Floquet engineering of interactions

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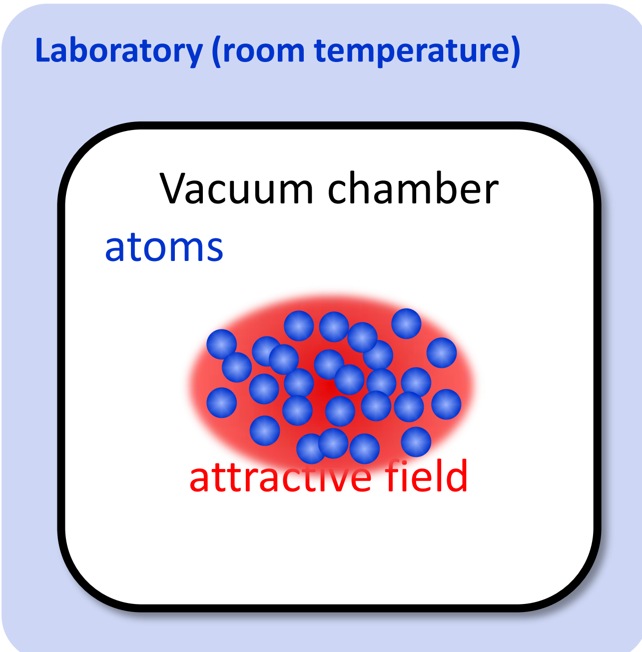
Dresden

Long-term workshop
“Novel Quantum States in Condensed Matter 2017”
Yukawa Institute for Theoretical Physics
November 23, 2017

Ultracold atomic quantum gases

Trap neutral atoms



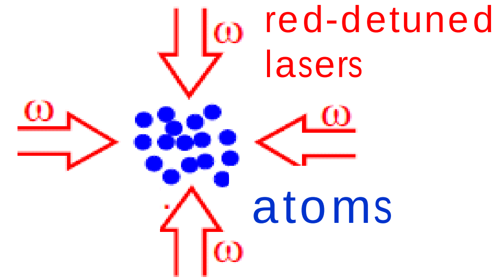
$T \sim$ nano Kelvin

$N \sim 1$ to 10^8

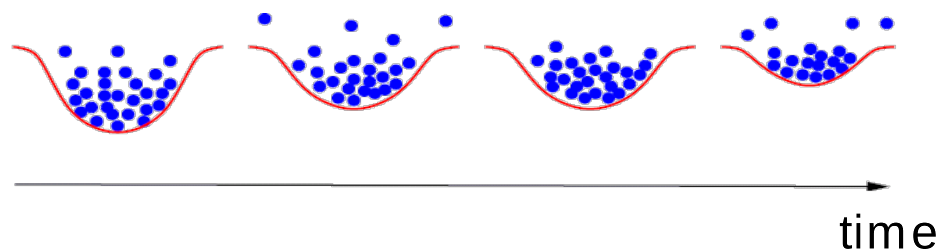
$\frac{N}{V} \sim 10^{13}$ to $10^{15} cm^{-3}$

(air: $10^{19} cm^{-3}$, solids: $10^{22} cm^{-3}$)

Laser cooling



Evaporative cooling

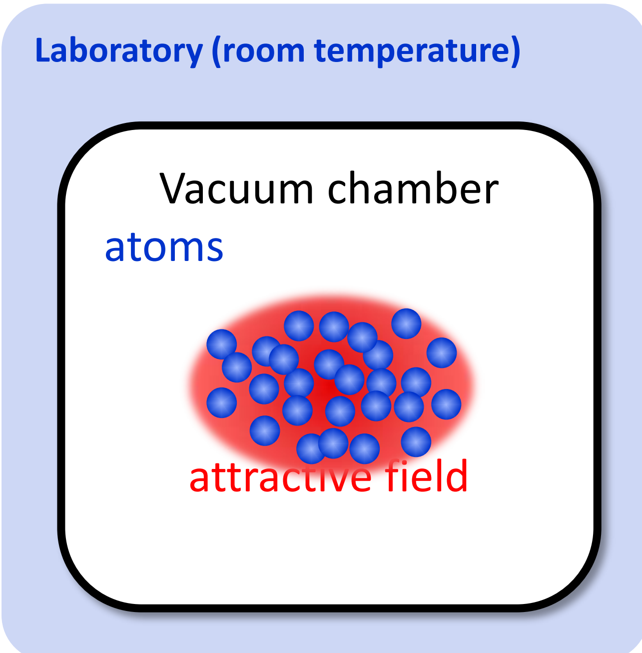


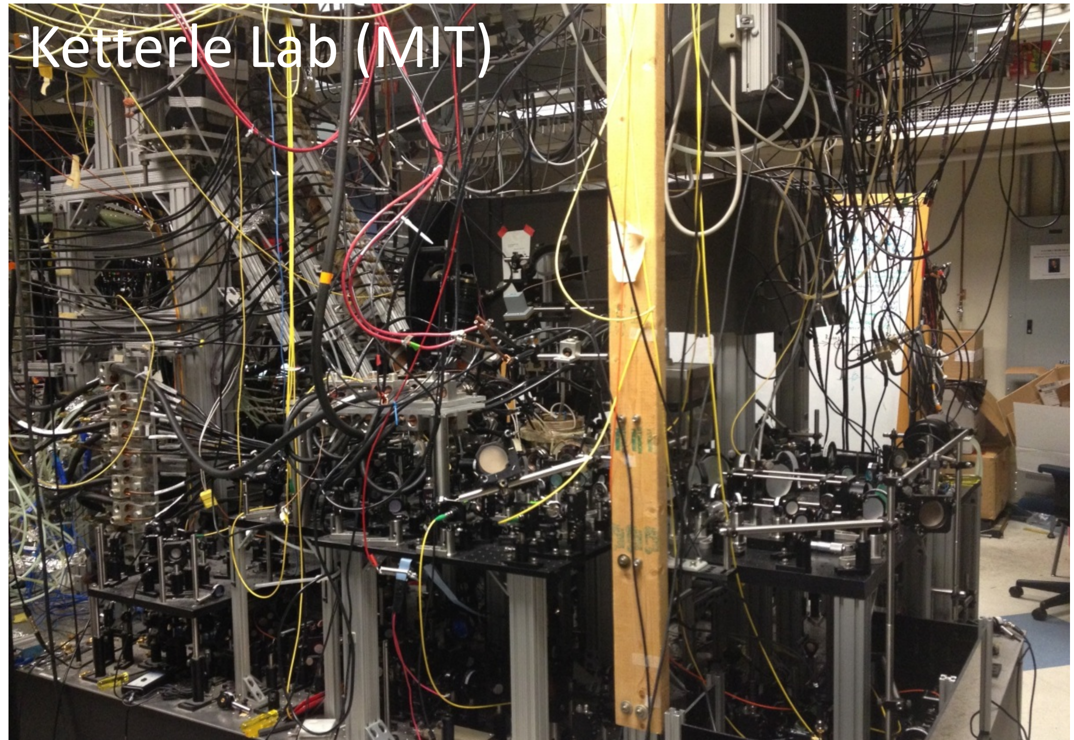
Quantum degeneracy

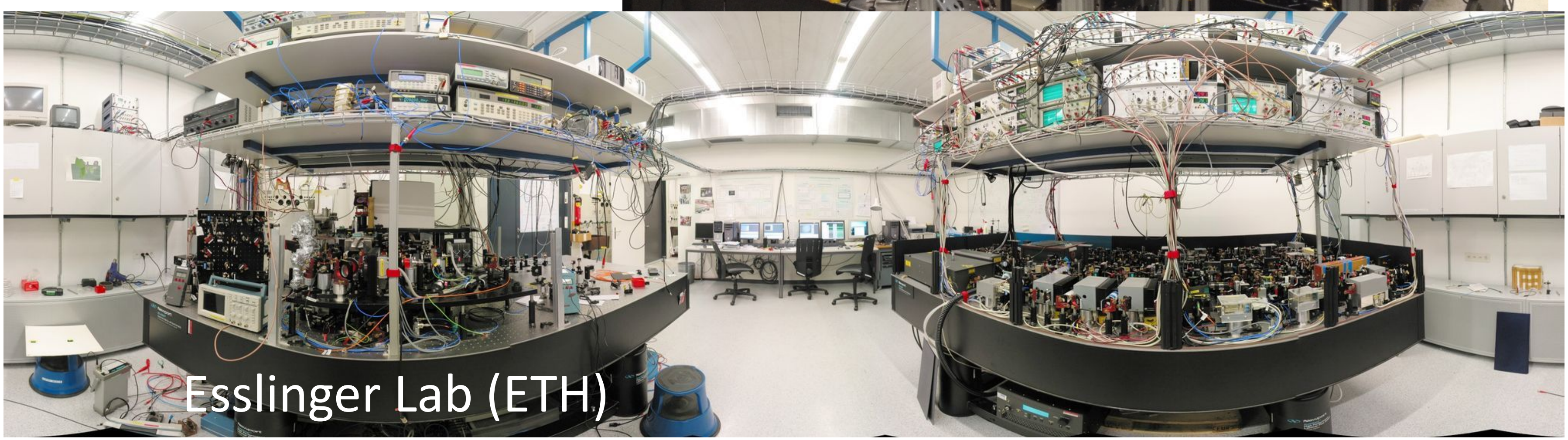
$$T_{\text{deg}} \sim \frac{\hbar^2 \pi^2}{k_B m} \left(\frac{N}{V}\right)^{2/3} \sim \begin{cases} 10^{-6}\,\text{K} & \text{trapped atoms} \\ 1\,\text{K} & \text{liquid Helium} \\ 10^4\,\text{K} & \text{electron gas} \end{cases}$$

Ultracold atomic quantum gases

Trap neutral atoms







Description

Spinless bosons:

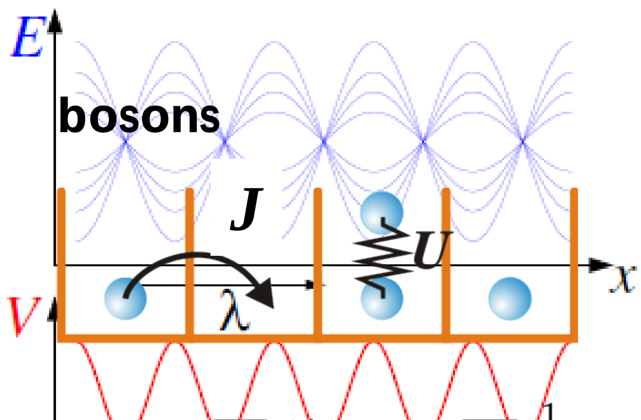
$$\hat{H} = \int \mathrm{d}\boldsymbol{r}\, \hat{\psi}^\dagger(\boldsymbol{r}) \left[\frac{-\hbar^2}{2m} \nabla^2 + V(\boldsymbol{r}) \right] \hat{\psi}(\boldsymbol{r}) + \frac{g}{2} \int \mathrm{d}\boldsymbol{r}\, \hat{\psi}^\dagger(\boldsymbol{r}) \hat{\psi}^\dagger(\boldsymbol{r}) \hat{\psi}(\boldsymbol{r}) \hat{\psi}(\boldsymbol{r})$$

- clean & well isolated from environment
- universal contact interactions $g = \frac{4\pi\hbar^2 a_s}{m}$
- Tailorable, control also during experiment

$$V(\boldsymbol{r}) \rightarrow V(\boldsymbol{r}, t) \qquad g \rightarrow g(t)$$

- additional “features” possible
  fermions, spin, dissipation, disorder, …,
  artificial magnetic fields, …

Optical Lattices



standing light wave

Described by Hubbard models

Jaksch et al., PRL (1998)

clean periodic potential

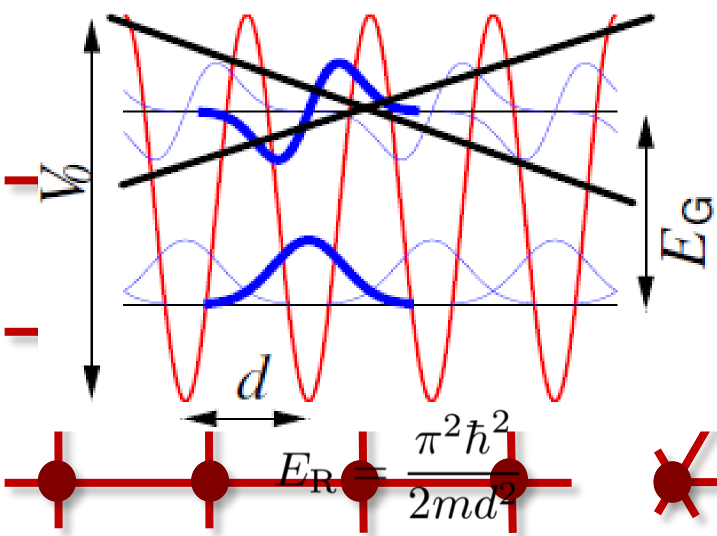
Ratio U/J tunable via lattice depth (laser power):

from weak to strong coupling regime

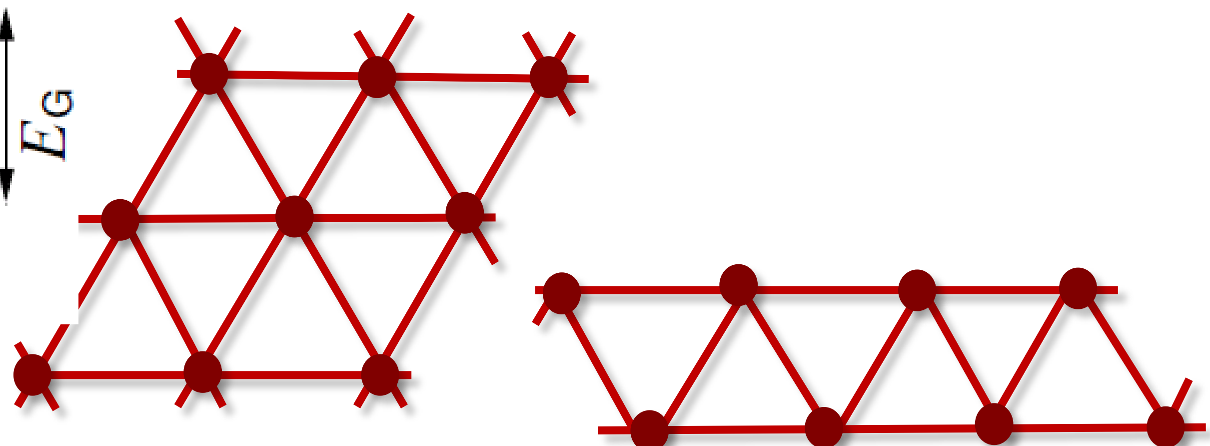
$$\hat{H} = -J \sum_{\langle ij \rangle} \hat{b}_i^\dagger \hat{b}_j + U \sum_i \frac{1}{2} \hat{n}_i (\hat{n}_i - 1)$$

s / reduction to 1D or 2D

Deep lattices



$$E_R = \frac{\pi^2 \hbar^2}{2 m d^2}$$

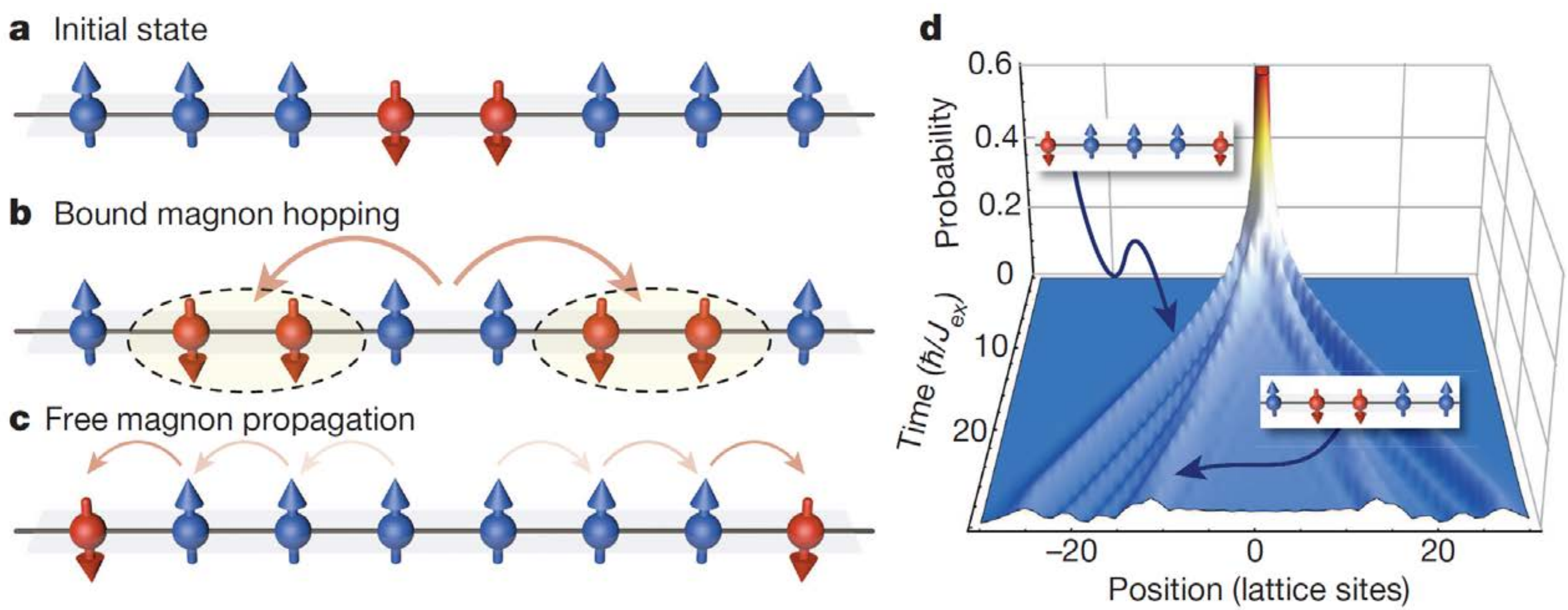


Large time and length scales

Lattice spacing $\sim 0.1 - 1\ \mu m$

Tunneling times $\sim 1 - 10$ ms

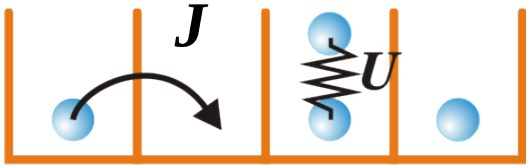
Measure and manipulate system
on its intrinsic length and time scales!



Fukuhara et al., Nature 502, 76 (2013)

Atomic quantum gas in optical lattice

- **clean & tunable** realizations of minimal many-body lattice models
- **strong interactions** possible
- **well isolated** from environment
- **time-dependent control/ time-resolved measurements**



$\Rightarrow$ **Ideal platform for studying coherent quantum many-body dynamics**

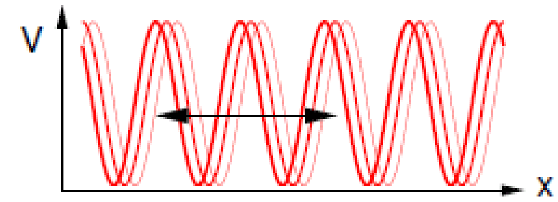
This talk:
Controlling atomic quantum gases via strong time-periodic forcing

Colloquium: Atomic quantum gases in periodically driven optical lattices,
A.E., Rev. Mod. Phys. **89**, 011004 (2017)

Periodically driven quantum systems

Hamiltonian

$$H(t) = H(t+T) = \sum_{m=-\infty}^{\infty} H_m e^{im\omega t}$$



One-cycle time-evolution operator defines an effective time-independent Hamiltonian H_{eff}

$$U(T,0) = \mathcal{T}\exp\left(-\frac{i}{\hbar}\int_0^T \mathrm{d}t\, H(t)\right) \equiv \exp\left(-\frac{i}{\hbar}\, T H_{\mathrm{eff}}\right)$$

Time evolution in steps of the driving period:

$$|\psi(nT)\rangle = \exp\left(-\frac{i}{\hbar}\, nT H_{\mathrm{eff}}\right)|\psi(0)\rangle$$

Useful concept? Yes! If H_{eff} has simple form (at least approximately on relevant time scale)

Floquet engineering

Engineer driving protocol that realizes the desired H_{eff} !

Periodically driven quantum systems

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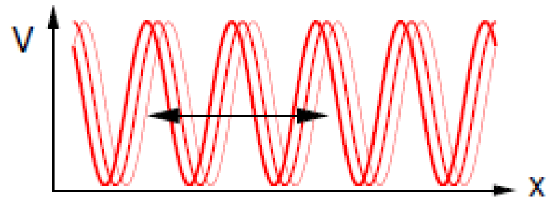

Quasi stationary states: **Floquet states** [Shirley 1965]

Quasienergy

$$|\psi_n(t)\rangle = |u_n(t)\rangle \; e^{-\frac{i}{\hbar}\varepsilon_n t} = |u_{nm}(t)\rangle \; e^{-\frac{i}{\hbar}\varepsilon_{nm} t}$$

Floquet mode $|u_n(t)\rangle = |u_n(t+T)\rangle$

$$\varepsilon_{nm} = \varepsilon_n + m\hbar\omega$$

$$|u_{nm}(t)\rangle = |u_n(t)\rangle e^{im\omega t}$$

Time evolution

$$|\psi(t)\rangle = \sum_n c_n \; |u_n(t)\rangle e^{-i\varepsilon_n t/\hbar}$$ with $c_n = \langle u_n(0)|\psi(0)\rangle$

Periodically driven quantum systems

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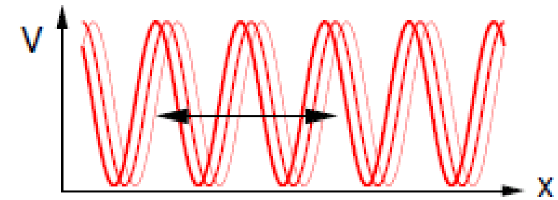

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$$|u_{nm}(t)\rangle = |u_n(t)\rangle e^{im\omega t}$$

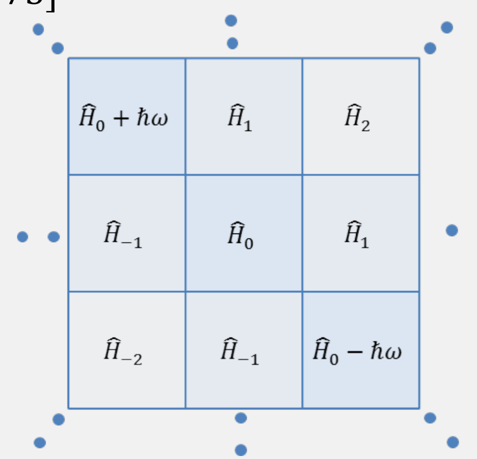
Eigenvalue problem of hermitian **quasienergy operator** Q [Sambe 73]

$$\underbrace{[H(t) - i\hbar\partial_t]}_{Q}\,|u_{nm}\rangle\rangle = \varepsilon_{nm}|u_{nm}\rangle\rangle$$

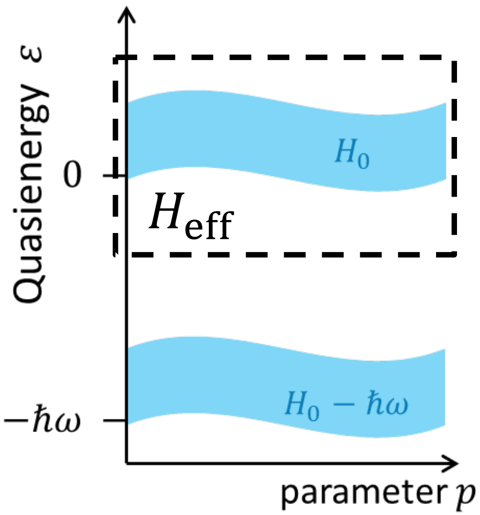
in Floquet space (space of time-periodic states):

basis $|\alpha m\rangle\rangle$: $|\alpha\rangle e^{im\omega t}$

$$\langle\langle\alpha' m'|Q|\alpha m\rangle\rangle = \langle\alpha'|H_{m-m'}|\alpha\rangle + \delta_{m'm}\delta_{\alpha'\alpha} m\hbar\omega$$

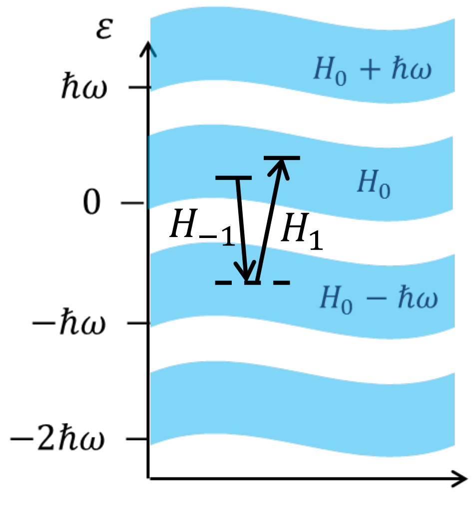


Single-particle finite-band picture



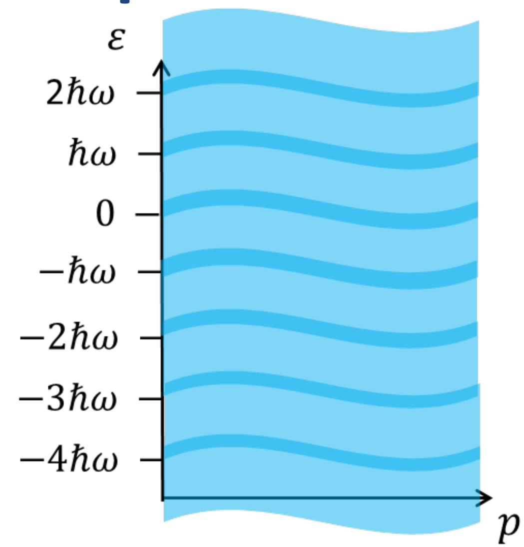
High frequencies

$$H_{\text{eff}} \simeq H_0 = \frac{1}{T}\int_0^T H(t)dt$$

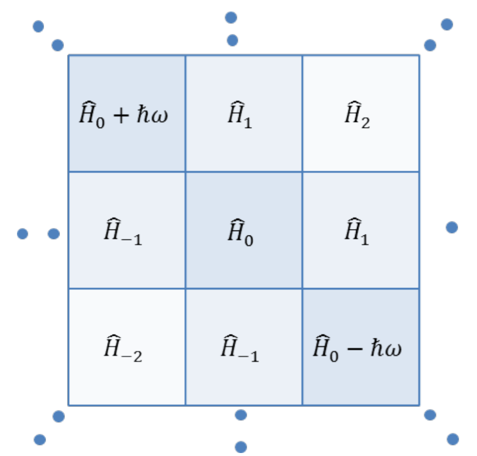


Moderately large freq.

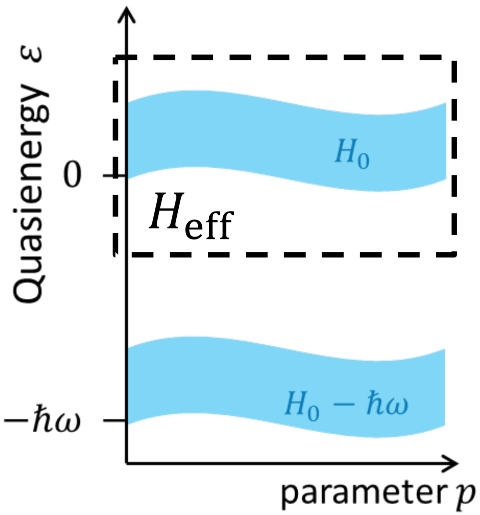
$$H_{\text{eff}} \simeq H_0 + \sum_{m\neq 0} \frac{H_m H_{-m}}{m\hbar\omega} + \cdots$$

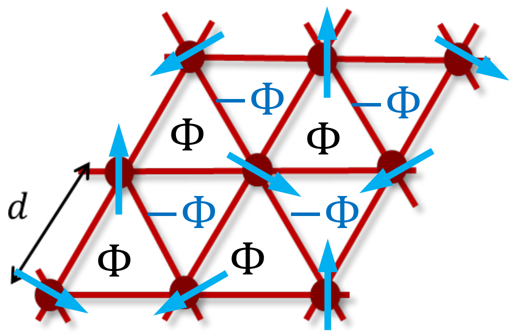


Intermediate freq.



Shaken optical lattice





Inertial force via circular lattice shaking

$$\boldsymbol{F}(t) = F_0\left[\cos(\omega t)\boldsymbol{e}_x + \sin(\omega t)\boldsymbol{e}_y\right]$$

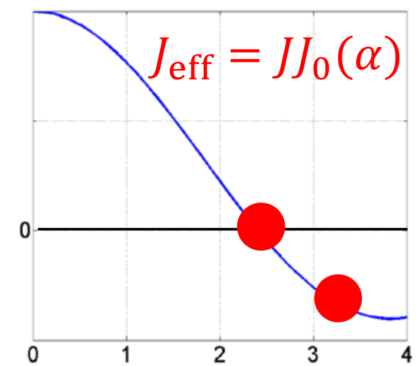
$$+\delta F_0 \sin(2\omega t)\,\boldsymbol{e}_y$$

$$H(t) = -\sum_{\langle ij\rangle} J e^{i\alpha \sin(\omega t - \varphi_{ij})} a_i^\dagger a_j$$

$$\alpha = \frac{F_0 d}{\hbar\omega}$$

High frequencies

$$H_{\rm eff} \simeq H_0 = \frac{1}{T}\int_0^T \mathrm{d}t\, H(t) = -\sum_{\langle ij\rangle} J_{\rm eff} a_i^\dagger a_j$$



Driving amplitude α

Dynamic localization (coherent destruction of tunneling): $J_{\rm eff} = 0$

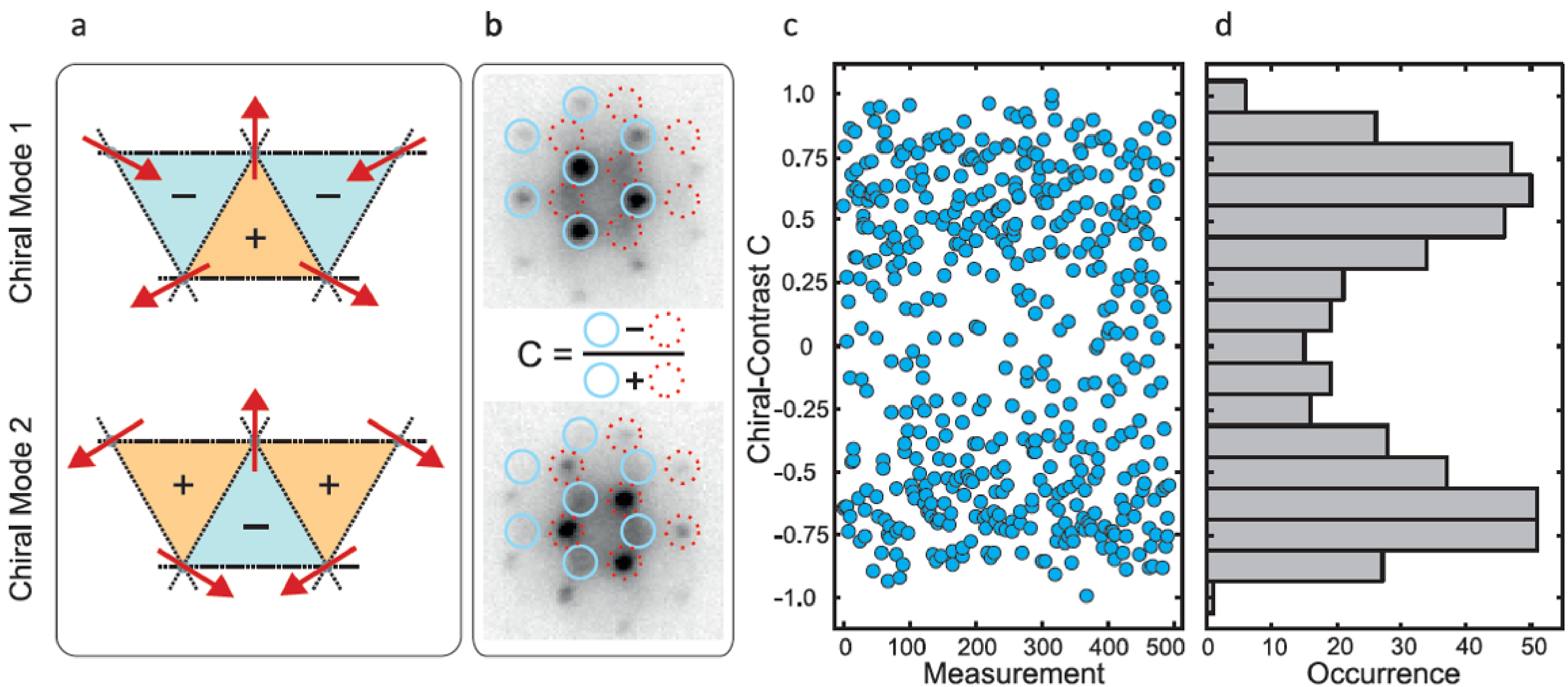
Dunlap & Kenkre 1986, Großmann & Hänggi 1991, Holthaus 1992

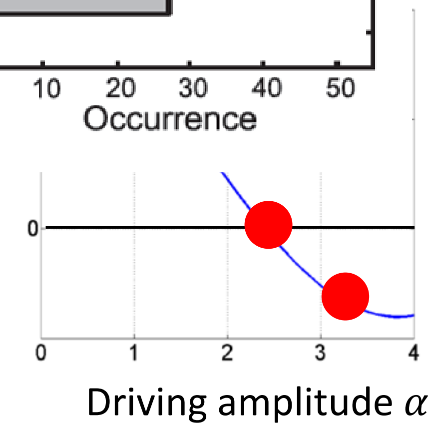
Kinetic Frustration: $J_{\rm eff} < 0$ in non-bipartite lattice

A.E. et al.: EPL 2010, Struck et al., Science 2011

Artificial magnetic fields: requires complex $J_{\rm eff}$

Struck et al. PRL 2012, Hauke et al. PRL 2012, Struck et al. Nat. Phys. 2013





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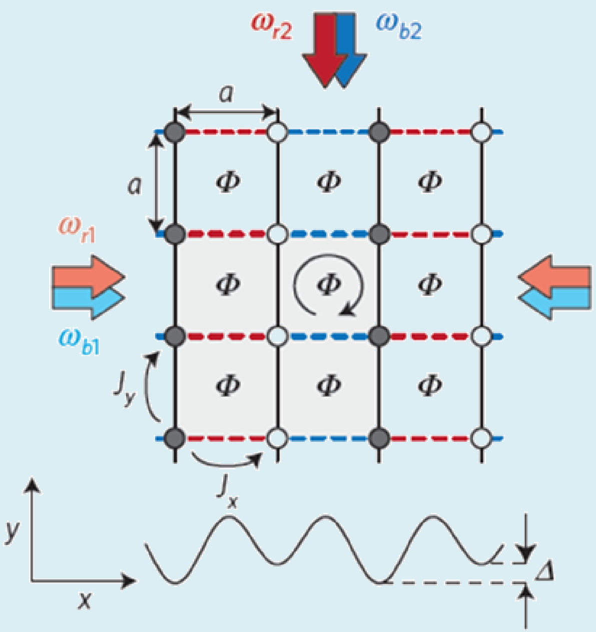
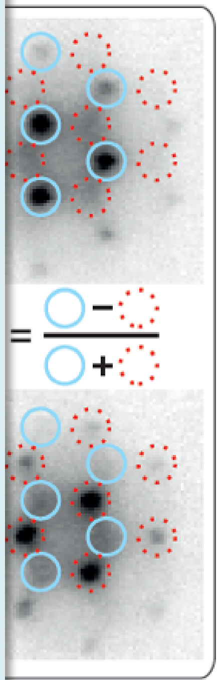
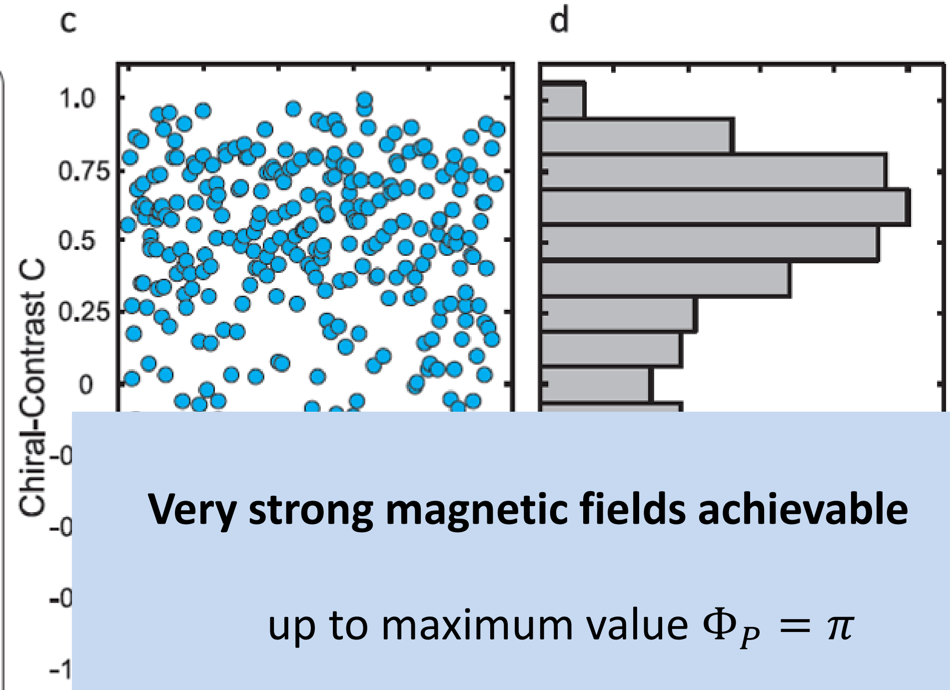
Harper model





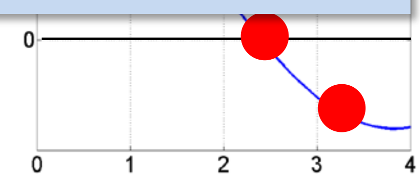


Bloch/Ketterle/Greiner

Very strong magnetic fields achievable

up to maximum value $\Phi_P = \pi$

Corresponds to 10^4 Tesla for graphene!



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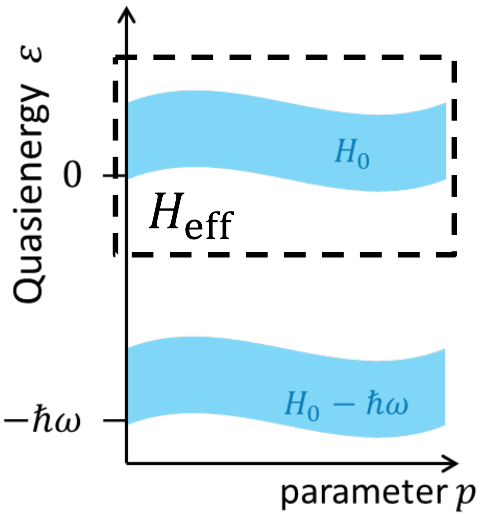
Single-particle finite-band picture



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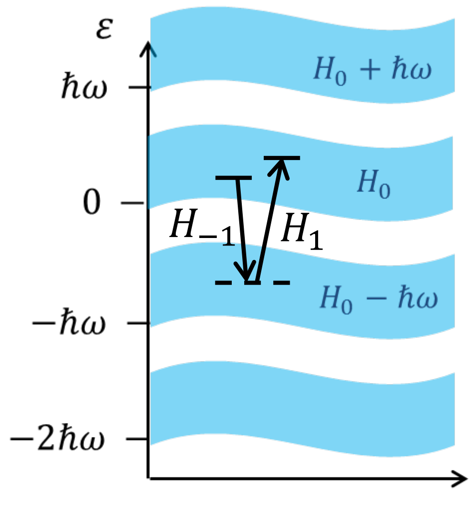

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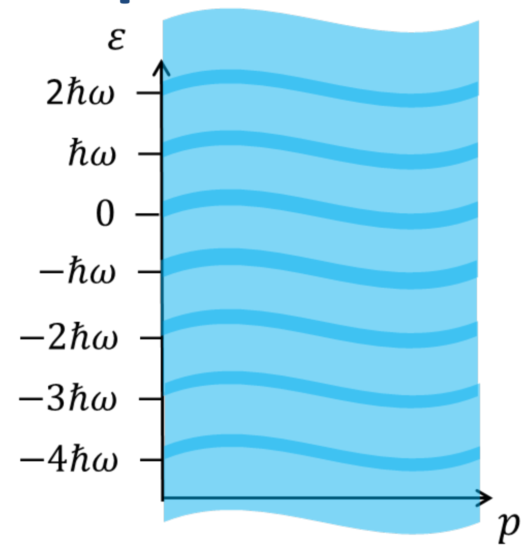


Intermediate freq.

Common strategy: control tunneling

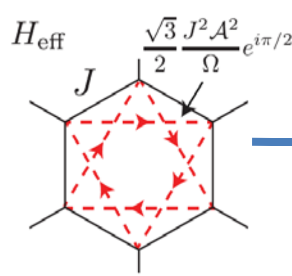
$$H(t) = -\sum_{\langle ij\rangle} J e^{i\theta_{ij}(t)} a_i^\dagger a_j$$

conservative forcing

$$H_{\text{eff}} \simeq -\sum_{\langle ij\rangle} J_{ij}^{\text{eff}} e^{i\theta_{ij}^{\text{eff}}} a_i^\dagger a_j$$

Quantum-gas experiments: Arimondo, Tino, Sengstock, Nägerl, Bloch, Ketterle, Greiner

Floquet topological insulator



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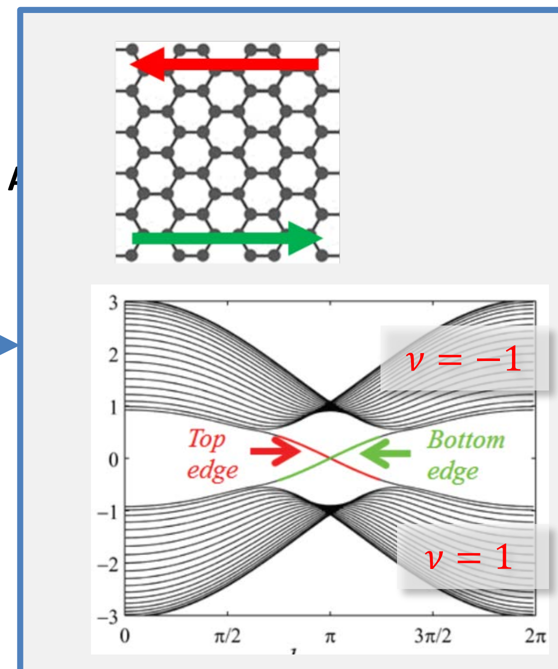

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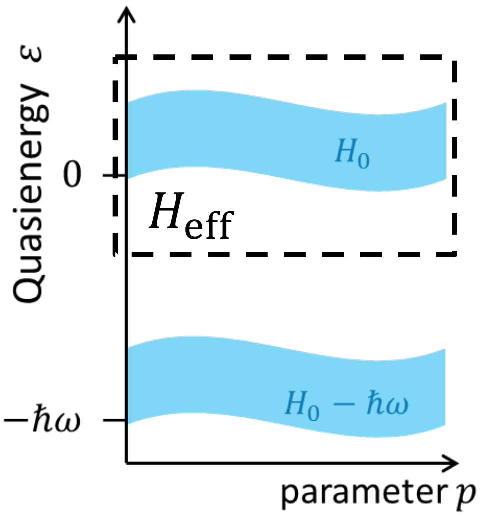

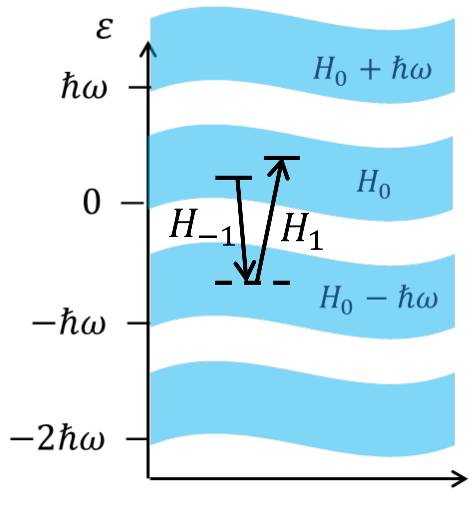

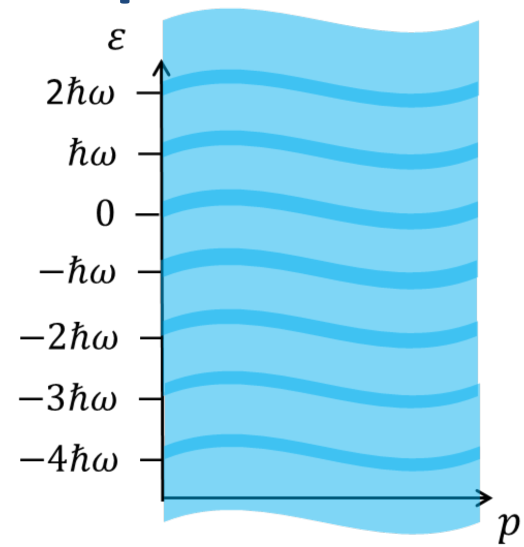

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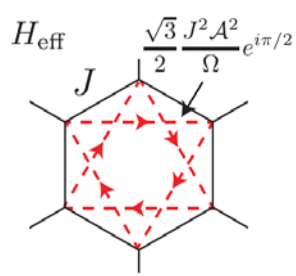

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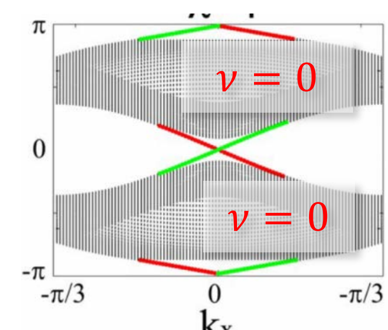
Anomalous topological edge states



Kitagawa et al. 2010, Jiang et al. 2011, Rudner et al 2013, ...
Wave-guides: Mukherjee et al. 2015, Maczewesky et al. 2016

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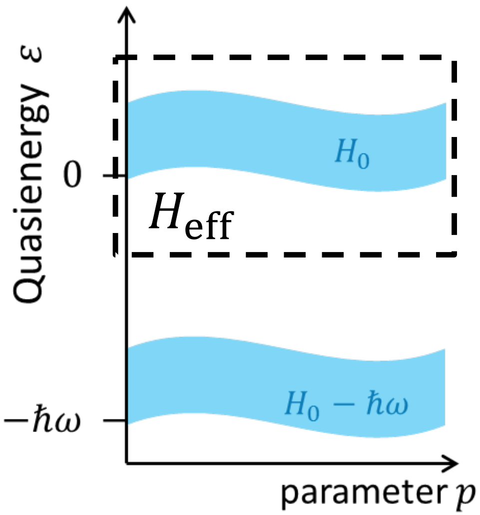

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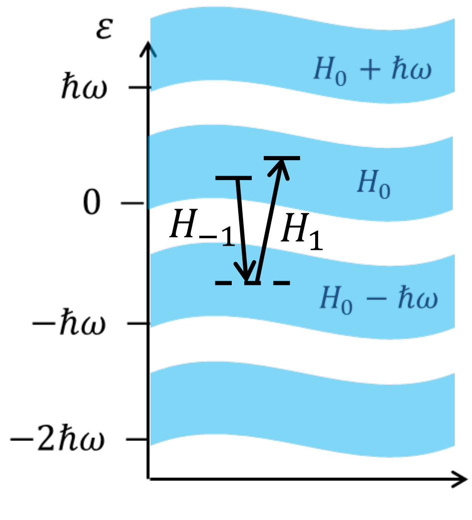

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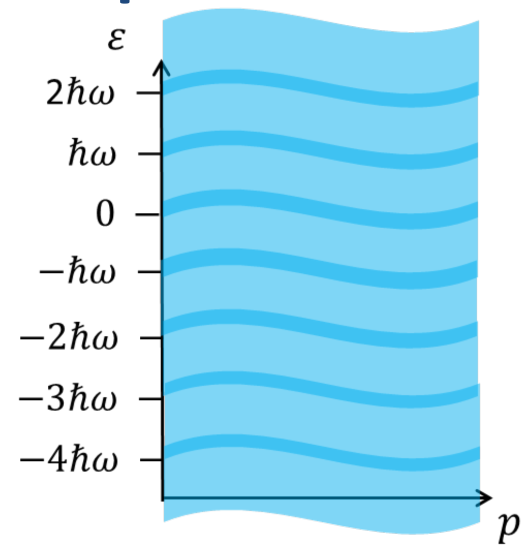
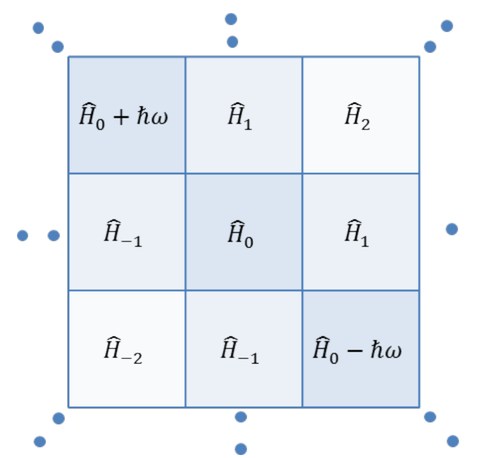


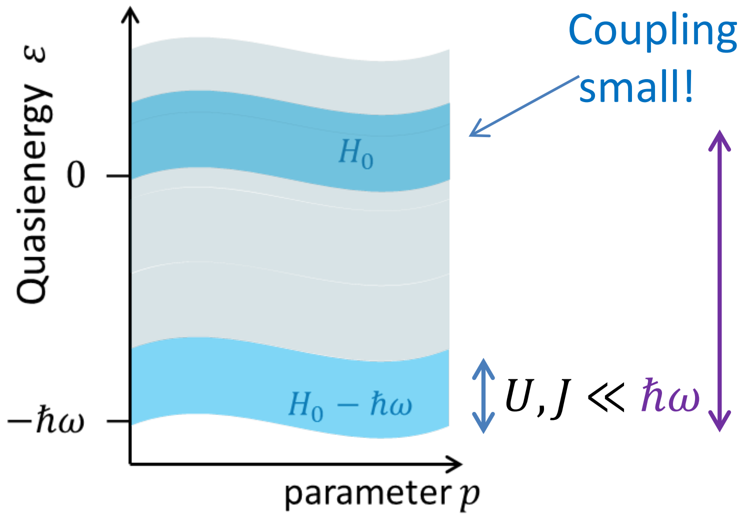
Intermediate freq.

Role of Interactions?

- **Heating**
- Interplay with modified kinetics
- Modification of interactions



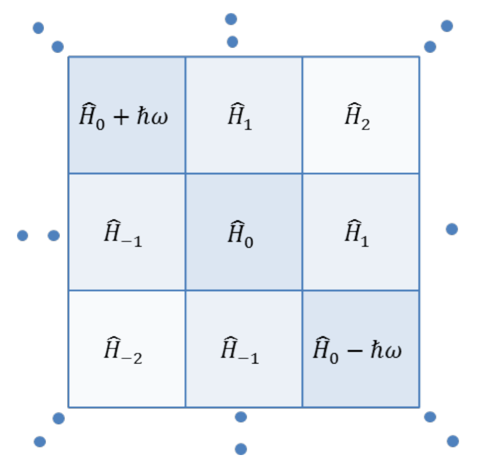
Heating



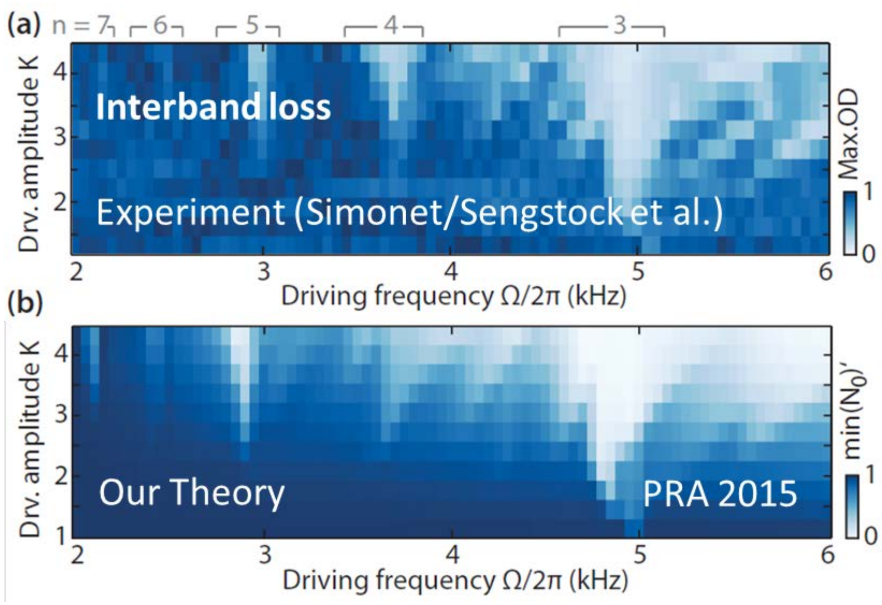
$H_{\text{eff}} \approx H_0$ before heating sets in on time scale t_h
[A.E. et al. 2005]

Exponential growth of t_h with ω:

- Perturbation theory for bosonic Mott state [A.E., Holthaus 2008]
- Proof for spin systems [Abanin et al. 2016, Kwahara et al. 2016]

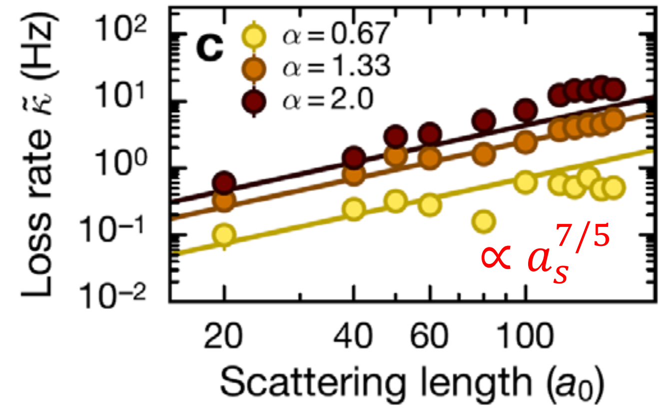


Heating

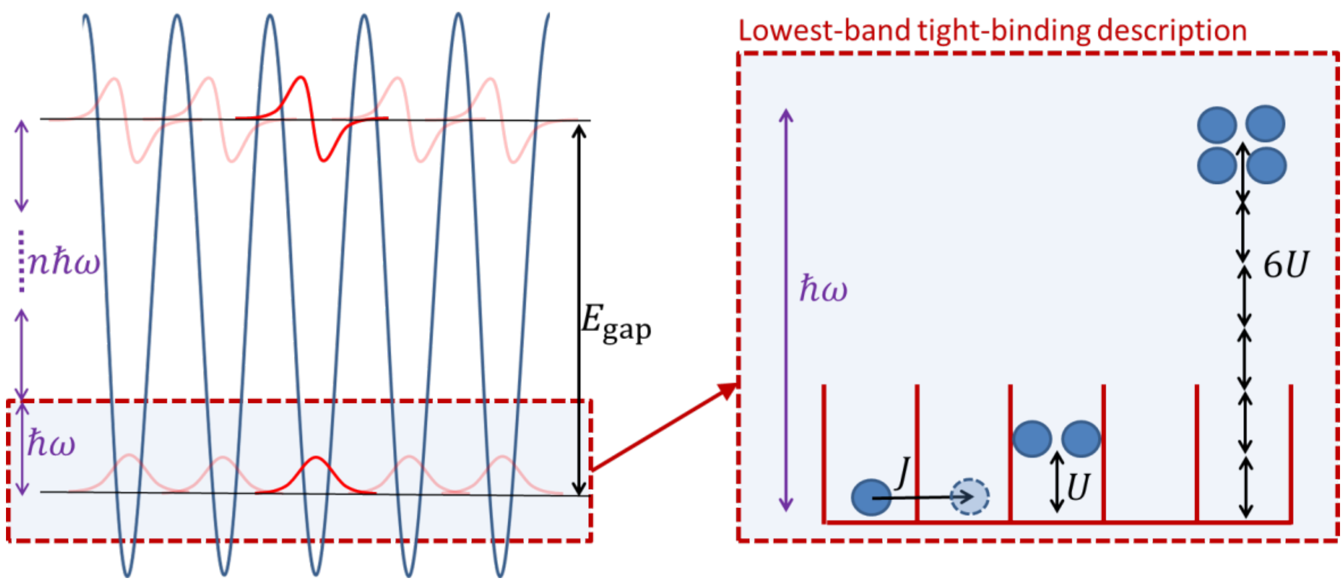


also Sträter & A. E., Z. Naturforsch. 2016

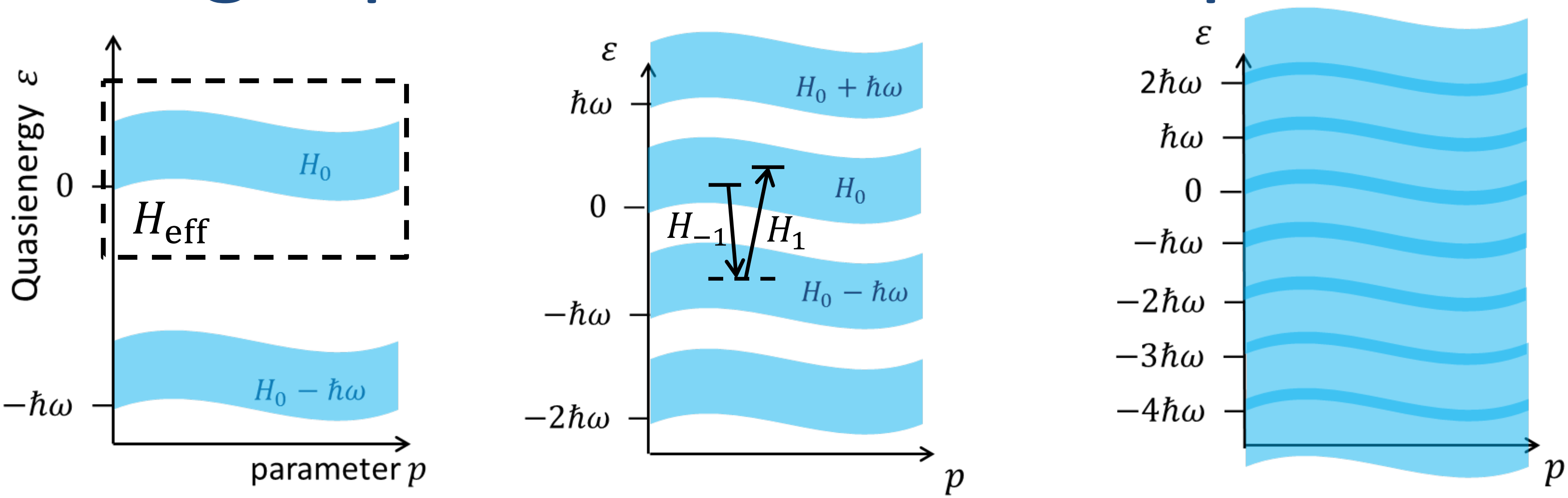
Intraband loss



Experiment (Schneider/Bloch) & Our Theory [arXiv:1706.04819]



Single-particle finite-band picture



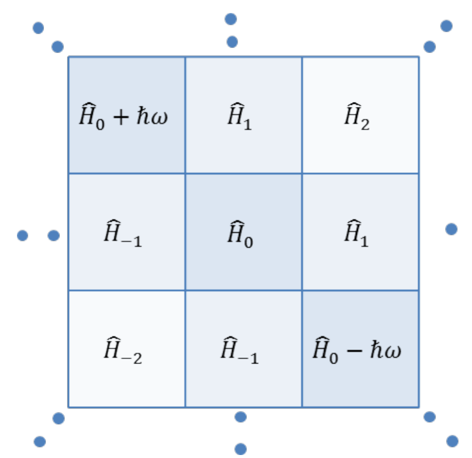
High frequencies | **Moderately large freq.** | **Intermediate freq.**

$$H_{\mathrm{eff}} \simeq H_0 = \frac{1}{T}\int_0^T H(t)dt$$

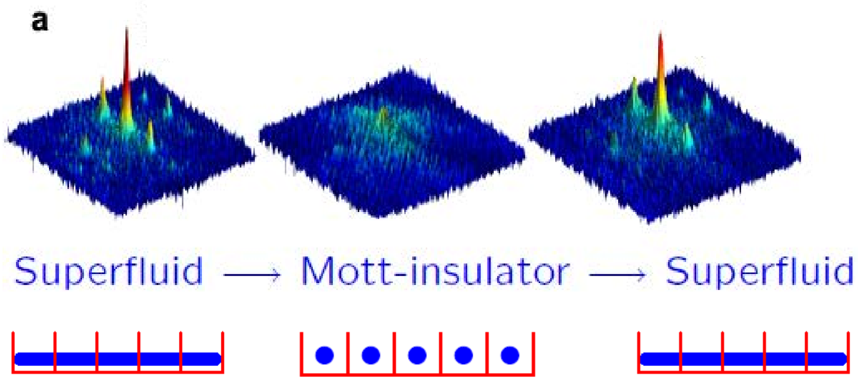
$$H_{\mathrm{eff}} \simeq H_0 + \sum_{m\neq 0} \frac{H_m H_{-m}}{m\hbar\omega} + \cdots$$

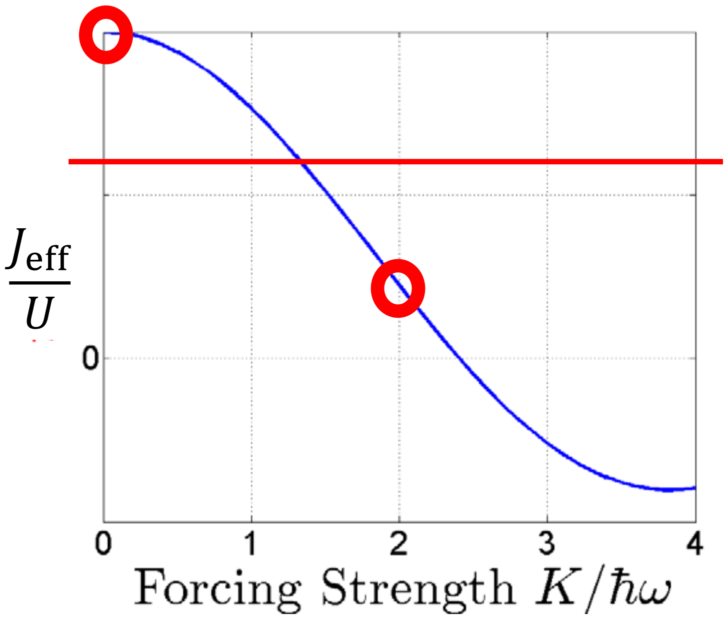
Role of Interactions?

- Heating
- **Interplay with modified kinetics**
- Modification of interactions



Dynamically induced quantum phase transition

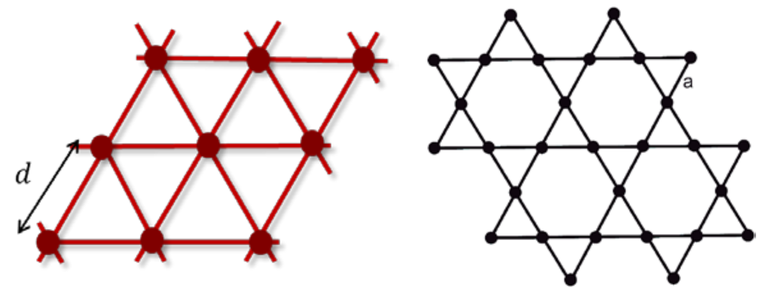




experiment: Zenesini et al., PRL (2009)
proposal: A.E. et al., PRL (2005)

Mimic quantum antiferromagnetism

$$H_{\text{eff}} \simeq +|J_{\text{eff}}| \sum_{\langle ij \rangle} a_i^\dagger a_j + \frac{U}{2} \sum_i n_i(n_i - 1)$$



Hard-core bosons ($U \gg J$) map to spin-1/2 model:

$$n_i = 1: \quad S_i^z = \uparrow$$
$$n_i = 0: \quad S_i^z = \downarrow$$

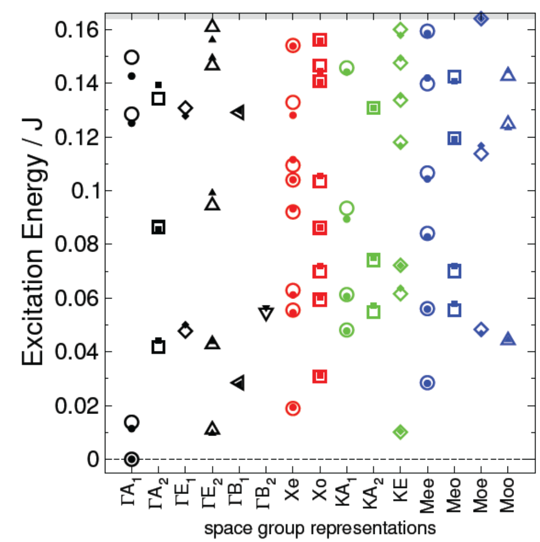
$$a_j \to S_j^+ \quad a_j^\dagger \to S_j^-$$

$$H_{\text{eff}} \simeq +|J_{\text{eff}}| \sum_{\langle ij \rangle} \left(S_i^x S_j^x + S_i^y S_j^y \right)$$

$\frac{|J_{\text{eff}}|}{\text{Temperature}}$ might be larger than for Heisenberg magnetism in Mott insualtor of fermionic atoms.

Interesting observation:

Frustrated XY and Heisenberg models can share low-energy properties:

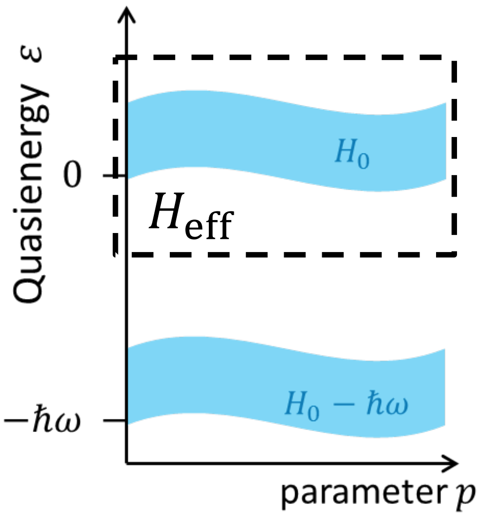


A.E. et al. EPL 2010

Heisenberg vs. XY for Kagomé lattice

[Läuchli & Moessner arXiv:1504.04380]

Single-particle finite-band picture



High frequencies

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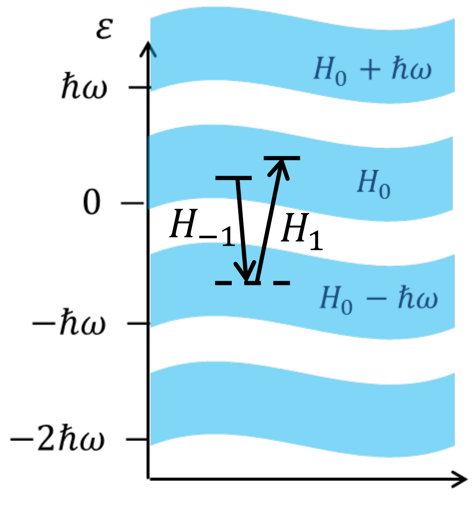
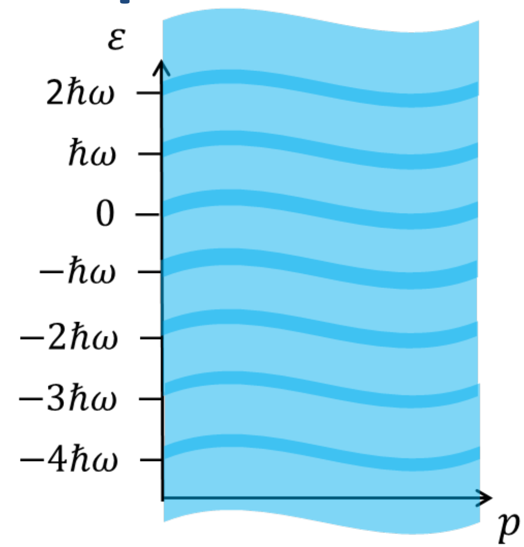


Moderately large freq.

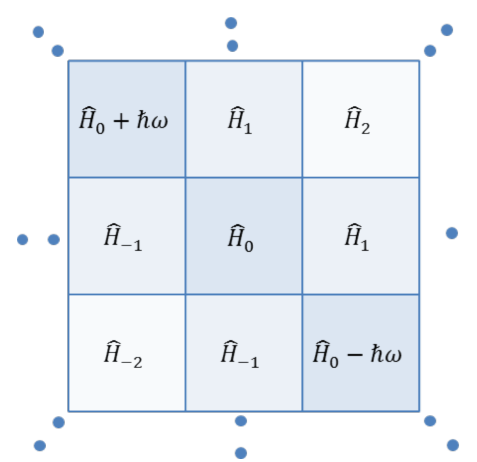
$$H_{\text{eff}} \simeq H_0 + \sum_{m\neq 0} \frac{H_m H_{-m}}{m\hbar\omega} + \cdots$$



Intermediate freq.

Role of Interactions?

- Heating
- Interplay with modified kinetics
- **Modification of interactions**

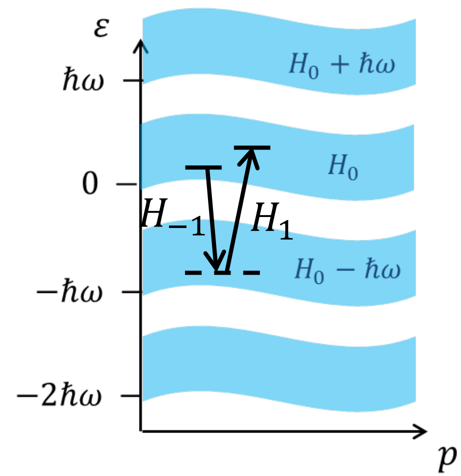


NON-LOCAL INTERACTIONS FROM REAL-SPACE MICROMOTION AND THEIR IMPACT ON A FRACTIONAL CHERN INSULATOR

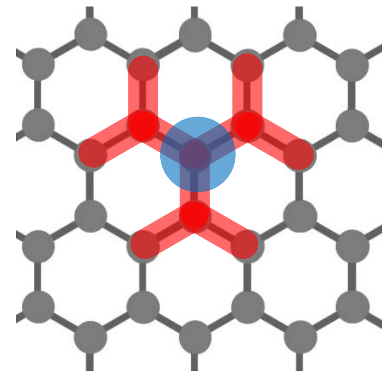
Floquet Fractional Chern insulator

Impact on stabilization of Fractional Chern insulator?

Bosonic case?



Moderately large freq.

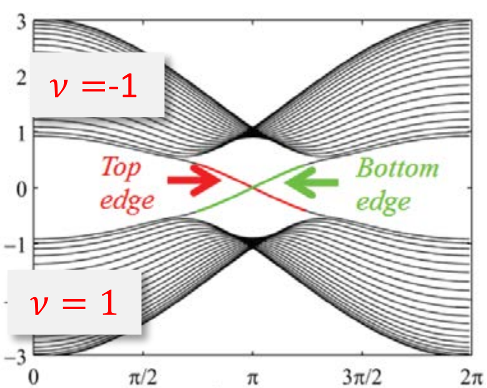


Bosons with on-site interactions

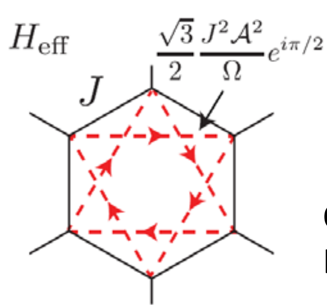
$$H_{\text{eff}} \simeq H_0 + \sum_{m\neq 0} \frac{H_m H_{-m}}{m\hbar\omega} + H_{\text{int}} + \sum_{m\neq 0} \frac{[H_{-m},[H_{\text{int}},H_m]]}{2(m\hbar\omega)^2}$$

Interaction corrections

A.E. & Anisimovas NJP 2015



Chern insulator



Oka & Aoki PRB 2009
Kitagawa et al. 2011:

Fermionic fractional Chern insualtor ($\nu = 1/3$)
(without interaction corrections)
Grushin et al. 2014:

Floquet Fractional Chern insulator

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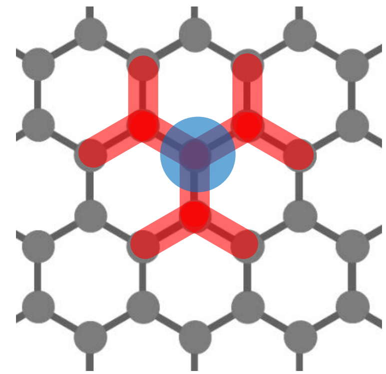

Bosons with on-site interactions

E.g. for spinless bosons:

$$\sum_{m\neq 0} \frac{[H_{-m},[H_{\text{int}},H_m]]}{2(m\hbar\omega)^2} = 8W_a \left[-z\sum_{\ell}\frac{1}{2}n_\ell(n_\ell - 1) + \sum_{\langle \ell'\ell\rangle} n_{\ell'}n_\ell\right] + W_b \sum_{\langle \ell'\ell\rangle} a^\dagger_{\ell'}a^\dagger_{\ell'}a_\ell a_\ell$$

$$-W_c \sum_{\langle \ell' k \ell\rangle} a^\dagger_{\ell'}(4n_k - n_{\ell'} - n_\ell)a_\ell - W_d \sum_{\langle \ell' k \ell\rangle}(a^\dagger_{\ell'}a^\dagger_\ell a_k a_k + h.c.)$$

$$W_x = \frac{UJ^2}{2\hbar\omega}J_1^2\left(\frac{K}{\hbar\omega}\right) + O\left(\left(\frac{K}{\hbar\omega}\right)^4\right)$$

A.E. & Anisimovas NJP 2015

8 bosons, $\nu = 1/2$,
from exact diagonalization + band projection

Anisimovas, Žlabys, Anderson, Juzeliūnas, A.E. PRB 2015,   Račiūnas, Žlabys, A.E., Anisimovas, PRA 2016

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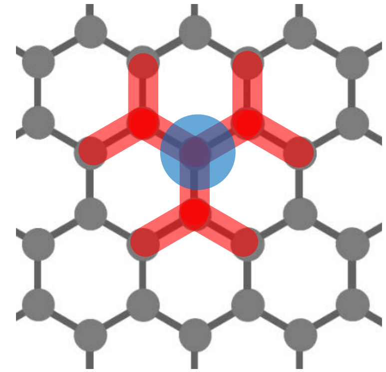
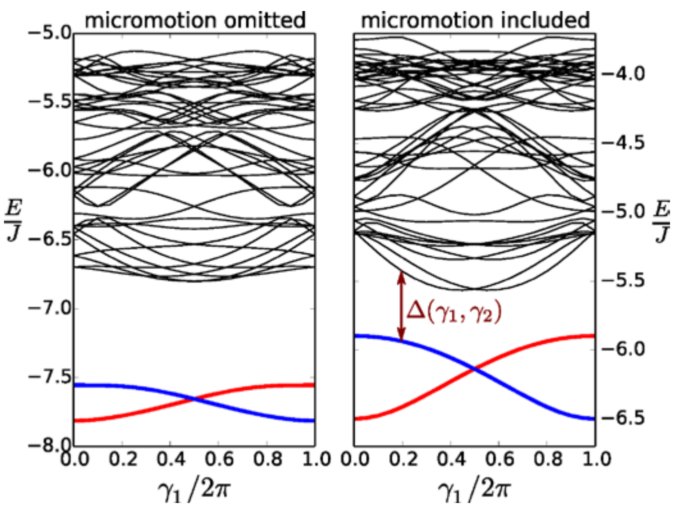
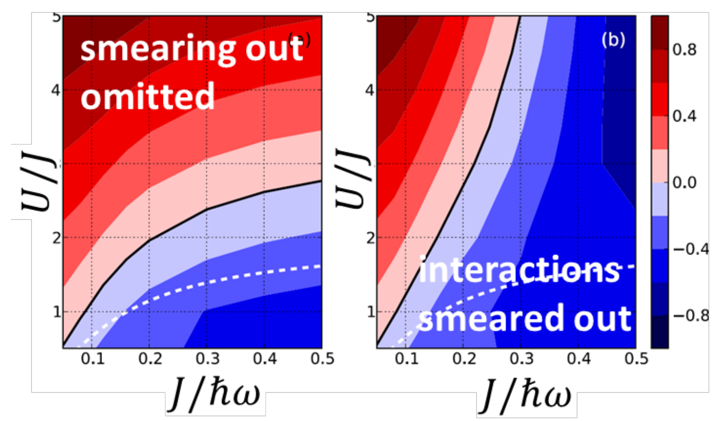


Bosons with on-site interactions

Spectrum and spectral flow



Topological gap



6 bosons, $\nu = 1/2$,
from exact diagaonal. + band projection

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Floquet Fractional Chern insulator

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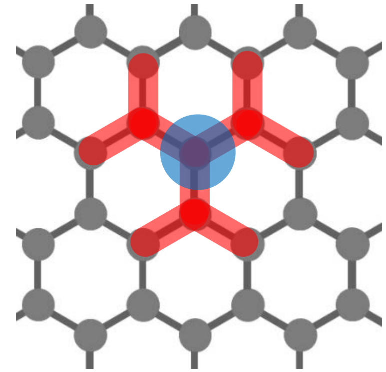

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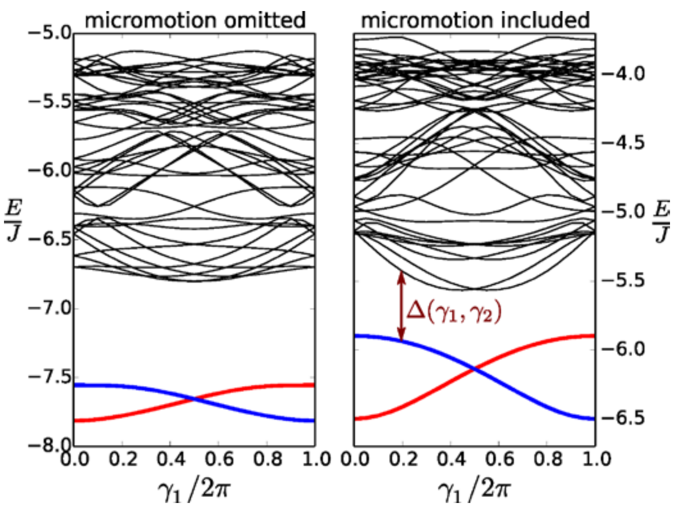

6 bosons, $\nu = 1/2$,
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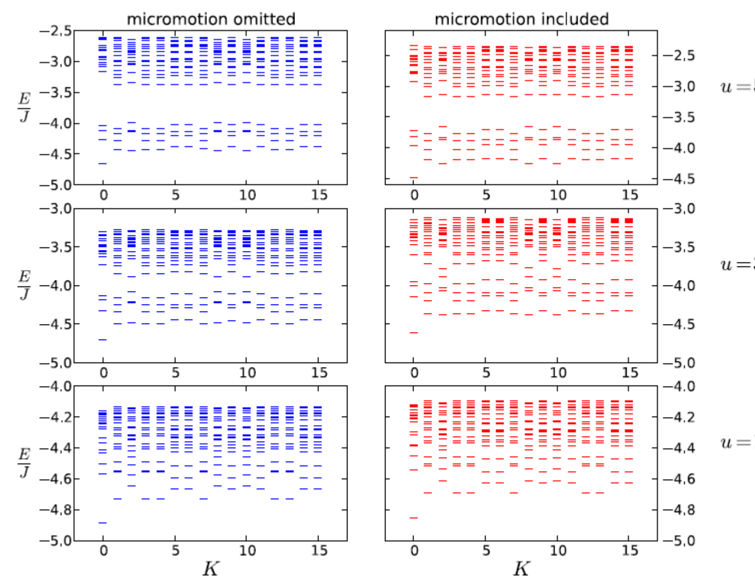
Quasihole spectrum



8 bosons, $\nu = 1/2$,
from exact diagonalization + band projection

Anisimovas, Žlabys, Anderson, Juzeliūnas, A.E. PRB 2015, Račiūnas, Žlabys, A.E., Anisimovas, PRA 2016

REALIZATION AND SIGNATURES OF 1D ANYONS

1D anyons on a lattice

Keilmann, Lanzmich, Mc Culloch, Roncaglia, Nat. Comm. 2011

Tight-binding chain

$$H = -J \sum_{j=2}^{M} \left(a_j^\dagger a_{j-1} + \mathrm{h.c.} \right)$$

Bosons

$$a_j a_i^\dagger - a_i^\dagger a_k = \delta_{kj}$$
$$a_j a_i - a_i a_k = 0$$

1D Anyons

$$a_j a_k^\dagger - e^{i\theta \operatorname{sgn}(k-j)} a_k^\dagger a_j = \delta_{kj}$$
$$a_j a_k - e^{i\theta \operatorname{sgn}(k-j)} a_k a_j = 0$$

interpolate between
Bosons ($\boldsymbol{\theta = 0}$) &
Pseudo-Fermions ($\boldsymbol{\theta = \pi}$)

Fermions

$$a_j a_k^\dagger + a_k^\dagger a_j = \delta_{kj}$$
$$a_j a_k + a_k a_j = 0$$

1D anyons represented by bosons $\boldsymbol{b_j}$ with number-dependent tunneling:

Jordan-Wigner transformation $a_j = b_j \exp(i\theta \sum_{i>k} n_i)$

$$H = -J \sum_{j=2}^{M} \left(b_j^\dagger b_{j-1} e^{i\theta n_j} + \mathrm{h.c.} \right)$$

How to realize 1D anyons?

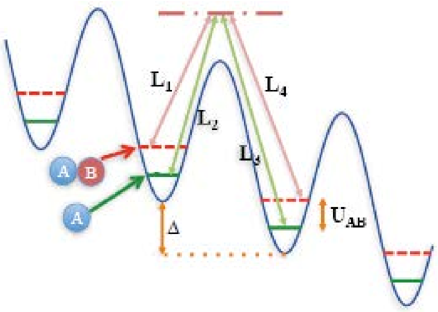
Bosonic representation of anyonic Hubbard model

$$H = -J \sum_{j=2}^{M} \left(b_j^{\dagger} b_{j-1} e^{i\theta n_j} + \mathrm{h.c.} \right) + \frac{U}{2} \sum_{j} n_j (n_j - 1)$$

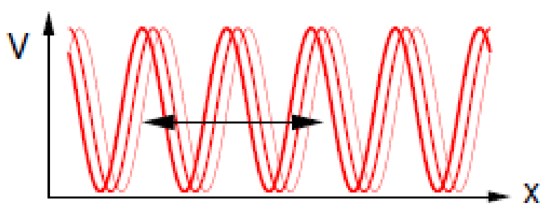
Proposals relying on Raman-assisted tunneling

[Keilmann et al. 2011, Greschner & Santos 2015]

experimentally involved (require additional lasers)



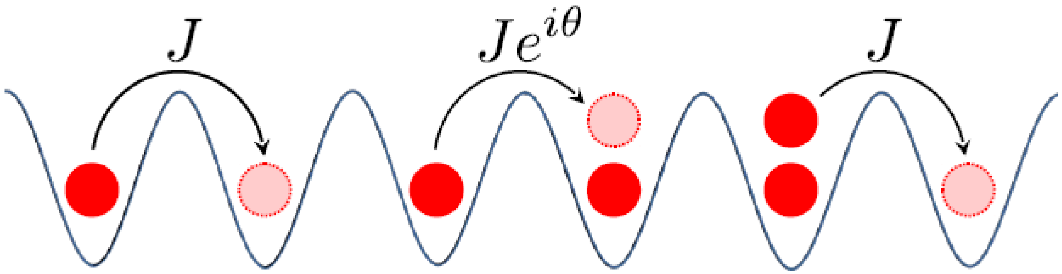
Here: implementation based on simple lattice-shaking



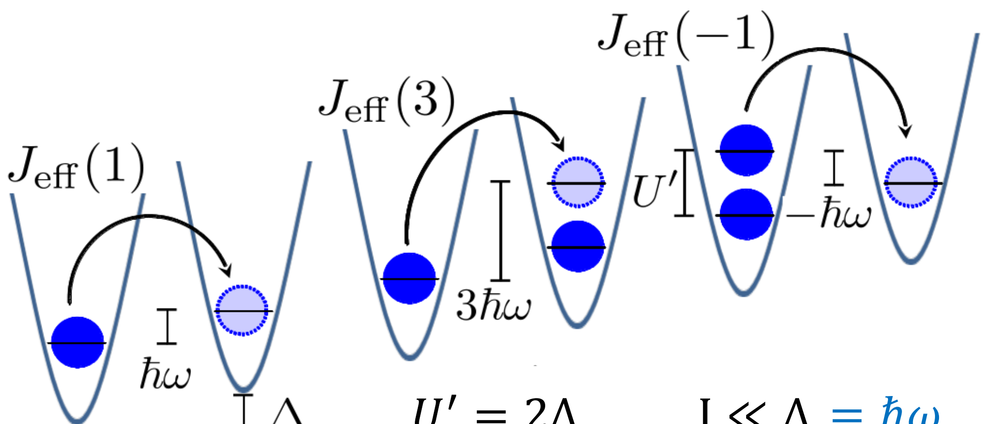
Sträter, Srivastava, A.E. PRL **117**, 205303 (2016)

See also scheme based on modulation of lattice depth.
Cardarelli, Greschner & Santos PRA 2016

Realization



Low-density regime:
three basic processes

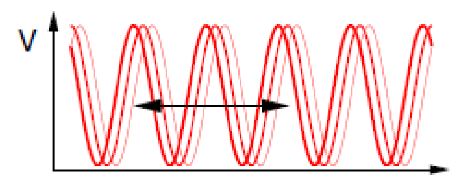


Δ $U' = 2\Delta$ $J \ll \Delta = \hbar\omega$

Suppress tunneling:
Strong lattice tilt + strong interactions

$$\Delta E^{\text{tun}}_{j,j-1} = \hbar\omega\hat{\nu}_{j,j-1}$$

$$\hat{\nu}_{j,j-1} = 2(\hat{n}_j - \hat{n}_{j-1}) + 3 = \pm 1, \pm 3, \ \ldots$$



Coherent tunneling via **resonant lattice shaking**

$$H(t) = -J\sum_{j=2}^{M}\left(\hat{b}_j^\dagger \hat{b}_{j-1} e^{i\omega t\hat{\nu}_{j,j-1} - i\chi(t)} + \text{h.c.}\right)$$

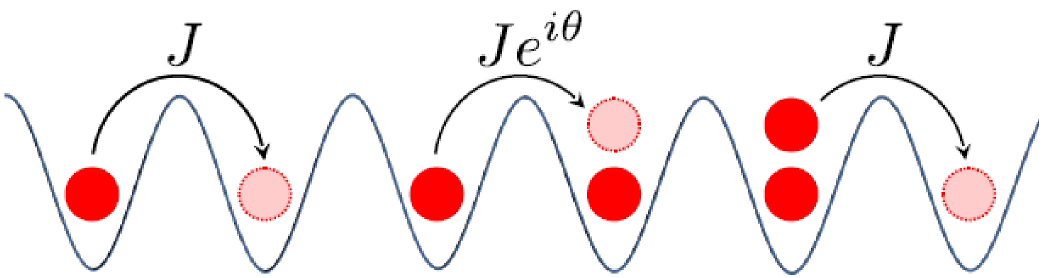
$$\chi(t) = \chi(t+T) \propto \text{lattice velocity}$$

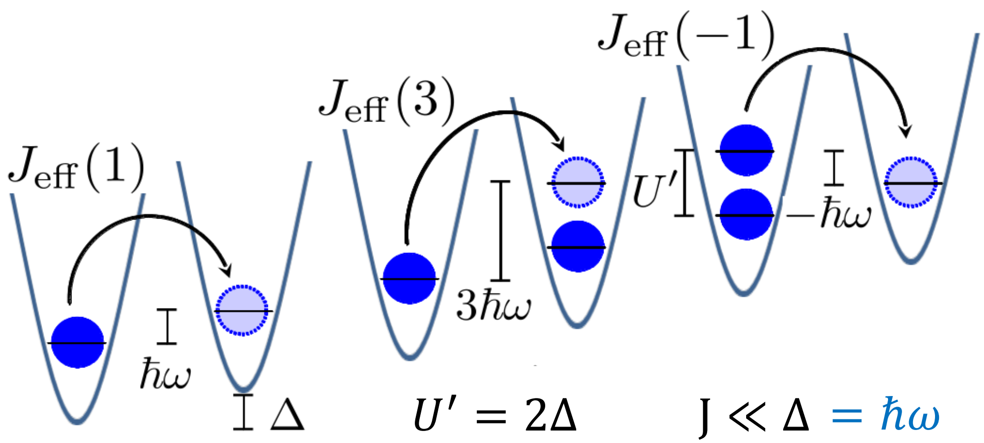
$$= A\cos(\omega t) + B\cos(2\omega t)$$

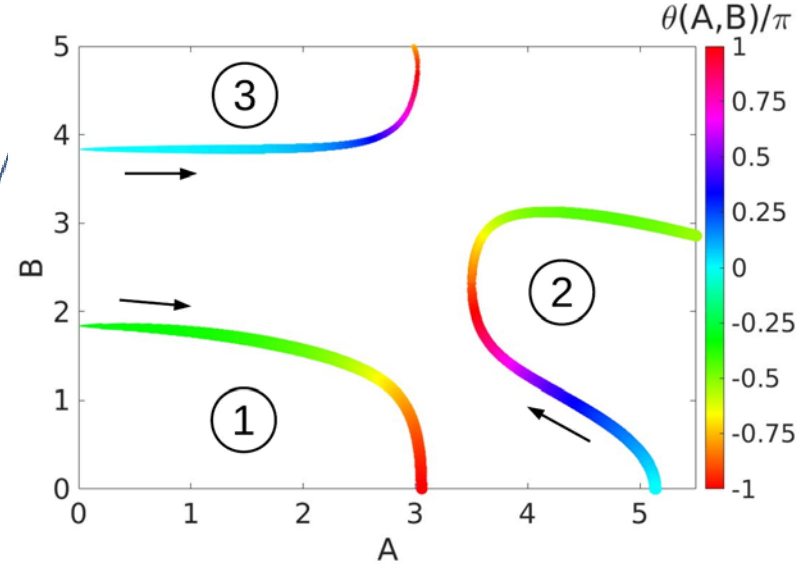
$$H_{\text{eff}} \approx H_0 = -\sum_{j=2}^{M}\left(\hat{b}_j^\dagger \hat{b}_{j-1} J_{\text{eff}}(\hat{\nu}_{j,j-1}) + \text{h.c.}\right)$$

$$J_{\text{eff}}(\nu) = \frac{J}{T}\int_0^T \mathrm{d}t\, e^{i\omega t\nu - i\chi(t)}$$

Realization







$$\hat{v}_{j,j-1} = 2(\hat{n}_j - \hat{n}_{j-1}) + 3$$
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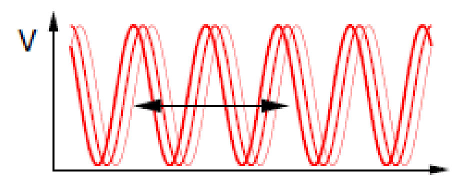

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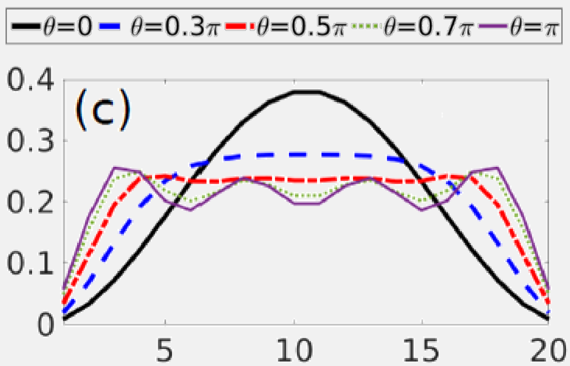
Signature of smooth fermionization

Anyonic momentum distribution not measurable (not invariant under Jordan-Wigner transf.)

Real-space density (identical for bosons and anyons)

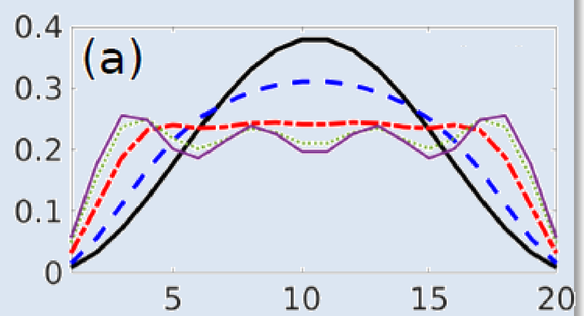
Anyonic ground state
(4 particles on 20 sites)



Fermionization reflected in **Friedel oscillations**

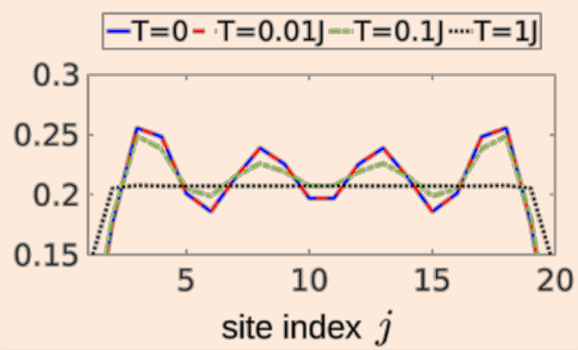
$\lambda_{\text{Friedel}} = \lambda_{\text{Fermi}}/2$

Ground state of H_{eff}

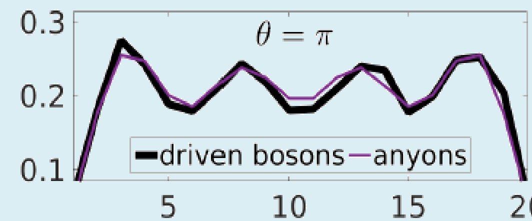


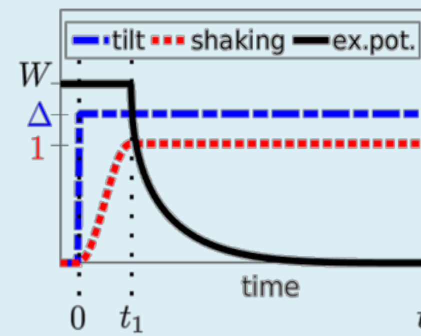
Confirms low-density approximation

Finite-temperature state of H_{eff}



Full time evolution of (a)diabatic preparation using time-dep. Ham. $H(t)$





Confirms also high-frequency approximation for H_{eff}

Thank you!