Progresses of observing spin-orbit coupling and topological physics for ultracold atoms

Xiong-Jun Liu（刘雄军）

International Center for Quantum Materials, Peking University

ICQM 国际量子材料科学中心
International Center for Quantum Materials

北京大學
PEKING UNIVERSITY

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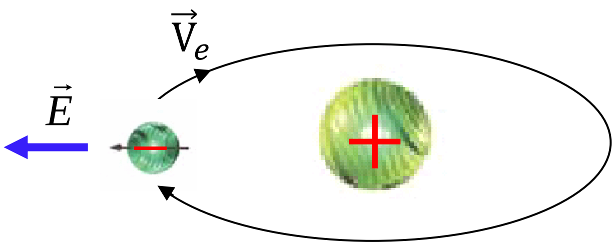
Outline

1. Basics of SO coupling: from 1D to high dimensional SOC
2. Optical Raman lattice schemes for SOC and realization
3. Topological phases for the SO coupled systems: bosons and fermions
4. Experimental studies: equilibrium vs non-equilibrium topological physics

Introduction: why spin-orbit coupling is interesting?

Spin-orbit coupling for electrons



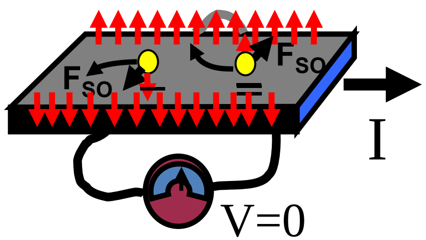
Pauli spin-orbit term:

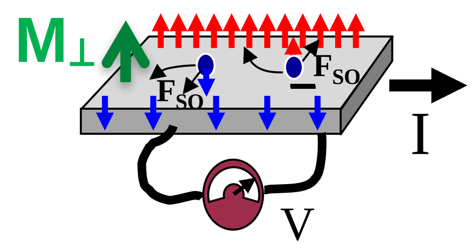
$$H_{so} = \frac{\hbar}{4m_0^2c^2}\boldsymbol{\nabla}V \cdot (\vec{P} \times \vec{\sigma})$$

1. Spintronics (Nagaosa, Sinova, Onoda, MacDonald, Ong, Rev. Mod. Phys. 2010)

A. Spin Hall effect

B. Anomalous Hall effect

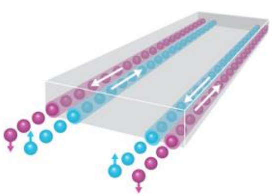


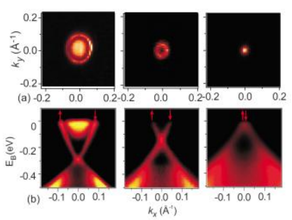


2. Topological insulator (Hasan, Kane, Rev. Mod. Phys. 2010; Qi, Zhang, Rev. Mod. Phys. 2011)

A. 2D Topological insulator

B. 3D Topological insulator



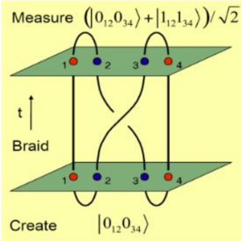


3. Topological superconductor (Alicea, Rep. Prog. Phys. 2012)

A. Majorana zero modes

B. Topological quantum computation

$$\boxed{\gamma = \gamma^\dagger}$$



Spin-orbit coupling for cold atoms

1D Scheme: XJL, M. F. Borunda, X. Liu, and J. Sinova, PRL, 102, 046402 (2009); arXiv: 0808.4137.
And some related works.

Experiments

^{87}Rb boson: I. Spielman group, 2011
Shuai Chen, Jianwei Pan group, 2012
P. Engels' group, Washington State U.
Y. P. Chen, Puedue U
^{40}K fermion: J. Zhang group, 2012
^{6}Li fermion: M. Zwierlein group, 2012.
^{161}Dy fermion: Lev, 2016; ^{173}Yb fermion: G.-B. Jo & XJL 2016; ….

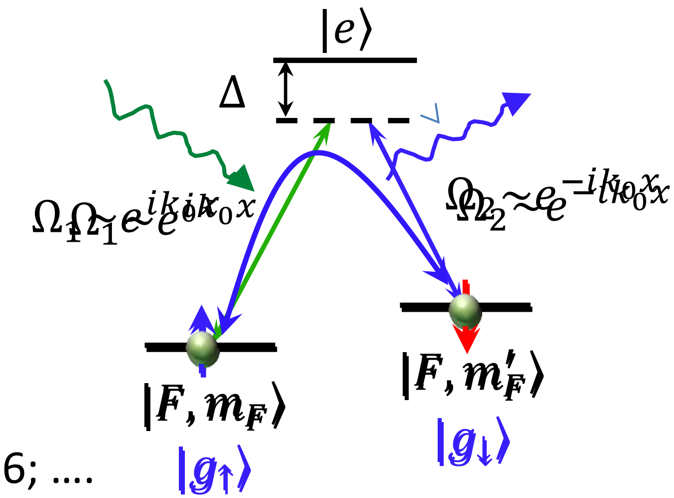
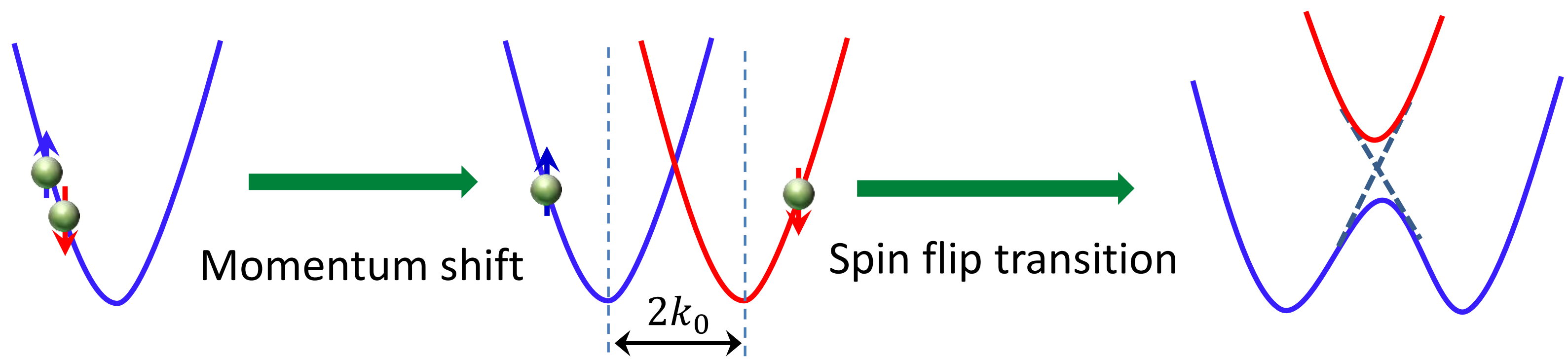


Illustration of 1D SO coupling:



1D spin-orbit coupling plus Zeeman coupling

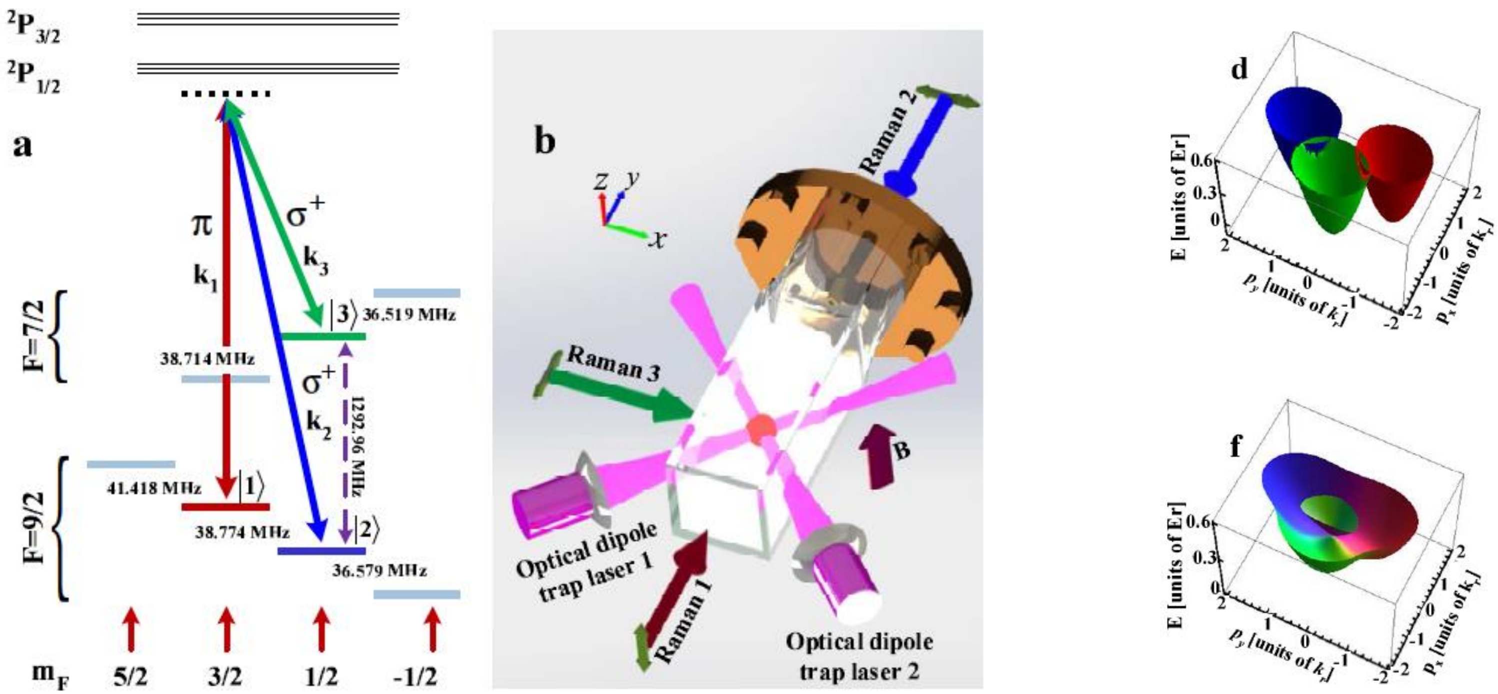
$$H_0 = \frac{\vec{p}^2}{2m} + \underline{\frac{k_0}{m} p_x \sigma_z + \frac{\Omega_R}{2} \sigma_x}$$

However, this is only a 1D SO coupling!

2D SO coupling for continuum gas: tripod scheme

RF spectrum measurement of a 2D Rashba SO coupling: J. Zhang group: Lianghui Huang, et.al., Nature Phys. 12, 540 (2016).

$$H_{SO} = (\lambda_{x1} p_x + \lambda_{y1} p_y)\sigma_x + (\lambda_{x2} p_x + \lambda_{y2} p_y)\sigma_z$$

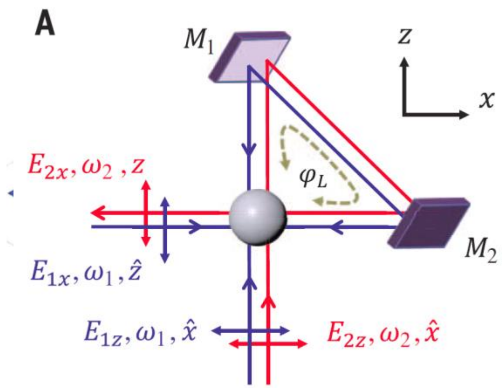


1) applying more than two ground states.
2) spin-orbit coupling for pseudospin-1/2 states (two dark states): $|\uparrow\rangle = |D_1\rangle, |\downarrow\rangle = |D_2\rangle$.

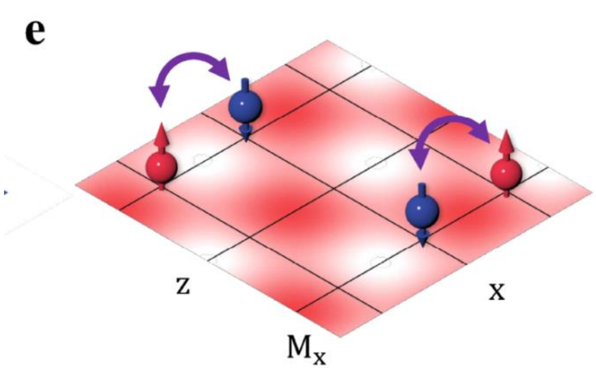
2D SO coupling for optical lattices: optical Raman lattice scheme

Realization of 2D spin-orbit coupling for cold atoms (PKU+USTC): Wu, Zhang, Sun, Xu, Wang, Deng, S. Chen, XJL & J.-W. Pan, Science, 354, 83-88 (2016).

$$H = \frac{p^2}{2m} + V_{\mathrm{latt}}(x,z) + m_z\sigma_z + (M_x - M_y\cos\delta\varphi_L)\sigma_x + M_y\sin\delta\varphi_L\sigma_y$$

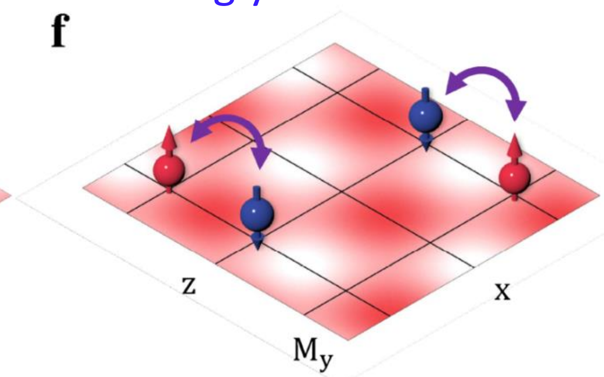


Spin-flipped hopping along x direction



$$M_x = M_0\cos k_0 x \sin k_0 z$$

Spin-flipped hopping along y direction



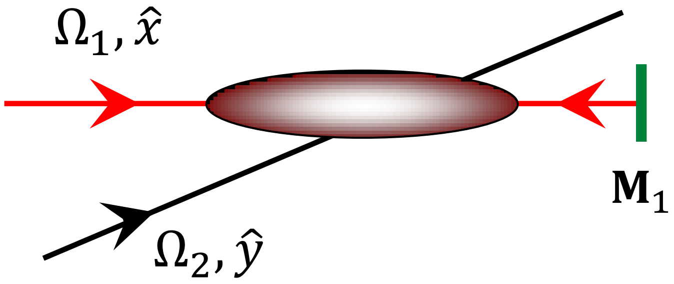
$$M_y = M_0\cos k_0 z \sin k_0 x$$

The realization of minimal SO coupled quantum anomalous Hall phase (XJL, K. T. Law, and T. K. Ng, PRL, 112, 086401 (2014); PRL, 113, 059901 (2014))

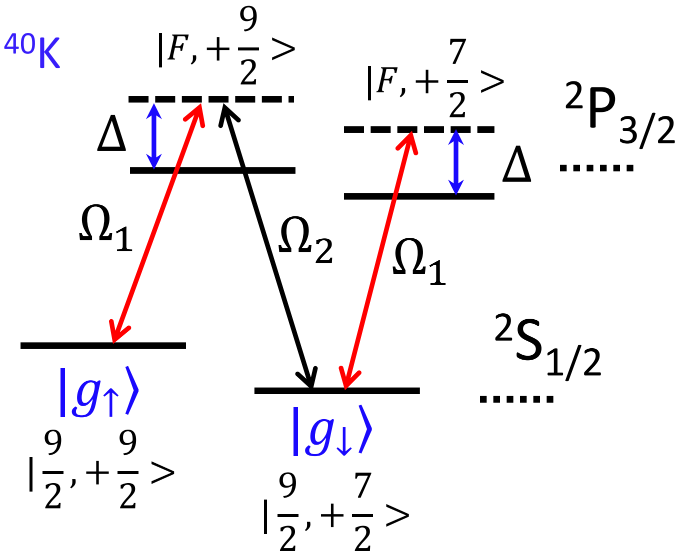
$$\begin{aligned} H_{\mathrm{TI}} &= \sum_{\mathbf{q}} [c^\dagger_\uparrow(\mathbf{q}), c^\dagger_\downarrow(\mathbf{q})]\mathcal{H}(\mathbf{q})[c_\uparrow(\mathbf{q}), c_\downarrow(\mathbf{q})]^T, \\ \mathcal{H}(\mathbf{q}) &= [m_z - 2t_0(\cos q_x a + \cos q_y a)]\sigma_z + 2t_{\mathrm{so}}\sin q_x a\sigma_y + 2t_{\mathrm{so}}\sin q_y a\sigma_x \end{aligned}$$

Optical Raman lattice: optical lattice + Raman lattice

1D model for spin-1/2 atoms



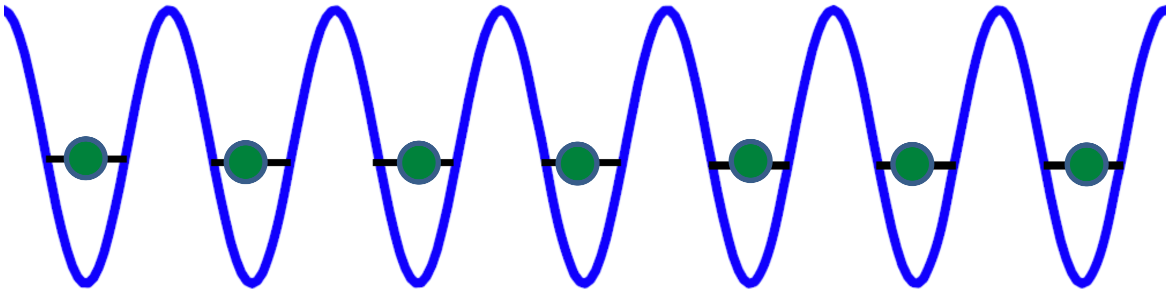
candidate: ^{40}K



Generated 1D lattice and Raman coupling potential:

$V_{\mathrm{latt}}(x) \propto \cos^2(k_0 x)$

$M(x) \propto \cos(k_0 x)\sigma_x$



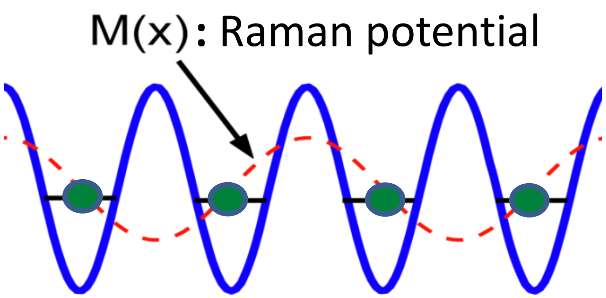
Key features:

- $M(x)$: anti-symmetric with respect to each lattice-site.
- $M(x)$ has half periodicity relative to the lattice.

XJL, Z.-X. Liu, M. Cheng, PRL, 110, 076401 (2013)

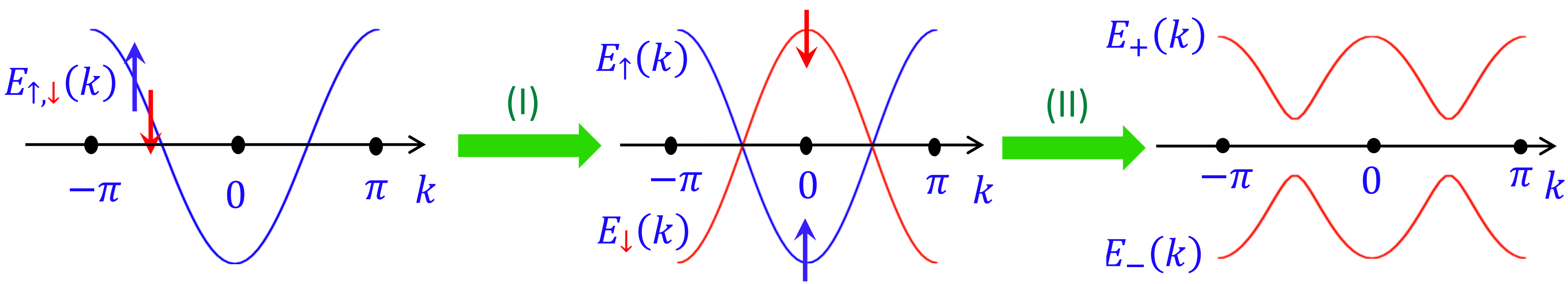
Band structure due to the Raman-lattice configuration

The realized Hamiltonian: $H = \frac{p_x^2}{2m} + V_0 \cos^2 k_0 x + M_0 \cos k_0 x \, \sigma_x + \frac{\delta}{2}\sigma_z$



Effects of the Raman coupling:

(I) $\frac{\pi}{a}$ momentum transfer; (II) SO Coupling.

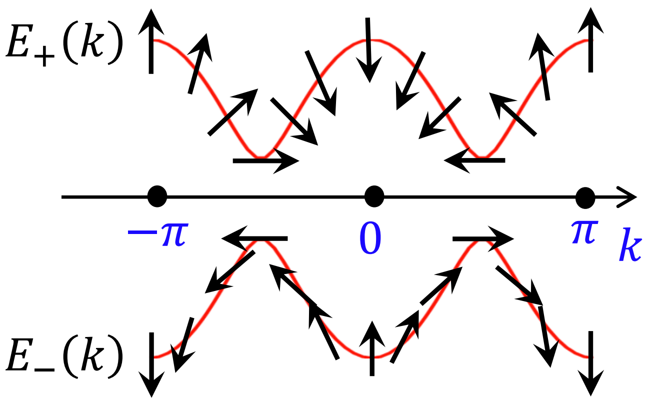


Tight-binding model with spin-orbit coupled hopping ($\Gamma_z = \frac{\delta}{2}$):

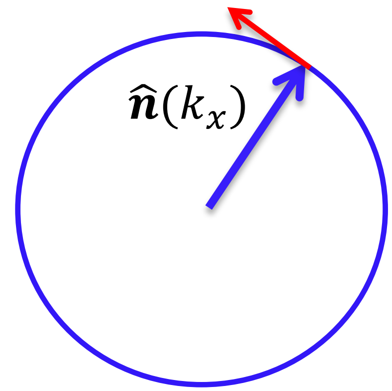
$$H = -t_s \sum_{<i,j>} (\hat{c}^\dagger_{i\uparrow}\hat{c}_{j\uparrow} - \hat{c}^\dagger_{i\downarrow}\hat{c}_{j\downarrow}) + \sum_i \Gamma_z (\hat{n}_{i\uparrow} - \hat{n}_{i\downarrow})$$
$$+ \big[\sum_j t^{(0)}_{\rm so} (\hat{c}^\dagger_{j\uparrow}\hat{c}_{j+1\downarrow} - \hat{c}^\dagger_{j\uparrow}\hat{c}_{j-1\downarrow}) + \text{H.c.} \big].$$

XJL, Z.-X. Liu, M. Cheng, PRL, 110, 076401 (2013).

Symmetry protected topological state: AIII class and Z invariant (chiral symmetry)



Mapping: $k_x \mapsto \hat{\boldsymbol{n}}$



$$\hat{\boldsymbol{n}}(k_x) = \frac{\vec{\boldsymbol{s}}}{|\vec{\boldsymbol{s}}|}$$

Topology: classified by integer winding numbers: Z

Discussions

1) Fractional charge, 1/4-spin states; topological classification: $Z \rightarrow Z_4$ (with interaction);
   XJL, Z.-X. Liu, M. Cheng, PRL, 110, 076401 (2013).
   X. Zhou, J.-S. Pan, Z.-X. Liu, W. Zhang, W. Yi, G. Chen, and S. Jia, arXiv:1612.08880 (2016).

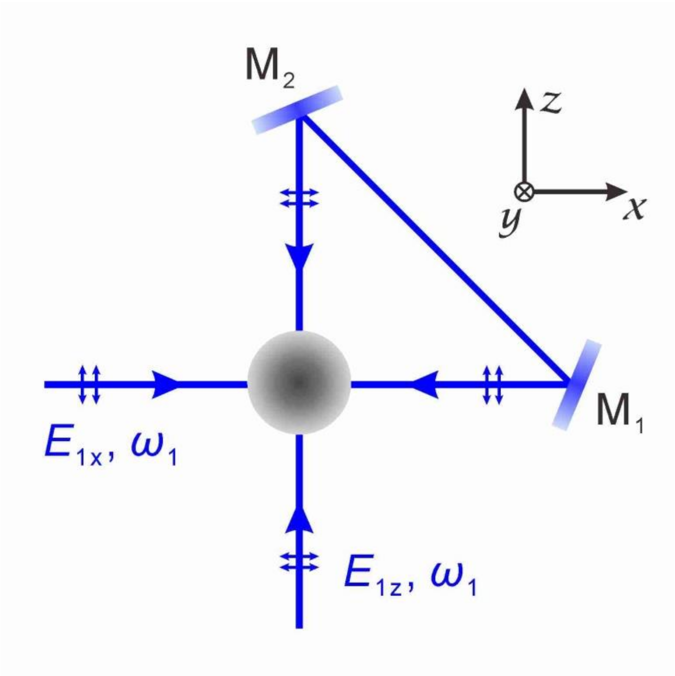
2) BDI class topological superconductivity/superfludity with s-wave pairing;
   He, Wu, Choy, XJL, Tanaka, Law, Nat. Comm. 5, 3232 (2014).

3) Topological superradiant phase by putting in the cavity;
   Pan, XJL*, Zhang*, Yi*, and Guo, PRL 115, 045303 (2015).

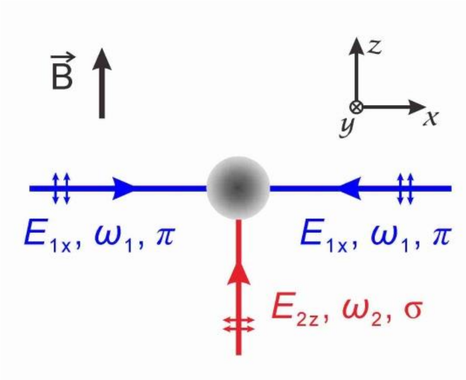
4) Hidden nonsymmorphic symmetry and band degeneracy;
   H. Chen, XJL, and X. C. Xie, PRA 93, 053610 (2016).

Generalization to 2D SOC: Experimental scheme

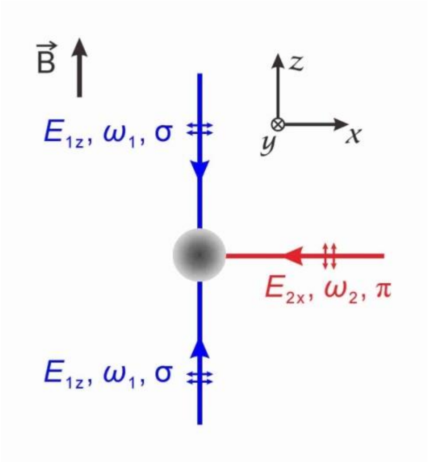
candidate: ^{87}Rb bosons, ^{40}K fermions



1) Raman coupling (I)



2) Raman coupling (II)



The phase difference of the two laser beams go through the loop:

Light ω_1 Light ω_2

$$\delta\varphi_L = \frac{\delta\omega}{c} L \qquad \delta\omega = \omega_2 - \omega_1$$

Proposal: *XJL etal @PKU.* Experiment: *S. Chen & J.-W. Pan etal @USTC*

RESEARCH ARTICLE

PKU Group & USTC Group

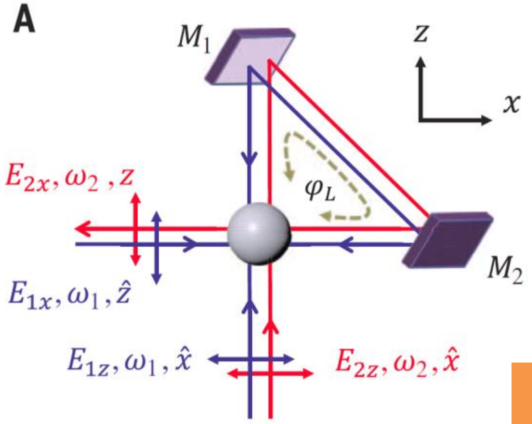
QUANTUM SIMULATION

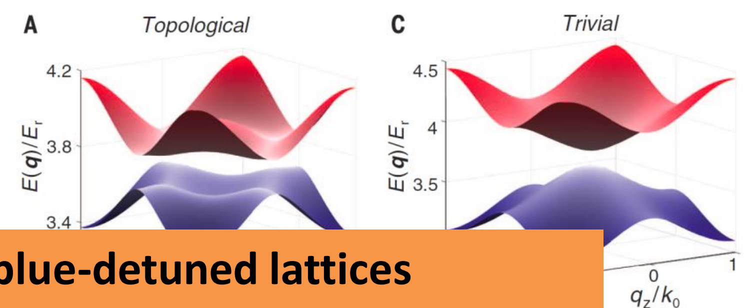
SCIENCE sciencemag.org

Science, 354, 83-88 (2016).

Realization of two-dimensional spin-orbit coupling for Bose-Einstein condensates

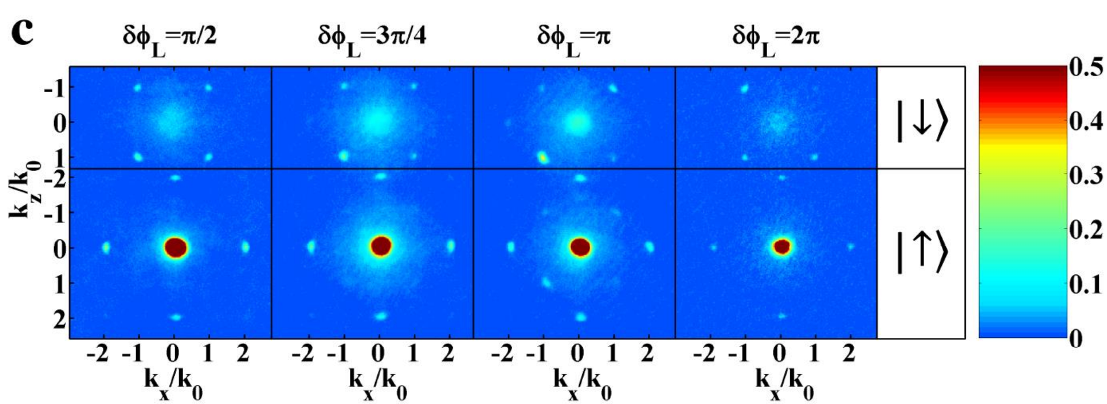
Zhan Wu,[1,2,3] Long Zhang,[1,4,5] Wei Sun,[1,2,3] Xiao-Tian Xu,[1,2,3] Bao-Zong Wang,[1,4,5] Si-Cong Ji,[1,2] Youjin Deng,[1,2,3] Shuai Chen,[1,2,3*] Xiong-Jun Liu,[4,5*] Jian-Wei Pan[1,2,3*]





1) have to be blue-detuned lattices
2) without exact inversion or C4 symmetry

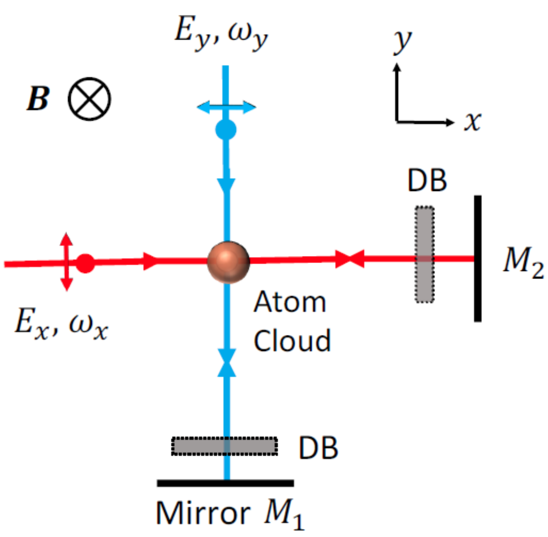
1D-2D crossover of SO coupling



The new scheme: a hierarchy set of optical Raman lattices

Motivation: A hierarchy set of schemes for realization of Dirac, Rashba, and Weyl type spin-orbit couplings, with high controllability and long lifetime.

I) 2D Dirac type SOC: spin-independent lattice + Raman lattice



Two standing wave beams:

$$\boldsymbol{E}_x = \hat{\boldsymbol{y}} E_{xy} \cos k_0 x + i \hat{\boldsymbol{z}} E_{xz} \sin k_0 x \qquad \omega_x$$

$$\boldsymbol{E}_y = \hat{\boldsymbol{x}} E_{yx} \cos k_0 y + i \hat{\boldsymbol{z}} E_{yz} \sin k_0 y \qquad \omega_y$$

The generated Raman couplings:

$$\hat{y} E_{xy} \times \hat{z} E_{yz} \rightarrow M_x \sigma_x$$

$$\hat{x} E_{yx} \times \hat{z} E_{xz} \rightarrow M_y \sigma_x$$

Key features:
1) With precisely controllable symmetries:
2) Valid for any type of detunings, blue or red.

B.-Z. Wang, Y.- H. Lu, W. Sun, S. Chen*, Y. Deng, XJL*, arXiv:1706.08961v2.

The realized effective Hamiltonian

The effective Hamiltonian can be write as $m_z = \delta/2$ two-photon detuning):

$$H = \frac{p^2}{2m} + V_{latt}(x,y) + (M_x - M_y \cos\delta\varphi)\sigma_x + M_y \sin\delta\varphi\,\sigma_y + m_z\sigma_z$$

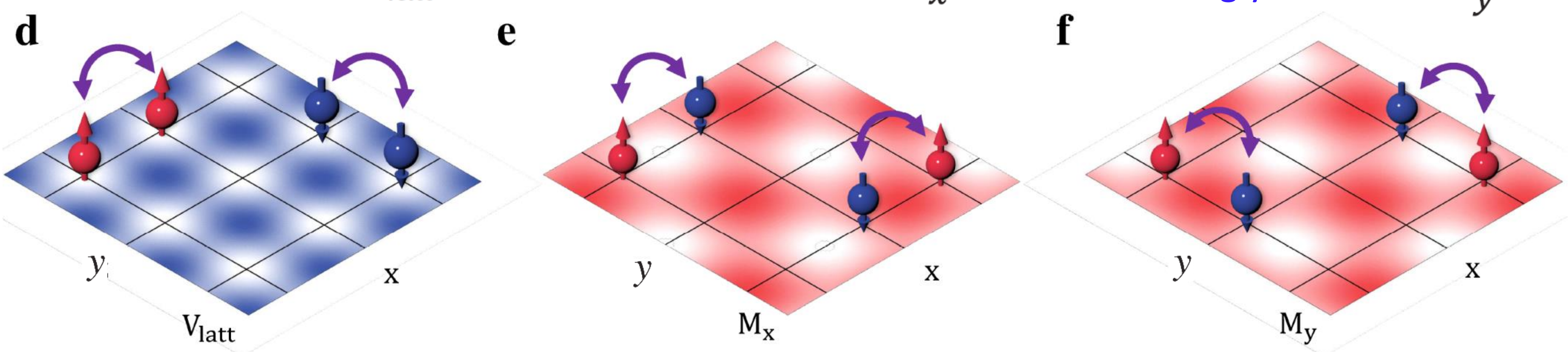
The Raman coupling potentials:

$$M_x = M_0 \cos k_0 x \sin k_0 y \qquad M_y = M_0 \cos k_0 y \sin k_0 x$$

Spin-conserved hopping by optical lattice V_{latt}

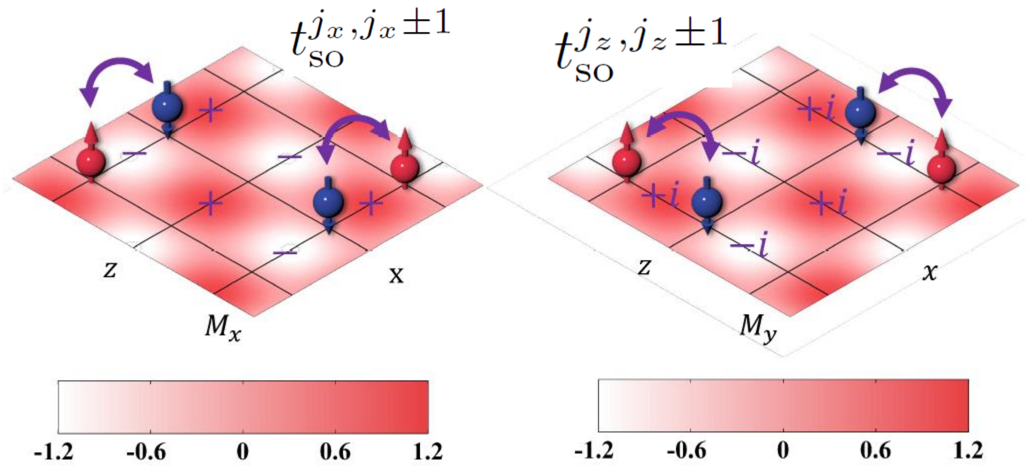
Spin-flipped hopping along x direction M_x

Spin-flipped hopping along y direction M_y



$$t_0^{i,j} = t_0 \qquad t_{\rm so}^{j_x, j_x \pm 1} = \pm(-1)^{j_x+j_y} t_{\rm so}^{(0)} \qquad t_{\rm so}^{j_y, j_y \pm 1} = \pm i(-1)^{j_x+j_y} t_{\rm so}^{(0)}$$

Topological phases for s-band model: Fermions



Spin-flip hopping:

$$t_{\text{SO}}^{j_x,j_x\pm1} = \pm(-1)^{j_x+j_y} t_{\text{SO}}^{(0)}$$

$$t_{\text{SO}}^{j_y,j_y\pm1} = \pm i(-1)^{j_x+j_y} t_{\text{SO}}^{(0)}$$

The staggered factor $(-1)^{j_x+j_y}$ implies the relative (π,π) momentum transfer between spin-up and spin-down Bloch states.

Tight-binding Hamiltonian (after shifting (π,π) momentum for spin-down states):

$$\begin{aligned} H_{\text{TI}} \; = \; & -t_s \sum_{<\vec{i},\vec{j}>} (\hat{c}^{\dagger}_{\vec{i}\uparrow}\hat{c}_{\vec{j}\uparrow} - \hat{c}^{\dagger}_{\vec{i}\downarrow}\hat{c}_{\vec{j}\downarrow}) + \sum_{\vec{i}} m_z(\hat{n}_{\vec{i}\uparrow} - \hat{n}_{\vec{i}\downarrow}) + \\ & + \Big[\sum_{j_x} t_{\text{SO}}^{(0)} (\hat{c}^{\dagger}_{j_x\uparrow}\hat{c}_{j_x+1\downarrow} - \hat{c}^{\dagger}_{j_x\uparrow}\hat{c}_{j_x-1\downarrow}) + \text{H.c.}\Big] + \\ & + \Big[\sum_{j_y} i t_{\text{SO}}^{(0)} (\hat{c}^{\dagger}_{j_y\uparrow}\hat{c}_{j_y+1\downarrow} - \hat{c}^{\dagger}_{j_y\uparrow}\hat{c}_{j_y-1\downarrow}) + \text{H.c.}\Big]. \quad (2) \end{aligned}$$

XJL, K. T. Law, and T. K. Ng, PRL, 112, 086401 (2014); PRL, 113, 059901 (2014)

I. Non-interacting: quantum anomalous Hall effect (Chern insulator)

$$H_{\mathrm{TI}} = \sum_{\mathbf{q}} [c^\dagger_\uparrow(\mathbf{q}), c^\dagger_\downarrow(\mathbf{q})]\mathcal{H}(\mathbf{q})[c_\uparrow(\mathbf{q}), c_\downarrow(\mathbf{q})]^T,$$

$$\mathcal{H}_{\vec{q}} = [m_z + 2t_s(\cos q_x + \cos q_y)]\sigma_z + 2t_{so}(\sin q_x \sigma_x + \sin q_y\, \sigma_y)$$

The Nobel Prize in Physics 2016

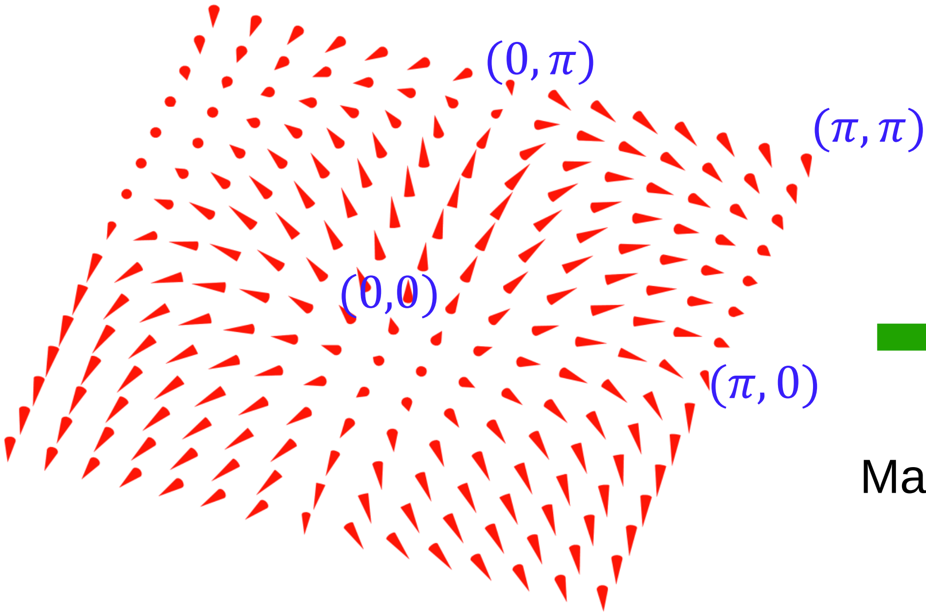
Photo: A. Mahmoud
David J. Thouless
Prize share: 1/2

Photo: A. Mahmoud
F. Duncan M. Haldane
Prize share: 1/4

Photo: A. Mahmoud
J. Michael Kosterlitz
Prize share: 1/4

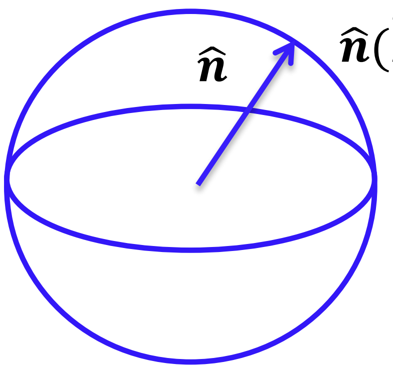
This is the minimal single-band SO coupled QAH model.

- 2D spin texture (magnetic skyrmion) in **k**-space:



- Chern number (Qi, Wu, Zhang, PRB 2006):

$$\mathrm{Ch}_1 = \begin{cases} \mathrm{sgn}(m_z), & \text{for } 0 < |m_z| < 4t_0, \\ 0, & \text{for } |m_z| \geq 4t_0, m_z = 0. \end{cases}$$

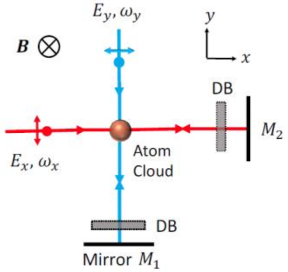


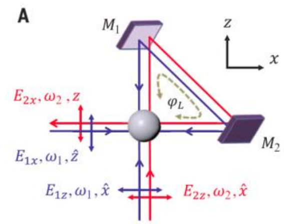
$$\hat{\boldsymbol{n}}(\vec{k}) = \frac{\vec{\boldsymbol{s}}}{|\vec{\boldsymbol{s}}|}$$

Mapping: $\vec{k} \mapsto \hat{\boldsymbol{n}}$

XJL, K. T. Law, and T. K. Ng, PRL, 112, 086401 (2014); PRL, 113, 059901 (2014)

Effect of inversion symmetry breaking

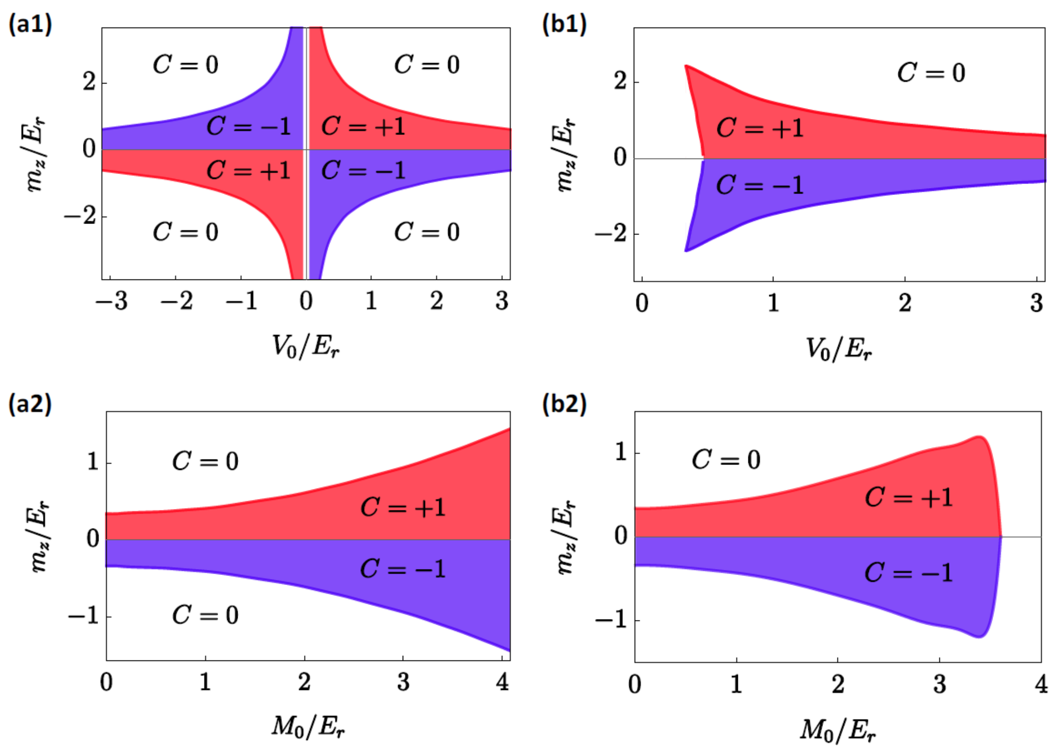


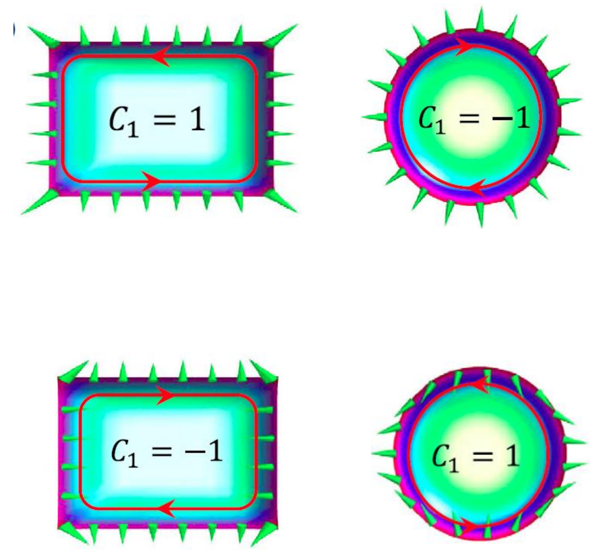


Topological spin texture for edge states in the boundary (Wu, Liu, and XJL, PRL 113, 136403 (2014))

C_4-symmetric

Without C_4-symmetry



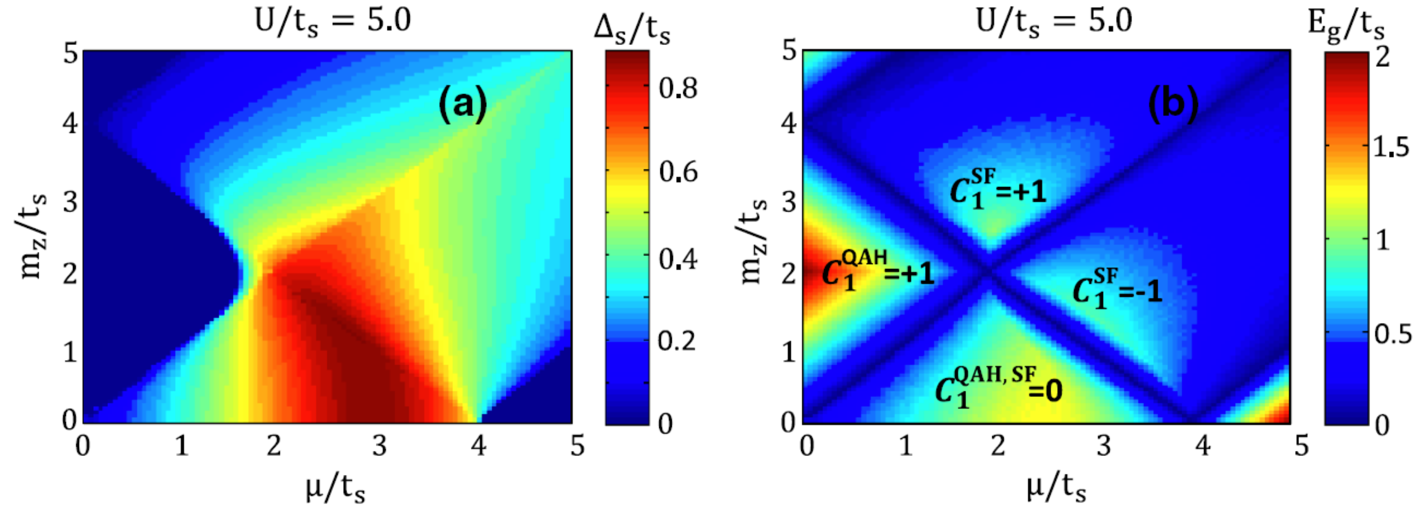


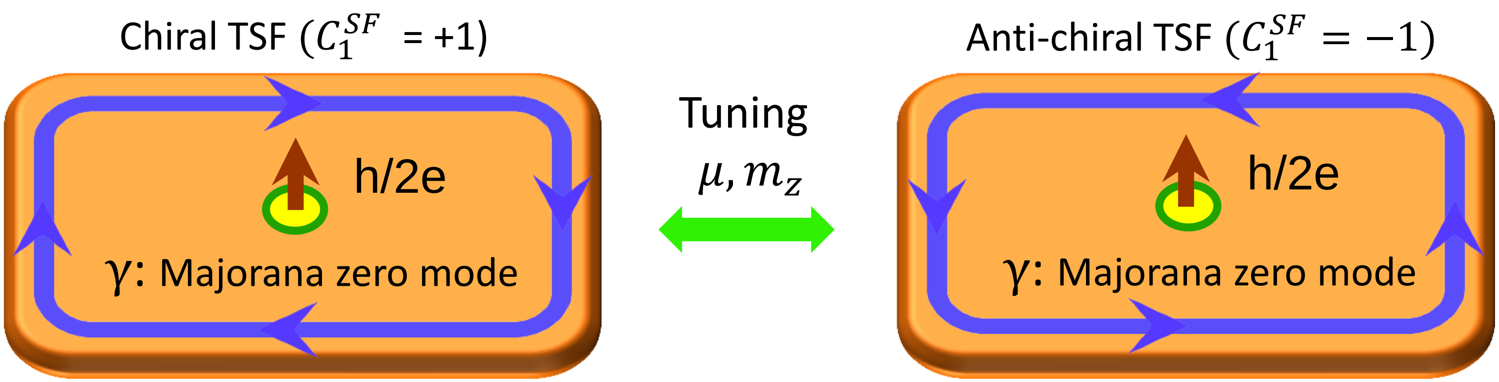
J.S. Pan, W. Zhang, W. Yi, and G.-C. Guo, PRA, 94, 043619 (2016);
B.-Z. Wang, Y.- H. Lu, W. Sun, S. Chen, Y. Deng, XJL, arXiv:1706.08961v2.

II. Interacting regime: Chiral topological superfluids

Attractive Hubbard model: $$H = \sum_{\vec{k}} C^{\dagger}(\vec{k})\,\mathcal{H}_s(\vec{k})\,C(\vec{k}) - \sum_i U n_{i\uparrow} n_{i\downarrow}$$

Phase diagram:



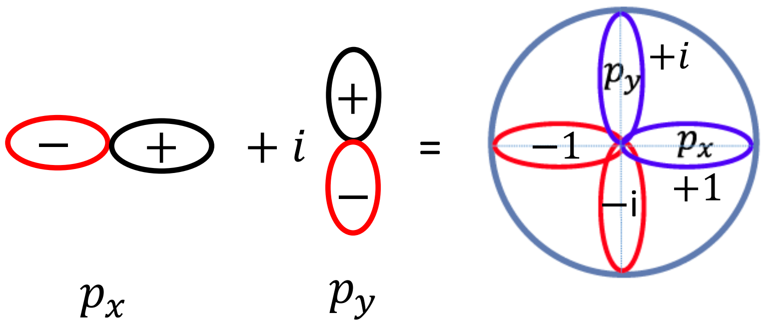


- One Majorana zero bound state $\gamma(E = 0)$ exists in each vortex core. Majorana bound modes obey non-Abelian statistics (Reed & Green, PRB, ’00; Ivanov, PRL, ’01; Alicea et al., Nat. Phys., ’11)

XJL, K. T. Law, and T. K. Ng, PRL, 2014.

Topological physics for p-band model: Bosons

Background: orbital order formed by spinless p-band bosons



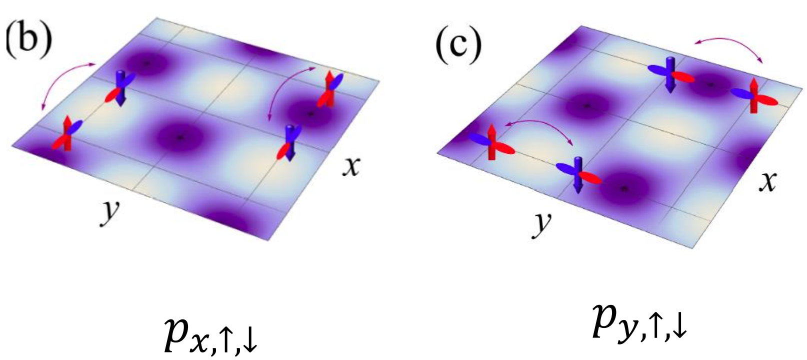
$$H_I = \frac{U}{2}\sum_{\vec{r}}\{n^2 - \frac{1}{3}L_z^2\}$$

- Break time-reversal symmetry spontaneously
- With SO(2) symmetry

(V. W. Liu, C. Wu, PRA, 2006; X. Li and V. W. Liu, Rep. Pro. Phys. 2016)

Motivation: competition between orbital degree freedom & spin degree of freedom with SOC。

Model: $H = \sum_\mu H_{0\mu} + H_I \qquad \mu = x, y$



$$H_{0\mu} = -\sum_{\langle\vec{i},\vec{j}\rangle_\mu,s,s'} t_\mu \hat{p}^\dagger_{\vec{i}s\mu}\sigma_z^{ss'}\hat{p}_{\vec{j}s'\mu} + \sum_{\vec{i}} m_z(\hat{n}_{\vec{i}\uparrow\mu} - \hat{n}_{\vec{i}\downarrow\mu})$$
$$+[\sum_{\langle\vec{i},\vec{j}\rangle_\mu} \delta_\mu t_{\rm so}(\hat{p}^\dagger_{\vec{i}\uparrow\mu}\hat{p}_{\vec{j}\downarrow\mu} - \hat{p}^\dagger_{\vec{i}\uparrow\mu}\hat{p}_{\vec{j}\downarrow\mu}) + {\rm H.c.}]. \qquad (1)$$

$$H_I = U_{\rm int}\sum_{\vec{i}}\{n_{\vec{i}}^2 + \frac{1}{2}\sum_{\mu s}(n^2_{\vec{i},\mu s} + p^\dagger_{\vec{i},\mu s}p^\dagger_{\vec{i},\mu s}p_{\vec{i},\bar{\mu}s}p_{\vec{i},\bar{\mu}s})\}$$

Y.-Q. Wang and XJL, arXiv:1710.02070v2.

Self-consistent solution for ground phase

$$\Delta_\uparrow = \langle p^\dagger_{x(y)\uparrow} p_{x(y)\uparrow}\rangle,\ \ \Delta_\downarrow = \langle p^\dagger_{x(y)\downarrow} p_{x(y)\downarrow}\rangle,\ \ \Delta_{xy\uparrow} = \langle p^\dagger_{x\uparrow} p_{y\uparrow}\rangle,\ \ \Delta_{xy\downarrow} = \langle p^\dagger_{x\downarrow} p_{y\downarrow}\rangle$$

Mean field Hamiltonian

$$H^{\mathrm{MF}} = H_0 + H^{\mathrm{MF}}_{\mathrm{I}}$$

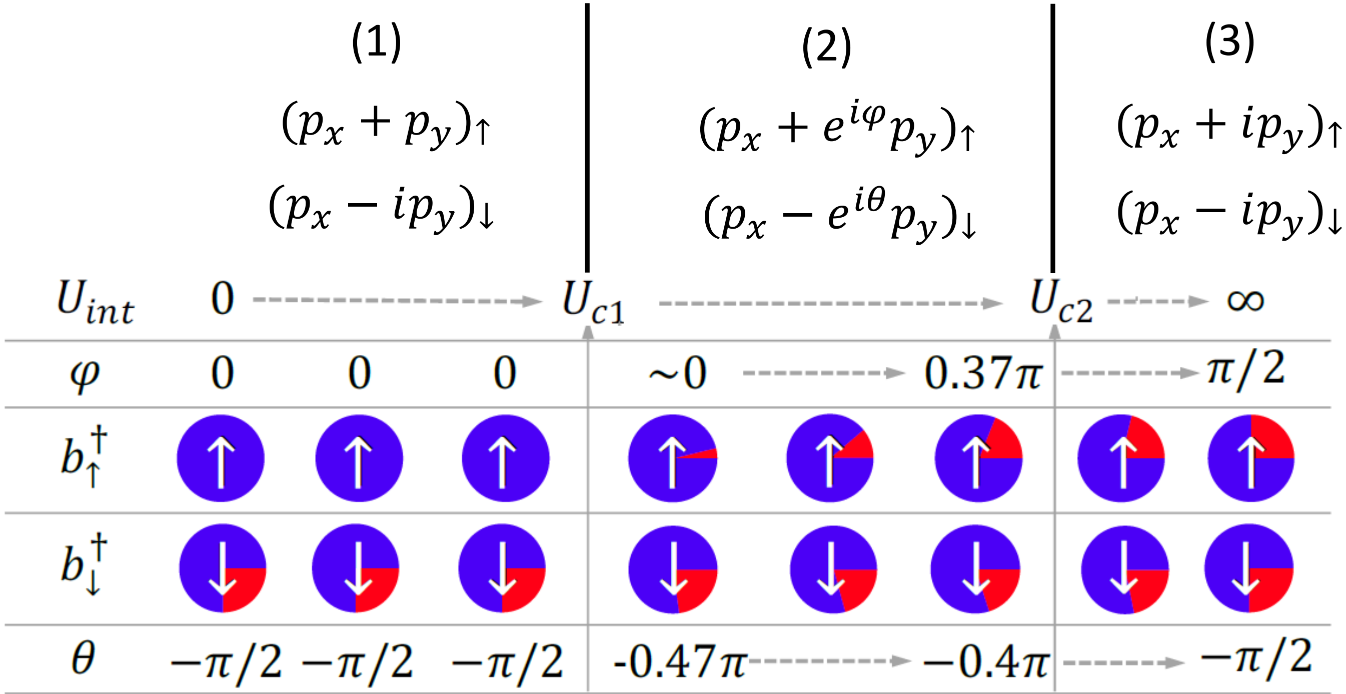
$$\begin{aligned} H^{\mathrm{MF}}_{\mathrm{I}} &= U_{\mathrm{int}} \sum_{\vec{r}} [3(n_{p_x\uparrow}\Delta_\uparrow + n_{p_x\downarrow}\Delta_\downarrow + n_{p_y\uparrow}\Delta_\uparrow + n_{p_y\downarrow}\Delta_\downarrow) \\ &+ (\Delta_{xy\uparrow} p^\dagger_{x\uparrow} p_{y\uparrow} + \Delta^*_{xy\uparrow} p^\dagger_{y\uparrow} p_{x\uparrow} + \Delta_{xy\downarrow} p^\dagger_{x\downarrow} p_{y\downarrow} + \Delta^*_{xy\downarrow} p^\dagger_{y\downarrow} p_{x\downarrow}) \\ &+ 2(\Delta_{xy\uparrow} p_{x\uparrow} p^\dagger_{y\uparrow} + \Delta^*_{xy\uparrow} p^\dagger_{x\uparrow} p_{y\uparrow} + \Delta_{xy\downarrow} p_{x\downarrow} p^\dagger_{y\downarrow} + \Delta^*_{xy\downarrow} p^\dagger_{x\downarrow} p_{y\downarrow}) \\ &+ 4(n_{p_x\uparrow}\Delta_\downarrow + \Delta_\uparrow n_{p_x\downarrow} + n_{p_y\uparrow}\Delta_\downarrow + \Delta_\uparrow n_{p_y\downarrow})]. \end{aligned}$$

Iteration steps:

$$\begin{tikzcd} |G_0\rangle \text{ of } H_0 \arrow[r, "\text{calculate } \Delta_\alpha\text{'s}"] & H_{MF} \arrow[r, "\text{solve } H_{MF}"] & |G_{MF}\rangle \text{ of } H_{MF} \arrow[l, bend left, "\text{calculate } \Delta_\alpha\text{'s}"] \end{tikzcd}$$

Until the solution converges

Ground state phase diagram



The condensate wave function

$$|\Phi_{\mathrm{BEC}}\rangle = \sin\alpha e^{i\beta} |\Phi_{p_x+e^{i\varphi}p_y}, \uparrow\rangle - \cos\alpha |\Phi_{p_x+e^{i\theta}p_y}, \downarrow\rangle.$$

This is generically an entangled order between spin and orbital states.

Y.-Q. Wang and XJL, arXiv:1710.02070v2.

Topological vs Dirac type excitations

$$H_I' = U_{\rm int} \sum_{\vec{i}} (n_{\vec{i}}^2 + \frac{1}{2} n_{\vec{i},\uparrow}^2 \cos^2 \varphi + \frac{1}{2} n_{\vec{i},\downarrow}^2 \cos^2 \theta)$$

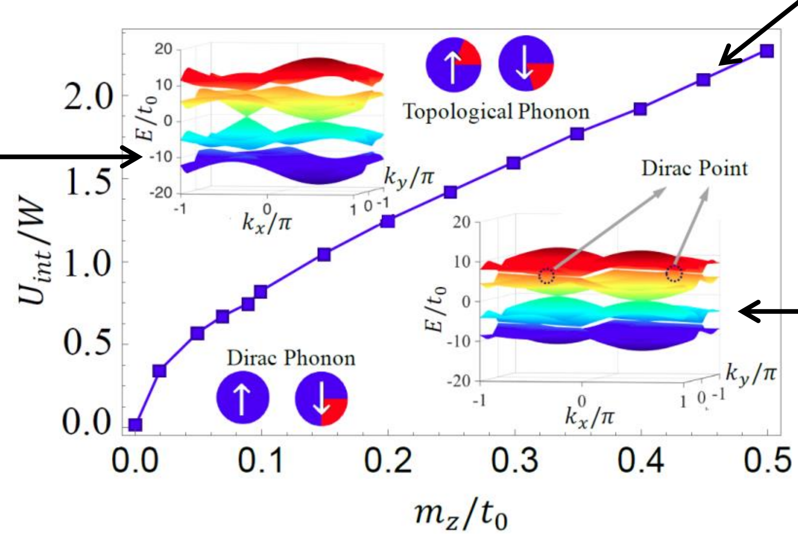
Chern number of the n-th band excitations (Shindou, Matsumoto, Murakami, Ohe, Saitoh, PRB, 2013; Furukawa, Ueda, NJP, 2015)

$$C_1^{(n)} = -\frac{1}{2\pi} \int_{\rm FBZ} dk_x dk_y \Omega_{n,\mathbf{k}}^{xy},$$

$$\Omega_{n,k}^{xy} = i(\sigma_3)_{n,n} \epsilon_{xy} (\frac{\partial}{\partial_{k_x}} \langle t_n(\boldsymbol{k})|) \sigma_3 (\frac{\partial}{\partial_{k_y}} |t_n(\boldsymbol{k})\rangle)$$

Phase diagram for band topology

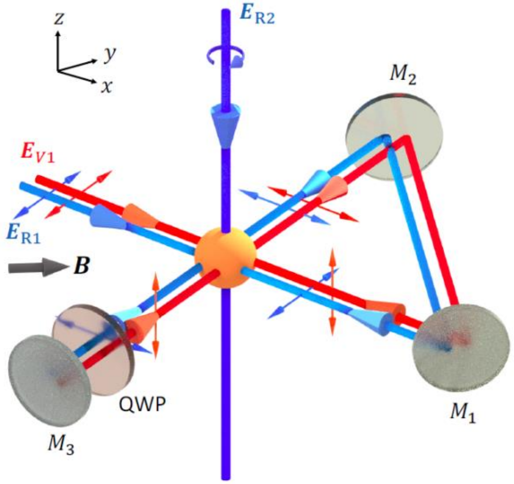
Both are complex local orders



One real & one complex local order

Y.-Q. Wang and XJL, arXiv:1710.02070v2.

II) 2D Rashba and 3D Weyl type SOCs: spin-dependent lattice + Raman lattice



$$V(r) = u_s|\boldsymbol{E}|^2 + iu_v\left(\boldsymbol{E}^* \times \boldsymbol{E}\right)\cdot\boldsymbol{S},$$

The realized Hamiltonian:

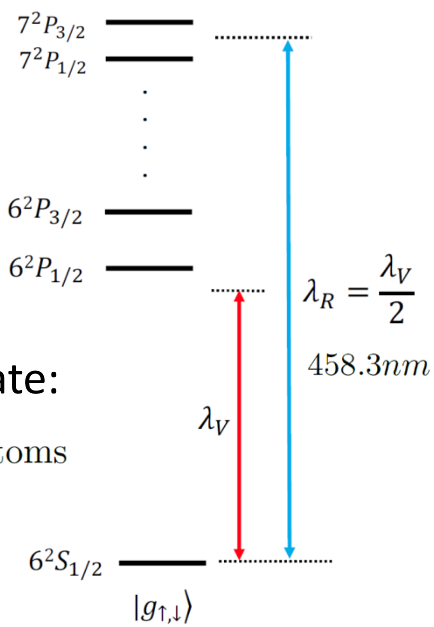
$$H_{3D} = \frac{\boldsymbol{p}^2}{2m} + m_z\sigma_z + \underline{V_0\left(\sin 2k_0x + \sin 2k_0y\right)\sigma_z} \quad \text{(a)}$$
$$+ M_0 e^{i2k_0z\sigma_z}\underline{\left(\cos 2k_0x\sigma_x + \cos 2k_0y\sigma_y\right).} \quad \text{(b)}$$

(a) spin-dependent lattice generated by:

$$E_{v1}, \omega$$

(b) spin-flip Raman coupling generated by laser beams with double frequency

$$E_{R1}, 2\omega$$

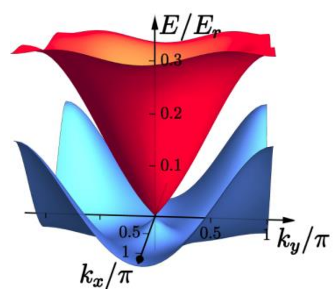


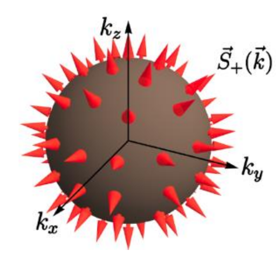
candidate:

$^{133}\mathrm{Cs}$ atoms

Band structure for s-band model

2D Rashba





3D Weyl cone

Wang, Lu, Sun, Chen, Deng, XJL, arXiv:1706.08961v2.

Experimental results: SPT phase in 1D optical Raman lattice

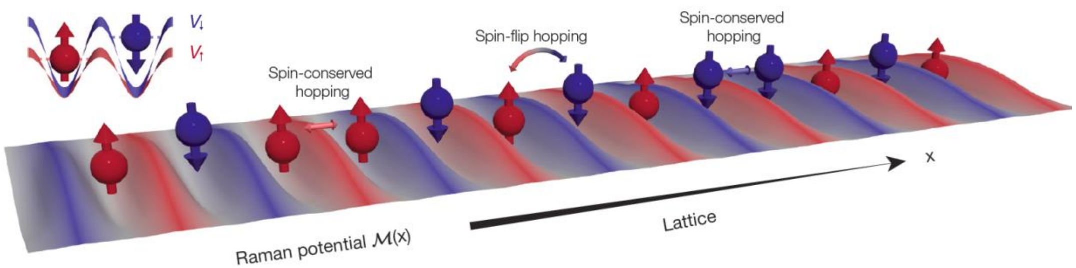
Hamiltonian realized in experiment: B. Song, L. Zhang, C. He, T. F. Jeffrey Poon, E. Hajiyev, S. Zhang, XJL, and G.-B. Jo, arXiv:1706.00768v2

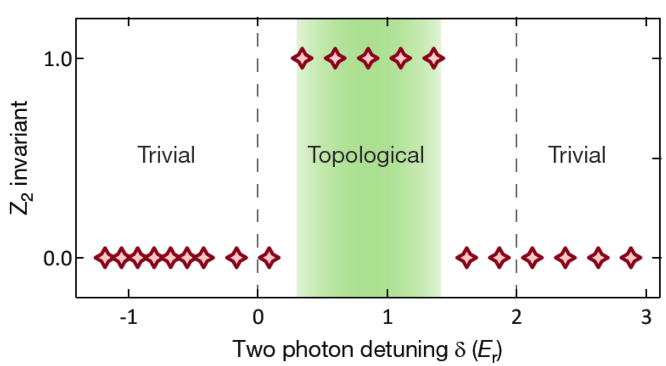
$$H = \left[\frac{p_x^2}{2m} + \frac{V_\uparrow^{\rm latt}(x) + V_\downarrow^{\rm latt}(x)}{2}\right] \otimes \hat{\mathbf{1}} + \left[\frac{\delta}{2} + \frac{V_\uparrow^{\rm latt}(x) - V_\downarrow^{\rm latt}(x)}{2}\right]\sigma_z + \mathcal{M}(x)\sigma_x.$$

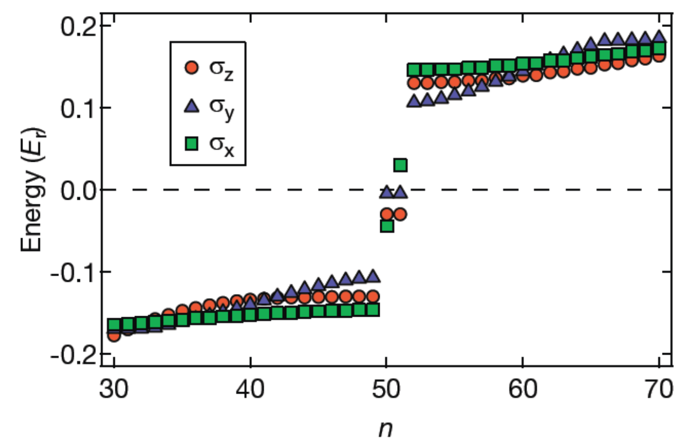
The s-band Hamiltonian realizes a SPT phase protected by magnetic group and non-local chiral symmetry:

$$M_x = \sigma_z K \otimes R_x \qquad \mathcal{S} = \sigma_z \otimes T_x(k_0) \otimes R_x$$

candidate: ^{173}Yb fermions



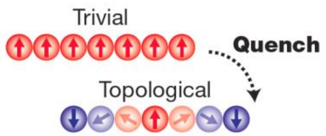


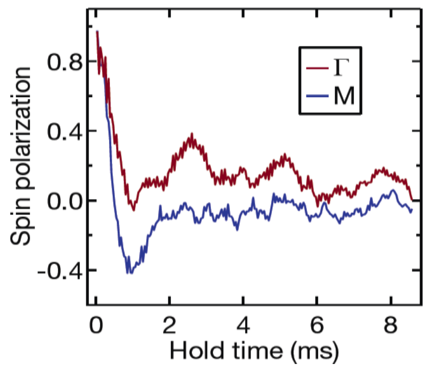


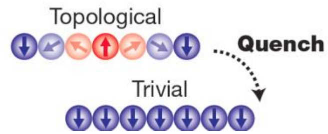
End in-gap states

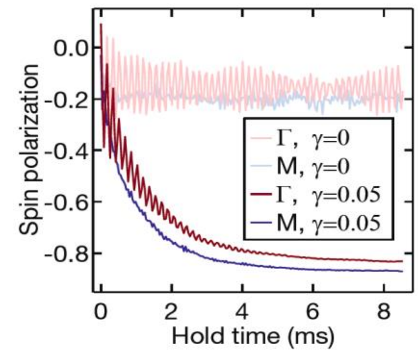
Quench dynamics: 1) trivial to topological; 2) topological to trivial

$$\dot{\rho} = -\frac{i}{\hbar}[H', \rho] + \gamma \left(L\rho L^\dagger - \frac{1}{2}\{L^\dagger L, \rho\} \right)$$

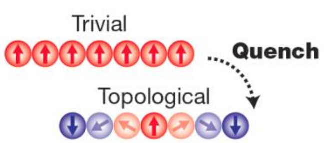


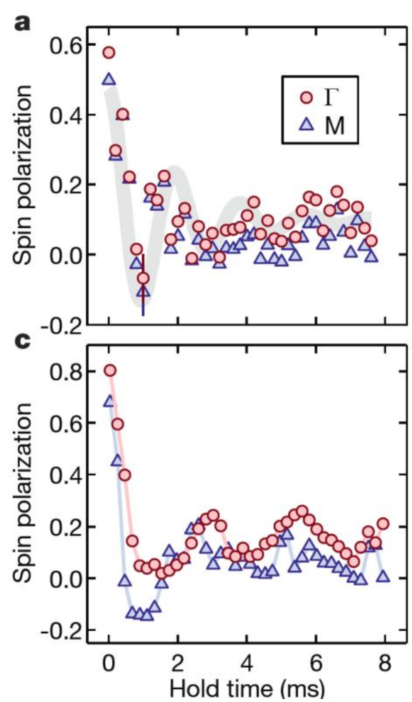


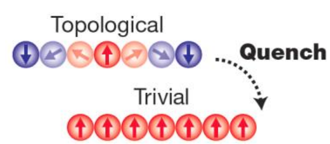


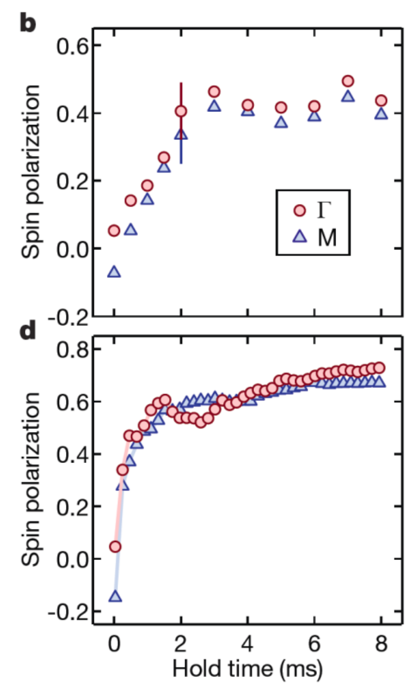


Experimental measurement





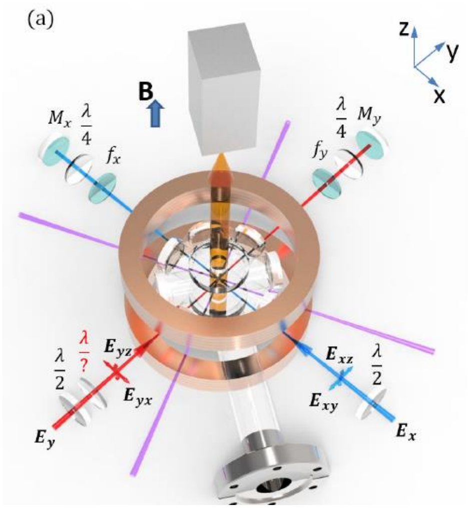


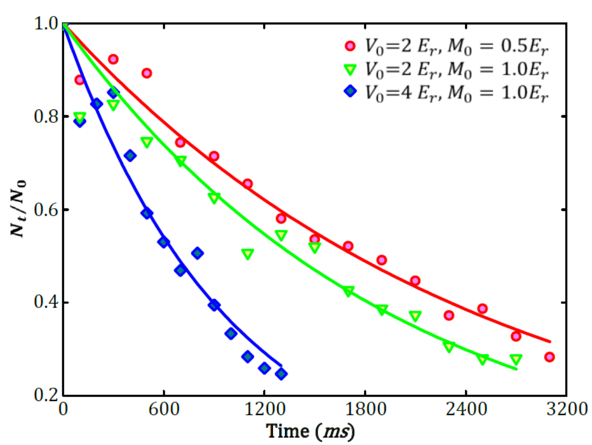


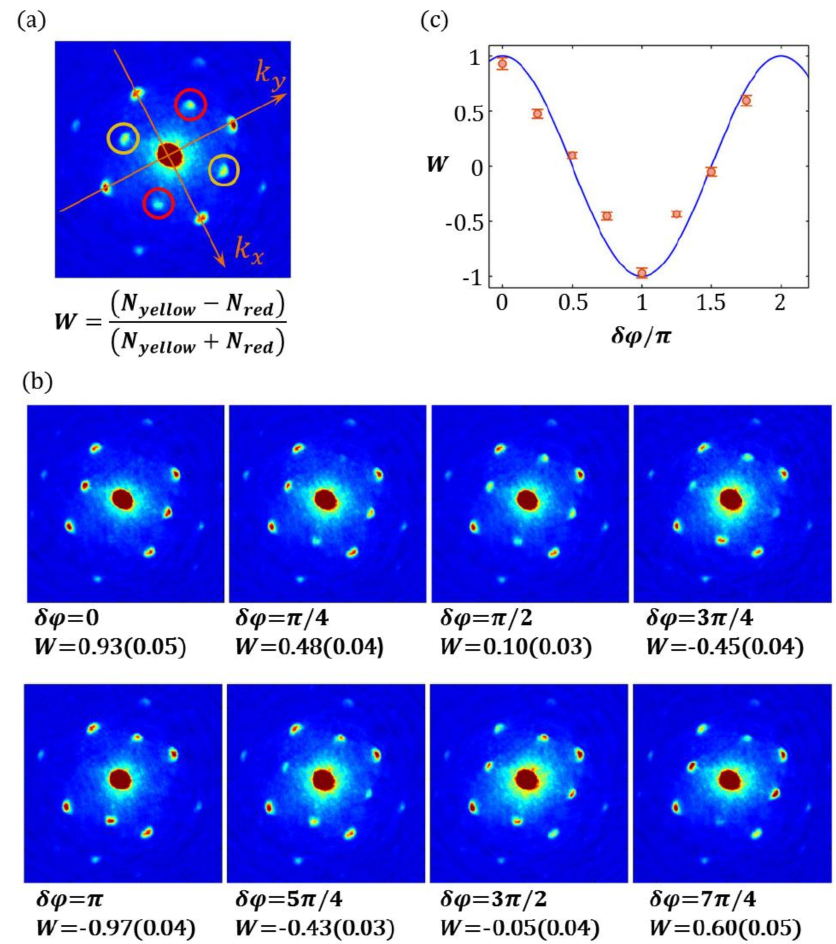
Experimental results: 2D Chern band

The new scheme: B.-Z. Wang, Y.- H. Lu, W. Sun, S. Chen, Y. Deng, XJL, arXiv:1706.08961v2.

New experiment: Sun, Wang, Xu, Yi, Zhang, Wu, Deng, XJL, S. Chen, J.-W. Pan, arXiv:1710.00717.





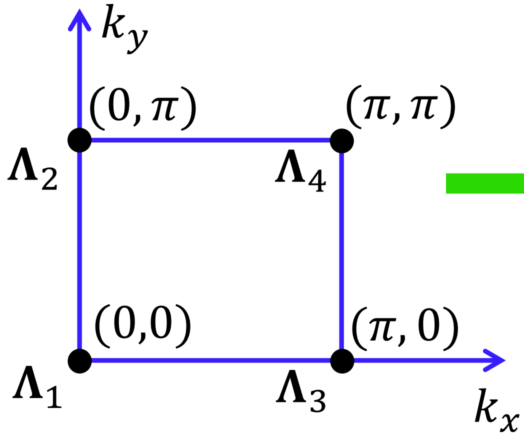


The measured lifetime can be $\tau_{\text{life}} > 1.0\ s$

A highly resolved 1D-2D SOC crossover

Measurement of phase diagram for band topology

Determining topology by Bloch states at symmetric momenta. XJL, Law, and Ng, and Patrick A. Lee, PRL, 111, 120402 (2013); XJL, Liu, Law, Vincent Liu, and Ng, New J. Phys. 18, 035004 (2016).

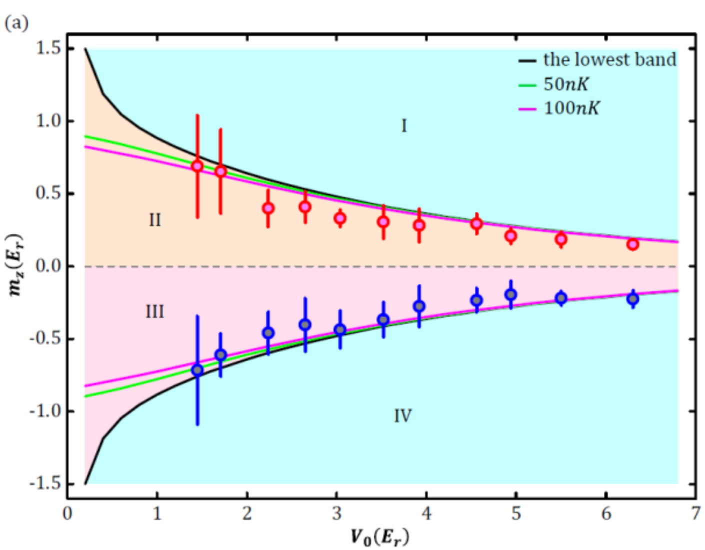


σ_z: "parity operator"

$$\sigma_z|u_m(\boldsymbol{\Lambda}_\mathrm{i})\rangle = \xi^{(m)}|u_m(\boldsymbol{\Lambda}_\mathrm{i})\rangle \;\Longrightarrow\; C_1^{(m)} = -\frac{1-\Theta}{4}\sum_{j=1}^{4}\mathrm{sgn}[\xi_m(\Lambda_j)].$$

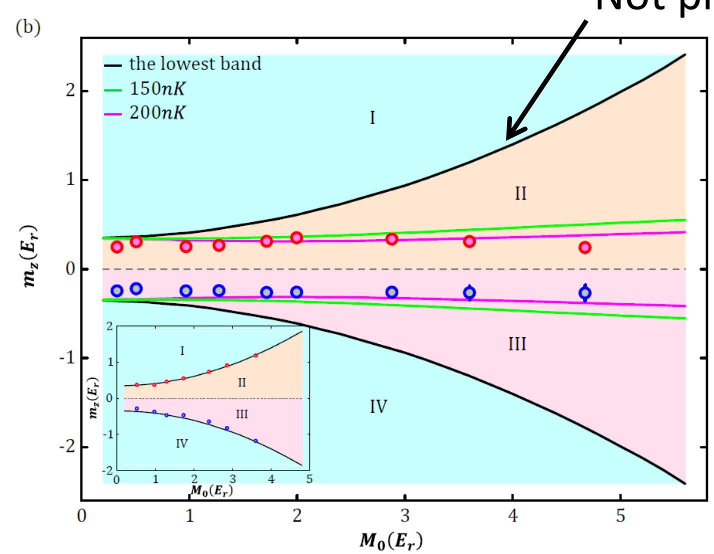
$$\xi^{(m)} = \pm 1 \qquad\qquad \Theta = \prod_i \xi^{(m)}(\boldsymbol{\Lambda}_i)$$

Phase diagram of band topology vs: 1) lattice depth; 2) Raman potential strength



1) lattice depth

Not precise!



2) Raman potential strength

Uncover topology by quench

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T.-F. Jeffrey Poon and XJL, arXiv:1701.01992v2

Summary and Discussions

- We proposed optical Raman lattice schemes to realize 1D/2D/3D SOC and topological phases.
- Successfully realize in experiment a symmetry protected topological phase for fermions with 1D SOC.
- Successfully realize in experiment 2D SO coupling with 87Rb quantum degenerate atom gas. The SO coupling effects and topological bands are observed.
- Topological equilibrium physics and non-equilibrium dynamics are also observed in the latest experiments.

Some latest references:

1) Wu, Zhang, Sun, Xu, Wang, Deng, S. Chen*, XJL* & J.-W. Pan*, Science, **354**, 83-88 (2016).
2) Song, Zhang, He, T. F. Jeffrey Poon, Zhang, XJL*, and G.-B. Jo*, arXiv:1706.00768v2.
3) B.-Z. Wang, Y.- H. Lu, W. Sun, S. Chen*, Y. Deng, XJL*, arXiv:1706.08961v2.
4) Sun, B.-Z. Wang, Xu, Yi, L. Zhang, Wu, Deng, XJL*, S. Chen*, J.-W. Pan*, arXiv:1710.00717.
5) Y.-Q. Wang and XJL, arXiv:1710.02070v2.

Discussions

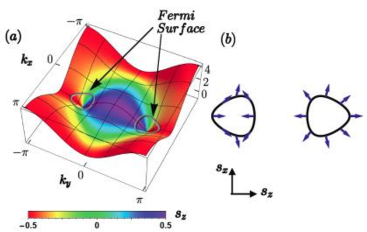
- Realization for fermions. New exotic topological phases.

----- New minimal scheme for 2D SOC and topological superfluids (T.-F. Jeffrey Poon and XJL, arXiv:1701.01992v2, to be published).

$$Ch_1 = n_L - n_U + \sum_i (-1)^{q_i} \left(n_F^{(i)} - \int_{\partial \mathcal{S}^{(i)}} \nabla_k \theta^{(i)}_{k[\mathcal{S}^{(i)}]} \cdot dl \right)$$

- Generic theory for non-Abelian Majorana zero modes.

C. Chan, L. Zhang, T.-F. Jeffrey Poon, Y.-P. He, Y.-Q. Wang and XJL, PRL **119**, 047001 (2017).



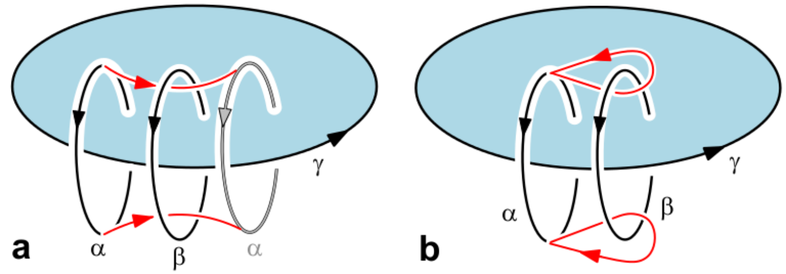
$$\boxed{\nu_3 = \sum_i^N n_i w_i \bmod 2,}$$

with $w_i \equiv \frac{1}{2\pi} \oint_{\mathrm{FS}_i} \nabla \arg \Delta_{\mathbf{Q}_i}(\mathbf{k}) \cdot d\mathbf{k}$

- Generalization to 3D systems

A new emergent 4D non-Abelian topological order (C. Chan and XJL, PRL. **118**, 207002 (2017)).

$$C_2 = \frac{1}{32\pi^2} \int_{\mathcal{M}} d^4\mathsf{p}\, \epsilon_{ijk\ell} \mathrm{Tr}[\mathsf{F}_{ij}\mathsf{F}_{k\ell}] \in \mathbb{Z},$$



Non-Abelian loop braiding statistics

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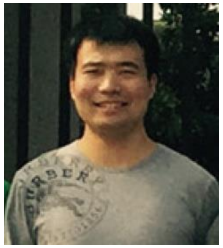
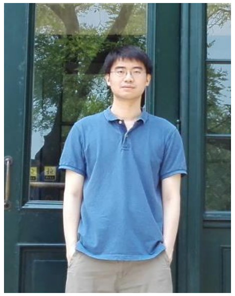
Group@PKU

Postdoctors

Dr. Long Zhang





Dr. Cheung Chan

Students

Yu-Qin Chen Ying-Ping He Xiang-Ru Kong Sen Niu Ting-Fung Jeffrey Poon Bao-Zong Wang (PKU/USTC) Lin Zhang

Yan-Qi Wang (undergraduate → UC Berkeley)

Thank you for your attention!

Collaborators

Prof. Jian-Wei Pan (USTC)

Prof. Shuai Chen (USTC)

Prof. Youjin Deng (USTC)

Prof. Gyu-Boong Jo (HKUST)

Prof. Jian Wang (ICQM, PKU)

Prof. Jian Wei (ICQM, PKU)