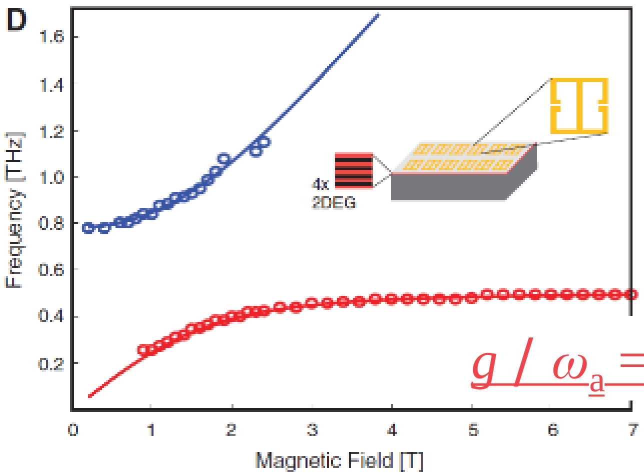
$g / \omega_a = 12\%$



Artificial atoms
in superconducting circuits (microwave)
T. Niemczyk, et al., Nature Phys. **6**, 772 (2010)



$g / \omega_a = 22\%$



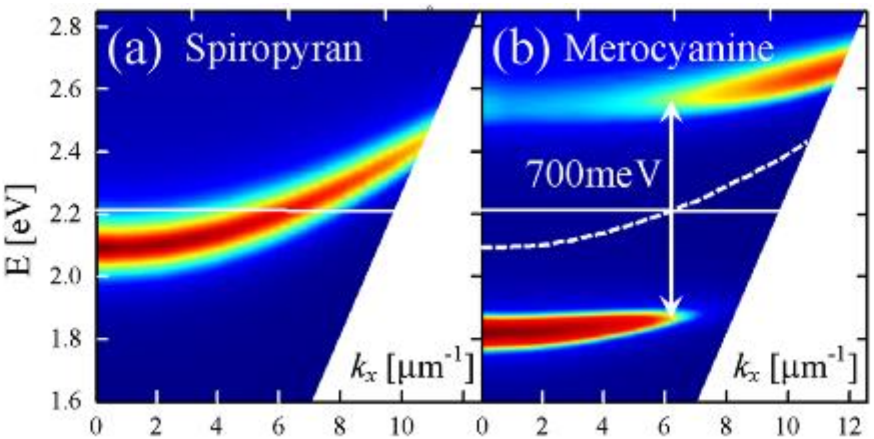
$g / \omega_a = 60\%$

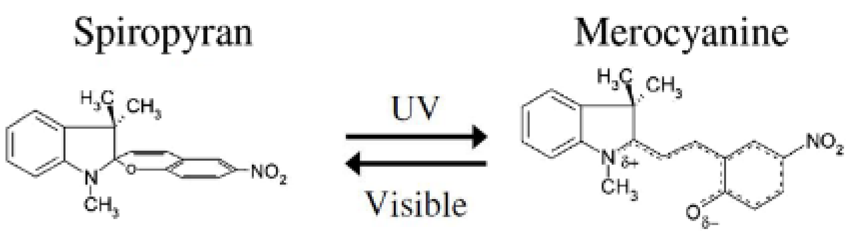
Intersubband transition in QWs (THz)
G. Gunter, et al., Nature **458**, 178 (2009)

Cyclotron transitions (THz)
G. Scalari, et al., Science **335**, 1323 (2012)

Materials showing ultra-strong interaction

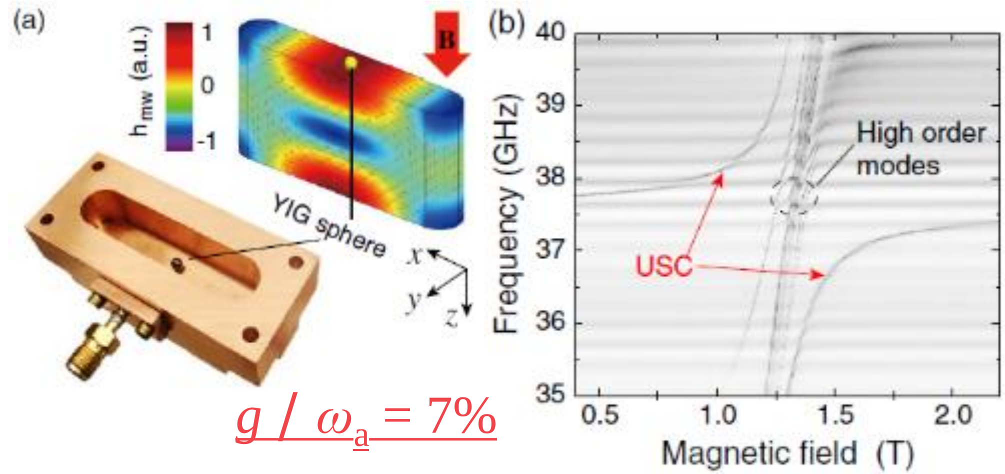
$g / \omega_a = 16\%$





Dye molecules (visible)

T. Schwartz, et al., PRL **106**, 196405 (2011)

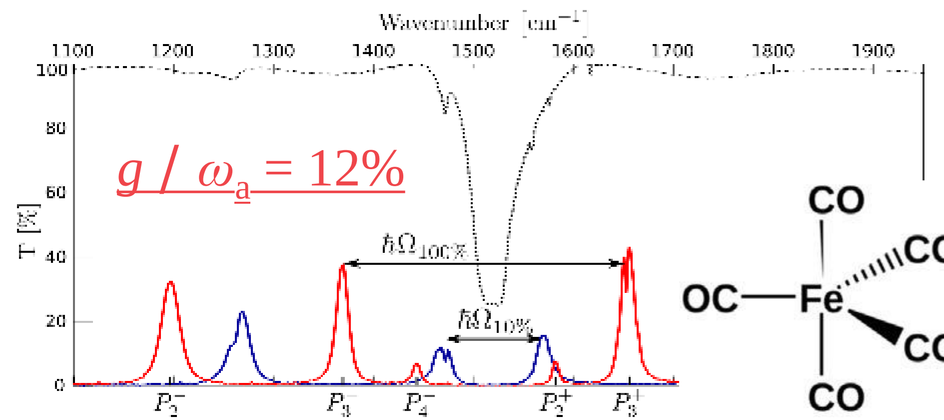


$g / \omega_a = 7\%$

Magnon in YIG sphere (microwave)

X. Zhang, et al., PRL **113**, 156401 (2014)

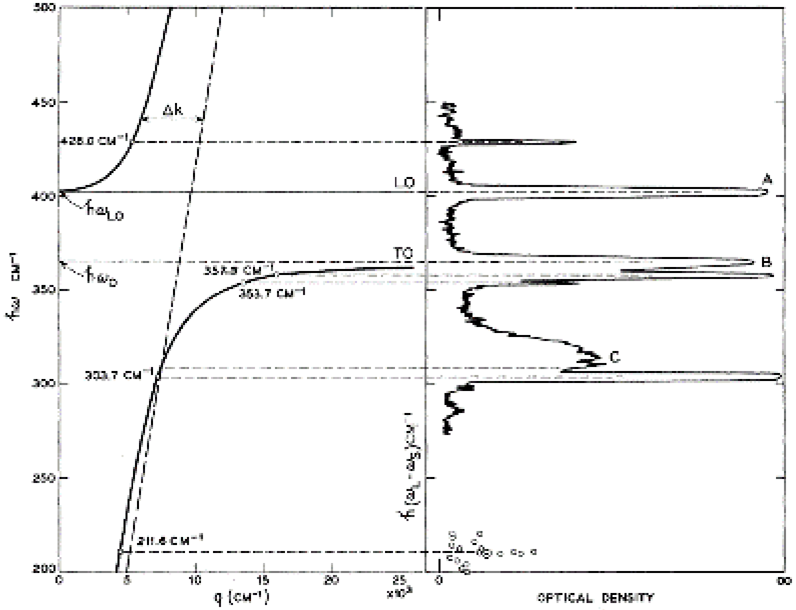
$g / \omega_a = 12\%$



Molecular vibration (infra-red)

J. George, et al., PRL **117**, 153601 (2016)

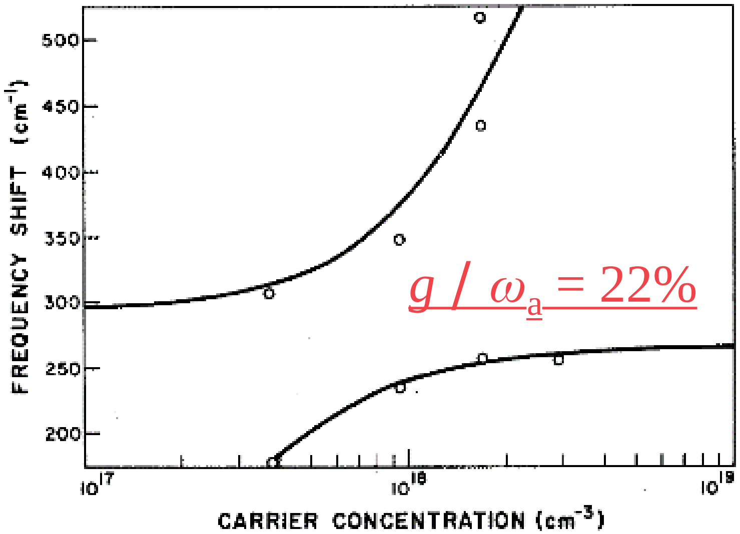
Traditional systems in ultra-strong regime



$g\ /\ \omega_a = 23\%$

Transverse optical phonon in GaP (THz)

W. L. Faust & C. H. Henry, PRL **17**, 1265 (1966)

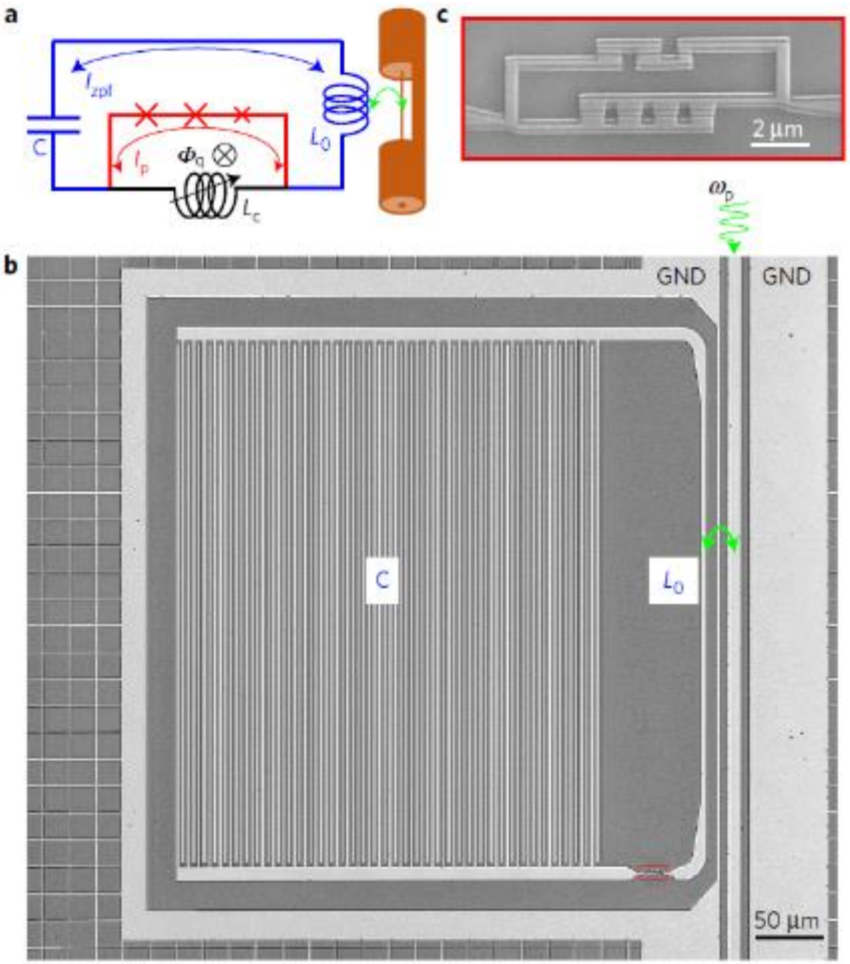


$g\ /\ \omega_a = 22\%$

Longitudinal optical phonon
- plasmon coupled (LOPC) mode in GaAs (THz)

A. Mooradian & G. B. Wright, PRL **16**, 999 (1966)

Beyond the critical interaction strength



$g\ /\ \omega_a = 134\%$

Microwave resonator (LC circuit)
+ an artificial atom (flux qubit)
F. Yoshihara, et al., Nat. Phys. **13**, 44 (2017)

However, the SRPT does not exist
even in the thermodynamic limit (many artificial atoms).

What is the problem?

The thermal SRPT has NEVER been observed,

since the first proposals in 1973

$$\hat{H}_{\mathrm{Dicke}} = \hbar\omega_{\mathrm{c}}\hat{a}^\dagger\hat{a} + \sum_{j=1}^{N}\frac{\hbar\omega_{\mathrm{a}}}{2}\hat{\sigma}_j^z + \frac{\hbar g}{\sqrt{N}}\sum_{j=1}^{N}(\hat{\sigma}_j^\dagger\hat{a} + \hat{a}^\dagger\hat{\sigma}_j)$$

- It is not the problem of the interaction strength.
- Unfortunately, many systems CAN NOT be described by the Dicke Hamiltonian in the ultra-strong regime & in thermal equilibrium.
- The Dicke Hamiltonian is a toy model, and we must start from more fundamental Hamiltonians.

Lacking term

K. Rzążewski, K. Wódkiewicz, & W. Żakowicz, Phys. Rev. Lett. **35**, 432 (1975)

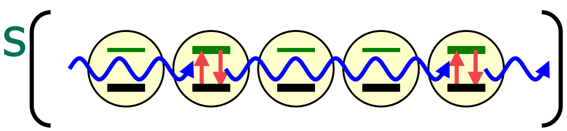
Minimal-coupling Hamiltonian

Electric field | Magnetic flux density | Momentum | Vector potential | Position

$$\hat{H}_{\min} = \int d\boldsymbol{r} \left[\frac{\varepsilon_0 \hat{\boldsymbol{E}}_\perp(\boldsymbol{r})^2}{2} + \frac{\hat{\boldsymbol{B}}(\boldsymbol{r})^2}{2\mu_0} \right] + \sum_\lambda \frac{1}{2m} \left[\hat{\boldsymbol{p}}_\lambda + e\hat{\boldsymbol{A}}(\hat{\boldsymbol{r}}_\lambda) \right]^2 + \hat{V}(\{\hat{\boldsymbol{r}}_\lambda\})$$

Kinetic energy | Coulomb interaction

For two-level atoms in a cavity



$$\hat{H}_{\min}^{\text{cavity}} = \hbar\omega_c \hat{a}^\dagger \hat{a} + \sum_{j=1}^{N} \frac{\hbar\omega_a}{2} \hat{\sigma}_j^z + \frac{\hbar g}{\sqrt{N}} \sum_{j=1}^{N} (\hat{\sigma}_j + \hat{\sigma}_j^\dagger)(\hat{a} + \hat{a}^\dagger) + \frac{\hbar g^2}{\omega_a} (\hat{a} + \hat{a}^\dagger)^2$$

$\boldsymbol{p}_\lambda \cdot \boldsymbol{A}$ | A^2 term

Does not show the SRPT

Neglecting A^2 term

Dicke Hamiltonian

$$\hat{H}'_{\text{Dicke}} = \hbar\omega_c \hat{a}^\dagger \hat{a} + \sum_{j=1}^{N} \frac{\hbar\omega_a}{2} \hat{\sigma}_j^z + \frac{\hbar g}{\sqrt{N}} \sum_{j=1}^{N} (\hat{\sigma}_j + \hat{\sigma}_j^\dagger)(\hat{a} + \hat{a}^\dagger)$$ shows the SRPT

Recognition in 1970s: The SRPT is an artifact due to the lack of A^2 term

More generally (classical analysis)

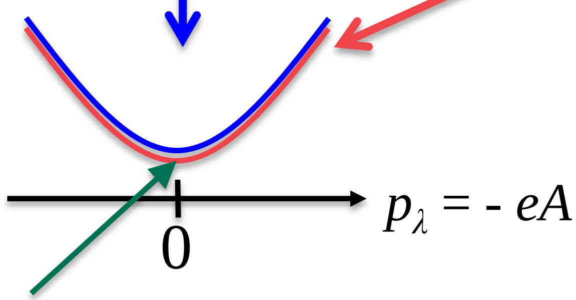
The SRPT does not exit in minimal-coupling Hamiltonian

$$\hat{H}_{\min} = \int \mathrm{d}\boldsymbol{r} \left[\frac{\varepsilon_0 \hat{\boldsymbol{E}}_\perp(\boldsymbol{r})^2}{2} + \frac{\hat{\boldsymbol{B}}(\boldsymbol{r})^2}{2\mu_0} \right] + \sum_\lambda \frac{1}{2m} \left[\hat{\boldsymbol{p}}_\lambda + e\hat{\boldsymbol{A}}(\hat{\boldsymbol{r}}_\lambda) \right]^2 + \hat{V}(\{\hat{\boldsymbol{r}}_\lambda\})$$

Electromagnetic energy

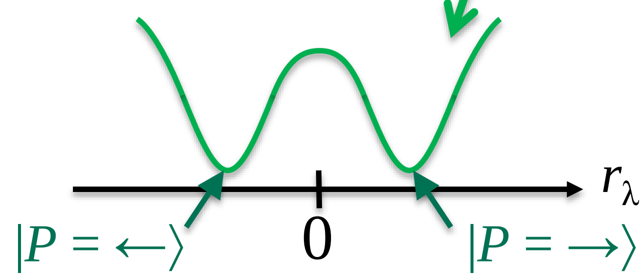
Kinetic energy minimized at $\boldsymbol{p}_\lambda = -e\boldsymbol{A}$

Coulomb energy



Magnetic flux density (A)
$=$ Current $(p_\lambda) = 0$

No amplitude in thermal equilibrium



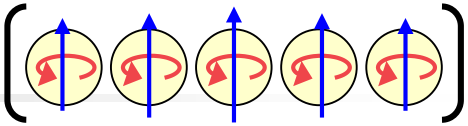
Electric polarization can be $P \neq 0$

But, phase transition of just matters

Quantum analysis (no-go theorem)

I. Bialynicki-Birula and K. Rzążewski, PRA **19**, 301 (1979)
K. Gawędzki and K. Rzążnewski, PRA **23**, 2134 (1981)

SRPT history



- In 1973, the SRPT was proposed for the Dicke Hamiltonian H_{Dicke}.
  K. Hepp and E. H. Lieb, Ann. Phys. **76**, 360 (1973)

- In 1975, it was pointed out that H_{Dicke} is not good in ultra-strong regime.
  K. Rzążewski, K. Wódkiewicz, and W. Żakowicz, PRL **35**, 432 (1975)

- In 1979-1981, it was pointed out that
  the SRPT does not exist in the minimal-coupling Hamiltonian.
  I. Bialynicki-Birula and K. Rzążewski, PRA **19**, 301 (1979)
  K. Gawędzki and K. Rzążnewski, PRA **23**, 2134 (1981)

- From 2009, many systems with ultra-strong interaction have been reported

- In 2010, a non-equilibrium analogue of the SRPT was reported in cold atoms driven by laser light.
  K. Baumann, *et al.*, Nature **464**, 1301 (2010)

- In 2010, discussion of thermal SRPT in superconducting circuit was started

- In 2016, we found a circuit showing the SRPT in thermal equilibrium
  M. Bamba, K. Inomata, and Y. Nakamura, PRL **117**, 173601 (2016)

SRPT in superconducting circuits (circuit QED systems)

Advantage of superconducting circuits

We have a large number of degrees of freedom in designing the Hamiltonian.

In contrast, real atoms are basically described by the minimal-coupling Hamiltonian (not showing SRPT).

$$\hat{H}_{\mathrm{min}} = \int \mathrm{d}\boldsymbol{r} \left[\frac{\varepsilon_0 \hat{\boldsymbol{E}}_\perp(\boldsymbol{r})^2}{2} + \frac{\hat{\boldsymbol{B}}(\boldsymbol{r})^2}{2\mu_0} \right] + \sum_\lambda \frac{1}{2m} \left[\hat{\boldsymbol{p}}_\lambda + e\hat{\boldsymbol{A}}(\hat{\boldsymbol{r}}_\lambda) \right]^2 + \hat{V}(\{\hat{\boldsymbol{r}}_\lambda\})$$

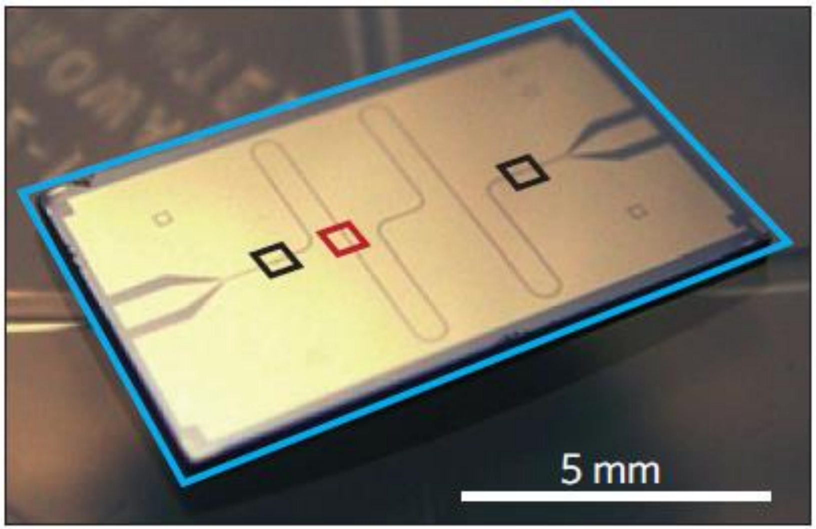
It is still contoversial whether the SRPT exists or not when we explicitly consider the spin degrees of freedom.

J. M. Knight, Y. Aharonov, and G. T. C. Hsieh, Phys. Rev. A **17**, 1454 (1978)

But, today's topic is superconducting circuits.

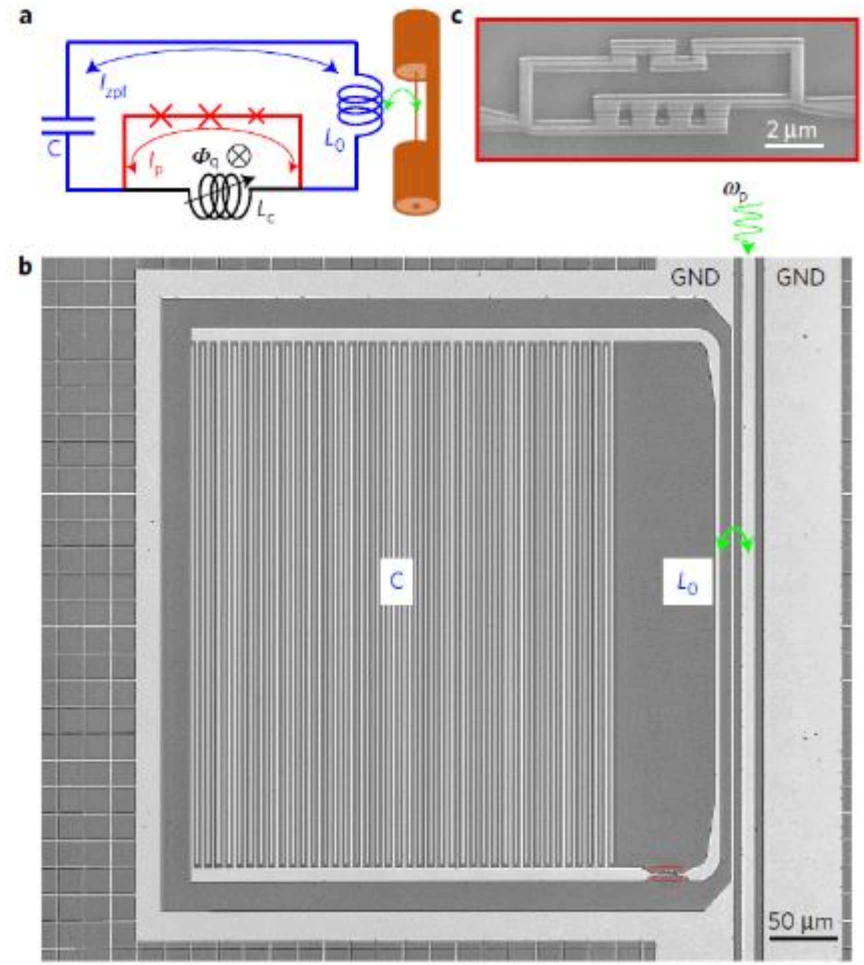
“Photons” in circuit

Microwave confined in a waveguide with a finite length



T. Niemczyk, et al., Nature Phys. **6**, 772 (2010)

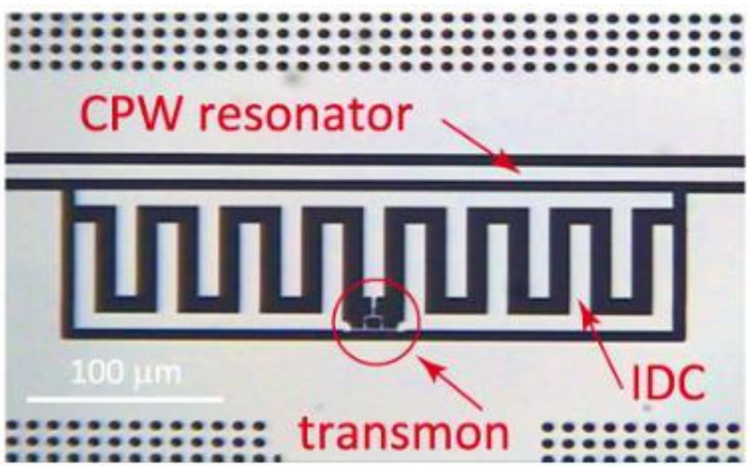
Oscillation of charge (current) in a LC circuit (resonator)



F. Yoshihara, et al., Nat. Phys. **13**, 44 (2017)

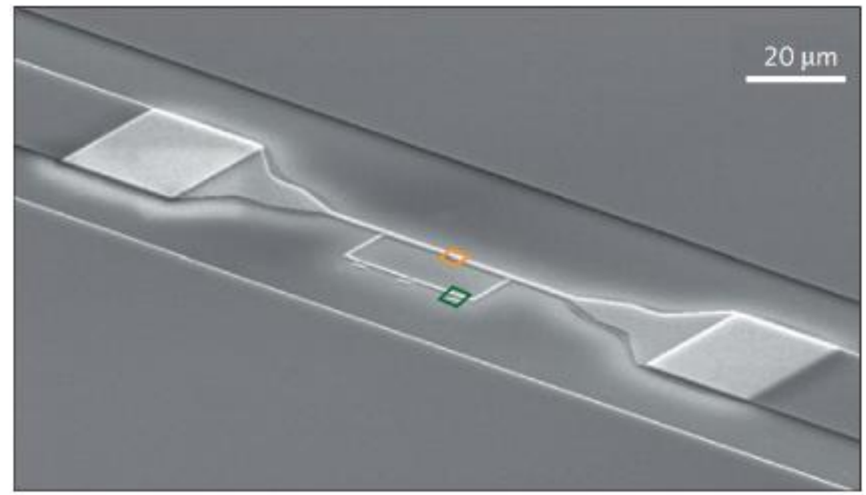
"Atoms" in circuit

Charge qubit (Transmon)

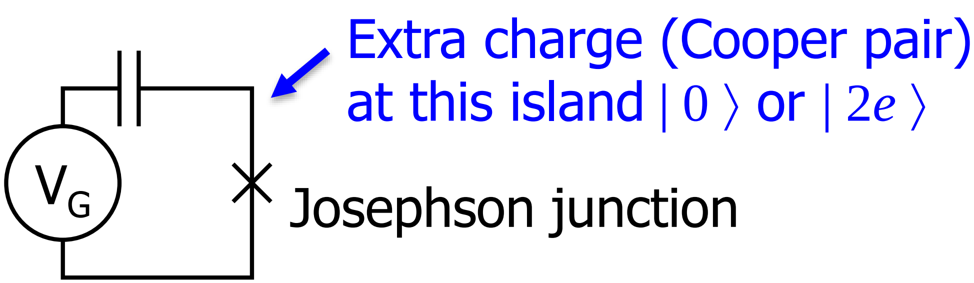


W. D. Oliver & P. B. Welander, MRS Bulletin **38**, 816 (2013)

Flux qubit

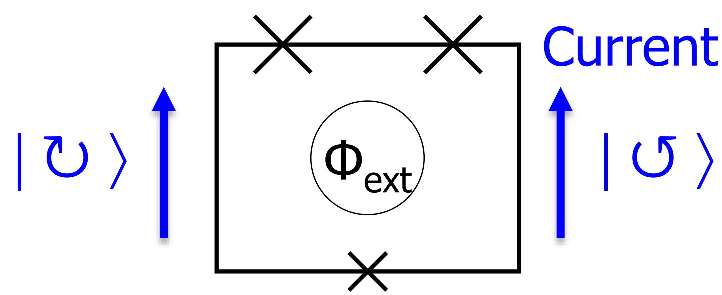


T. Niemczyk, et al., Nature Phys. **6**, 772 (2010)



Superposition of charge:

$|\,0\,\rangle \pm |\,2e\,\rangle$

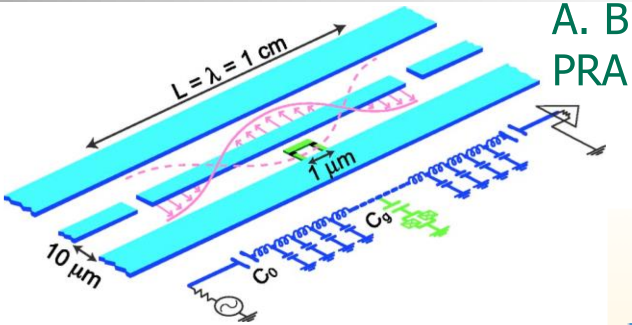


Superposition of current:

$|\,\circlearrowleft\,\rangle \pm |\,\circlearrowright\,\rangle$

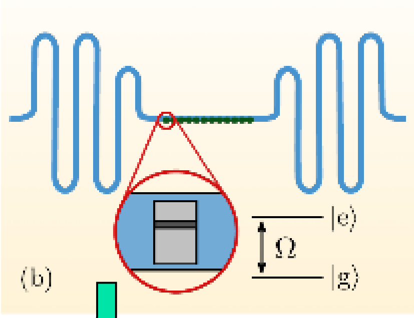
Early theories on SRPT in circuits

- <u>Possible by capacitive coupling</u>
  - Probably this kind of circuit →
  - P. Nataf and C. Ciuti,
    Nature Commun. **1**, 72 (2010)



A. Blais, et al.,
PRA **69**, 062320 (2004)

- <u>Impossible by capacitive coupling</u>
  - Above result is an artifact
    by a failure of estimating A^2 term
  - O. Viehmann, J. von Delft, and F. Marquardt, PRL **107**, 113602 (2011)



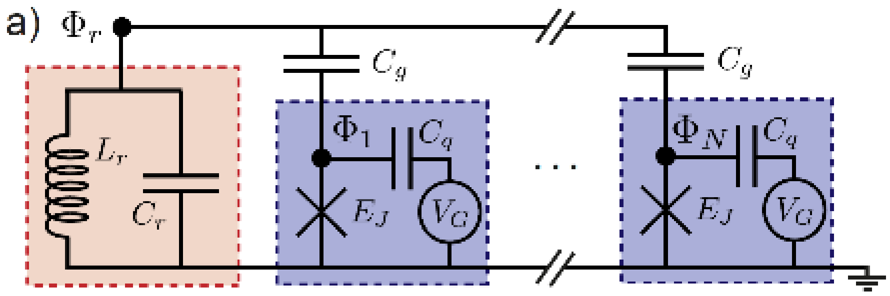
- <u>Possible by three-level artificial atoms with capacitive coupling</u>
  - Because the sum rule (for estimating A^2 term) is modified
  - C. Ciuti and P. Nataf, PRL **109**, 179301 (2012)

However, <u>they did not show any circuit diagrams</u>,
which are inevitable for examining how "photons" and "atoms" interact (e.g., whether A^2 term exists or not).

Absence of SRPT in a particular circuit

The following circuit diagram with capacitive coupling between LC resonator and charge qubits was examined.

T. Jaako, *et al.*, PRA **94**, 033850 (2016)



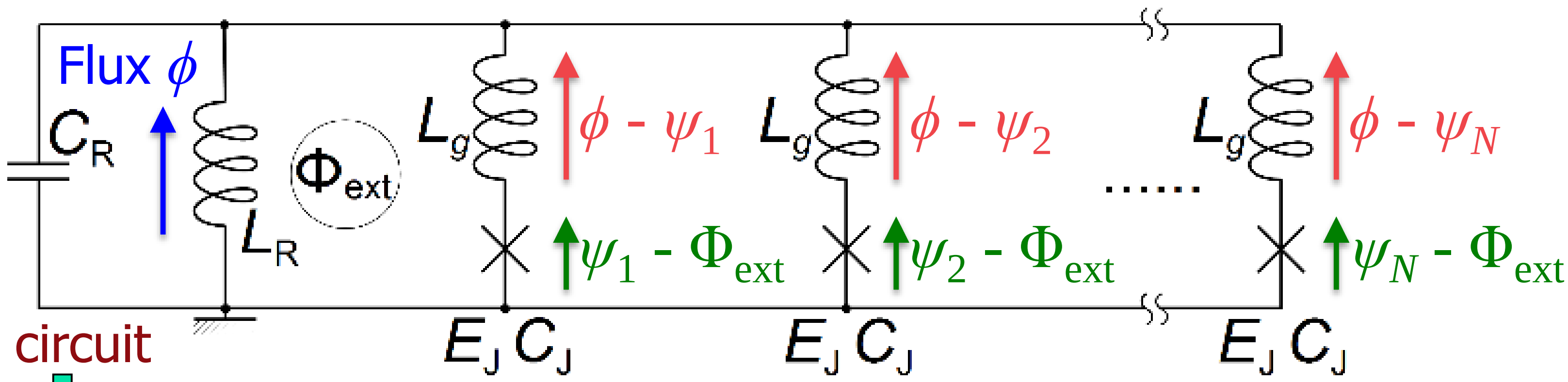
Hamiltonian was derived by the standard quantization procedure from this circuit diagram, and the absence of SRPT was confirmed in this circuit (but only for this circuit).

In contrast, we consider a different circuit with coupling through inductance

Superconducting circuit showing SRPT

M. Bamba, K. Inomata, and Y. Nakamura, PRL **117**, 173601 (2016)

Current $I = \phi / L_R$



LC circuit

"Photonic" resonator

Nonlinearity of Josephson junctions → Anharmonic "atomic" levels

Charge | Flux | Flux difference | Charge | Effective flux

$$\hat{H} = \frac{\hat{q}^2}{2C_R} + \frac{\hat{\phi}^2}{2L_R} + \sum_{j=1}^{N} \left[\frac{(\hat{\phi} - \hat{\psi}_j)^2}{2L_g} + \frac{\hat{\rho}_j{}^2}{2C_J} + E_J \cos \frac{2\pi \hat{\psi}_j}{\Phi_0} \right]$$

"Photonic" variables $[\hat{\phi}, \hat{q}] = i\hbar$

"Atomic" variables $[\hat{\psi}_j, \hat{\rho}_{j'}] = i\hbar\delta_{j,j'}$

External magnetic flux $\Phi_{ext} = \Phi_0/2$ flips the sign ($\Phi_0 = h/2e$)

Origin of SRPT

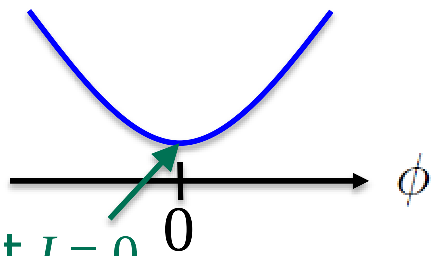
Interaction

Anharmonicity

$$\hat{H} = \frac{\hat{q}^2}{2C_{\mathrm{R}}} + \frac{\hat{\phi}^2}{2L_{\mathrm{R}}} + \sum_{j=1}^{N} \left[\frac{(\hat{\phi} - \hat{\psi}_j)^2}{2L_g} + \frac{\hat{\rho}_j{}^2}{2C_{\mathrm{J}}} + E_{\mathrm{J}} \cos \frac{2\pi \hat{\psi}_j}{\Phi_0} \right]$$

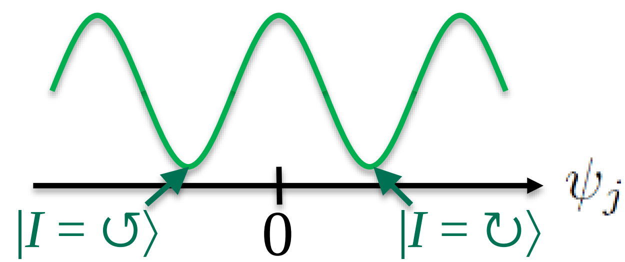
minimized at $\phi = \psi_j$

"Photonic" flux energy



Current $I = 0$

"Atomic" flux energy



Current $I \neq 0$

Competition

SRPT in the thermodynamic limit $N \to \infty$

M. Bamba, K. Inomata, and Y. Nakamura, PRL **117**, 173601 (2016)

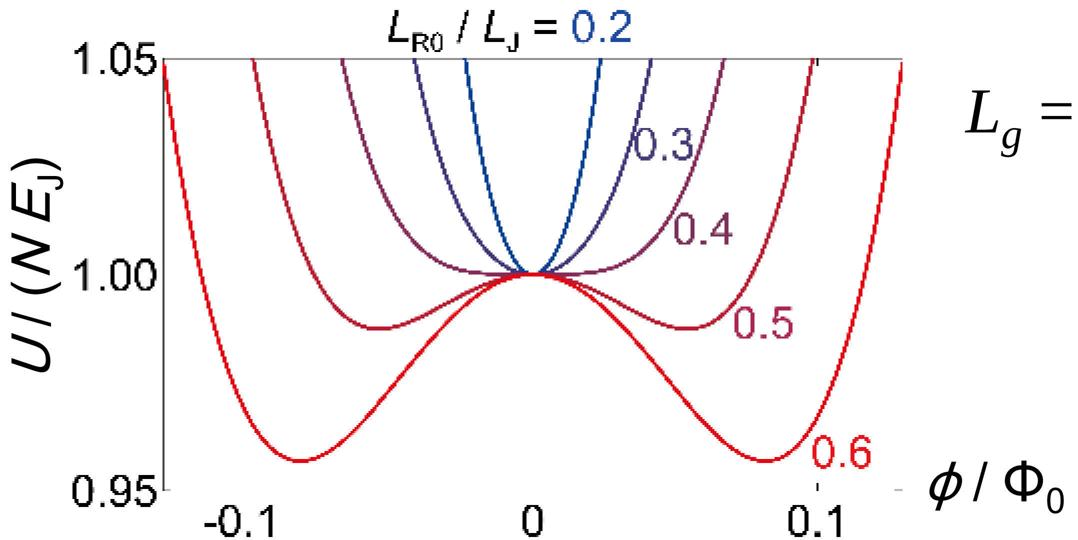
Inductive energy & minimum

$$U(\phi, \{\psi_j\}) = \frac{\phi^2}{2L_\mathrm{R}} + \sum_{j=1}^{N}\left[\frac{(\phi-\psi_j)^2}{2L_g} + E_\mathrm{J}\cos\frac{2\pi\psi_j}{\Phi_0}\right]$$

Minimized at $\psi_j = [1 + L_g/(NL_\mathrm{R})]\phi$

$L_\mathrm{J} \equiv [\Phi_0/(2\pi)]^2/E_\mathrm{J}$

$L_\mathrm{R} = L_\mathrm{R0} / N$ ⇒ U/N is N-independent



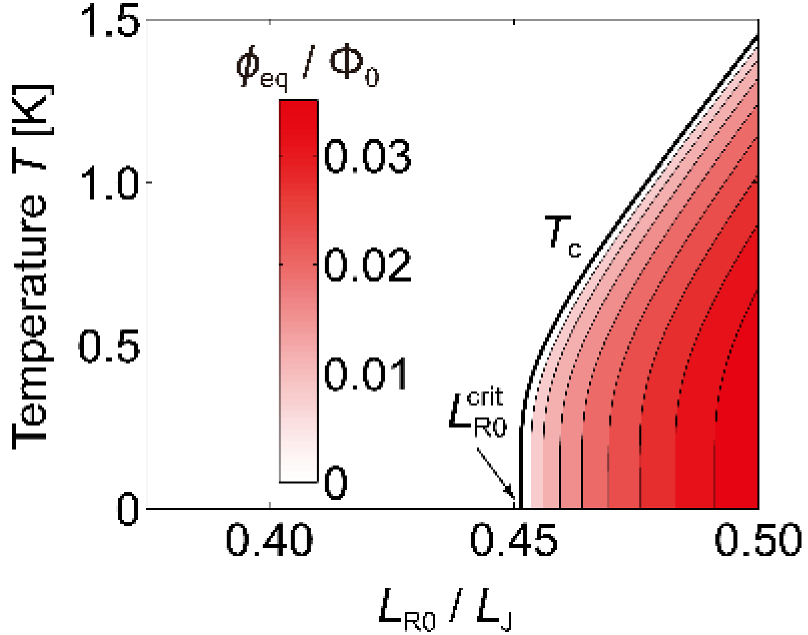
$L_g = 0.6L_\mathrm{J}$

Non-zero flux amplitude $\phi_\mathrm{eq} \neq 0$ for $L_\mathrm{R0} > L_\mathrm{J} - L_g$ ($= 0.4L_\mathrm{J}$)

⇒ SRPT (“quantum” phase transition)

Phase diagram of superconducting current

Expected flux amplitude ϕ_{eq} is calculated through $\mathcal{Z}(T) = \mathrm{Tr}[e^{-\hat{H}/k_B T}]$
for $T > 0$ in thermodynamic limit ($N \to \infty$)



Parameters

$$L_J = 0.75 \text{ nH}$$
$$L_g = 0.6 L_J = 0.45 \text{ nH}$$
$$C_J = 24 \text{ fF}$$
$$C_{R0} = 2 \text{ fF} = C_R/N$$
$$L_{R0} = N L_R$$

"Photonic" flux $\phi_{eq} \neq 0$ (i.e., current) get an amplitude spontaneously for

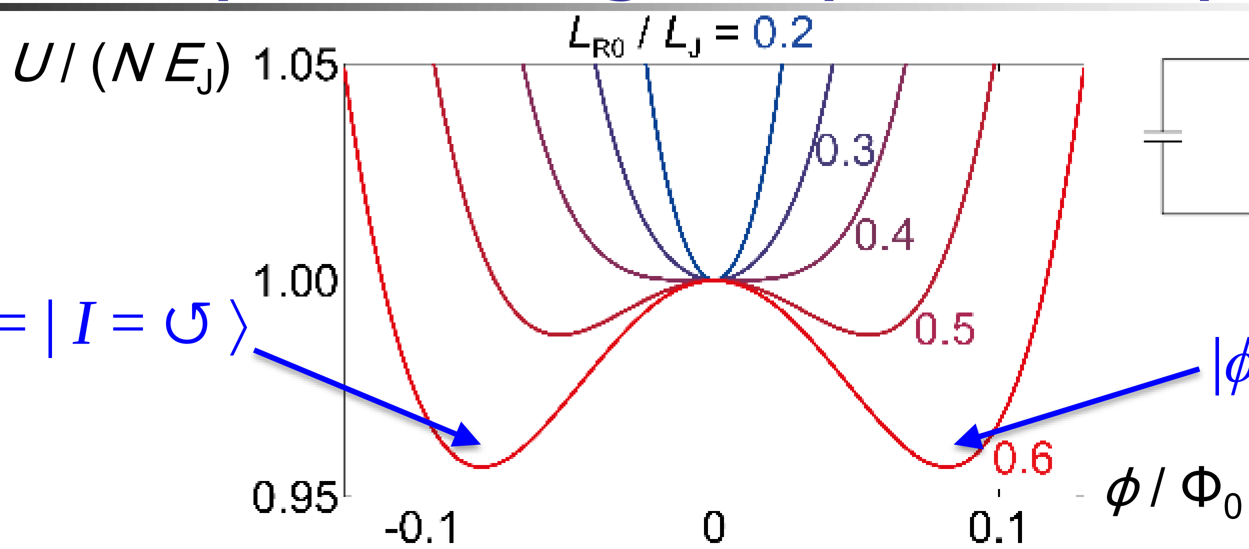
$L_{R0} > L_{R0}{}^{crit} \cong 0.45\, L_J$ & $T < T_c$ ⇒ Thermal phase transition

⇓

"Quantum" phase transition (by the change of a parameter)

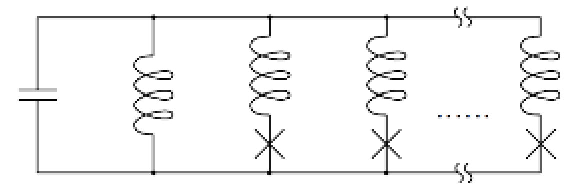
J. Larson & E. K. Irish, J. Phys. A **50**, 174002 (2017)

Symmetry breaking in quantum physics



$|-\phi_{eq}\rangle = |\, I = ↺ \,\rangle$

$|\phi_{eq}\rangle = |\, I = ↻ \rangle$



Quantum theory for finite number of junctions

$$|g\rangle = \frac{|\phi_{eq}\rangle + |-\phi_{eq}\rangle}{\sqrt{2}}$$ Superposition of the two minima

$$\Rightarrow \langle g|\hat{\phi}|g\rangle = \langle g|\hat{\psi}_j|g\rangle = 0$$ No coherent amplitude

Thermodynamic limit $(N \to \infty)$ justifies the classical analysis

$$|g\rangle = |\phi_{eq}\rangle \text{ or } |-\phi_{eq}\rangle$$

$$\langle g|\hat{\phi}|g\rangle = \pm\phi_{eq}$$ Non-zero ϕ_{eq} = (parity) symmetry breaking

Essential difference from real atoms

Mixing and anharmonicity are described by $\boldsymbol{p}_\lambda$ and $\boldsymbol{r}_\lambda$, respectively

→ No-go theorem

I. Bialynicki-Birula and K. Rzążewski, PRA **19**, 301 (1979);
K. Gawędzki and K. Rzążnewski, PRA **23**, 2134 (1981)

$$\hat{H}_{\min} = \int \mathrm{d}\boldsymbol{r} \left[\frac{\varepsilon_0 \hat{\boldsymbol{E}}_\perp(\boldsymbol{r})^2}{2} + \frac{\hat{\boldsymbol{B}}(\boldsymbol{r})^2}{2\mu_0} \right] + \sum_\lambda \frac{1}{2m} \left[\hat{\boldsymbol{p}}_\lambda + e\hat{\boldsymbol{A}}(\hat{\boldsymbol{r}}_\lambda) \right]^2 + \hat{V}(\{\hat{\boldsymbol{r}}_\lambda\})$$

$\phi \longrightarrow \boldsymbol{A}$

$\psi_j \longrightarrow \boldsymbol{p}_\lambda$

$\rho_j \longrightarrow \boldsymbol{r}_\lambda$

Mixing

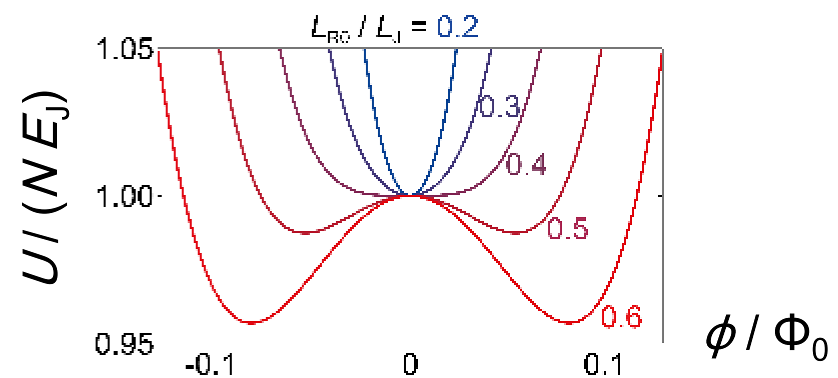
Anharmonicity

$$\hat{H} = \frac{\hat{q}^2}{2C_\mathrm{R}} + \frac{\hat{\phi}^2}{2L_\mathrm{R}} + \sum_{j=1}^{N} \left[\frac{(\hat{\psi}_j - \hat{\phi})^2}{2L_g} + \frac{\hat{\rho}_j{}^2}{2C_\mathrm{J}} + E_\mathrm{J} \cos \frac{2\pi\hat{\psi}_j}{\Phi_0} \right]$$

Mixing and anharmonicity are both described by ψ_j

↓

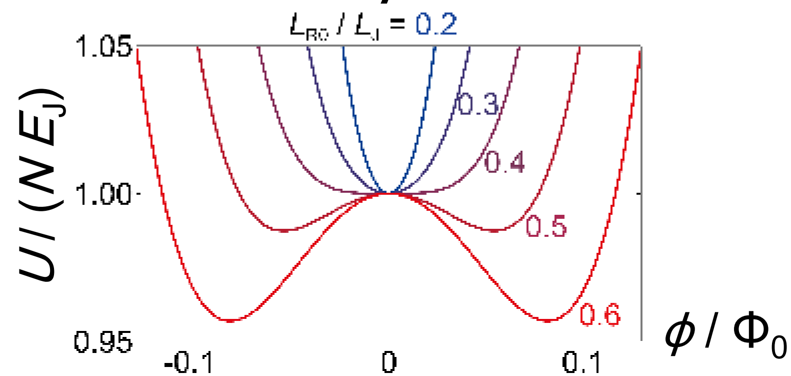
The no-go theorem is not applied



Summary

M. Bamba, K. Inomata, and Y. Nakamura, PRL **117**, 173601 (2016);
M. Bamba and N. Imoto, arXiv:1703.03533 [quantum-ph]
to be published in PRA

- We propose a circuit showing a SRPT in thermal equilibrium
  - It has not been realized since the first proposals in 1973
- Our SRPT is a natural transition in classical analysis of circuit
  - In this sense, our proposal is reliable



Reliability of theoretical proposals
has long been the main issue

- Future directions
  - Experimental observation (excitation spectra, SQUID, etc.)
  - SRPT by spins, by replacing photons with phonons, etc.
  - Phenomena (and energy conservation technologies) involving the heat, light, current, spins, etc.
  - Non-equilibrium statistical physics by comparing SRPT and laser.
  - Quantum information technologies by the entanglement between atoms and photons obtained even in the thermal equilibrium.